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8	**Heat Exchangers** 8.1 Elements of shell and tube heat exchanger 8.2 Thermal design of heat exchangers 8.2.1 Overall heat transfer coefficient 8.2.2 Fouling factor or dirt factor 8.2.2 Temperature profiles in heat exchangers 8.2.4 Why multi-pass exchangers 8.2.5 LMTD correction factor 8.2.6 Individual heat transfer coefficient 8.2.7 Pressure drop in the heat exchanger 8.2.7.1 Correlation for tube side pressure drop 8.2.7.2 Correlation for shell side pressure drop 8.2.8 Heat transfer effectiveness and number of transfer units 8.2.9 Calculation and designing of the heat exchanger 8.2.9.1 Double-pipe heat exchanger 8.2.9.1 Shell and tube heat exchanger	5
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Preface

The course material on Heat Transfer Operations is designed for the undergraduate students of Chemical Engineering. The course provides an elementary knowledge of the heat transfer operations. The course is primarily developed for the students those who need the fundamental knowledge in the heat transfer operations. The course contains nine modules divided into forty lectures. A couple of solved examples are provided during discussion in the lecture. At the end of each of the module, a few of the frequently asked questions along with problems for practice are provided.

In the first module, mode of heat transfer is discussed along with some of the material properties relevant to heat transfer is discussed. The second module contains one dimensional conduction process through constant and variable heat transfer area. The heat conduction in bodies with heat sources is also covered for different geometries like slab, cylinder and sphere. The third module is devoted to one dimensional convective heat transfer, which covers principle of heat flow in fluids and concept of heat transfer coefficient. The concept of overall heat transfer coefficient with the temperature profile is also discussed in detail. The concept of fins is introduced and general relations are elaborated for different cases using suitable boundary conditions and assumptions. The brief knowledge of thermal insulation is also covered in the third module. The module four is about the forced convection heat transfer, where the flow of the fluid is in or on the surface of different geometries along with various correlation. The module also covers the various useful analogies. The module five contains heat transfer by natural convection and covers the correlation for different geometries. The module six is devoted to boiling and condensation. The principle of boiling and condensation is discussed in detail. The condensation in and on the surface of different geometries are discussed and correlation are provided. The module seven is on radiation heat transfer, covering emissive power, radiosity, and irradiation along with the various laws. The definition of blackbody and grey body is discussed. The view factor, relation between view factor, and radiation shield are discussed. At the end of the module, the introduction to radiation combined with conduction and convection is provided. The module eight and nine are devoted to the heat transfer equipment. Heat exchangers are discussed in the module eight. The various elements of the shell and tube heat exchanger are described along with the thermal design of the heat exchanger. Calculation of the pressure drop, NTU method, as well as calculation and designing of the heat exchanger are described in this module. The module nine contains the discussion on the evaporators. Different solution properties are discussed along with the different kind of evaporators. Performance of the steam heated tubular evaporators and the temperature profile in the evaporator are discussed. Different methods of feeding are discussed as well as calculation and procedure of enthalpy balance is described in details.

The different correlations used in the study material are developed over the time by many scientists and researchers. The references to these famous correlations are given in the standard books on heat transfer and are listed in the reference link of this course material.

1. Introduction

Heat transfer is a science, which deals with the flow of heat from a higher temperature to lower temperature. Heat cannot be stored and it is defined as the energy in transit due to the difference in the temperatures of the hot and cold bodies. The study of heat transfer not only explains how the heat energy transports but also predicts about the rate of heat transfer. When a certain amount of water is evaporated or condensed, the amount of heat transferred in either of the processes is same. However, the rate of heat transfer in both the cases may be different.

At this point, it is very important to understand about the basic information that the phases of a substance (solid, liquid, and gas) are associated with its energy content. In the solid phase, the molecules or atoms are very closely packed to give a rigid structure (fig.1.1a). In the liquid phase, sufficient thermal energy is present, which keeps the molecules sufficiently apart and as a result the rigidity looses (fig.1.1b). In the gas phase, the presence of additional energy results in a complete separation and the molecules or atoms are free to move anywhere in the space (fig.1.1c). It must be noticed that whenever a change in phase occurs, a large amount of energy involves in the transition.

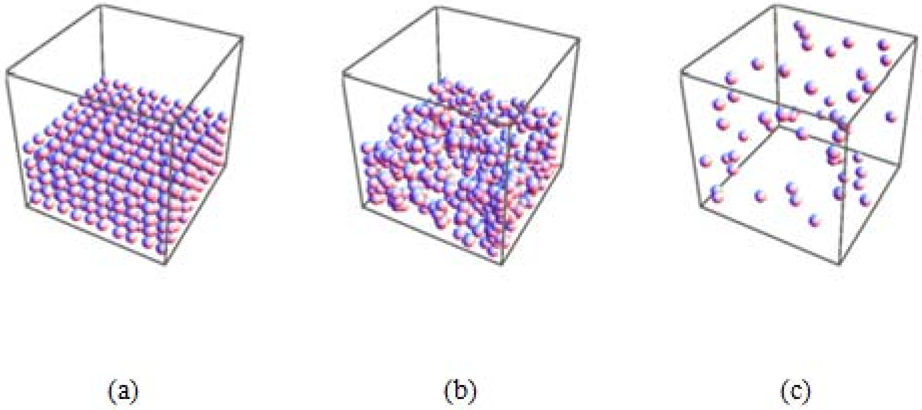


Fig. 1.1: Relative molecular distance of different phases of a substance at a fixed temperature (a) gas/vapour, (b) liquid, and (c) solid

As we are dealing with the heating and cooling of materials in almost our all the processes, the heat transfer is an indispensable part of any of the industries. Therefore, heat transfer is a common subject in many engineering disciplines, especially mechanical and chemical engineering. Study of heat transfer has a vital role in the chemical process industries. Chemical engineers must have a thorough knowledge of heat transfer principles and their applications.

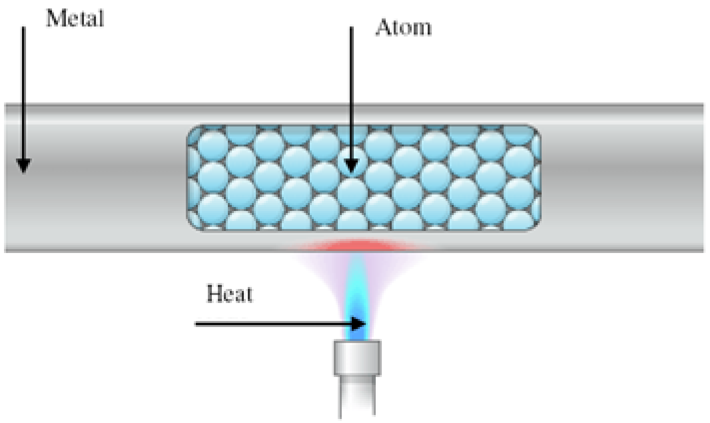
There are three different modes in which heat may pass from a hot body to a cold one. These modes are conduction, convention, and radiation. It should be noted that the heat transfer takes place in combination of two or three modes in any of the real engineering application. In this chapter, we will briefly discuss about the different modes of heat transfer along with the various basic information that will help us as a building block for further study.

1.1 Mode of heat transfer

In this section, we will discuss about the three different modes of heat transfer. The discussion will help us to understand about the conduction, convection, and radiation. Moreover, we would be able to understand the basic difference between the three modes of heat transfer.

1.1.1Conduction

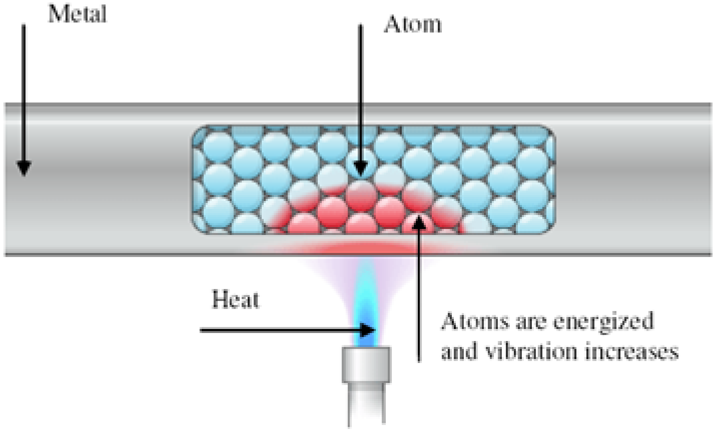
Conduction is the transfer of heat in a continuous substance without any observable motion of the matter. Thus, heat conduction is essentially the transmission of energy by molecular motion. Consider a metallic rod being heated at the end and the other end of the rod gets heated automatically. The heat is transported from one end to the other end by the conduction phenomenon. The molecules of the metallic rod get energy from the heating medium and collide with the neighbouring molecules. This process transfers the energy from the more energetic molecules to the low energetic molecules. Thus, heat transfer requires a temperature gradient, and the heat energy transfer by conduction occurs in the direction of decreasing temperature. Figure 1.2 shows an illustration for the conduction, where the densely packed atoms of the rod get energized on heating and vibration effect transfers the heat as described in fig.1.2.



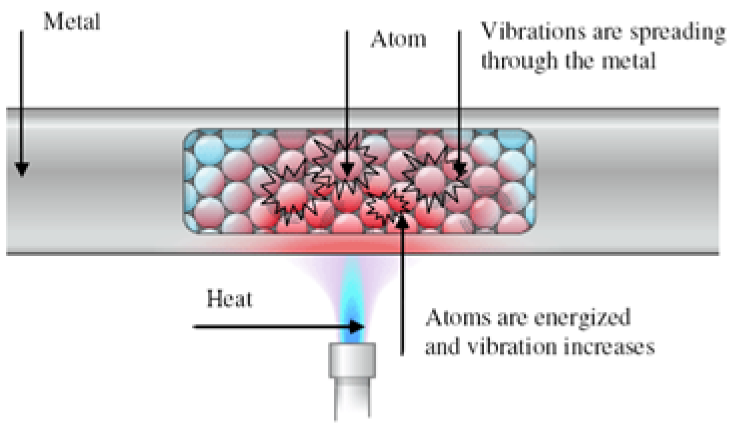
Step 1: The metallic rod is being heated

(a)

Step 2: The energy is transferred to the atoms and vibration of the atoms started



(b)



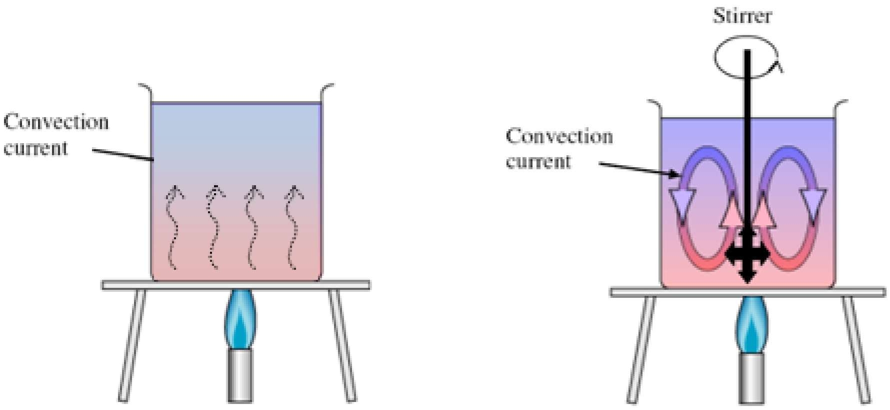
(c)

Step 3: As more energy is transferred to the atoms, the vibration increases as well as transferred to the other atoms

Fig.1.2: Different stages during conduction in a metallic rod

1.1.2 Convection

When a macroscopic particle of a fluid moves from the region of hot to cold region, it carries with it a definite amount of enthalpy. Such a flow of enthalpy is known as convection. Convection may be natural or forced. In natural convection, the movement of the fluid particles is due to the buoyancy forces generated due to density difference of heated and colder region of the fluid as shown in the fig.1.3a. Whereas, in forced convection the movement of fluid particles from the heated region to colder region is assisted by some mechanical means too (eg., stirrer) as shown in fig.1.3b.



(a) (b)

Fig.1.3: Heat transfer through convection (a) natural, and (b) forced

1.1.3Radiation

We have seen that a medium is required for the heat transfer in case of conduction and convection. However, in case of radiation, electromagnetic waves pass through the empty space. Electromagnetic waves travel at the velocity of light in vacuum. These waves are absorbed, reflected, and/or transmitted by the matter, which comes in the path of the wave. We will limit our discussion (in this NPTEL course) to the thermal radiation. Thermal radiation is the term used to describe the electromagnetic radiation, which is observed to be emitted by the surface of the thermally excited body. The heat of the Sun is the most obvious example of thermal radiation.

There will be a continuous interchange of energy between two radiating bodies, with a net exchange of energy from the hotter to the colder body as shown in the fig.1.4.

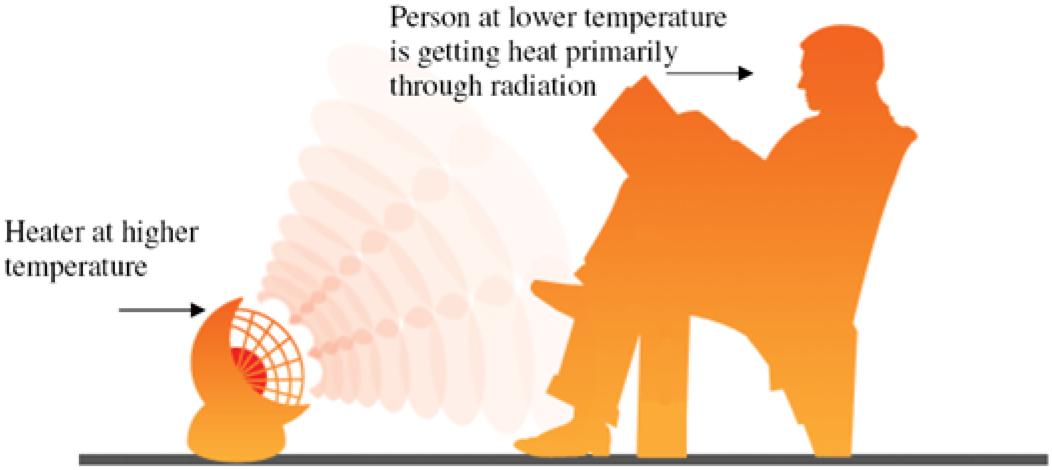


Fig.1.4: Heat transfer through radiation

1.2 Material properties of importance in heat transfer

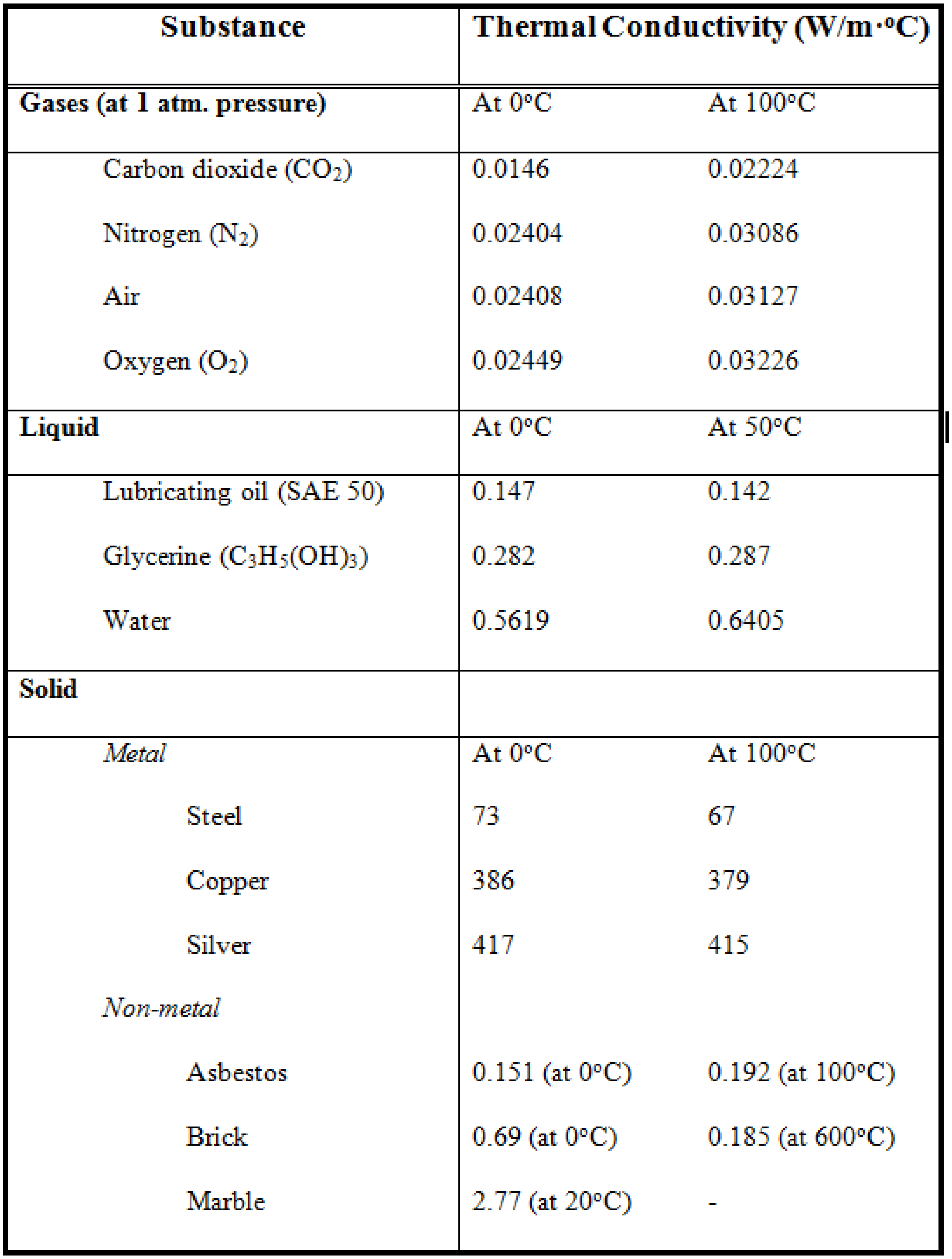
Before understanding heat transfer laws, we have to understand various properties of the material. This section is devoted to a brief discussion of some of the important properties of the material.

1.2.1Thermal conductivity

As discussed earlier, the heat conduction is the transmission of energy by molecular action. Thermal conductivity is the property of a particular substance and shows the ease by which the process takes place. Higher the thermal conductivity more easily will be the heat conduction through the substance. It can be realized that the thermal conductivity of a substance would be dependent on the chemical composition, phase (gas, liquid, or solid), crystalline structure (if solid), temperature, pressure, and its homogeneity.

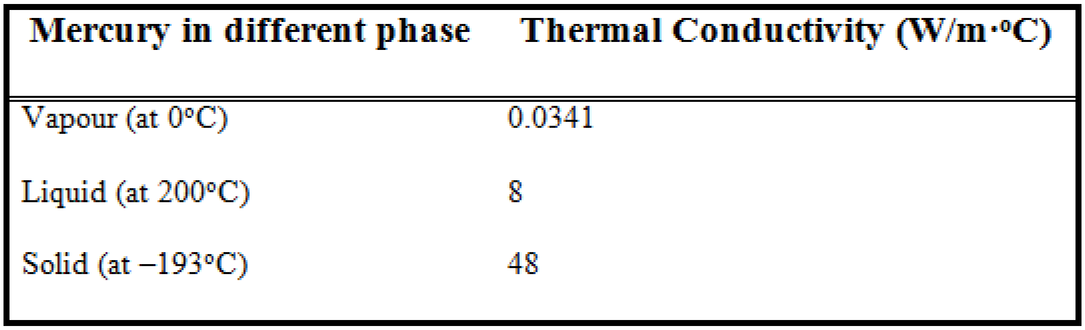
The thermal conductivity of various substances is shown in table-1.1 and table 1.2.

Table-1.1: Thermal conductivities of various substances at 0oC



Substance	**Thermal Conductivity (W/m·°C)**	
Gases (at 1 atm. pressure)	At 0°C	At 100°C
Carbon dioxide (CO_2)	0.0146	0.02224
Nitrogen (N_2)	0.02404	0.03086
Air	0.02408	0.03127
Oxygen (O_2)	0.02449	0.03226
Liquid	At 0°C	At 50°C
Lubricating oil (SAE 50)	0.147	0.142
Glycerine ($C_3H_5(OH)_3$)	0.282	0.287
Water	0.5619	0.6405
Solid		
Metal	At 0°C	At 100°C
Steel	73	67
Copper	386	379
Silver	417	415
Non-metal		
Asbestos	0.151 (at 0°C)	0.192 (at 100°C)
Brick	0.69 (at 0°C)	0.185 (at 600°C)
Marble	2.77 (at 20°C)	-

Table-1.2: Thermal conductivity of mercury at three different phases



Mercury in different phase	Thermal Conductivity (W/m·°C)
Vapour (at 0°C)	0.0341
Liquid (at 200°C)	8
Solid (at −193°C)	48

The general results of the careful analysis of the table-1.1 and 1.2 are as follows,

- Thermal conductivity depends on the chemical composition of the substance.
- Thermal conductivity of the liquids is more than the gasses and the metals have the highest.
- Thermal conductivity of the gases and liquids increases with the increase in temperature.
- Thermal conductivity of the metal decreases with the increase in temperature.
- Thermal conductivity is affected by the phase change.

These differences can be explained partially by the fact that while in gaseous state, the molecules of a substance are spaced relatively far away and their motion is random. This means that energy transfer by molecular impact is much slower than in the case of a liquid, in which the motion is still random but in liquids the molecules are more closely packed. The same is true concerning the difference between the thermal conductivity of the liquid and solid phases. However, other factors are also important when the solid state is formed.

Solid having a crystalline structure has high thermal conductivity than a substance in an amorphous solid state. Metal, crystalline in structure, have greater thermal conductivity than non-metal (refer table-1.1). The irregular arrangement of the molecules in amorphous solids inhibits the effectiveness of the transfer of the energy by molecular impact. Therefore, the thermal conductivity of the non-metals is of the order of liquids. Moreover, in solids, there is an additional transfer of heat energy resulting from vibratory motion of the crystal lattice as a whole, in the direction of decreasing temperature.

Many factors are known to influence the thermal conductivity of metals, such as chemical composition, atomic structure, phase changes, grain size, temperature, and pressure. Out of the above factors, the temperature, pressure, and chemical composition are the most important. However, if we are interested in a particular material then only the temperature effects has to be accounted for.

As per the previous discussion and the table it is now clear that the thermal conductivity of the metal is directly proportional to the absolute temperature and mean free path of the molecules. The mean free path decreases with the increase in temperature so that the thermal conductivity decreases with the temperature. It should be noted that it is true for the pure metal, and the presence of impurity in the metal may reverse the trend. It is usually possible to represent the thermal conductivity of a metal by a linear relation $k = k_o(1 + bT)$, where k_o is the thermal conductivity of the metal at 0°C, T is the absolute temperature, and b is a constant.

In general the thermal conductivity of the liquids is insensitive to the pressure if the pressure is not very close to the critical temperature. Therefore, in liquids (as in solids) the temperature effects on the thermal conductivity are generally considered. Liquids, in general, exhibit a decreasing thermal conductivity with temperature. However, water is a notable exception. Water has the highest thermal conductivity among the non-metallic liquids, with a maximum value occurring at 450°C.

The thermal conductivity of a gas is relatively independent of pressure if the pressure is near 1 atm. Vapours near the saturation point show strong pressure dependence. Steam and air are of great engineering importance. Steam shows irregular behaving rather showing a rather strong pressure dependence for the thermal conductivity as well as temperature dependence.

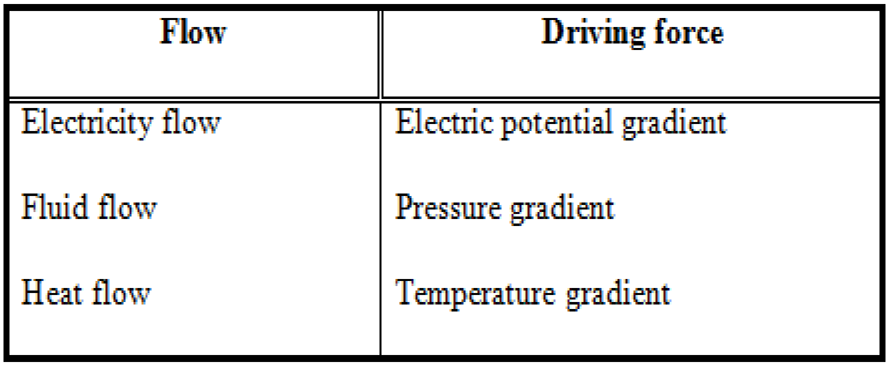
The above discussions concerning thermal conductivity were restricted to materials composed of homogeneous or pure substances. Many of the engineering materials encountered in practice are not of this nature like building material, and insulating material. Some material may exhibit non-isotropic conductivities. The non-isotropic material shows different conductivity in different direction in the material. This directional preference is primarily the result of the fibrous nature of the material like wood, asbestos etc.

1.2.2 Specific heat capacity

Now we know that the thermal conductivity facilitates the heat to propagate through the material due to the temperature gradient. Similarly, specific heat capacity or specific heat is the capacity of heat stored by a material due to variation in temperature. Thus the specific heat capacity (unit: kJ/kg·°C) is defined as the amount of thermal energy required to raise the temperature of a unit amount of material by 1°C. Since heat is path dependent, so is specific heat. In general, the heat transfer processes used in the chemical process plant are at constant pressure; hence the specific heat capacity (c_o) is generally used.

The fundamentals of heat conduction were established over one and a half century and its contribution goes to a French mathematician and physicist, Jean Baptiste Joseph Fourier. You may be aware that any flow whether it is electricity flow, fluid flow, or heat flow needs a driving force. The flow is proportional to the driving force and for various kinds of flows the driving force is shown in the table 2.1.

Table 2.1. Various flows and their driving forces



Flow	Driving force
Electricity flow	Electric potential gradient
Fluid flow	Pressure gradient
Heat flow	Temperature gradient

Thus the heat flow per unit area per unit time (heat flux, $\dot{q}'$) can be represented by the following relation,

$$\dot{q}' \propto \frac{dT}{dx} \tag{2.1}$$

$$\dot{q}' = k\frac{dT}{dx} \tag{2.2}$$

where, proportionality constant *k* is the thermal conductivity of the material, *T* is the temperature and *x* is the distance in the direction of heat flow. This is known as Fourier's law of conduction.

The term steady-state conduction is defined as the condition which prevails in a heat conducting body when temperatures at fixed points do not change with time. The term one-dimensional is applied to a heat conduction problem when only one coordinate is required to describe the distribution of temperature within the body. Such a situation hardly exists in real engineering problems. However, by considering one-dimensional assumption the real problem is solved fairly upto the accuracy of practical engineering interest.

2.1 Steady-state conduction through constant area

A simple case of steady-state, one-dimensional heat conduction can be considered through a flat wall as shown in the fig.2.1.

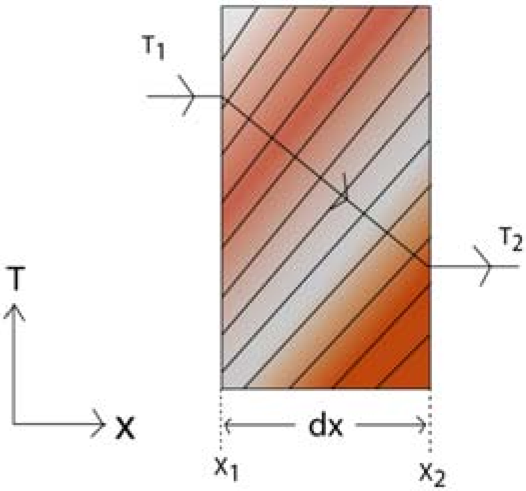


Fig.2.1: Steady-state conduction through a slab (constant area)

Illustration 2.1

The two sides of a wall (2 mm thick, with a cross-sectional area of 0.2 m2) are maintained at 30°C and 90°C. The thermal conductivity of the wall material is 1.28 W/(m·°C). Find out the rate of heat transfer through the wall?

Solution 2.1

Assumptions
1. Steady-state one-dimensional conduction
2. Thermal conductivity is constant for the temperature range of interest

3. The heat loss through the edge side surface is insignificant
4. The layers are in perfect thermal contact
Given,

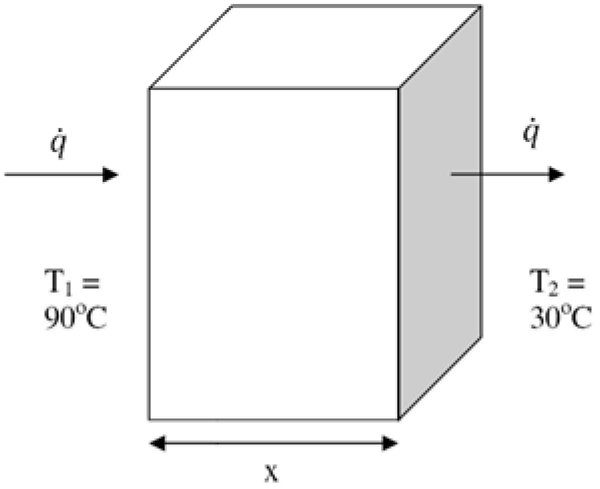


Fig. 2.4: Illustration 2.1

$k = 1.28$ W/(m·°C) $A = 0.2$ m^2 $x = 2$ mm$=0.002$ m

The rate of heat transfer can be written as,

$$\dot{q} = \frac{\Delta T}{x/_{kA}}$$

$$\dot{q} = \frac{90 - 30}{0.002/_{1.28 \times 0.2}} = 7680\ W$$

Illustration 2.2

One side of a 1 cm thick stainless steel wall (k_1 = 19 W/m·°C) is maintained at 180°C and the other side is insulated with a layer of 4 cm fibreglass (k_2 = 0.04 W/m·°C). The outside of the fibreglass is maintained at 60°C and the heat loss through the wall is 300 W. Determine the area of the wall?

Solution 2.2

Assumptions:

1. Steady-state one-dimensional conduction.
2. Thermal conductivity is constant for the temperature range of interest.
3. The heat loss through the edge side surface is insignificant.
4. The layers are in perfect thermal contact.

Given, $k_1 = 19$ W/m·°C $k_2 = 0.04$ W/m·°C $\dot{q} = 300$ W

$x_1 = 1$ cm $= 0.01$ m $x_2 = 4$ cm $= 0.04$ m

The resistance of the above composite,

$$R = \frac{x_1}{k_1 A} + \frac{x_2}{k_2 A}$$

Illustration 2.3

Consider a composite wall containing 5-different materials as shown in the fig. 2.7. Calculate the rate of heat flow through the composite from the following data?

$x_1 = 0.1\ m$ $x_2 = 0.2\ m$ $x_3 = 0.15\ m$

$k_1 = 15\ W/m{\cdot}^oC$ $k_2 = 25\ W/m{\cdot}^oC$ $k_3 = 30\ W/m{\cdot}^oC$

$k_4 = 20\ W/m{\cdot}^oC$ $k_5 = 35\ W/m{\cdot}^oC$

$h_2 = 1\ m$ $h_3 = 3\ m$ $h_4 = 2.5\ m$

$h_5 = 1.5\ m$ $T_A = 120^oC$ $T_B = 50^oC$

Solution 2.3

Assumptions:

1. Steady-state one-dimensional conduction.
2. Thermal conductivity is constant for the temperature range of interest.
3. The heat loss through the edge side surface is insignificant.
4. The layers are in perfect thermal contact.
5. Area in the direction of heat flow is 1 m^2.

The height of the first layer is 4 m ($h_1 = h_2 + h_3$).
The equivalent circuit diagram of the above composite is,

$$R = R_1 + \frac{1}{{}^{1}/_{R_2} + {}^{1}/_{R_3}} + \frac{1}{{}^{1}/_{R_4} + {}^{1}/_{R_5}}$$

On calculating equivalent resistance with the given data (Note: thickness of layer 2 = thickness of layer 3 and thickness of layer 4 = thickness of layer 5, in the heat flow direction),

$$R = \frac{0.1}{(15)(1)} + \frac{1}{\frac{1}{\left(\frac{0.2}{25\left(\frac{1}{4}\right)}\right)} + \frac{1}{\left(\frac{0.2}{30\left(\frac{3}{4}\right)}\right)}} + \frac{1}{\frac{1}{\left(\frac{0.15}{20\left(\frac{2.5}{4}\right)}\right)} + \frac{1}{\left(\frac{0.15}{35\left(\frac{1.5}{4}\right)}\right)}}$$

$$R = 0.0195\ \frac{°\mathrm{C}}{W}$$

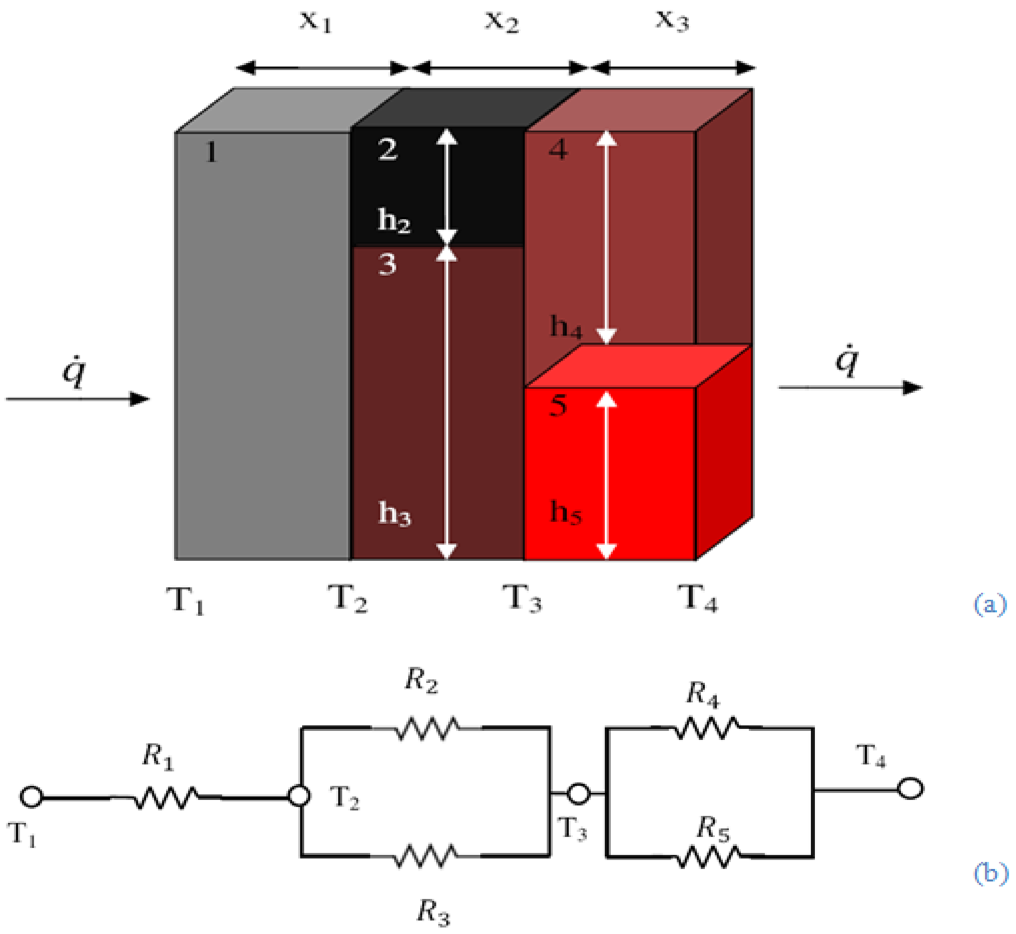


Fig. 2.7: Composite of illustration 2.3; (a) composite, (b) corresponding electrical circuit

Thus the heat flow rate through the composite,

$$\dot{q} = \frac{T_A - T_B}{R}$$

$$\dot{q} = \frac{120 - 50}{0.0195} = 3.59\ kW$$

2.3 Steady-state heat conduction through a variable area

It was observed in the previous discussion that for the given plane wall the area for heat transfer was constant along the heat flow direction. The plane solid wall was one of the geometries but if we take some other geometry (tapered plane, cylindrical body, spherical body etc.) in which the area changes in the direction of heat flow. Now we will consider geometrical configuration which will be mathematically simple and also of great engineering importance like hollow cylinder and hollow sphere. In these cases the heat transfer area varies in the radial direction of heat conduction. We will take up both the cases one by one in the following sections.

2.3.1 Cylinder

Consider a hollow cylinder as shown in the fig.2.9a. The inner and outer radius is represented by r_i and r_o, whereas T_i and T_o ($T_i > T_o$) represent the uniform temperature of the inner and outer wall, respectively.

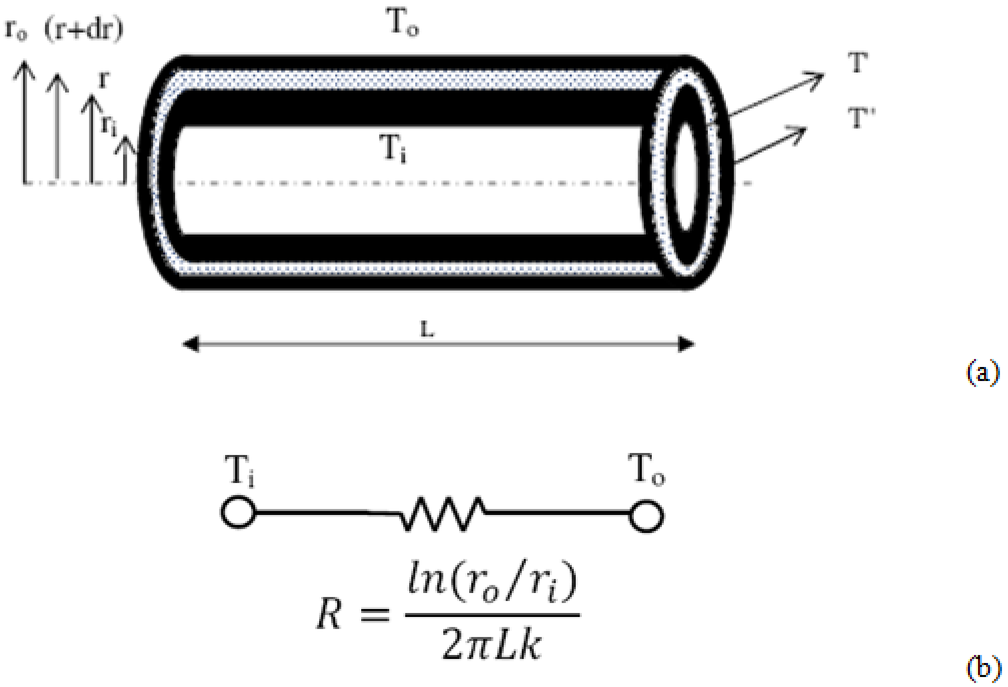


Fig. 2.9. (a) Hollow cylinder, (b) equivalent electrical circuit

Consider a very thin hollow cylinder of thickness d_r in the main geometry (fig.2.9a) at a radial distance r. If d_r is small enough with respect to r, then the area of the inner and outer surface of the thin cylinder may be considered to be of same area. In other words, for very small d_r with respect to *r*, the lines of heat flow may be considered parallel through the differential element in radial outward direction.

We may ignore the heat flow through the ends if the cylinder is sufficiently large. We may thus eliminate any dependence of the temperature on the axial coordinate and for one dimensional steady state heat conduction, the rate of heat transfer for the thin cylinder,

$$\dot{q} = -k\frac{dT}{dr}\ 2\pi rL$$

Where *dT* is the temperature difference between the inner and outer surface of the thin cylinder considered above and *k* is the thermal conductivity of the cylinder.
On rearranging,

$$\frac{dr}{r} = -\frac{2\pi Lk}{\dot{q}}dT$$

To get the heat flow through the thick wall cylinder, the above equation can be integrated between the limits,

$$\int_{r_i}^{r_o}\frac{dr}{r} = -\frac{2\pi Lk}{\dot{q}}\int_{T_i}^{T_o}dT$$

2.4 Heat conduction in bodies with heat sources

The cases considered so far have been those in which the heat conducting solid is free of internal heat generation. However, the situations where the internal heat is generated are very common cases in chemical industries for example, the exothermic reaction in the solid pallet of a catalyst.

We have learnt that how the Fourier equation is used for the steady-state heat conduction through the composites of three different geometries that were not having any heat source in it. However, the heat generation term would come into the picture for these geometries. It would not be always easier to remember and develop heat conduction relations for different standard and non-standard geometries. Therefore, at this point we should learn how to develop a general relation for the heat conduction that should be applicable to the entire situation such as steady-state, unsteady state, heat source, different geometry, heat conduction in different direction, etc. Again here we will consider that the solid is isotropic in nature, which means the thermal conductivity of the material is same in all the direction of heat flow.

To get such a general equation the differential form of the heat conduction equation is most important. For simplicity, we would consider an infinitesimal volume element in a Cartesian coordinate system. The dimensions of the infinitesimal volume element are d_x, d_y, and d_z in the respective direction as shown in the fig.2.11.

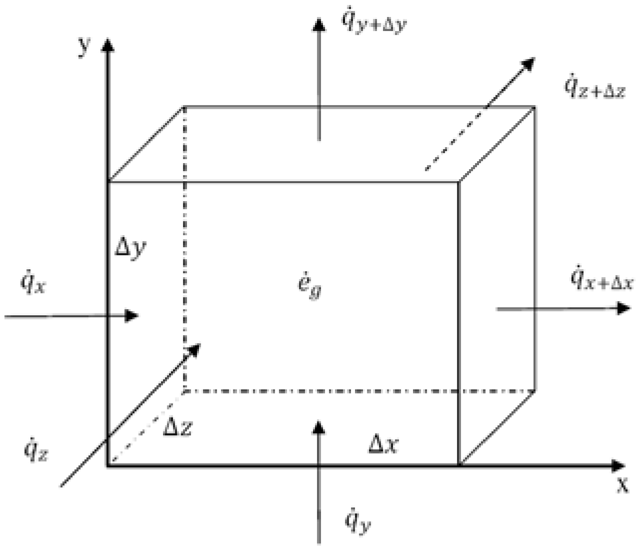


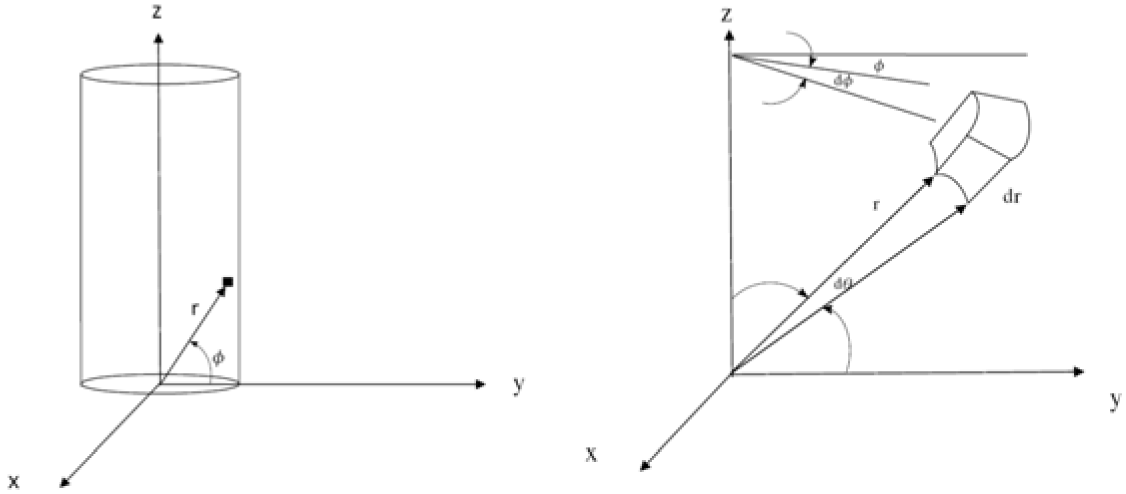
Fig.2.11. Volume element for deriving general equation of heat conduction in cartesian coordinate

The fig.2.11 shows that the heat is entering into the volume element from three different faces of the volume element and leaving from the opposite face of the control element. The heat source within the volume element generates the volumetric energy at the rate of $\dot{e}_g$(W/m^3). According to Fourier's law of heat conduction, the heat flowing into the volume element from the left (in the x-direction) can be written as,

$$\dot{q}_x = -k\,dy\,dz\,\frac{\partial T}{\partial x}$$

The heat flow out from the right surface (in the x-direction) of the volume element can be obtained by Taylor series expansion of the above equation. As the volume element is of infinitesimal volume, we may retain only first two element of the Taylor series expansion with a reasonable approximation (truncating the higher order terms). Thus,

General heat conduction relation in cylindrical coordinate system (fig. 2.12) is derived (briefly) below.



