Contents

Ch-1-Knowing Our Numbers

Number

Numbers help us to identify which collection of objects is bigger and arrange them in order.

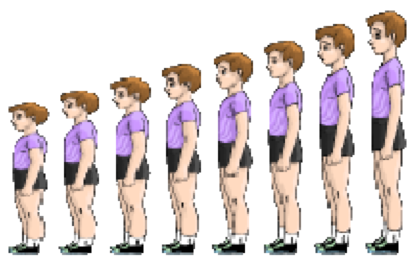
We must note that the number should not start with zero. If the number starts with zero, then remove the zero and follow the steps because a number cannot begin with zero.

Comparison and Ordering Of Numbers

With the help of numbers, we can count living and non-living objects. For example, number of chocolates, number of marbles, number of birds, etc. Numbers help us to identify which collection of objects is bigger and arrange them in order.

We must note that the number should not start with zero. If the number starts with zero, then remove the zero and follow the steps because a number cannot begin with zero.

Now, when you stand in a line for the morning assembly in your school, you must have observed that you often stand according to your heights the taller students stand at the back of the line and the shorter ones in the front. This is nothing but standing in the line in increasing order of your heights. On the contrary, if one starts counting from the back, then you are standing in decreasing order of your heights.



Therefore, there are two ways of arrangement of numbers (objects).

1. Ascending order**:** The arrangement of numbers from the **smallest to the greatest** is known as ascending order. Ascending order means in increasing order.

For example, if we have to write the numbers 81, 18, 26, and 47 in ascending order, then they will be written as 18, 26, 47, and 81.

2. Descending order**:** The arrangement of numbers from the **greatest to the smallest** is known as descending order. Descending order means in decreasing order.

For example, if we have to write the numbers 54, 12, 98, and 4 in descending order, then they will be written as 98, 54, 12, 4.

Ascending and descending orders are reverse of each other.

Now, let us solve some examples to understand this topic better.

Example 1:

Which number are greater, 426 or 4378?

Solution:

The first number is a three-digit number and the second number is a four-digit number. Therefore, 4378 is greater than 426.

Example 2:

Find the greatest and the smallest number among the following numbers.

1018, 1081, 1801, 1011, 1065

Solution:

Here, all the given numbers are four-digit numbers. First digit is same for all the numbers. Second digit from the left is the greatest in third number (which is 8) while the remaining numbers have the same third digit (which is 0). Therefore, 1801 is the greatest number.

The third digit is same in the first and fourth numbers (which is 1), while the second and fifth numbers have 8 and 6 as their third digit. Among 1018 and 1011, the fourth digit is smaller for the latter number. Therefore, 1011 is the smallest number.

Example 3:

Arrange the following numbers in ascending and descending order.

1253, 7876, 1982, 1574

Solution:

Ascending order means the arrangement of numbers from the smallest to the greatest.

Therefore, the above numbers in ascending order are 1253, 1574, 1982, and 7876.

Descending order means arrangement of numbers from the greatest to the smallest.

Therefore, the above numbers in descending order are 7876, 1982, 1574, and 1253.

Formation of Numbers from Given Digits

A number is a combination of digits and interestingly, we can form various numbers by combining the same set of digits differently. A number will be smaller or larger than the other number depending upon the combination. You will realize this as you go through the following video.

Example 1:

What will be the greatest and the smallest five-digit number that can be formed using the digits 5, 7, 0, 2, and 9 without repetition?

Solution:

If we arrange the numbers in descending order, then we will obtain the greatest number. Therefore, the greatest number using the given digits is 97520.

If we arrange the numbers in ascending order, then we will obtain the smallest number. Therefore, the smallest number is 20579 (since a number cannot begin with 0, it cannot be placed at the first place).

Example 2:

Write the greatest and the smallest four-digit number.

Solution:

The greatest four-digit number should contain the maximum number of nines (as 9 is the largest digit among the numbers 0 - 9).

Therefore, the greatest four-digit number is 9999.

The smallest four-digit number should contain the maximum number of zeroes. However, we cannot put zero at the first place from the left. Therefore, 1 should come at the first place followed by three zeroes.

Therefore, the smallest four-digit number is 1000.

Example 3:

Form the greatest and the smallest four-digit number using the digits 9, 3, 8, and 1 without repetition, such that 1 is at the third place of the number.

Solution:

If we arrange the digits in descending order, then we will obtain the digits in the order 9, 8, 3, 1 and if we arrange the digits in ascending order, then we will obtain the digits in the order 1, 3, 8, 9.

Now, we have to form the greatest and the smallest four-digit number using these numbers such that they have 1 at their third place.

Therefore, the greatest number is 9813 and the smallest number is 3819.

Read, Write And Identify Large Numbers

Do you know how many students are there in your class?

The number of students in your class would be a two-digit number or, at the maximum, a three-digit number. These are smaller numbers.

But if you are asked the number of students in your school, then it would be a bigger number.

The total number of students in a city would be a large number, i.e. at least a five-digit number.

And, if we count the total number of students in the whole country, then we would have to use very large numbers (like eight or nine-digit numbers).

Therefore, here, we will learn about large numbers. We are already aware of the numbers up to four digits. Now, if we add 1 to the greatest 4-digit number (i.e., 9999), then we will obtain the smallest five-digit number. This number is ten thousand (10,000).

i.e. 9,999 + 1 = 10,000 (ten thousand)

Similarly,

99,999 + 1 = 1,00,000 (One lakh)

9,99,999 + 1 = 10,00,000 (Ten lakh)

99,99,999 + 1 = 1,00,00,000 (One crore)

9,99,99,999 + 1 = 10,00,00,000 (Ten crore)

and so on.

We observe that,

Greatest 4-digit number + 1 = smallest 5-digit number

Greatest 5-digit number + 1 = smallest 6-digit number

Greatest 6-digit number + 1 = smallest 7-digit number

and so on.

Try to read the number 7,86,790.

Is it difficult?

This number will be read as seven lakh eighty six thousand seven hundred and ninety.

Similarly, we can write the numeral value of a given number.

We must remember the following conversions which will be helpful in reading and writing numbers.

1 hundred = 10 tens

1 thousand = 10 hundreds

= 100 tens

1 lakh = 100 thousands

= 1000 hundreds

1 crore = 100 lakhs

= 10,000 thousands

Let us now see some examples to understand the concept better.

Example 1:

Write the following numbers in words and answer the questions given below.

432079, 5601729 and 1794805

(1) Which is the smallest number?

(2) Which is the largest number?

(3) Arrange these numbers in ascending and descending order.

Solution:

(1) 4,32,079 = Four lakh thirty two thousand and seventy nine

56,01,729 = Fifty six lakh one thousand seven hundred and twenty nine

17,94,805 = Seventeen lakh ninety four thousand eight hundred and five

(2) 4,32,079 is the smallest number.

(3) 56,01,729 is the largest number.

(4) Therefore, the numbers in the ascending order are 4,32,079, 17,94,805, and

56,01,729.

Numbers in descending order are 56,01, 729, 17,94,805, and 4,32,079.

Expansion of Numbers and Place Value Tables

Example 1:

Write 6<u>5</u>0<u>8</u>92<u>7</u> in expanded form and write the place value of underlined digits.

Solution:

The number 6508927 can be written in expanded form as

$6508927 = 6 \times 1000000 + 5 \times 100000 + 0 \times 10000 + 8 \times 1000 + 9 \times 100 + 2 \times 10 + 7 \times 1$

Place value of 7 = one

Place value of 8 = thousand

Place value of 5 = lakh

Example 2:

Write the expanded number [8000000 + 500000 + 70000 + 900 + 40 + 2] in the numeral form.

Solution:

The given number is 8000000 + 500000 + 70000 + 900 + 40 + 2.

In addition, the number can be written in complete expanded form as

$8 \times 1000000 + 5 \times 100000 + 7 \times 10000 + 0 \times 1000 + 9 \times 100 + 4 \times 10 + 2 \times 1$

Thus, the numeral form is 8570942.

Indian and International System of Numeration

A teacher asked his students to write the number 800000000 in words.

Two of his students, Kunal and Arpit, wrote the number in different ways but the teacher said both are correct.

Kunal wrote the given number as eighty crore and Arpit wrote as eight hundred million.

Do you know why both are correct?

Kunal wrote the number according to the Indian number system and Arpit wrote the number according to the international number system.

Now, we will discuss two types of number systems according to which the numbers can be written in an easier way. The two types of number systems are

1. Indian number system
2. International number system

Let us now study about these.

Indian Number System

According to the Indian number system, the place value chart is divided into four groups - hundred, thousand, lakh, and crore. Each group is divided into subgroups as shown in the following table.

<table>
<tr><th>Group</th><th>Sub group</th></tr>
<tr><td rowspan="2">Crore</td><td>Ten crore</td></tr>
<tr><td>Crore</td></tr>
<tr><td rowspan="2">Lakh</td><td>Ten lakh</td></tr>
<tr><td>Lakh</td></tr>
<tr><td rowspan="2">Thousand</td><td>Ten thousand</td></tr>
<tr><td>Thousand</td></tr>
<tr><td rowspan="2">Hundred</td><td>Hundred</td></tr>
<tr><td>Ten</td></tr>
</table>

	One

International Number System

According to the International number system, the place value chart is divided into groups and subgroups as follows.

Group	Sub group
Million	Hundred million
	Ten Million
	Million
Thousand	Hundred thousand
	Ten thousand
	Thousand
Hundred	Hundred
	Ten
	One

Differences between the numerals according to the Indian and the International number system

Indian number system

In this system, the first comma comes after the hundredth digit (i.e. after first three digits from the right), the second comma comes after the ten thousandth digit (i.e. after five digits from the right), the third comma comes after the ten lakh digit (i.e. after seven digits from the right), and so on.

For example, the number 234567890 can be written according to the Indian number system as 23,45,67,890 and can be read as twenty three crore forty five lakh sixty seven thousand eight hundred and ninety.

International number system

In this system, the first comma comes after the hundredth digit (after three digits from right), the second comma comes after the hundred thousandth digit (after six digits from the right), the third comma comes after the thousand million digit (after nine digits), and so on.

For example, according to the international number system, the number 234567890 can be written as 234,567,890 and read as two hundred thirty four million five hundred sixty seven thousand eight hundred and ninety.

Let us now look at some examples to understand this concept better.

Example 1:

Write 39784012 in words using both the Indian and the International number system.

Solution:

In the Indian number system, the number can be written according to the place value of each digit as follows.

Crore	Ten lakh	Lakh	Ten thousand	Thousand	Hundred	Ten	One
3	9	7	8	4	0	1	2

Therefore, the number can be written as three crore ninety seven lakh eighty four thousand and twelve.

In the International number system, the number can be written according to the place value of each digit as follows.

Ten million	Million	Hundred thousand	Ten thousand	Thousand	Hundred	Ten	One
3	9	7	8	4	0	1	2

Therefore, the number can be written as thirty nine million seven hundred eighty four thousand and twelve.

Example 2:

Insert commas between the numbers according to both the Indian and the International number system.

(a) 88500784 (b) 32098175

Solution:

(a) According to the Indian number system, the number will be written as

8,85,00,784.

According to the International number system, the number will be written as

88,500,784.

(b) According to the Indian number system, the number will be written as

3,20,98,175.

According to the International number system, the number will be written as

32,098,175.

Mathematical Operations

Sonia goes to a shop and asks for 10,000 grams of potatoes. The shopkeeper tells her to tell the quantity she wants in kilograms because it is very difficult for him to weigh that many grams.

Can you tell how many kilograms of potatoes does Sonia want?

For this, we will first learn the various units of measurement and their appropriate usages.

If we have to measure the length of a pen, then we use centimetre as the unit of measurement. However, if we have to measure the length of a pole, then centimetre will be a smaller unit of measurement. Hence, we will use metre for measuring the length of the pole.

Similarly, we will use kilometre for measuring larger distances such as the distance between two cities.

In the same manner, kilogram, gram, and milligram are used for weighing items of different weights, whereas kilolitre, litre, and millilitre are used for measuring the capacities of containers of different sizes.

Let us see some conversions of unit.

1 centimetre = 10 millimetres

1 metre = 100 centimetres

1 kilometre = 1000 metres

We can convert the units of weight and capacity in the same manner.

For weight

1 kilogram = 1000 grams

1 gram = 100 centigrams

1 centigram = 10 milligrams

For capacity

1 kilolitre = 1000 litres

1 litre = 100 centilitres

1 centilitre = 10 millilitres

Now, we know that 1 kilogram = 1000 grams, i.e. 1000 grams is equivalent to 1 kilogram.

Thus, Sonia should have asked for 10 kg of potatoes instead of 10,000 grams of potatoes.

Let us now look at some examples to understand the concept better.

Example 1:

Convert 2 kilometres into metres.

Solution:

We know that, 1 kilometre = 1000 metres

∴ 2 kilometres = (2 × 1000) metres

= 2000 metres

Example 2:

How many grams are there in 50,000 centigrams?

Solution:

We know that, 1 gram = 100 centigrams

∴ 50,000 centigrams = (50,000 ÷ 100) grams

= 500 grams

Example 3:

Isha has a 2 m long ribbon. If she cuts 1 m and 25 cm off it, then what is the length of the remaining ribbon?

Solution:

Initial length of the ribbon = 2 m

= 200 cm (1 m = 100 cm)

Length of the ribbon cut by Isha = 1 metre and 25 cm

= 100 cm + 25 cm

= 125 cm

Length of the remaining ribbon = (200 – 125) cm

= 75 cm

Example 4:

Gagan bought 2 kg potatoes, 2500 gm carrots, and 4 packets of peas, each packet containing 550 gm peas. What is the total weight of the vegetables bought by him?

Solution:

Weight of potatoes = 2 kg

Weight of carrots = 2500 gm = 2.5 kg

Weight of peas = 4 $\times$550 gm

= 2200 gm

= 2.2 kg

∴ Total weight of vegetables = 2 + 2.5 + 2.2

= 6.70 kg or 6 kg and 700 gm

Example 5:

If 16 kg and 200 g of rice can be packed in one bag, then how many bags will be required to pack 405 kg of rice?

Solution:

We know that 1 kg = 1000 g

405 kg = 405×1000 g

= 4,05,000 g

16 kg and 200 g = 16 kg + 200 g

= 16 × 1000 g + 200 g

= 16,200 g

16 kg and 200 g of rice can be packed in 1 bag.

Number of bags required to pack 405 kg of rice = 4,05,000 ÷ 16,200

$$= \frac{4,05,000}{16,200} = 25$$

Therefore, 25 bags are required to pack 405 kg of rice.

Estimation of Numbers to Given Place Values

Many a times, instead of reporting an exact figure, an approximate value of the actual figure is used. This is called rounding off and is an important concept.

An important point to note regarding rounding off numbers is that the number 5 (which is equidistant from 0 and 10) is rounded off to 10.

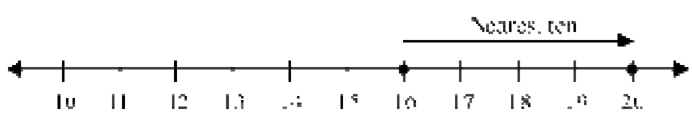
Similarly, while rounding off to the nearest hundreds, 1250 (which is equidistant from 1200 and 1300) is rounded off to 1300.

Let us now look at some more examples to understand this concept better.

Example 1:

Radhika has 16 pencils in her pencil box. Round off the number of pencils to the nearest tens. Plot the number on the number line.

Solution:



It can be seen that 16 is closer to 20. Therefore, 16 is rounded off to 20.

Example 2:

Round off 234 to the nearest ten.

Solution:

The number 234 lies between 230 and 240. However, 234 is closer to 230.

Therefore, 234 is rounded off to 230.

Example 3:

Estimate 4504 by rounding off to the nearest thousand.

Solution:

4000 and 5000 are two multiples of thousand between which the given number lies. Now, 4504 is closer to 5000. Therefore, the rounded off value of 4504 is 5000.

Example 4:

Estimate 7896 by ounding off to the nearest hundred.

Solution:

7800 and 7900 are the two multiples of hundred between which the number 7896 lies, but 7900 is closer to 7896. Therefore, the rounded off value of 7896 is 7900.

Example 5:

In the given table, some numbers and their rounded forms are given. Find out whether the given rounded forms are correct or not.

Given number	Approximate to nearest	Rounded form
7893	Tens	7890
527	Thousands	500
62987	Hundreds	63000
4213	Tens	4200
90909	Thousands	90000

Solution:

(i) 7893 lies between 7890 and 7900. However, it is closer to 7890. Thus, the rounded form should be 7890. Therefore, the given rounded form is correct.

(ii) 527 lie between 0 and 1000. It is closer to 1000. Thus, the rounded form should be 1000. Therefore, the given rounded form is incorrect.

(iii) 62987 lie between 62900 and 63000. The given number is closer to 63000. Thus, the rounded form should be 63000. Therefore, the given rounded form is correct.

(iv) 4213 lies between 4210 and 4220. However, it is closer to 4210. Thus, the rounded form should be 4210. Therefore, the given rounded form is incorrect.

(v) 90909 lie between 90000 and 91000. It is closer to 91000. Thus, the rounded form should be 91000. Therefore, the given rounded form is incorrect.

Estimation Strategies to Add, Subtract, and Multiply Numbers

The estimation of numbers proves quite useful in real life.

Suppose we want to purchase some clothes from the market. Before leaving home, we find a rough estimate of the amount we require to spend on the clothes and take that much money with us. But how will we estimate the amount we have to carry?

Here, we will learn some estimation strategies to make our calculations easier.

When we estimate a number, we should have an idea of the place value to which we are going to round off the number.

In case of addition or subtraction, we first round off the numbers and then the numbers are added or subtracted.

For example, let us estimate the sum 4369 + 263.

On rounding off the numbers to the nearest thousands, we obtain

4369 rounds off to 4000	4000
263 rounds off to 0	+0
Estimated sum	4000

However, we can observe that the actual answer is 4632, which is not very close to the estimated sum which is 4000. Therefore, we should estimate the numbers to the nearest hundreds to find a more accurate answer.

On rounding off the numbers to the nearest hundreds, we obtain

4369 rounds off to 4400	4400
263 rounds off to 300	+300
Estimated sum	4400

This is a better estimation as it is closer to the actual sum 4632.

Similarly, we can find the product using estimation strategies.

Here is a general rule for rounding off the product of two or more numbers.

First round off each factor of the product to its greatest place value, and then multiply the rounded off values.

For example, let us estimate the product 248 × 63.

Here, 248 would be rounded off to the nearest hundred (200) and 63 to the nearest tens (60).

248 rounds off to 200	200
63 rounds off to 60	×60
Estimated product	12000

Therefore, the estimation of 248 × 63 is 12000.

To get a more reasonable estimate, we try rounding off 63 to the nearest tens, i.e. 60, and 248 to the nearest tens, i.e. 250. We will get the answer as 250 × 60 = 15000 which is the more accurate estimation.

Let us now solve some more examples.

Example 1:

The marks of a student in five subjects are 98, 62, 59, 87, and 93. Find his total marks using estimation strategies.

Solution:

Rounding off the numbers to the nearest tens,

98 rounds off to 100

62 rounds off to 60

59 rounds off to 60

87 rounds off to 90

93 rounds off to 90

The estimated sum comes out to be 400. Therefore, the total marks of the student are 400.

Example 2:

If a bicycle manufacturing company produced 4873 bicycles in the last two years (2006 and 2007) and if 2209 bicycles were produced in the last year (2007), then find the estimated number of bicycles produced by the company in the year 2006.

Solution:

Estimated number of bicycles produced in two years (2006 and 2007) = 4900

(Rounding off 4873 to nearest hundred)

Estimated number of bicycles produced in the year 2007 = 2200

(Rounding off 2209 to nearest hundred)

Therefore, estimated number of bicycles produced in the year 2006 = 4900 – 2200

= 2700

Example 3:

Harry has Rs 400 with him. He wants to buy 17 bats which cost Rs 33 each. Will he be able to buy the bats?

Solution:

On rounding off the numbers 17 and 33, we obtain

17 rounds off to 20	20
33 rounds off to 30	$\times 30$
Estimated price of bats	Rs 600

But Harry has only Rs 400 with him. Therefore, he cannot buy 17 bats.

Example 4:

Estimate the value of the following expression.

298 + 1902 + 2387 – 567

Solution:

Firstly, we round off the numbers to nearest hundred. Therefore, we obtain

298 rounds off to 300

1902 rounds off to 1900

2387 rounds off to 2400

567 rounds off to 600

Estimated value = 300 + 1900 + 2400 – 600 = 4000

Thus, the estimated value of the given expression is 4000.

Use of Brackets to Simplify Calculations

Consider the following problem.

Let us now understand the use of brackets by solving some more examples.

Example 1:

Write the expressions for each of the following statements.

i. **Ten is multiplied with the sum of 9 and 3.**
ii. **Nine is added to the product of 5 and 2.**
iii. **Twice the sum of 4 and 5 is divided by 2.**

Solution:

i. The expression for the given statement is $10 \times (9 + 3)$.
ii. The expression for the given statement is $9 + (5 \times 2)$.
iii. The expression for the given statement is $(2(4 + 5))/2$.

Example 2:

Write a situation for the expression $3 \times (8 - 5)$ where brackets are necessary.

Solution:

The situation of the given expression can be written as follows.

Sahil bought 8 apples. Out of them, 5 apples were rotten. The cost of one apple was Rs 3. What was the cost of good apples?

Remove Brackets to Simplify Calculations

Let us now solve some examples to understand this concept better.

Example 1:

4 is multiplied with the difference of 100 and 70. Write the expression for the given statement using brackets and solve that expression.

Solution:

The expression is $4 \times (100 - 70)$.

Now, $4 \times (100 - 70) = 4 \times 30$

$= 120$

Example 2:

Solve the expression 203×207 using brackets.

Solution:

$203 \times 207 = (200 + 3) \times (200 + 7)$

= 200 × 200 + 3 × 200 + 200 × 7 + 3 × 7

= 40000 + 600 + 1400 + 21

= 42021

Example 3:

Solve the following expressions.

i. **3 × 7 × (80 + 26 - 61)**
ii. (3(94 - 56) + 2(25 + 16) + (4 x 3))/ 2

Solution:

i. 3 × 7 × (80 + 26 – 61) = 3 × 7 × (106 – 61)

= 3 × 7 × (45)

= 3 × 7 × 45

= 945

ii. (3(94 - 56) + 2(25 + 16) + (4 x 3))/ 2

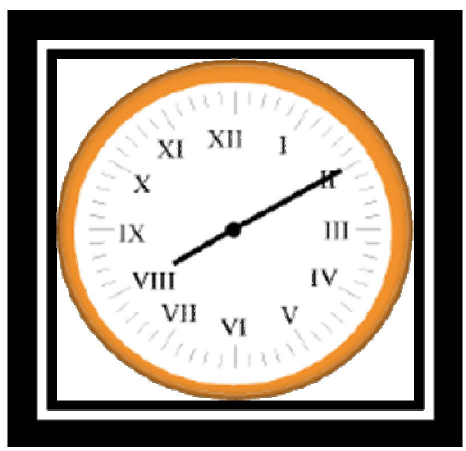
= (3(38) + 2(41) + (12))/ 2

= (114 + 82 + 12)/ 2

= 208/ 2

= 104

Conversion between Hindu-Arabic and Roman Numerals



There are various ways of writing numerals. We use Hindu-Arabic numeral system according to which numerals are written as 1, 2, 3, 4…etc.