(a) (b)

Fig.2.12. Cylindrical coordinate system (a) and an element of the cylinder

The energy conservation for the system is written as,

I + II = III + IV (2.21)

where,

I : Rate of heat energy conducted in

II : Rate of heat energy generated within the volume element

III : Rate of heat energy conducted out

IV : Rate of energy accumulated (IV)

and the above terms are defines as,

I : $\dot{q}_r + \dot{q}_\emptyset + \dot{q}_z$

II : $\dot{e}_g \, dr \, dz \, r \, d\emptyset$

III : $\dot{q}_{r+dr} + \dot{q}_{\emptyset+d\emptyset} + \dot{q}_{z+dz}$

IV : $\rho \, dr \, dz \, r d\emptyset \, c_p \frac{\partial T}{\partial t}$ (T is a function of space and time)

Thus,

(I) - (III) $: (-\frac{\partial \dot{q}_r}{\partial r}\, dr) + (-\frac{\partial \dot{q}_\emptyset}{\partial \emptyset}\, d\emptyset) + (-\frac{\partial \dot{q}_z}{\partial z}\, dz)$

Using Fourier's law

$$\dot{q}_r = -k(r\, d\emptyset\, dz)\frac{\partial T}{\partial r}$$

$$\dot{q}_\emptyset = -k(dr\, dz)\frac{\partial T}{r\partial \emptyset}$$

$$\dot{q}_z = -k(r\, d\emptyset\, dr)\frac{\partial T}{\partial z}$$

(I) - (III) $: \frac{\partial}{\partial r}(k\, r\, d\emptyset\, dz \frac{\partial T}{\partial r})dr + \frac{\partial}{\partial \emptyset}(k\, dr\, dz \frac{\partial T}{r\partial \emptyset})d\emptyset + \frac{\partial}{\partial z}(k\, r\, d\emptyset\, dr \frac{\partial T}{\partial z})dz$

If k is isotropic,

$$= k\, d\emptyset\, dz\, dr \frac{\partial}{\partial r}(r \frac{\partial T}{\partial r}) + k\, d\emptyset\, dz\, dr \frac{\partial}{\partial \emptyset}(\frac{\partial T}{r\partial \emptyset}) + r\, k\, d\emptyset\, dr\, dz \frac{\partial}{\partial z}(\frac{\partial T}{\partial z})$$

$$= k\, r\, d\emptyset\, dz\, dr\, [\frac{1}{r}\frac{\partial}{\partial r}(r \frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial \emptyset}(\frac{\partial T}{\partial \emptyset}) + \frac{\partial}{\partial z}(\frac{\partial T}{\partial z})]$$

$$= k\, r\, d\emptyset\, dz\, dr\, [\frac{1}{r}\frac{\partial}{\partial r}(r \frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \emptyset^2} + \frac{\partial^2 T}{\partial z^2}]$$

On putting the values in equation 2.21,

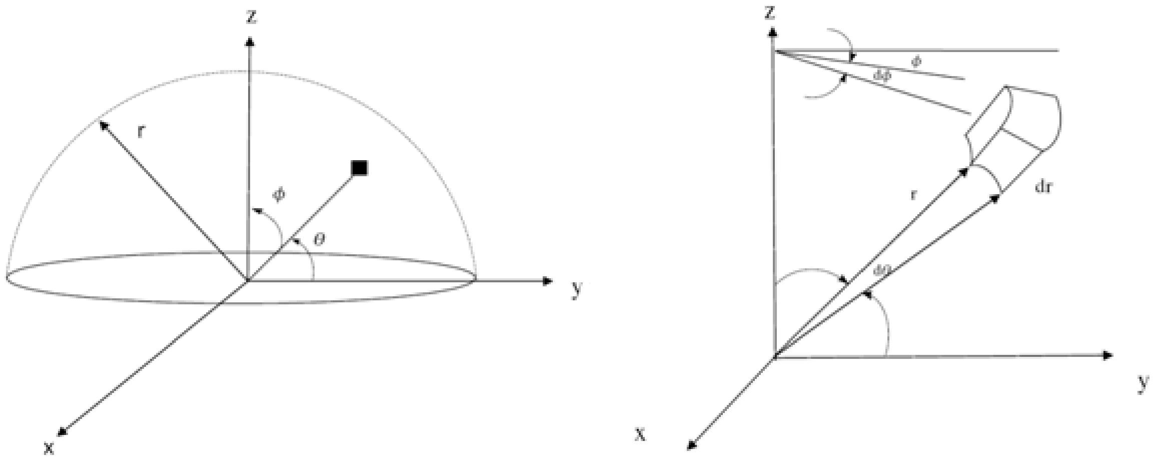
$$k\, r\, d\emptyset\, dz\, dr\, [\frac{1}{r}\frac{\partial}{\partial r}(r \frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \emptyset^2} + \frac{\partial^2 T}{\partial z^2}] + \dot{e}_g\, dr\, dz\, r\, d\emptyset = \rho\, dr\, dz\, r d\emptyset\, C_p \frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r \frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \emptyset^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_g}{k} = \frac{\rho C_P}{k}\, \frac{\partial T}{\partial t}$$

$$\nabla^2 T + \frac{\dot{e}}{k} = \frac{1}{\propto}\frac{\partial T}{\partial t} \qquad (\text{where } \alpha = \frac{k}{\rho C_P}) \qquad (2.22)$$

Thus the Laplacian operator is,

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r \frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial \emptyset^2} + \frac{\partial^2}{\partial z^2} \qquad (2.23)$$



(a) (b)

Fig.2.13. Spherical coordinate system (a) and an element of the sphere

In a similar way the general expression for the conduction heat transfer in spherical body with heat source can also be found out as per the previous discussion. The Laplacian operator for the spherical coordinate system (fig.2.13) is given below and the students are encouraged to derive the expression themselves.

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\emptyset}\frac{\partial}{\partial\emptyset}\left(sin\emptyset\,\frac{\partial}{\partial\emptyset}\right) + \frac{1}{r^2\sin^2\emptyset}\frac{\partial^2}{\partial\theta^2} \qquad (2.24)$$

The rate of heat transfer in a solid body or medium can be calculated by Fourier's law. Moreover, the Fourier law is applicable to the stagnant fluid also. However, there are hardly a few physical situations in which the heat transfer in the fluid occurs and the fluid remains stagnant. The heat transfer in a fluid causes convection (transport of fluid elements) and thus the heat transfer in a fluid mainly occurs by convection.

3.1 Principle of heat flow in fluids and concept of heat transfer coefficient

It is learnt by day-to-day experience that a hot plate of metal will cool faster when it is placed in front of a fan than exposed to air, which is stagnant. In the process, the heat is convected away, and we call the process convective heat transfer. The term convective refers to transport of heat (or mass) in a fluid medium due to the motion of the fluid. Convective heat transfer, thus, associated with the motion of the fluid. The term convection provides an intuitive concept of the heat transfer process. However, this intuitive concept must be elaborated to enable one to arrive at anything like an adequate analytical treatment of the problem.

It is well known that the velocity at which the air blows over the hot plate influences the heat transfer rate. A lot of questions come into the way to understand the process thoroughly. Like, does the air velocity influence the cooling in a linear way, i.e., if the velocity is doubled, will the heat transfer rate double. We should also suspect that the heat-transfer rate might be different if we cool the plate with some other fluid (say water) instead of air, but again how much difference would there be? These questiones may be answered with the help of some basic analysis in the later part of this module.

The physical mechanism of convective heat transfer for the problem is shown in fig.3.1.

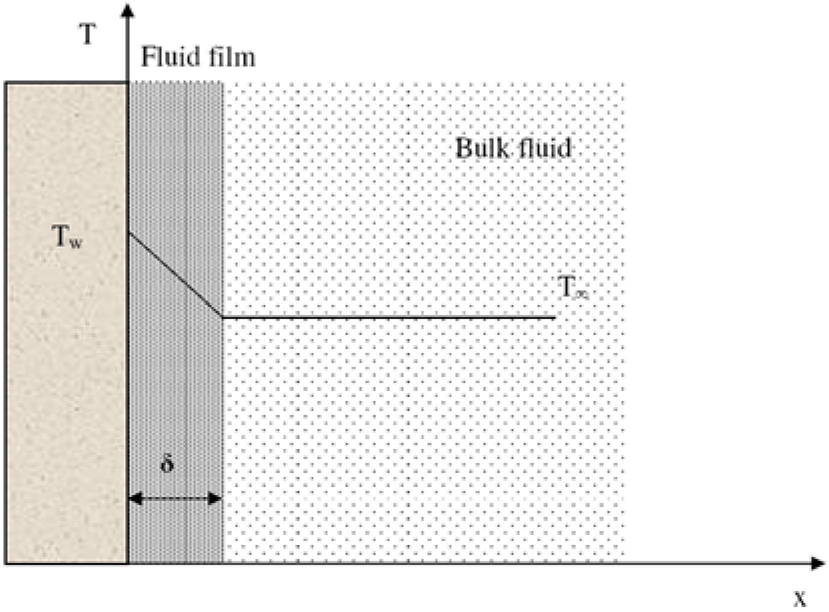
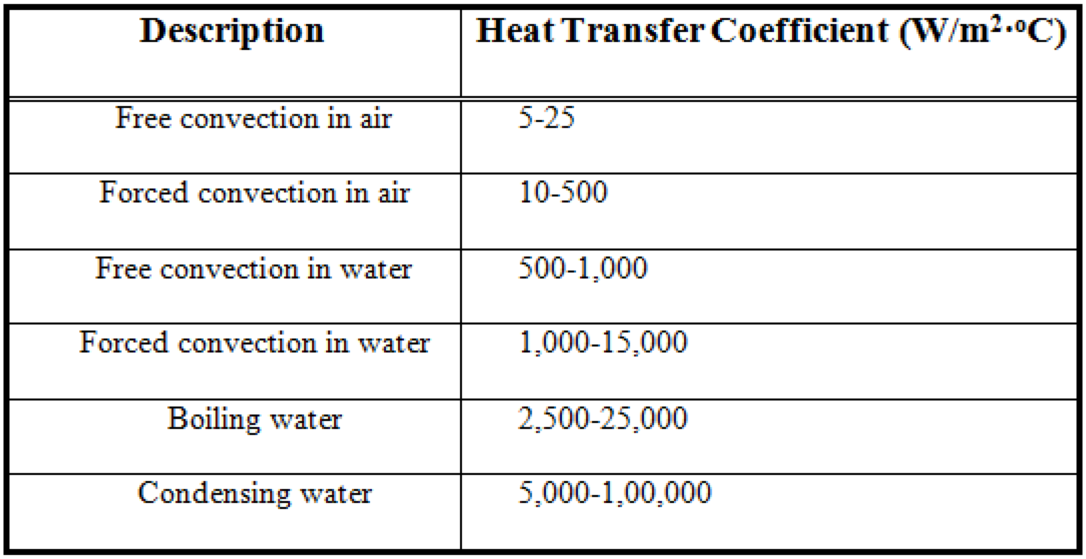


Fig. 3.1: Convective heat transfer from a heated wall to a fluid

Consider a heated wall shown in fig.3.1. The temperature of the wall and bulk fluid is denoted by T_w, and T_∞, respectively. The velocity of the fluid layer at the wall will be zero. Thus the heat will be transferred through the stagnant film of the fluid by conduction only. Thus we can compute the heat transfer using Fourier's law if the thermal conductivity of the fluid and the fluid temperature gradient at the wall is known. Why, then, if the heat flows by conduction in this layer, do we speak of convective heat transfer and need to consider the velocity of the fluid? The answer is that the temperature gradient is dependent on the rate at which the fluid carries the heat away; a high velocity produces a large temperature gradient, and so on. However, it must be remembered that the physical mechanism of heat transfer at the wall is a conduction process.



Description	Heat Transfer Coefficient (W/m²·°C)
Free convection in air	5-25
Forced convection in air	10-500
Free convection in water	500-1,000
Forced convection in water	1,000-15,000
Boiling water	2,500-25,000
Condensing water	5,000-1,00,000

3.2 Individual and overall heat transfer coefficient

If two fluids are separated by a thermally conductive wall, the heat transfer from one fluid to another fluid is of great importance in chemical engineering process plant. For such a case the rate of heat transfer is done by considering an overall heat transfer coefficient. However, the overall heat transfer coefficient depends upon so many variables that it is necessary to divide it into individual heat transfer coefficients. The reason for this becomes apparent if the above situation can be elaborated as discussed in the following sub-sections.

3.2.1 Heat transfer between fluids separated by a flat solid wall

As shown in fig.3.2, a hot fluid is separated by solid wall from a cold fluid. The thickness of the solid wall is l, the temperature of the bulk of the fluids on hot and cold sides are T_h and T_c, respectively. The average temperature of the bulk fluid is T_1 and T_4, for hot and cold fluid, respectively. The thicknesses of the fictitious thin films on the hot and cold sides of the flat solid are shown by δ_1 and δ_2. It may be assumed that the Reynolds numbers of both the fluids are sufficiently large to ensure turbulent flow and the surfaces of the solid wall are clean.

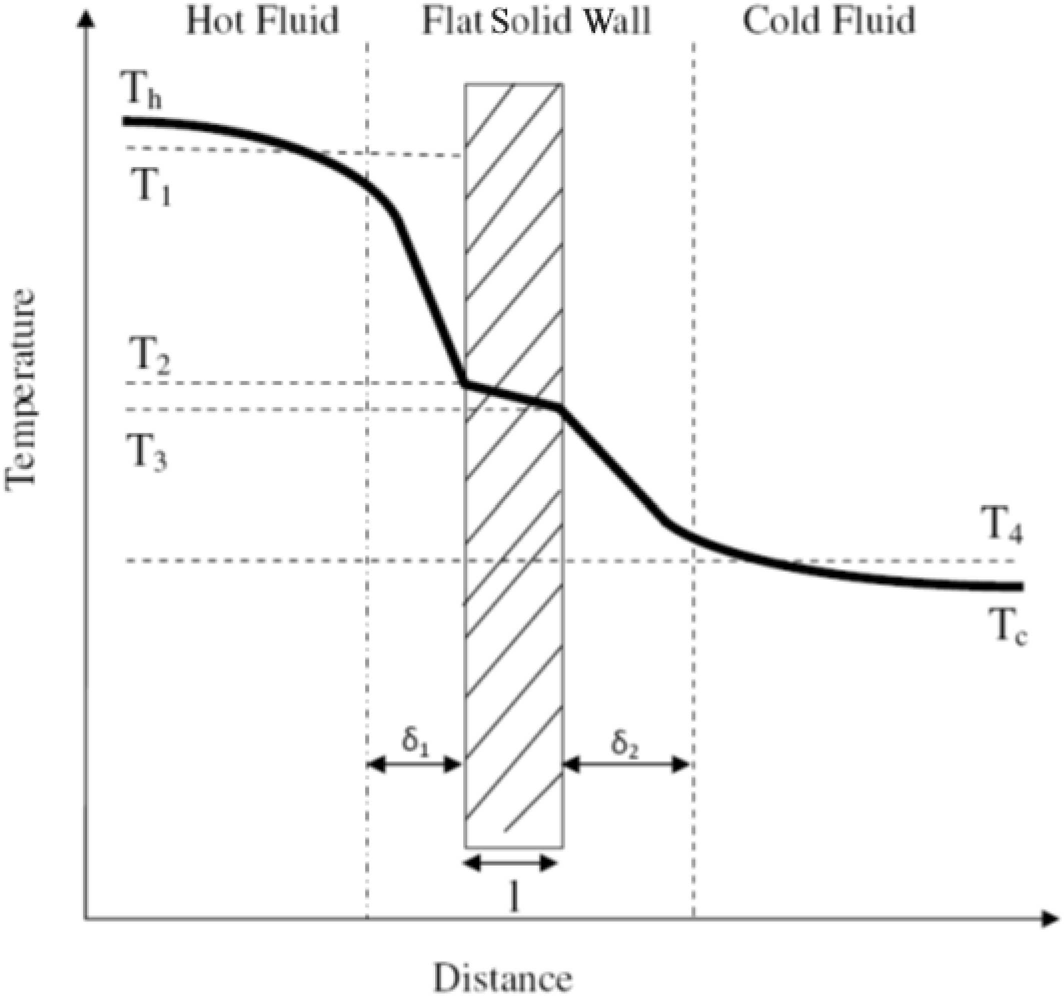


Fig.3.2. Real temperature profile

It can be seen that the temperature gradient is large near the wall (through the viscous sublayer), small in the turbulent core, and changes rapidly in the buffer zone (area near the interface of sublayer and bulk fluid). The reason was discussed earlier that the heat must flow through the viscous sublayer by conduction, thus a steep temperature gradient exists because of the low temperature gradient of most of the fluids.

The average temperatures of the warm bulk fluid and cold bulk fluids are slightly less than the maximum temperature T_h (bulk temperature of hot fluid) and slightly more than the minimum temperature T_c (bulk temperature of cold fluid), respectively. The average temperatures are shown by T_1 and T_4, for the hot and cold fluid streams, respectively.

Figure 3.3 shows the simplified diagram of the above case, where *T2* and *T3* are the temperatures of the fluid wall interface.

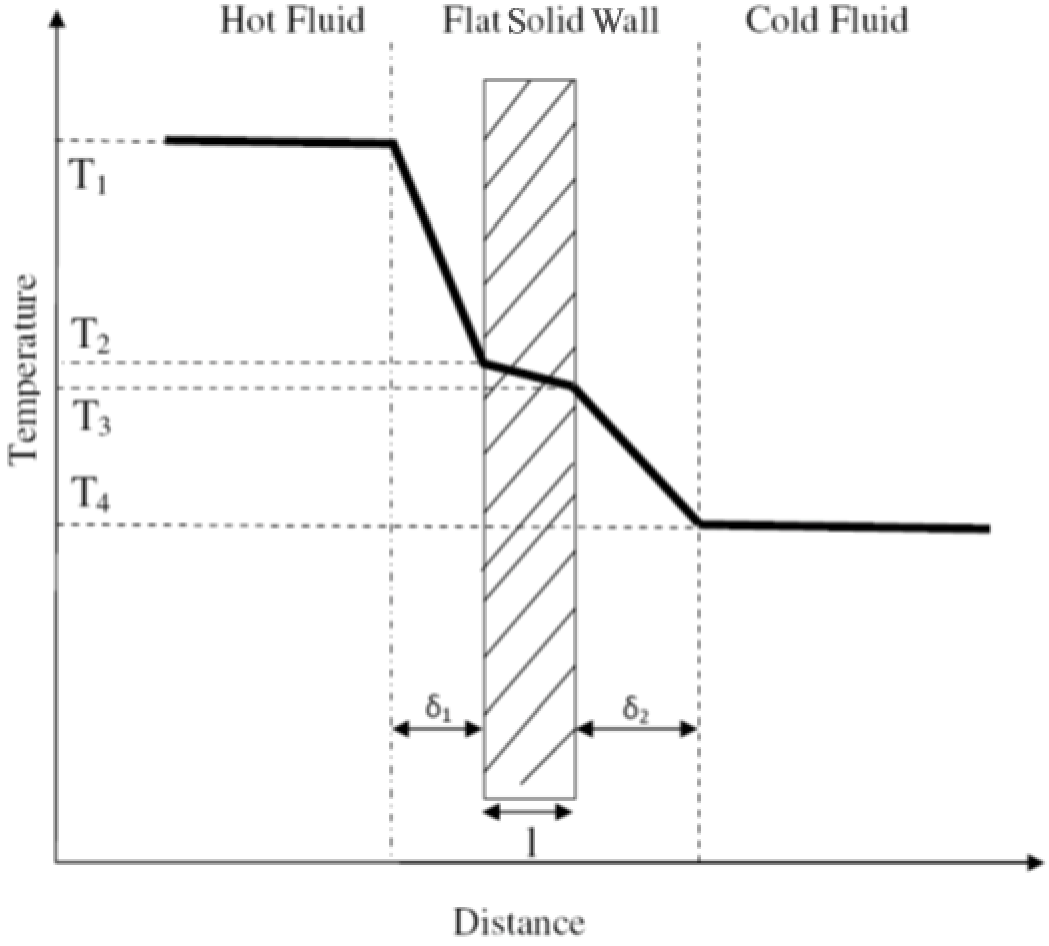


Fig.3.3. Simplified temperature profile for fig.3.2

If the thermal conductivity of the wall is k, and the area of the heat transfer is A, the electrical analogy of the fig.3.3 can be represented by fig.3.4, where h_1 and h_2 are the individual heat transfer coefficient of the hot and cold side of the fluid.

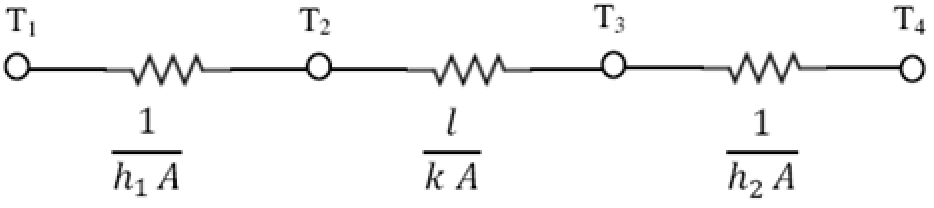


Fig.3.4. Equivalent electrical circuit for fig. 3.3

Considering that the heat transfer is taking place at the steady-state through a constant area and the heat loss from other faces are negligible, then the rate of heat transfer on two sides of the wall will be represented by eq. 3.4-3.6.

Rate of heat transfer from the hot fluid to the wall, Rate of heat transfer from the wall to cold fluid,

$$\dot{q}_3 = h_2\, A\, (T_3 - T_4) \tag{3.6}$$

At steady state, the rate of heat transfers $(\dot{q}_1, \dot{q}_2,$ and $\dot{q}_3)$ are same and can be represented by $\dot{q}$. Therefore,

$$T_1 - T_2 = \frac{\dot{q}}{h_1\, A} \tag{3.7}$$

$$T_2 - T_3 = \frac{\dot{q}}{A\, k/l} \tag{3.8}$$

$$T_2 - T_3 = \frac{\dot{q}}{h_2\, A} \tag{3.9}$$

On adding equations (3.7 to 3.9)

$$T_1 - T_4 = \frac{\dot{q}}{A}\left(\frac{1}{h_1} + \frac{1}{k/l} + \frac{1}{h_3}\right)$$

$$T_1 - T_4 = \frac{\dot{q}}{A}\,\frac{1}{U} \tag{3.10}$$

where,

$$\frac{1}{U} = \frac{1}{h_1} + \frac{l}{k} + \frac{1}{h_3}$$

Thus,

$$\dot{q} = UA(T_1 - T_4) \tag{3.11}$$

The quantity U is called the overall heat transfer coefficient (can be calculated if the $h_1, h_2, k,$ and l are known). Thus from the system described is established that the overall heat transfer coefficient is the function of individual heat transfer coefficient of the fluids on the two sides of the wall, as well as the thermal conductivity of the flat wall. The overall heat transfer coefficient can be used to introduce the controlling term concept. The controlling resistance is a term which possesses much larger thermal resistance compared to the sum of the other resistances.

At this point it may be noted that in general the resistance offered by the solid wall is much lower. Similarly, if a liquid and a gas are separated by a solid wall the resistance offered by the gas film may generally be high.

$$\dot{q}_1 = h_1 A \left(T_1 - T_2\right) \qquad (3.4)$$

Rate of heat transfer through the wall,

$$\dot{q}_2 = \frac{k A \left(T_2 - T_3\right)}{l} \qquad (3.5)$$

Illustration 3.1.

The steady state temperature distribution in a wall is , where x (in meter) is the position in the wall and T is the temperature (in °C). The thickness of the wall is 0.2 m and the thermal conductivity of the wall is 1.2 (W/m·°C). The wall dissipates the heat to the ambient at 30 °C. Calculate the heat transfer coefficient at the surface of the wall at 0.2 m.

Solution 3.1

The rate of heat transfer through the wall by conduction will be equal to the rate of heat transfer from the surface to the ambient by convention at steady state,

Rate of heat transfer by conduction at x=0.2 is given by,

$$-kA\frac{dT}{dx} = h\,A\left(T_x - T_a\right)$$

where T_a is the ambient temperature.

$$T = 300 - 3050x^2$$

$$T_{x=0.2} = 300 - 3050(0.2^2) = 178\ ^{\circ}\mathrm{C}$$

$$\frac{dT}{dx} = -6100x$$

$$h = \frac{-kA\frac{dT}{dx}}{A\left(T_x - T_a\right)}$$

$$= \frac{(-k)(-6100x)}{T_x - T_a}$$

On putting the values and solving,

$$= \frac{(1.2)(6100)(0.2)}{178-30}$$

$$h = 9.89 \text{ W/(m}^2\text{·°C)}$$

3.2.2 Heat transfer between fluids separated by a cylindrical wall

In the above section we have seen that how the rate of heat transfer is calculated when the two fluids are separated by a flat wall. Another commonly encountered shape in the chemical engineering plant is the heat transfer between fluids separated by a cylindrical wall. Therefore, we will see them to understand the overall heat transfer coefficient in such a system. Consider a double pipe heat exchanger which consists of two concentric pipes arrange as per the fig. 3.4.

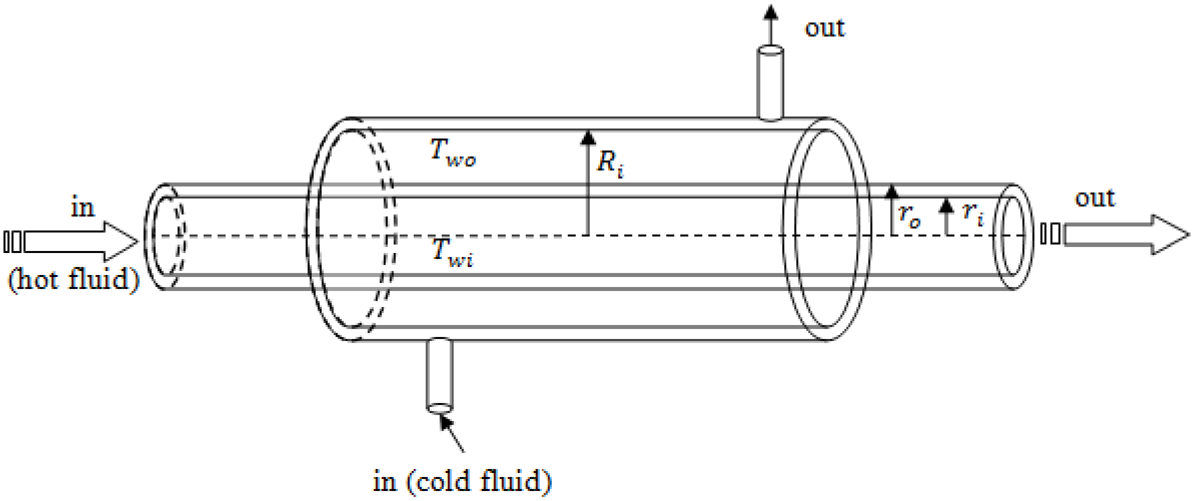


Fig. 3.5: Schematic of a co-current double pipe heat exchanger

The purpose of a heat exchanger is to increase the temperature of a cold fluid and decrease that of the hot fluid which is in thermal contact, in order to achieve heat transfer.

The fig. 3.5 shows that the hot fluid passes through the inner tube and the cold fluid passes through the outer tube of the double pipe heat exchanger. The inner and the outer radii of the inner pipe are **r_i and r_o,** respectively, whereas the inner radius of the outer tube is R_i. The heat transfer coefficient of the fluid in the inner pipe is h_i and the heat transfer coefficient of the fluid over the inner pipe is h_o. T_{wi} and T_{wo} are the inner and outer wall temperatures of the inner pipe. The bulk fluid temperatures of the hot and cold fluids are T_o and T_i , respectively, at steady state condition and assumed to be fairly constant over the length of the pipe (say *L*). The construction in fig. 3.6 provides a better understanding.

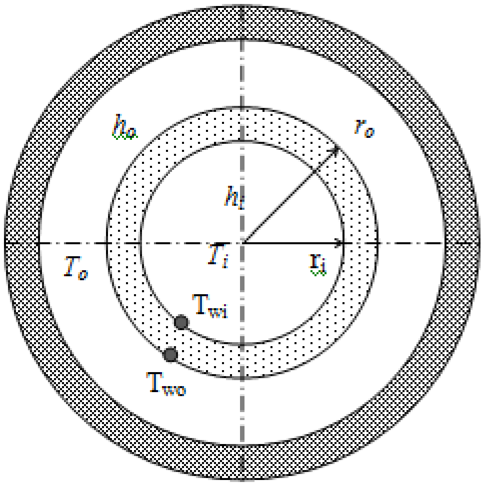


Fig. 3.6: Cross-section of the double pipe heat exchanger shown in fig. 3.5

The rate of heat transfer from the hot fluid to the inner surface which is at temperature T_{wi}

$$\dot{q}_1 = A_i h_i (T_i - T_{wi}) \qquad (3.12)$$

The rate of heat transfer through the pipe wall is,

$$\dot{q}_1 = A_i h_i (T_i - T_{wi}) \qquad (3.13)$$

(Refer to the section, heat conduction through varying area.)

The rate of heat transfer from the outer surface of the inner pipe to the cold fluid is,

$$\dot{q}_2 = A_o h_o (T_{wo} - T_o) \qquad (3.14)$$

The rate of heat transfers will be same, thus

$$T_i - T_{wi} = \frac{\dot{q}}{h_i A_i} \qquad (3.15)$$

$$T_{wi} - T_{wo} = \frac{\dot{q}}{2\pi L k / \ln\,(r_o/r_i)} \qquad (3.16)$$

$$T_{wo} - T_o = \frac{\dot{q}}{h_o A_o} \qquad (3.17)$$

Thus on rearranging above equations,

$$T_i - T_o = \dot{q}\left(\frac{1}{h_i A_i} + \frac{\ln(r_0/r_i)}{2\pi k L} + \frac{1}{h_o A_o}\right)$$

$$\dot{q} = \frac{T_i - T_o}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_0/r_i)}{2\pi k L} + \dfrac{1}{h_o A_o}}$$

$$\dot{q} = UA(T_i - T_o) \qquad (3.18)$$

where,

$$UA = \frac{1}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_0/r_i)}{2\pi k L} + \dfrac{1}{h_o A_o}}$$

If we compare the overall heat transfer coefficient shown above with the overall heat transfer coefficient discussed in eq.3.11 (for flat plate). It can be seen that due to the different inside and outside radii of the pipe, the overall heat transfer coefficient will be different. Therefore, the overall heat transfer coefficient can be defined either by U_i (overall heat transfer coefficient based on inside surface area) or U_o (overall heat transfer coefficient based on outside surface area). But it should be noted that the rate of heat transfer and the driving force remain the same. Therefore, we have

$$\dot{q} = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o) \qquad (3.19)$$

where,

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi k L} + \dfrac{A_i}{h_o A_o}}$$

or,

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{r_i \ln(r_o/r_i)}{k} + \dfrac{r_i}{h_o r_o}}$$

Similarly,

$$U_o = \frac{1}{\frac{r_o}{r_i h_i} + \frac{r_o \ln(r_o/r_i)}{k} + \frac{1}{h_o}}$$

In terms of thermal resistance, we can use eq. 3.19

$$\dot{q} = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o)$$

$$\frac{T_i - T_o}{R_T} = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o)$$

$$\Rightarrow R_T = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

$$\Rightarrow U_i A_i = U_o A_o \qquad (3.20)$$

Illustration 3.2.

Warm methanol flowing in the inner pipe of a double pipe heat exchanger is being cooled by the flowing water in the outer tube of the heat exchanger. The thermal conductivity of the exchanger, inner and outer diameter of the inner pipe are 45 W/(m·°C), 26 mm, and 33 mm, respectively. The individual heat transfer coefficients are:

	Coefficient (W/(m^2·°C))
Methanol, h_i	**1000**
Water, h_o	**1750**

Calculate the overall heat transfer coefficient based on the outside area of the inner tube.

Solution 3.2

Using following equation,

$$U_o = \frac{1}{\frac{r_o}{r_i h_i} + \frac{r_o \ln(r_o/r_i)}{k} + \frac{1}{h_o}}$$

It is apparent that all the values are known. Thus, on putting the values the U_o is 519 W/(m^2·°C).

3.3 Enhanced heat transfer: concept of fins

In the previous discussion, we have seen that the heat transfer from one fluid to another fluid needs a solid boundary. The rate of heat transfer depends on many factors including the individual heat transfer coefficients of the fluids. The higher the heat transfer coefficients the higher will be the rate of heat transfer. There are many situations where the fluid does not have a high heat transfer coefficient. For example, the heat lost by conduction through a furnace wall must be dissipated to the surrounding by convection through air. The air (or the gas phase in general) has very low heat transfer coefficient, since the thermal conductivities of gases are very low, as compared to the liquid phase. Thus if we make heat transfer device for gas and a liquid (of course separated by a heat conducting wall), the gas side film will offer most of the thermal resistance as compared to the liquid side film. Therefore, to make the heat transfer most effective we need to expose higher area of the conductive wall to the gas side. This can be done by making or attaching fins to the wall of the surface. A fin (in general) is a rectangular metal strip or annular rings to the surface of heat transfer. Thus, a fin is a surface that extends from an object to increase the rate of heat transfer to or from the environment by increasing convections. Fins are sometimes known as extended surface. Figure 3.7 shows photographs of an electric motor with the fins on the motor body and a computer processor with the fins to dissipate the generated heat into the environment. Figure 3.8 shows the different types of finned surfaces.

Fig. 3.7. Cooling fins of (a) electric motor, (b) computer processor

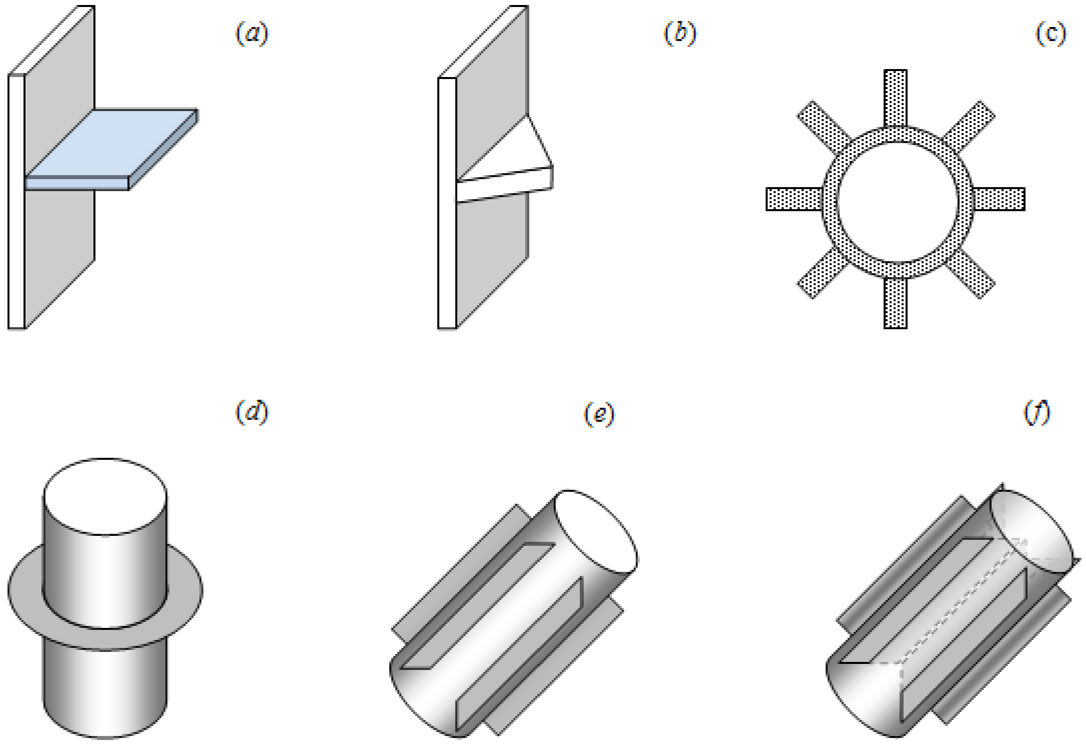


Figure 3.8: Different types of finned surface (*a*) straight rectangular fin, (*b*) straight triangular fin, (c) straight rectangular fin on circular tube, (*d*) ring type fin on pipe, (*e*) external longitudinal fin of rectangular profile, and (*f*) internal and external longitudinal fin of rectangular profile

Figure 3.9 shows a simple straight rectangular fin on plane wall. The fin is protruded a distance l from the wall. The temperature of the plane wall (in fact the base of the fin) is T_w and that of the ambient is T_∞. The distances of the fin are: length l; thickness t; and the breadth b. The heat is conducted through the body by conduction and dissipates to the surrounding by convection. The heat dissipation to the surrounding occurs from both top, bottom, and side surfaces of the fin. Here, it is assumed that the thickness of the fin is small and thus the temperature does not vary in the Y-direction. However, the fin temperature varies in the X-direction only.

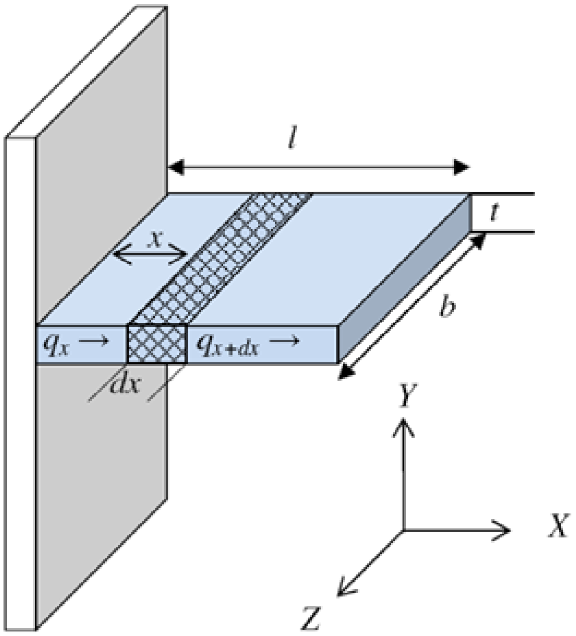


Fig. 3.9. 1-D heat conduction and convection through a rectangular fin

Consider a thin element of thickness d_x of the fin at a distance *x* from the fin base. The energy balance on the fin element at steady state is discussed below.

Rate of heat input at x: $btq_{x|x}$

Rate of heat output at $x+dx$: $btq_{x|x+dx}$

Rate of heat loss from the exposed surface: $Pdx \cdot h(T - T_\infty)$

where, *P* is the perimeter [2(*b*+t)] of the element, *T* is the local temperature of the fin, *h* is the film heat transfer coefficient, and *bt* is the fin area (*A*) perpendicular to the direction of heat transfer.

$$q_{x|x} = -k\frac{dT}{dx}\text{, and}$$

$$q_{x|x+dx} = -k\frac{dT}{dx}|_{x+dx} = -k(\frac{dT}{dx} + \frac{d^2T}{dx^2}dx)$$

Thus, at steady state,
Rate of heat input – Rate of heat output – Rate of heat loss = 0

$$-kA\frac{dT}{dx} - \left[-kA\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}dx\right)\right] - hPdx(T - T_\infty) = 0$$

$$kA\frac{d^2T}{dx^2}dx - hPdx(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0 \qquad (3.21)$$

Defining $\bar{T} = T - T_\infty$, the above equation reduces to,

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}\bar{T} = 0 \qquad (3.22)$$

Boundary conditions are,

$$\bar{T} = \overline{T_w} = T_w - T_\infty \text{ at } x=0.$$

However, the other boundary conditions depend on the physical situation of the problem. A few of the typical cases are,

Case I: The fin is very long and thus the temperature at the end of the fin is same as that of the ambient fluid.

Case II: The fin is of finite length and looses heat from its end by convection.

Case III: The end of the fin is insulated so that at $\frac{dT}{dx} = 0$ at $x=l$.

3.3.1 Analytical solution of the above cases

Case I:

The boundary conditions will be

B.C.1: $\bar{T} = \overline{T_w}$ at $x = 0$

B.C.2: $\bar{T} = 0$ at $x = \infty$

and if $\frac{hP}{kA} = m^2$, the eq. 3.22 reduces to,

$$\frac{d^2\bar{T}}{dx^2} = m^2\bar{T}$$

$$\bar{T} = C_1 e^{-mx} + C_2 e^{mx} \qquad (3.23)$$

Using boundary conditions, the solution of the equation 3.23 becomes,

$$\frac{\bar{T}}{\overline{T_W}} = \frac{T - T_W}{T_W - T_\infty} = e^{-mx}$$

All of the heat lost by the fin must have conducted from the base at *x*=0. Thus, we can compute the heat loss by the fin using the equation for temperature distribution,

$$q = -kA\frac{dT}{dx}\Big|_{x=0}$$

$$q = -kA\left(-m\overline{T_w}e^{-m(0)}\right) = \sqrt{hPkA}(T_w - T_\infty) \qquad (3.24)$$

Similarly, for **Case – II**, the boundary conditions are:

B.C.1: $\bar{T} = \overline{T_w} = T_w - T_\infty,$ at $x = 0$, and

B.C.2: $-k\frac{dT}{dx} = h(T - T_\infty),$ i.e., $-k\frac{d\bar{T}}{dx} = h\bar{T}$ at $x=l$.

The second boundary condition is a convective boundary condition which implies that the rate at which heat is conducted from inside the solid to the boundary is equal to the rate at which it is transported to the ambient fluid by convection.

The temperature profile is,

$$\bar{T} = C_1 e^{-mx} + C_2 e^{mx}$$

or we can write,

$$\bar{T} = A_1 cosh(mx) \; + \; A_2 sinh(mx)$$

Using B.C.1: $\overline{T_w} \; = \; A_1$

Using B.C.2: $-k[A_1 m \, sinh(ml) \; + \; A_2 m \, cosh(ml)] = h \, \bar{T}$

$$-k[A_1 m \, sinh(ml) \; + \; A_2 m \, cosh(ml)] = h \; [A_1 cosh(ml) \; + \; A_2 sinh(ml)]$$

$$A_1 \left[sinh(ml) \; + \frac{h}{mk} \; cosh(ml) \right] = \; -A_2 \left[cosh(ml) + \frac{h}{mk} \; sinh(ml) \right]$$

Thus,

$$\overline{T_w} \left[sinh(ml) \; + \frac{h}{mk} \; cosh(ml) \right] = \; -A_2 \left[cosh(ml) + \frac{h}{mk} \; sinh(ml) \right]$$

$$A_2 \; = \; \overline{T_w} \, \frac{sinh(ml) \; + \frac{h}{mk} \; cosh(ml)}{cosh(ml) + \frac{h}{mk} \; sinh(ml)}$$

Therefore, the boundary conditions led to the following solution to the eq.3.23.

$$\frac{T - T_\infty}{T_W - T_\infty} = \frac{\cosh m(l - x) + \frac{h}{mk} \sinh m(l - x)}{\cosh(ml) + \frac{h}{mk} \sinh(ml)} \qquad (3.25)$$

Thus, the heat loss by the fin, using the equation for temperature distribution can be easily found out by the following equation,

$$q = -kA \frac{dT}{dx} \Big|_{x=0} \qquad (3.26)$$

In a similar fashion we can solve the **case – III** also.

The boundary conditions are,

$$\bar{T} = \overline{T_w} \qquad \text{at} \qquad x=0\text{, and}$$

$$\frac{d\bar{T}}{dx} = 0 \qquad \text{at } x=l$$

Thus, on solving eq.3.23,

$$\frac{\bar{T}}{\overline{T_w}} = \frac{e^{-mx}}{1+e^{-2ml}} + \frac{e^{mx}}{1+e^{2ml}}$$

$$= \frac{\cosh[m(l-x)]}{\cosh(ml)}$$

Thus the heat loss by the fins, using the equation for temperature distribution,

$$q = -kA\frac{dT}{dx}\Big|_{x=0}$$

$$q = \sqrt{hPkA}(T_w - T_\infty)\tanh(ml) \qquad (3.27)$$

It is to be noted that the general expression for the temperature gradient (eq.3.23) was developed by assuming the temperature gradient in the x-direction. It is really applicable with very less error, if the fin is sufficiently thin. However, for the practical fins the error introduced by this assumption is less than 1% only. Moreover, the practical fin calculation accuracy is limited by the uncertainties in the value of *h*. It is because the *h* value of the surrounding fluid is hardly uniform over the entire surface of the fin.

3.3.2 Fin efficiency

It was seen that the temperature of the fin decreases with distance x from the base of the body. Therefore, the driving force (temperature difference) also decreases with the length and hence the heat flux from the fin also decreases. It may also be visualized that if the thermal conductivity of the fin material is extremely high. Its thermal resistance will be negligibly small and the temperature will remain almost constant (T_w) throughout fin. In this condition the maximum heat transfer can be achieved and of-course it is an ideal condition. It is therefore, interesting and useful to calculate the efficiency of the fins.