Another way of writing numerals is Roman numeral system in which 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are written as I, II, III, IV, V, VI, VII, VIII, IX, X respectively.

Roman numbers are used in clocks; they are also used in timetables to represent periods in school.

Some standard symbols of Roman numerals are as follows.

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

Now, let us understand the rule to convert Roman numerals into Hindu-Arabic numerals.

(i) When a symbol of Roman numeral system is repeated, its value is added as many times as it is repeated.

For example, XXX = 10 + 10 + 10

= 30

(ii) When we write a symbol of smaller value to the right of a symbol of greater value, the value of the smaller symbol is added to the value of the greater symbol.

For example, XII = 10 + 2 = 12

(iii) If we write a symbol of smaller value to the left of a symbol of greater value, then the value of the smaller symbol is subtracted from the value of the greater symbol.

For example, IX = 10 – 1 = 9

We must remember some points while writing roman numerals.

(i)A symbol can never be repeated more than three times.

For example, 40 is written as XL = 50 – 10 = 40 and not as XXXX.

(ii) The symbols V, L, and D are never repeated.

For example, we cannot write VV, LL, DD to represent numbers like 10, 100, 1000.

(iii) We can subtract I only from V and X.

For example, we cannot write IL or IC to represent 49 or 99 respectively. 49 is represented by XLIX (XL for 40 and IX for 9). Similarly, 99 is represented by XCIX (XC for 90 and IX for 9).

(iv) We can subtract X only from L, M, and C.

For example, we cannot write XD to represent 490. The number is represented by CDXC (CD for 400 and XC for 90).

(v) V, L, and D can never be subtracted from any symbol.

For example, we never write VXX to represent 20 or LCCC to represent 250.

Let us now look at some examples to understand this concept better.

Example 1:

Write the following numbers in Roman numerals.

(a) 29 (b) 45 (c) 67 (d) 540

Solution:

(a) 29 = 20 + 9

= 10 + 10 + 9

= XX + IX

= XXIX

(b) 45 = 40 + 5

= (50 – 10) + 5

= XL + V

= XLV

(c) 67 = 60 + 7

= (50 + 10) + 7

= LX + VII

= LXVII

(d) 540 = 500 + 40

= 500 + (50-10)

= D + XL

= DXL

Example 2:

Write the following Roman numerals in Hindu-Arabic numerals.

(i) LXXVII (ii) CXCIX (iii) DCLXIV

Solution:

(i) LXXVII = L + XX + VII

= 50 + 20 + 7

= 77

(ii) CXCIX = C + XC + IX

= 100 + 90 + 9

= 199

(iii) DCLXIV = D + C + L + X + IV

= 500 + 100 + 50 + 10 + 4

= 664

Ch-2-Whole Numbers

Predecessor and successor

Successor:

Successor is the number that comes just after a given number. Here are some examples.

Predecessor:

Predecessor is the number that comes just before a given number. Here are some examples.

Example 1:

Find the successor and predecessor of each of the following whole numbers:

(i) 1000
(ii) 11999
(iii) 400099
(iv) 1000001
(v) 99999

Solution:
(i) 1000
The successor of 1000 is (1000 + 1) = 1001.
The predecessor of 1000 is (1000 - 1) = 999.

(ii) 11999
The successor of 11999 is (11999 + 1) = 12000.
The predecessor of 11999 is (11999 - 1) = 11998.

(iii) 400099
The successor of 400099 is (400099 + 1) = 400100.
The predecessor of 400099 is (400099 - 1) = 400098.

(iv) 1000001
The successor of 1000001 is (1000001 + 1) = 1000002.
The predecessor of 1000001 is (1000001 - 1) = 1000000.

(v) 99999
The successor of 99999 is (99999 + 1) = 100000.
The predecessor of 99999 is (99999 - 1) = 99998.

Example 2:

What is Successor and Predecessor of a giver number?

Solution:

PREDECESSOR	NUMBER	SUCCESSOR
(7148 - 1) **7147**	**7148**	(7148 + 1) **7149**
(8950 - 1) **8949**	**8950**	(8950 + 1) **8951**
(7620 - 1) **7619**	**7620**	(7620 + 1) **7621**
(12499 - 1) **12498**	**12499**	(12499 + 1) **12500**

Whole number:

The numbers in the set {0, 1, 2, 3, 4, 5, 6, 7 . . .} are called whole numbers.
In other words, whole numbers is the set of all counting numbers plus zero.

Number line:

A number line is a line that has points to represent a real number at every point and they are all equally spaced.
The distance between these points labeled as 0 and 1 is called unit distance. On this line, mark a point to the right of 1 and at unit distance from 1 and label it 2. In this way go on labeling points at unit distances as 3, 4, 5... on the line. You can go to any whole number on the right in this manner. This is a number line for the whole numbers.

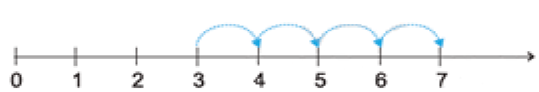
Addition on the number line:

Addition of whole numbers can be shown on the number line. Here are some examples.

Example 1:

Let us find 3 + 4 on a number line?

Solution:
Let us see the addition of 3 and 4. Start from 3 since we add 4 to this number so we make 4 jumps to the right; from 3 to 4, 4 to 5, 5 to 6 and 6 to 7 as shown below. The tip of the last arrow in the fourth jump is at 7. Hence, the sum of 3 and 4 is 7, i.e. 3 + 4 = 7.

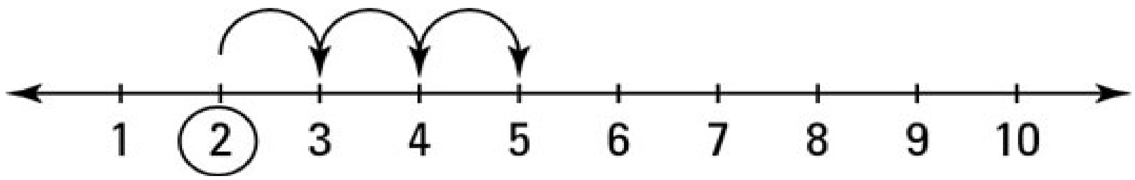


Example 2:

Let us find 2 + 3 on a number line?

Solution:

Start from 2 since we add 3 to this number so we make 3 jumps to the right; from 2 to 3, 3 to 4, and 4 to 5 as shown below. The tip of the last arrow in the third jump is at 5. Hence, the sum of 2 and 3 is 5, i.e. 2 + 3 = 5.



Subtraction on the number line:

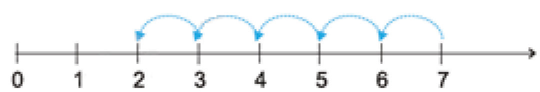
Subtraction of whole numbers can be shown on the number line. See examples.

Example 1:

Let us find 7 - 5 on the number line?

Solution:

Let us subtract 5 from 7.Start from 7. Since 5 is being subtracted, so move towards left with 1 jump of 1 unit. Make 5 such jumps. We reach the point 2. We get 7 – 5 = 2.

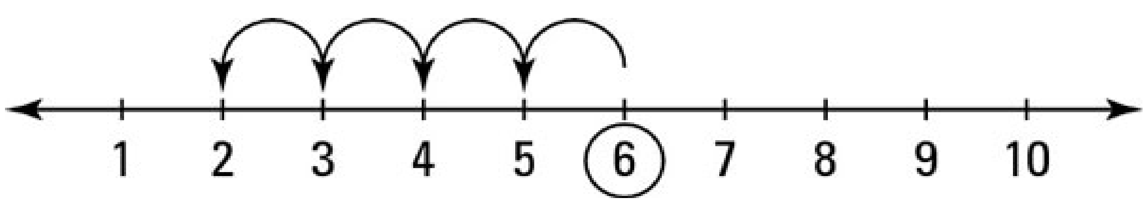


Example 2:

Let us find 6 - 4 on the number line?

Solution:

Start from 6, since 4 is being subtracted, so move towards left with 1 jump of 1 unit. Make 4 such jumps. We reach the point 2. We get 6 – 4 = 2.



Multiplication on the number line:

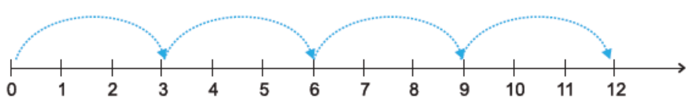
Multiplication of whole numbers can be shown on the number line. Here are some examples.

Example 1:

Let us find 4 X 3 on the number line?

Solution:

Start from 0 and move 3 units at a time to the right and make such 4 moves. You will reach 12. So, we say, $3 \times 4 = 12$.



Properties of whole numbers:

Closure property:

The closure property of whole number addition states that when we add whole numbers to other whole numbers the result is also whole number.

For example:
$3 + 6 = 9$
Here, 3, 6, and 9 are whole numbers.

Multiplication property:

The closure property of whole number multiplication states that when we multiply whole numbers with other whole numbers the result is also whole number.

For example:
$5 \times 8 = 40$
Here, 8, and 40 are whole numbers.

Whole numbers are not closed under subtraction:

It states that when we subtract whole numbers to other whole numbers the result is not a whole number.

For example:
Consider two whole numbers 7 and 8.
$7 - 8 = -1$

Negative 1 (-1) is not a whole number.
So, closure property doesn't work here. Therefore, the set of whole numbers is not closed under subtraction.

Whole numbers are not closed under division:

It states that when we divide whole numbers to other whole numbers the result is not a whole number.

For example:
Consider the two whole numbers 5 and 7.
$5 \div 7 = 5/7$
So, 5/7 is not a whole number.
So, closure property doesn't work here. Therefore, the set of whole numbers is not closed under division.

Division by Zero:

Any whole numbers divided by zero is undefined.

Examples:
$5132 \div 0$ it can not be found.
$232 \div 0$ it is undefined.

Commutative Property of Addition:

The Commutative Property of Addition states that changing the order of addends does not change the sum, i.e. if a and b are two whole numbers, then

$$\mathbf{a + b = b + a}$$

Example:

Add two whole numbers 5 and 14.

Solution:
The expression $5 + 14 = 19$ can be written as $14 + 5 = 19$.
Hence, $5 + 14 = 14 + 5$

Similarly,
$9 + (-5) = 4$ can be written as $(-5) + 9 = 4$.
Hence, $9 + (-5) = (-5) + 9$

Commutative Property of Multiplication:

The Commutative Property of Multiplication states that changing the order of factors does not change the product, i.e. if a and b are two whole numbers, then

$$a \times b = b \times a$$

Examples:

Multiply two whole numbers 3 and 9.

Solution:

The expression $3 \times 9 = 27$ can be written as $9 \times 3 = 27$.
Hence, $3 \times 9 = 9 \times 3$

Similarly,
$(-7) \times 6 = -42$ can be written as $6 \times (-7) = -42$.
Hence, $(-7) \times 6 = 6 \times (-7)$

Associative Property of addition:

The associative property of addition state that when we add more than two whole numbers the order of the addends does not change the sum. In general, the associative property of addition can be written as:

$$(a + b) + c = a + (b + c)$$

For example: if we add 3, 6 and 8. We observe that

$(3 + 6) + 8 = 3 + (6 + 8)$
$9 + 8 = 3 + 14$
$17 = 17$
Therefore, LHS = RHS.

Associative Property of Multiplication:

The associative property of multiplication says that when we multiply more than two numbers the grouping of the factors does not change the product. In general, the associative property of multiplication can be written as:

$$(a \times b) \times c = a \times (b \times c)$$

For example: if we multiply 2, 4 and 3 then we can observe that

$(2 \times 4) \times 3 = 2 \times (4 \times 3)$
$8 \times 3 = 2 \times 12$
$24 = 24$

Therefore, LHS = RHS.

Distributive Property:

Distributive Property states that the product of a number and a sum is equal to the sum of the individual products of the addends and the number. This is also known as distributivity of multiplication over addition. That is,

$$a \times (b + c) = a \times b + a \times c.$$

For example: let us check, 5 x (3 + 1) = 5 × 3 + 5 × 1
Consider LHS: 5(3 + 1) = 5(4) = 20
Consider RHS: 5 × 3 + 5 × 1 = 15 + 5 = 20
Here, we see that, LHS = RHS

Identity Properties of Addition:

Identity property of addition states that the sum of zero and any number is the number itself.

For example:
4 + 0 = 4,
- 11 + 0 = - 11,
7 + 0 = 7
These examples illustrating the identity property of addition

Identity Properties of Multiplication:

Identity property of multiplication states that the product of 1 and any number is the number itself.

For example:
4 × 1 = 4,
- 11 × 1 = - 11,
8 × 1 = 8
These examples illustrating the identity property of multiplication

Patterns in Whole Numbers:

We shall try to arrange numbers in elementary shapes made up of dots. The shapes we take are (1) a line (2) a rectangle (3) a square and (4) a triangle. Every number should be arranged in one of these shapes. No other shape is allowed.

Every number can be arranged as a **line**;
The number 2, 3,4 and so on is shown as

Every number can be arranged as a Line.

Number	Pattern
2	● ●
3	● ● ●
4	● ● ● ●
5	● ● ● ● ●

Some numbers can be shown also as **rectangles**.
For example,
The number 6 can be shown as a rectangle. Note there are 2 rows and 3 columns.

Some numbers can be shown as Rectangles.

Number	Pattern
4 (2+2)	● ● ● ●
6 (4 + 2)	● ● ● ● ● ●
8 (4+2+2)	● ● ● ● ● ● ● ●
10(4+2+2+2)	● ● ● ● ● ● ● ● ● ●

Some numbers like 4 or 9 can also be arranged as **squares**.

Some numbers can be shown as Squares.

Number	Pattern
4 (1 + 3)	● ● ● ●
9 (1+3+5)	● ● ● ● ● ● ● ● ●
16 (1+3+5+7)	● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●
25(1+3+5+7+9)	● ●

Some numbers can also be arranged as **triangles**.
For example,
Note that the triangle should have its two sides equal. The number of dots in the rows starting from the bottom row should be like 4, 3, 2, and 1.
The top row should always have 1 dot

Some numbers can be shown as Triangles.

Number	Pattern
3 (1 + 2)	● ● ●
6 (1+2+3)	● ● ● ● ● ●
10 (1+2+3+4)	● ● ● ● ● ● ● ● ● ●
15(1+2+3+4+5)	● ● ● ● ● ● ● ● ● ● ● ● ● ● ●

Ch-3-Playing with Numbers

Factors and Multiples

Factors:

An integer that divides into another integer exactly is called a **Factor**.

Example 1:

2 is a factor of 8, because 2 divide evenly into 8.

Example 2:

$15 \times 4 = 60$ implies that 15 and 4 are the factors of 60.

Example 3: What are the factors of 30?

Solution:

Step 1: The number 30 can be written as 5×6.
Step 2: $5 \times 6 = 5 \times 3 \times 2$
Step 3: So, the factors of 30 are 5, 3, and 2 or $30 = 5 \times 3 \times 2$.

Multiple:

The product of a number with any integer is called the **multiple** of that number. Or If a number p is multiplied with an integer q, then its multiple (product) n is given as $n = p \times q$

Example 1:

Find the first seven multiples of 3.

Solution:

Step 1: A multiple of a number is the product of the number with an integer.

Step 2:
$1 \times 3 = 3$
$2 \times 3 = 6$
$3 \times 3 = 9$
$4 \times 3 = 12$
$5 \times 3 = 15$
$6 \times 3 = 18$
$7 \times 3 = 21$

Step 3: So, the first seven multiples of 3 are 3, 6, 9, 12, 15, 18, and 21.

Example 2:

What are the multiples of 6 and 4?

Solution:

The multiples of 6 are 6, 12, 18, 24, 30, 36 . . .
The multiples of the number 4 are 4, 8, 12, 16, 20, 24...

Let us see what we conclude about factors and multiples:

1. Is there any number which occurs as a factor of every number? Yes. It is 1.

For example:

$6 = 1 \times 6$,
$18 = 1 \times 18$
We say **1 is a factor of every number.**

2. Can 7 be a factor of itself? Yes. You can write 7 as $7 = 7 \times 1$. You will find that every number can be expressed in this way. We say that **every number is a factor of itself.**

3. What are the factors of 16? They are 1, 2, 4, 8, and 16. Out of these factors do you find any factor which does not divide 16? NO. You will find that **every factor of a number is an exact divisor of that number.**

4. What are the factors of 34? They are 1, 2, 17 and 34. Out of these which are the greatest factor? It is 34. The other factors 1, 2 and 17 are less than 34.
We say that **every factor is less than or equal to the given number.**

5. The number 76 has 5 factors. How many factors do 136 or 96 have? You will find that you are able to count the number of factors of each of these. Even if the numbers are as large as 10576, 25642 etc. or larger, you can still count the number of factors of such numbers, (though you may find it difficult to factorize such numbers).
We say that **number of factors of a given number is finite.**

6. What are the multiples of 7? Obviously, 7, 14, 21, 28... You will find that each of these multiples is greater than or equal to 7. Will it happen with each number? Check this for the multiples of 6, 9 and 10. We find that **every multiple of a number is greater than or equal to that number.**

7. Write the multiples of 5. They are 5, 10, 15, 20 ... Do you think this list will end anywhere? No! The list is endless. Try it with multiples of 6, 7 etc.
We find that **the number of multiples of a given number is infinite.**

8. Can 7 be a multiple of itself? Yes, because 7 = 7×1. Will it be true for other numbers also? Yes. Try it with 3, 12 and 16. You will find that **every number is a multiple of itself.**

Perfect Number:

A number for which sum of all its factors is equal to twice the number is called a perfect number.

For example: Consider the number 6. The proper divisors of 6 are 1, 2, and 3. Sum of these divisors = 1 + 2 + 3 = 6. As the sum of the divisors is 6 and the number is also 6, so 6 is a perfect number.

Example: Is 28 a perfect number?

Solution: Yes.

Step 1: A perfect number is equal to the sum of all its factors.

Step 2: The factors of 28 are 1, 2, 4, 7, and 14.

Step 3: Sum of the factors = 1 + 2 + 4 + 7 + 14 = 28

Step 4: So, according to the definition, 28 is a perfect number.

Prime and Composite numbers

Prime number:

A prime number is a positive integer that has exactly two factors, 1 and the number itself.

Example:

2, 3, 5, 7, 11, 13, 17, 19, etc. are all prime numbers. There are infinitely many prime numbers.

Example: Is 41 a prime number?

Solution: Yes.
Because the only divisors of 41 are 1 and 41 so, 41 is a prime number.

Composite Number:

A whole number that has factors other than 1 and the number itself is a Composite Number

For Example:

4, 6, 9, 15, 32, 45 are some examples of composite numbers.

Example:

Is 121 a composite number?

Solution:

Step 1: Here, only 121 has factors other than 1 and itself.
Step 2: The factors of 121 are 1, 11, and 121. So, 121 is a composite number.

Note: 1 is neither a prime nor a composite number.

Even and Odd numbers

Even number:

An Even Number is a number that is divisible by 2. For example: {0, 2, 4, 6, 8 . . .}

A number with 0, 2, 4, 6, and 8 at the ones place is an even number.
So, 350, 4862, 59246 are even numbers.

Odd Number:

An odd number is a whole number that is not divisible by 2. For example: {1, 3, 5, 7, 9 . . .} is the set of odd numbers.

An odd number is a whole number that has 1, 3, 5, 7, and 9 in the ones place. The following are few examples of odd numbers. 27, 55, 89, 45, 99, 221, 999, 100, 375

Let us try to find some interesting facts:

(a) Which is the smallest even number? It is 2. Which is the smallest prime number? It is again 2.
Thus, **2 is the smallest prime number which is even.**

(b) The other prime numbers are 3, 5, 7, 11, 13 . . . Do you find any even number in this list? Of course not, they are all odd.
Thus, we can say that **every prime number except 2 is odd.**

Tests for Divisibility of Numbers

Divisibility by 10:

If a number has 0 in the ones place then it is divisible by 10.

Example:

1,470 is divisible by 10 since the last digit is 0.

Divisibility by 5:

A number which has either 0 or 5 in its ones place is divisible by 5.

Example:

195 is divisible by 5 since the last digit is 5.

Divisibility by 2:

A number is divisible by 2 if it has any of the digits 0, 2, 4, 6 or 8 in its ones place.

Example:

168 is divisible by 2 since the last digit is 8.

Divisibility by 3:

If the sum of the digits is a multiple of 3, then the number is divisible by 3.

Example:

168 is divisible by 3 since the sum of the digits is 15 (1+6+8=15), and 15 is divisible by 3.

Divisibility by 6:

If a number is divisible by 2 and 3 both then it is divisible by 6 also.

Example:

168 is divisible by 6 since it is divisible by 2 and 3 both.

Divisibility by 4:

A number with 3 or more digits is divisible by 4 if the number formed by its last two digits (i.e. ones and tens) is divisible by 4.

Example:

316 is divisible by 4 since 16 is divisible by 4.

Divisibility by 8:

A number with 4 or more digits is divisible by 8, if the number formed by the last three digits is divisible by 8.

Example:

7120 is divisible by 8 since 120 is divisible by 8.

Divisibility by 9:

If the sum of the digits of a number is divisible by 9, then the number itself is divisible by 9.

Example:

549 is divisible by 9 since the sum of the digits is 18 (5+4+9=18), and 18 is divisible by 9.

Divisibility by 11:

First find the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number.
If the difference is either 0 or divisible by 11, then the number is divisible by 11.

Example:

Is 60258 divisible by 11?

Solution:
Sum of odd places, 6 + 2 + 8 = 16
Sum of even places, 0 + 5 = 5
Difference of 16-5 = 11
So, 60258 is divisible by 11.

Common Factors and Common Multiples

Common Factors:

A Common Factor is a number that divides two or more numbers exactly.

Example:

Find the common factor of 6 and 8?

The factors of 6 are 1, 2, 3, and 6.

The factors of 8 are 1, 2, 4, and 8.

So, the common factors of 6 and 8 are 1, 2.

Example:

Find all the common factors of 16, 28, and 32.

Solution: We have,

$16 = 1 \times 16$,
$16 = 2 \times 8$,
$16 = 4 \times 4$
The factors of 16 are 1, 2, 4, 8, and 16.

$28 = 1 \times 28$,
$28 = 2 \times 14$,
$28 = 4 \times 7$
The factors of 28 are 1, 2, 4, 7, 14, and 28.

$32 = 1 \times 32$,
$32 = 2 \times 16$,
$32 = 4 \times 8$
The factors of 32 are 1, 2, 4, 8, 16, and 32.
So, the common factors of 16, 28, and 32 are 1, 2, and 4.

Common Multiple:

A common multiple is a number that is a multiple of two or more other numbers.

For Example:

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48...

Multiples of 8 are 8, 16, 24, 32, 40, 48...
So, the common multiples of 6 and 8 are 24, 48.

Example:

Find the common multiples of 5 and 10.

Solution:

The multiples of 5 are 5, 10, 15, 20, 25...

The multiples of 10 are 10, 20, 30, 40, 50...

We observe that all the multiples of 10 are also the multiples of 5.Therefore, 10, 20, 30, 40. . . are the common multiples of 5 and 10.

Co-prime number:

Two numbers having only 1 as a common factor are called co-prime numbers.

For Example:

4 and 15 are co-prime numbers.

Some More Divisibility Rules

Let us observe a few more rules about the divisibility of numbers:

(i) Can you give a factor of 18? It is 9. Name a factor of 9? It is 3. Is 3 a factor of 18? Yes it is. Take any other factor of 18, say 6. Now, 2 is a factor of 6 and it also divides 18. Check this for the other factors of 18. Consider 24. It is divisible by 8 and the factors of 8 i.e. 1, 2, 4 and 8 also divide 24. So, we may say that **if a number is divisible by another number then it is divisible by each of the factors of that number.**

(ii) The number 80 is divisible by 4 and 5. It is also divisible by 4 × 5 = 20, and 4 and 5 are co-primes. Similarly, 60 is divisible by 3 and 5 which are co-primes. 60 is also divisible by 3 × 5 = 15. **If a number is divisible by two co-prime numbers then it is divisible by their product also.**

(iii) The numbers 16 and 20 are both divisible by 4. The number 16 + 20 = 36 is also divisible by 4. Check this for other pairs of numbers.
If two given numbers are divisible by a number, then their sum is also divisible by that number.

(iv) The numbers 35 and 20 are both divisible by 5. Is their difference 35 – 20 = 15 also divisible by 5? Try this for other pairs of numbers also. **If two given numbers are divisible by a number, then their difference is also divisible by that number.**

Prime Factorization:

Prime factorization is to write a composite number as a product of its prime factors.

Examples:

Find the prime factorization of 48.

Solution:

The prime factorization of 48 is $2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$.

Example:

Find the prime factorization of 108.

Solution:

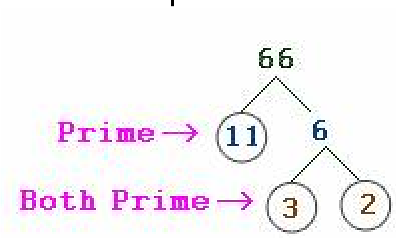
The prime factorization of 108 = $3 \times 3 \times 3 \times 2 \times 2$

Factor Tree:

Factor Tree is a hierarchical structure used to represent the prime factors of a number. All the composite numbers can be written as the product of the prime numbers using the factor tree.

For example:

To find the prime factors of 66, we use here a factor tree.



The circled numbers in the factor tree are the prime factors of 66.

$66 = 11 \times 3 \times 2$.

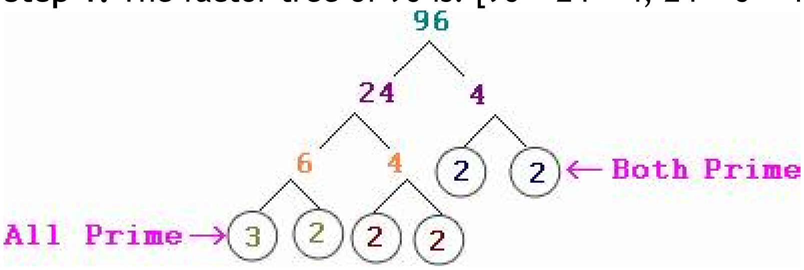
The prime factorization of 66 is $11 \times 3 \times 2$.

Example:

Find the prime factorization of 96.

Solution:

Step 1: The factor tree of 96 is: [96 = 24 × 4, 24 = 6 × 4, 4 = 2 × 2, 6 = 3 × 2]



Step 2: The circled numbers in the factor tree are the prime factors of 96.

Step 3: 96 = 3 × 2 × 2 × 2 × 2 × 2 = 31 × 25.

Highest Common Factor (HCF):

The Highest Common Factor (HCF) of two or more numbers is the highest number that divides the numbers exactly.

Example:

To find the HCF of 12, 24, and 36, first we list out the factors of the three numbers.

12 – 1, 2, 3, 4, 6, and **12**

24 – 1, 2, 3, 4, 6, 8, **12**, and 24

36 - 1, 2, 3, 4, 6, 8, **12**, 18, and 36

So, the HCF of the numbers 12, 24, and 36 is 12.

In another method, we need to write all the prime factors of the three numbers 12, 24, and 36.

12 – **2** × **2** × **3**
24 – **2** × **2** × 2 × **3**
36 **-** **2** × **2** × **3** × 3

Then we list out all the common prime factors. The common prime factors of the three numbers are 2 × 2 × 3. Then we have to multiply the common prime factors.
2 × 2 × 3 = 12
so, the HCF of the three numbers 12, 24, and 36 is 12.

Example:

Find the HCF of 18 and 35.

Solution: First we list the factors of 18 and 35.

18 = 1, 2, 3, 6, 9, and 18

35 = 1, 5, 7, and 35

The common factor of 18 and 35 is 1.
So, the HCF of 18 and 35 is 1.

Least Common Multiple (LCM):

Least common multiple is the smallest nonzero number that is a common multiple of two or more numbers considered.

Example:

The LCM of 6, 9, and 15 is 90.

6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90 . . .

9 = 9, 18, 27, 36, 45, 54, 63, 72, 81, 90 . . .

15 = 15, 30, 45, 60, 75, 90 . . .

Example:

Find the least common multiple of 13 and 11.

Solution:

As 13 and 11 are co-primes; the least common multiple is nothing but their product.
So, least common multiple of 13 and 11 is 143.

Some Problems on HCF and LCM

Example:

Two tankers contain 850 litres and 680 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

Solution:

The required container has to measure both the tankers in a way that the count is an exact number of times. So its capacity must be an exact divisor of the capacities of both the tankers. Moreover, this capacity should be maximum Thus; the maximum capacity of such a container will be the HCF of 850 and 680.

$850 = 2 \times 5 \times 5 \times 17 = 2 \times 5 \times 17 \times 5$ and

$680 = 2 \times 2 \times 2 \times 5 \times 17 = 2 \times 5 \times 17 \times 2 \times 2$

The common factors of 850 and 680 are 2, 5 and 17.

Thus, the HCF of 850 and 680 is $2 \times 5 \times 17 = 170$.
Therefore, maximum capacity of the required container is 170 litres. It will fill the first container in 5 and the second in 4 refills.

Example:

In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Solution:

The distance covered by each one of them is required to be the same as well as minimum. The required minimum distance each should walk would be the lowest common multiple of the measures of their steps. Can you describe why? Thus, we find the LCM of 80, 85 and 90. The LCM of 80, 85 and 90 is 12240.
The required minimum distance is 12240 cm.

Example:

Find the least number which when divided by 12, 16, 24 and 36 leaves a remainder 7 in each case.

Solution:

We first find the LCM of 12, 16, 24 and 36 as follows:
LCM of 12, 16, 24, 36 = $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$
144 is the least number which when divided by the given numbers will leave remainder 0 in each case. But we need the least number that leaves remainder 7 in each case.
Therefore, the required number is 7 more than 144. The required least number = $144 + 7 = 151$.

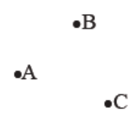
Ch-4-Basic Geometrical Ideas

Points:

A point determines a location.

If you mark three points on a paper, you would be required to distinguish them. For this they are denoted by a single capital letter like A, B, C. These points will be read as point A, point B and point C.

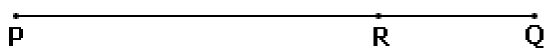
In the figure shown, A, B and C are points.



Line Segment:

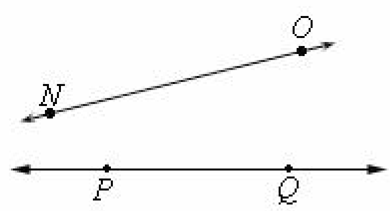
Line segment is the part of a line consisting of two endpoints and all points between them.

In the figure shown, PQ, PR, and RQ are line segments.



Line:

A Line is a straight path that is endless in both directions. A line does not have any thickness.

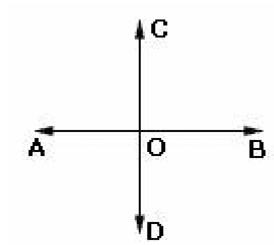


In the above figure, *NO* and *PQ* extend endlessly in both directions. So, NO and PQ represent lines.

Intersecting Lines:

Lines that have one and only one point in common are known as intersecting lines.

The common point where all the intersecting lines meet is called the Point of Intersection.

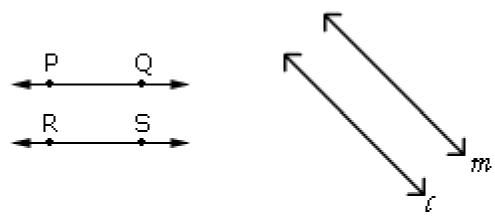


In figure, there are two lines **AB** and **CD** which intersect each other at **O**.

Parallel Lines:

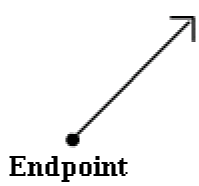
Parallel Lines are distinct lines lying in the same plane and they never intersect each other. Parallel lines have the same slope.

In the figure below, lines PQ and RS are parallel and the lines l and m are parallel.



Ray:

A ray is a part of a line that begins at a particular point (called the endpoint) and extends endlessly in one direction. A ray is also called half-line.



How do we name a Ray?
A ray is named based on the direction in which it extends.

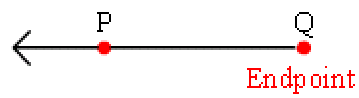
A ray is named with its endpoint in the first place, followed by the direction in which it's moving.

In the example shown below, P is the endpoint and Q is the point towards which the ray extends. So, the ray PQ is represented as:

$\overrightarrow{PQ}$

Look at another example. This ray will be called QP as it starts at Q and extends towards P. So, the ray QP is represented as:

$\overrightarrow{QP}$



Curve:

Curve is a line that is not straight.

A curve can be classified into two categories, an open curve or a closed curve.

Examples of Curve

Figure 1 Figure 2

Figure 1 is a closed curve and Figure 2 is an open curve.

Position in a figure

A court line in a tennis court divides it into three parts: inside the line, on the line and outside the line. You cannot enter inside without crossing the line.

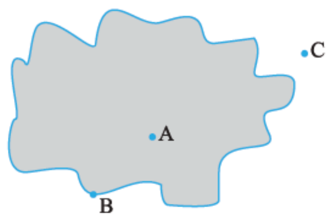
A compound wall separates your house from the road. You talk about ‘inside’ the compound, ‘on’ the boundary of the compound and ‘outside’ the compound.
In a closed curve, thus, there are three parts.

(i) Interior (‘inside’) of the curve

(ii) Boundary (‘on’) of the curve and

(iii) Exterior (‘outside’) of the curve.

In the figure below, A is in the interior, C is in the exterior and B is on the curve.



The interior of a curve together with its boundary is called its "**region**".

Polygon:

A polygon is a closed plane figure made up of 3 or more line segments.

Polygons have special names depending on the number of lines forming their boundary.

For example, a polygon with three sides is called **a triangle**.

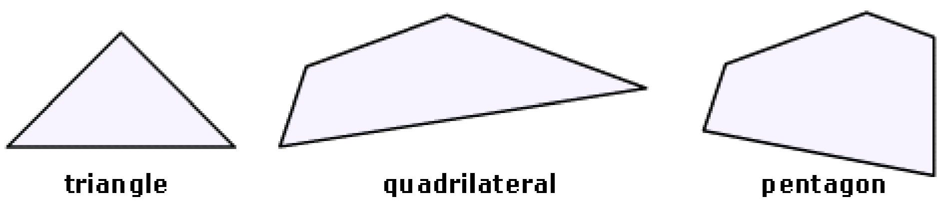
A polygon with four sides is called **a quadrilateral.**

A polygon with five sides is called **a pentagon.**

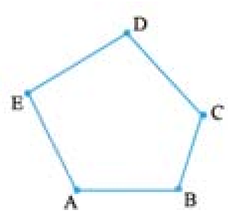
Polygons that have all sides measure the same are called **regular polygons.**

Examples of Polygon:

The figure shows a few polygons.



Sides, vertices and diagonals



Sides:

The line segments forming a polygon are called its **sides.**

The sides of polygon ABCDE are AB, BC, CD, DE and EA.

Vertex:

The meeting point of a pair of sides is called its **vertex.**

Sides AE and ED meet at E, so E is a vertex of the polygon ABCDE. Points B and C are its other vertices.

Adjacent sides:

Any two sides with a common end point are called the **adjacent sides** of the polygon.

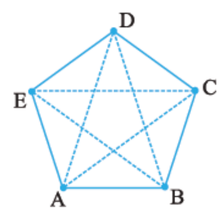
Adjacent vertices:

The end points of the same side of a polygon are called the **adjacent vertices**. Vertices E and D are adjacent, whereas vertices A and D are not adjacent vertices.

Diagonals:

Consider the pairs of vertices which are not adjacent. The joins of these vertices are called the diagonals of the polygon.

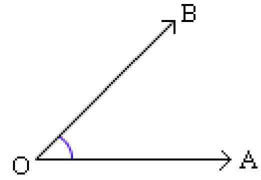
In the figure below, AC, AD, BD, BE and CE are diagonals.



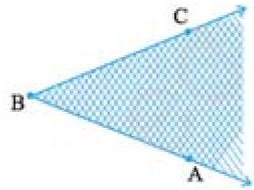
Angle:

An angle is formed by two rays with a common endpoint (called the vertex).

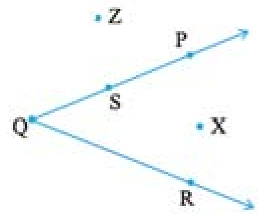
In the figure shown, angle AOB is formed by the rays OA and OB with a common endpoint O.



The portion common to both shadings is called the interior of ∠ABC. (Note that **the interior** is not a restricted area; it extends indefinitely since the two sides extend indefinitely).



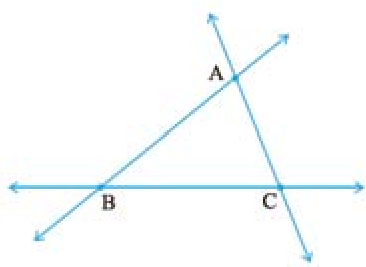
In the below diagram, X is in the interior of the angle, Z is not in the interior but in the exterior of the angle; and S is on the PQR. Thus, the angle also has three parts associated with it.



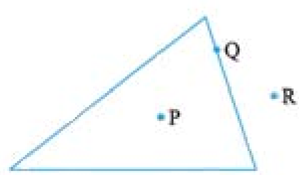
Triangle:

A triangle is a three-sided polygon.

In fact, it is the polygon with the least number of sides. We write DABC instead of writing "Triangle ABC". The three sides of the triangle are AB, BC and CA. The three angles are ∠BAC, ∠BCA and ∠ABC. The points A, B and C are called the vertices of the triangle.



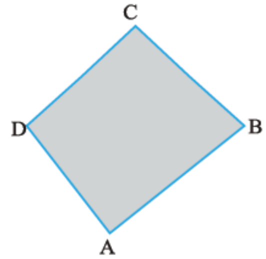
Being a polygon, a triangle has an exterior and an interior. In the figure, P is in the interior of the triangle, R is in the exterior and Q on the triangle.



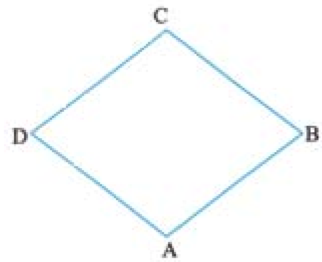
Quadrilateral:

A four sided polygon is a **quadrilateral.**

This quadrilateral ABCD has four sides AB, BC, CD and DA. It has four angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$.

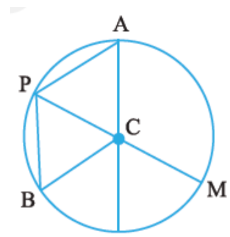


In any quadrilateral ABCD, AB and BC are adjacent sides. AB and DC are opposite sides. $\angle A$ and $\angle C$ are said to be opposite angles. Similarly, $\angle D$ and $\angle B$ are opposite angles. Naturally $\angle A$ and $\angle B$ are adjacent angles.



Circles:

A Circle is the locus of all points that are at an equal distance from a given point (on the plane) called the center.



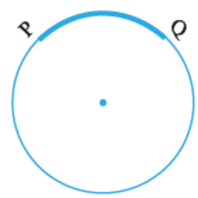
Parts of a circle

Here is a circle with **centre** C. Where A, P, B, M are points on the circle.

You will see that CA = CP = CB = CM. Each of the segments CA, CP, CB, and CM is **radius** of the circle. The radius is a line segment that connects the centre to a point on the circle.

CP and CM are radii (plural of ‘radius’) such that C, P, M are in a line. PM is known as **diameter** of the circle. Is a diameter double the size of a radius? Yes.

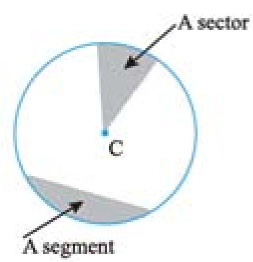
PB is a **chord** connecting two points on a circle.
Is PM also a chord? Yes.



An **arc** is a portion of circle.

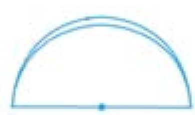
If P and Q are two points on a circle then you get the arc PQ.

As in the case of any simple closed curve you can think of the **interior** and **exterior** of a circle. A region in the interior of a circle enclosed by an arc on one side and a pair of radii on the other two sides is called **a sector.**



A region in the interior of a circle enclosed by a chord and an arc is called **a segment** of the circle.

The distance around a circle is its **circumference.**



The diameter of a circle divides it into **two semi-circles**. A semi-circle is half of a circle, with the end points of diameter as part of the boundary.

Ch-5-Understanding Elementary Shapes

Line Segment:

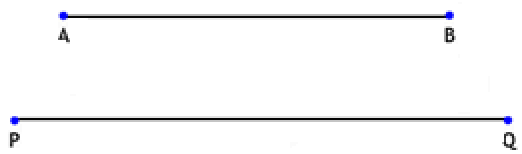
A line segment is a fixed portion of a line. This makes it possible to measure a line segment. This measure of each line segment is a unique number called its "**length**".

To compare any two line segments, we find a relation between their lengths.
This can be done in several ways.

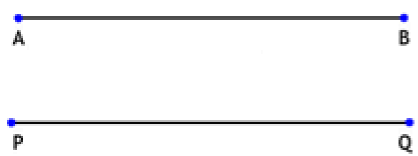
Comparison by observation:

We can compare line segments simply by looking at them.

For example, here the line segment PQ is greater than AB.



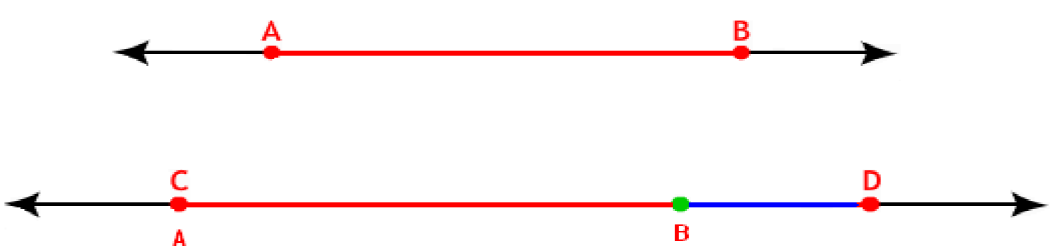
But sometimes we cannot be sure about our judgment. As shown in the figure:



Comparison by Tracing

Another method of comparing line segments is by tracing.

Consider two line segments and to compare AB and CD, we use a tracing paper, trace CD and place the traced segment on AB.

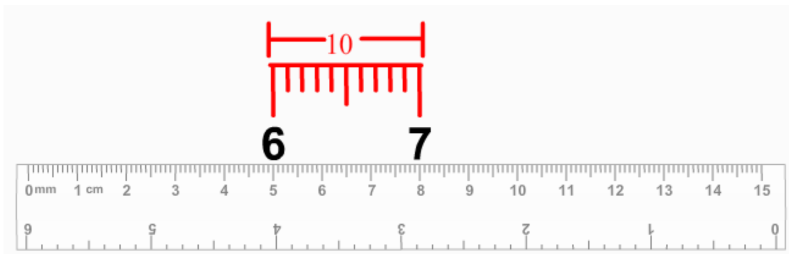


Now we can easily tell that CD is greater than AB.

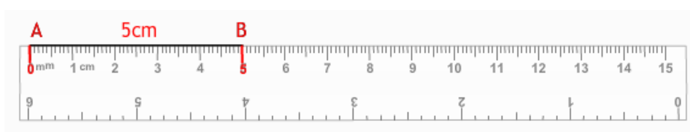
Accuracy of this method depends upon the accuracy of the trace. Also, it is hideous to trace a line segment every time we want to compare it.

Comparison using Ruler and a Divider

Let us know about a ruler and a divider. A ruler is divided into 15 parts, each of which is of length 1cm. Each centimeter is divided into 10 subparts, each of which is of length 1mm.

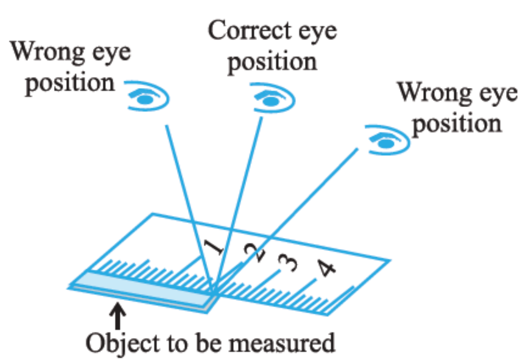


Now let us measure the line segment AB. Place the zero mark on the ruler at one end of the line segment say A. Read the mark at B. It is 5cm. So, the length of AB is 5cm.



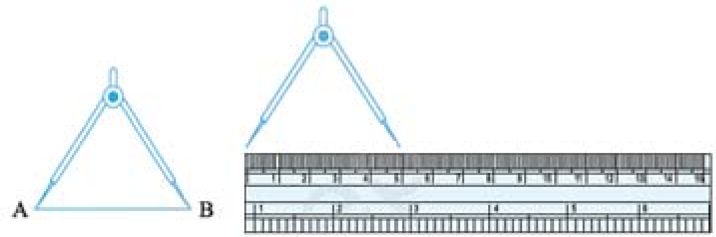
AB = 5 cm

Place your eye vertically above the mark to get a correct measure.



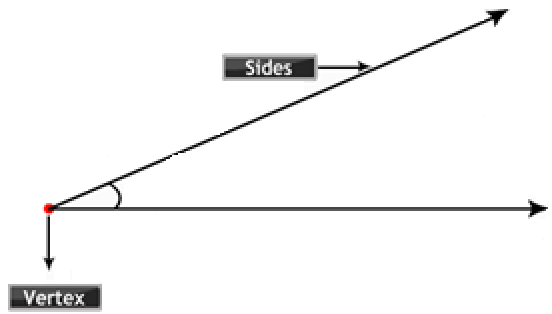
Let us use the divider to measure length.

To measure a line segment by a divider, open it and place the end point of one of its arms at A and the end point of the second arm at B. Now, without disturbing the opening of the divider, lift the divider and place it on a ruler. Ensure that one end point is at the zero mark of the ruler. Now read the mark against the other end point.



Angles – 'Right' and 'Straight'

An angle is the figure formed by two rays, called the **sides of the angle**, having a common endpoint, called the **vertex.**

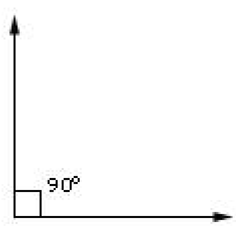


Right Angle

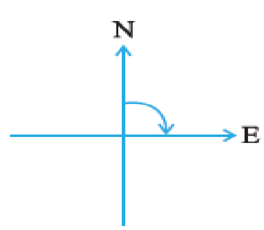
Right angle is an angle that has a measure of 90°.