The fin efficiency may be define as,

$$\text{Fin efficiency } (\eta_f) =$$

$$\frac{\text{Actual heat transferred}}{\text{Heat which would be transferred if entire fin area were at base temperature of the body}}$$

Thus depending upon the condition, the actual heat transfer can be calculated as shown previously. As an example, for case – III (end of the fin is insulated), the rate of heat transfer was

$$q = \sqrt{hPkA}(T_w - T_\infty)\tanh(ml) \qquad (3.28)$$

The maximum heat would be transferred from the fin in an ideal condition in which the entire fin area was at T_w. In this ideal condition the heat transferred to the surrounding will be,

$$q' = hpl(T_w - T_\infty) \qquad (3.29)$$

Therefore, under such conditions, the efficiency of the fin will be;

$$\eta_f = \frac{q}{q'} = \frac{\sqrt{hPkA}\,(T_w - T_\infty)}{hPl\,(T_w - T_\infty)} \tanh ml$$

$$\eta_f = \frac{\tanh ml}{ml} \qquad (3.30)$$

$$\text{where,} \qquad m = \sqrt{\frac{hP}{kA}}$$

$$= \sqrt{\frac{h\,(2b+2t)}{kbt}}$$

If the fin is quite deep as compared to the thickness, the term 2*b* will be very large as compared to 2*t*, and

$$m = \sqrt{\frac{h2b}{kbt}} = \sqrt{\frac{2h}{kt}}$$

$$\Rightarrow ml = \sqrt{\frac{2h}{kt}}\; l \qquad (3.31)$$

The equation shows that the efficiency (from eq.3.30) of a fin which is insulated at the end can be easily calculated, which is the case-III discussed earlier. The efficiency for the other cases may also be evaluated in a similar fashion.

The above derivation is approximately same as of practical purposes, where the amount of heat loss from the exposed end is negligible.

It can be noted that the fin efficiency is maximum for the zero length of the fin ($l = 0$) or if there is no fin. Therefore, we should not expect to be able to maximize fin efficiency with respect to the fin length. However, the efficiency maximization should be done with respect to the quantity of the fin material keeping economic consideration in mind.

Sometimes the performance of the fin is compared on the basis of the rate of heat transfer with the fin and without the fin as shown,

$$Fin\ performance\ or\ effectiveness = \frac{\text{Rate of heat transfer with fin}}{\text{Rate of heat transfer without fin}}$$

Illustration 3.3.

A steel pipe having inner diameter as 78 mm and outer diameter as 89 mm has 10 external longitudinal rectangular fins of 1.5 mm thickness. Each of the fins extends 30 mm from the pipe. The thermal conductivity of the fin material is 50 W/m °C. The temperature of the pipe wall and the ambient are 160 °C, and 30 °C, respectively, whereas the surface heat transfer coefficient is 75 W/m² °C. What is the percentage increase in the rate of heat transfer after the fin arrangement on the plane tube?

Solution 3.3

As the fins are rectangular, the perimeter of the fin, $P = 2(b + t)$. The thickness (t) of the fin is quite small as compared to the width (b) of the fin. Thus, $P = 2b$ as well as we may assume that there is no heat transfer from the tip of the fin. Under such condition we can treat it as case-III, where there was no heat transfer to the atmosphere due to insulated fin tip. Using eq. 3.30,

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{75 \times 2\,b}{50\ \times t \times b}} = \quad 44.7$$

Thus,

$$\eta_f = \frac{\tanh ml}{ml} = \frac{\tanh(44.7 \times 0.03)}{44.7 \times 0.03} = 0.65$$

As the pipe length is not given, we will work-out considering the length of the pipe as 1 m and henceforth the breadth of the fins should also be considered as 1 m. We have to consider the area of the fins in order to consider the heat dissipation from the fins. However, we may neglect the fin area at the y-z plane and x-y plane (refer fig. 3.8) as compared to the area of x-z plane.

The area of all the fins = (number of fins) (2 faces) (1) (0.03) = 0.6 m^2

The maximum rate of heat transfer from the fins

$$h\,A\,\Delta T = 75 \times 0.6\ \times (160 - 30) = 5850\ W$$

Actual rate of heat transfer = $\eta_f \times 5850 = 3802.5\ W$

The total rate of heat transfer from the finned tube will be the sum of actual rate of heat transfer from the fins and the rate of heat transfer from the bare pipe, the pipe portion which is not covered by the fins. Therefore, the remaining area will be calculated as follows,

The remaining area = Total pipe area - base area covered by the 10 fins

Pipe are = $2\pi\ outer\ radius \times 1 = 2\pi \left(\frac{89}{2} 10^{-3}\right)(1) = 0.28\ m^2$

Attached area of 10 fins = (10) (1) (0.0015) = 0.015 m^2

The remaining area comes out to be (0.28 – 0.015) = 0.265 m^2

The corresponding heat transfer = (75) (0.265) (160-30) = 2583.75 W

The total heat transfer from the finned tube = 3802.5 + 2583.75 = 6386.25 W

Rate of heat transfer from the tube if it does not have any fins = (75) (0.28) (160-30) = 2730 W

The percentage increase in the heat transfer = $\frac{6386.25-2730}{2730} = 134\%$

3.4 Thermal insulation

We have seen how heat transfer is important in various situations. Previous discussion indicates that we are all the time interested in the flow of the heat from one point to another point. However, there are many systems; in fact it is a part of the system, in which we are interested to minimize the losses through heat transfer. For example, in a furnace we want to have high heat transfer inside the furnace; however we do not want any heat loss through the furnace wall. Thus to prevent the heat transfer from the furnace to the atmosphere a bad heat conductor or a very good heat insulator is required. In case of furnace the wall is prepared by multiple layers of refractory materials to minimize the heat losses. Therefore, wall insulation is required in various process equipment, reactors, pipelines etc. to minimize the heat loss from the system to the environment or heat gain from the environment to the system (like cryogenic systems). However, there are situations in which we want to maximize the losses for example, insulation to electric wires.

The petroleum conservation research association (PCRA) provides a good database on the properties and applications of industrial thermal insulations

(*http://www.pcra.org/English/education/literature.htm*). The table 3.2 shows some common insulations used in chemical process industries for various process equipment and pipelines.

Table-3.2: Thermal properties of a few of the insulations being used in the chemical process industries

Material	Temperature (°C)	Approximate thermal conductivity(W/(m°C))	Density (kg/m³)

Asbestos	-200 to 0	0.074	469
Glass wool	−7 to 38 38 to 93	0.031 0.041	64 64
Fibre insulating board	21	0.049	237
Hard rubber	0	0.151	2000
Polyurethane foam	−170 to 110	0.018	32

An interesting application of the heat loss from a surface of some practical significance is found in the case of insulation of cylindrical surfaces like small pipes or electrical wires. In many a cases we desire to examine the variation in heat loss from the pipe with the change in insulation thickness, assuming that the length of the pipe is fixed. As insulation is added to the pipe, the outer exposed surface temperature will decrease, but at the same time the surface area available to the convective heat dissipation will increase. Therefore, it would be interesting to study these opposing effects.

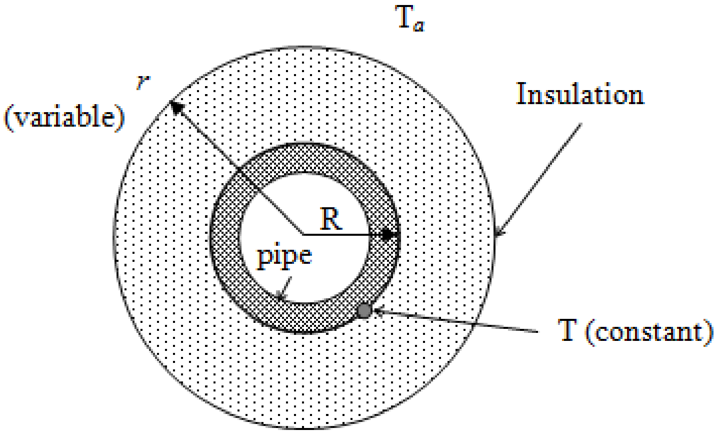


Fig. 3.10: Heat dissipation from an insulated pipe

Let us consider a thick insulation layer which is installed around a cylindrical pipe as shown in fig. 3.10 (equivalent electrical circuit is shownin figure 3.11). Let the pipe radius be R and the insulation radius is r. This (r-R) will represent the thickness of the insulation. If the fluid carried by the pipe is at a temperature T and the ambient temperature is T_a. The insulation of the pipe will alter pipe surface temperature T in the radial direction. That is the temperature of the inner surface of the pipe and the outer surface (below insulation) of the pipe will be different. However, if the thermal resistance offered by the pipe is negligible, it can be considered that the temperature (T) is same across the pipe wall thickness and it is a common insulation case (please refer previous discussion). It can also be assumed that the heat transfer coefficient inside the pipe is very high as compared to the heat transfer coefficient at the outside of the insulated pipe. Therefore, only two major resistances in series will be available (insulation layer and gas film of the ambient).

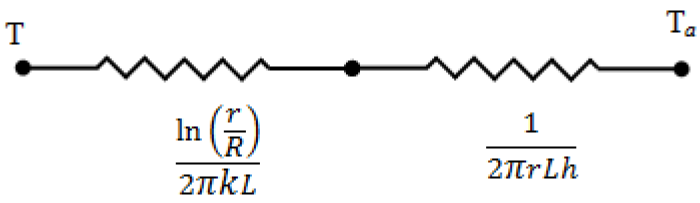


Fig.3.11: Resistance offered by the insulation and ambient gas film

Therefore,

$$\dot{q} = \frac{2\pi L(T - T_a)}{\frac{1}{k}\ln r/R + \frac{1}{rh}}$$

where, k is the thermal conductivity of the material.

On differentiating above equation with respect to *r* will show that the heat dissipation $\dot{q}/_L$ reaches a maximum,

$$\frac{\mathrm{d}\dot{\mathrm{q}}}{\mathrm{dr}}\Big|_{r_c} = 2\pi L(T - T_a)[\frac{-\left(\frac{1}{k}\right)\left(\frac{1}{\frac{r}{R}}\right)\left(\frac{1}{R}\right)-\left(\frac{1}{h}\right)(-1)(r^{-2})}{(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh})^2}]\Big|_{r_c}$$

$$= 2\pi L(T - T_a)\left[\frac{\frac{1}{hr^2}-\frac{1}{kr}}{(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh})^2}\right]\Big|_{r_c} = 0$$

$$\frac{\mathrm{d}^2\dot{\mathrm{q}}}{\mathrm{dr}^2}\Big|_{r_c} = 2\pi L(T - T_a)[\frac{\left\{\left(\frac{1}{h}\right)(-2)\left(\frac{1}{r^3}\right)-\left(\frac{1}{k}\right)(-1)(r^2)\right\}^{\square}-2\left(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh}\right)\{\frac{1}{k}\left(\frac{1}{\frac{r}{R}}\right)\left(\frac{1}{R}\right)+\left(\frac{1}{h}\right)(-1)(r^{-2})\}}{(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh})^4}]\Big|_{r_c}$$

$$= \frac{2\pi L(T-T_a)\{-\frac{h^2}{k^3}\}}{(\frac{1}{k}ln\frac{r}{R}+\frac{1}{rh})^4}\Big|_{r_c} < 0$$

So it is maxima, where the insulation radius is equal to

$$r = r_c = \frac{k}{h}$$

where, r_c denotes the critical radius of the insulation. The heat dissipation is maximum at r_c which is the result of the previously mentioned opposing effects.

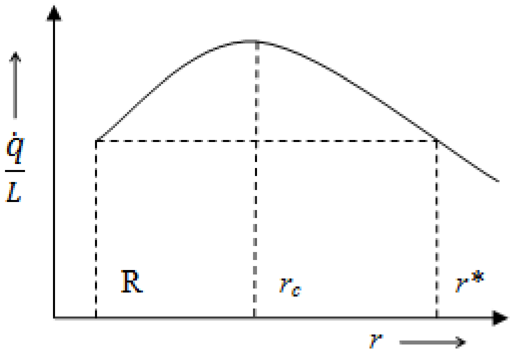


Fig. 3.12: The critical insulation thickness of the pipe insulator

Therefore, the heat dissipation from a pipe increases by the addition of the insulation. However, above r_c the heat dissipation reduces. The same is shown in fig. 3.12.

The careful analysis of the r_c reveals that it is a fixed quantity determined by the thermal properties of the insulator. If $R < r_c$, then the initial addition of insulation will increase the heat loss until $r = r_c$ and after which it begins to decrease. The same heat dissipation which was at bare pipe radius is again attained at r^*. The critical insulation thickness may not always exist for an insulated pipe, if the values of k and h are such that the ratio k/h turns out to be less than R.

It is clear from the above discussion that the insulation above rc reduces the heat dissipation from the cylindrical surface. However, if we keep on increasing the insulation the cost of insulation also increases. Thus again there are two opposing factors that must be considered to obtain the optimum thickness. It should be calculated that what is the pay-back period, that is in how many years the cost of insulation is recovered by the cost of energy saving.

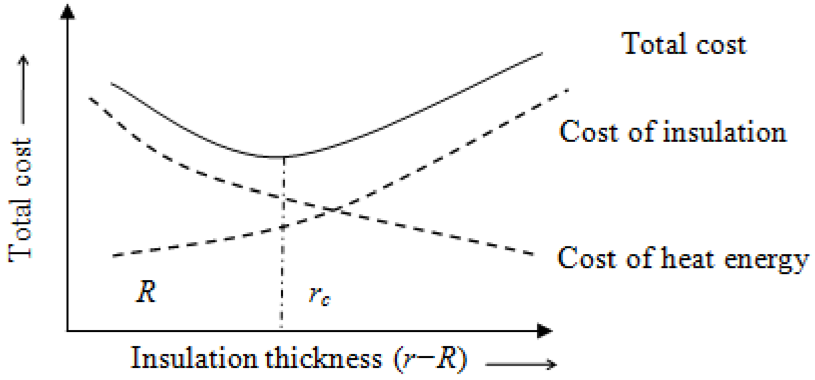


Fig. 3.13: Optimum insulation thickness

The optimum insulation thickness (fig. 3.13) can be determined at which the sum of the insulation cost and the cost of the heat loss is minimum.

4.1 Principle of convection

Till now we have understood and solved the problem where heat transfer coefficient was known.

Now, we would emphasize on finding the convective heat transfer coefficient *h*. Finding out the heat transfer coefficient is not so easy in a given situation as it is a very complex term and depends in many physical properties of the fluid like, thermal conductivity, viscosity, density etc. Moreover, the flow field has a great impact on the convective heat transfer. The convective heat transfer requires a good knowledge of fluid dynamics, and we expect that the reader has adequate knowledge of that. We have seen that it is easy to form a differential heat balance equations. However, for practical cases, it becomes tedious or impossible to solve the differential equations. Therefore, for practical situations, the heat transfer coefficient is calculated using the empirical correlations, which are developed over the years. Though these correlations have a good amount of error in most of the situations but these are indispensible for the estimation of heat transfer coefficients.

The convective heat transfer may be

- forced convection: in which the motion in the fluid medium is generated by the application of an external force, e.g. by a pump, blower, agitator etc.
- natural convection: in which the motion in the fluid is generated due to a result of density difference caused by the temperature difference.

Moreover, in many applications, heat is transferred between fluid streams without any phase change in the fluid. However, in some situations, the heat transfer is accompanied by phase change. In this chapter we will consider the forced convection. Before we analyse the forced convection we need to know its physical mechanism.

4.2 Forced convection mechanism: flow over a flat horizontal plate

Consider the flow of a fluid over an immersed flat horizontal plate of very large width, as shown in fig. 4.1. The fig. shows that the flow is fully developed with no entrance losses. The bulk flow or free stream velocity is represented by v. Velocity of the fluid at the surface of the flat plate is zero at no-slip condition. The velocity of the fluid in the y-direction from the plate increases from zero to free stream velocity. The region above the plate surface within which this change of velocity from zero to the free stream value occurs is called the boundary layer. The thickness of this region is called the boundary layer thickness and is generally denoted by δ. It can be seen in the fig.4.1 that the boundary layer thickness increased with the distance x from the body edge. The boundary layer thickness, δ, usually is defined as the distance from the surface to the point where the velocity is within 1% of the free shear velocity. It should be noted that the y-axis of the fig. 4.1 is exaggerated greatly to have a clear picture.

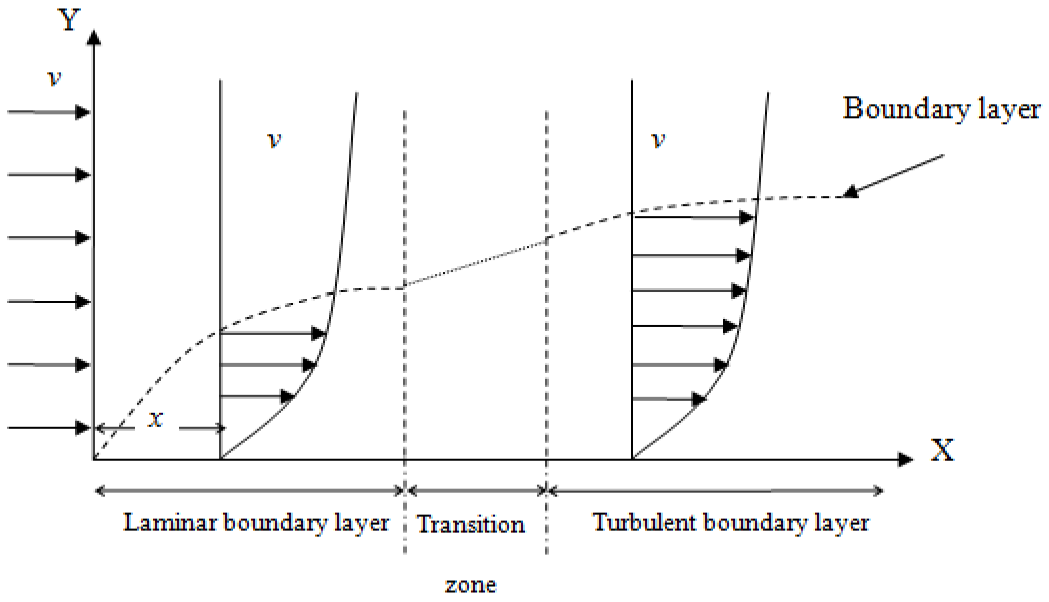


Fig.4.1. Boundary layer flow past a flat plate

The velocity profile merges smoothly and asymptotically in the free shear and the boundary layer thickness is difficult to measure. However, the theoretical laminar boundary layer thickness can be calculated using the eq.4.1.

$$\text{For } Re<5\times10^5, \qquad \delta = \frac{5.0x}{\sqrt{Re_x}} \qquad (4.1)$$

where, Re_x is the local Reynold number at a distance x. The derivative of δ can be found in any stated boundary.

The turbulent boundary layer thickness can be calculated using eq.4.2.

$$\text{For } 5\times10^5< Re < 10^7, \ \delta = \frac{0.382x}{{Re_x}^{1/5}} \qquad (4.2)$$

Figure 4.2 shows the velocity boundary layer for the turbulent and laminar zone.

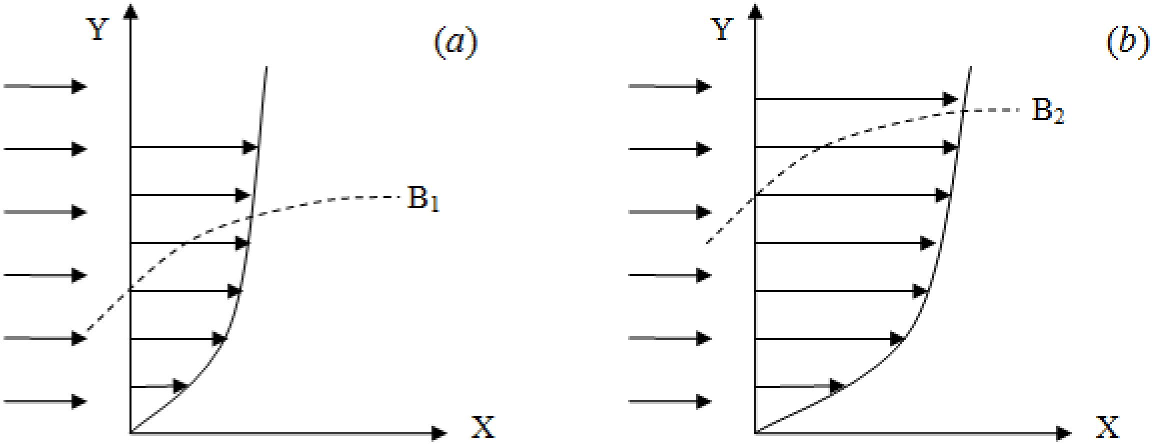


Fig.4.2: Boundary layer flow past a flat surface (a) turbulent, and (b) laminar

In continuation to the above discussion and understanding, if the solid surface is maintained at a temperature, T_s, which is different from the fluid temperature, T_f, measured at a point far away from the surface, a variation of the temperature of the fluid is observed, which is similar to the velocity variation described. That is, the fluid temperature varies from T_s at the wall to T_f far away from the wall, with most of the variation occurring close to the surface.

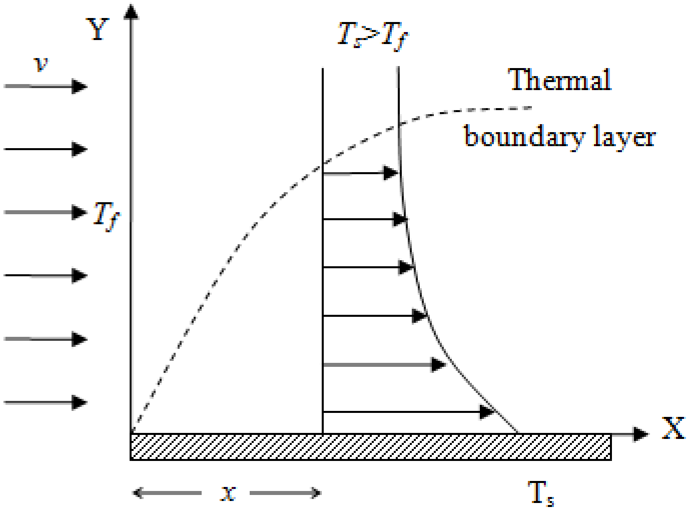


Fig.4.3: Thermal boundary layer flow past a flat surface

If $T_s > T_f$, the fluid temperature approaches asymptotically and the temperature profile at a distance x is shown in fig.4.3. However, a thermal boundary may be defined (similar to velocity boundary) as the distance from the surface to the point where the temperature is within 1% of the free stream fluid temperature (T_f). Outside the thermal boundary layer the fluid is assumed to be a heat sink at a uniform temperature of T_f. The thermal boundary layer is generally not coincident with the velocity boundary layer, although it is certainly dependant on it. That is, the velocity, boundary layer thickness, the variation of velocity, whether the flow is laminar or turbulent etc are all the factors which determine the temperature variation in the thermal boundary layer. The thermal boundary layer and velocity boundary layer are related by the Prandtl number, $Pr = \frac{\nu}{\alpha}$; where $\nu\ (= \frac{\mu}{\rho})$ is called the momentum diffusivity and $\alpha(= \frac{\kappa}{\rho C_p})$ is called the thermal diffusivity; is less than unity, the momentum boundary layer (or velocity boundary layer) remains within the thermal boundary layer. If $P_r > 1$, the boundary layers will be reversed as shown in the fig.4.4. The thermal boundary layer and velocity boundary layer coincides at $P_r = 1$.

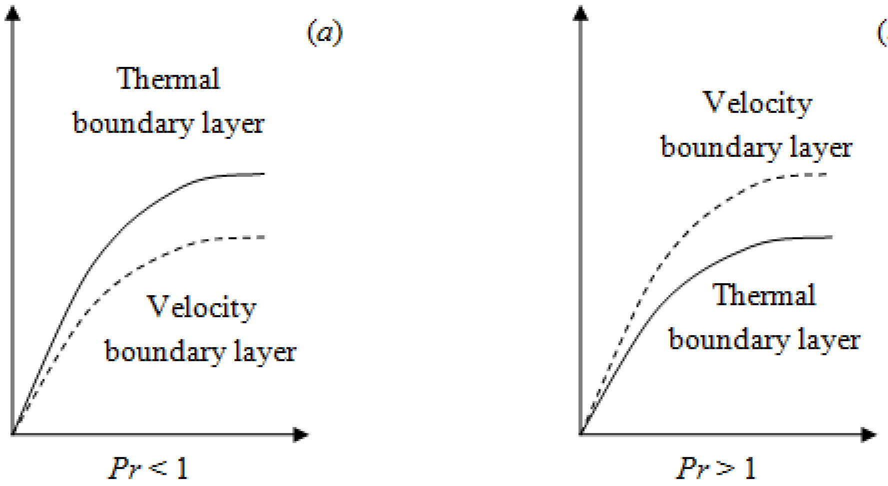


Fig.4.4: The relation of two boundary layers at different Pr numbers

The above boundary layer theory will be helpful to understand the heat transfer in the process. Through the boundary layers heat transfer is covered in a separate chapter, but the detailed derivation and development of all the relationships having engineering importance for the prediction of forced convection heat transfer coefficient is beyond the scope of the course. The reader may consult any standard fluid mechanics and heat transfer books for detailed knowledge.

The purpose of this chapter is to present a collection of the most useful of the existing relations for the most frequently encountered cases of forced convection. Some of these relations will be having theoretical bases, and some will be empirical dimensionless correlations of experimental data. In some situations, more than one relation will be given.

The discussion on heat transfer correlations consists of many dimensionless groups. Therefore, before we discuss the importance of heat transfer coefficients, it is important to understand the physical significance of these dimensionless groups, which are frequently used in forced convection heat transfer. The table 4.1 shows some of the dimensionless numbers used in the forced convection heat transfer.

Table-4.1: Some important dimensionless numbers used in forced heat transfer convection

Dimensionless no	**Physical significance**	**Expression**
Nusselt	$\frac{\text{Wall temperature gradient}}{\text{Temperature gradient across the fluid in the pipe}}$	$Nu = \frac{hl}{k}$
Reynolds	$\frac{\text{Inertial force}}{\text{Viscous force}}$	$\frac{lv\rho}{\mu}$
Prandtl	$\frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$	$Pr = \frac{c_p\mu}{k} = \frac{\nu}{\alpha}$ where, $\nu = \frac{\mu}{\rho}, \alpha = \frac{k}{\rho C_p}$
Stanton	$\frac{\text{Heat flux by convection}}{\text{Heat flux by bulk flow}}$	$St = \frac{h}{u\rho c_p} = \frac{Nu}{Re\,Pr}$
Peclet	$\frac{\text{Heat flux by bulk flow}}{\text{Heat flux by conduction}}$	$Pe = \frac{lu\rho}{\mu} \cdot \frac{c_p\mu}{k}$

Graetz	Same as Peclet number, however, used in connection with analysis of heat transfer in laminar flow in pipes of length L at distance *d*	$Gz = Pe \cdot \frac{d}{L}$

Notations:

h = heat transfer coefficient

l = characteristic length

k = thermal conductivity

ν = Momentum diffusivity

u = velocity

ρ = density

μ = viscosity

c_p = specific heat at constant pressure

L = length of the pipe (in Graetz number)

d = diameter of the pipe (in Graetz number)

4.3 Flow through a pipe or tube

4.3.1 Turbulent flow

A classical expression for calculating heat transfer in fully developed turbulent flow in smooth tubes/pipes of diameter (d) and length (L) is given by Dittus and Boelter

$$Nu = 0.023Re^{0.8}Pr^{n} \tag{4.3}$$

where,

n = 0.4, for heating of the fluid

n = 0.3, for cooling of the fluid

The properties in this equation are evaluated at the average fluid bulk temperature. Therefore, the temperature difference between bulk fluid and the wall should not be significantly high.

Application of eq. 4.3 lies in the following limits

$$0.7 \leq Pr \leq 160; \qquad \frac{d}{L} < 0.1; \qquad Re \geq 10{,}000$$

Gnielinski suggested that better results for turbulent flow in smooth pipe may be obtained from the following relations

$$Nu = 0.0214(Re^{0.8} - 100)Pr^{0.4} \tag{4.4}$$

Application conditions for eq. 4.4;

$$0.5 \leq Pr \leq 1.5; \qquad 10^4 < Re < 5 \times 10^6$$

$$Nu = 0.012(Re^{0.87} - 280)Pr^{0.4} \tag{4.5}$$

Application condition for eq. 4.5,

$$1.5 < Pr < 500; \qquad 3000 < Re < 10^6$$

When the temperature difference between bulk fluid and wall is very high, the viscosity of the fluid and thus the fluid properties changes substantially. Therefore, the viscosity correction must be accounted using Sieder – Tate equation given below

$$\mathrm{Nu} = 0.027Re^{0.8}Pr^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{4.6}$$

Application conditions for eq. 4.6,

$$0.7 \leq Pr \leq 16{,}700; \quad Re \geq 10{,}000; \; \frac{d}{L} \leq 0.1$$

However, the fluid properties have to be evaluated at the mean bulk temperature of the fluid except μ_w which should be evaluated at the wall temperature. The earlier relations were applicable for fully developed flow when entrance length was negligible. Nusselt recommended the following relation for the entrance region when the flow is not fully developed.

$$Nu = 0.036Re^{0.8}Pr^{0.33}\left(\frac{d}{L}\right)^{0.055} \tag{4.7}$$

where, L is the tube length and d is the tube diameter.

The fluid properties in eq. 4.7 should be evaluated at mean bulk temperature of the fluid.

Applicability conditions, $10 < \frac{L}{d} < 400$.

As different temperature terms will appear in the course therefore to understand these terms see the following details.

Bulk temperature/mixing cup temperature: Average temperature in a cross-section.

Average bulk temperature: Arithmetic average temperature of inlet and outlet bulk temperatures.

Wall temperature: Temperature of the wall.

Film temperature: Arithmetic average temperature of the wall and free stream temperature.

Free stream temperature: Temperature free from the effect of wall.

Log mean temperature difference: It will be discussed in due course of time

Illustration 4.1

Pressurized air is to be heated by flowing into a pipe of 2.54 cm diameter. The air at 200°C and 2 atm pressure enters in the pipe at 10 m/s. The temperature of the entire pipe is maintained at 220°C. Evaluate the heat transfer coefficient for a unit length of a tube considering the constant heat flux conditions are maintained at the pipe wall. What will be the bulk temperature of the air at the end of 3 m length of the tube?

The following data for the entering air (at 200°C) has been given,

Pr number	0.681
Viscosity	2.57×10^{-5} kg/m s
Thermal conductivity	0.015 W/m °C
Density	1.493 kg/m^3
c_p	1.025 kJ/kg °C

Solution 4.1

Reynolds number can be calculated from the above data,

$$Re = \frac{d\ v\ \rho}{\mu} = 14756$$

The value of Reynolds number shows that the flow is in turbulent zone. Thus the Dittus-Boelter equation (eq.4.3) should be used,

$$Nu = 0.023Re^{0.8}Pr^{n}$$

$n = 0.4$ as the air is being heated,

$$\frac{h\ d}{k} = 0.023\ (14756)^{0.8}\ (0.681)^{0.4} = 42.67$$

Thus *h* can be calculated for the known values of *k*, and *d*, which comes out to be

$$h = 25.20\ W/m^2\ °C$$

Energy balance is required to evaluate the increase in bulk temperature in a 3 m length of the tube,

$$h(\pi d\ dL)(220 - T) = \dot{m}c_p dT$$

$$\int_{200}^{T} \frac{dT}{220-T} = \frac{h\pi d}{\dot{m}c_p}\int_{0}^{L} dL$$

$$= \frac{h\ \pi\ d\ L}{\rho v\frac{\pi d^2}{4}c_p}$$

$$= \frac{4hL}{\rho c_p v d}$$

$$= \frac{(4)(25.2)(3)}{(1.493)(1025)(10)(0.0254)}$$

$$= 0.778$$

$$[ln\ (220 - T)]_{T}^{200} = 0.778$$

$$ln 20 - \ln(220 - T) = 0.778$$

$$T = 210.81\ °C$$

Therefore the temperature of the air leaving the pipe will be at 210.81°C.

4.3.2 *Laminar flow*

Hausen presents the following empirical relations for fully developed laminar flow in tubes at constant wall temperature.

$$Nu = 3.66 + \frac{0.0668\left(\frac{d}{L}\right)Re\,Pr}{1+0.04\left[\left(\frac{d}{L}\right)Re\,Pr\right]^{0.67}} \qquad (4.8)$$

The heat transfer coefficient calculated from eq. 4.8 is the average value over the entire length (including entrance length) of tube $Nu = \frac{\bar{h}\,d}{k}$. Sieder and Tate suggested a simple relation for laminar heat transfer in tubes.

$$Nu = 1.86\left(\frac{Re\,Pr}{\frac{L}{d}}\right)^{0.33}\left(\frac{\mu}{\mu_W}\right)^{0.14} \qquad (4.9)$$

The condition for applicability of eq. 4.9:

$$0.48 < Pr < 16{,}700; \quad 0.0044 < \frac{\mu}{\mu_W} < 9.75; \quad Gz > 10$$

where, μ is the viscosity of the fluid at the bulk temperature and μ_w is that at the wall temperature T_w. The other fluid properties are at mean bulk temperature of the fluid. Here also the heat transfer coefficient calculated from eq. 4.9 is the average value over the entire length (including entrance length) of tube $Nu = \frac{\bar{h}\,d}{k}$.

The empirical relations shown in eq. 4.2-4.9 are for smooth pipe. However, it case of rough pipes, it is sometimes appropriate that the Reynolds analogy between fluid friction and heat transfer be used to effect a solution under these conditions and can be expressed in terms of Stanton number.

In order to account the variation of the thermal properties of different fluids the following equations may be used (i.e. Stanton number multiplied by $\mathrm{Pr}^{2/3}$),

$$StPr^{2/3} = \frac{f}{8} \qquad (4.10)$$

Where, *f* is, fanning friction factor defined by,

$$\Delta P = \frac{fL}{d}\rho\frac{\bar{v}^2}{2} \qquad (4.11)$$

where, $\bar{v}$ is the mean free velocity. The friction factor can be evaluated from Moody's chart.

4.3.3 Flow through non-circular ducts

The same co-relations as discussed in section 4.4.1 can be used for the non-circular ducts. However, the diameter of the tube has to be replaced by the hydraulic diameter or equivalent diameter for the non-circular ducts. The hydraulic diameter is defined as

$$D_h = 4 \times \frac{\text{Cross-sectional area of flow}}{\text{Wetted perimeter}} = 4 \times r_h \qquad (4.12)$$

Where r_h is hydraulic radius.

4.3.4. Flow over a flat plate

Heat transfer in flow over a plate occurs through the boundary layer formed on the plane. Therefore at any location the heat transfer coefficient will depend on the local Reynolds and Prandtl number. For local heat transfer coefficient in laminar boundary layer flow, the following correlation can be used to find the local Nusselt number. It depends upon the distance from the leading edge (*x*) of the plate.

$$Nu_x = 0.332Re_x^{1/2}Pr^{1/3}, \qquad (0.5 \le Pr \le 50) \qquad (4.13)$$

where, $Nu_x = \frac{xh}{k}$ and $Re_x = \frac{xv\rho}{\mu}$ and are the local Nusselt and Reynold numbers, respectively. An average value of the heat transfer coefficient over a distance *l* may be obtained by,

$$h_{av} = \frac{1}{l}\int_0^l h_x dx$$

$$\Rightarrow Nu_{av} = 0.664 Re_l^{1/2} Pr^{1/3} \qquad (4.14)$$

A simple correlation exists for the heat transfer in turbulent boundary layer flow,

$$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3} \qquad (4.15)$$

Applicability of eq. 5.15 is

$$5 \times 10^5 < Re_x < 10^7 \; ; \qquad 0.6 < Pr < 60$$

4.3.5 Flow across cylinders and spheres

4.3.5.1 Flow across a cylinder

The heat transfer coefficient can be found out by the correlations given by many researchers

i. *Ford correlation*

$$Nu = (0.35 + 0.56) Re^{0.52} Pr^{0.3} \qquad (4.16)$$

Applicability of eq. 4.16: $0.1 < Re < 10^5$

ii. *Eckert and Drake correlation*

$$Nu = (0.43 + 0.5 Re^{0.5}) \left(\frac{Pr}{Pr_w}\right)^{0.25} \qquad (4.17)$$

and, $$Nu = 0.25 Re^{0.6} \left(\frac{P_r}{Pr_w}\right)^{0.25} \qquad (4.18)$$

The eq. 4.17 and 4.18 are applicable for $1 < Re < 10^3$, and $10^3 < Re < 2 \times 10^3$, respectively. The Pr_w in the above equations are the Prandtl number at the wall temperature.

iii. *Churchill and Burnstein correlation*

$$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{1/4}} \left[1 + \left(\frac{Re}{2.82 \times 10^5}\right)^{5/8}\right]^{4/5} \qquad (4.19)$$

Applicability of eq. 4.19: $10^2 < Re < 10^7$, and $Re\,Pr > 0.2$.

However, the following equation (eq. 4.20) is more accurate for the condition where 20,000 < *Re* < 4,00,000 and $Re\,Pr > 0.2$.

$$Nu = 0.3 + \frac{0.62Re^{1/2}Pr^{1/3}}{\left[1+\left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{1/4}}\left[1 + \left(\frac{Re}{2.82\times10^5}\right)^{1/2}\right] \qquad (4.20)$$

iv. *Whitakar correlation*

$$Nu = \left[0.4Re^{\frac{1}{2}} + 0.06Re^{2/3}\right]Pr^{0.4} \times \left(\frac{\mu}{\mu_w}\right)^{0.25} \qquad (4.21)$$

Applicability of eq. 4.21: $40 < Re < 10^5$, and $0.65 < Pr < 300; 0.25 < \frac{\mu}{\mu_w} < 5.2$.

4.3.5.2. Flow across a sphere

i. *Kramers correlation for flow of liquids past a sphere*

$$NuPr^{-1/3} = 0.97 + 0.68Re^{0.5} \qquad (4.23)$$

Applicability of the eq. 4.23 is $1 < Re < 2000$.

ii. *Whitaker correlation*

$$Nu = 2 + \left[0.4Re^{1/2} + 0.06Re^{2/3}\right]Pe^{0.4}\left(\frac{\mu}{\mu_w}\right)^{0.25} \qquad (4.24)$$

The above correlation is applicable to both gases and liquids.

4.3.5.3 Flow over a bank of tubes

Flow over bank of tubes is one of the very important phenomena in chemical process industries. Heat exchanger, air conditioning for cooling and heating etc. involve a bank or bundle of tube over which a fluid flows. The two most common geometric arrangements of a tube bank are shown in fig. 4.5.

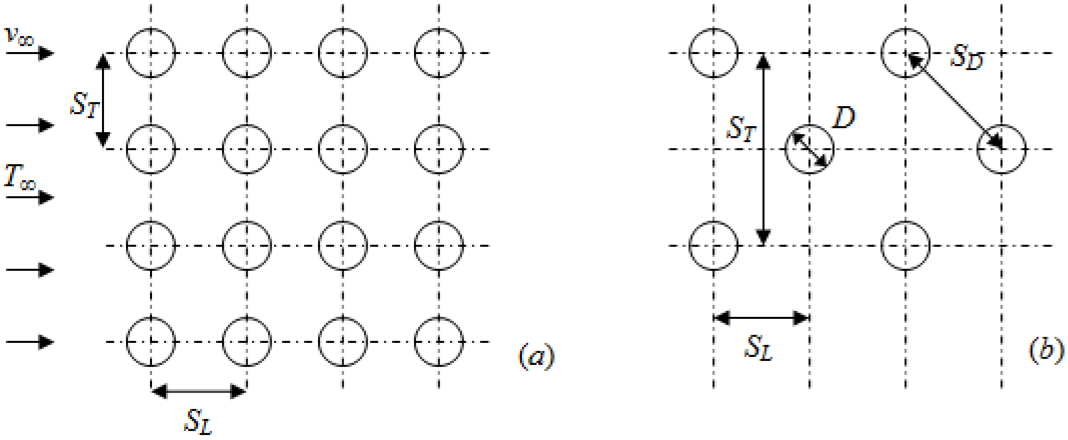


Fig.4.5: Tube banks: (*a*) aligned; (*b*) staggered

In any of the arrangements, D is the diameter of tube, S_L is the longitudinal spacing, and S_T is the transverse tube spacing.

The flow over a tube is quite different than the flow over bank of tubes. In case of bank of tube, the flow is influenced by the effects such as the "shading" of one tube by another etc. Moreover, the heat transfer for any particular tube thus not only determined by the incident fluid conditions, -$v_\infty$and T_∞, but also by D, S_L, and S_T and the tube positions in the bank. It is now clear that the heat transfer coefficient for the first row of tubes is much like that for a single cylinder in cross flow. However, the heat transfer coefficient for the tubes in the inner rows is generally larger because of the wake generation by the previous tubes.

For the heat transfer correlations, in tube banks, the Reynolds number is defined by

$$Re = \frac{v_m D}{\nu}$$

where v_m is the maximum fluid velocity occurring at the minimum vacant area of the tube bank. For the aligned tube arrangement,

$$v_m = \frac{v_\infty S_T}{S_T - D} \qquad (4.25)$$

For the staggered arrangements, where $S_D = \sqrt{S_L^2 + \left(\frac{S_T}{2}\right)^2}$,

a. if $2(S_D - D) > (S_T - D)$, then eq. 4.25 can be used.

b. if $2(S_D - D) < (S_T - D)$, then

$$v_m = \frac{v_\infty (S_T/2)}{(S_D - D)} \qquad (4.26)$$

In case of bank of tubes, generally we are interested for a single tube but interested to know the average heat transfer coefficient for the entire bank of tubes. Zukauskas has summarized his extensive for the heat transfer coefficients for fluid past a bank of tubes,

$$Nu = \frac{h_{av}D}{k} = CRe^{m}Pr^{0.36}\left(\frac{Pr}{Pr_w}\right)^{0.25} \tag{4.27}$$

The applicability of eq. 4.27: $0.7 < Pr < 500 \text{ and } 10 < Re < 10^6$, and number of tubes are atleast 20.
The constants *C* and *m* of co-relation 5.26 can be found out from any standard book on heat transfer. It may be noted that the above relation is for the inner rows of bank, or for banks of many rows.

4.4 Momentum and heat transfer analogies

Consider a fluid flows in a circular pipe in a laminar low (fig.6.6). The wall of the pipe is maintained at T_w temperature, which is higher than the flowing fluid temperature. The fluid being in relatively lower temperature than the wall temperature will get heated as it flows through the pipe. Moreover, the radial transport of the momentum in the pipe occurs as per the Newton's law of viscosity. For a circular pipe momentum transport and heat transport may be written in a similar way as shown in the eq. 4.28,

Momentum flux = momentum diffusivity × gradient of concentration of momentum

$$\tau = -\nu \frac{d}{dr}(\rho u_z) \tag*{4.28(a)}$$

It may be noted that the fluid velocity (u_z) is a function of radius of the pipe.

Heat flux= thermal diffusivity × gradient of concentration of heat energy

$$q = \alpha \frac{d}{dy}(\rho c_p T) \tag*{4.28(b)}$$

where $\alpha = \text{Thermal diffusion} = \frac{k}{\rho c_p}$

Now, the question comes, why are we discussion about the similarities? The answer is straight forward that it is comparatively easy to experimentally/theoretically evaluate the momentum transport under various conditions. However, the heat transport is not so easy to find out. Therefore, we will learn different analogies to find the heat transport relations.

Equation 4.28 is for the laminar flow but if the flow is turbulent, eddies are generated. Eddy is a lump/chunk of fluid elements that move together. Thus it may be assumed that the eddies are the molecules of the fluid and are responsible for the transport of momentum and heat energy in the turbulent flow. Therefore, in turbulent situation the momentum and heat transport is not only by the molecular diffusion but also by the eddy diffusivities.

Thus, turbulent transport of momentum and turbulent transport of heat may be represented by eq. 4.29a and 4.29b, respectively.

$$\tau = -(\nu + \epsilon_m)\frac{d}{dr}(\rho u_z) \qquad (4.29a)$$

$$q = (\alpha + \epsilon_h)\frac{d}{dr}(\rho c_p T) \qquad (4.29b)$$

The terms ϵ_m, and ϵ_h represent the eddy diffusivities for momentum and heat, respectively. At the wall of the pipe, the momentum equation (eq. 4.29a) becomes,

$$\tau_w = -(\nu + \epsilon_m)\frac{d}{dr}(\rho u_z)\Big|_{r=R} \qquad (4.30)$$

$$\tau_w = \frac{1}{2} f \rho \overline{u_z} \qquad (4.31)$$

Where f is the fanning friction factor (ratio of shear force to inertial force) and $\overline{u_z}$ is the average fluid velocity.

Equation eq.4.30 can be rearranged as,

$$-\frac{d\hat{u}_z}{dr}\Big|_{r=R} = \frac{\tau_w}{(\nu + \epsilon_m)\rho\bar{u}_z}$$

where, $\hat{u}_z$ = dimensionless velocity $\frac{u_z(r)}{\bar{u}_z}$

Using eq.4.31 in the above equation,

$$-\frac{d\hat{u}_z}{dr}\Big|_{r=R} = \frac{f\bar{u}_z}{2(\nu+\epsilon_m)} \qquad (4.32)$$

The eq.4.32 is the dimensionless velocity gradient at the wall using momentum transport. We may get the similar relation using heat transport as shown below.