Example:

The figure below shows a right angle.

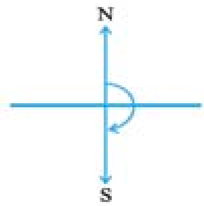


The turn from north to east is also a right angle.

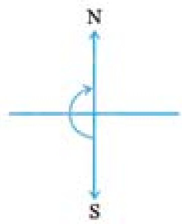


Straight angle

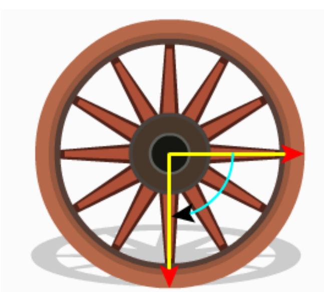
The turn from north to south is by two right angles; it is called a **straight angle**. (NS is a straight line)



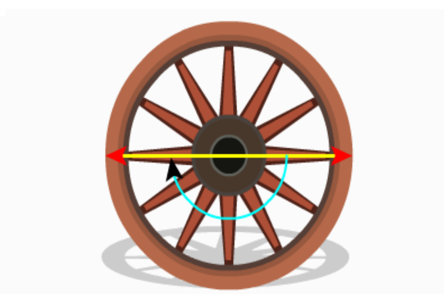
Stand facing south. Turn by a straight angle. Which direction do you face now? You face north.



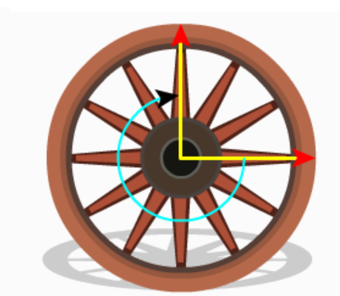
Turning by two straight angles (or four right angles) in the same direction makes a full turn. This one complete turn is called **one revolution**. The angle for one revolution is a **complete angle**.



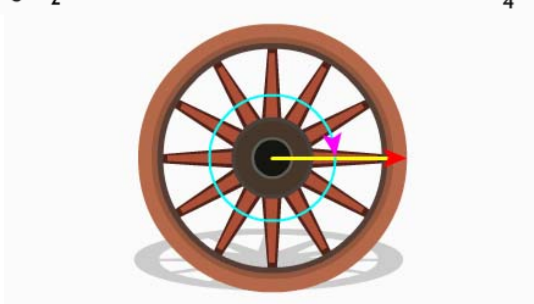
$\frac{1}{4}$ of the revolution = 1 right angle



$\frac{1}{2}$ of the revolution = 2 right angle = 1 straight angle



$\frac{3}{4}$ of the revolution = 3 right angles

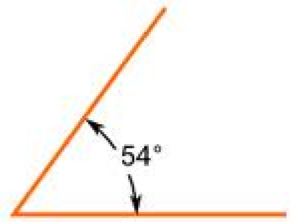


1 complete revolution = 4 right angles = 1 complete angle

Angles – ‘Acute’, ‘Obtuse’ and ‘Reflex’

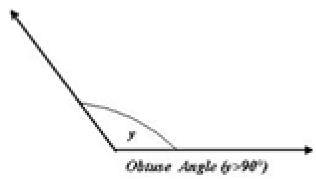
Acute angle:

An angle smaller than a right angle (90˚) is called an **acute angle**



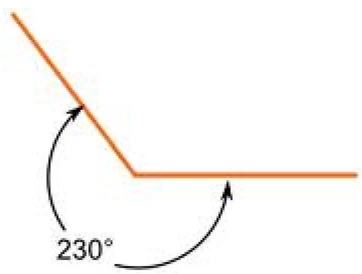
Obtuse Angle:

Any angle greater than 90˚ and smaller than 180˚ is called an **Obtuse Angle.**



Reflex Angle:

Reflex Angle is an angle that lies between 180˚ and 360˚.



Measuring Angles

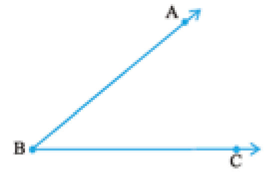
The measure of angle

We call our measure, 'degree measure'. One complete revolution is divided into 360 equal parts. Each part is a **degree**. We write 360˚ to say 'three hundred sixty degrees'.

The Protractor

You can find a readymade protractor in your 'instrument box'. The curved edge is divided into 180 equal parts. Each part is equal to a 'degree'. The markings start from 0˚ on the right side and ends with 180˚ on the left side, and vice-versa.

Suppose you want to measure an angle ABC.



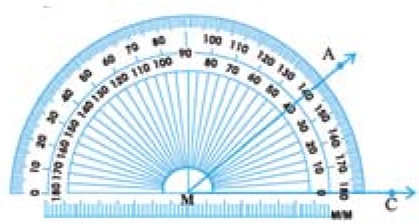
Given ∠ABC

Step 1: Place the protractor so that the mid point (M in the figure) of its straight edge lies on the vertex B of the angle.

Step 2: Adjust the protractor so that BC is along the straight-edge of the protractor.

Step 3: There are two 'scales' on the protractor: read that scale which has the 0° mark coinciding with the straight-edge (i.e. with ray BC).

Step 4: The mark shown by BA on the curved edge gives the degree measure of the angle.



Measuring ∠ABC

We write m ∠ABC= 40°, or simply ∠ABC= 40°.

Perpendicular Lines:

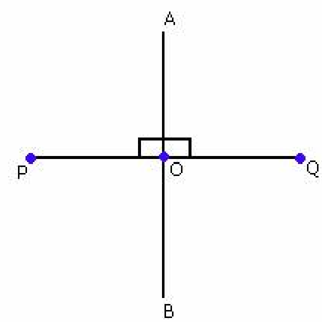
When two lines intersect and the angle between them is a right angle, then the lines are said to be **perpendicular**. If a line AB is perpendicular to CD, we write AB ⊥ CD.

Perpendicular Bisector

Perpendicular Bisector is a perpendicular line or a segment that passes through the midpoint of a line.

Example:

In the figure shown, AB is the perpendicular bisector of the line segment PQ passing through its midpoint 'O'.



Classification of Triangles

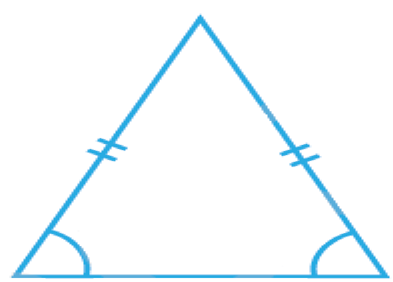
Naming triangles based on sides

Scalene Triangle:

A triangle having all three unequal sides is called a **Scalene Triangle.**

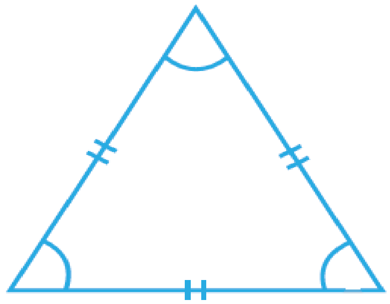
Isosceles Triangle:

A triangle having two equal sides is called an **Isosceles Triangle.**



Equilateral Triangle:

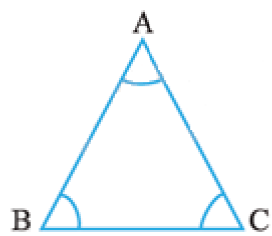
A triangle having three equal sides is called an **Equilateral Triangle.**



Naming triangles based on angles

Acute angled triangle:

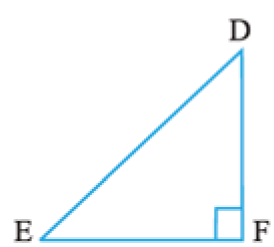
If each angle is less than 90°, then the triangle is called an **acute angled triangle**.



Acute Angled Triangle

Right angled triangle:

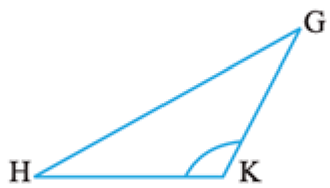
If any one angle is a right angle then the triangle is called a **right angled triangle**.



Right Angled Triangle

Obtuse angled triangle:

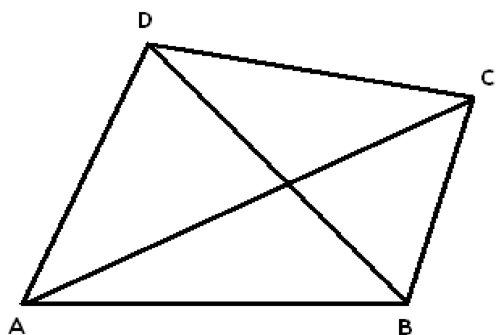
If any one angle is greater than 90°, then the triangle is called an **obtuse angled triangle**.



Obtuse Angled Triangle

Quadrilaterals

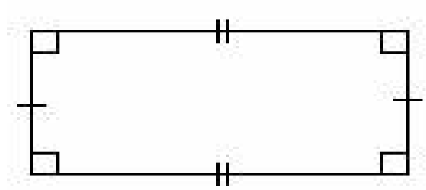
A quadrilateral is a polygon which has four sides. It is a quadrilateral, like the one you see here. The sides of the quadrilateral are AB, BC, CD, and DA. There are 4 angles for this quadrilateral. They are given by ∠BAD, ∠ADC, ∠DCB and ∠CBA.
BD and AC are the diagonals.



Quadrilateral

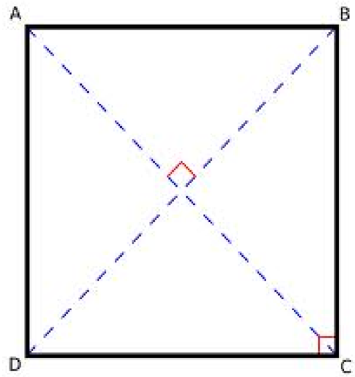
Rectangle:

A rectangle is a quadrilateral with four right angles. The either sides of a rectangle are parallel and of equal length.



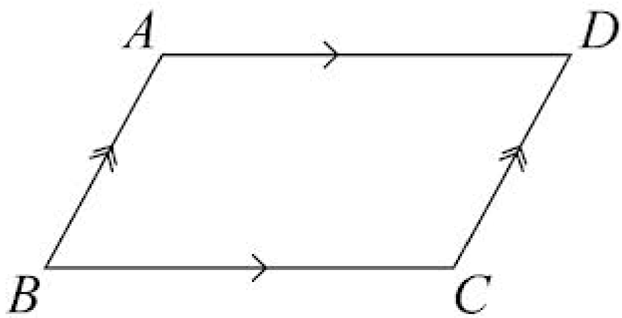
Square:

All the sides are of equal length.



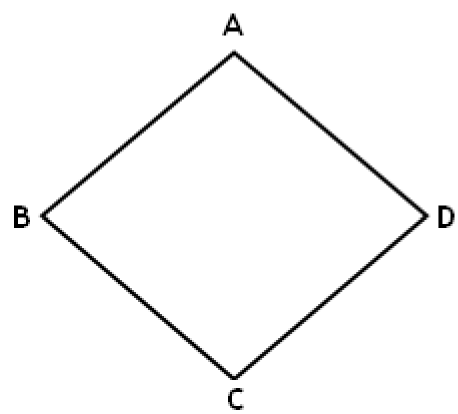
Parallelogram:

A Parallelogram is a quadrilateral whose opposite sides are parallel and equal.



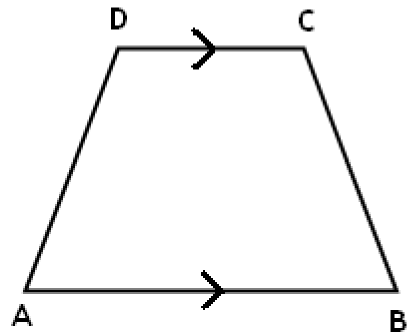
Rhombus:

Rhombus is a parallelogram with four equal sides.



Trapezium:

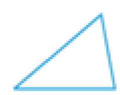
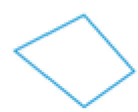
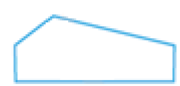
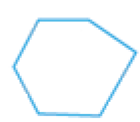
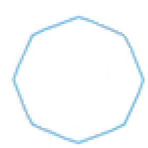
Trapezium is a quadrilateral with only one pair of parallel sides.



Polygons:

A polygon is a closed plane figure made up of 3 or more line segments.

Polygons are named based on their sides.

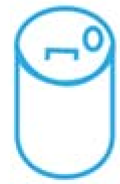
Number of sides	Name	Illustration
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
8	Octagon	

Three Dimensional Shapes

We see around us many three dimensional shapes like Cubes, cuboids, spheres, cylinders, cones, prisms and pyramids are some of them.

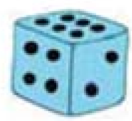
The ball is
a sphere.

The ice-cream is in
the form of a cone.

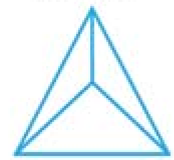


This can is a
cylinder.

The box is
a cuboid.



The playing die
is a cube.

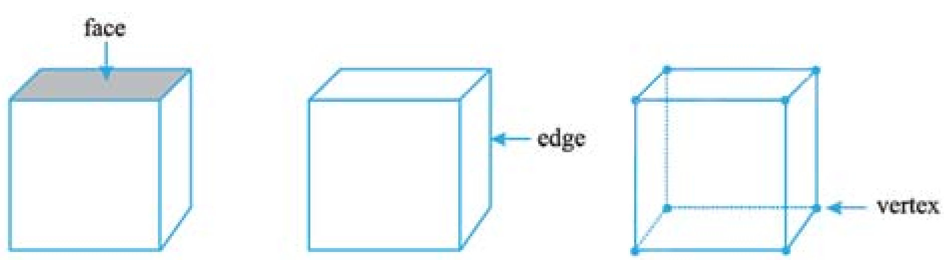


This is the shape
of a pyramid.

Faces, edges and vertices

In case of many three dimensional shapes we can distinctly identify their faces, edges and vertices. (Note 'Vertices' is the plural form of 'vertex').

Consider a cube each side of the cube is a flat surface called a flat **face** (or simply a **face**)**.** Two faces meet at a line segment called an **edge.** Three edges meet at a point called a **vertex.**

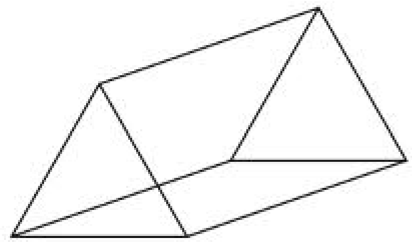


Prism:

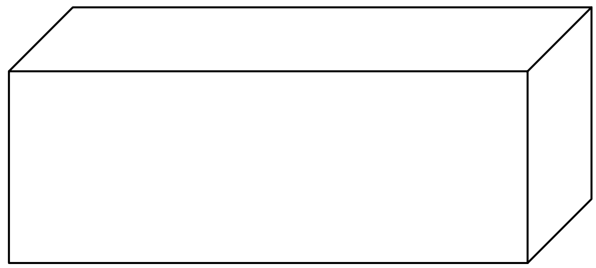
A prism is a polyhedron consisting of two parallel, congruent faces called bases.

One of its faces is a triangle. So it is called a **Triangular prism**.

The triangular face is also known as its base. A prism has two identical bases; the other faces are rectangles.

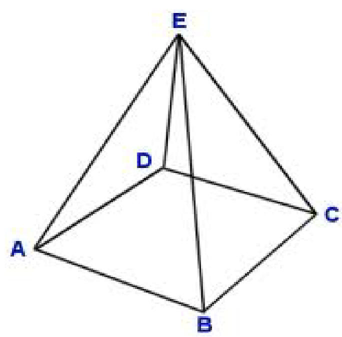


If the prism has a rectangular base, it is a **rectangular prism**.



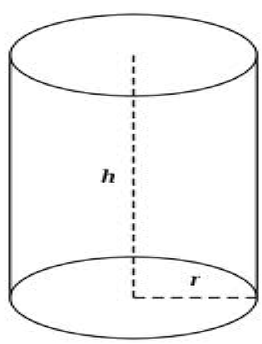
Pyramid:

A pyramid is a polyhedron with a polygonal base and triangles for sides.



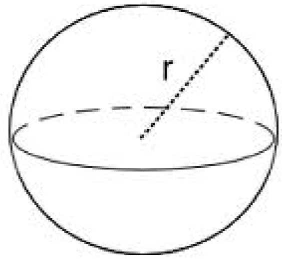
Cylinder:

A Cylinder is a three-dimensional geometric figure that has two congruent and parallel bases.



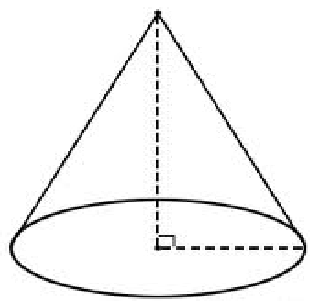
Sphere:

A three-dimensional surface, all points of which are equidistant from a fixed point



Cone:

Cone is a three-dimensional figure that has one circular base and one vertex.



Ch-6-Integers

Integers:

Integers are the set of whole numbers and their opposites. {. . . -3, -2, -1, 0, 1, 2, 3 . . .} is the set of integers.

Following are some examples of integers: -12, 315, 733, 751, 10, and 121.

Positive Integer:

The numbers greater than zero are called **Positive integers**

Positive integers are represented towards right of zero (0) on a number line. In this collection, 1, 2, 3 ... are said to be positive integers

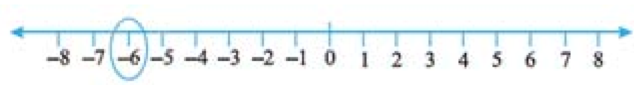
Negative Integer:

A number which is less than zero but not a fraction or a decimal is called a **Negative Integer**. It is represented by putting '-' sign before the number. It is shown to the left of zero on a number line. In this collection, - 1, – 2, – 3 ... are said to be negative integers.

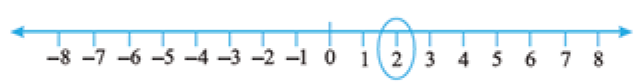
Representation of integers on a number line

Draw a line and mark some points at equal distance on it as shown in the figure.

Mark a point as zero on it. Points to the right of zero are positive integers and are marked + 1, + 2, + 3, etc. or simply 1, 2, 3 etc. Points to the left of zero are negative integers and are marked – 1, – 2, – 3 etc. In order to mark – 6 on this line, we move 6 points to the left of zero.



In order to mark + 2 on the number line, we move 2 points to the right of zero.



Ordering of integers:

Let us once again observe the integers which are represented on the number line.

We know that 7 > 4 and from the number line shown above, we observe that 7 is to the right of 4. Similarly, 4 > 0 and 4 is to the right of 0. Now, since 0 is to the right of –3 so, 0 > – 3. Again, – 3 is to the right of – 8 so, – 3 > – 8.

Thus, we see that on a number line the number increases as we move to the right and decreases as we move to the left.

Therefore, – 3 < – 2, – 2 < – 1, – 1 < 0, 0 < 1, 1 < 2, 2 < 3 so on. Hence, the collection of integers can be written as..., –5, –4, – 3, – 2, – 1,
0, 1, 2, 3, 4, 5...

Example 1:

Which integers lie between – 10 and – 4? Which is the largest integer and the smallest integer among them?

Solution:

Integers between – 10 and – 6 are – 9, – 8, – 7, -6, -5. The integer – 5 is the largest and – 9 is the smallest.

Addition of Integers:

Let us perform additions with the help of two positive integers.

You add When you have two positive integers like (+5) and (+7) then (+5) + (+7) = (+12) [=5+7]

Example:

Find the addition of
(a) (+ 10) + (+ 4)
(b) (+ 23) + (+ 40)

Solution:

(a) (+ 10) + (+ 4) = (10 + 4) = +14
(b) (+ 23) + (+ 40) = (23 + 40) = (+63)

Similarly, you also add when you have two negative integers, but remember that the answer will take a minus (-) sign like (-8) + (-2) = - (8+2) = -10.

Example:

Find the solution of the following:
(a) (– 11) + (– 12)
(b) (– 32) + (– 25)

Solution:

(a) (– 11) + (– 12) = - (11 + 12) = -23
(b) (– 32) + (– 25) = - (32 + 25) = -57

Now, when you have one positive and one negative integer, you must subtract, but answer will take the sign of the bigger integer.

Example: Fine the addition of

(a) (+12) + (– 7)
(b) (+ 7) + (– 10)

Solution:

(a) (+12) + (– 7) = (+12 -7) = (12– 7) = 5
(b) (+ 7) + (– 10) = (+ 7) + (– 10) = (+7 – 10) = (7 – 10) = -3

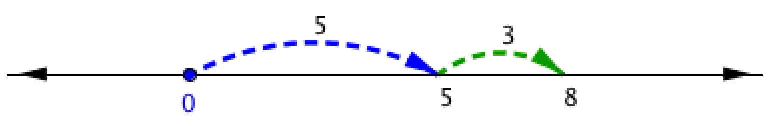
Addition of integers on a number line:

Example 1:

Let us add 5 and 3 on number line.

Solution:

On the number line, we first move 5 steps to the right from 0 reaching 5, and then we move 3 steps to the right of 5 and reach 8. Thus, we get 5 + 3 = 8

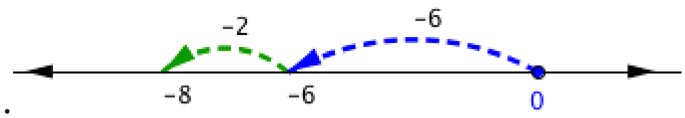


Example 2:

Let us add – 6 and – 2 on the number line.

Solution:

On the number line, we first move 6 steps to the left of 0 reaching – 6, then we move 2 steps to the left of – 6 and reach – 8. Thus, (– 6) + (– 2) = – 8



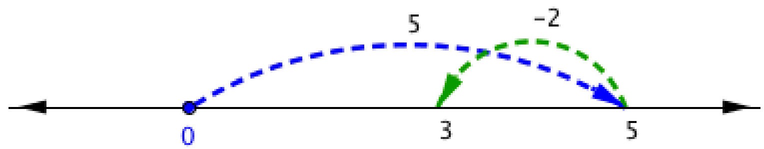
Note: We observe that when we add two positive integers, their sum is a positive integer. When we add two negative integers, their sum is a negative integer.

Example 3:

Find the sum of (+ 5) and (– 2) on the number line.

Solution:

First we move to the right of 0 by 5 steps reaching 5. Then we move 3 steps to the left of 5 reaching 3. Thus, (+ 5) + (– 2) = 3

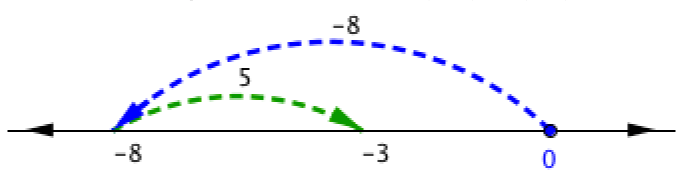


Example 4:

Let us find the sum of (– 8) and (+ 5) on the number line.

Solution:

First we move 8 steps to the left of 0 reaching – 8 and then from this point we move 5 steps to the right. We reach the point – 3. Thus, (- 8) + (+5) = – 3.



Additive Inverse:

The Additive Inverse of a number is the opposite of the number. A number and its opposite add up to give zero. They are called **additive inverse** of each other.

Example:

Find the additive inverse of
(a) 7

(b) -2?

Solution:

(a)The additive inverse of 7 is - 7.
7 + (- 7) = 0

(b)The additive inverse of - 2 is 2.
- 2 + 2 = 0

Subtraction of Integers with the help of a Number Line:

Example: 1

Find the value of – 8 – (–10) using number line.

Solution:

– 8 – (– 10) is equal to – 8 + 10 as additive inverse of –10 is 10.
On the number line, from – 8 we will move 10 steps towards right.

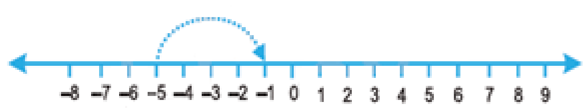
We reach at 2. Thus, –8 – (–10) = 2 Hence, to subtract an integer from another integer it is enough to add the additive inverse of the integer that is being subtracted, to the other integer.

Example: 2

Let us now find the value of –5 – (–4) using a number line.

Solution:

We can say that this is the same as –5 + (4), as the additive inverse of –4 is 4. We move 4 steps to the right on the number line starting from –5. We reach at –1 i.e. –5 + 4 = –1. Thus, -5 – (-4) = -1.



Some more examples of addition and subtraction:

Example 1:

Find the sum of (– 9) + (+ 4) + (– 6) + (+ 3)

Solution:

We have,

(– 9) + (+ 4) + (– 6) + (+ 3)

= (– 9) + (– 6) + (+ 4) + (+ 3)

= (– 15) + (+ 7)

= – 8

Example 2:

Find the value of (30) + (– 23) + (– 63) + (+ 55)

Solution:

(30) + (+ 55) + (– 23) + (– 63)

= 85 + (– 86)

= – 1

Example 3:

Find the sum of (– 10), (92), (84) and (– 15)

Solution:

(– 10) + (92) + (84) + (– 15)

= (– 10) + (– 15) + 92 + 84

= (– 25) + 176 = 151

Example 4:

Subtract (– 4) from (– 10)

Solution:

(– 10) – (– 4)

= (– 10) + (additive inverse of – 4)

= –10 + 4 = – 6

Example 5:

Subtract (+ 3) from (– 3)

Solution:

(– 3) – (+ 3)

= (– 3) + (additive inverse of + 3)

= (– 3) + (– 3) = – 6

Ch-7-Fraction

Fraction

A fraction is a number that represents part of a whole. A fraction is written in the form p/q, where $q \neq 0$.

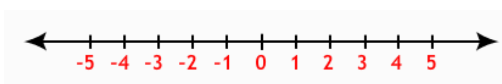
The top number of a fraction is called the **numerator**. The bottom number of a fraction is called the **denominator**.

Example:

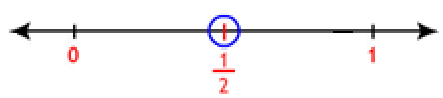
½ is a fraction, with numerator 1 and denominator 2.

Fractions on a number line:

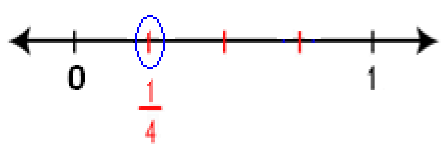
Let us draw a number line and try to mark ½ on it.



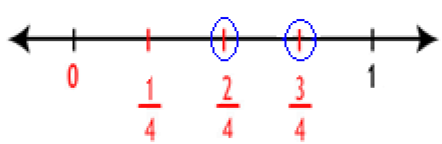
We know that ½ is greater than 0 and less than 1, so it should lie between 0 and 1. Since we have to show ½ we divide the gap between 0 and 1 into two equal parts and show 1 part as ½.



Now, we want to show ¼ on a number line. We divide the length between 0 and 1 into 4 equal parts and show one part as ¼.



Similarly, we can also show 2/4 and ¾ on a number line as shown in the figure below.



Proper Fractions:

A fraction having the numerator less than the denominator is called a **Proper Fraction**. The value of a proper fraction is always less than 1.
For Example:

2/5, 4/5, 7/11, 2/13 are proper fractions.

Improper Fraction:

An improper fraction is a fraction in which the number in the numerator is greater than or equal to the number in the denominator.

For Example:

17/15, 9/1, 32/12, 4/2 are improper fraction.

Mixed Fraction:

A Mixed Fraction is a number with a combination of an integer and a proper fraction. Mixed fraction is also called as mixed number.

The mixed fraction can also be written as Quotient = Remainder/Divisor.

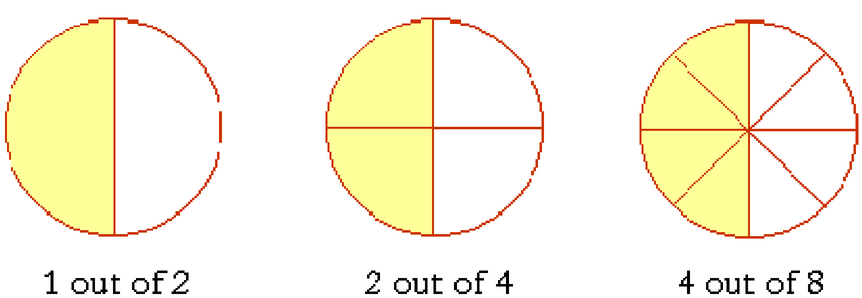
For Example:

2(3/4), 5(7/8), 3(9/10) are mixed fractions.

Equivalent Fractions:

Equivalent Fractions of a fraction are those fractions whose numerator and denominator are in the same ratio as that of the original fraction.

Equivalent fraction can be generated by multiplying or dividing the numerator and denominator by the same number.



Example:

In the given figure, shaded parts of the figures represent the equivalent fractions as $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.

Understanding equivalent fractions:

To find an equivalent fraction of a given fraction, you may multiply both the numerator and the denominator of the given fraction by the same number.

For example:

1/5 = (1x2) / (5x2) = 2/10.

Similarly, 2/3 = (2x3) / (3x3) = 6/9.

Now we see another way,

To find an equivalent fraction, we may divide both the numerator and the denominator by the same number.

For example:

15/35 = (15÷5) / (35÷5) = 3/7

Similarly, 16/20 = (16÷4) / (20÷4) = 4/5

Simplest Form of a Fraction:

A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.

Example:
Find the simplest form of 10/18?

Solution:

We have,

36/54= (36÷2) / (54÷2) = 18/27

But 18 and 27 also have common factors other than one.
The common factors are 1, 3, 9; the highest is 9.

Therefore, 18/27 = (18÷9) / (27÷9) = 2/3

Now 2 and 3 have no common factor except 1; we say that the fraction 2/3 is in the simplest

The shortest way, The shortest way to find the equivalent fraction in the simplest form is to find the HCF of the numerator and denominator, and then divide both of them by the HCF.

For example:

Find the simplest form of 26/39?

Solution:

We have, 26/39

The HCF of 26 and 39 is 13.

So, (26÷13) / (39÷13) = 2/3. The fraction 2/3 is in the lowest form. Thus, HCF helps us to reduce a fraction to its lowest form.

Like Fractions:

The different fractions with the same denominator are **Like Fractions**.

Example:

3/7, 5/7, 11/7, 8/7 are like fractions, as they have the same denominator 7.

Unlike Fractions:

Fractions whose denominators are not the same are called **unlike fractions**.

Example:

10/7, 5/4, 11/8, 5/2 are unlike fractions, as they have the different denominator.

Comparing Fractions

Comparing like fractions:

When comparing two fractions with like denominators, the larger fraction is the one with the greater numerator. Let's look at some examples of comparing fractions with like denominators.

Example 1:

Compare between 2/5 and 8/5.

Solution:

Given fractions are like fractions, so the fraction with greater numerator is larger. Hence, 8/5 is greater than 2/5.

Example 2:

Find among the given two fraction which is greater 24/87 and 10/87.

Solution:

Given fractions have same denominator, so the fraction with greater numerator is larger.
Hence, 24/87 > 10/87

Example 3:

Compare between 23/10 and 13/10.

Solution:

Given fractions are like fractions, so the fraction with greater numerator is larger.
Hence, 23/10 > 13/10

Example 4: Find among the given two fraction which is greater 11/90 and 77/90.

Solution:

Given fractions have same denominator, so the fraction with greater numerator is larger.
Hence, 77/90 is greater than 11/90.

Comparing unlike fractions (with same numerator)

When you need to compare unlike fraction with same numerator, the fraction with smallest denominator is greatest. Let's look at some examples of comparing fractions with same numerator

Example 1:

Compare 15/6 and 15/8

Solution:

The given fractions have same numerator i.e. 15, so we compare them on the basis of their denominator.
The fraction which have smallest denominator is greatest.
In this, denominator 6 is smaller than denominator 8.
Hence, 15/6 > 15/8

Example 2:

Compare between 9/8 and 9/2

Solution:

The given fractions have same numerator i.e. 9, so we compare them on the basis of their denominator.
The fraction which have smallest denominator is greatest.
In this, denominator 2 is smaller than denominator 8.
Hence, 9/2 > 9/8

Example 3:

Compare 77/32 and 77/23

Solution:

The given fractions have same numerator i.e. 77, so we compare them on the basis of their denominator.
The fraction which have smallest denominator is greatest.
In the given example denominator 23 is smaller than denominator 32.
Hence, 77/23 > 77/32

Example 4:

Compare between 243/100 and 243/150

Solution:

The given fractions have same numerator i.e. 243, so we compare them on the basis of their denominator.
The fraction which have smallest denominator is greatest.
In this, denominator 100 is smaller than denominator 150.
Hence, 243/100 > 243/150

Comparing unlike fractions (with different numerator and denominator):

When you need to compare two Unlike Fractions whose numerators are different;
First find equivalent fractions of both the given unlike fractions with a common

denominator. Then compare equivalent fractions and the fraction with larger numerator is the biggest.

Example 1:

Compare 1/3 and 2/5.

Solution:

The given unlike fractions have different numerators i.e. 1 and 2
So, first find a common denominator with the help of LCM of denominator i.e. 3 and 5 and we get; LCM of 3 and 5 = 15.
Now, find equivalent fractions with common denominator(15) of given unlike fraction;
Equivalent Fraction of 1/3 = (1 × 5) / (3 × 5) = 5/15
And, Equivalent Fraction of 2/5 = (2 × 3) / (5 × 3) = 6/15
Now, compare equivalent fractions i.e. 5/15 and 6/15;
As numerator 6 is larger than numerator 5 of given fractions,
So 6/15 is greater than 5/15
or corresponding fractions; 2/5 is greater than 1/3.

Example 2:

Compare 7/6 and 1/15.

Solution:

The given unlike fractions have different numerators i.e. 7 and 1,
So, first find a common denominator with the help of LCM of denominator i.e. 6 and 15 and we get;
LCM of 6 and 15 = 30
Now, find equivalent fractions with common denominator(30) of given unlike fraction;
Equivalent Fraction of 7/6 = (7 × 5) / (6 × 5) = 35/30
And, Equivalent Fraction of 1/15 = (1 × 2) / (15 × 2) = 2/30
Now, compare equivalent fractions i.e. 35/30 and 2/30;
As numerator 35 is larger than numerator 2 of given fractions,
So 35/30 is greater than 2/30
or corresponding fractions; 7/6 is greater than 1/15.

Example 3:

Compare 7/10 and 3/4.

Solution:

The given unlike fractions have different numerators i.e. 7 and 4,
So, first find a common denominator with the help of LCM of denominator i.e. 10 and 4 and we get;
LCM of 10 and 4 = 20.
Now, find equivalent fractions with common denominator(20) of given unlike fraction;
Equivalent Fraction of 7/10 = (7 × 2) / (10 × 2) = 14/20
And, Equivalent Fraction of 3/4 = (3 × 5) / (4 × 5) = 15/20
Now, compare equivalent fractions i.e. 14/20 and 15/20;
As numerator 15 is larger than numerator 14 of given fractions,
So 15/20 > 14/20
or corresponding fractions; 3/4 is greater than 7/10

Adding or subtracting like fractions

Addition of fractions (with Same Denominator):

When we have to add fractions with same denominators, we simply add the numerators and denominator remains same.

Example: 1

Add fractions 1/10 and 5/10

Solution:

In the given fractions the denominators are same, so addition proceeds as:
Add the numerators and denominator remains same and we get:
(1+5) / 10 = 6 / 10 = 3 / 5 (in Lowest Term)

Example: 2

Add 10/3 and 22/3

Solution:

In the given fractions the denominators are same, so addition proceeds as:
Add the numerators and denominator remains same and we get:
(10+22) / 3 = 32 / 3.

Example: 3

Add 2/101 and 4/101

Solution:

In the given fractions the denominators are same, so addition proceeds as:
Add the numerators and denominator remains same and we get:
(2+4) / 101 = 6 / 101.

Example: 4

Add 25/39 and 40/39

Solution:

In the given fractions the denominators are same, so addition proceeds as:
Add the numerators and denominator remains same and we get:
(25+40) / 101 = 65 / 39.

Addition of Fraction (with Different Denominators):

It involves following steps:

Step 1 = LCM of Denominators of given Fraction

Step 2 = Divide LCM (obtained in step 1) with the denominators of given fractions.

Step 3 = Multiply quotient (obtained from step 2) with the numerators of given fractions.

Example:

Add (2/7) and (3/5).

Solution:

The addition proceeds as (2/7) + (3/5)

LCM of denominators 7 and 5 = 35
Now, divide the LCM by denominator & multiply its quotient with numerators and we get:
= [(2×5) + (3×7)] / 35
= (10 + 21) / 35
= 31 / 35

Subtraction of Fractions (with Same Denominator):

When we have to subtract fractions with same denominators (or subtract like fractions). We simply subtract the numerators and denominator remains same.

Example: 1

Subtract 10/2 from 15/2.

Solution:

(15/2) - (10/2)
In the given fractions the denominators are same, so subtraction proceeds as:
Subtract the numerators and denominator is kept common and we get:
= (15 - 10) / 2
= 5 / 2.

Example: 2

Subtract 10/11 from 8/11.

Solution:

(8/11) - (10/11)
In the given fractions the denominators are same, so subtraction proceeds as:
Subtract the numerators and denominator is kept common and we get:
= (8 - 10) / 11
= -2 / 11.

Example: 3

Solve (29/100) - (28/100).

Solution:

In the given fractions the denominators are same, so subtraction proceeds as:
Subtract the numerators and denominator is kept common and we get:
= (29 - 28) / 100
= 1 / 100

Subtraction of Fractions (with Different Denominator):

To subtract fractions with different denominators, we must follow these steps:

Step 1 = Find equivalent fractions of given fraction with common denominator

Step 2 = Follow the process of subtraction of fractions with same denominator

Example 1:

Subtract 3/4 from 5/6.

Solution:

We need to find equivalent fractions of 3/4 and 5/6, which have the same denominator. This denominator is given by the LCM of 4 and 6. The required LCM of 4 and 6 is 12.
Therefore, 5/6 – 3/4 = (5 x 2)/ (6 x 2) – (3 x 3)/ (4 x 3) = 10/12 – 9/12 = 1/12

Example 2:

Subtract 2/5 from 5/7.

Solution:

We need to find equivalent fractions of 2/5 and 5/7, which have the same denominator. This denominator is given by the LCM of 5 and 7. The required LCM of 5 and 7 is 35.
Therefore, 5/7 – 2/5 = (5 x 5)/ (7 x 5) – (2 x 7)/ (5 x 7) = 25/35 – 14/35 = 11/35.

Addition of Mixed Fraction:

Example: 1

Add 2(4/5) and 3(5/6).

Solution:

2(4/5) + 3(5/6) = 2 + 4/5 + 3+ 5/6 = 5 + 4/5 + 5/6.

Now 4/5 +5/6 = (4 x 6)/ (5 x 6) + (5 x 5)/ (6 x 5) (Since LCM of 5 and 6 = 30)
= 24/30 + 25/30 = 49/30 = (30 + 19)/ 30 = 1+ 19/30.
Thus, 5 + 4/5 + 5/6 = 5 + 1 + 19/30 = 6 + 19/30
= 6(19/30)

And, therefore, 2(4/5) + 3 (5/6) = 6(19/30)

Example: 2

Add 4(2/3) and 3(1/4).

Solution:

4(2/3) + 3(1/4) = 4 + 2/3 + 3 + 1/4 = 7 + 2/3 + 1/4.

Now 2/3 +1/4 = (2 x 4)/ (3 x 4) + (1 x 3)/ (4 x 3)
= 8/12 + 3/12 = 11/12
Thus, 7 + 11/12 = 7(11/12)

And, therefore, 4(2/3) + 3 (1/4) = 7(11/12)

Subtraction of Mixed Fraction:

Example: 1

Subtract 1(3/4) from 2(1/5).

Solution:

2(1/5) - 1(3/4)
= (2 + 1/5) – (1+ 3/4)
= [(2 x 5) +1]/5 – [(4 x 1) +3]/4
Now, (10+1)/5 – (4 +3)/4 = 11/5 – 7/4
= (11 x 4)/ (5 x 4) - (7 x 5)/ (4 x 5)
= 44/20 - 35/20 = 9/20
And, therefore, 2(1/5) - 1(3/4) = (9/20)

Example: 2

Subtract 2(2/5) from 3(5/6).

Solution:

3(5/6) - 2(2/5)

= 3(5/6) - 2(2/5) = 3 + 5/6 - 2 - 2/5 = 1 + 5/6 - 2/5.
Now, (5/6) - (2/5) = (5 x 5)/ (6 x 5) - (2 x 6)/ (5 x 6)
= 25/30 - 12/30 = 13/30
Thus, 1 + (5/6 - 2/5) =1 + 13/30
And, therefore, 3(5/6) - 2(2/5) = 1(13/20)

Ch-8-Decimals

Decimal:

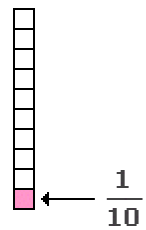
A number that uses place value and a decimal point to show tenths, hundredths, thousandths and so on.

For example:

The decimal 0.1 = $^{1}/_{10}$, 0.12 = $^{12}/_{100}$, 0.003 = $^{3}/_{1000}$

Tenths:

One out of ten equal parts is called a **tenth**.



The fractional form of one tenth is $\frac{1}{10}$ and the decimal equivalent is 0.1.

In a decimal, the first digit after the decimal point denotes tenth.

For example:

There are nine tenths in the number 2543.978.

Example:

Identify the digit in hundredths place of 345.235

Solution:

Step 1: Represent the places of the digits in the number as shown below.

Hundreds (100)	Tens (10)	Ones 1	Tenths (1/10)	Hundredths (1/100)	Thousandths (1/1000)
3	4	5	2	3	5

Decimal Line

Step 2: From the above figure, we learn that the digit in the tenth place is 2.

Representing Decimals on number line:

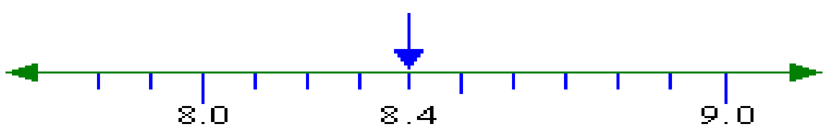
To represent a decimal on a number line, divide each segment of the number line into ten equal parts.

Example:

To represent 8.4 on a number line,

Solution:

First divide the segment between 8 and 9 into ten equal parts. The arrow is four parts to the right of 8 where it points at 8.4.

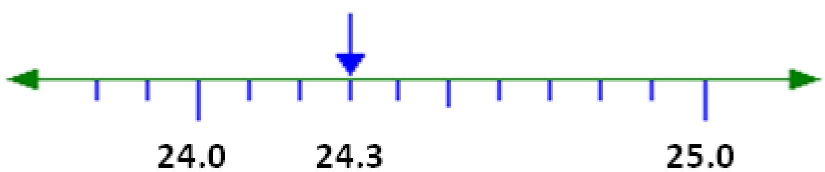


Example:

To represent 24.3 on a number line,

Solution:

First divide the segment between 24 and 25 into ten equal parts. The arrow is third part to the right of 8 where it points at 24.3.



Fractions as decimals:

We have seen how a fraction with denominator 10 can be represented using decimals. So Now, Let us try to find decimal representation of a fraction with the following examples:

Example: 1

Write in decimal notation.

(a) 3/2
(b) 4/5
(c) 8/5

Solution:

For writing in decimal notation, the denominator should be 10.

(a) 3/2

In 3/2, the denominator is 2. We first make an equivalent fraction of 3/2 which have denominator 10. So,

3/2 = (3x5) / (2x5) = 15/ 10 = 1.5

Therefore, 3/2 is **1.5** in decimal notation.

(a) 4/5

In 4/5, the denominator is 5. We first make an equivalent fraction of 4/5 which have denominator 10. So,

4/5 = (4x2) / (5x2) = 8/10 = 0.8

Therefore, 4/5 is **0.8** in decimal notation.

(c) 8/5

In 8/5, the denominator is 5. We first make an equivalent fraction of 8/5 which have denominator 10. So,

8/5 = (8x2) / (5x2) = 16/10 = 1.6

Therefore, 8/5 is **1.6** in decimal notation.

Decimals as fractions:

Any decimal can be converted to a fraction by counting the number of decimal places, and putting the decimal's digits over 1followed by the appropriate number of zeroes.

Example:

Write as fractions in lowest terms.

(a) 0.46
(b) 1.5
(c) 10.2
(d) 0.0003

Solution:

(a) 0.46 = 46/100 = 23/50

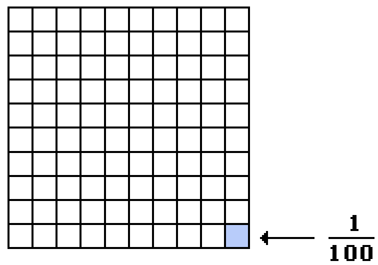
(b) 1.5 = 15/10 = 3/2

(c) 10.2 = 102/10 = 51/5

0.0003 = 3 / 10000

Hundredths:

One out of one hundred equal parts is called a **hundredths**



The fractional form of one hundredth is $\frac{1}{100}$ and the decimal equivalent is 0.01.

In a decimal, the second digit after the decimal point denotes hundredth.

For example: T

here are seven hundredths in the number 2543.978.

Example:

Identify the digit in hundredths place of 123.456.

Solution:

Step 1: Represent the places of the digits in the number as shown below.

Hundreds (100)	Tens (10)	Ones 1	Tenths (1/10)	Hundredths (1/100)	Thousandths (1/1000)
1	2	3	4	5	6

Decimal Line

Step 2: From the above figure, we learn that the digit in the hundredth place is 5.

Thousandth:

One part out of 1,000 equal parts of a whole
The fractional form of one hundredth is 1/1000 and the decimal equivalent is 0.001.

Comparing Decimals:

To compare two decimal numbers, compare digits in corresponding positions from left to right until the larger number is identified.

For example:

Compare 1.57 and 1.6.

Solution:

The unit's digit is 1 for both numbers and the tenths digit is 5 and 6 for the respective numbers.

Clearly, $5/10 < 6/10$

So, 1.57 is less than 1.6.
Thus, 1.6 is the larger number.