Wall heat flux can be written as,

$$(\alpha + \epsilon_{\mathrm{H}})\frac{d}{dr}(\rho c_p T)\Big|_{r=R} = q'_w = \mathrm{h}\,(\mathrm{T_w} - \mathrm{T_{av}})$$

Where T_{av} is the wall temperature and the T_{av} is the average temperature of the fluid. Thus, the dimensionless temperature gradient at the wall using heat transfer will be,

$$\frac{d\hat{T}}{dr}\Big|_{\mathrm{r=R}} = \frac{h}{\rho c_p(\alpha+\varepsilon_{\mathrm{H}})} \qquad (4.33)$$

Where the heat transfer coefficient is represented by *h* and dimensionless temperature is represented by $\hat{T} = \frac{T}{T_w - T_{av}}$.

Based on the above discussion many researchers have given their analogies. These analogies are represented in the subsequent section.

4.4.1 Reynolds analogy

Reynolds has taken the following assumptions to find the analogy between heat and momentum transport.

1. Gradients of the dimensionless parameters at the wall are equal.
2. The diffusivity terms are equal. That is

Thus if we use the above assumptions along with the eq.4.32 and 4.33,

$$(\nu + \epsilon_{\mathrm{M}}) = (\alpha + \epsilon_{\mathrm{H}})$$

Thus if we use the above assumptions along with the eq.4.32 and 4.33,

$$\frac{h}{\rho c_p \bar{v}_z} = \frac{f}{2}$$

On simplifying

$$\frac{\frac{hd}{k}}{\left(\frac{\mu C_P}{k}\right)\left(\frac{d\rho\bar{v}_z}{\mu}\right)} = \frac{f}{2}$$

$$\frac{Nu_d}{Pr\ Re_d} = \frac{f}{2}$$

$$St_d = \frac{f}{2} \qquad (4.34)$$

Equation 4.34 is known as Reynolds's analogy.

The above relation may also be written in terms of the Darcy's friction factor (fD) instead of fanning friction factor ($f_D = 4f$)

Where Stanton number (St) is defined as,

$$St_d = \frac{h}{\rho C_p \bar{v}_z} = \frac{h\,\Delta T}{\rho C_p \bar{v}_z\,\Delta T} = \frac{heat\ transfer\ by\ convection}{heat\ transfer\ by\ bulk\ flow}$$

The advantage of the analogy lies in that the *h* may not be available for certain geometries/situations however, for which *f* value may be available as it is easier to perform momentum transport experiments and then to calculate the *f*. Thus by using the eq.4.34 the *h* may be found out without involving into the exhaustive and difficult heat transfer experiments.

4.4.2 The Chilton-Colburn analogy

The Reynolds analogy does not always give satisfactory results. Thus, Chilton and Colburn experimentally modified the Reynolds' analogy. The empirically modified Reynolds' analogy is known as Chilton-Colburn analogy and is given by eq.4.35,

$$St = \frac{\frac{f}{2}}{Pr^{\frac{2}{3}}} \qquad (4.35a)$$

or

$$(St)(Pr^{\frac{2}{3}}) = \frac{f}{2} \qquad (4.35b)$$

It can be noted that for unit Prandtl number the Chilton-Colburn analogy becomes Reynolds analogy.

4.4.3 The Pradntl analogy

In the turbulent core the transport is mainly by eddies and near the wall, that is laminar sub-layer, the transport is by molecular diffusion. Therefore, Prandtl modified the above two analogies using universal velocity profile while driving the analogy (eq. 4.36).

$$St = \frac{\frac{f}{2}}{1+5\sqrt{\frac{f}{2}}(Pr-1)} \qquad (4.36)$$

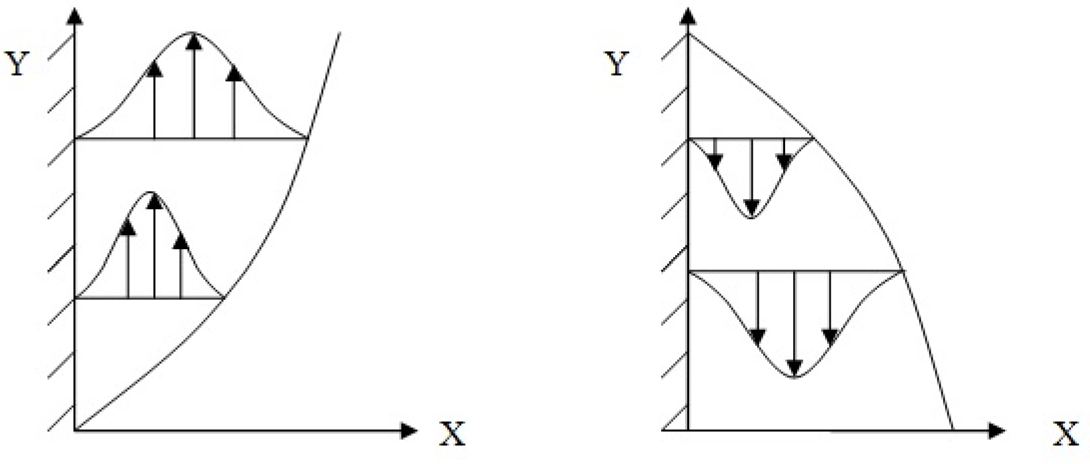
4.4.4 The Van Karman analogy

Though Prandtl considered the laminar and turbulent laminar sublayers but did not consider the buffer zone. Thus, Van Karman included the buffer zone into the Prandtl analogy to further improve the analogy.

$$St = \frac{\frac{f}{2}}{1+5\sqrt{\frac{f}{2}}\left[(Pr-1)+\ln\left\{1+\frac{5}{6}(Pr-1)\right\}\right]} \qquad (4.37)$$

5.1 Introduction

In the previous chapter, we have discussed about the forced convective heat transfer when the fluid motion relative to the solid surface was caused by an external input of work by means of pump, fan, blower, stirrer, etc. However, in this chapter we will discuss about the natural or free convection. In natural convection, the fluid velocity far from the solid body will be zero. However, near the solid body there will be some fluid motion if the body is at a temperature different from that of the free fluid. In this situation there will be a density difference between the fluid near the solid surface and that far away from the system. There will be a positive or negative buoyancy force due to this density difference. Hot surface will create positive buoyancy force whereas the cold surface will create the negative buoyancy force. Therefore, buoyancy force will be the driving force which produce and maintain the free convective process. Figure 5.1 shows the natural convective process for a hot and cold vertical surface.



(a) (b)

Fig.5.1: Free convection boundary layer for vertical (a) hot surface and (b) cold surface

Consider a vertical flat plate with contact of a fluid (say liquid) on one side of the plate. Now assume that we raise the temperature of the plate to Ts, a natural convective boundary layer forms as shown in fig. 5.2. The velocity profile in this boundary layer is slightly different as compared to forced convection boundary layer. At the wall the velocity is zero because of no slip condition. The velocity increases to maximum and then reduces to zero at the end of the boundary layer because the fluid is at rest in the bulk. Initially the laminar flow is achieved in the boundary layer, but at some distance from the leading edge, depending on the fluid properties and the temperature difference between plate and bulk fluid, turbulent eddies are found thus laminar to transition region comes. On further away from the leading edge the boundary layer may become turbulent and the boundary layer instability comes in to picture. Instability of the boundary layer is quite complex and does not fall into the scope of this study material.

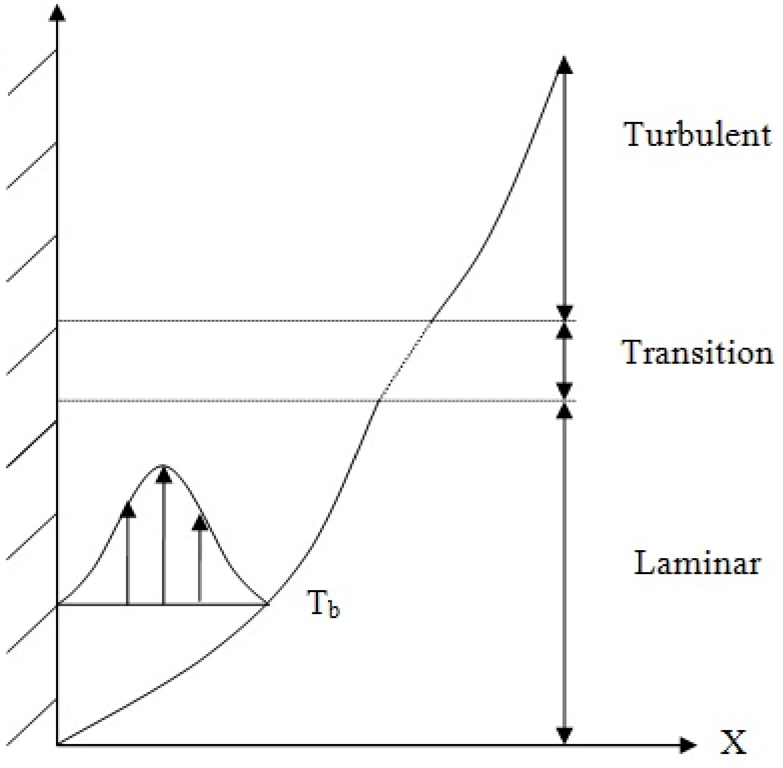


Fig. 5.2: Boundary layer on a hot vertical flat plate (T_s: surface temperature; T_b: bulk fluid temperature)

It has been found over the years that the average Nusselt number (or the average heat transfer coefficient) for convective heat transfer can be represented by the following functional dependence (say viscous flow past a hard body).

$$Nu = f(Re, Gr, Ec, Pr) \tag{5.1}$$

The Reynolds number (*Re*) is the ratio of inertia forces in the fluid to the viscous forces. The Grashof number (*Gr*) is the ratio of buoyant forces to the viscous forces. The Eckert number (*Ec*) is a measure of the thermal equivalent of kinetic energy of the flow to the imposed temperature differences. The Eckert number arises due to the inclusion of viscous dissipation. Thus *Ec* is absent where dissipation is neglected. The Prandtl number, *Pr*, is the ratio of the momentum diffusivity (kinematic viscosity) to the thermal diffusivity. In other words, Prandtl number is a measure of the relative magnitude of the diffusion of momentum, through viscosity, and the diffusion of heat through conduction, in the fluid.

In case of perfect natural-convection and in absence of heat dissipation, the eq. 5.1 reduces to,

$$Nu = f(Gr, Pr) \tag{5.2}$$

It is to be noted that in case of perfect natural convection, the main fluid stream is absent, thus Reynolds number is no longer significant.

The dimensionless numbers involved in eq. 5.2 evaluated at the average film temperature, $\bar{T} = \frac{T_s + T_b}{2}$. It can be easily found that in case of the forced convection and in absence of heat dissipation the function for average heat transfer will be,

$$Nu = f(Re, Pr) \tag{5.3}$$

On comparing eq. 5.2 and 5.3, one can see that the Grashof number will perform for free convection in a same way as the Reynolds number for forced convection.

Another parameter, the Rayleigh number is also used for perfect natural-convection is defined as,

$$Ra = Gr \,.\, Pr \tag{5.4}$$

Thus the functional relation is eq. 5.2 can be written as,

$$Nu = f(Ra, Pr) \tag{5.5}$$

As discussed earlier that all free convection flows are not limited to laminar flow. If instability occurs, the problem becomes complex. A general rule one may expect that transition will occur for critical Rayleigh number of

$$Ra_{x,c} \cong 10^9 \tag{5.6}$$

The Grashof number is defined as

$$Gr_L = \frac{g\beta(T_s - T_b)L^3}{\nu^2} = \frac{\text{Buoyancy force}}{\text{Viscous force}}$$

where,

g = acceleration due to gravity

β = coefficient of volume expansion = $-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$

T_s = surface temperature

T_b = bulk fluid temperature

L = characteristic length
v = Momentum diffusivity (kinematic viscosity)

5.2 Empirical relations for natural-convective heat transfer

5.2.1 *Natural convection around a flat vertical surface*

Churchill and Chu provided the correlation for average heat transfer coefficient for natural convection for different ranges of Rayleigh number.

Case I: If $Ra_L < 10^9$

$$\overline{Nu} = 0.68 + \frac{0.67 R_a^{1/4}}{\left[1+\left(\frac{0.492}{P_r}\right)^{\frac{9}{16}}\right]^{4/9}} \tag{5.7}$$

Case II: If $10^{-1} < Ra_L < 10^{12}$

$$Nu^{1/2} = \left[0.825 + \frac{0.387 Ra^{1/6}}{\left[1+\left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{8/27}}\right]^2 \tag{5.8}$$

It should be noted that the eq. 5.7 and 5.8 are also applicable for an inclined surface upto less than inclination from the vertical plane.

The above relations can be used for the vertical cylinder if the boundary layer thickness is quite small as compared to the diameter of the cylinder. The criteria to use the above relation for vertical cylinder is,

$$\frac{D}{L} \geq \frac{35}{Gr_L^{1/4}} \tag{5.9}$$

where, is the diameter and is the height of the cylinder.

5.2.2 *Natural convection around a horizontal cylinder*

Churchill and Chu has provided the following expression for natural-convective heat transfer.

$$\overline{Nu} = [0.6 + 0.387 \frac{Ra_d^{1/6}}{\{1+(0.559/Pr)^{\frac{9}{16}}\}^{8/27}}]^2 \tag{5.10}$$

Condition of applicability of the eq.5.10:

5.2.3 *Natural convection around a horizontal flat surface*

In the previous case of vertical flat surface, the principal body dimension was in-line with the gravity (i.e. vertical). Therefore, the flow produced by the free convection was parallel to the surface regardless of whether the surface was hotter or cooler compared to the bulk fluid around. However, in case of horizontal flat plate the flow pattern will be different and shown in fig. 5.3.

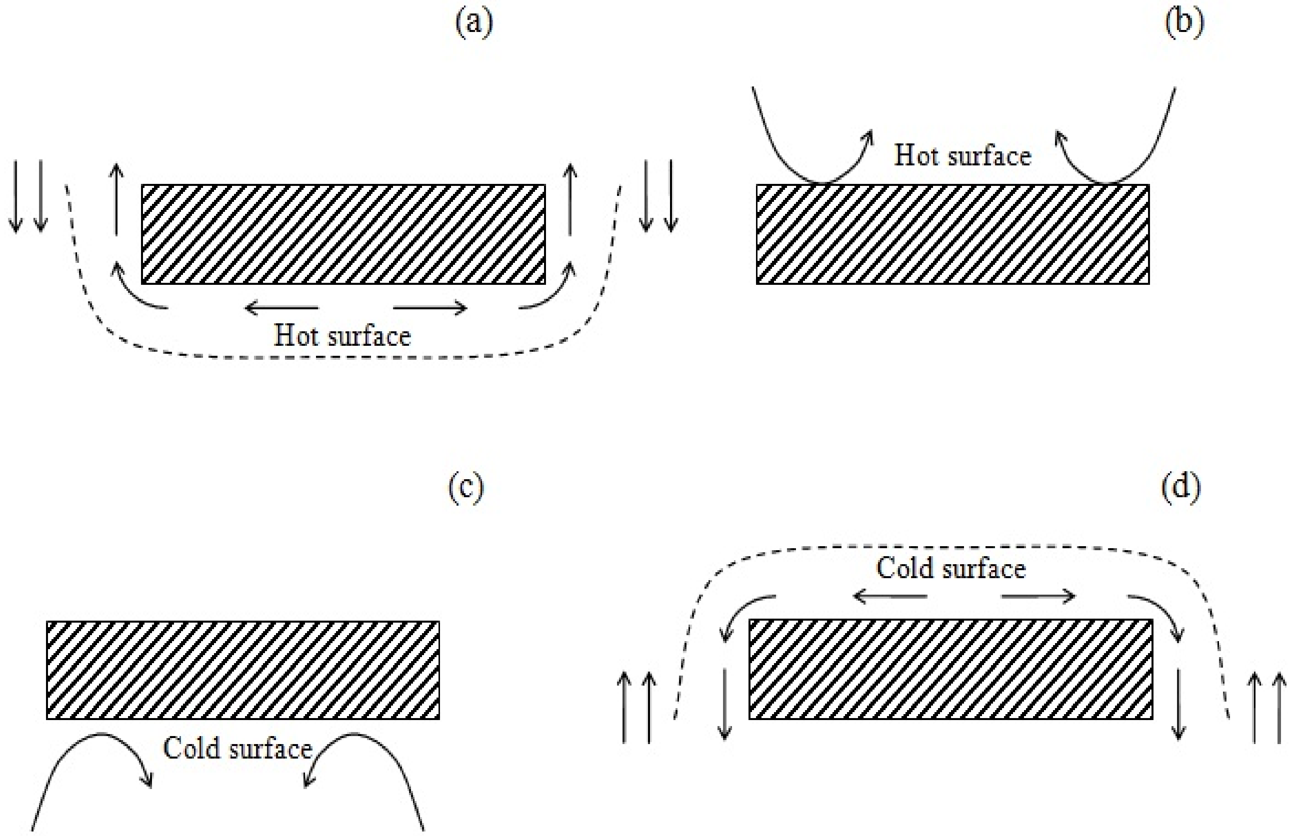


Fig. 5.3: A representative flow pattern (natural convection) for (a) hot surface down, (b) hot surface up, (c) cold surface down, and (d) cold surface up

Thus from fig. 5.3 it is understood that there are in fact two cases (i) when the heated plate facing up or cooled plate facing down, and (ii) heated plate facing down or cooled plate facing up.

Case I: *Heated plate facing up, cooled plate facing down*

$$Nu_{Lc} = 0.54 {Ra_{Lc}}^{1/4} \text{ for, } 2.6 \times 10^4 < Ra_{Lc} < 10^7 \tag{5.11}$$

$$Nu_{Lc} = 0.15 {Ra_{Lc}}^{1/3} \text{ for, } 10^7 < Ra_{Lc} < 3 \times 10^{10} \tag{5.12}$$

Case II: *Heated plate facing down, cooled plate facing up*

$$Nu_{Lc} = 0.27 {Ra_{Lc}}^{1/4} \text{ for, } 3 \times 10^5 < Ra_{Lc} < 10^{10} \tag{5.13}$$

where, L_c is characteristic length defined as below.

$$L_c = \frac{\text{Plate area}}{\text{Plate perimeter}}$$

5.2.4 Natural convection around sphere

Churchill proposed,

$$\overline{Nu} = 2 + \frac{0.589\, Ra^{1/4}}{[1+(0.469/Pr)^{\,9/16}]^{4/9}} \tag{5.14}$$

Condition for applicability: $Pr \geq 0.7$; $Ra \leq 10^{11}$

5.2.5 Natural convection in enclosure

It is another class of problems for which there are many cases and their corresponding correlations are also available in the literature. Here two cases will be discussed, (i) in which a fluid is contained between two vertical plates separated by a distance d, (ii) the other where the fluid is in an annulus formed by two concentric horizontal cylinders.

In the case first, the plates are at different temperature, T_1 and T_2. Heat transfer will be from higher temperature (T_1) to lower temperature (T_2) through the fluid.

The corresponding Grashof number will be

$$Gr_d = \frac{g\beta(T_1 - T_2)d^3}{\nu^2}$$

McGregor and Emery proposed the following correction for free convection heat transfer in a vertical rectangular enclosure, where the vertical walls are heated or cooled and the horizontal surfaces many be assumed adiabatic,

$$Nu_d = 0.42(Gr_d Pr)^{\frac{1}{4}} Pr^{0.012}\left(\frac{L}{d}\right)^{-0.3} \tag{5.15}$$

Applicability conditions for the above equation are,

$$10 < \frac{L}{d} < 40; 1 < Pr < 2 \times 10^4; 10^4 < Gr_d Pr < 10^7$$

or,

$$Nu_d = 0.046(Gr_d Pr)^{\frac{1}{3}} \tag{5.16}$$

Applicability conditions are,

$$1 < \frac{L}{d} < 40; 1 < Pr < 20; 10^6 < Gr_d Pr < 10^9$$

Here *L/d* is known as the aspect ratio.

At steady state condition, the heat flux (q_x) is equal thus,

$$q_x = h(T_1 - T_2)$$

or, $$q_x = h\left(\frac{d}{k}\frac{k}{d}\right)(T_1 - T_2)$$

or, $$q_x = \frac{Nu_x k}{d}(T_1 - T_2)$$

or, $$q_x = (Nu_x k)\frac{T_1 - T_2}{d}$$

or, $$q_x = k_c\frac{T_1 - T_2}{d}$$

where, $k_c(= Nu_x k)$ is known as the apparent thermal conductivity.

In the second case the heat transfer is involved in the enclosure formed by two concentric cylinders in horizontal position, the correlation given by Raithby and Holland,

$$\frac{k_e}{k} = 0.386\left(\frac{Pr}{0.861 + Pr}\right)^{0.25}(Ra_c{}^*)^{0.25} \tag{5.17}$$

$Ra_c{}^*$ is the modified Rayleigh number given by,

$$Ra_c{}^* = \frac{\{\ln\left(\frac{d_o}{d_i}\right)\}^4}{l^3\left(d_i^{-0.6} - d_o^{-0.6}\right)^5} Ra_l$$

where, d_i and d_0 are the outer and inner diameter of the inner and outer cylinders, respectively. The enclosure characteristic length l is defined as (d_0 - d_i).

The applicability of the eq. 5.17 is $10^2 <$ $Ra_c{}^*$ $> 10^7$.

It should be noted that the rate of heat flow by natural convection per unit length is same as that through the annular cylindrical region having effective thermal conductivity k_e for the case,

$$q = \frac{2\pi k_e (T_1 - T_2)}{\ln(d_o/d_i)}$$

where, T_1 and T_2 are the temperatures of the inner and outer cylindrical walls, respectively.

Illustration 5.1

A hot oven is maintained at 180 °C having vertical door 50 cm high is exposed to the atmospheric air at 20°C. Calculate the average heat transfer coefficient at the surface of the door.

The various air properties at the average temperature [(180+20)/2 = 100°C] are,

k = 0.032 W/m °C; Pr = 0.7; Kinematic viscosity = 24 x 10^{-6} m^2/s

At T_b = 20°C, $\beta = \frac{1}{293\,K}$

Solution 5.1

First we have to find the Grashof number,

$$G_{r_L} = \frac{g\beta(T_s - T_b)L^3}{\nu^2} = \frac{9.8 \times \left(\frac{1}{293}\right) \times (180 - 20) \times 0.5^3}{(24 \times 10^{-6})^2} = 1.16 \times 10^8$$

With the help of *Gr* and *Pr*, we can estimate the R_a number,

$$Ra = G_r P_r = 1.16 \text{ X } 10^8 \text{ X } 0.7 = 8.12 \text{ X } 10^7$$

As *Ra* < 10^9, the eq.5.7 can be used,

$$\overline{N_u} = 0.68 + \frac{0.67R_a^{1/4}}{\left[1+\left(\frac{0.0492}{P_r}\right)^{\frac{9}{16}}\right]^{4/9}} = 0.68 + \frac{0.67(8.12\times 10^7)^{1/4}}{\left[1+\left(\frac{0.0492}{0.7}\right)^{\frac{9}{16}}\right]^{4/9}} = 58.8$$

$$\overline{N_u} = \frac{\bar{h}L}{k} = 58.8$$

Thus, $\bar{h} = 3.76\frac{W}{m^2\ °C}$

5.3 Combined natural and forced convection

As we know that there is hardly any situation in which only natural or forced convective heat transfer occurs. Generally, in all the processes natural and forced convection heat transfer occur but depending upon the contribution made, the process may be approximated as either natural or forced convection problem. However, certain situation needs to be addressed as combined natural and forced convection problem. The following is a thumb rule to determine the individual situations.

1. Forced convection region i.e. negligible natural convection contribution $\frac{Gr_L}{{Re_L}^2} \ll 1$
2. Natural convection region i.e. negligible forced convection contribution $\frac{Gr_L}{{Re_L}^2} \gg 1$
3. Mixed convection i.e., significant contribution by both natural and forced convections $\frac{Gr_L}{{Re_L}^2} \cong 1$

In this situation, the following equation may be used,

$$Nu^m = {Nu_n}^m \pm {Nu_f}^m$$

where, Nu is the Nusselt number due to mixed convection, Nu_n is the Nusselt number due to natural convection, and Nu_f is the Nusselt number due to forced convection. The value of m is generally taken as 3, whereas positive and negative signs can be used for the convection in the same and opposite directions, respectively.

Illustration 5.2

In the oven door described in illustration 5.1 is subjected to an upward flow of air (that is forced convection). What would be the minimum free stream velocity for which natural convection may be neglected?

Solution 5.2.

Section 5.3 above shows that for the following condition the effect of natural convection may be neglected,

$$\frac{Gr_L}{{Re_L}^2} \ll 1$$

The value of *Gr* number calculated in the previous illustration was 1.16 X 10^8

Thus,

$$\frac{1.16 \times 10^8}{(\frac{LU}{v})^2} \ll 1$$

U >> 0.24 m/s

Therefore, the bulk velocity of the air should be far greater that 0.24 m/s.

Heat transfer in boiling and condensation

In the previous chapter we have discussed about the convective heat transfer in which the homogeneous single phase system was considered. The heat transfer processes associated with the change of fluid phase have great importance in chemical process industries. In this chapter, we will focus our attention towards the phase change from liquid to vapour and vice-versa.

6.1 Heat transfer during boiling

The conversion of a liquid into a vapour is one of the important and obvious phenomena. It has been found that if water (say) is totally distilled and degassed so that it does not have any impurity or dissolved gases, it will undergo liquid to vapour phase change without the appearance of bubbles, when it is heated in a clean and smooth container. However, in normal situation, as can be understood, the presence of impurities, dissolved gases, and surface irregularities causes the appearance of vapour bubble on the heating surface, when the rate of heat input is significantly high.

The boiling may be in general of two types. The one in which the heating surface is submerged in a quiescent part of liquid, and the heat transfer occur by free convection and bubble agitation. The process is known as pool boiling. The pool boiling may further be divided into sub-cooled or local boiling and saturated or bulk boiling. If the temperature of the liquid is below the saturation temperature, the process is known as sub-cooled, or local, boiling. If the liquid is maintained at saturation temperature, the process is known as saturated or bulk boiling.

The other form of the boiling is known as forced convective boiling in which the boiling occurs simultaneously with fluid motion induced by externally imposed pressure difference. In this chapter, we will mostly consider the pool boiling.

As generally the bubbles are formed during boiling, we will first refresh the following basic information. Consider a spherical bubble of radius in a liquid as shown in fig. 6.1

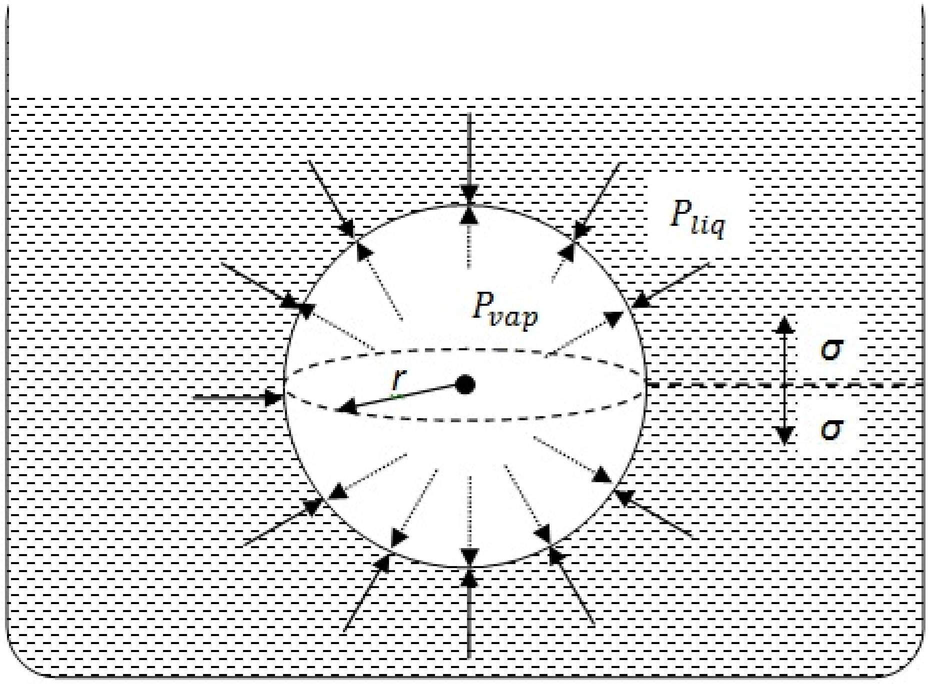


Fig. 6.1: Force balance on a submerge spherical bubble in a liquid

The pressure of vaporisation inside the bubble, P_{vap}, must exceed that in the surrounding liquid, P_{liq}, because of the surface tension (σ) acting on the liquid-vapour interface.

The force balance on the equatorial plane

$$\pi r^2(P_{vap} - P_{liq}) = 2\pi r\sigma$$

$$P_{vap} - P_{liq} = \frac{2\sigma}{r} \tag{6.1}$$

The eq. 6.1 shows that to create a bubble of small radius, it would be necessary to develop very large pressure in the vapour. In other word, a high degree super heat is necessary for the generation of a tiny bubble (or nucleus) in the bulk liquid. This is the reason, the bubble are usually formed at bits existing in the surface irregularities, where a bubble of finite initial radius may form, or where gasses dissolved in the system of the liquid come out of the solution.

6.2 Boiling of saturated liquid

In this section, we will discuss about the boiling curve which is a log-log plot between heat flux (q/A) or heat transfer coefficient (h) and excess temperature (ΔT). Excess temperature ($\Delta T = T_w - T_{sat}$) is the temperature difference between heating surface (T_w) and saturated temperature of the liquid (T_{sat}).

Figure 6.2 shows a typical representative pool boiling curves for water contained in a container where the water is heated by an immersed horizontal wire. Consider we are measuring the heat flux (thus, h) and the temperature difference (ΔT) between the boiling water (T_{sat}) and the wall temperature of the heater wire (T_w). The temperature of the heater wire is gradually raised while measuring the heat flux between heated surface and boiling water until a large value of ΔT reaches. The corresponding plot is prepared at the log-log scale. The plot shows six different sections in the boiling curve shown in the fig.6.2.

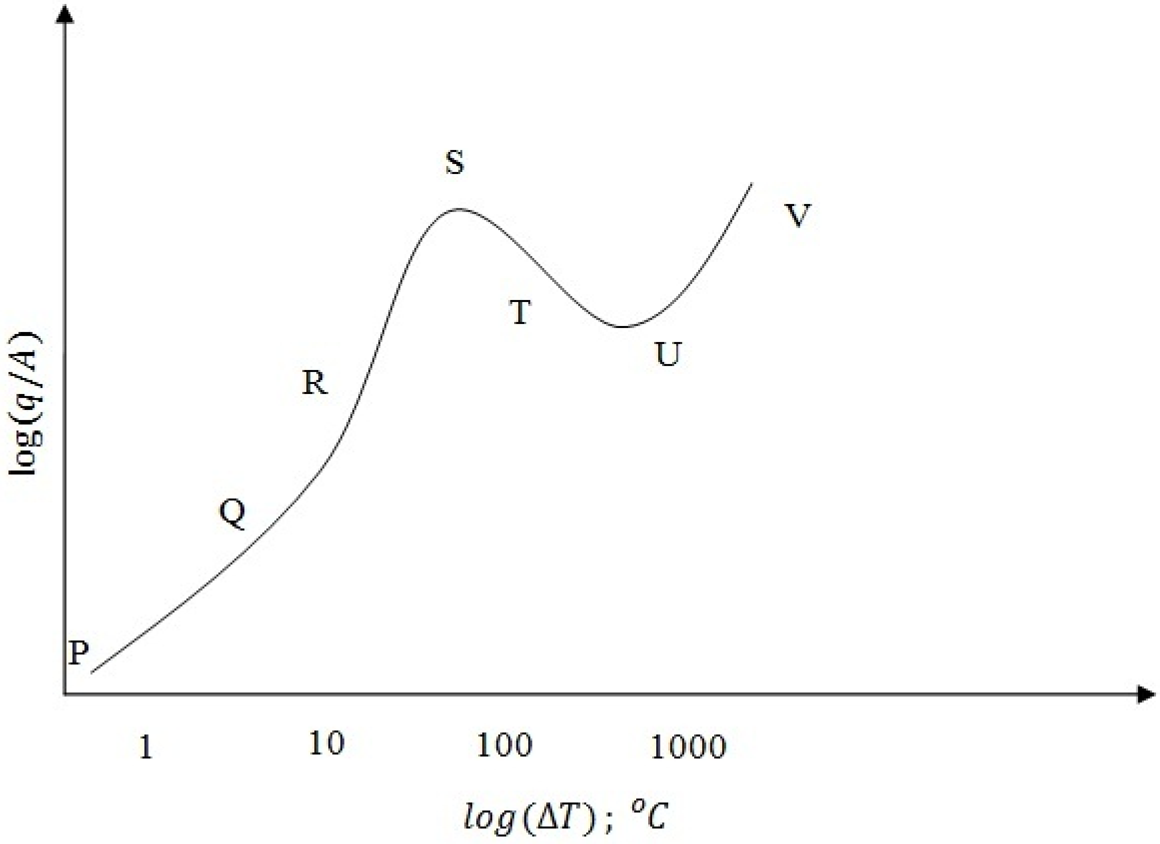


Fig. 6.2: Saturated water boiling curve

The different regimes of the boiling plot (fig. 6.2) have different mechanism. We will see those mechanisms in-brief in the following section.

Section PQ: In section *PQ*, initially when the wire temperature is slightly above the saturation temperature of the liquid, the liquid in contact with the heating surface get slightly superheated. The free convection of this heated fluid element is responsible for motion of the fluid, and it subsequently evaporated when it rises to the surface. This regime is called the interfacial evaporation regime.

Section QS: The section *QS* is composed of section *QR* and section *RS*. In *QR* section, bubbles begin to form on the surface of the wire and are dissipated in the liquid after detaching from the heating surface. If the excess temperature (further increases, bubbles form rapidly on the surface of the heating wire, and released from it, rise to the surface of the liquid, and are discharged into the top of the water surface (fig 6.3). This particular phenomenon is shown in section *RS*. Near the point *S*, the vapour bubbles rise as columns and bigger bubbles are formed. The vapour bubbles break and coalesce thus an intense motion of the liquid occurs which in-turn increases the heat transfer coefficient or heat transfer flux to the liquid from the heating wire. The section *QS* is known as nucleate boiling.

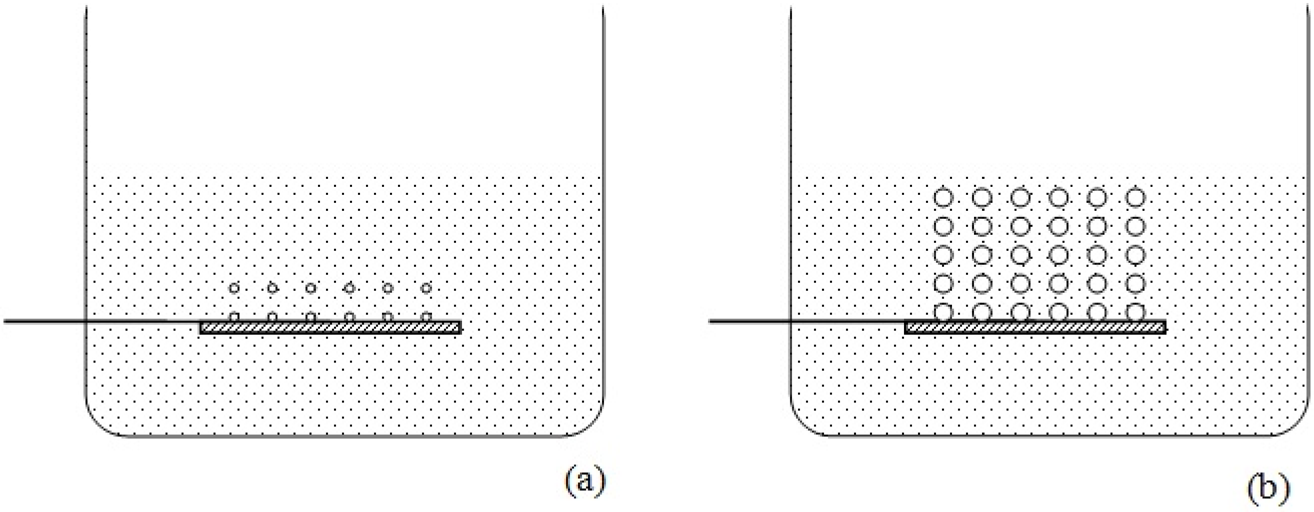


Fig. 6.3: (a) Formation of tiny bubbles, and (b) Grown up bubbles

Section *ST*: At the beginning of the section *ST* or at the end of the section , the maximum number of bubbles are generated from the heating surface. The bubbles almost occupy the full surface of the heating wire. Therefore, the agitation becomes highest as they discharge from the surface. Thus, maximum heat transfer coefficient is obtained at point *S*. However, once the population of the bubbles reaches to maximum, the nearby bubbles coalesce and eventually a film of vapour forms on the heating surface. This layer is highly unstable and it forms momentarily and breaks. This is known as transition boiling (nucleate to film). In this situation the vapour film (unstable) imparts a thermal resistance and thus the heat transfer coefficient reduces rapidly.

Section *TU*: If the excess temperature is further increased, the coalesced bubbles form so rapidly that they blanket the heating surface (stable vapour film) and prevent the inflow of fresh liquid from taking their place. The heat conducts only by the conduction through this stable vapour film. As a result the flux of heat transfer decreases continuously and reaches a minimum at point *U*. All the resistance to the heat transfer is imposed by this layer stable layer of vapour film.

Section *UV*: At very high excess temperature the heat transfer is facilitated by the radiation through the vapour film and thus the heat transfer coefficient start increasing. Infact the excess temperature in this regime is so high that the heating wire may get melted. This situation is known as boiling crises. The combine regime of *ST*, *TU*, and *UV* is known as film boiling regime.

At this stage it would be interesting to know the Leidenfrost phenomenon, which was observed by Leidenfrost in 1756. When water droplets fall on a very hot surface they dance and jump on the hot surface and reduces in size and eventually the droplets disappear. The mechanism is related to the film boiling of the water droplets. When water droplet drops on to the very hot surface, a film of vapour forms immediately between the droplet and the hot surface. The vapour film generated provide and up-thrust to the droplet. Therefore, the droplet moves up and when again the droplet comes in the contact of the hot surface, the vapour generated out of the water droplet and the phenomenon continues till it disappears.

The effectiveness of nucleate boiling depends primarily on the ease with which bubbles form and free themselves from the heating surface. The important factor in controlling the rate of bubble detachment is the interfacial tension between the liquid and the heating surface. If this interfacial tension is large the bubbles tends to spread along the surface and blocked the heat transfer area, rather than leaving the surface, to make room for other bubbles. The heat transfer coefficient obtained during the nucleation boiling is sensitive to the nature of the liquid, the type and condition of the heating surface, the composition and purity of the liquid, agitation, temperature and pressure.

Fact: Film boiling is not normally desired in commercial equipment because the heat transfer rate is low for such a large temperature drop.

6.2.1 Nucleation boiling

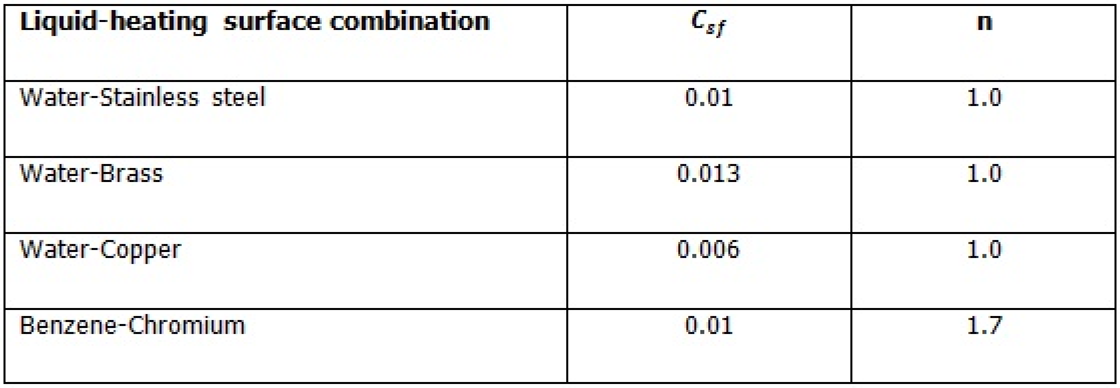
Rohsenow correlation may be used for calculating pool boiling heat transfer

$$q = \mu_l \lambda \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} \left[\frac{C_{pl} T_e}{C_{sf}\, \lambda (Pr_l)^n}\right]^3 \tag{6.2}$$

where,

q is the heat flux (W/m^2)
μ_l is the liquid viscosity (Pa.s)
λ is the enthalpy of liquid vaporisation (J/kg)
ρ_l and ρ_v are the liquid and vapour density, respectively, (kg/m^3)
c_{p1} is the specific heat of liquid (J/kg/°C)
σ is the surface tension (N/m)
T_e is the excess temperature of the boiling surface, T_w - T_{sat}, (K)
Pr_1 is the liquid Prandtl number

C_{sf} and n are the constants and depend on the liquid and heating surface combination for boiling operation, for example,



Liquid-heating surface combination	C_{sf}	n
Water-Stainless steel	0.01	1.0
Water-Brass	0.013	1.0
Water-Copper	0.006	1.0
Benzene-Chromium	0.01	1.7

All the properties are to be evaluated at film temperature.

6.2.2 Maximum heat flux

Maximum heat flux corresponding to the point *S* in the fig.6.2 can be found by Leinhard correlation,

$$q_{max} = 0.149\lambda\rho_v\left[\sigma g\,\frac{\rho_l-\rho_v}{\rho_v{}^2}\right]^{1/4} \tag{6.3}$$

The notations are same as for eq.6.2.

6.2.3 Film boiling

$$h = 0.62\left[\frac{k_v{}^3\rho_v(P_l-P_v)g(L+0.4C_{pv}T_e)}{d\mu_v T_e}\right]^{1/4} \tag{6.4}$$

where, k_v is the thermal conductivity of the vapour, μ_v is the viscosity of the vapour, *d* is the characteristic length (tube diameter or height of the vertical plate), other notations are same as for eq. 6.2.

If the surface temperature is high enough to consider the contribution of radiative heat transfer, the total heat transfer coefficient may be calculation by,

$$h_T = h\left(\frac{h}{h_T}\right)^{1/3} + h_r \tag{6.5}$$

where, h_r is the radiative heat transfer coefficient and is given in eq.6.4.

Upto this section, we have discussed about the boiling phenomenon where the liquid phase changes to vapour phase. In the subsequent sections, we will study the opposite phenomena of boiling that is condensation, where the vapour phase changes to the liquid phase.

6.3 Heat transfer during condensation

Condensation of vapours on the surfaces cooler than the condensing temperature of the vapour is an important phenomenon in chemical process industries like boiling phenomenon. It is quite clear that in condensation the phase changes from vapour to liquid. Consider a vertical flat plate which is exposed to a condensable vapour. If the temperature of the plate is below the saturation temperature of the vapour, condensate will form on the surface and flows down the plate due to gravity. It is to be noted that a liquid at its boiling point is a saturated liquid and the vapour in equilibrium with the saturated liquid is saturated vapour. A liquid or vapour above the saturation temperature is called superheated. If the non-condensable gases will present in the vapour the rate of condensation of the vapour will reduce significantly.

Condensation may be of two types, film condensation and dropwise condensation. If the liquid (condensate) wets the surface, a smooth film is formed and the process is called film type condensation. In this process, the surface is blocked by the film, which grows in thickness as it moves down the plate. A temperature gradient exists in the film and the film represents thermal resistance in the heat transfer. The latent heat is transferred through the wall to the cooling fluid on the other side of the wall. However, if the liquid does not wet the system, drops are formed on the surface in some random fashion. This process is called dropwise condensation. Some of the surface will always be free from the condensate drops (for a reasonable time period).

Now, with the help of the above discussion one can easily understand that the condensate film offers significant heat transfer resistance as compared to dropwise condensation. In dropwise condensation the surface is not fully covered by the liquid and exposed to the vapour for the condensation. Therefore, the heat transfer coefficient will be higher for dropwise condensation. Thus the dropwise condensation is preferred over the film condensation. However, the dropwise condensation is not practically easy to achieve. We have to put some coating on the surface or we have to add some additive to the vapour to have dropwise condensation. Practically, these techniques for dropwise condensation are not easy for the sustained dropwise condensation. Because of these reasons, in many instances we assume film condensation because the film condensation sustained on the surface and it is comparatively easy to quantify and analyse.

6.4 Film condensation on a vertical flat plate

Figure 6.4 shows a vertical wall very long in z-direction. The wall is exposed to a condensable vapour. The condensate film is assumed to be fully developed laminar flow with zero interfacial shear and constant liquid properties. It is also assumed that the vapour is saturated and the heat transfer through the condensate film occurs by condensation only and the temperature profile is assumed to be linear.

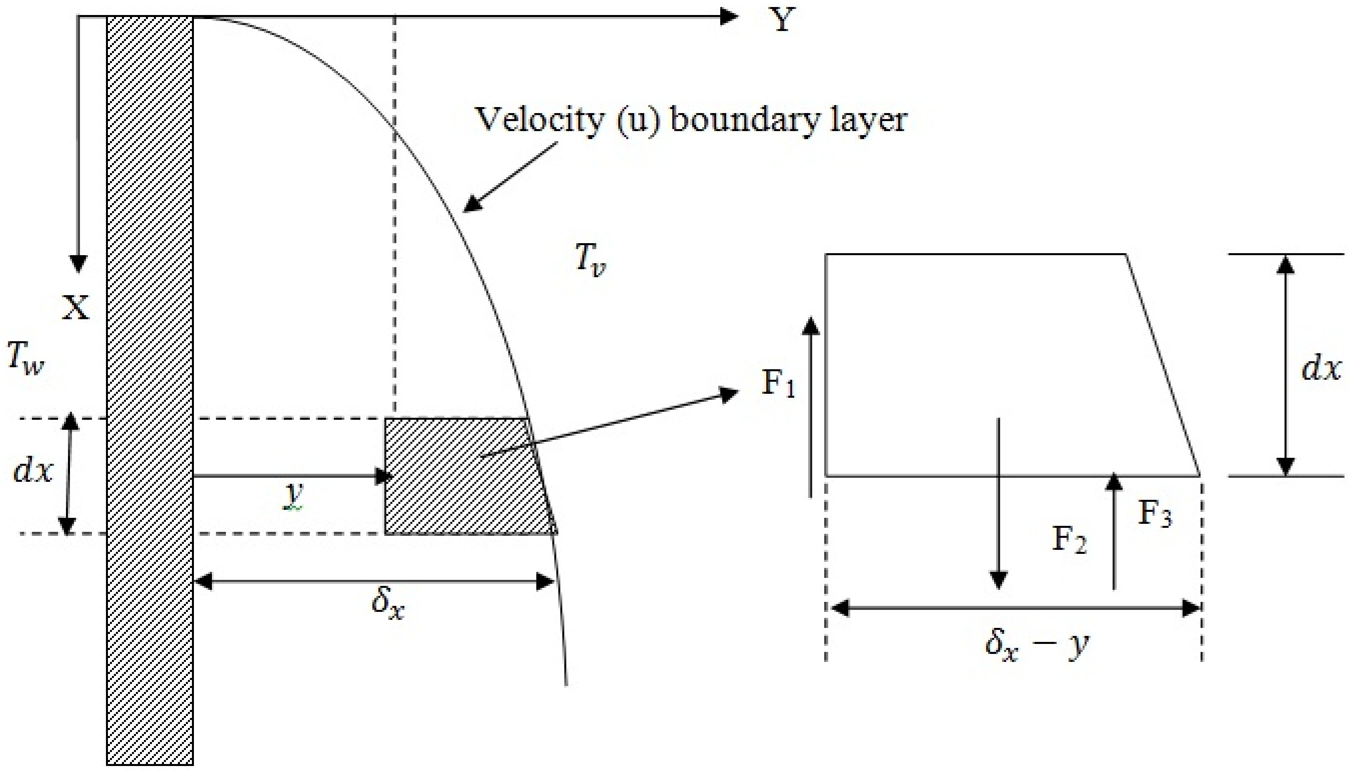


Fig. 6.4: Condensation of film in laminar flow

The wall temperature is maintained at temperature T_w and the vapour temperature at the edge of the film is the saturation temperature T_v. The condensate film thickness is represented by δ_x, a function of x. A fluid element of thickness dx was assumed with a unit width in the z-direction. The force balance on the element provides,

$$F_1 = F_2 - F_3$$

where, shear force $F_1 = \mu_l \frac{du}{dy} dx$; μ_l is the viscosity of the condensate (liquid). In the subsequent sections of this module, the subscripts *l* and *v* will represent liquid and vapour phase.

Gravity force, $F_2 = \rho_l g (\delta_x - y)dx$; and

Buoyancy force, $F_3 = \rho_v g (\delta_x - y)dx$

Thus,

$$\mu_l \frac{du}{dy} dx = \rho_l g(\delta_x - y)dx - \rho_v g(\delta_x - y)dx$$

On integrating for the following boundary condition, u = 0 at y = 0; no slip condition.

$$u = \frac{(\rho_l - \rho_v)g}{\mu_l}\left[y\delta_x - \frac{y^2}{2}\right] \qquad (6.6)$$

Equation 6.6 shows the velocity profile in the condensate falling film.

The corresponding mass flow rate of the condensate for *dy* thickness and unit width of the film,

$$\dot{m} = \int_0^{\delta} u(dy \cdot 1)\rho_l$$

$$\dot{m} = \frac{\rho_l(\rho_l - \rho_v)g\delta_x{}^3}{3\mu_l} \qquad (6.7)$$

where *dy* is the length of the volume element at y distance. The rate of condensation for *dx*.1 (over element surface) area exposed to the vapour can be obtained from the rate of heat transfer through this area.

The rate of heat transfer $= k_l \frac{T_v - T_w}{\delta_x} dx \cdot 1$

The thermal conductivity of the liquid is represented by k_l. The above rate of heat transfer is due to the latent heat of condensation of the vapour. Thus,

$$d\dot{m} = k_l \frac{T_v - T_w}{\delta_x \, \lambda} dx \tag{6.8}$$

The specific latent heat of condensation is represented by λ. On solving eqs.6.7 and 6.8, for boundary layer conditions ($x = 0$; $\delta_x = 0$)

$$\delta_x = \left[\frac{4\mu_l k_l x \,(T_v - T_w)}{g\lambda\rho_l(\rho_l - \rho_v)}\right]^{1/4} \tag{6.9}$$

The eq. 6.9 gives the local condensate film thickness at any location x. If h is the film heat transfer coefficient for the condensate film, heat flux through the film at any location is,

$$h = k_l \left[\frac{4\mu_l k_l x \,(T_v - T_w)}{g\lambda\rho_l(\rho_l - \rho_v)}\right]^{-\frac{1}{4}} \tag{6.10a}$$

The local Nusselt number will be,

$$Nu_x = \frac{hx}{k_l}\left[\frac{g\lambda\rho_l(\rho_l - \rho_v)x^3}{4\mu_l \, k_l(T_v - T_w)}\right]^{1/4}$$

We can also calculate the average heat transfer coefficient along the length of the surface,

$$h_{av} = \bar{h} = \frac{1}{L}\int_0^L h dx = 0.943\left[\frac{g\lambda\rho_l(\rho_l - \rho_v)k_l^3}{4\,\mu_l L(T_v - T_w)}\right]^{1/4} \;\; ; \;\; \bar{h} = \frac{4}{3}\, h_{x=L} \tag{6.10b}$$

In eq. 6.10, the liquid properties can be taken at the mean film temperature $\left(\frac{T_v + T_w}{2}\right)$. The equation 6.10 is applicable for Pr > 0.5 and $\frac{c_{p\,T}}{\lambda} \le 1.0$

It can also be understood that at any location on the plate the liquid film temperature changes from T_v to T_w. It indicates that apart from latent heat some amount of sensible heat will also be removed. Thus, to take this into account and to further improve the accuracy of Nusselt's equation (eq. 6.10), a modified latent heat term $\dot{\lambda} = \lambda(1 + 0.68\, J_a)$ can be used in place of λ. The term J_a is called the Jacob number as is defined by eq. 6.11. All the properties are to be evaluated at film temperature.

$$J_a = c_{pl}\frac{T_v - T_w}{\lambda} \tag{6.11}$$

In the previous discussion we have not discussed about the ripples or turbulent condition of the condensate film as it grows while coming down from the vertical wall. The previous discussion was applicable only when the flow in the condensate film was 1-D and the velocity profile was half parabolic all along the length of the wall. However, if the rate of condensation is high or the height of the condensing wall is more, the thickness of the condensate film neither remains small nor the flow remains laminar.

The nature of the flow is determined by the film Reynolds number (Re_f). The local average liquid velocity in the film can be obtained by eq. 6.6.

$$\bar{u} = \frac{1}{\delta_x}\int_0^{\delta_x} u dy = \frac{(\rho_l - \rho_v)g\ \delta^2}{3\mu_l}$$

Now, the Re_f can be calculated by,

$$Re_f = \frac{D\bar{u}\,\rho}{\mu_l} = \frac{4\,\dot{m}}{\mu_l} \tag{6.12}$$

where D is the hydraulic diameter of the condensate film. The hydraulic diameter can be calculated by the flow area (δ_x.1) and wetted perimeter (unit breadth, thus 1). It has been found that, if

Case 1: $Re_f \leq 30$; the film remains laminar and the free surface of the film remains wave free.
Case 2: $30 < Re_f < 1600$; the film remains laminar but the waves and ripples appear on the surface.
Case 3: $Re_f \geq 1600$; the film becomes turbulent and surface becomes wavy.

The corresponding average heat transfer coefficient can be calculated by the following correlation,

$$\overline{Nu}(= Co) = \frac{\bar{h}\delta_x}{k_l} = \frac{\bar{h}}{k_l}\left[\frac{\mu_l^2}{\rho_l(\rho_l - \rho_v)g}\right]^{1/3} = 1.47\left(Re_f\right)^{-1/3} \quad \text{: for Case 1}$$

(It is same as eq. 6.10 if Re_f is taken at the bottom of the wall.)

$$\overline{Nu} = \frac{Re_f}{1.08\left(Re_f\right)^{1.22} - 5.2} \quad \text{: for Case 2}$$

$$\overline{Nu} = \frac{Re_f}{8750 + 58\left[\left(Re_f\right)^{0.75} - 253\right]Re_f^{-0.5}} \quad \text{: for Case 3}$$

The Nusselt number in case-1 is defined as Modified Nusselt number or condensation number (*Co*).

The above relations may also be used for condensation inside or outside of a vertical tube if the tube diameter is very large in comparison to condensate film thickness. Moreover, the relations are valid for the tilted surfaces also. If the surface make an angle "*θ*" from the vertical plane the "*g*" will be replaced by "*g.cosθ*" in the above equations

Illustration 6.1

Saturated steam at 70.14 kPa is condensing on a vertical tube 0.5 m long having an outer diameter of 2.5 cm and a surface temperature of 80°C. Calculate the average heat-transfer coefficient.

Solution 6.1

It is a problem of condensation on a vertical plate, thus eq.6.10b can be used,

$$h_{av} = \frac{1}{L}\int_0^L h dx = 0.943\left[\frac{g\lambda\rho_l(\rho_l - \rho_v){k_l}^3}{\mu_l L(T_v - T_w)}\right]^{1/4}$$

where, different liquid and steam properties are evaluate at average film temperature,

$$\frac{T_v + T_w}{2}$$

Using steam table, the temperature of the steam corresponding to 70.14kPa pressure is 90°C. The average film temperature will then be the average of 80 °C and 90 °C and it comes out to be 85 °C,

Using given data the different properties can be found using steam table and other relevant tables given in the standard literature. The data is tabulates below at 85°C,

Latent heat of steam (λ_v)	2651.9 kJ/kg	$\lambda = \lambda_v - \lambda_l$ = 2296 kJ/kg
Latent heat of water (λ_l)	355.9 kJ/kg	
Density of vapour (ρ_v)	1/ sp.vol.= 1/ 2.828 kg/m^3	= 0.354 kg/m^3
Density of water (ρ_l)	1/ sp.vol. = 1/ 0.0010325 kg/m^3	= 968.5 kg/m^3
Viscosity of water (μ_w)	0.335 cP = .335 x 10^{-3}Pa s	
Thermal conductivity of water (k_l)	0.67	

On putting the above values in the above equation,

h_{av} = 1205.2 W/m^2 °C

6.5 Condensation for horizontal tube

6.5.1 Condensation outside horizontal tube or bank of tube

This type of condensation is very common especially for the shell and tube heat exchanger. In case of condensation outside the vertical array of horizontal tubes the condensate flows as a film along the cylindrical surface or it may drop down. In case of another tube below, the condensate film flows down from the bottom edge of the upper tube to the upper edge of the bottom tube. As it goes on to the lower tubes, the thickness of the condensate film increases. Some of the correlations are given below.

6.5.1.1 Condensation on a single horizontal tube

$$\bar{h} = 0.728\left[\frac{g\lambda\rho_l(\rho_l-\rho_v)k_l^3}{d\mu(T_v-T_w)}\right]^{1/4} \tag{6.13}$$

6.5.1.2 Condensation on a vertical tube of N horizontal tubes

$$\bar{h} = 0.728\left[\frac{g\lambda\rho_l(\rho_l-\rho_v)k_l^3}{Nd\mu(T_v-T_w)}\right]^{1/4} \tag{6.14}$$

6.5.1.3 Condensation inside a horizontal tube

Figure 6.5 shows the physical picture of the condensation inside a horizontal tube (like an open channel flow).

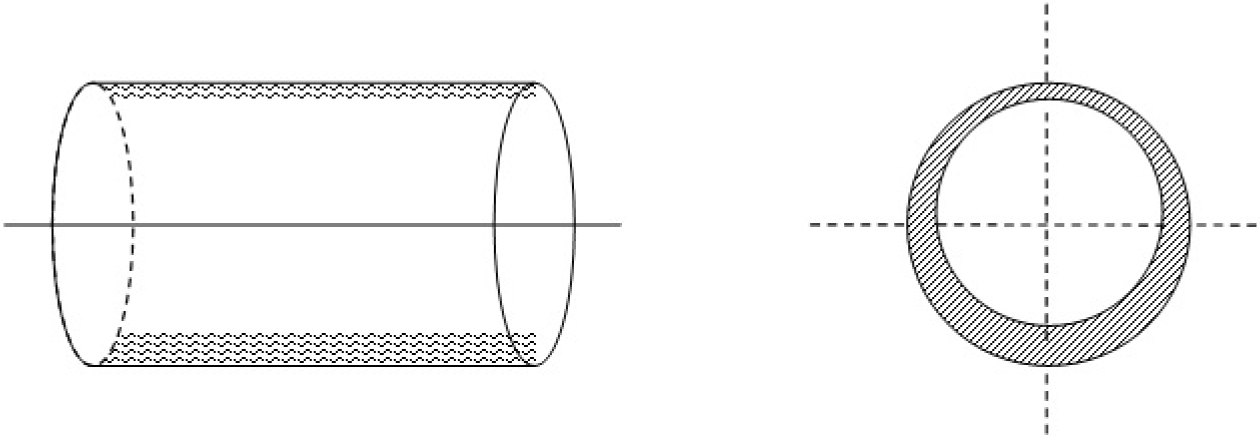


Fig. 6.5: Film condensation inside a horizontal tube

Case 1: The length is small or the rate of condensation is low.

This situation will have small thickness of the flowing condensate layer at the bottom of the tube and the following coefficient can be used,

$$\bar{h} = 0.555\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3\acute{\lambda}}{d_i\mu_i(T_v-T_w)}\right]^{1/4} \tag{6.15}$$

where, $\acute{\lambda} = \lambda\left[1+\frac{3}{8}J_a\right]$, and the vapour Reynolds number (Re_v) should be less than 35,000. The Re_v is calculated based on inlet condition of vapour and inside diameter of tube.

Case 2: The length is high or the rate of condensation is high.

In this the following relation can be used.

$$\frac{\bar{h}d_i}{k} = 0.026(Re_{in})^{0.8}{Pr_l}^{1/3} \tag{6.16}$$

where,

$$Re_{in} = \frac{d}{\mu_l}\left[G_l + G_v\left(\frac{\rho_l}{\rho_v}\right)^{\frac{1}{2}}\right]$$

(Condition: Re_l >5000, Re_v > 20,000)

where, G_l and G_v are the liquid and vapour mass velocities calculated on the basis of the cross-section $\left(=\frac{\pi d_i^2}{4}\right)$ of the tube.

6.6 Correlations for packed and fluidized bed

6.6.1 Packed bed

The heat transfer correlation for gas flow through a packed bed is given as,

$$\epsilon\,(\mathrm{St})\,(Pr)^{\frac{2}{3}} = 2.06\,(Re)_{dp}^{-0.575} \tag{6.17}$$

Conditions to use eq.6.12 are,

$\mathrm{Pr} \approx 0.7$ $\qquad$ $90 \leq \mathrm{Re}|_{\mathrm{dp}} \geq 4000$

where,

$\mathrm{St} = \frac{Nu}{Re\ Pr}$ is the Stanton number.

$\mathrm{Re}|_{\mathrm{dp}} = \frac{d_P v_0 \rho}{\mu}$ is the particle Reynolds number

d_p = Diameter or the effective diameter of a particle

v_0 = Superficial fluid velocity. It is the velocity based on the cross-section of the bed).

$\in$: Bed porosity or void fraction

$\in$: 0.3 → 0.5

Theoretically: $\in$ = 0.69 for uniform shape

= 0.71 bed of cubes

= 0.79 bed of cylinder $(\frac{L}{d} = 1)$

$(\mathrm{St})(Pr)^{\frac{2}{3}} =$ Colburn factor

6.6.2 Fluidized bed

The heat transfer coefficient to or from particles in a fluidized bed can be estimated with the help of following correlation,

$$\frac{h d_P}{k} = 2 + 0.6\,[\frac{d_P v_0 \rho}{\mu}]^{\frac{1}{2}}\,[\frac{C_P \mu}{k}]^{\frac{1}{3}} \tag{6.18}$$

$$Nu_P = 2 + 0.6\,[Re_{dP}]^{\frac{1}{2}}\ \ [Pr]^{\frac{1}{3}}$$

where, v_0 is the superficial velocity.

In the previous chapters it has been observed that the heat transfer studies were based on the fact that the temperature of a body, a portion of a body, which is hotter than its surroundings, tends to decrease with time. The decrease in temperature indicates a flow of energy from the body. In all the previous chapters, limitation was that a physical medium was necessary for the transport of the energy from the high temperature source to the low temperature sink. The heat transport was related to conduction and convection and the rate of heat transport was proportional to the temperature difference between the source and the sink.

Now, if we observe the heat transfer from the Sun to the earth atmosphere, we can understand that there is no medium exists between the source (the Sun) and the sink (earth atmosphere). However, still the heat transfer takes place, which is entirely a different energy transfer mechanism takes place and it is called thermal radiation.

Thermal radiation is referred when a body is heated or exhibits the loss of energy by radiation. However, more general form "radiation energy" is used to cover all the other forms. The emission of other form of radiant energy may be caused when a body is excited by oscillating electrical current, electronic bombardment, chemical reaction etc. Moreover, when radiation energy strikes a body and is absorbed, it may manifest itself in the form of thermal internal energy, a chemical reaction, an electromotive force, etc. depending on the nature of the incident radiation and the substance of which the body is composed.

In this chapter, we will concentrate on thermal radiation (emission or absorption) that on radiation produced by or while produces thermal excitation of a body.

There are many theories available in literature which explains the transport of energy by radiation. However, a dual theory is generally accepted which enables to explain the radiant energy in the characterisation of a wave motion (electromagnetic wave motion) and discontinuous emission (discrete packets or quanta of energy).

An electromagnetic wave propagates at the speed of light (3×10^8 m/s). It is characterised by its wavelength λ or its frequency v related by

$$c = \lambda v \tag{7.1}$$

Emission of radiation is not continuous, but occurs only in the form of discrete quanta. Each quantum has energy

$$E = hv \tag{7.2}$$

where, = 6.6246×10^{-34} J.s, is known as Planck's constant.

Table 7.1 shows the electromagnetic radiation covering the entire spectrum of wavelength

Table 7.1: Electromagnetic radiation for entire spectrum of wavelength

Type	Band of wavelength (μm)
Cosmic rays	upto 4×10^{-7}
Gamma rays	4×10^{-7} to 1.4×10^{-4}
X-rays	1×10^{-5} to 2×10^{-2}
Ultraviolet rays	5×10^{-3} to 3.9×10^{-1}
Visible light	$3.9\times10^{-1}$to 7.8×10^{-1}

Infrared rays	7.8×10^{-1} to 1×10^{3}
Thermal radiation	1×10^{-1} to 1×10^{2}
Microwave, radar, radio waves	1×10^{3} to 5×10^{10}

It is to be noted that the above band is in approximate values and do not have any sharp boundary.

7.1 Basic definition pertaining to radiation

Before we further study about the radiation it would be better to get familiarised with the basic terminology and properties of the radiant energy and how to characterise it.

As observed in the table 7.1 that the thermal radiation is defined between wavelength of about 1×10^{-1} and 1×10^{2} μm of the electromagnetic radiation. If the thermal radiation is emitted by a surface, which is divided into its spectrum over the wavelength band, it would be found that the radiation is not equally distributed over all wavelength. Similarly, radiation incident on a system, reflected by a system, absorbed by a system, etc. may be wavelength dependent. The dependence on the wavelength is generally different from case to case, system to system, etc. The wavelength dependency of any radiative quantity or surface property will be referred to as a spectral dependency. The radiation quantity may be monochromatic (applicable at a single wavelength) or total (applicable at entire thermal radiation spectrum). It is to be noted that radiation quantity may be dependent on the direction and wavelength both but we will not consider any directional dependency. This chapter will not consider directional effect and the emissive power will always used to be (hemispherical) summed overall direction in the hemisphere above the surface.

7.1.1 Emissive power

It is the emitted thermal radiation leaving a system per unit time, per unit area of surface. The total emissive power of a surface is all the emitted energy, summed over all the direction and all wavelengths, and is usually denoted as E. The total emissive power is found to be dependent upon the temperature of the emitting surface, the subsystem which this system is composed, and the nature of the surface structure or texture.

The monochromatic emissive power E_λ, is defined as the rate, per unit area, at which the surface emits thermal radiation at a particular wavelength λ. Thus the total and monochromatic hemispherical emissive power are related by

$$E = \int_0^\infty E_\lambda d\lambda \tag{7.3}$$

and the functional dependency of E_λ on λ must be known to evaluate E.

7.1.2 Radiosity

It is the term used to indicate all the radiation leaving a surface, per unit time and unit area.

$$J = \int_0^\infty J_\lambda \, d\lambda \tag{7.4}$$

where, J and J_λ are the total and monochromatic radiosity. The radiosity includes reflected energy as well as original emission whereas emissive power consists of only original emission leaving the system. The emissive power does not include any energy leaving a system that is the result of the reflection of any incident radiation.

7.1.3 Irradiation

It is the term used to denote the rate, per unit area, at which thermal radiation is incident upon a surface (from all the directions). The irradiative incident upon a surface is the result of emission and reflection from other surfaces and may thus be spectrally dependent.

$$G = \int_0^\infty G_\lambda \, d\lambda \tag{7.5}$$

where, G and G_λ are the total and monochromatic irradiation.

Reflection from a surface may be of two types specular or diffusive as shown in fig.7.1.

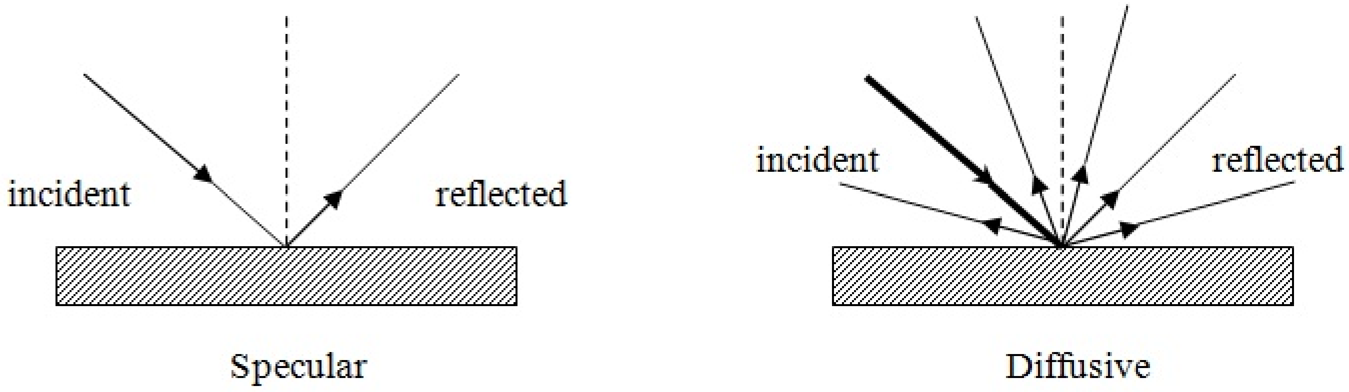


Fig. 7.1: (a) Specular, and (b) diffusive radiation

Thus,

$$J = E + \rho G \tag{7.6}$$

7.1.4 Absorptivity, reflectivity, and transmitting

The emissive power, radiosity, and irradiation of a surface are inter-related by the reflective, absorptive, and transmissive properties of the system.

When thermal radiation is incident on a surface, a part of the radiation may be reflected by the surface, a part may be absorbed by the surface and a part may be transmitted through the surface as shown in fig.7.2. These fractions of reflected, absorbed, and transmitted energy are interpreted as system properties called reflectivity, absorptivity, and transmissivity, respectively.

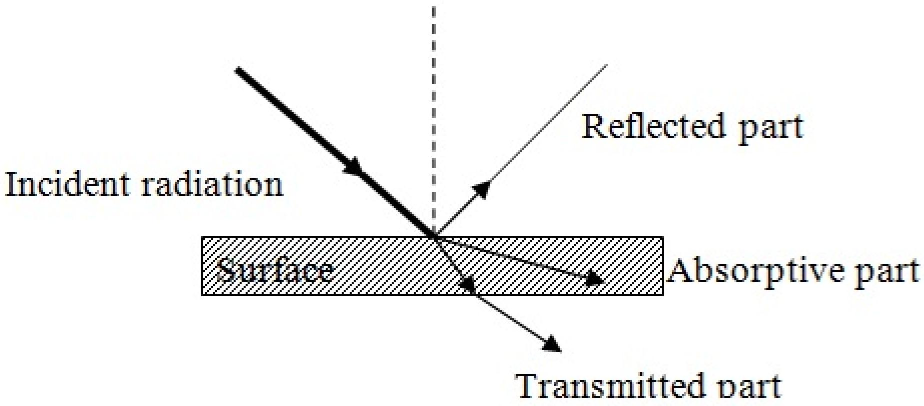


Fig. 7.2: Reflection, absorption and transmitted energy

Thus using energy conservation,

$$\rho + \alpha + \tau = 1 \tag{7.7}$$

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \tag{7.7}$$

where, ρ , α and τ are total reflectivity, total absorptivity, and total transmissivity. The subscript λ indicates the monochromatic property.

In general the monochromatic and total surface properties are dependent on the system composition, its roughness, and on its temperature.

Monochromatic properties are dependent on the wavelength of the incident radiation, and the total properties are dependent on the spectral distribution of the incident energy.

Most gases have high transmissivity, i.e. $\tau \approx 1$ and $\rho = \alpha = 0$ (like air at atmospheric pressure). However, some other gases (water vapour, CO_2 etc.) may be highly absorptive to thermal radiation, at least at certain wavelength.

Most solids encountered in engineering practice are opaque to thermal radiation $(\tau \approx 0)$. Thus for thermally opaque solid surfaces,

$$\rho + \alpha = 1 \tag{7.6}$$

Another important property of the surface of a substance is its ability to emit radiation. Emission and radiation have different concept. Reflection may occur only when the surface receives radiation whereas emission always occurs if the temperature of the surface is above the absolute zero. Emissivity of the surface is a measure of how good it is an emitter.

7.2 Blackbody radiation

In order to evaluate the radiation characteristics and properties of a real surface it is useful to define an ideal surface such as the perfect blackbody. The perfect blackbody is defined as one which absorbs all incident radiation regardless of the spectral distribution or directional characteristic of the incident radiation.

$$\alpha = \alpha_\lambda = 1$$

$$\rho = \rho_\lambda = 0$$

A blackbody is black because it does not reflect any radiation. The only radiation leaving a blackbody surface is original emission since a blackbody absorbs all incident radiation. The emissive power of a blackbody is represented by , and depends on the surface temperature only.

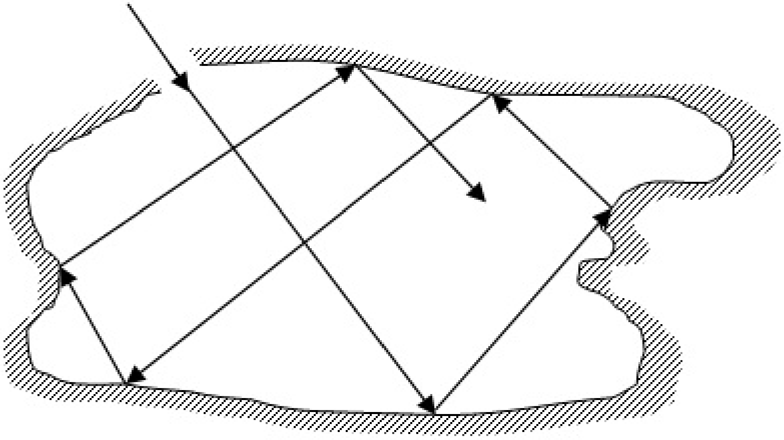


Fig. 7.3: Example of a near perfect blackbody

It is possible to produce a near perfect blackbody as shown in fig.7.3.

Figure 7.2 shows a cavity with a small opening. The body is at isothermal state, where a ray of incident radiation enters through the opening will undergo a number of internal reflections. A portion of the radiation absorbed at each internal reflection and a very little of the incident beam ever find the way out through the small hole. Thus, the radiation found to be evacuating from the hole will appear to that coming from a nearly perfect blackbody.

7.2.1 Planck's law

A surface emits radiation of different wavelengths at a given temperature (theoretically zero to infinite wavelengths). At a fixed wavelength, the surface radiates more energy as the temperature increases. Monochromatic emissive power of a blackbody is given by eq.7.10.

$$E_{b,\lambda} = \frac{2\pi h c^2 \lambda^{-5}}{\exp\left(\frac{hc}{\lambda k T}\right) - 1} \tag{7.7}$$

where, h = 6.6256 X 10^{-34} JS; Planck's constant

c = 3 X 10^8 m/s; speed of light

T = absolute temperature of the blackbody

λ = wavelenght of the monochromatic radiation emitted

k = Boltzmann constant

Equation 7.10 is known as Planck's law. Figure 7.4 shows the representative plot for Planck's distribution.

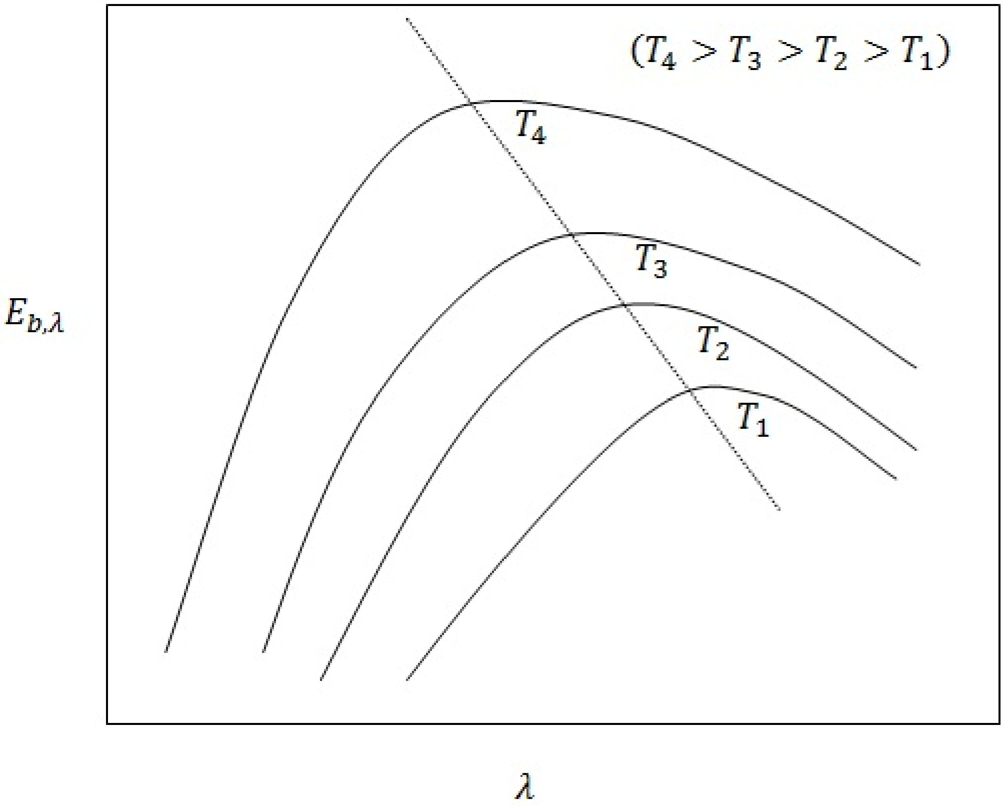


Fig. 7.4: Representative plot for Planck's distribution

7.2.2 Wien's law

Figure 7.4 shows that as the temperature increases the peaks of the curve also increases and it shift towards the shorter wavelength. It can be easily found out that the wavelength corresponding to the peak of the plot (λ_{max}) is inversely proportional to the temperature of the blackbody (Wein's law) as shown in eq. 7.11.

$$\lambda_{max} T = 2898 \tag{7.11}$$

Now with the Wien's law or Wien's displacement law, it can be understood if we heat a body, initially the emitted radiation does not have any colour. As the temperature rises the λ of the radiation reach the visible spectrum and we can able to see the red colour being height λ (for red colour). Further increase in temperature shows the white colour indicating all the colours in the light.

7.2.3 The Stefan-Boltzmann law for blackbody

Josef Stefan based on experimental facts suggested that the total emissive power of a blackbody is proportional to the fourth power of the absolute temperature. Later, Ludwig Boltzmann derived the same using classical thermodynamics. Thus the eq. 7.12 is known as Stefan-Boltzmann law,

$$E_b = \int_0^{\infty} E_{b\lambda}(\lambda, T)\, d\lambda$$

$$E_b = \sigma T^4 \qquad (7.12)$$

where, E_b is the emissive power of a blackbody, T is absolute temperature, and σ (= 5.67 X 10^{-8}W/m^2/K^4) is the Stefan-Boltzmann constant.

The Stefan-Boltzmann law for the emissive power gives the total energy emitted by a blackbody defined by eq.7.3.

7.2.4 Special characteristic of blackbody radiation

It has been shown that the irradiation field in an isothermal cavity is equal to E_b. Moreover, the irradiation was same for all planes of any orientation within the cavity. It may then be shown that the intensity of the blackbody radiation, I_b, is uniform. Thus, blackbody radiation is defined as,

$$E_b = \pi I_b \qquad (7.13)$$

where, $I_b = \int_0^{\infty} I_{b\lambda}\, d\lambda$ is the total intensity of the radiation and is called the spectral radiation intensity of the blackbody.

7.2.5 Kirchhoff's law

Consider an enclosure as shown in fig.7.2 and a body is placed inside the enclosure. The radiant heat flux (q) is incident onto the body and allowed to come into temperature equilibrium. The rate of energy absorbed at equilibrium by the body must be equal to the energy emitted.

$$EA = \alpha q A$$

$$E = \alpha q \qquad (7.14)$$

where, E is the emissive power of the body, α is absorptivity of the of the body at equilibrium temperature, and A is the area of the body.

Now consider the body is replaced by a blackbody i.e. $E \rightarrow E_b$ and $\alpha = 1$, the equation 7.14 becomes

$$E_b = q \tag{7.15}$$

Dividing eq. 7.14 by eq.7.15,

$$\frac{E}{E_b} = \alpha \tag{7.16}$$

At this point we may define emissivity, which is a measure of how good the body is an emitter as compared to blackbody. Thus the emissivity can be written as the ratio of the emissive power to that of a blackbody,

$$\frac{E}{E_b} = \epsilon \tag{7.17}$$

On comparing eq.7.16 and eq.7.17, we get

$$\epsilon = \alpha \tag{7.18}$$

Equation 7.18 is the Kirchhoff's law, which states that the emissivity of a body which is in thermal equilibrium with its surrounding is equal to its absorptivity of the body. It should be noted that the source temperature is equal to the temperature of the irradiated surface. However, in practical purposes it is assumed that emissivity and absorptivity of a system are equal even if it is not in thermal equilibrium with the surrounding. The reason being the absorptivity of most real surfaces is relatively insensitive to temperature and wavelength. This particular assumption leads to the concept of grey body. The emissivity is considered to be independent of the wavelength of radiation for grey body.

7.3 Grey body

If grey body is defined as a substance whose monochromatic emissivity and absorptivity are independent of wavelength. A comparative study of grey body and blackbody is shown in the table 7.2.

Table-7.2: Comparison of grey and blackbody

Blackbody	**Grey body**
Ideal body	Ideal body
Emissivity ($\in$) is independent of wavelength	Emissivity ($\in$) is independent of wavelength
Absorptivity (α) is independent of wavelength	Absorptivity (α) is independent of wavelength
$\varepsilon = 1$	$\varepsilon < 1$
$\alpha = 1$	$\alpha < 1$

Illustration 7.1

The surface of a blackbody is at 500 K temperature. Obtain the total emissive power, the wavelength of the maximum monochromatic emissive power.

Solution 7.1

Using eq. 7.12, the total emissive power can be calculated,

$$E_b = \sigma T^4$$

where, σ (= 5.67 X 10^{-8} W/m^2/K^4) is the Stefan-Boltzmann constant. Thus at 500 K,

$$E_b = (5.67 \text{ X } 10^{-8})(5000^4) \ W/m^2$$

$$E_b = 354.75 \ W/m^2$$

The wavelength of the maximum monochromatic emissive power can be obtained from the Wien's law (eq. 7.11),

$$\lambda_{max} T = 2898$$

$$\lambda_{max} = \frac{2898}{500} = 5.796 \, \mu m$$

7.4 Radiative heat exchanger between surfaces

Till now we have discussed fundamental aspects of various definitions and laws. Now we will study the heat exchange between two or more surfaces which is of practical importance. The two surfaces which are not in direct contact, exchanges the heat due to radiation phenomena. The factors those determine the rate of heat exchange between two bodies are the temperature of the individual surfaces, their emissivities, as well as how well one surface can see the other surface. The last factor is known as view factor, shape factor, angle factor or configuration factor.

7.4.1 View factor

In this section we would like to find the energy exchange between two black surfaces having area $A_1$and A_2, respectively, and they are at different temperature and have arbitrary shape and orientation with respect to each other. In order to find the radiative heat exchange between the bodies we have to first define the view factor as

F_{12} = fraction of the energy leavings surface 1 which reaches surface 2

F_{21} = fraction of the energy leaving surface 2 which reaches surface 1 or in general,

F_{mn} = fraction of the energy leaving surface m which raches surface n

Thus the energy leaving surface 1 and arriving at surface 2 is $E_{b1}A_1F_{12}$ and the energy leaving surface 2 and arriving at surface 1 is $E_{b2}A_2F_{21}$. All the incident radiation will be absorbed by the blackbody and the net energy exchange will be,

$$Q = E_{b1}A_1F_{12} - E_{b2}A_2F_{21}$$

At thermal equilibrium between the surfaces $Q_{12} = 0$ and $E_{b1} = E_{b2}$, thus

$0 = E_{b1} (A_1F_{12} - A_2F_{21})$

$$A_1F_{12} - A_2F_{21} \qquad (7.19)$$

Equation 7.19 is known as reciprocating relation, and it can be applied in general way for any blackbody surfaces.

$$A_iF_{ij} - A_jF_{ji} \qquad (7.20)$$

Though the relation is valid for blackbody it may be applied to any surface as long as diffuse radiation is involved.

7.4.1.1 Relation between view factors

In this section we will develop some useful relation of view factor considering fig. 7.5

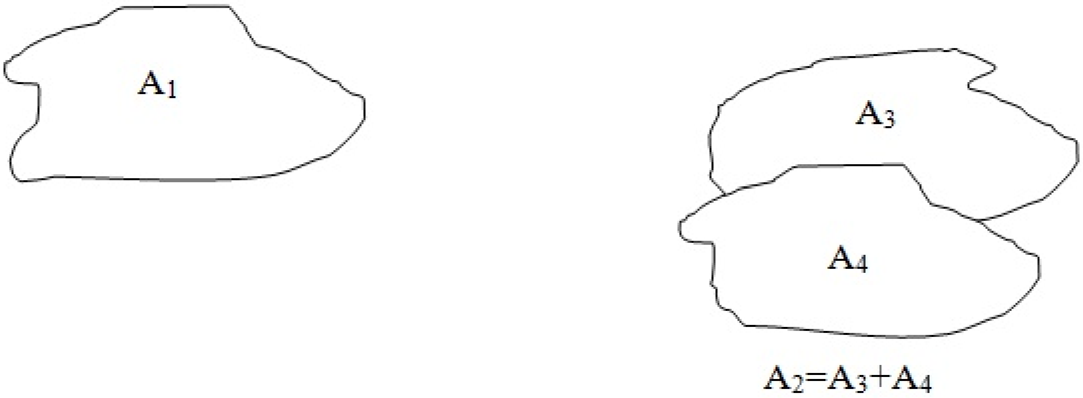


Fig. 7.5: Exchange of energy between area A1 and A2 (A is area of blackbody)

View factor for radiation from A_1 to the combined area A_2,

$$F_{12} = F_{13} + F_{14} \qquad (7.21)$$

and using the reciprocating relations for surface 1 and 4,

$$A_1F_{14} = A_4F_{41} \qquad (7.22)$$

Using eq. 7.21 and 7.22,

$$F_{41} = \frac{A_1}{A_4} F_{14}$$

$$F_{41} = \frac{A_1}{A_4} (F_{12} - F_{13})$$

Thus the unknown view factor F_{14} can be estimated if the view factors F_{12} and F_{13}, as well as their areas are (A_1, A_2) known.

Now, consider a flat plate (for eg.) which is emitting the radiation, it can be understood that the radiation of the flat plat cannot fall on its own surface (partly or fully). Such kind or surfaces are termed as "not able to see itself". In such situations,

$$F_{11} = F_{22} = F_{33} = F_{44} = 0$$

However, if the surface can see itself like concave curved surfaces, which may thus see themselves and then the shape factor will not be zero in those cases.

Another property of the shape factor is that when the surface is enclosed, then the following relation holds,

$$\sum_{j=1}^{n} F_{ij} = 1 \tag{7.23}$$

where, F_{ij} is the fraction of the total energy leaving surface i which arrives at surface j.

In case of N-walled enclosure, some of the view factors may be evaluated from the knowledge of the rest and the total N^2 view factors may be represented in square matrix form shown below,

$$\begin{bmatrix} F_{11} & F_{12} & \cdots F_{1N} \\ F_{21} & F_{22} & \ldots F_{2N} \\ \cdots & \cdots & \cdots\cdots \\ F_{N1} & F_{N2} & \cdots F_{NN} \end{bmatrix}$$

Illustration 7.2

A pipe having 10 cm of diameter is carrying saturated steam at 8 bar of absolute pressure. The pipe runs through a room. The wall of the room is at 300 °K. A portion around 1 m of the pipe insulation is damaged and exposed to the room atmosphere. Calculate the net rate of heat loss from the pipe by radiation.

Solution 7.2

The emissivity of the pipe surface is not given so it may be considered black. Moreover, since the room may be big compared to the surface area of the pipe, the room may also be considered to be a blackbody.

We can write $F_{11} + F_{12} = 1$.

The value of $F_{11} = 0$, as the pipe cannot see itself.

Thus F_{12}, the view factor (1-pipe, 2-room) will be 1.

The net rate heat loss due to radiation,

$$Q_{12} = A_{pipe}\epsilon_{pIpe}F_{12}\sigma\left(T_{pipe}^{4} - T_{room}^{4}\right)$$

T_{pipe} can be obtained by the temperature of the steam at the prevailing pressure with the help of steam table = 450 K.

σ (= 5.67×10^{-8} W/m^2/K^4)

On putting the value,

$Q_{12} = \{\pi(0.1)(1)\}(1)(1)(5.67 \times 10^{-8})\{450^4 - 300^4\}$

$Q_{12} = 586$ *W*

7.5 Heat exchange between non blackbodies

Evaluation of radiative heat transfer between black surfaces is relatively easy because in case of blackbody all the radiant energy which strikes the surface is absorbed. However, finding view factor is slightly complex, but once it can be done, finding heat exchange between the black bodies is quite easy.

When non blackbodies are involved the heat transfer process becomes very complex because all the energy striking on to the surface does not get absorbed. A part of this striking energy reflected back to another heat transfer surface, and part may be reflected out from the system entirely. Now, one can imagine that this radiant energy can be reflected back and forth between the heat transfer surfaces many times.

In this section, we will assume that all surfaces are in the analysis are diffuse and uniform in temperature and that the reflective and emissive properties are constant over all surfaces.