Let us see some more examples:

Example:

Find the larger number in each pair:

a. 3.6 and 3.63
b. 5.738 and 5.74
c. 29.86527 and 29.869613

Solution:

(a). 3.6 is the same as 3.60. Now to compare 3.60 and 3.63, compare the digits in corresponding positions from left to right until the larger number is identified.

The unit's digit is 3 and the tenths digit is 6 for both numbers. The hundredths digit is 0 and 3 for the respective numbers.

Clearly, $0/100 < 3/100$

So, 3.6 is less than 3.63.
Thus, 3.63 is the larger number.

(b). 5.74 is the same as 5.740. Now to compare 5.738 and 5.740, compare the digits in corresponding positions from left to right.

The unit's digit is 5 and the tenths digit is 7 for both numbers. The hundredths digit is 3 and 4 for the respective numbers.

Clearly, $3/100 < 4/100$

So, 5.738 is less than 5.74.
Thus, 5.74 is the larger number.

(c). To compare 29.86527 and 29.869613, compare the digits in corresponding positions from left to right.

The tens digit, units digit, tenths digit and hundredths digit are the same for both numbers. The thousandths digit is 5 and 9 for the respective numbers.

Clearly, $5/1000 < 9/1000$

So, 29.86527 is less than 29.869613.
Thus, 29.869613 is the larger number.

Conversion Using Decimals

Money:

We know that, 100 paise = Rs 1

Therefore, 1 paise = Rs 1/100 = Rs 0.01

So, 65 paise = Rs 65/ 100 = Rs 0.65

And 5 paise = Rs5/100 = Rs 0.05

Example:

What is 105 paise in Rs?

Solution:

1 paise = Rs 1/100 = Rs 0.01

105 paise = 100 paise +5 paise = 1Rs + (5/100) Rs = Rs (1 + 0.05) =Rs 1.05

Length:

In order to compare lengths (or distances), it is essential that we express them in the **same units**.

Example 1:

Convert 74 mm to cm.

Solution:

We know that 1cm = 10mm or 1mm =1/10cm

So, 74 = 74 x 1/10cm

= (74/10) cm

= 7.4cm.

Example 2:

Convert 38 cm to m.

Solution:

We know that 1m = 100cm or 1cm =1/100m

So, 38cm = 38 x 1/100m

= 0.38 m

Example 3:

Convert 10m to km.

Solution:

We know that 1km = 1000m or 1m = 1/1000km

So, 10m = 10 x 1/1000km

= 0.01km

Weight:

1000 g = 1 kg

Therefore, 1 g = 1/1000kg = 0.001 kg

Example: 1

Write 2kg 9g in 'kg' using decimals?

Solution:

We know that, 1000 g = 1 kg or 1 g = 1/1000kg = 0.001 kg

2kg 9g = 2kg + 9/1000kg
= 2 kg + 0.009kg
= 2.009kg

Example 2:

Convert 1086g to kg.

Solution:

We know that 1kg = 1000m or 1g = 1/1000kg

So, 1086g = 1086 x 1/1000kg

=1086/1000kg

= 1.086kg

Addition of Numbers with Decimals:

To add decimal numbers, write them with decimal points below one another.

Example: 1

Add 6.7 + 3.9 + 4.6?

Solution:

we write them with decimal points below one another.

$$
\begin{array}{r}
2 \\
6.7 \\
3.9 \\
+\ 4.8 \\
\hline
15.4 \\
\hline
\end{array}
$$

Example: 2

Calculate 5.84 + 8 + 12.79.

Solution:

To add the given decimal numbers, insert zeros so that all of the numbers have the same number of decimal places and write the numbers with decimal points below one another.

$$
\begin{array}{r}
{\scriptstyle 1\,1\,1} \\
5.84 \\
8.00 \\
+\,12.79 \\
\hline
26.63 \\
\hline
\end{array}
$$

Example: 3

Mohan bought 9 kg 40 g of apples, 4 kg 70 g of grapes and 12 kg 500 g of mangoes. Find the total weight of all the fruits he bought.

Solution:

Weight of apples = 9 kg 40 g = 9.040 kg
Weight of grapes = 4 kg 70 g = 4.070 kg
Weight of mangoes = 12 kg 500 g = 12.500 kg

Therefore, the total weight of the fruits bought is

$$
\begin{array}{r}
{\scriptstyle 1\;1\;1\qquad\quad} \\
9.040\text{ kg} \\
4.070\text{ kg} \\
+\,12.500\text{ kg} \\
\hline
25.610\text{ kg} \\
\hline
\end{array}
$$

Total weight of the fruits bought = 25.610 kg

Subtraction of Decimals:

To subtract a small decimal number from a larger decimal number, write them down with the larger one on top and the decimal points underneath one another. Then calculate the subtraction as you would for whole numbers and line up the decimal point in the answer.

Example:

Calculate 3.67 – 1.83?

Solution:

$$
\begin{array}{r}
{}^{2}\cancel{3}.{}^{1}67 \\
-\,1.83 \\
\hline
1.84 \\
\hline
\end{array}
$$

Thus, 3.67 – 1.83 = 1.84

Example:

Calculate 83.47 – 57.684?

Solution:

$$\begin{array}{r} 83.470 \\ -\ 57.684 \\ \hline 25.786 \\ \hline \end{array}$$

Thus, 83.47 - 57.684 = 1.84

Example:

Sonia's school is at a distance of 5 km 350 m from her house.
She travels 1 km 70 m on foot and the rest by bus. How much distance does she travel by bus?

Solution:

Total distance of school from the house = 5.350 km
Distance travelled on foot = 1.070 km
Therefore, distance travelled by bus = 5.350 km – 1.070 km
= 4.280 km

Thus, distance travelled by bus = 4.280 km or 4 km 280 m

Ch-9-Data Handling

Data

Data can be defined as a collection of facts or information from which conclusions may be drawn.

Example:

The data shown below are Mark's scores on five Math tests conducted in 10 weeks.
45, 23, 67, 82, 71

Recording Data:

A data collection sheet is a way of recording information.

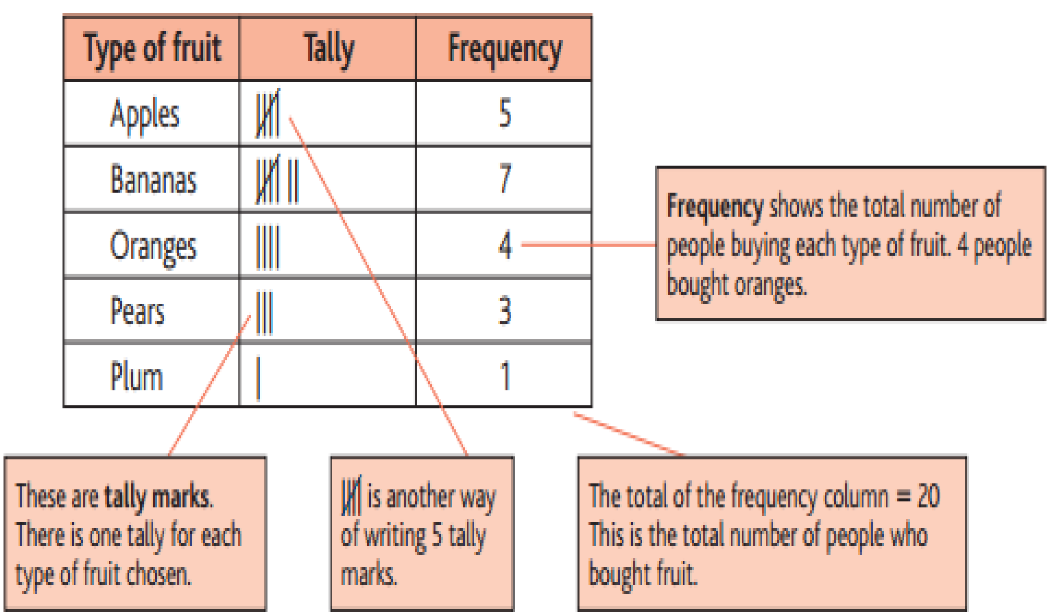
Example:

Twenty people bought the following types of fruit.

Apples	Bananas	Plum	Apples	Bananas
Oranges	Oranges	Apples	Pears	Pears
Bananas	Bananas	Oranges	Bananas	Apples
Bananas	Bananas	Pears	Apples	Oranges

Draw a completed data collection sheet to show this information.

Solution:



Organisation of Data:

Example:

20 students took a math test. Their test marks are shown below

17, 9, 6, 12, 19,

4, 13, 12, 15, 16,

20, 14, 5, 10, 11,

17, 8, 14, 12, 18

Complete the grouped frequency table.

Mark	Tally	Frequency
1–5		
6–10		
11–15		
16–20		

Solution:

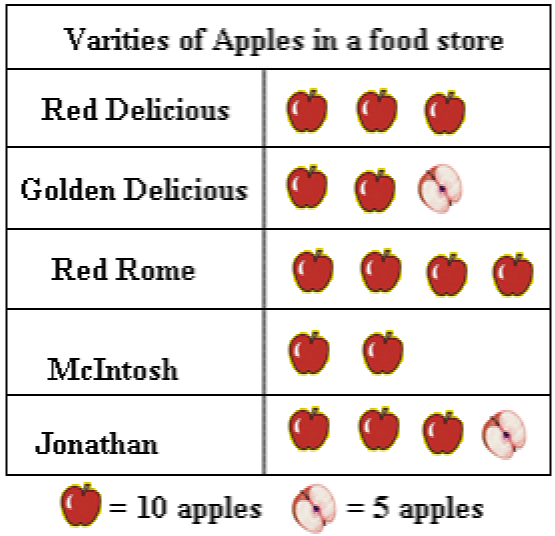
Mark	Tally	Frequency
1–5	||	2
6–10	||||	4
11–15	~~||||~~ |||	8
16–20	~~||||~~ |	6

Pictograph:

Pictograph is a way of representing statistical data using symbolic figures to match the frequencies of different kinds of data.

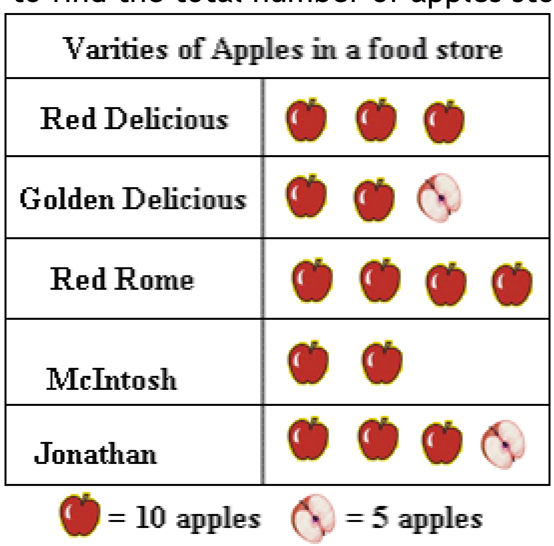
For Example:

The pictograph shows the number of varieties of apples stored at a supermarket.



Example:

Use the pictograph to find the total number of apples stored in the supermarket.



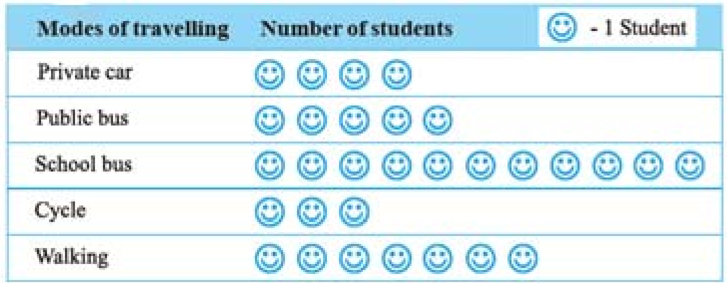
Solution:

Step 1: The pictograph shows 14 full apples and 2 half apples.

Step 2: So, there are 140 + 10 = 150 apples stores in the supermarket.

Example:

A survey was carried out on 30 students of class VI in a school. Data about the different modes of transport used by them to travel to school was displayed as pictograph.



Solution:

From the pictograph we find that:
(a) The number of students coming by private car is 4.
(b) Maximum number of students uses the school bus. This is the most popular way.
(c) Cycle is used by only three students.
(d) The number of students using the other modes can be similarly found.

Drawing of pictograph:

To Draw a Pictograph:

1. Decide what symbol to use and what it will equal. (For example, @ = 20 people)
2. Set up the graph in rows and columns.
3. Draw in the appropriate number of symbols for each category.

Example:

The following table shows how many university students in each year volunteer at a hospital. Create a pictograph to display the data.

Year	1st	2nd	3rd	4th
Number of Students	75	50	100	25

Symbol: @ = 25

Solution:

Year	Number of Students
1st	@ @ @
2nd	@ @
3rd	@ @ @ @
4th	@

Bar Graph:

Let us see some other way of representing data visually. Bars of **uniform width** can be drawn horizontally or vertically with **equal spacing** between them and then the length of each bar represents the given number. Such method of representing data is called a **bar diagram** or a **bar graph.**

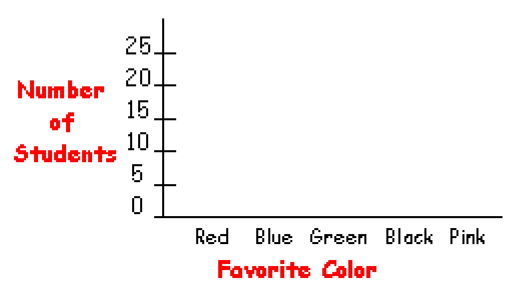
To draw a bar graph

For the following example, we will make a bar graph of the data set to the right, giving information about a group of children's favorite color.

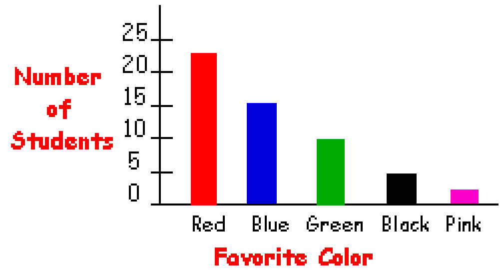
Favorite Color	Number of Students
Red	22
Blue	15
Green	11
Black	5
Pink	2

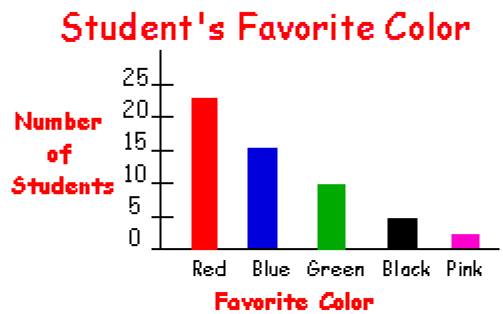
1. Look at your data to determine how big your bar graph should be and whether horizontal or vertical bars will fit better on your paper. Decide the scale your bar graph will have. This is determined by the biggest and the smallest numbers in your data set. In the data from our example, the biggest number is 22; the smallest is 2. In this case, a scale showing multiples of 5 makes creating and reading the graph easier. Label the scale on your graph.

2. Decide how wide the other axis should be to show all of the type of data (5 colors in this case). Label this axis of your graph.



3. Draw the rectangles the right length to represent the data. Pick a good width for the data bars. Color coding can make a graph easier to read.





4. Give your graph a title.
Let us see some more examples.

Example:

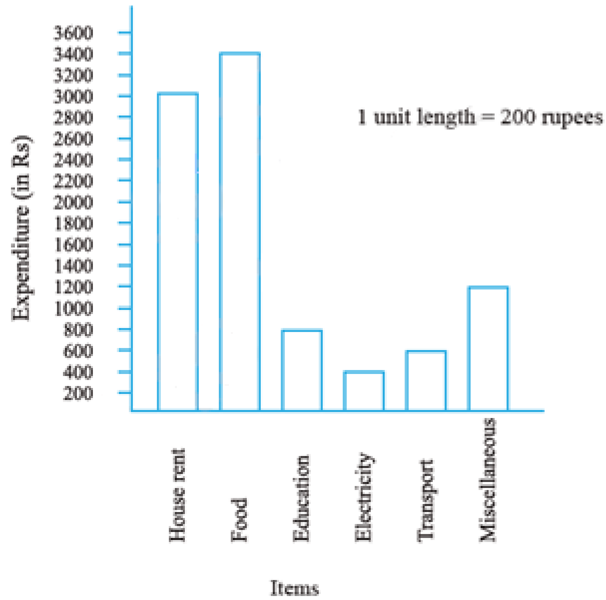
Following table shows the monthly expenditure of imran's family on various items.

Items	Expenditure (in Rs)
House rent	3000
Food	3400
Education	800
Electricity	400
Transport	600
Miscellaneous	1200

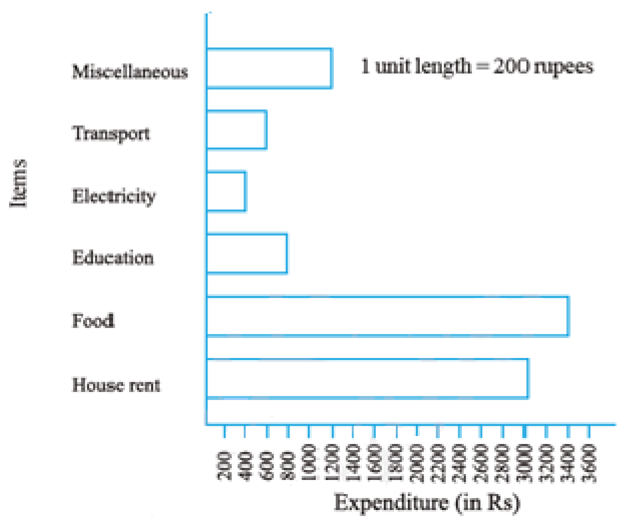
To represent this data in the form of a bar diagram, here are the steps.
(a) Draw two perpendicular lines, one vertical and one horizontal.
(b) Along the horizontal line, mark the 'items' and along the vertical line, mark the corresponding expenditure.
(c) Take bars of same width keeping uniform gap between them.
(d) Choose suitable scale along the vertical line. Let 1 unit length = Rs 200 and then mark the corresponding values.

Calculate the heights of the bars for various items as shown below.

House rent	:	3000 ÷ 200 = 15 units
Food	:	3400 ÷ 200 = 17 units
Education	:	800 ÷ 200 = 4 units
Electricity	:	400 ÷ 200 = 2 units
Transport	:	600 ÷ 200 = 3 units
Miscellaneous	:	1200 ÷ 200 = 6 units



Same data can be represented by interchanging positions of items and expenditure as shown below:



Ch-10-Mensuration

Mensuration is the branch of mathematics which deals with the study of Geometric shapes, their area, Volume and different parameters in geometric objects.

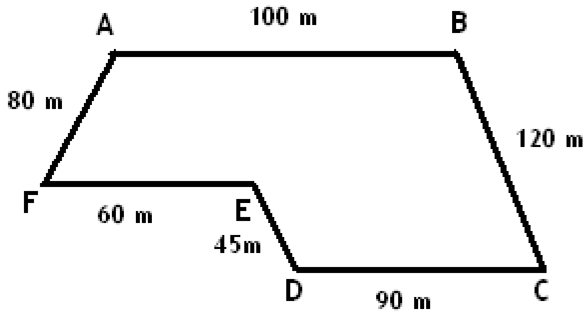
Perimeter:

Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.

The perimeter of any polygon is the sum of the lengths of all the sides.

Example:

Find out the perimeter of given figures:



Solution:

Perimeter of enclosed figure: sum of length of all sides

= AB+ BC+ CD+ DE+ EF +FA

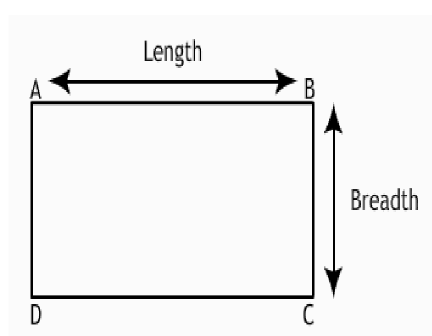
=100+ 120+ 90+ 45+ 60 + 80 m

=495 m

Perimeter of a rectangle:

Perimeter of a rectangle = sum of the length of its four sides.

Perimeter of a rectangle= 2(length + breadth)



Example1:

An athlete takes 10 rounds of a rectangular park, 60 m long and 30 m wide. Find the total distance covered by him.

Solution:

Length of the rectangular park = 60 m

Breadth of the rectangular park = 30 m

Total distance covered by the athlete in one round will be the perimeter of the park

Now, perimeter of the rectangular park

= 2 × (length + breadth)

= 2 × (60 m + 30 m)

= 2 × 90 m = 180 m

So, the distance covered by the athlete in one round is 180 m.

Therefore, distance covered in 10 rounds = 10 × 180 m = 1800m

The total distance covered by the athlete is 1800

Example2:

Find the perimeter of a rectangle whose length and breadth are 150 cm and 1 m respectively.

Solution:

Length = 150 cm

Breadth = 1m = 100 cm

Perimeter of the rectangle

= 2 × (length + breadth)

= 2 × (150 cm + 100 cm)

= 2 × (250 cm) = 500 cm = 5m

Example3:

Find the cost of fencing a rectangular park of length 250 m and breadth 175 m at the rate of Rs 12 per meter.

Solution:

Length of the rectangular park = 250 m

Breadth of the rectangular park = 175 m

To calculate the cost of fencing we require perimeter.

Perimeter of the rectangle = 2 × (length + breadth)

= 2 × (250 m + 175 m)

= 2 × (425 m) = 850 m

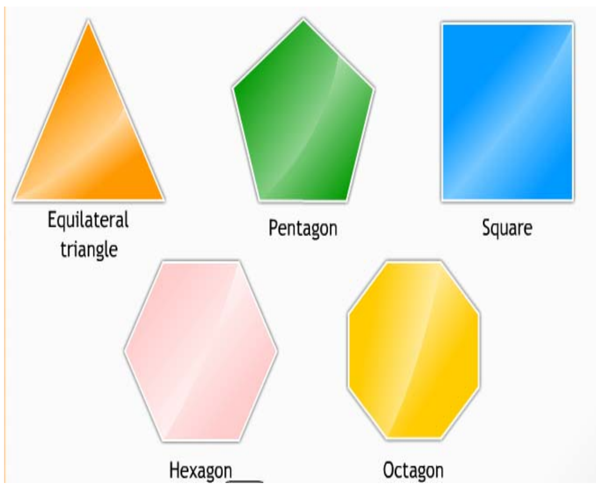
Cost of fencing 1m of park = Rs 12

Therefore, the total cost of fencing the park

= Rs 12 × 850 = Rs 10200

Perimeter of Regular shapes

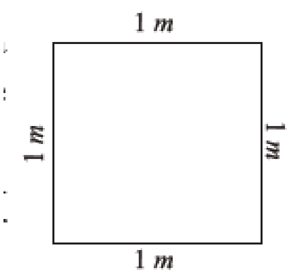
Figures having all the sides of equal length and all the angles of equal measure are known as regular closed figures. For example, an equilateral triangle, a square, a regular hexagon, etc.



Square

Perimeter of a square = 4 × length of a side

= 4 x 1= 4 m



Example:

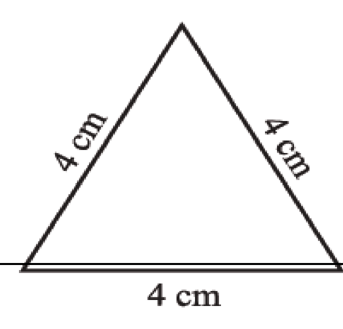
Find the distance travelled by Sonia if she takes three rounds of a square park of side 70 m.

Solution:

Perimeter of the square park = 4 × length of a side = 4 × 70 m = 280 m

Distance covered in one round = 280 m

Therefore, distance travelled in three rounds =3×280m = 840m



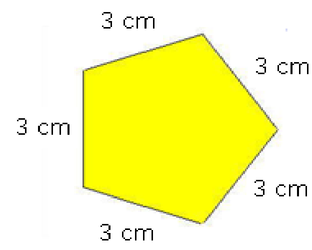
Equilateral triangle:

Perimeter of an equilateral triangle = 3 × length of a side

= 3 x 4=12 cm

Pentagon

Perimeter of pentagon= 5 x length of a side (if all sides are equal) if not then add all length of all sides of pentagon.



Pentagon

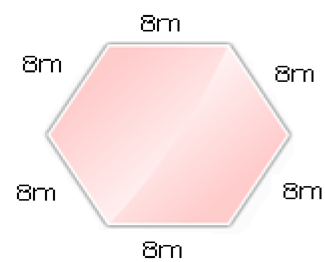
Perimeter of pentagon= 5 x length of a side

= 5 x 3 cm =15 cm

Hexagon

Perimeter of Hexagon= 6 x length of a side

Find the perimeter of a regular hexagon with each side measuring 8 m.



Hexagon

Solution:

Perimeter of Hexagon= 6 x 8 m= 48 m

Area

The amount of surface enclosed by a closed figure is called its **area**.

To calculate the area of a figure using a squared paper, the following conventions are adopted:

1) The area of one full square is taken as 1 sq unit. If it is a centimeter square sheet, then area of one full square will be 1 sq cm.

2) Ignore portions of the area that are less than half a square.

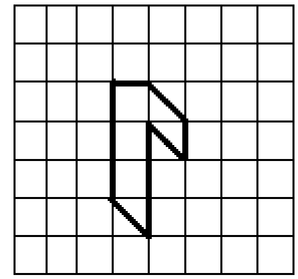
3) If more than half of a square is in a region, just count it as one square.

4) If exactly half the square is counted, take its area as 1/2 sq unit.

Example:

Find the area of the shape shown in the figure.

Solution:



This figure is made up of line-segments.

Moreover, it is covered by full squares and half squares only. This makes our job simple.

(i) Fully-filled squares = 3

(ii) Half-filled squares = 3

Area covered by full squares

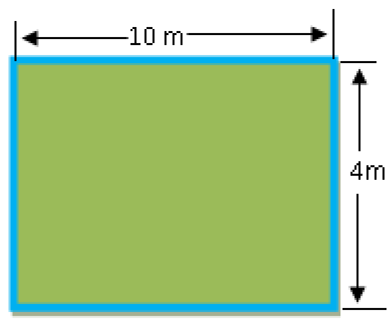
= 3 × 1 sq units = 3 sq units

Area covered by half squares

= 3 × 1/2 sq units = 3/2 sq units

Total area = 3 + 3/2 = 9/2 = 4 (1/2) sq units.

Area of a rectangle:



Area of a rectangle = length x breath

= 10 m x 4 m= 40 m^2

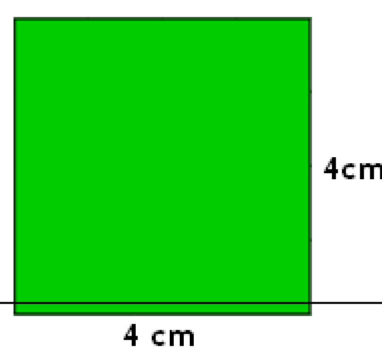
Example:

Find the area of a rectangle whose length and breadth are 10 cm and 3 cm respectively.

Solution:

Area of a rectangle = length x breath

= 10 cm x 3 cm= 30 cm^2



Area of a square:

Area of a square = $(\text{side})^2$

Area of a square = $(\text{side})^2$

$= (4\ \text{cm})^2$

$=16\ \text{cm}^2$

Example:

Find the area of a square plot of side 9 m.

Solution:

Side of the square = 9 m

Area of the square = side × side

= 9 m × 9 m = 81 sq m.

Ch-11-Algebra

Algebra

Algebra is a branch of mathematics that deals in representing numbers through variables.

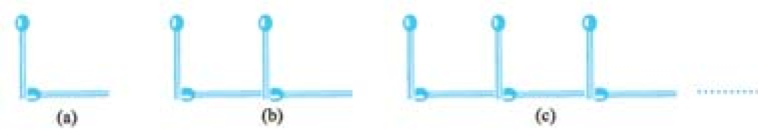
Algebra also deals with symbols, relations, functions, and equations.

For Example:

$5 + a = 12$
Here, the variable 'a' represents a whole number and it can be solved to find its actual value. 'a' must be equal to 7 as $5 + 7 = 12$.

Matchstick Patterns:

Sonal and Minal are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Sonal takes two matchsticks and forms the letter L as shown in Fig (a)



Then Minal also picks two sticks, forms another letter L and puts it next to the one made by Sonal in Fig (b). Then Sonal adds one more L and this goes on as shown by the dots in Fig (c). Their friend Mohan comes in. He looks at the pattern. Mohan always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Sonal and Minal are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....
Number of matchstick required	2	4	6	8	10	12	14	16	....

Mohan gets the answer to his question from the Table 1; 7Ls require 14 matchsticks. While writing the table, Sonal realizes that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required = 2 × number of Ls.

For convenience, let us write the letter n for the number of

Ls. If one L is made, $n = 1$; if two Ls are made, $n = 2$ and so on; thus, n can be any natural number 1, 2, 3, 4, 5 ... We then write,
Number of matchsticks required = $2 \times n$.
Instead of writing $2 \times n$, we write 2n. Note that 2n is same as $2 \times n$.

Sonal tells her friends that her rule gives the number of matchsticks required for forming any number of Ls.
Thus, for $n = 1$, the number of matchsticks required = $2 \times 1 = 2$
For $n = 2$, the number of matchsticks required = $2 \times 2 = 4$
For $n = 3$, the number of matchsticks required = $2 \times 3 = 6$ etc.
These numbers agree with those from Table 1.

The Idea of a Variable:

Variables are (usually) letters or other symbols that represent unknown numbers or values. The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was:

Number of matchsticks required = 2n

Here, n is the number of Ls in the pattern, and n takes values 1, 2, 3, 4...

In the table, the value of n goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

n is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4 ... We wrote the rule for the number of matchsticks required using the variable n.

Note: One may use any letter as m, l, p, x, y, z etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3.
It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not a variable. But n in the examples we have looked is a variable. It takes on various values 1, 2, 3, 4...

Examples of Variable:

The following are examples of algebraic expressions and equations containing variables.
$2x + 5 = 10$, the variable here is x

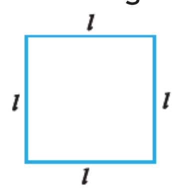
7y + 10 = 24, the variable here is y
a + b, the variables here are a and b

Use of Variables in Common Rules

Rules from geometry:

Perimeter of a square:

A square has 4 sides and they are equal in length.

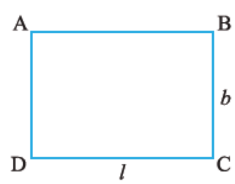


Therefore, The perimeter of a square = Sum of the lengths of the sides of the square
= 4 times the length of a side of the square
= 4 × **l** = 4**l**.

Thus, we get the rule for the perimeter of a square. The use of the variable **l** allows us to write the general rule in a way that is concise and easy to remember.
We may take the perimeter also to be represented by a variable, say **p**.
Then the rule for the perimeter of a square is expressed as a relation between the perimeter and the length of the square, **p** = 4**l**

Perimeter of a rectangle:

The rectangle ABCD has four sides AB, BC, CD and DA. The opposite sides of any rectangle are always equal in length.



Thus, in the rectangle ABCD, let us denote by **l**, the length of the sides AB or CD and, by **b**, the length of the sides AD or BC.

Therefore, Perimeter of a rectangle = length of AB + length of BC + length of CD + length of AD
= 2 × length of CD + 2 × length of BC = 2**l** + 2**b**

The rule, therefore, is that the perimeter of a rectangle = 2**l** + 2**b** where, **l** and **b** are respectively the length and breadth of the rectangle. If we denote the perimeter of the rectangle by the variable **p**, the rule for perimeter of a rectangle becomes **p = 2l + 2b**

Note: Here, both **l** and **b** are variables. They take on values independent of each other. i.e. the value one variable takes does not depend on what value the other variable has taken.

Rules from arithmetic:

Commutativity of addition of two numbers:

The Commutative Property of Addition states that changing the order of addends does not change the sum, i.e. Let a and b be two variables which can take any number value. Then,

$$\mathbf{a + b = b + a}$$

Once we write the rule this way, all special cases are included in it. If **a** = 7 and **b** = 8, we get 7 + 8 = 8 + 7. If **a** = 12 and **b** = 34, we get 12 + 34 = 34 + 12 and so on.

Commutativity of multiplication of two numbers:

The Commutative Property of Multiplication states that changing the order of factors does not change the product, i.e. Let a and b be two variables which can take any number value. Then,

$$\mathbf{a \times b = b \times a.}$$

For example:

3 × 9 = 9 × 3 and 37 × 73 = 73 × 37 follow from the general rule.

Distributivity of numbers:

Distributive Property states that the product of a number and a sum is equal to the sum of the individual products of the addends and the number. Let *a*, *b* and *c* be three variables each of which can take any number.
That is,

$$\mathbf{a \times (b + c) = a \times b + a \times c.}$$

For example:

7 × 38 = 7 × (30 + 8) = 7 × 30 + 7 × 8 = 210 + 56 = 266

Expressions with Variables:

An **expression** consists of terms that are written with arithmetical signs between them such as the addition, subtraction, division and multiplication signs.

For example:

$2n$, $5m$, $x + 10$, $x - 3$ etc. where the expression $2n$ is formed by multiplying the variable n by 2; the expression $(x + 10)$ is formed by adding 10 to the variable x and so on.

Example: 1

How these expressions have been formed?

(a) y + 5
(b) r + 25
(c) 10 y + 7
(d) – 5 q
(e) x/3

Solution:

(a) 5 added to y
(b) 25 added to r
(c) y multiplied by 10 and then 7 added to the product
(d) q multiplied by –5
(e) x divided by 3

Examples: 2

Give expressions in the following cases.

(a) n multiplied by 2 and 1 subtracted from the product.
(b) a multiplied by 10.
(c) 7 subtracted from – m
(d) 7 subtracted from p
(e) y is multiplied by – 8 and then 5 is added to the result.

Solution:

(a) 2n -1
(b) 10a
(c) –m-7
(d) P-7
(e) -8y +5

Equation:

An Equation is a mathematical sentence that uses the equal sign (=) to show that two expressions are equal.

Example:

The following are some examples of equation.

$10 + 2 = 12$
$4a - 3 = 1$
$5x + 8 = 40$

Note that an equation has an **equal sign** (=) between its two sides. The equation says that the value of the left hand side (LHS) is equal to the value of the right hand side (RHS). If the LHS is not equal to the RHS, we do not get an equation.

For example:

The statement 2n is greater than 10, i.e. $2n > 10$ is not an equation. Similarly, the statement 2n is smaller than 10 i.e. $2n < 10$ is not an equation. Also, the statements $(x - 3) > 11$ or $(x - 3) < 11$ are not equations.

Example:

Check which of the following are equations (with a variable)?

(a) $2n + 1 = 11$
(b) $(3/2)\ q < 5$
(c) $7 - x = 5$
(d) $(t - 7) > 5$

Solution:

(a) **Yes,** $2n + 1 = 11$ are an equation because it has a variable n and equal to sign.
(b) **No,** $(3/2)\ q < 5$ are not an equation because it has a variable q but not equal to sign.
(c) **Yes,** $7 - x = 5$ are an equation because it has a variable x and equal to sign.
(d) **No,** $(t - 7) > 5$ are not an equation because it has a variable t but not equal to sign.

Solution of an Equation:

The value of the variable in an equation which satisfies the equation is called a solution to the equation.

For example:
Let us take the equation $x - 3 = 11$
This equation is satisfied by $x = 14$, because for $x = 14$,
LHS of the equation $= 14 - 3 = 11 =$ RHS.

Example:

Complete the table and Find the solution to the equation x – 7 = 3.

X	3	2	6	10	16	5
X -7						

Solution:

X	3	2	6	10	16	25
X -7	-4	-5	-1	3	9	18

We have x – 7 = 3,
Add 7 both sides, we get
X – 7 + 7 = 3 + 7
X = 3 + 7 = 10
X = 10

Hence, the solution of the equation x – 7 = 3 is x = 10.

Ch-12-Ratio and Proportion

Ratio

When two quantities of same unit are compared by division it is known as **Ratio.**
Ratios are denoted using a symbol **':'**

For example:

4: 7, 1: 6, 10: 3 etc. are examples of ratio.
Any ratio a: b can also be written as 'a is to b' or a/b.

Example:

A town, whose total population is 100, has 60 males, 40 females, and 20 children.
Find the ratio of

a) Number of males to number of females
b) Number of females to number of children
c) Number of males to total population of town
d) Number of males to number of children
e) Number of females to total population of town
f) Number of children to total population of town

Solution:

In the example given is:

Total Population = 100
Number of Males = 60
Number of Females = 40
Number of Children = 20

The Required Ratio are calculated as :-

a) Number of males to number of females = Number of Males : Number of Females = 60 : 40
and its lowest term = 3 : 2
Hence ,ratio of number of males to number of females = 3 : 2

b) Number of females to number of children = Number of Females : Number of Children = 40 : 20
and its lowest term = 2 : 1
Hence, ratio of number of females to number of children = 2 : 1

c) Number of males to total population of town = Number of Males : Total Population

of Town = 60 : 100
and its lowest term = 3 : 5
Hence, ratio of number of males to total population of town = 3 : 5

d) Number of males to number of children = Number of Males : Number of Children = 60 : 20
and its lowest term = 3 : 1
Hence, ratio of number of males to number of children = 3 : 1

e) Number of females to total population of town = Number of Females : Total Population of Town = 40 : 100
and its lowest term = 2 : 5
Hence, ratio of number of females to total population of town = 2 : 5

f) Number of children to total population of town = Number of Children : Total Population of Town = 20 : 100
and its lowest term = 1 : 5
Hence, ratio of number of children to total population of town = 1 : 5

In ratio, the two quantities can be compared only if they are in the same unit. Let us look some examples.

Example:

Length of a pencil is 18 cm and its diameter is 8 mm. What is the ratio of the diameter of the pencil to that of its length?

Solution:

Since the length and the diameter of the pencil are given in different units, we first need to convert them into same unit.

Thus, length of the pencil = 18 cm
= 18 x 10 mm = 180 mm.

The ratio of the diameter of the pencil to that of the length of the pencil = 8/ 180
= 2/45 = 2 : 25

Example:

Cost of a toffee is 50 paise and cost of a chocolate is Rs 10. Find the ratio of the cost of a toffee to the cost of a chocolate.

Solution:

Since the cost of toffee and chocolate are given in different units, we first need to convert them into same units.

Thus, the cost of the chocolate = Rs 10
= 10 x 100 paise = 1000 paise.

The ratio of the cost of a toffee to the cost of a chocolate = 50/ 1000 = 1/20 = 1 : 20

Example:

Give two equivalent ratios of 6 : 4.

Solution:

Ratio 6 : 4 = 6/4
= (6 x 2)/ (4 x 2)
= 12/8
Therefore, 12: 8 is an equivalent ratio of 6 : 4

Similarly,
The ratio 6: 4 = 6/4
= (6 ÷ 2)/ (4 ÷ 2) = 3/2 = 3 : 2
So, 3:2 is another equivalent ratio of 6 : 4.

Therefore, we can get equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

Example:

Divide Rs 120 in the ratio 2: 4 between Mohan and Karan.

Solution:

The two parts are 2 and 4.

Therefore, sum of the parts = 2 + 4 = 6.

This means if there are Rs 6, Mohan will get Re 2 and Karan will get Rs 4.

Or, we can say that Mohan gets 2 parts and Karan gets 4 parts out of every 6 parts.

Therefore, Mohan’s share = (2/6) x 120 = Rs 40

And Karan’s share = (4/6) x 120 = Rs 80

Proportion

When two Ratios are equal, they are said to be in **Proportion**. When two Ratios are in Proportion we use the symbol **':: ' or ' = '** to denote them.

Example:

Check and discuss whether the following ratios are in proportion.

1) 2 : 5 and 5 : 7
2) 7 : 3 and 56 : 24
3) 12 : 22 and 15 : 27

Solution:

The proceeds are as:

(1) 2 : 5 and 5 : 7
Here, both the ratios are in lowest form and
2 : 5 ≠ 5 : 7

So, we can say that the given **Ratios are not in proportion.**

(2) (2) 7 : 3 and 56 : 24
Here, First Ratio(7 : 3) is in Lowest From: but Second Ratio(56:24) needs to be converted into Lowest Form:
56 : 24 = 7 : 3 (Lowest Form)
Lowest Forms of both ratios are equal,

So, **7 : 3 :: 56 : 24 (both are in proportion)**

(3) 12 : 22 and 15 : 27
Firstly, convert the ratios into Lowest Form
12 : 22 = 6 : 11(Lowest Form)
15 : 27 = 5 : 9 (Lowest Form)
Lowest Forms of both ratios are not equal,

So, **12 : 22 ≠ 15 : 27 (both are not in proportion)**

If two ratios are not equal, then we say that they are not in proportion. In a statement of proportion, the four quantities involved when taken in order are known as respective terms. First and fourth terms are known as **extreme terms.** Second and third terms are known as **middle terms.**

Example:

Are the ratios 25g : 30g and 40 kg : 48 kg in proportion?

Solution:

25 g : 30 g = 25/30 = 5 : 6

40 kg : 48 kg = 40/48 = 5 : 6

So, 25 : 30 = 40 : 48.

Therefore, the ratios 25 g : 30 g and 40 kg : 48 kg are in proportion,

i.e. 25 : 30 :: 40 : 48

The **middle terms** in this are 30, 40 and the **extreme terms** are 25, 48.

Example:

Are 30, 40, 45 and 60 in proportion?

Solution:

Ratio of 30 to 40 =30/40 = 3 : 4.

Ratio of 45 to 60 =45/60 = 3 : 4.

Since, 30 : 40 = 45 : 60.

Therefore, 30, 40, 45, 60 are in proportion.

Example:

Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?

Solution:

Ratio of 15 cm to 2 m = 15 : 2 × 100 (1 m = 100 cm)
= 3 : 40

Ratio of 10 sec to 3 min = 10 : 3 × 60 (1 min = 60 sec)
= 1 : 18

Since, 3 : 40 ≠ 1 : 18, therefore, the given ratios do not form a proportion.

Unitary Method

The method in which first we find the value of one unit and then the value of required number of units is known as **Unitary Method**.

Here are some examples:

Example:

5 buses carry 200 passengers. How many passengers can 20 buses carry?

Solution: The process is as:-

5 buses carry passengers = 200

1 bus carry passenger = (200 ÷ 5)

20 bus carry passenger = (200 ÷ 5) × 20 = 40 × 20 = 800.

Example:

If the cost of 8 cans of juice is Rs 240, then what will be the cost of 15 cans of juice?

Solution:

Cost of 8 cans of juice = Rs 240

Therefore, cost of one can of juice =240 ÷ 8 = Rs 30

Therefore, cost of 15 cans of juice = Rs 30 × 15 = Rs 450.

Thus, cost of 15 cans of juice is Rs 450.

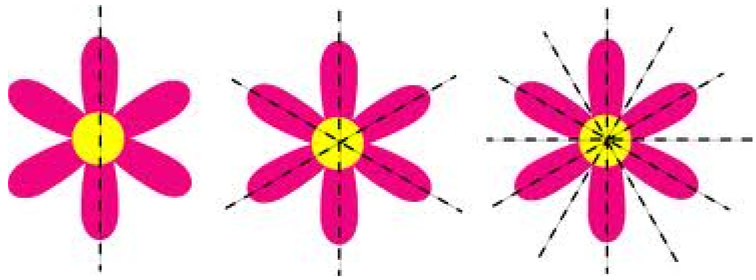
Ch-13-Symmetry

Symmetry:

The quality of being made up of exactly similar parts facing each other or around an axis of symmetry.

Or

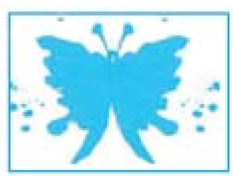
Symmetry is when one shape becomes exactly like another if you flip, slide or turn it.



In this figures, dotted lines are line of symmetry.

Making Symmetric Figures: Ink-blot Devils

Take a piece of white paper. Fold it in half. Spill a few drops of ink on one half side. Now press the halves together. The resulting figure is symmetric. Look at this figure.



Inked- string pattern

Fold a paper in half. On one half-portion, arrange short lengths of string dipped in a variety of coloured inks or paints. Now press the two halves. You will get the symmetrical figure.

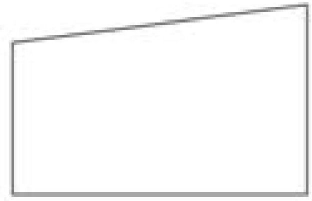
Figures which are not symmetrical

There are figures which are not symmetrical.

For example: Hockey, Smiley and Quadrilateral

Hockey

smiley

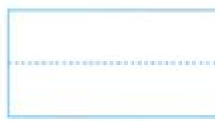


quadrilateral

Figures with Two Lines of Symmetry

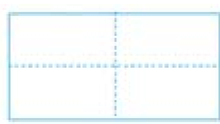
A rectangle

Take a rectangular sheet. Fold it once lengthwise so that one half fits exactly over the other half. You will get one line of symmetry. As shown below:



1st fold

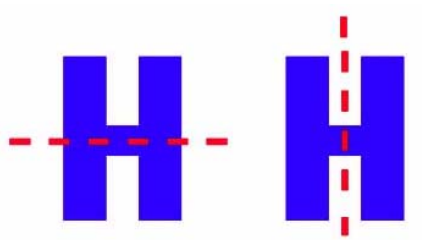
Open it up now and again fold on its width in the same way. Now you will get two line of symmetry.



2nd fold

Hence, a rectangle has two lines of symmetry. Similarly, an alphabet H has also two lines of symmetry.

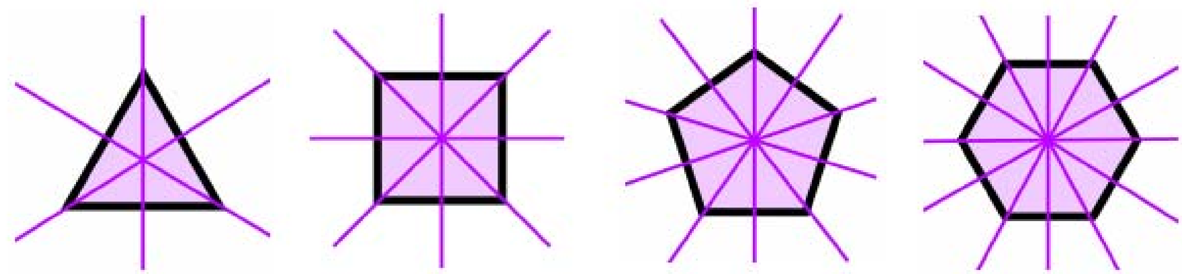
Look at this figure:



Figures with Multiple (more than two) Lines of Symmetry

There are figures which have more than one line of symmetry. Let us consider the following examples.