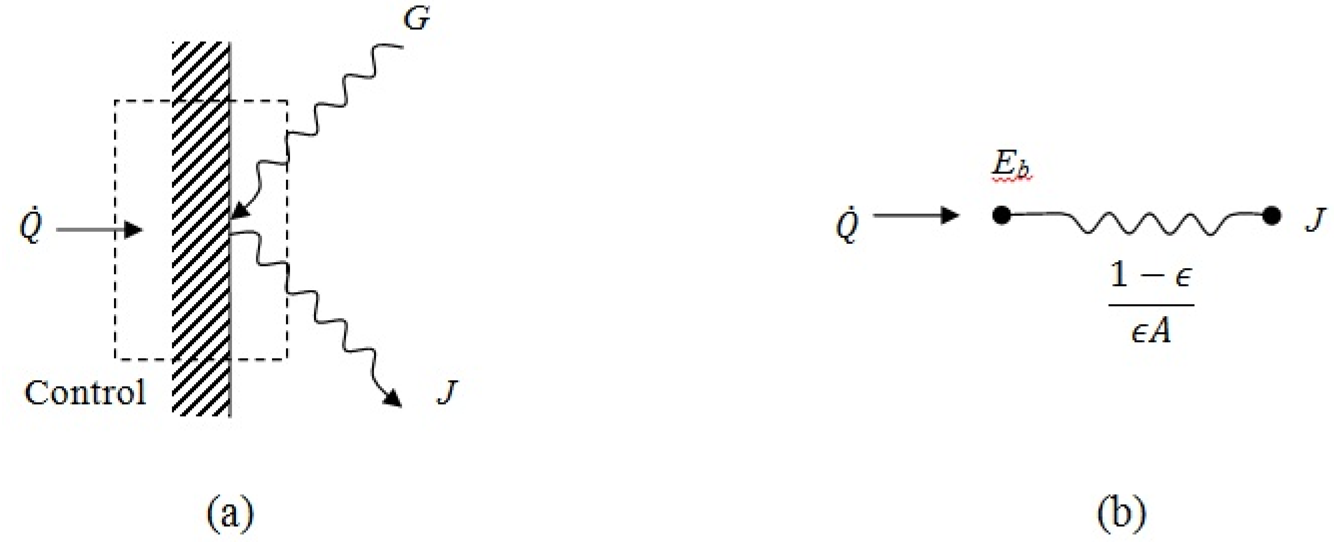


Fig. 7.6: (a) Surface energy balance for opaque surface (b) equivalent electrical circuit

It is also assumed that the radiosity and irradiation are uniform over each surface. As we have already discussed that the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted (as opaque body), or

$$J = \epsilon E_b + \rho G \tag{7.24}$$

where, ϵ is the emissivity and E_b is the blackbody emissive power. Because the transmissivity is zero due to opaque surface and absorptivity of the body (grey) will be equal to its emissivity by Kirchhoff's law.

$$\rho = 1 - \alpha = 1 - \epsilon$$

Thus, eq.7.24 becomes

$$J = \epsilon E_b + (1 - \epsilon)G \tag{7.25}$$

The net energy leaving the surface is the difference between the radiosity and the irradiance (fig.7.6a),

$$\frac{\dot{Q}}{A} = J - G = \epsilon E_b + (1-\epsilon)G - G$$

$$\frac{\dot{Q}}{A} = \frac{\epsilon A}{1-\epsilon}(E_b - J)$$

$$\dot{Q} = \frac{E_b - J}{(1-\epsilon)/\epsilon A} \tag{7.26}$$

The eq.7.26 can be analogous to the electrical circuit as shown in fig.7.6(b). The numerator of the eq.7.26 is equivalent to the potential difference, denominator is equivalent to the surface resistance to radiative heat, and left part is equivalent to the current in the circuit.

In the above discussion we have considered only one surface. Now we will analyse the exchange of radiant energy by two surfaces, A_1 and A_2, as shown in the fig.7.7a.

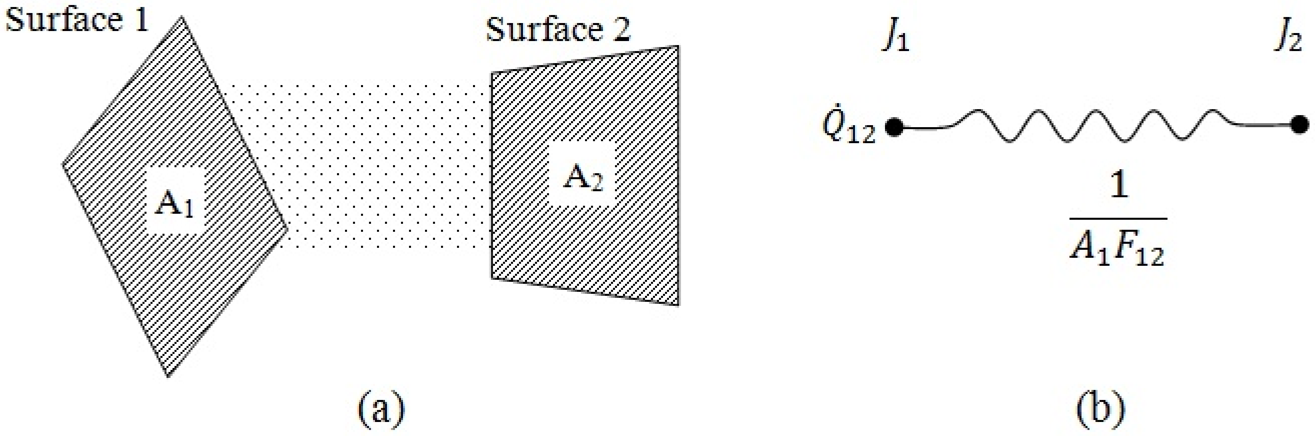


Fig. 7.7: (a) Energy exchange between two surfaces, (b) equivalent circuit diagram

The radiation which leaves surface 1, the amount that reaches surface 2 is

$$J_1A_1F_{12}$$

Similarly, the radiation which leaves system 2, the amount that reaches surface 1 is

$$J_2A_2F_{21}$$

The net energy transfer between the surfaces,

$$\dot{Q}_{12} = J_1A_1F_{12} - J_2A_2F_{21}$$

Reciprocity theorem states that

$$A_1F_{12} = A_2F_{21}$$

$$\Rightarrow \dot{Q}_{12} = (J_1 - J_2)A_1F_{12} = (J_1 - J_2)A_2F_{21}$$

$$\Rightarrow \dot{Q}_{12} = \frac{(J_1 - J_2)}{1/\,A_1F_{12}} \tag{7.27}$$

It also resembles an electrical circuit shown in fig.7.7b. The difference between eq.7.26 and 7.27 is that in eq.7.27 the denominator term is space resistance instead of surface resistance.

Now, to know, the net energy exchange between the two surfaces we need to add both the surface resistances along with the overall potential as shown in the fig.7.8. Here the surfaces see each other and nothing else.

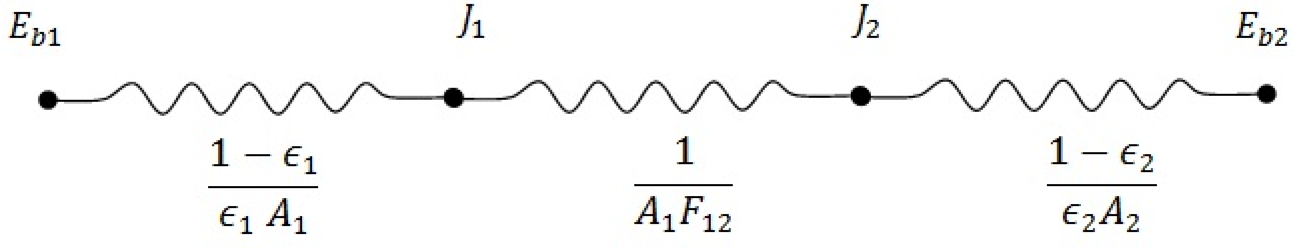


Fig. 7.8: Radiative nature for two surfaces which can see each other nothing else

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}} \tag{7.28}$$

7.6 Radiation shield

Till now we have discussed about the radiative heat transfer from one surface to another without any interfering surface in between. Here we will discuss about an interfering shield in between, which is termed as radiation shield. A radiation shield is a barrier wall of low emissivity placed between two surfaces which reduce the radiation between the bodies. In fact, the radiation shield will put additional resistance to the radiative heat transfer between the surfaces as shown in fig.7.9.

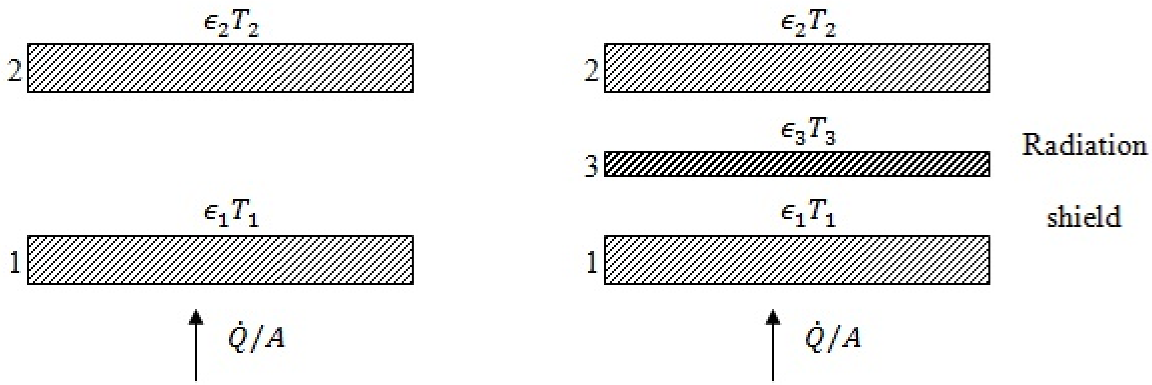


Fig. 7.9: Radiation between two large infinite plates (a) without and (b) with radiation shield

Considering fig.7.9(b) and the system is at steady state, and the surfaces are flat (F_{ij} because each plate is in full view of the other). Moreover, the surface are large enough and $\frac{A_1}{A_2} \approx 1$ may be considered and the equivalent blackbody radiation energy may be written as $E_b = \sigma T^4$.

Thus, eq. 7.28 becomes

$$\left.\frac{\dot{Q}}{A}\right|_{net} = \left.\frac{\dot{Q}_{13}}{A_1}\right|_{net} = \left.\frac{\dot{Q}_{32}}{A_3}\right|_{net} = \frac{\sigma\left(T_1^4 - T_3^4\right)}{\frac{1}{\epsilon_1}+\frac{1}{\epsilon_3}-1} = \frac{\sigma\left(T_3^4 - T_2^4\right)}{\frac{1}{\epsilon_3}+\frac{1}{\epsilon_2}-1} \tag{7.29}$$

In order to have a feel of the role of the radiation shield, consider that the emissivities of all the three surfaces are equal.

$$\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon$$

Then it can be seen that the heat flux is just one half of that which would be experienced if there were no shield present.
In similar line we can deduce that when n-shields are arranged between the two surfaces then,

$$\left(\frac{\dot{Q}}{A}\right)_{net_{with\ shield}} = \frac{1}{n+1}\left(\frac{\dot{Q}}{A}\right)_{without\ shield} \tag{7.30}$$

7.7 Electrical network for radiation through absorbing and transmitting medium

The previous discussions were based on the consideration that the heat transfer surfaces were separated by a completely transparent medium. However, in real situations the heat transfer medium absorbs as well as transmits. The examples of such medium are glass, plastic film, and various gases.

Consider two non-transmitting surfaces (same as in fig. 7.8) are separated by a transmitting and absorbing medium. The medium may be considered as a radiation shield which see themselves and others. If we distinguish the transparent medium by *m* and if the medium is non-reflective (say gas) then using Kirchhoff's law,

$$\alpha_m + \tau_m = 1 = \epsilon_m + \tau_m \tag{7.31}$$

The energy leaving surface 1 which is transmitted through the medium and reaches the surface 2 is,

$$J_1 A_1 F_{12} \tau_m$$

and that which leaves surface 2 and arrives at surface 1 is,

$$J_2 A_2 F_{21} \tau_m$$

Therefore, the net exchange in the transmission process is therefore,

$$\dot{Q}_{12} = A_1 F_{12} \tau_m (J_1 - J_2)$$

Using eq. 7.31,

$$\dot{Q}_{12} = \frac{(J_1 - J_2)}{\left(\frac{1}{A_1 F_{12}(1-\epsilon_m)}\right)}$$

Thus the equivalent circuit diagram is shown in fig. 7.9

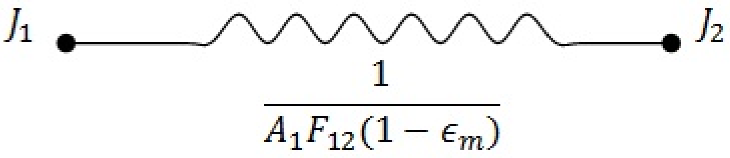


Fig. 7.9. Equivalent electrical circuit for radiation through gas

7.8 Radiation combined with conduction and convection

In industrial processes, in general, the heat transfer at higher temperature has significant portion of radiation along with conduction and convection. For example, a heated surface is shown in the fig. 7.10 with all the three mechanism of heat transfer.

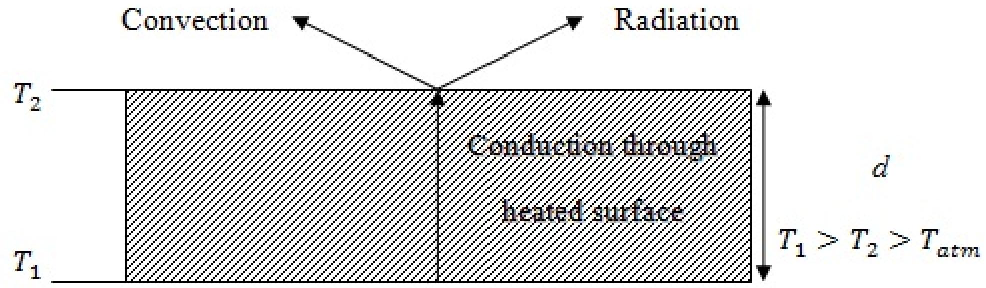


Fig. 7.11: Radiation combined with conduction and convection

At steady state

Heat flux by conduction = heat flux by convention + heat flux by radiation

$$\frac{K}{d}(T_1 - T_2) = h(T_2 - T_{atm}) + \epsilon\sigma(T_2^4 - T_{atm}^4)$$

where, h is the heat transfer coefficient at the surface in contact (outer surface) with atmosphere due to natural and forced convection combined together, ϵ is the emissivity of the outer surface, and T_{atm} is the atmospheric temperature.

In any process industry, we need to transfer heat for different operations (like cooling, heating, vaporizing, or condensing) to or from various fluid streams in various equipment like condensers, water heaters, re-boilers, air heating or cooling devices etc., where heat exchanges between the two fluids. In a chemical process industry, the heat exchanger is frequently used for such applications. A heat exchanger is a device where two fluids streams come into thermal contact in order to transfer the heat from hot fluid to cold fluid stream.

In this chapter, we will discuss about the technical analysis of the heat exchangers along with the method for predicting heat exchanger performance and operational parameters. Moreover the discussion on heat exchanger size will also be discussed. However, we will not discuss the economics (though discuss the heat exchanger size) of the heat exchanger. We will consider that the heat transfer will primarily be taken by conduction and convection only. We will describe the commonly used heat exchangers and their important characteristics.

In general heat exchangers may be categorized into two general classes depending on the relative orientation of the flow direction of the two fluid streams. If the two streams cross one another in space, usually at right angles, the heat exchangers are known as cross flow heat exchanger as shown in the fig. 8.1(a).

In the second class of heat exchanger the two streams move in parallel direction in space. The usual shell and tube heat exchanger or concentric pipe exchanger or double tube exchanger is the most frequently used exchanger in the class. Two situations may arise when the fluid flow in the parallel direction, one in which the fluids flow in same direction and the other in which the fluids flow in opposites direction. "Parallel –flow" or "Co-current flow" is used when the flow is in same direction and counter current is used when the fluid flow is in the opposite direction.

Before understanding the principle of heat exchanger we would first understand it from the point of construction.

8.1 Elements of shell and tube heat exchanger

We will discuss shell and tube heat exchanger as they or the most commonly used heat exchangers in the chemical process industries. Schematic of a typical shell and tube heat exchanger is shown in fig. 8.2.

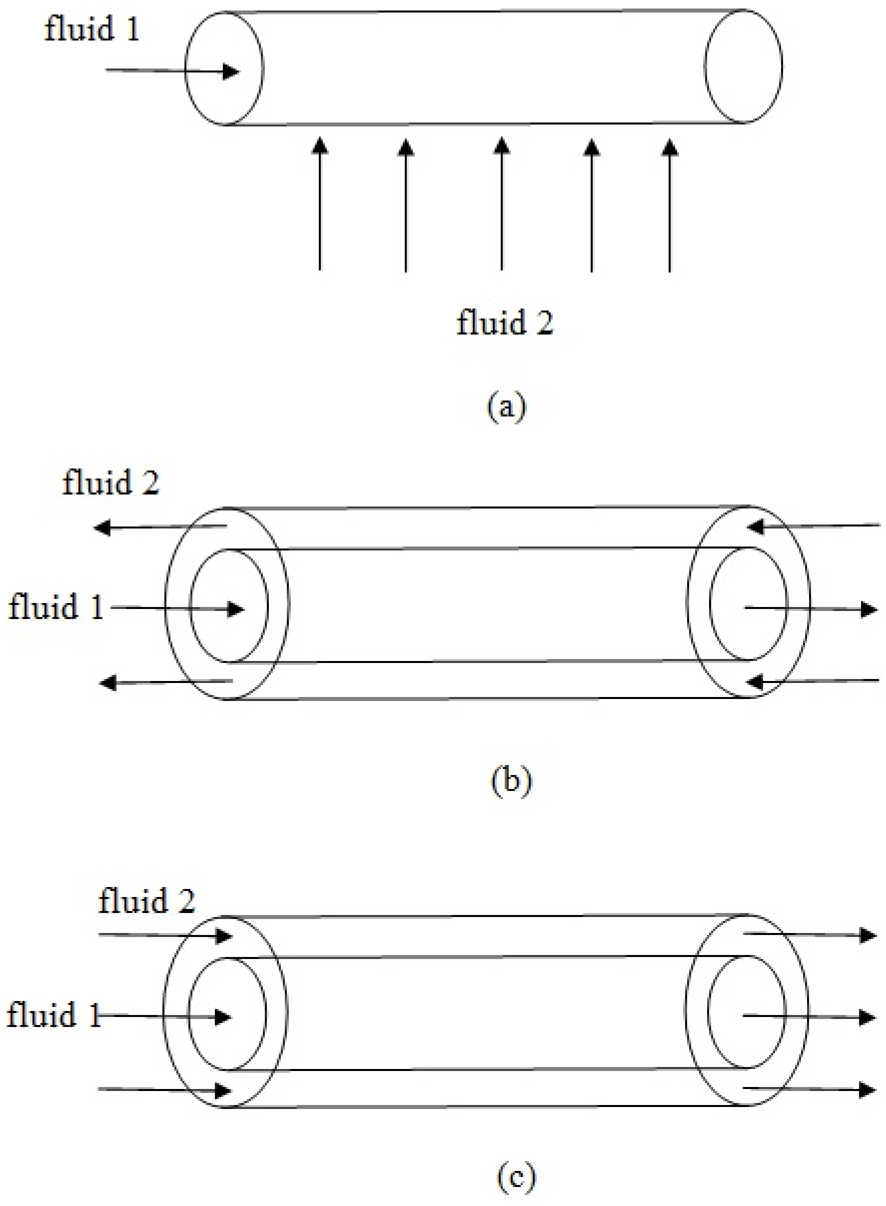
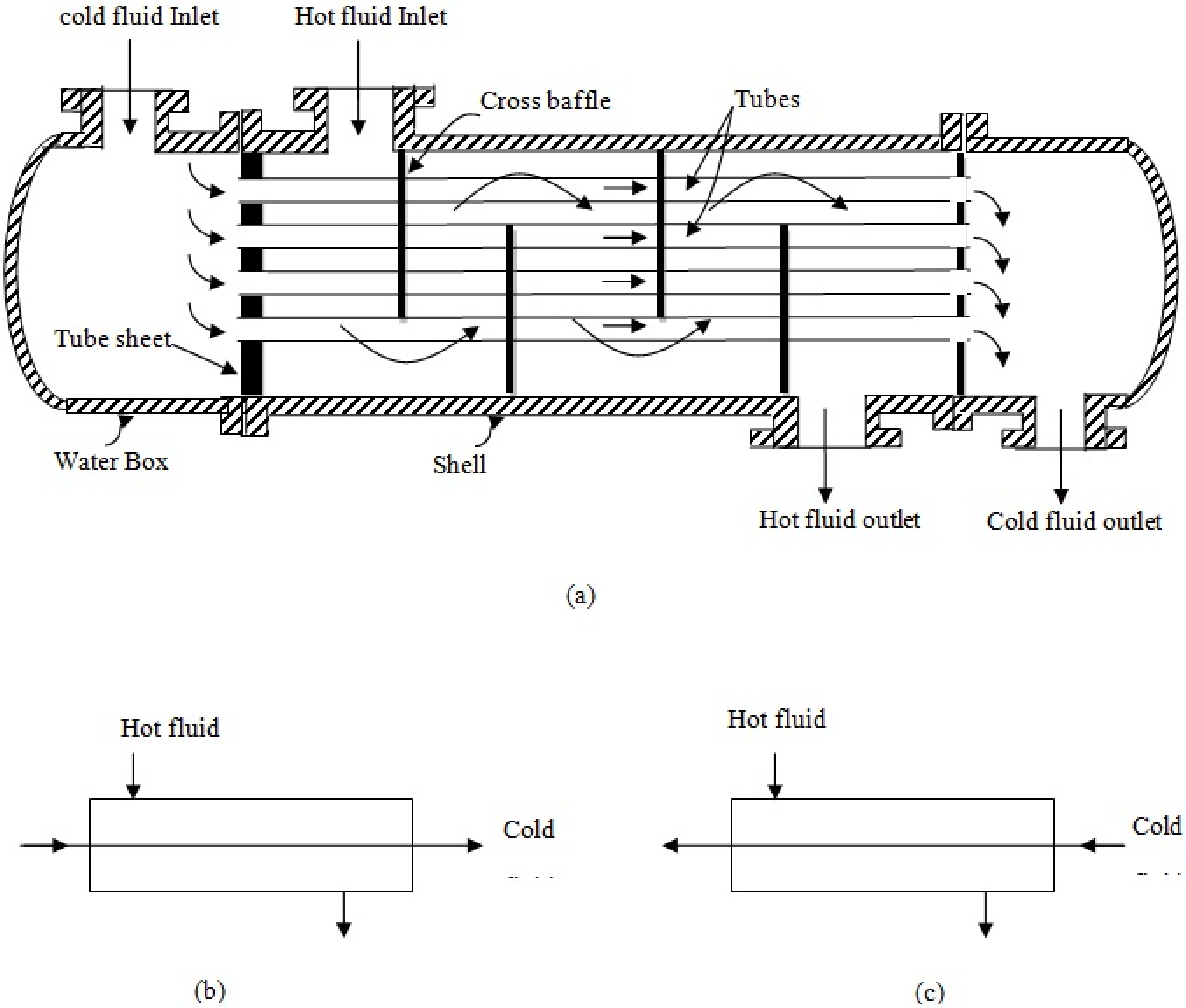


Fig. 8.1: Orientation of fluid stream in heat exchanger (a) cross flow (b) counter current flow (c) parallel flow

The heat exchanger as shown in fig. 8.2 consists of a bundle of tubes properly secured at either ends in tube sheets. The tube sheets are drilled plates into which the tubes are fixed up using different technique to have leak proof joints. The entire tube bundles shown in the fig. 8.3 is placed inside a closed shell, which seals around the tube sheet periphery to form the two immiscible zones for hot and cold fluids are shown in fig.8.4.



(a)

(b)

(c)

Fig.8.2: A schematic of (a) one-shell pass, one tube pass heat exchanger; (b) parallel flow; and (c) counter flow

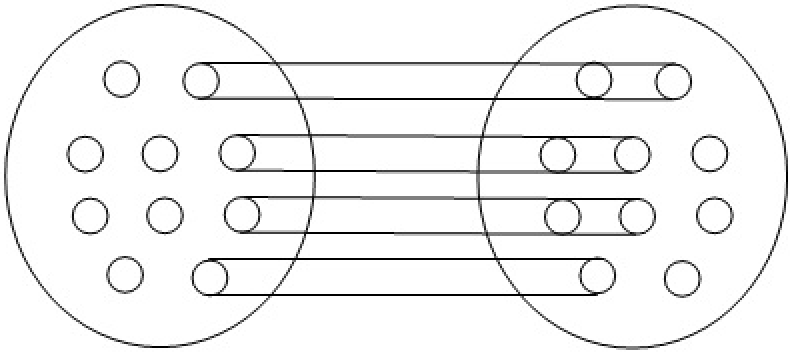


Fig. 8.3 Tube bundle fitted in two sheets

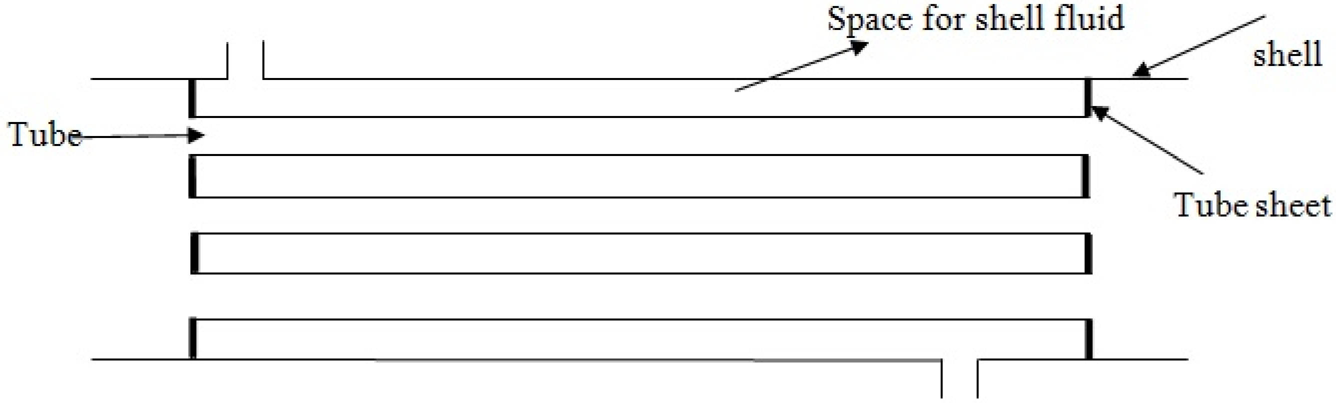


Fig. 8.4: Tube bundle inside a shell

One fluid flows through the tubes while the other fluid flows around the outside of the tubes, it is the space between the tube sheets and enclosed by the outer shell.

For a thorough distribution of the shell side fluid, baffles are placed normal to the tube bundle. This baffle creates turbulence in the shell side fluid and enhances the transfer coefficients for the shell side flow.

Fig. 8.2 shows the simplified diagram of a shell and tube heat exchanger, showing a few of the important components. Infact, the present heat exchanger used in the process industry are quite complex and have improved design such as factors for thermal expansion stresses, tube fouling due to contaminated fluids, ease of assembly and disassembly, size, weight, etc.

The heat exchanger shows in fig. 8.2 is having one shell and one tube pass since both the shell and tube side fluid make a single traverse through the heat exchanger. Thus, this type of shell-and-tube heat exchangers is designated as 1-1 exchanger. If we desire to pass the tube fluid twice, then it is designated as 1-2 exchangers. Similarly, if there are 2 shell pass and 4 tube pass, the designation will be 2-4 exchanger. The number of pass in tube side is done by the pass partition plate. A pass particular plate or pass divider as shown in fig.8.5. The shell side pass can be creator by a flat plate as shown in fig.8.6.

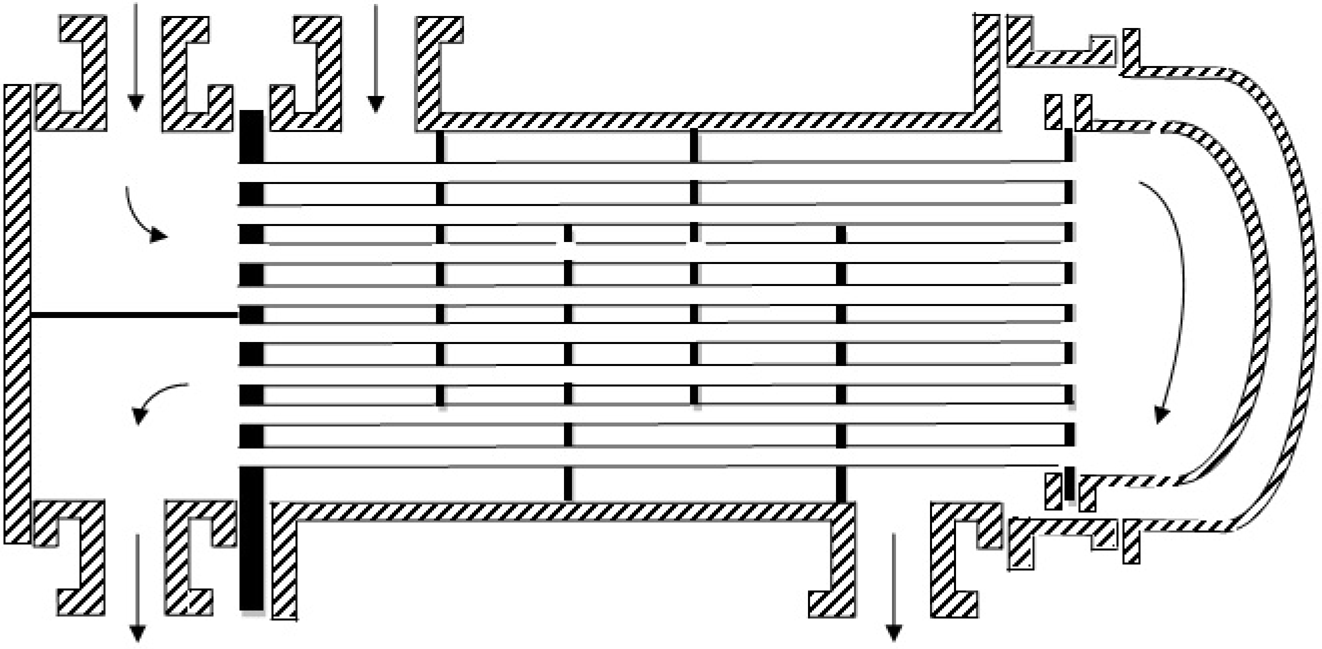


Fig. 8.5: 1-2 exchanger showing pass partition plate

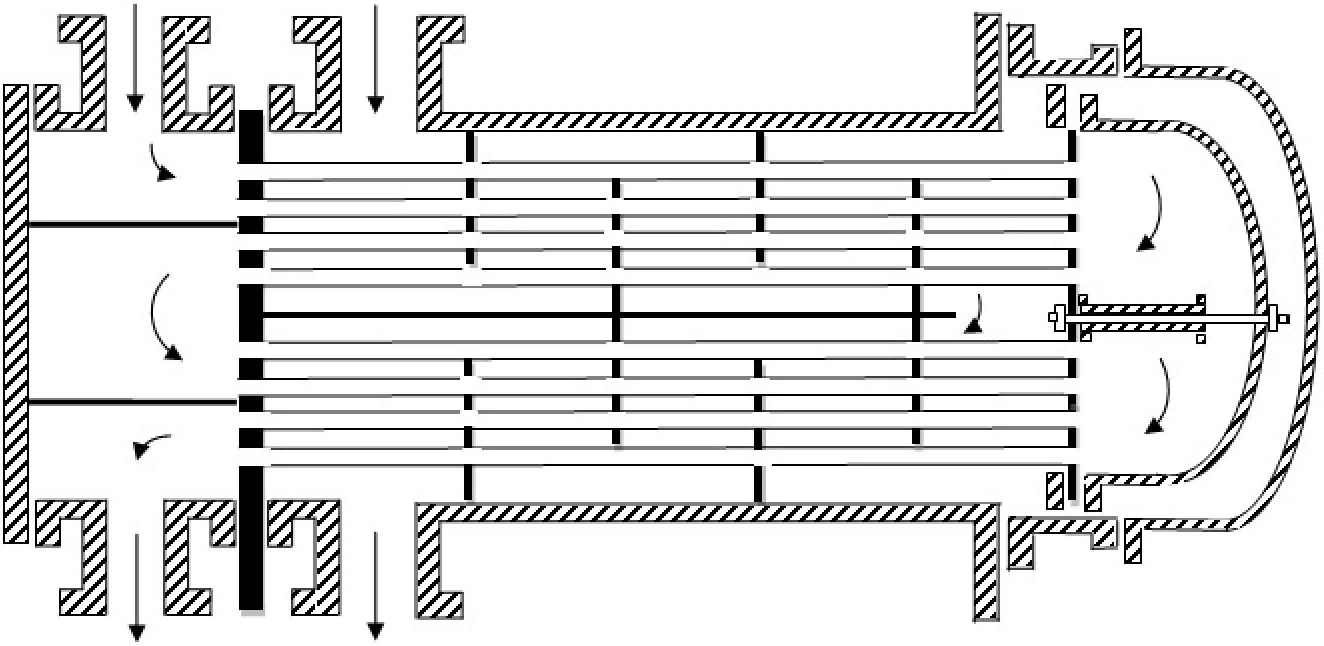
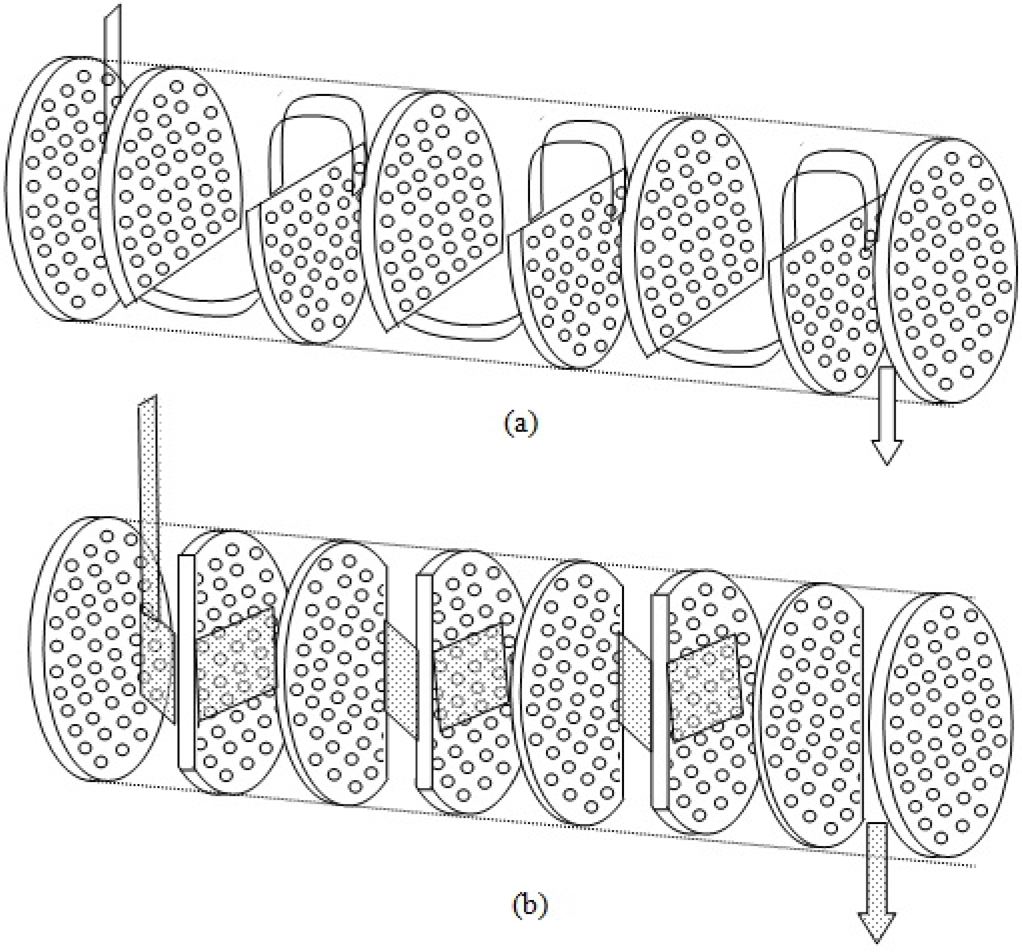
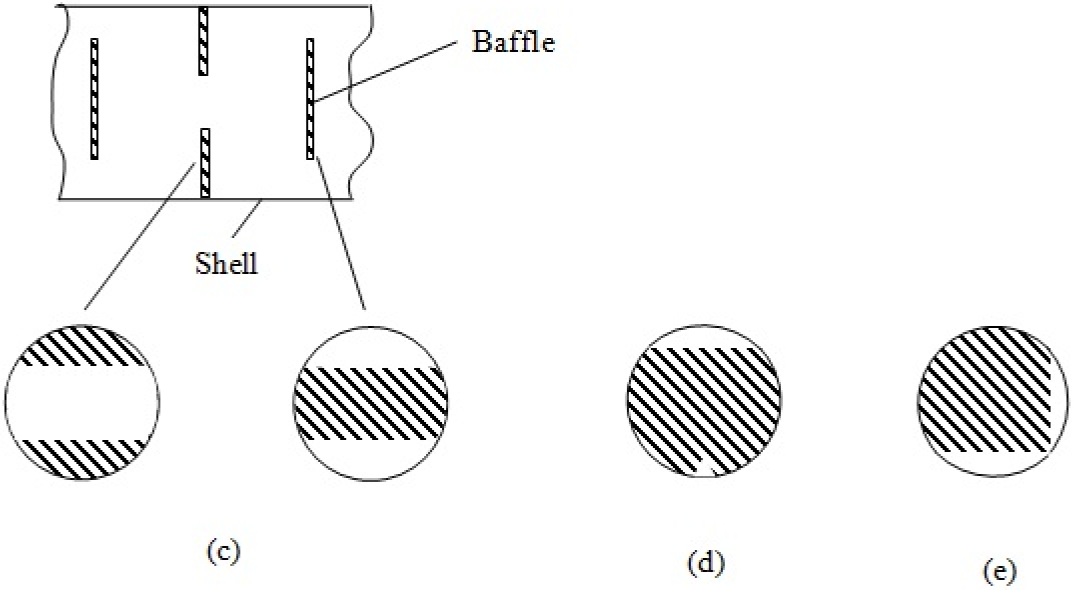


Fig. 8.6: 2-4 exchanger showing shell and tube passes

It can be understood that for a given number of tubes; the area available for flow of the tube side fluid is inversely proportional to the number of passes. Thus, on increasing the pass the area reduces and as a result the velocity of fluid in the tube increases and henceforth the Reynolds number increases. It would result in increased heat transfer coefficient but at the expense of high pressure drop. Generally, even numbers of tube passes are used for the multi-pass heat exchangers.

Figure 8.2 shows some baffles. These baffles (or shell-side baffles) are a metal plate usually in the form of the segment of a circular disc having holes to accommodate tubes. Shell-side baffles have two functions. The first is to create turbulence in the shell side fluid by changing the flow pattern parallel or cross flow to the tube bundles and thus increases the shell side heat transfer coefficient. The second major function of these baffles is to support the tube all along its length otherwise the tube may bend. Moreover, these baffles may have horizontal or vertical cuts (segmental baffle) as shown in fig.8.7.





(c) (d) (e)

Fig.8.7: Baffles; (a) horizontal cut baffles; (b) Vertical cut baffles; (c, d and e)the shaded region show the baffle area

The cut portion of the baffle, which is called baffle window, provides the area for flow of the shell fluid. The baffle window area ranges from 15% to 50%. At 25% cut segmental baffle means that the area of the cut-out portion is 25% of the area of the baffle. The spacing between the baffles is an important aspect. A larger baffles spacing reduces the shell side pressure drop but at the same time decreases the turbulence and heat transfer coefficient. Smaller baffle spacing increases the turbulence and heat transfer coefficient. However, the pressure drop may increase significantly, thus the advantage attained due to the higher heat transfer coefficient may be nullified. Therefore baffle spacing is selected considering the allowable shell side pressure drop and the heat transfer coefficient desired. A rule of thumb is that the minimum spacing of segmental baffles is one by fifth of the shell diameter or 5 cm, whichever is larger.

8.2 Thermal design of heat exchangers

The mechanical design is done by the mechanical engineers on the inputs of chemical engineers and using the codes. The most widely used code in Tubular Exchanger Manufactures Associations (TEMA). This USA code along with ASME selection VIII (unfired pressure vessel) code is used together for the mechanical design of the heat exchanger. The Indian code for the heat exchanger design IS 4503.

Here we would discuss about the process design (or thermal design) leading to the sizing of the heat exchanger. Before understanding design steps, it is necessary to understand the following for the heat exchanger.

8.2.1 Overall heat transfer coefficient

As understood by the previous discussion that generally heat exchangers are tubular in nature (Note: we are not discussing about plate type heat exchangers). Thus we can easily find out the overall heat transfer coefficient based on our previous knowledge. Figure 8.8 shows a simplest form (double pipe heat exchanger) of tubular heat exchanger, where fluid A is being heated by fluid B in a co-current flow pattern. The inside and outside radii of the inner tube is represented r_i and r_o. The length of the exchanger for heat transfer is considered as L for section 1 to 2.

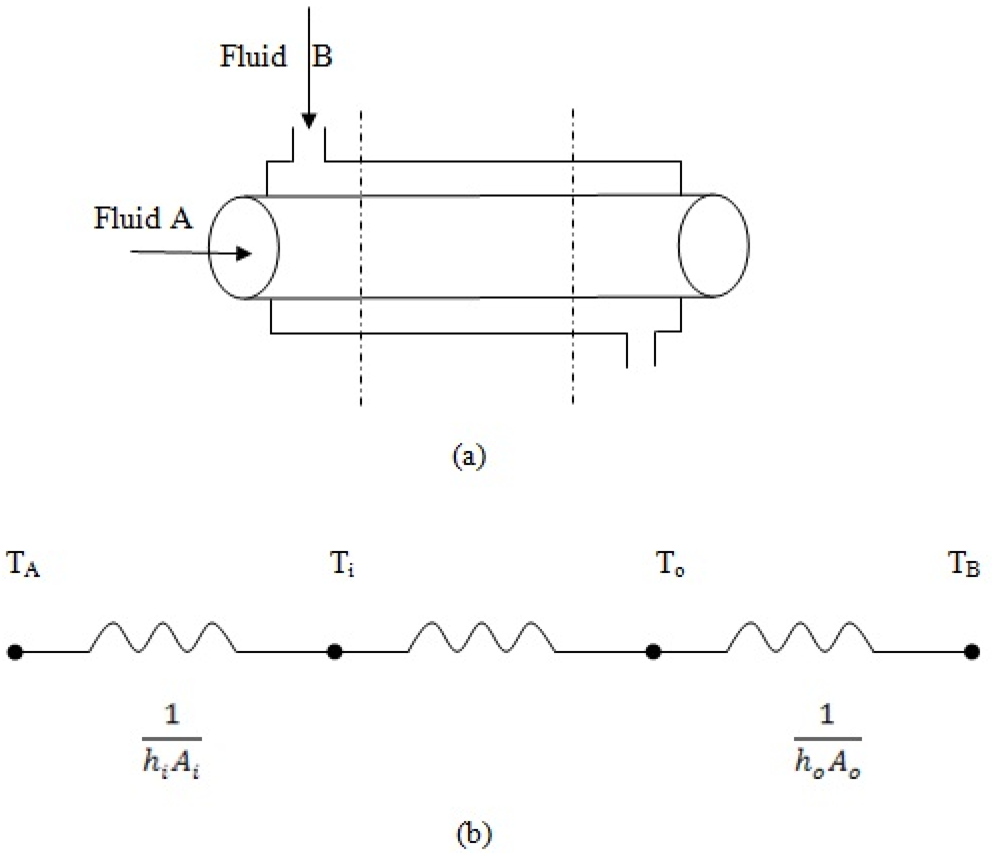


Fig.8.8: (a) Schematic of a double pipe heat exchanger (b) thermal resistance network for overall heat transfer

Thus the rate of heat transfer from the hot fluid to the cold fluid will be represented by eq.8.1,

$$q = \frac{T_A - T_B}{\frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o A_o}} \tag{8.1}$$

The overall heat transfer coefficient;

Based on inside area of the inner pipe (eq.8.2)

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{A_i \ln(r_o/r_i)}{2\pi k L} + \frac{A_i}{h_o A_o}} \tag{8.2}$$

Based on outer side area of the outer pipe (eq.8.3)

$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o \ln(r_o/r_i)}{2\pi k L} + \frac{1}{h_o}} \quad (8.3)$$

8.2.2 Fouling factor or dirt factor

Over a time period of heat exchanger operation the surface of the heat exchanger may be coated by the various deposits present in the flow system. Moreover, the surfaces may become corroded or eroded over the time. Therefore, the thickness of the surface may get changed due to these deposits. These deposits are known as scale. These scales provide another resistance and usually decrease the performance of the heat exchangers. The overall effect is usually represented by dirt factor or fouling factor, or fouling resistance, R_f (Table 8.1) which must have included all the resistances along with the resistances due to scales for the calculation of overall heat transfer coefficient.

The fouling factor must be determined experimentally using eq. 8.4,

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \quad (8.4)$$

Thus to determine the R_f, it is very important to know U_{clean} for the new heat exchanger. The U_{clean} must be kept securely to obtain the R_f, at any time of the exchanger's life.

Table-8.1 Fouling factor of a few of the industrial fluids

	Fouling factor (or resistance) $\frac{hm^2\,{}^\circ C}{k\,cal}\times 10^3$
Liquid	
Fuel oil	1.024
Refrigerant liquids	0.102
Mono-and di-ethanolamine solution	0.409
Gasoline , naphtha and kerosene	0.205
Light gas oil	0.409
Heavy gas oil	0.615
Gases and Vapour	
Solvent vapour	0.205
Air	0.102-0.205
Flue gases	0.205-0.615
Steam (Saturated, oil free)	0.102-0.307
Water	
River water (treated, velocity > 0.6 m/s)	0.205-0.409
Treated boiler feed water	0.102-0.205
Process water	0.205-0.409

8.2.3 Temperature profiles in heat exchangers

Fig. 8.9 shows the temperature profile along the length of a 1-2 exchangers and 2-4 exchangers.

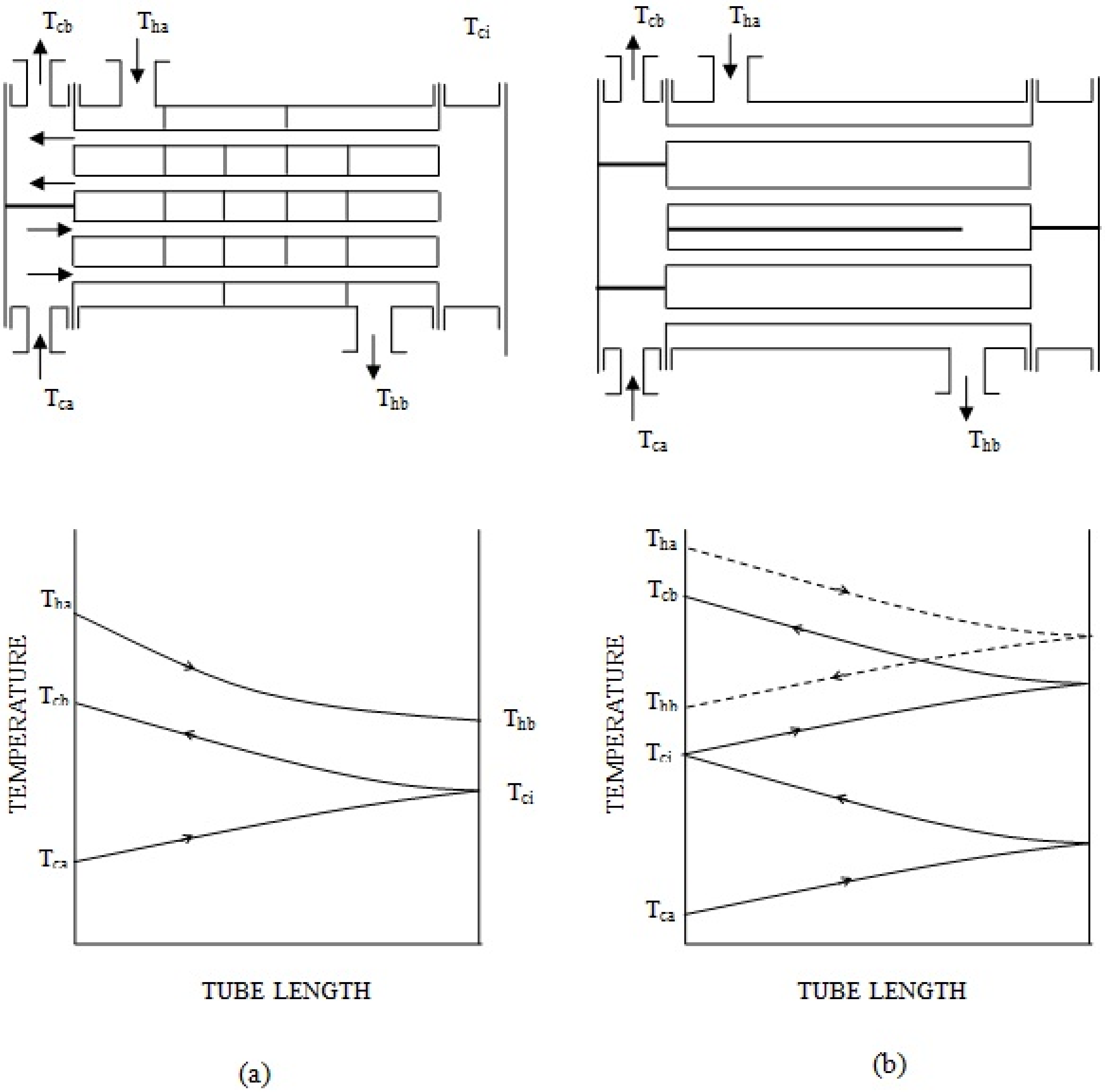


Fig.8.9 Temperature-length curve corresponding to (a) 1-2 exchanger ; (b) 2-4 exchanger

The nomenclature used in the fig.8.9 is described below

T_{ha}: Inlet temperature of hot fluid

T_{hb}: Outlet temperature of hot fluid

T_{ca}: Inlet temperature of cold fluid

T_{cb}: Outlet temperature of cold fluid

T_{ci}: Intermediate temperature of cold fluid

In the above arrangement it is assumed that the hot fluid is flowing in the shell side and cold fluid is flowing in the tube side of the exchangers. The fig.8.9 (a) shows the 1-2 exchangers in which the hot fluid enter into the exchanger from the left side and exits from the right side. The cold fluid enters concurrently that is from the left side to the tube of the exchangers and goes up to right end of the exchangers and returns back to make two tube pass, and exits from the left end of the exchangers. The temperature profile all along the length of the exchanger is shown in the corresponding temperature length profile. Figure8.9 (b) shows the flow direction and corresponding temperature length profile for 2-4 exchangers. The shell side fluid two passes and the tube side fluid has 4-passes in the exchangers.

It can be easily understood that whenever the number of passes is more than one, the flow cannot be truly co-current or counter current. Thus it will be a mix of co-current and counter current flows in any multi pass heat exchangers.

Though the temperature profile of the hot and cold streams can be easily predictable for single pass heat exchangers but for the complex flow modes, the prediction of temperature distribution will be difficult as shown in fig.8.9. As can be seen when 1-2 exchangers was (fig.8.9 (a)) used in co-current mode, the temperature profile was given in the figure. However, if the fluid streams enter in counter current mode a temperature cross may occur sometimes. Temperature cross is described as the positive temperature difference between the cold and the hot fluid, when these fluids leave the exchangers. In that case the cold fluid will attain the maximum temperature inside the exchanger instead of at the exit (fig.8.10).

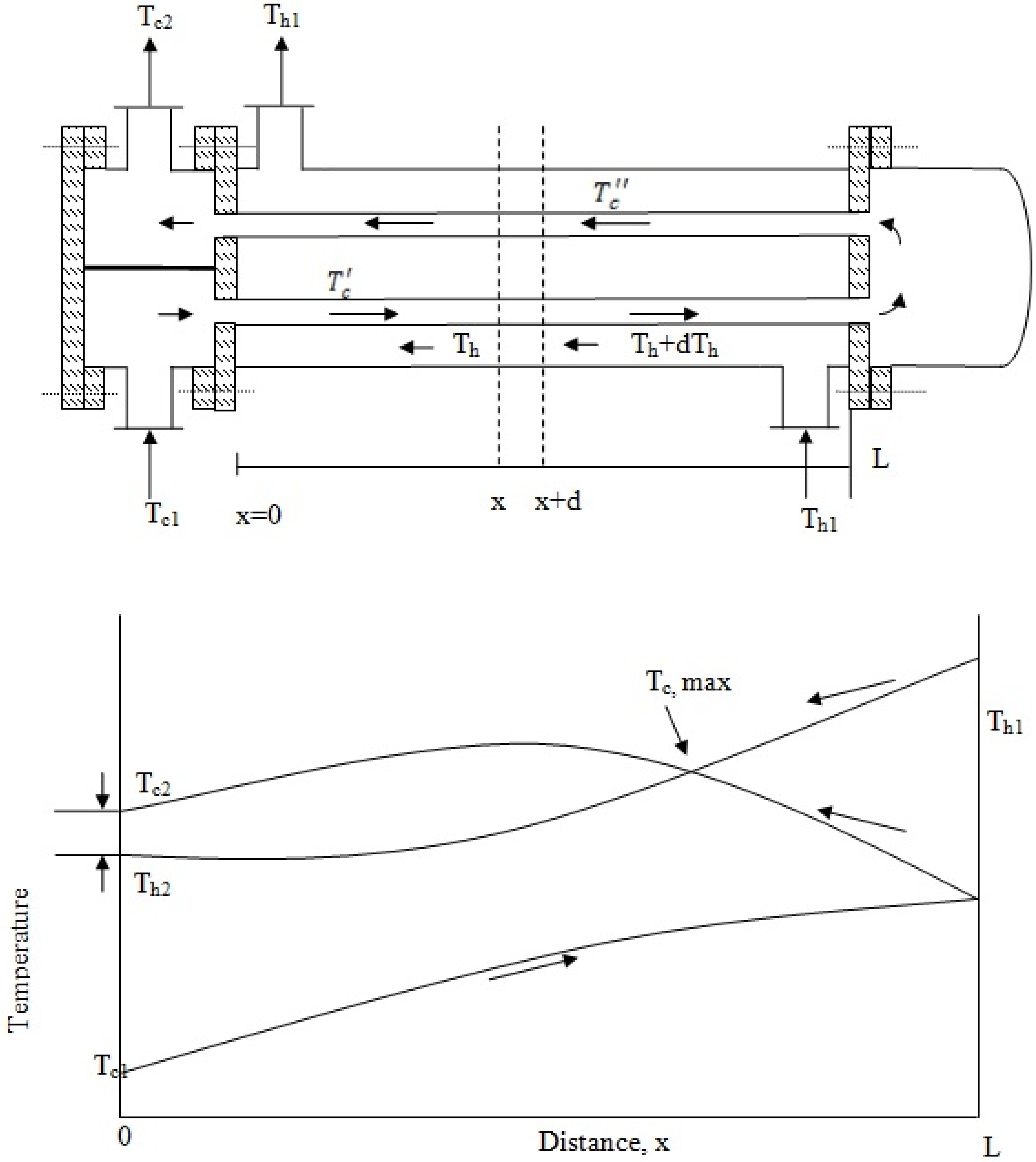


Fig. 8.10: 1-2 flow pattern and temperature profile in exchanger showing cross flow

At this temperature cross, the cold fluid temperature reaches the maximum at a point inside the exchanger and not at its exits. This temperature cross point also coincides with the point of intersection of the temperature profile of the hot fluid and the co-current zone of the cold fluid. The difference (T_{c2} - T_{h2}) is called the temperature cross of the exchanger. However, if the temperature cross does not appear then the (T_{c2} - T_{h2}) is called the approach. Moreover, on careful evaluation it can be seen that for the multi shell side pass a significant length of the exchanger have cross flow pattern in the tube flow when the shell side fluid is migrating from one shell pass to another shell pass. Thus calculating heat transfer co-efficient for shell side becomes little challenging and will be explained in section 8.2.6. Although the parallel flow or counter flow arequite similar, the parallel flow and counter flow heat exchangers differ greatly in the manner in which the fluid temperatures vary as the fluid pass through. The difference can be understood in the figure 8.11.

The fig.8.11 shows an important parameter, mc_p, the product of mass flow rate (m) and the specific heat, c_p, of the fluids. The product mc_p is called the rate of heat capacity. The overall energy balance of the heat exchanger gives the total heat transfer between the fluids, q, expressed by eq.8.5,

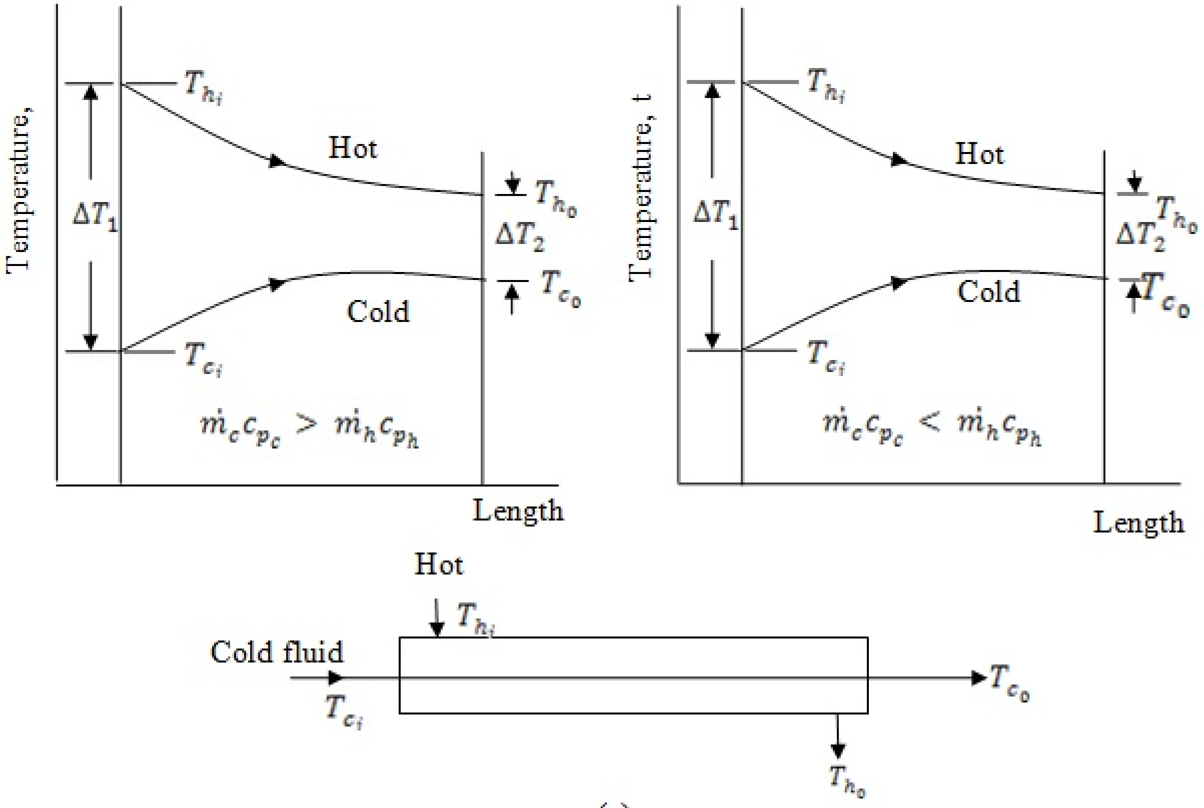
$$q = \dot{m}_c c_{pc}(T_{co} - T_{ci}) = \dot{m}_h c_{ph}(T_{hi} - T_{ho}) \tag{8.5}$$

$$\Rightarrow \frac{\dot{m}_c c_{pc}}{\dot{m}_h c_{ph}} = \frac{(T_{hi} - T_{ho})}{(T_{co} - T_{ci})}$$

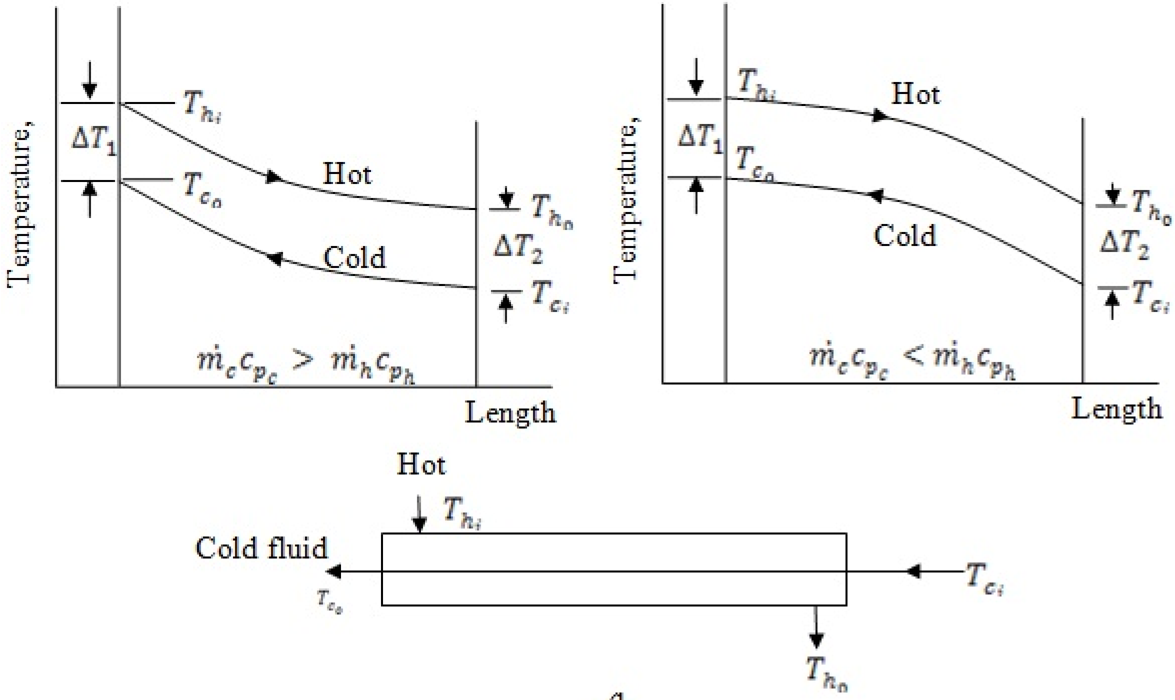
The fig.8.11 shows the relative variation of the two fluid temperatures through the heat exchanger, which is influenced by whether $\dot{m}_c c_{pc}$ is greater or less than $\dot{m}_h c_{ph}$. In particular, for counter flow, examination of the sketches in fig.8.11 shows that limiting condition for maximum heat transfer is determine by whether $\dot{m}_c c_{pc}$ is greater or less than $\dot{m}_h c_{ph}$. When, $\dot{m}_c c_{pc} > \dot{m}_h c_{ph}$ the maximum possible heat transfer is determined by the fact that the hot fluid can be cooled to the temperature of the cold fluid inlet. Thus, for $\dot{m}_c c_{pc} > \dot{m}_h c_{ph}$

$$T_{ho} \rightarrow T_{ci}$$

$$\begin{aligned} q_{max} &= \dot{m}_h c_{ph}(T_{hi} - T_{ho}) \\ &= \dot{m}_h c_{ph}(T_{hi} - T_{ci}) \end{aligned}$$


Temperature,
T_{h_i}
Hot
T_{h_o}
ΔT_1
ΔT_2
T_{c_o}
Cold
T_{c_i}
$\dot{m}_c c_{p_c} > \dot{m}_h c_{p_h}$
Length
Temperature, t
T_{h_i}
Hot
ΔT_1
T_{h_o}
ΔT_2
Cold
T_{c_o}
T_{c_i}
$\dot{m}_c c_{p_c} < \dot{m}_h c_{p_h}$
Length
Hot
T_{h_i}
Cold fluid
T_{c_i}
T_{c_o}
T_{h_o}

(a)


Temperature,
T_{h_i}
ΔT_1
T_{c_o}
Hot
Cold
T_{h_o}
ΔT_2
T_{c_i}
$\dot{m}_c c_{p_c} > \dot{m}_h c_{p_h}$
Length
Temperature,
T_{h_i}
Hot
ΔT_1
T_{c_o}
Cold
T_{h_o}
ΔT_2
T_{c_i}
$\dot{m}_c c_{p_c} < \dot{m}_h c_{p_h}$
Length
Hot
T_{h_i}
Cold fluid
T_{c_o}
T_{c_i}
T_{h_o}

(b

Fig.8.11: Temperature profiles of (a) parallel flow, and (b) counter flow, for different $\dot{m}c_p$ inequalities

For the other case when the limit is determined as the cold fluid is heated to the inlet temperature of the hot fluid:

For,

$$\dot{m}_c c_{pc} < \dot{m}_h c_{ph} : \; T_{co} \rightarrow T_{ho}$$

$$q_{max} = \dot{m}_c c_{pc}(T_{co} - T_{ci})$$

$$= \dot{m}_c c_{pc}(T_{ho} - T_{hi})$$

Thus for the counter flow exchanger, the above two set equations show that the maximum possible heat exchanger is determined in terms of the inlet parameters. The maximum possible heat exchange may be determined (eq.8.6) by the fluid stream having low heat capacity rate;

$$q_{max,ccf} = \left(\dot{m}c_p\right)_{min}(T_{hi} - T_{ci}) \tag{8.6}$$

The subscript *ccf* denotes counter current flow. The $\left(\dot{m}c_p\right)_{min}$ is for the fluid having lower value of $\left(\dot{m}c_p\right)$.

In case of parallel flow, regardless of the relative sizes of the two stream the limiting heat transfer condition is determine by the fact that the two fluid streams approach the same outlet temperature. Thus,

$T_{ho} \rightarrow T_{co}$ condition can be found out by the weighted average of the inlet stream.

$$T_{ho} \rightarrow T_{co} \rightarrow \frac{\dot{m}_c c_{pc} T_{ci} + \dot{m}_h c_{ph} T_{hi}}{\dot{m}_c c_{pc} + \dot{m}_h c_{ph}}$$

Thus, the maximum possible heat transfer may be,

$$q_{max} = \dot{m}_c c_{pc}(T_{co} - T_{ci})$$

$$= \frac{1}{\dfrac{1}{\dot{m}_c c_{pc}} + \dfrac{1}{\dot{m}_h c_{ph}}}(T_{hi} - T_{ci})$$

or,

$$q_{max,pf} = \frac{(\dot{m}c_p)_{min}(T_{hi} - T_{ci})}{1 + \left[(\dot{m}c_p)_{min} / (\dot{m}c_p)_{max}\right]} \tag{8.7}$$

The subscript *pf* represents parallel flow (co-current). From the above discussion and q_{max} equations (8.6 and 8.7) it can be calculated for a given inlet conditions the counter current flow arrangement always has a better potential for heat transfer as compared to parallel flow arrangement.

8.2.4 Why multi-pass exchangers?

The simplest type of heat exchangers is double pipe heat exchangers, which is inadequate for flow rates that cannot readily be handled in a few tubes. If several double pipes are used in parallel, the metal weight required for the outer tubes becomes so large that the shell and tube construction, such as 1-1 exchanger will be helpful. In that one shell serves for many tubes, is economical. The heat transfer coefficient of tube side and shell side fluid is very important and the individual heat transfer coefficients must be high enough to attain high overall heat transfer coefficient. As the shell would be quite large as compared to the tubes, the velocity and the turbulence of the shell side fluid is important.

In contrast, the 1-1 exchanger has limitations also. When the tube side flow is divided evenly among all the tubes, the velocity may be quite low, resulting in low heat transfer coefficient. There it may be required to increase the area to have the desired heat exchange for this low heat transfer coefficient. The area may be increased by increasing the length of the tube. However, the tube length requirement may be impractical for a given situation. Thus the number of tubes should be increased without increased the tube length. The increased number of tubes would also provide the increased velocity in the shell side resulting in the higher heat transfer coefficient. Therefore, multi-pass construction is needed, which would permit to use the practical and standard tube lengths. However, the disadvantages are that,

1. The construction of the exchangers become complex.
2. Parallel flow cannot be avoided.
3. Additional friction losses may occur.

It should be noted that generally even number of tube passes are used in multi pass exchanger.

8.2.5 LMTD correction factor

In the earlier chapter, we have seen for co-current or counter current flow system. The average driving force for heat transfer was defined by log mean temperature difference (LMTD). Thus the LMTD can be used for 1-1 exchangers for co-current and counter current. However, for multi pass exchangers (1-2, 2-4, etc.) the fluids are not always in co-current or counter current flow. The deviation for co-current or counter current flow causes a change in the average driving force. Therefore, in order to use true heat transfer driving force, a correction factor is required into the LMTD. Thus, the heat transfer rate can be written as (eq.8.8),

$$q = U_d A(F_T \Delta T_m) \tag{8.8}$$

where,

U_d = overall heat transfer coefficient including fouling/dirt

A = heat transfer area

$F_T \Delta T_m$ = true average temperature difference.

F_T = LMTD correction factor

It is to be noted that the following assumption have been considered for developing LMTD,

1. The overall heat transfer coefficient is constant throughout the exchanger
2. In case any fluid undergoes for phase change (e.g., in condenser), the phase change occurs throughout the heat exchanger and the constant fluid temperature prevails throughout the exchanger.
3. The specific heat and mass flow rate and hence the heat capacity rate, of each fluid is constant.
4. No heat is lost in to the surroundings.
5. There is no conduction in the direction of flow neither in the fluids nor in the tube or shell walls.
6. Each of the fluids may be characterized by a single temperature, at any cross section in the heat exchanger that is ideal transverse mixing in each fluid is presumed.

F_T, the LMTD correction factor can be directly obtained from available charts in the literature. These charts were prepared from the results obtained theoretically by solving the temperature distribution in multi-pass heat exchangers.

Figures 8.12 and 8.13 show the two generally used heat exchangers and their corresponding plots for finding F_T. It may be noted that the given figures have the representative plots and any standard book on heat transfer may be consulted for the accurate results.

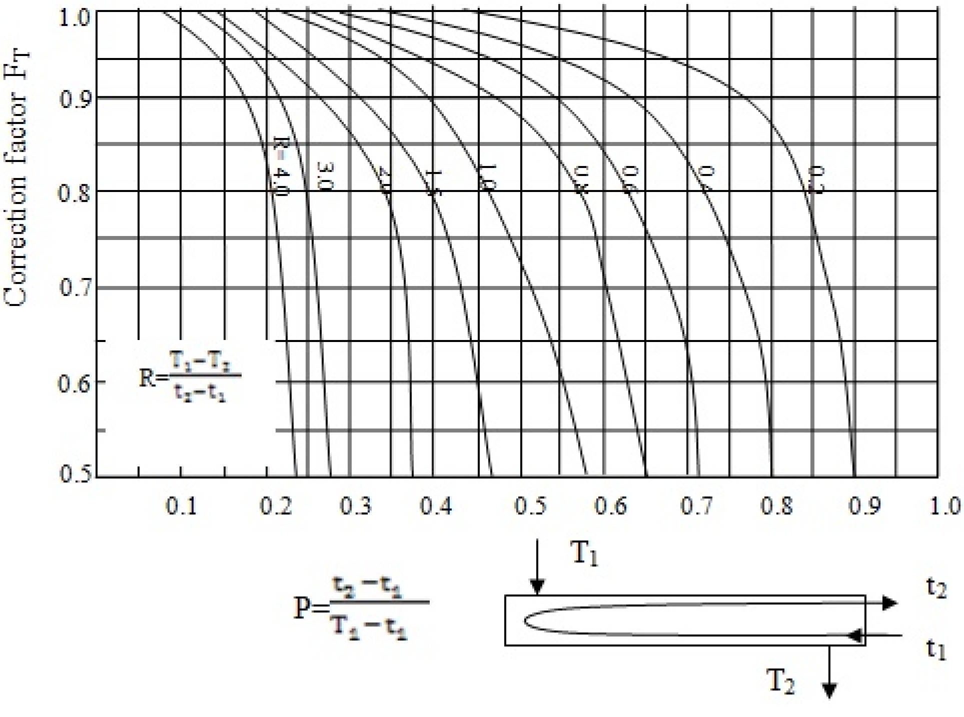


Fig. 8.12: F_T plot for 1-2 exchanger; t: cold fluid in the tube; T: hot fluid in the shell; 1: inlet; 2: outlet

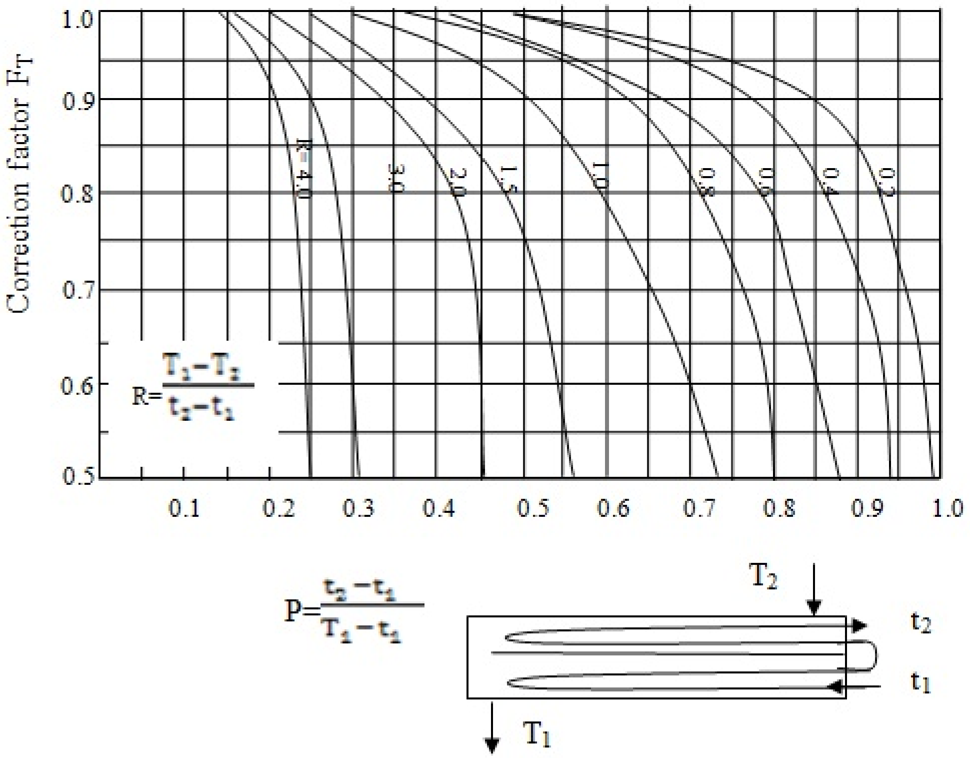


Fig. 8.13: F_T plot for 2-4 exchanger; t: cold fluid in the tube; T: hot fluid in the shell; 1: inlet; 2: outlet

It should be noted that in case of condensation or evaporation the correction factor becomes unity (F_T =1). While designing a heat exchanger, the rule of thumb is that the F_T should not be less than 0.8.

8.2.4 Why multi-pass exchangers?

The simplest type of heat exchangers is double pipe heat exchangers, which is inadequate for flow rates that cannot readily be handled in a few tubes. If several double pipes are used in parallel, the metal weight required for the outer tubes becomes so large that the shell and tube construction, such as 1-1 exchanger will be helpful. In that one shell serves for many tubes, is economical. The heat transfer coefficient of tube side and shell side fluid is very important and the individual heat transfer coefficients must be high enough to attain high overall heat transfer coefficient. As the shell would be quite large as compared to the tubes, the velocity and the turbulence of the shell side fluid is important.

In contrast, the 1-1 exchanger has limitations also. When the tube side flow is divided evenly among all the tubes, the velocity may be quite low, resulting in low heat transfer coefficient. There it may be required to increase the area to have the desired heat exchange for this low heat transfer coefficient. The area may be increased by increasing the length of the tube. However, the tube length requirement may be impractical for a given situation. Thus the number of tubes should be increased without increased the tube length. The increased number of tubes would also provide the increased velocity in the shell side resulting in the higher heat transfer coefficient. Therefore, multi-pass construction is needed, which would permit to use the practical and standard tube lengths. However, the disadvantages are that,

1. The construction of the exchangers become complex.
2. Parallel flow cannot be avoided.
3. Additional friction losses may occur.

It should be noted that generally even number of tube passes are used in multi pass exchanger.

8.2.5 LMTD correction factor

In the earlier chapter, we have seen for co-current or counter current flow system. The average driving force for heat transfer was defined by log mean temperature difference (LMTD). Thus the LMTD can be used for 1-1 exchangers for co-current and counter current. However, for multi pass exchangers (1-2, 2-4, etc.) the fluids are not always in co-current or counter current flow. The deviation for co-current or counter current flow causes a change in the average driving force. Therefore, in order to use true heat transfer driving force, a correction factor is required into the LMTD. Thus, the heat transfer rate can be written as (eq.8.8),

$$q = U_d A(F_T \Delta T_m) \tag{8.8}$$

where,

U_d = overall heat transfer coefficient including fouling/dirt

A = heat transfer area

$F_T \Delta T_m$ = true average temperature difference.

F_T = LMTD correction factor

It is to be noted that the following assumption have been considered for developing LMTD,

1. The overall heat transfer coefficient is constant throughout the exchanger
2. In case any fluid undergoes for phase change (e.g., in condenser), the phase change occurs throughout the heat exchanger and the constant fluid temperature prevails throughout the exchanger.
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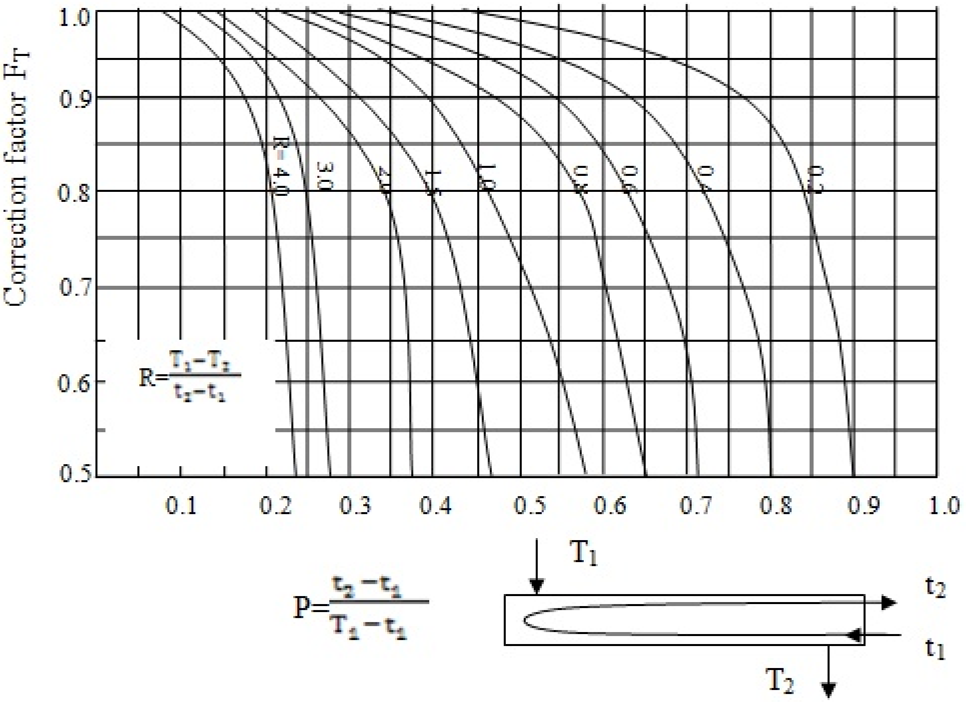

Fig. 8.12: F_T plot for 1-2 exchanger; t: cold fluid in the tube; T: hot fluid in the shell; 1: inlet; 2: outlet

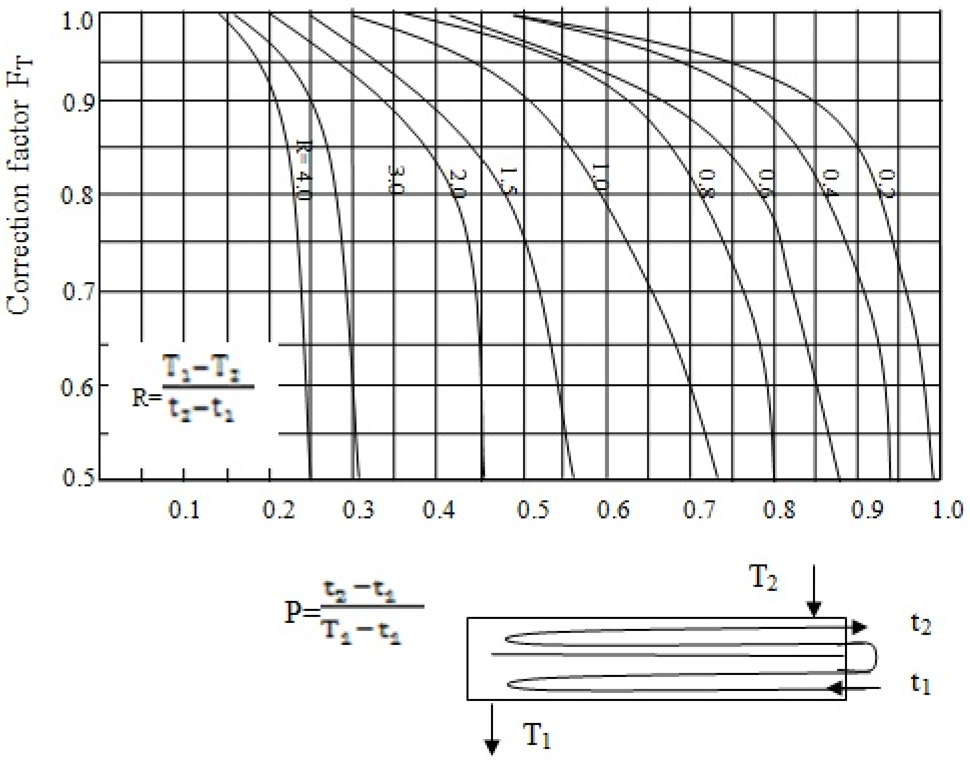

Fig. 8.13: F_T plot for 2-4 exchanger; t: cold fluid in the tube; T: hot fluid in the shell; 1: inlet; 2: outlet

It should be noted that in case of condensation or evaporation the correction factor becomes unity (F_T =1). While designing a heat exchanger, the rule of thumb is that the F_T should not be less than 0.8.

8.2.6 Individual heat transfer coefficient

In section 8.2.1, we have seen that the overall heat transfer coefficient can be calculated provide the parameters are known including individual heat transfer coefficients. In this, section we will discuss how to find out the individual heat transfer coefficient, which is basically based on the well-established correlations and discussed earlier also.

The heat transfer coefficient (h_i) for the tube side fluid in a heat exchanger can be calculated either by Sieder-Tate equation or by Colburn equation discussed in earlier chapter.

However, the shell side heat transfer coefficient (h_o) cannot be so easily calculated because of the parallel, counter as well as cross flow patterns of the fluid. Moreover, the fluid mass velocity as well as cross sectional area of the fluid streams vary as the fluid crosses the tube bundle. The leakages between baffles and shell, baffle and tubes, short circuit some of the shell fluid thus reduces the effectiveness of the exchanger.

Generally, modified Donohue equation (eq.8.9) (suggested by D.Q. Kern) is used to predict the h_o,

$$\frac{h_0 D_H}{k_0} = 0.36\left(\frac{D_H G_s}{\mu}\right)^{0.55}\left(\frac{\mu}{k_0}\right)^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14} \quad (8.9)$$

where,

h_0= shell side heat transfer coefficient

D_h= hydraulic diameter of the shell side

k_0= thermal conductivity of the shell side fluid

G_s= mass flow rate of the shell side

The D_h and G_s can be easily calculated if the geometry of the tube arrangement in the shell is known. The tubes may be generally arranged as a square or triangular pitch, as shown in figure 8.14.

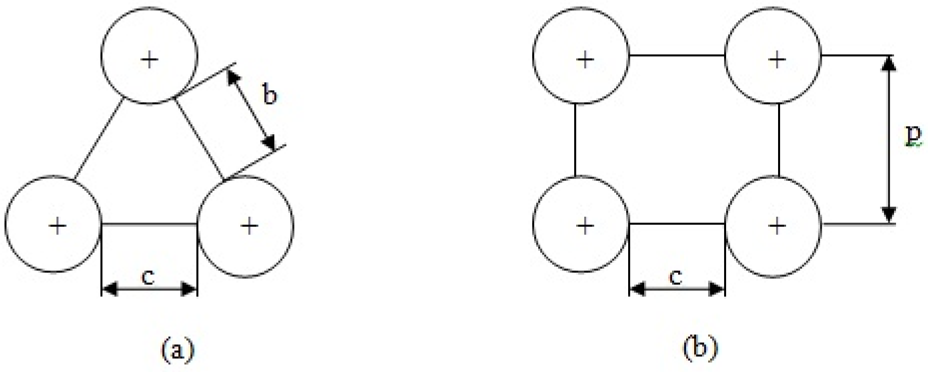

Fig.8.14: Tube arrangement in the shell (a) triangular pitch (b) square pitch

The hydraulic diameter (D_h) for tubes on square pitch $\frac{4\left(p^2-\pi\frac{d_o^2}{4}\right)}{\pi d_o}$

D_h For 60° triangular pitch= $\frac{4\left[(0.5p)(0.86p)-\pi\frac{d_o^2}{8}\right]}{\pi d_o/2}$

where,

d_o = outer diameter of tube

p = tube pitch

$$G_s = \frac{\dot{v}_s}{a_s}$$

where,

$\dot{v}_s$ = flow rate of shell fluid

a_s = shell side flow area

Shell side flow area can be calculated using baffle information number of tubes in the shell and tube arrangement. If 25% cut baffles are used, that means the shell side flow will be from this 25% area. However we have to reduce the area of the pipes which are accumulated in this opening. So depending upon the information we may determine the shell side fluid flow area. It may also be found out by the following way,

$$a_s = \frac{CBD_s}{p}$$

where,

C = tube clearance

B = baffle spacing

D_s= inside diameter of shell

p = pitch of the tube

8.2.7 Pressure drop in the heat exchanger

Pressure drop calculation is an important task in heat exchanger design. The pressure drops in the tube side as well as shell side are very important and quite a few co-relations are available in the literature. One such co-relation is given below in the subsequent subsection.

8.2.7.1 Correlation for tube side pressure drop (eq. 8.10)

$$\Delta P_{t,f} = \frac{f\, G_f\, Ln}{2g\rho_t d_t(\mu/\mu_w)^m} \tag{8.10}$$

where,

$\Delta P_{t,f}$ = total pressure drop in the bundle of tube

f = friction factor (can be found out from Moody's chart)

G_t = mass velocity of the fluid in the tube

L = tube length

n = no of tube passes
g = gravitational acceleration
ρ_t = density of the tube fluid
d_i = inside diameter of the tube
m =0.14 for Re > 2100
0.25 for Re < 2100

The above correlation is for the pressure drop in the tubes owing to the frictional losses. However in case of multi pass flow direction of the flow in the tube changes when flow is from 1-pass to another pass and the pressure losses due to the change in direction is called return-loss. The return-loss ($\Delta P_{t,r}$) is given by eq.8.11,

$$\Delta p_{t,r} = 4\left(\frac{v_t^2}{2g}\right)\rho_t \times n \tag{8.11}$$

n = no of tube pass
v_t = velocity of the tube fluid
ρ_t = density of the tube fluid

Therefore, the total tube side pressure drop will be,

$\Delta p_t = \Delta P_{t,f} + \Delta P_{t,r}$

8.2.7.2 Correlation for shell side pressure drop

The following correlation (eq.8.12) may be used for an unbaffled shell,

$$\Delta p_s = \frac{f_s G_s^2 L n_s}{2g\rho_s D_h (\mu/\mu_w)^{0.14}} \tag{8.12}$$

The above equation can be modified to the following form (eq.8.13) for a baffled shell,

$$\Delta p_s = \frac{f_s G_s^2 D_{si}(n_b + 1)}{2g\rho_s D_h (\mu/\mu_w)^{0.14}} \tag{8.13}$$

where

L = shell length
n_s = no of shell pass
n_b = no of baffles

ρ_s = shell side fluid density

G_s = shell side mass velocity

D_h = hydraulic diameter of the shell

D_{si} = inside diameter of shell

f_s = shell side friction factor

The hydraulic diameter (D_h) for the shell can be calculated by the following equation (eq. 8.14),

$$D_h = \frac{4\left[\pi D_S^2/4 - \pi d_o^2 n_t/4\right]}{\pi d_o n_t + \pi D_S} \tag{8.14}$$

where,

n_t = number of tubes in the shell

d_o = outer diameter of the tube

The friction factor (f_s) can be obtained by the Moody's chart for the corresponding Reynolds number $\left(R_e = {D_h G_s}/{\mu}\right)$.

8.2.8. Heat transfer effectiveness and number of transfer units (NTU)

The LMTD is required to be calculated for the evaluation of heat exchanger performance. However, the LMTD cannot be directly calculated unless all the four terminal temperatures ($T_{c,i}$, $T_{c,o}$, $T_{h,i}$, $T_{h,o}$) of both the fluids are known.

Sometimes the estimation of the exchanger performance (q) is required to be calculated on the given inlet conditions, and the outlet temperature are not known until q is determined. Thus the problem depends on the iterative calculations. This type of problem may be taken care of using performance equivalent in terms of heating effectiveness parameter (η), which is defined as the ratio of the actual heat transfer to the maximum possible heat transfer. Thus,

$$\eta = \frac{q}{q_{\max}} \tag{8.15}$$

For an infinite transfer area the most heat would be transferred in counter-current flow and the q_{max}will be dependent on the lower heat capacity fluid as such,

$$q_{max} = \dot{m}_c c_{pc}(T_{hi} - T_{ci}) \text{for } \dot{m}_c c_{pc} < \dot{m}_h c_{ph}$$

$$q_{max} = \dot{m}_h c_{ph}(T_{hi} - T_{ci}) \text{for } \dot{m}_c c_{pc} > \dot{m}_h c_{ph}$$

The actual heat transfer

$$q = \dot{m}_c c_{pc}(T_{co} - T_{ci}) = \dot{m}_h c_{ph}(T_{ho} - T_{hi})$$

$$\eta = \frac{(T_{co} - T_{ci})}{(T_{hi} - T_{ci})} \text{for } \dot{m}_c c_{pc} < \dot{m}_h c_{ph}$$

$$\eta = \frac{(T_{hi} - T_{ho})}{(T_{hi} - T_{ci})} \text{for } \dot{m}_c c_{pc} > \dot{m}_h c_{ph}$$

The capacity ratio, which is the relative thermal size of the two fluid streams, is defined as,

$$C_R = \frac{(\dot{m}c_p)_{min}}{(\dot{m}c_p)_{max}}$$

On careful analysis, we can say that

U·A: Heat exchange capacities per unit temperature difference.