A regular polygon has all sides equal, and all angles equal:



An Equilateral Triangle | A Square | A Regular Pentagon | A Regular Hexagon

An **Equilateral Triangle** (3 sides) has 3 Lines of Symmetry

A **Square** (4sides) has 4 Lines of Symmetry

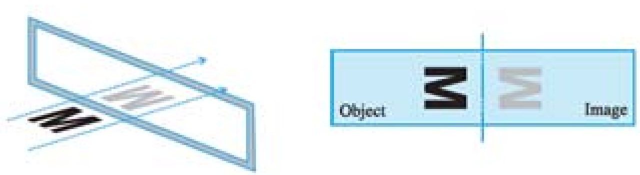
A **Regular Pentagon** (5 sides) has 5 Lines of Symmetry

A **Regular Hexagon** (6sides) has 6 Lines of Symmetry

Reflection and Symmetry

Line symmetry and mirror reflection are naturally related and linked to each other.

Here is a picture showing the reflection of the English letter M. You can imagine that the mirror is invisible and can just see the letter M and its image.

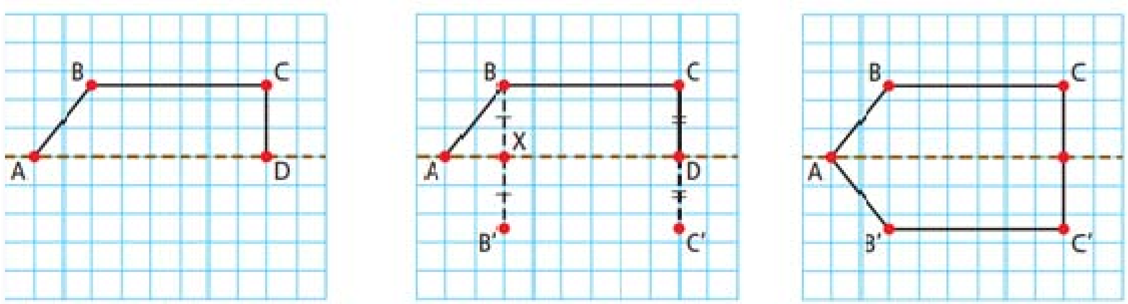


Let us see some examples:

The object and its image are symmetrical with reference to the mirror line. If the paper is folded, the mirror line becomes the line of symmetry.

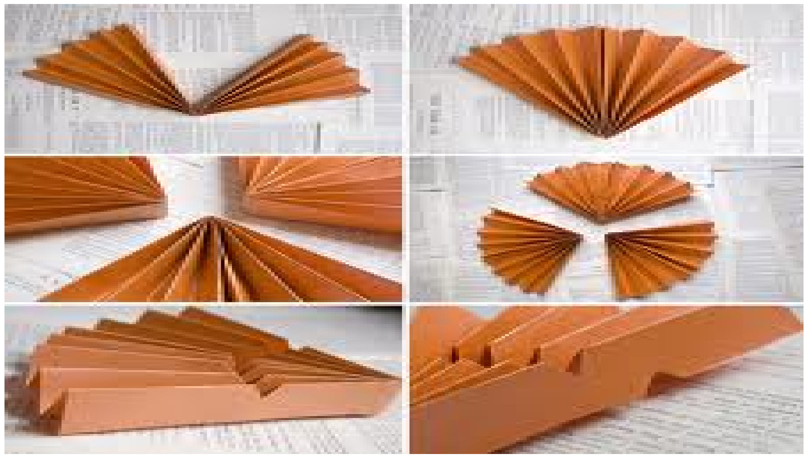
Use Measurement or Counting

Draw the half fi gure onto a grid, and label the vertices A, B, C, and D. All points not on the line of symmetry are refl ected on the opposite side of the line. In this fi gure, this is points B and C. The refl ected points are drawn the same perpendicular distance from the fold line so that BX = B’X and CD = C’D. Join A to B’, B’ to C’, and C’ to D to complete the figure.



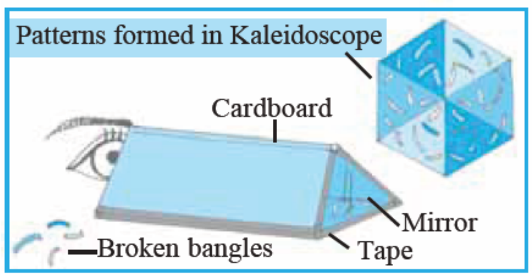
Paper decoration

Use thin rectangular coloured paper. Fold it several times and create some intricate patterns by cutting the paper, like the one shown here. Identify the line symmetries in the repeating design.



Kaleidoscope

A kaleidoscope uses mirrors to produce images that have several lines of symmetry (as shown here for example). Usually, two mirrors strips forming a V-shape are used. The angle between the mirrors determines the number of lines of symmetry.



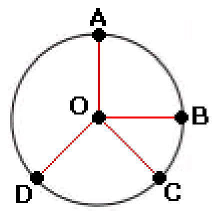
Ch-14-Practical Geometry

Circle

A Circle is the locus of all points that are at an equal distance from a given point (on the plane) called the center.

Example:

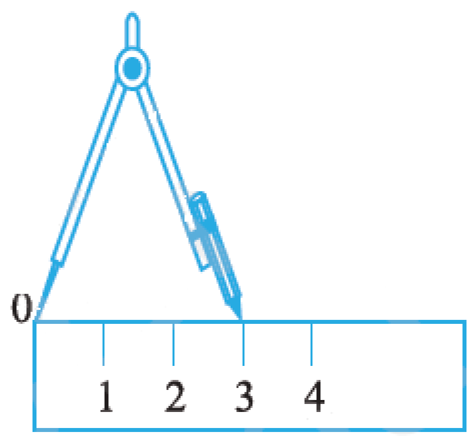
In the figure shown, O is the center of the circle. All points on the black ring, such as A, B, C, and D, are equidistant from O, the center. The length of the black ring is the circumference of the circle.



Construction of a circle when its radius is known

Suppose we want to draw a circle of radius 3 cm. We need to use our compass. Here are the steps to follow.

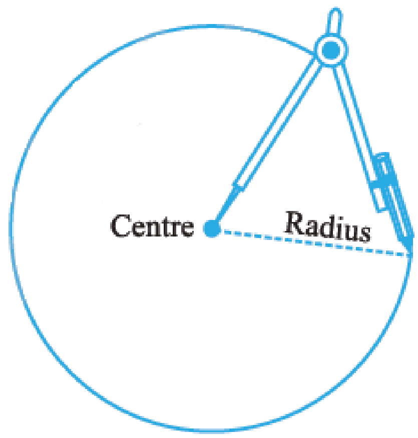
Step 1: Open the compass for the required radius of 3cm.



Step 2: Mark a point with a sharp pencil where we want the centre of the circle to be. Name it as O.

Step 3: Place the pointer of the compass on O.

Step 4: Turn the compass slowly to draw the circle. Be careful to complete the movement around in one instant.

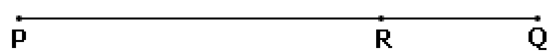


Line Segment:

Line segment is the part of a line consisting of two endpoints.

Example:

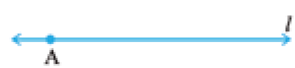
In the figure shown, PQ, PR, and RQ are line segments.



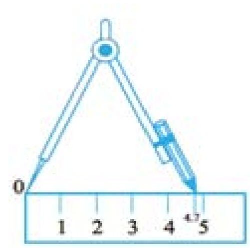
Construction of a line segment of a given length

By use of ruler and compasses

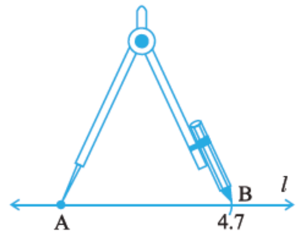
Step 1: Draw a line l. Mark a point A on a line l.



Step 2: Place the compasses pointer on the zero mark of the ruler. Open it to place the pencil point up to the 4.7cm mark.



Step 3: Taking caution that the opening of the compasses has not changed, place the pointer on A and swing an arc to cut l at B.



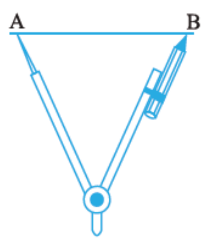
Step 4: AB is a line segment of required length.

Constructing a copy of a given line segment

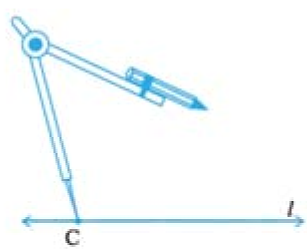
To make a copy of line segment AB

Steps1: Given line AB whose length is not known.

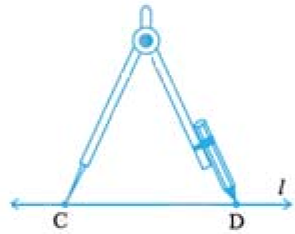
Step 2: Fix the compasses pointer on A and the pencil end on B. The opening of the instrument now gives the length of AB.



Step 3: Draw any line l. Choose a point C on l. Without changing the compasses setting, place the pointer on C.



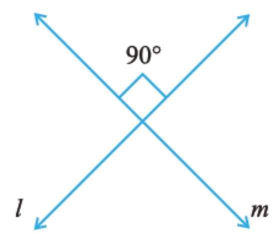
Step 4: Swing an arc that cuts l at a point, say, D. Now **CD** is a copy of AB.



Perpendiculars

Two lines (or rays or segments) are said to be perpendicular if they intersect such that the angles formed between them are right angles.

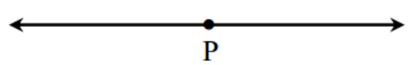
In the figure below, the lines l and m are perpendicular.



Perpendicular to a line through a point on it

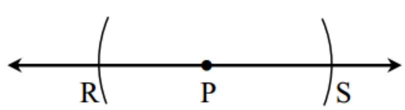
To construct a line perpendicular to a given line through a point on the line

Given: Point P on line l Construct QP such that QP ⊥ l.



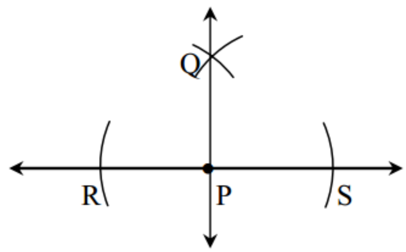
Step 1: Place compass point on P and construct arcs that intersect line l on both sides of P. Locate R & S.

Step 2: Set compass to distance greater than half of RS. Put point of compass on R and swing arc above l.



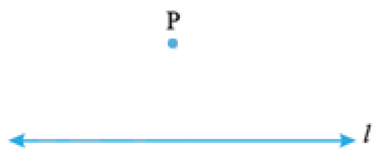
Step 3: Using same compass setting put compass on S and swings an arc intersecting arc from step 2. Locate Q.

Step 4: Draw QP.

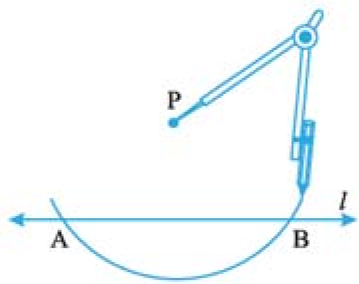


Perpendicular to a line through a point not on it

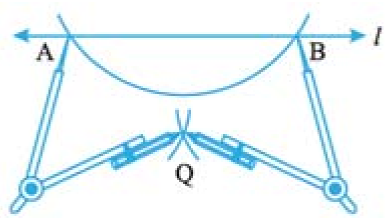
Step 1: Given a line l and a point P not on it.



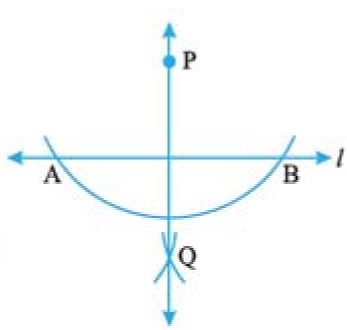
Step 2: With P as centre, draw an arc which intersects line l at two points A and B.



Step 3: Using the same radius and with A and B as centers, construct two arcs that intersect at a point, say Q, on the other side.



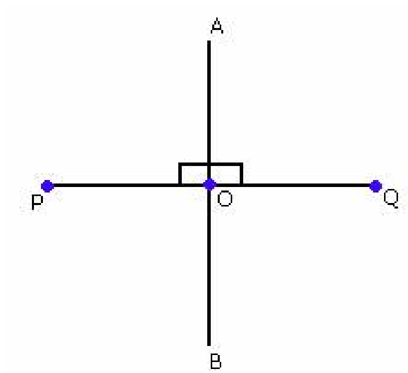
Step 4: Join PQ. Thus, PQ is perpendicular to l.



Perpendicular Bisector:

Perpendicular Bisector is a perpendicular line or a segment that passes through the midpoint of a line. A perpendicular bisector divides a line segment into two equal segments.

Example:

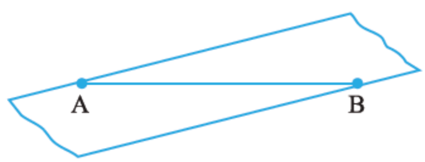


In the figure shown, AB is the perpendicular bisector of the line segment PQ passing through its midpoint 'O'.

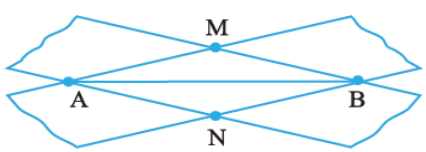
Perpendicular bisector of a line segment

Step 1: Draw a line segment AB.

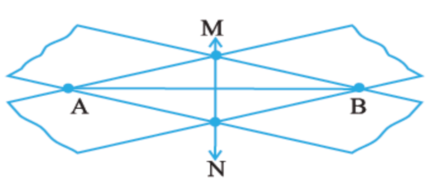
Step 2: Place a strip of a transparent rectangular tape diagonally across AB with the edges of the tape on the end points A and B, as shown in the figure.



Step 3: Repeat the process by placing another tape over A and B just diagonally across the previous one. The two strips cross at M and N.

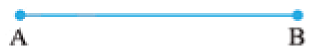


Step 4: Join M and N.

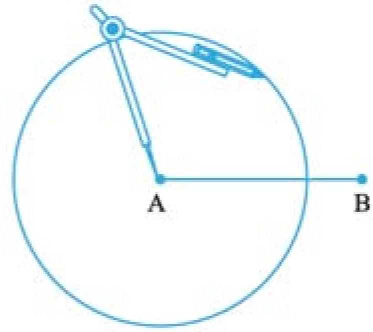


Construction using ruler and compasses

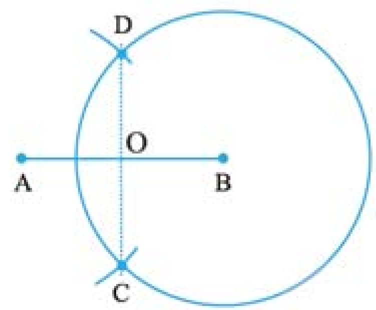
Step 1: Draw a line segment AB of any length.



Step 2: With A as centre, using compass draw a circle. The radius of your circle should be more than half the length of AB.



Step 3: With the same radius and with B as centre, draw another circle using compasses. Let it cut the previous circle at C and D.



Step 4 Join CD. It cuts AB at O. Use your divider to verify that O is the midpoint of AB. Also verify that $\angle COA$ and $\angle COB$ are right angles. Therefore, CD is the perpendicular bisector of AB.

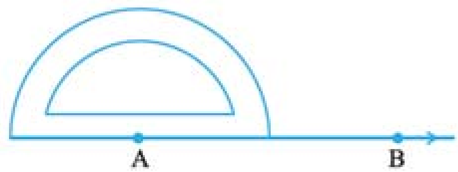
Angles

Suppose we want an angle of measure 40°.

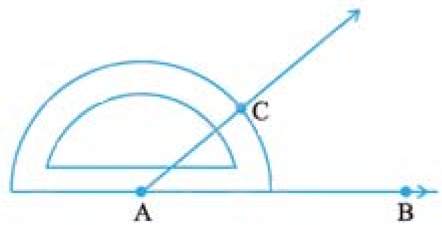
Here are the steps to follow:

Step 1: Draw AB of any length.

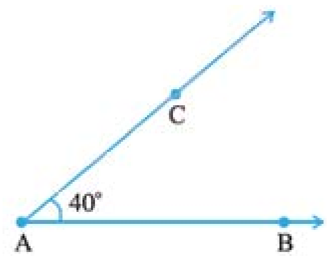
Step 2: Place the centre of the protractor at A and the zero edge along AB.



Step 3: Start with zero near B. Mark point C at 40˚.

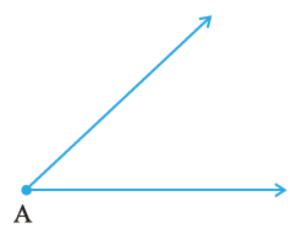


Step 4: Join AC. ∠BAC is the required angle.



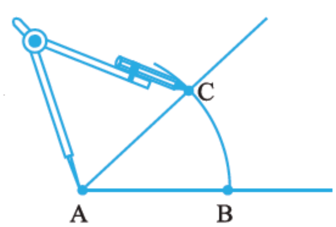
Constructing a copy of an angle of unknown measure

Given ∠A, whose measure is not known.

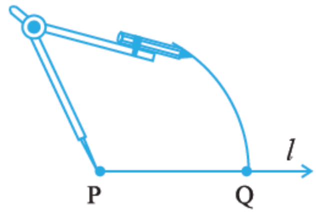


Step 1: Draw a line l and choose a point P on it.

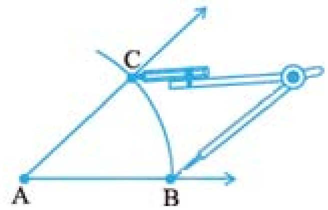
Step 2: Place the compasses at A and draw an arc to cut the rays of ∠A at B and C.



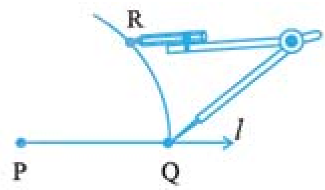
Step 3: Use the same compasses setting to draw an arc with P as centre, cutting l in Q.



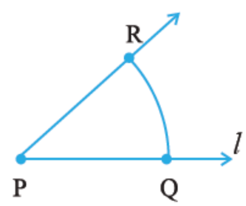
Step 4: Set your compasses to the length BC with the same radius.



Step 5: Place the compasses pointer at Q and draw the arc to cut the arc drawn earlier in R.



Step 6: Join PR. This gives us $\angle$P. It has the same measure as $\angle$A.
This means $\angle$QPR has same measure as $\angle$BAC.

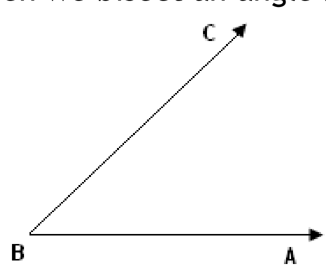


Bisector of an angle

Bisect means to divide into two equal sections or two equal halves.

Example:

Following steps are involved when we bisect an angle by using ruler and compass:

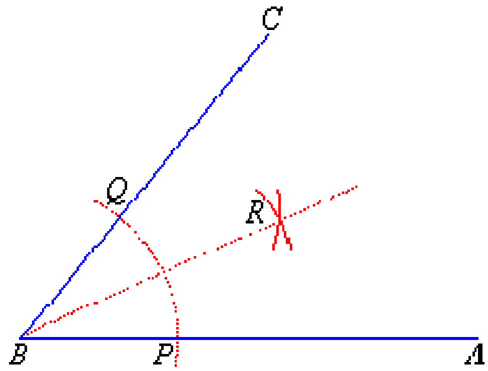


Draw an arc with B as the centre to cut the arms, BA and BC, of the angle at P and Q respectively.

Using the same radius, draw an arc centered at P.

Again, using the same radius, draw an arc centered at Q to cut the arc previously drawn at R.

Join, B, the vertex of the angle, to the point R.



BR bisects the angle ABC, and is called the **bisector of the angle ABC.**

Angles of special measures

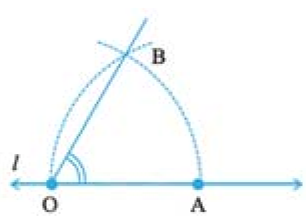
Constructing a 60° angle

Step 1: Draw a line l and mark a point O on it.

Step 2: Place the pointer of the compasses at O and draw an arc of convenient radius which cuts the line PQ at a point say, A.

Step 3: With the pointer at A (as centre); now draw an arc that passes through O.

Step 4: Let the two arcs intersect at B. Join OB. We get ∠BOA whose measure is 60°.



Constructing a 30° angle

We know that:

1/2 of 60° = 30°

So, to construct an angle of 30º, first construct a 60º angle and then bisect it. Often, we apply the following steps.

Step 1: Draw the arm PQ.

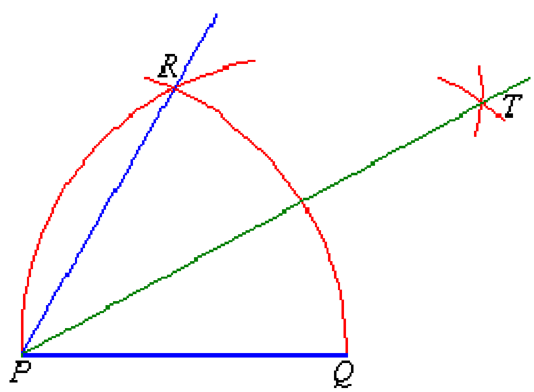
Step 2: Place the point of the compass at P and draw an arc that passes through Q.

Step 3: Place the point of the compass at Q and draw an arc that cuts the arc drawn in Step 2 at R.

Step 4: With the point of the compass still at Q, draw an arc near T as shown.

Step 5: With the point of the compass at R, draw an arc to cut the arc drawn in Step 4 at T.

Step 6: Join T to P. The angle QPT is 30º.



Constructing a 120° angle

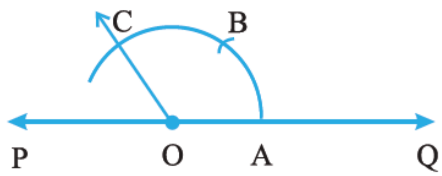
An angle of 120° is nothing but twice of an angle of 60°. Therefore, it can be constructed as follows:

Step 1: Draw any line PQ and take a point O on it.

Step 2: Place the pointer of the compasses at
O and draw an arc of convenient radius which cuts the line at A.

Step 3 Without disturbing the radius on the compasses, draw an arc with A as centre which cuts the first arc at B.

Step 4 Again without disturbing the radius on the compasses and with B as centre, draw an arc which cuts the first arc at C.



Constructing a 90° angle

We can construct a 90º angle either by bisecting a straight angle or using the following steps.

Step 1: Draw the arm PA.

Step 2: Place the point of the compass at P and draw an arc that cuts the arm at Q.

Step 3: Place the point of the compass at Q and draw an arc of radius PQ that cuts the arc drawn in Step 2 at R.

Step 4: With the point of the compass at R, draw an arc of radius PQ to cut the arc drawn in Step 2 at S.

Step 5: With the point of the compass still at R, draw another arc of radius PQ near T as shown.

Step 6: With the point of the compass at S, draw an arc of radius PQ to cut the arc drawn in step 5 at T.

Step 7: Join T to P. The angle APT is 90º.