This thermal sizing (*U·A*) can be non-dimensionalised by dividing it to the storage capacity of one of the fluid streams. Given $(\dot{m}c_p)_{min}$ limits the maximum heat transfers. The non-dimensional term obtained is known as the number of transfer units (NTU)

$$NTU = \frac{UA}{(\dot{m}c_p)_{min}}$$

It should be noted that

$$\eta = \eta(C_R NTU)$$

The actual determination of this function may be done using heat balances for the streams. For a parallel flow exchanger the relation is shown below

$$\eta = \frac{1 - exp[-(C_R + 1)\, NTU]}{C_R + 1}$$

$$NTU = \frac{-\ln[1 - (C_R + 1)\eta]}{C_R + 1}$$

The above relation is true for both the condition $\text{(i) } \dot{m}_c c_{pc} < \dot{m}_h c_{ph} \text{ (ii) } \dot{m}_c c_{pc} > \dot{m}_h c_{ph}$

Similarly the functional relationship for counter –current exchanger is

$$\eta = \frac{1-exp[-(1-C_R)\,NTU]}{1-C_R exp[-(1-C_R)\,NTU]} \tag{8.16}$$

$$NTU = \frac{\ln[(1-\eta)/(1-\eta C_R)]}{C_R-1} \tag{8.17}$$

The previous relation (eq. 8.16 and 8.17) were for 1-1 exchanger. The relation for 1-2 exchanger (counter current) is given by eq. 8.18, 8.19),

$$\eta = 2\left[(1+C_R) + (1+C_R^2)^{1/2}\,\frac{1+exp\left[-(1+C_R^2)^{1/2}\,NTU\right]}{1-exp\left[-(1-C_R^2)^{1/2}\,NTU\right]}\right]^{-1} \tag{8.18}$$

$$NTU = -(1+C_R^2)^{-1/2}\,\ln\left[\frac{2/\eta-1-C_R-(1+C_R^2)^{1/2}}{2/\eta-1-C_R+(1+C_R^2)^{1/2}}\right] \tag{8.19}$$

When the fluid streams are condensing in a 1-1 pass exchanger (fig. 8.15) as shown below,

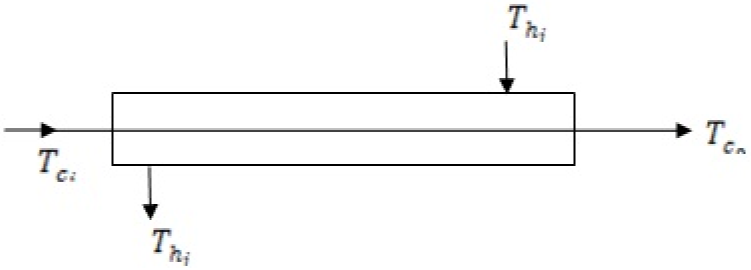

Fig.8.15: Condenser with the temperature nomenclature

the following relation arrives.

$$\eta = \frac{T_{co}-T_{ci}}{T_{hi}-T_{ci}} = 1-\exp(-NTU)$$
$$NTU = -\ln(1-\eta)$$

8.2.9 Calculation and designing of the heat exchanger

8.2.9.1 Double-pipe heat exchanger

The following steps may be used to design a double-pipe heat exchanger

1. Calculate LMTD from the known terminal temperatures.

2. Diameter of the inner and outer pipes may be selected from the standard pipes from the literature (generally available with the vendor and given in the books). The selection thumb rule is the consideration of higher fluid velocity and low pressure drop in the pipe.
3. Calculate the Reynolds number and evaluate the heat transfer coefficient, hi, using the co-relations given in the chapter.
4. Similarly, calculate the Reynolds number of the fluid flowing through annulus. Calculation the equivalent diameter of the annulus and find the outside heat transfer coefficient, ho.
5. Using hi and ho, calculate the overall heat transfer coefficients. Note that it will be a clean overall heat transfer coefficient. In order to find design outside heat transfer coefficient using a suitable dirt factor or fouling factor. The tube fouling factor is suggested by TEMA (table 8.1).

The calculations are based on trial and error. If the heat transfer coefficient comes out to be very small or the pressure drop comes out to be very high, this procedure to be redone for different set of diameters in the step1.

8.2.9.2 Shell and tube heat exchanger

The shell and tube heat exchanger also involves trial and error but it is not as simple as in case of double pipe heat exchanger.

The design of shell and tube heat exchanger includes,
a: heat transfer required for the given heat duty
b: tube diameter, length, and number,
c: shell diameter,
d: no of shell and tube passes,
e: tube arrangement on the tube sheet and its layout, and
f: baffle size, number and spacing of the baffles.

The calculation of LMTD can be done if the terminal temperatures are known. However, the design heat transfer co-efficient (i.e., heat transfer co-efficient including fouling factor) and the area are dependent on each other and thus challenges involve for the estimation. The also depends upon Reynolds number, which depends upon the liquid flow rate, sizes and the number of tubes. Therefore, is a function of diameter and the no of tubes and the parameter provides the area.

Moreover, can also be calculated is based on shell side co-efficient but then it requires tube number, diameters and pitch. Thus, the above discussion shows that and *A* are not fully explicit and requires trial and error method of calculation.
The guideline for shell-and-tube calculation is shown in below,

1. energy balance and exchanger heat duty calculation,
2. find all the thermo-physical properties of the fluid,
3. take initial guess for shell-and-tube passes,
4. calculate LMTD and F_T,
5. assume (or select) U_{dirty}, based on the outside tube area. Calculate corresponding heat transfer area, *A*.

6. Select tube diameter, wall thickness and the tube length. Based on this values and heat transfer area, find out the no of the tubes required.
7. Assume the tube pitch and assume diameter of the shell, which can accumulate the no of tube. Now, select the tube-sheet layout.
8. Select the baffle design.
9. Estimate h_i and h_o, if the estimated shell-side heat transfer coefficient (h_o) appears to be small; the baffles at a close distance may be tried. If the tube side co-efficient (h_i) is low, the number of tube passes to be reconsidered such that the Reynolds number increases (for a reasonable Δp) and henceforth h_i.
10. Evaluate U_{clean} on the outside tube area basis. Select a suitable fouling factor (R_d) and find U_{dirty}
11. Compare U_{dirty} and A values with the values assumed in step (5). If $A_{calculate} \geq A_{assumed}$, it may be acceptable. Otherwise a new configuration in terms of the size and no of the tubes and tube passes, shell diameter is assumed and recalculation be done
12. Calculate the tube-side and shell side Δp. If Δp is more than the allowable limit, the re-calculate after suitable adjustment has to be done.

Illustration

A heat transfer fluid is leaving a reactor at a rate of 167 kg/s at 85°C. The fluid is to be cooled to 50°C before it can be recycled to the reactor. Water is available at 30°C to cool the fluid in a 1-2 pass heat exchanger having heat transfer area of 15 m2. The water, which is being used to cool the fluid, must not be heated to above 38°C at the exit of the heat exchanger. The overall heat transfer co-efficient of 400 Kcal/hm2°C can be used for the heat exchanger. The water flows through the shell and the oil flows through the tubes. The specific heat of the fluid may be taken as 0.454 kcal/kg°C. Find out whether the heat exchanger would be suitable for the given heat duty?

Solution:

It is given,

$$m_f = 667\ kg/s \approx 10{,}000\ kg/h$$

$$C_{pf} = 0.454\ kcal/kg \cdot °C$$

13. $$T_{f1} = 85°C \qquad T_{f2} = 50°C$$

f : hot stream (fluid)

c : cold stream (water)

Energy balance across the heat exchanger will be,

$$\dot{m}_w(1)(38-30) = 10{,}000(0.454)(85-50) = 19{,}862\ kg/h$$

$$\dot{m}_f c_{pf} \text{for hot stream} = (10{,}000)(0.454) = 4{,}540 \text{ kcal/h°C}$$

$$\dot{m}_w c_{pw} \text{for cold stream} = (19{,}862)(1) = 19{,}862 \text{ kcal/h°C}$$

Thus the minimum stream will be the hot stream.

$$C_r = \frac{(\dot{m}c_p)_{min}}{(\dot{m}c_p)_{max}} = \frac{4{,}540}{19{,}862} = 0.2286$$

$$\eta = \frac{T_{f1} - T_{f2}}{T_{f1} - T_{c2}} = \frac{85-50}{85-30} = 0.636$$

Putting the values in the eq. 8.19,

$$NTU = -(1 + C_R^2)^{-1/2} \ln\left[\frac{2/\eta - 1 - C_R - (1 + C_R^2)^{1/2}}{2/\eta - 1 - C_R + (1 + C_R^2)^{1/2}}\right]$$

$$NTU = 1.1652$$

$$NTU = UA/(mC_p)_{min}$$

$$\frac{UA}{(mC_{p)})_{min}} = 1.1652$$

$$A = \frac{1.1652 \times 4540}{400} = 13.2\ m^2$$

The area 13.2 m^2 found is less than the available area (15 m^2). Therefore, the given heat exchanger will perform the required heat duty.

Evaporation is the vaporization of a liquid. Chemical process industries, in general, use evaporator for the vaporization of a solvent from a solution. We have already discussed the heat transfer for boiling liquids in early chapter. However the evaporation is so important operation in chemical process industry that it is considered an individual operation. In this chapter we will focus on the evaporation with an objective to concentrate a solution consisting of a non-volatile solute and a volatile solvent. If we continue the evaporation process, the residual mater will be solid, which is known as drying. However, our aim is not to dry but to concentrate the solution, moreover, we will also not deal with the crystallization, in which the evaporation leads to formation of crystal in the solution. It is suggested that reader should learn the difference between evaporator, drying and crystallization.

As we will deal with the solution for the evaporation process, a few of the facts must be known about the solution properties.

9.1 Solution properties

Knowledge of solution properties is important for the design of the equipment for evaporation. Some of the important properties of the solution are given below,

9.1.1 Concentration

Initially, the solution may be quite dilute and the properties of the solution may be taken as the properties of solvent. As the concentration increases, the solution becomes viscous and heat transfer resistance increases. The crystal may grow on the heating coil or on the heating surface. The boiling points of the solution also rise considerably. Solid or solute contact increases and the boiling temperature of the concentrated solution became higher than that of the solvent as the same pressure (i.e. elevation in boiling point).

9.1.2 Foaming

Many of the materials like organic substance may foam during vaporization. If the foam is stable, it may come out along the vapor known as entrainment. Heat transfer coefficient changes abruptly for such systems.

9.1.3 Degradation due to high temperature

The products of many chemical, food, pharmaceutical industries etc. are very temperature sensitive and they may get damaged during evaporation. Thus special case or technique is required for concentrating such solution.

9.1.4 Scaling

Many solution have tendency to deposit the scale on the heating surface, which may increase the heat transfer resistance. These scales produce extra thermal resistance of significant value. Therefore, scaling in the equipment should not be ignored thus de-scaling becomes an important and routine matter.

9.1.5 Equipment material

The material of the equipment must be chosen considering the solution properties so that the solution should neither be contaminated nor react with the equipment material.

9.2 Evaporator

Equipment, in which evaporation is performed, is known as evaporator. The evaporators used in chemical process industries are heated by steam and have tubular surface. The solution is circulated in the tube and the tubes are heated by steam. In general the steam is the saturated steam and thus it condenses on the outer tube surface in order to heat the tube. The circulation of the solution in the tube have reasonable velocity in order to increase the heat transfer coefficient and remove of scales on the inner surface of the tube. The steam heated tubular evaporators may be classified as natural and forced circulation evaporators.

9.2.1 Natural circulation evaporator

In this category the main evaporators are,

1. Calandria type or short tube evaporator
2. Long tube vertical evaporator

As the name indicates, the circulation of the solution is natural and the density difference derives it. The solution gets heat up and partially vaporized as it flows up the tubes. The heated liquid flows up because of the density difference. Vapor-liquid disengagement occurs above the tube. Thick liquor comes down from this down comer and withdrawn from the bottom. The natural-circulation evaporators may be used if the solution is quite dilute. In the dilute solution the natural circulation will be at sufficient speed. It may also be used when the solution does not have suspended solid particles. As the solution stays in the tube for larger time, the solution should not be heat sensitive.

The Calandria type or short-tube evaporators have short tubes as compared to the long tube evaporators. The short-tube evaporation uses circulation and solution flows many times in the evaporators. However, in case of the long tube evaporator the flow is once through.

9.2.2 Forced circulation evaporator

Natural circulation evaporators have many limitations (as mentioned earlier) through they are economical as compared to forced circulation evaporator. A forced circulation evaporator has a tubular exchanger for heating the solution without boiling. The superheated solution flashes in the chamber, where the solution gets concentrated. In forced circulation evaporator horizontal or vertical both type of design is in- practice. The forced circulation evaporators are used for handling viscous or heat sensitive solution.

9.2.3 Falling film evaporator

Highly heat sensitive materials are processed in falling film evaporators. They are generally once-through evaporator, in which the liquid enters at the top, flows downstream inside the heater tubes as a film and leaves from the bottom. The tubes are heated by condensing steam over the tube. As the liquid flows down, the water evaporates and the liquid gets concentrated. To have a film inside of the tube, the tube diameter is kept high whereas the height low to keep the residence time low for the flowing liquid. Therefore, these evaporators, with non-circulation and short resistance time, handle heat sensitive material, which are very difficult to process by other method. The main problem in falling film evaporator is the distribution of the liquid uniformly as a thin film inside the tube.

9.3 Performance of steam heated tubular evaporators

The performance of a steam heated tubular evaporator is evaluated by the capacity and the economy.

9.3.1 Capacity and economy

Capacity is defined as the no of kilograms of water vaporized per hour. Economy is the number of kg of water vaporized per kg of steam fed to the unit. Steam consumption is very important to know, and can be estimated by the ratio of capacity divided by the economy. That is the steam consumption (in kg/h) is

Steam Consumption = Capacity / Economy

9.3.2 Single and multiple effect evaporators

In single effect evaporator, as shown in fig. 9.1, the steam is fed to the evaporator which condenses on the tube surface and the heat is transferred to the solution. The saturated vapor comes out from the evaporator and this vapor either may be vented out or condensed. The concentrated solution is taken out from the evaporator.

Now we can see if we want the further concentrate, the solution has to be sent into another similar evaporator which will have the fresh steam to provide the necessary heat.

It may be noted that in this process the fresh steam is required for the second evaporator and at the same time the vapor is not utilized. Therefore it can be said the single effect evaporator does not utilized the steam efficiently. The economy of the single effect evaporator is thus less than one. Moreover, the other reason for low economy is that in many of the cases the feed temperature remains below the boiling temperature of the solution. Therefore, a part of the heat is utilized to raise the feed temperature to its boiling point.

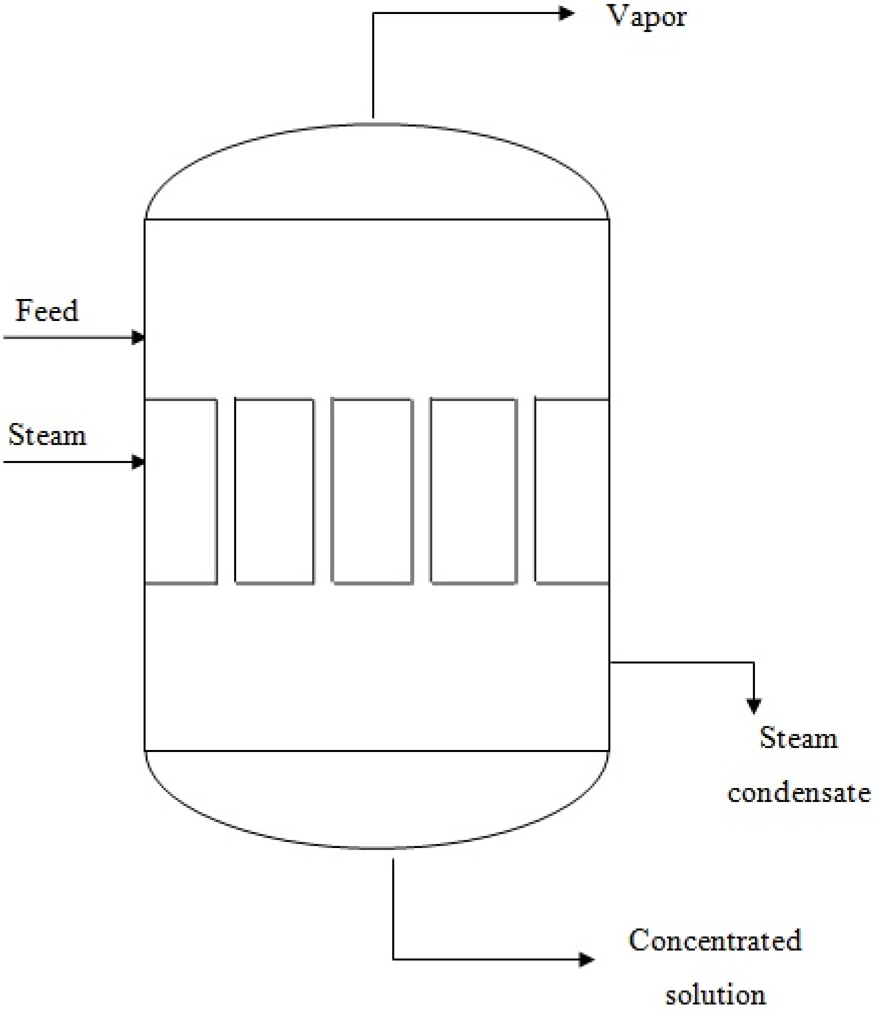

Fig.9.1: Single effect evaporator

In order to increase the economy we may consider the arrangement of the two evaporators as shown in the fig. 9.2.

The figure 9.2 shows that the two evaporators are connected in series. The saturated vapor coming out from the evaporator-1 is used as steam in the second evaporator. Partially concentrated solution works as a feed to the second evaporator. This arrangement is known as double effect evaporator in forward feed scheme. A few of the important point that we have to note for this scheme is that the vapour leaving evaporator-2 is at the boiling temperature of the liquid leaving the first effect. In order to transfer this heat from the condensing vapor from the evaporator-1 to the boiling liquid in evaporator-2, the liquid in evaporator-2 must boil at a temperature considerable less than the condensation temperature of the vaporization, in order to ensure reasonable driving force for heat transfer. A method of achieving this is to maintain a suitable lower pressure in the second effect so that the liquid boils at a lower temperature. Therefore, if the evaporator-1 operates at atmospheric pressure, the evaporator-2 should be operated at same suitable vacuum.

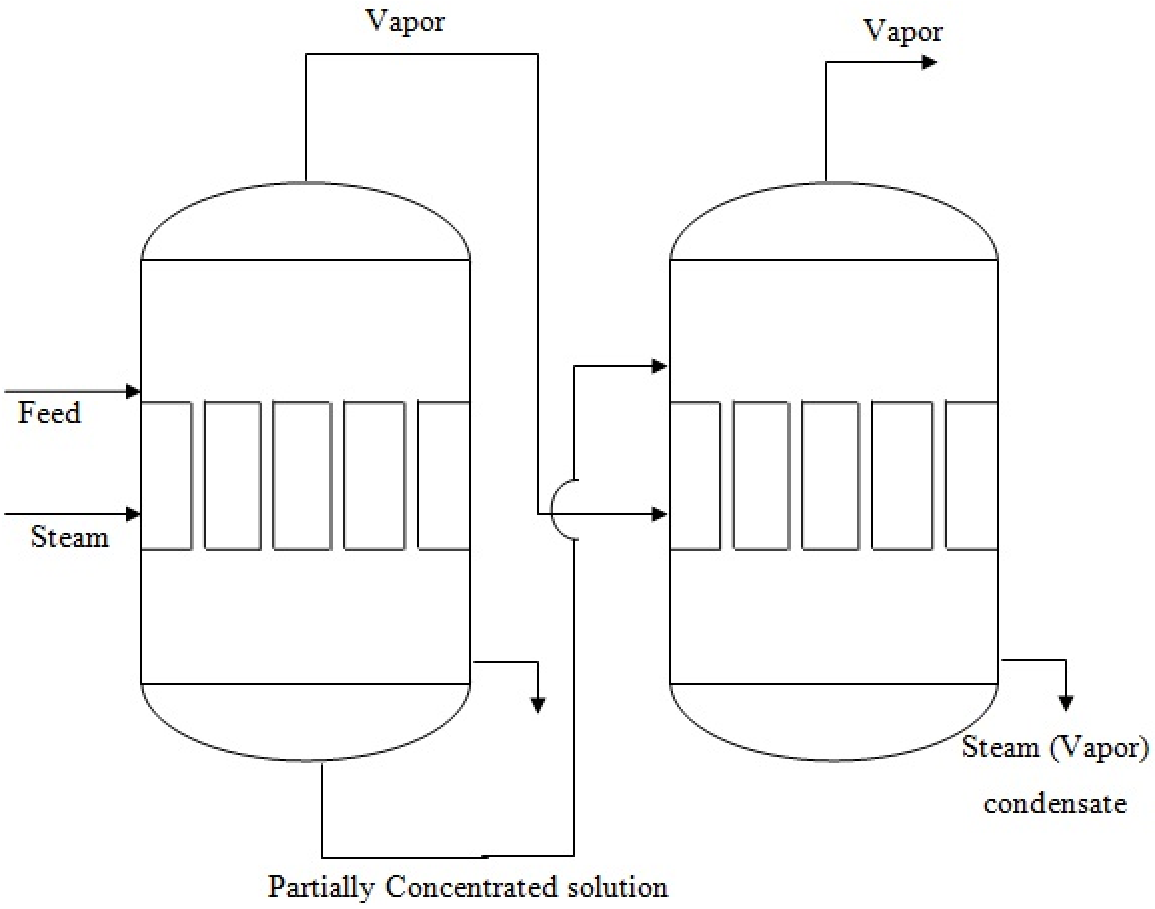

Fig.9.2: Double effect evaporator with forward feed scheme

The benefit of the use of multiple effect evaporators is that in this arrangement multiple reuse of heat supplied to the first effect is possible and results in improved steam economy.

9.3.3 Boiling point elevation

The evaporators produce concentrated solution having substantially higher boiling point than that of the solvent (of the solution) at the prevailing pressure. The increase in boiling point over that of water is known as boiling point elevation (BPE) of the solution. As the concentration increases the boiling point of the solution also increases. Therefore, in order to get the real temperature difference (or driving force) between the steam temperature and the solution temperature, the BPE must be subtracted from the temperature drop. The BPE may be predicted from the steam table (in case water is a solvent).

An empirical rule known as Dühring rule is suitable for estimating the BPE of strong solution. The Dühring rule states that the boiling point of a given solution is a linear function of the boiling point of the pure water at the same pressure. Therefore, if the boiling point of the solution is plotted against that of the water at the same pressure, a straight line results. Different lines are obtained at different concentrations. The fig. 9.3 shows representative Dühring plots for a solution (non-volatile solute in water).

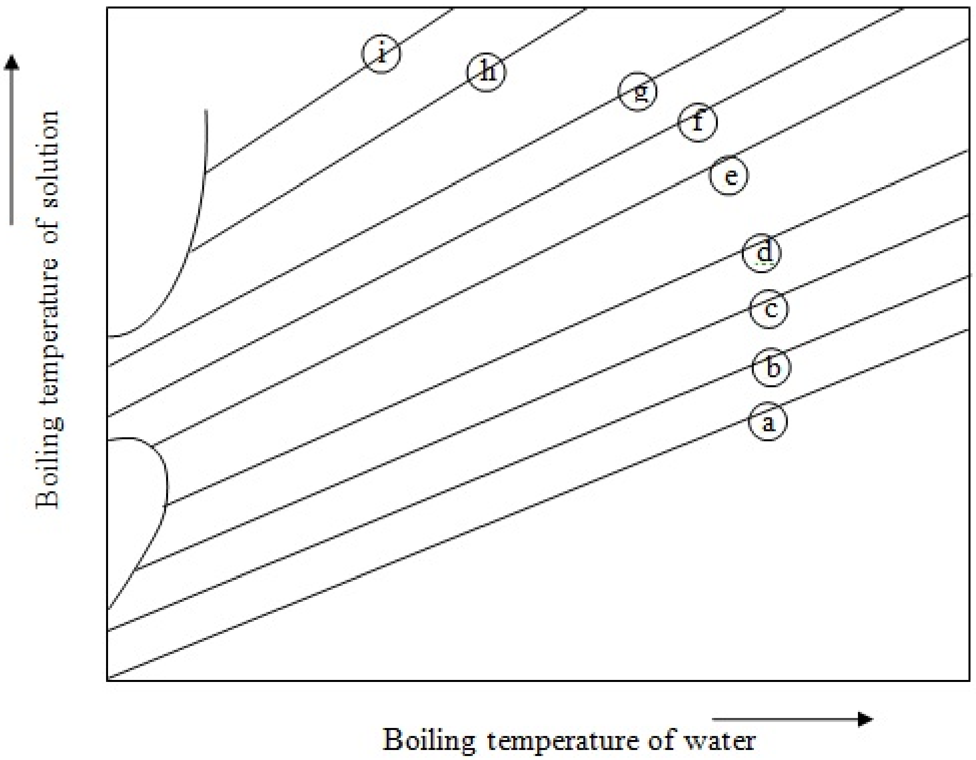

Fig.9.3: Representative Dühring lines for a system (non-volatile solute in water) mole fraction of solute in the solution (a) 0.1 (b) 0.2 (c) 0.25 (d) 0.39 (e) 0.35 (f) 0.45 (g) 0.5 (h) 0.6 (i) 0.7

The fig.9.3 helps to find out the boiling point of solution at moderate pressure. For example if a solution having 'x' mole fractions of solute have a pressure over it such that water boils at T° C, by reading up from the x-axis at T °C to the line for the x mole fraction solution and then moving horizontally to the y-axis, the boiling point of the solution can be found at that pressure.

9.4 Temperature profile in an evaporator

Let us consider the case of long-tube vertical evaporator heated by steam. After boiling and flashing of the superheated liquid, the disengagement of the vapor and liquid occur in vapor space of the evaporator and the recycled liquid flows down the external pipe. A part of this concentrated liquid is withdrawn as a product and the remaining part get mixed with a feed and again enter the evaporator tube. If T_{BP} is the boiling of the liquid in the evaporator as the prevailing pressure, then the temperature of the liquid in the tube will be T_{BP}. The temperature of the recycled stream entering the tubes will then also be T_{BP}, if the feed is sufficiently hot. Now, we will imagine how the temperature is changing in the tube. Let us see that when the liquid flows up in the tube, its temperature rises because at the bottom of the tube the pressure is higher (vapor chamber pressure + hydrostatic pressure + frictional loss) as compared to the top of the tube. Therefore, a liquid starts boiling at a level when its temperature rises to its saturation temperature at the pressure at that point. After the boiling in between the tube, as liquid goes up in the tube, the local temperature drops because of the reduction in the local pressure. It may also be mentioned that as the liquid moves up it gets concentrated and thus the boiling point of the solution also increases as the liquid traversed up in the tube. The liquid temperature profile in the tube is shown in the fig.9.4 for low (plot i) and high (plot ii) liquid velocity. The liquid temperature in the tubes increases up to certain height and then the temperature decreases due to the loss of superheat. At higher velocity the temperature raise is less and the liquid boils near the top of the tube. The plot (iii) shows the shell side temperature profile where steam is heating the tube. As can be seen, the slightly superheated steam enters the shell and soon the temperature of the steam losses its sensible heat and then condenses on the tubes and provide the latent heat of condensation (at temperature T_{steam}) to the tube and before boiling from the shell may get slightly sub-cooled. The plot (iv) is the boiling temperature of the water (T_w) at the pressure in the vapor chamber. Thus, the BPE= T_{BP}-T_w and the true temperature during force is the difference between the plot (iii) and the plot (i) or (ii).

It can be understood with the help of the discussion and fig.9.4 that the temperature changes all along the length of the tube. Thus, the real temperature driving force will be the difference in steam temperature and liquid temperature always the high. However, it is practically not easy to determine the temperature profile in the tube. Therefore, the driving force can be taken as (T_{steam} - T_{BP}) for the design purpose.

9.5 Heat transfer coefficient

The correlation used in the boiling and condensation may be used here. If the evaporator operates at very high liquid velocity so that the boiling occurs at the top end of the tube, the following correlation (eq. 9.1) may be used,

$$\frac{h_t D}{k} = 0.0278 \mathrm{Re}^{0.8} \mathrm{Pr}^{0.4} \tag{9.1}$$

where, D is the inner diameter of the tube, k is the thermal conductivity of the liquid or solution.

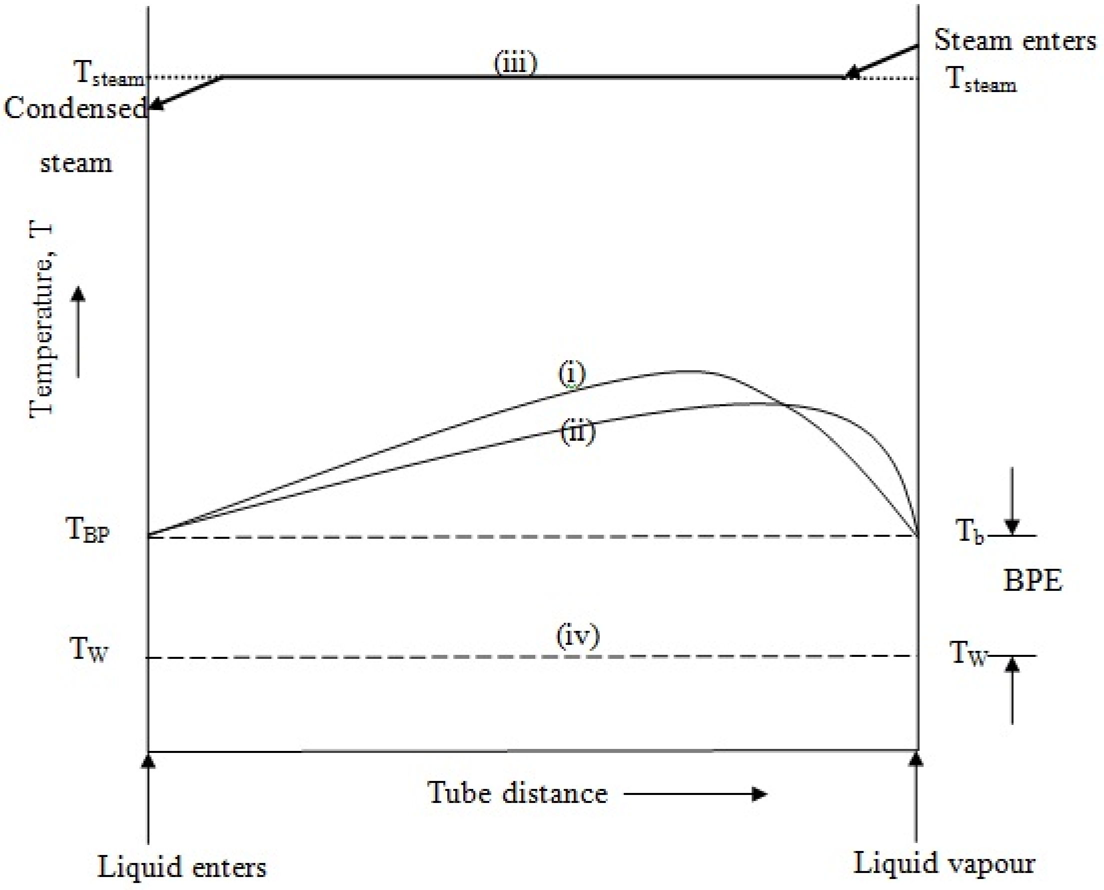

Fig.9.4: Temperature profiles in an evaporator

Fouling is a concern in the evaporator; therefore the following equation (eq.9.2) may be used for the overall heat transfer coefficient with time,

$$\frac{1}{U_{dirty}^2} = \frac{1}{U_{clean}^2} + \alpha t \tag{9.2}$$

where, t is the time for where the evaporator is the operation, α is a constant for a particular liquid, U_{dirty} and U_{clean} all the overall heat transfer coefficient of the dirty and clean evaporator.

9.6 Method of feeding: Multiple effect evaporators

The fig.9.5, 9.6, 9.7, and 9.8 show the four different feeding arrangement of feed to the evaporators. In the fig.9.5 the liquid feed is pumped into the first effect and the partially concentrated solution is sent to the second effect and so on. The heating steam is also sent through the first effect to another effect. This particular strategy is known as forward feed. In the forward feed the concentration of the liquid increases from first effect to the subsequent effects till the last effect. It may be noted that the first effect is that in which the fresh steam is fed, whereas the vapour generated in the first effect is fed to the next evaporator (connected in series with the first effect) is known as second effect and so on.

The forward feed requires a pump for feeding dilute solution to the first effect. The first effect is generally at atmospheric pressure and the subsequent effects are in decreasing pressure. Thus, the liquid may move without the pump from one effect to another effect in the direction of decreasing pressure. However, to take out the concentrated liquid from the last effect may need a pump.

The backward feed arrangement is very common arrangement. A triple-effect evaporator in backward arrangement is shown in the fig.9.6. In this arrangement the dilute liquid is fed to the last effect and then pumped through the successive effects to the first effect. The method requires additional pumps (generally one pump in between two effects) as shown in the fig. 9.6. Backward feed is advantageous and gives higher capacity than the forward feed when the concentrated liquid is viscous, because the viscous fluid is at higher temperature being in the first effect. However, this arrangement provides lower economy as compared to forward feed arrangement.

The combination of forward-feed and backward-feed is known as mixed feed arrangement. In mixed feed the dilute liquid enters in between effects, flows in forward feed to the end of the effect and then pumped back to the first effect for final concentration. Figure 9.7 shows triple effect mixed feed arrangement. This mixed feed arrangement eliminates the need of a few of the pumps. Moreover, it still passes the most concentrated liquid through the first effect, which is having higher temperature among all the effect (being at highest pressure compared to other effects).

Another common evaporator arrangements, which is more common in crystallization is parallel feed where feed is admitted individually to all the effects. Figure 9.7 shows such arrangement.

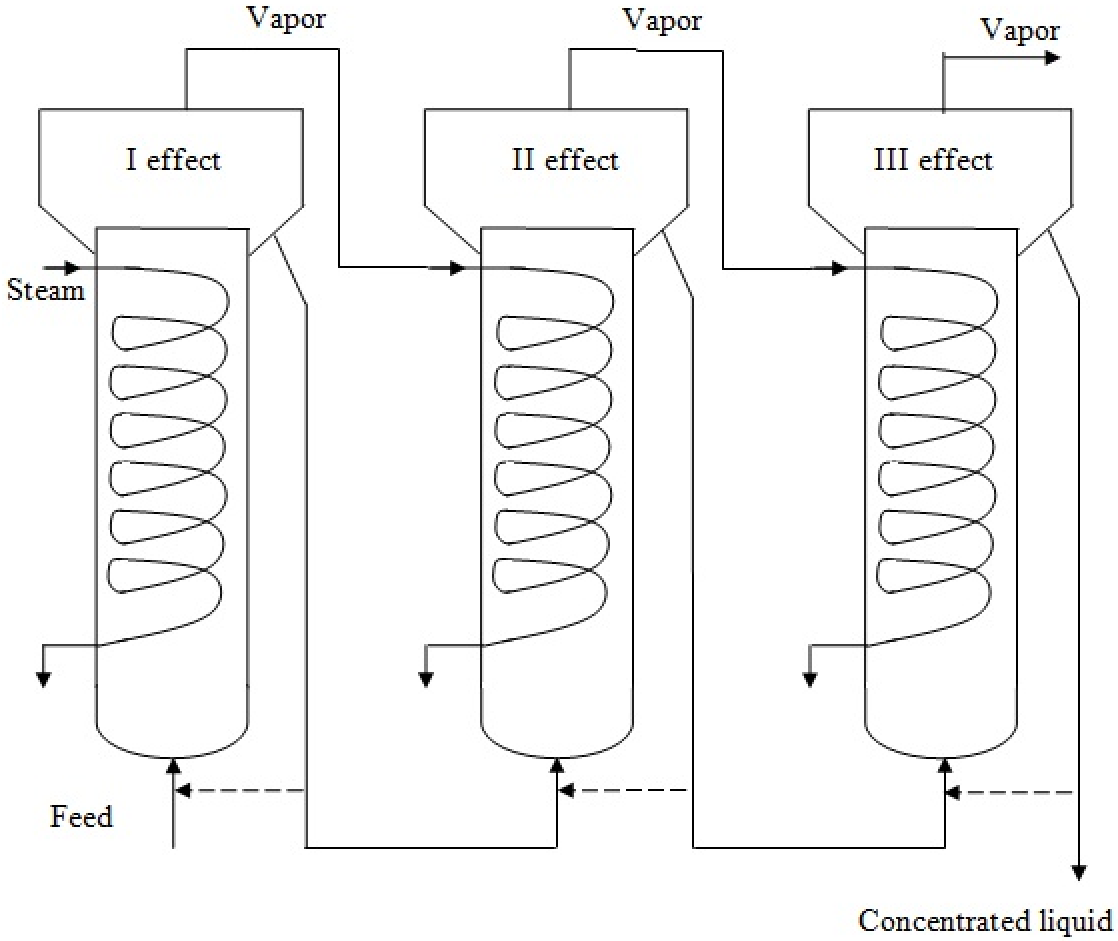

Fig.9.5: Forward feed arrangement in triple-effect evaporator (dotted line: recycle stream)

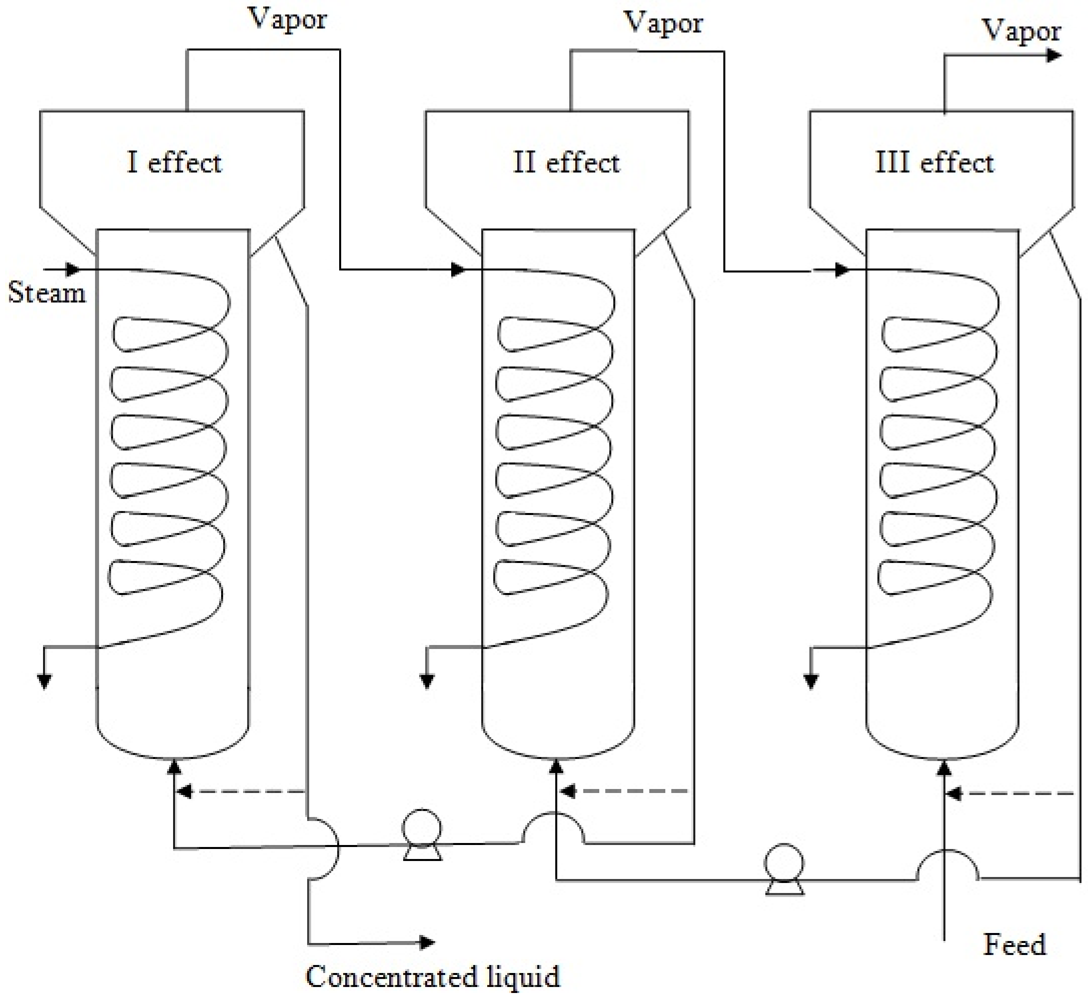

Fig.9.6: Backward feed arrangement in triple-effect evaporator (dotted line: recycle stream)

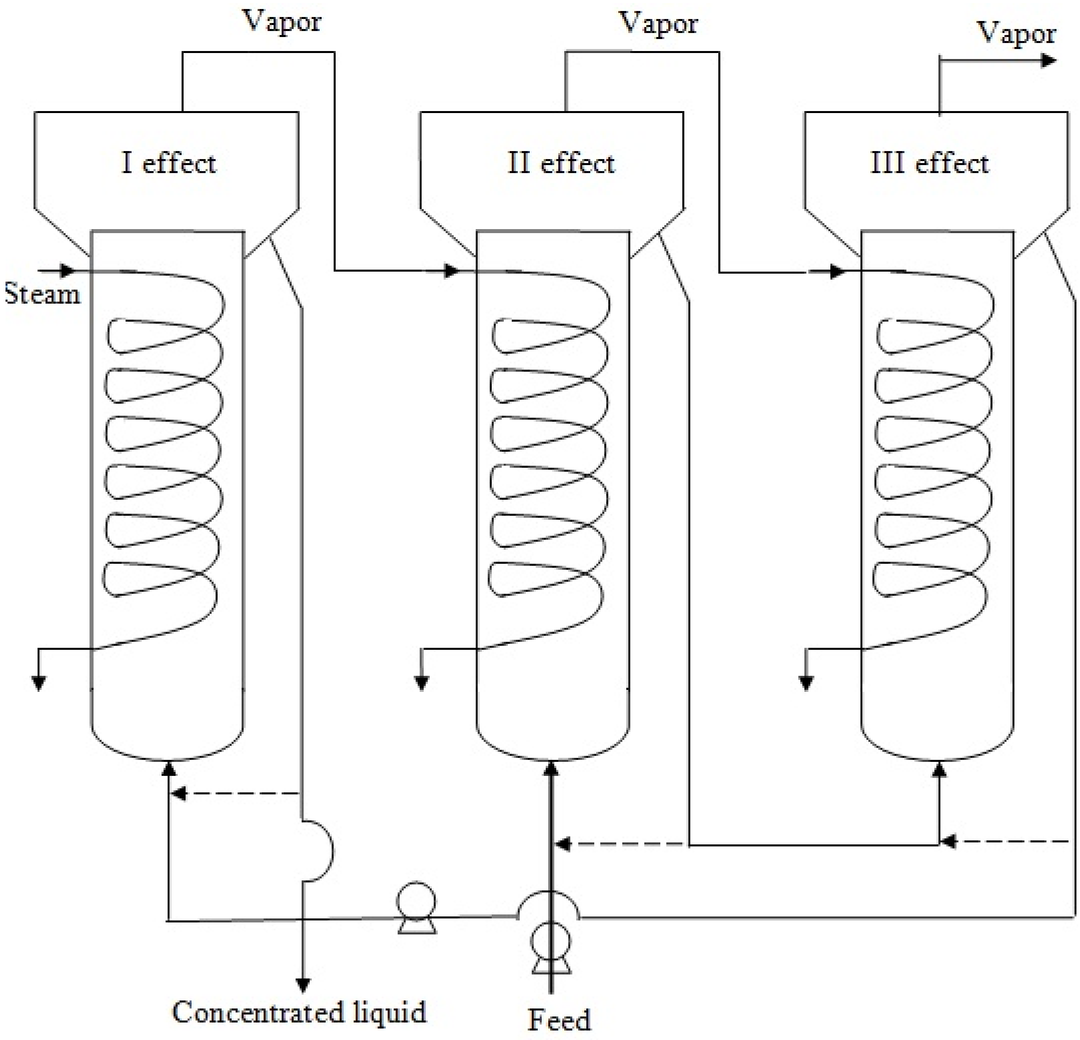

Fig.9.7: Mixed feed arrangement in triple-effect evaporator (dotted line: recycle stream)

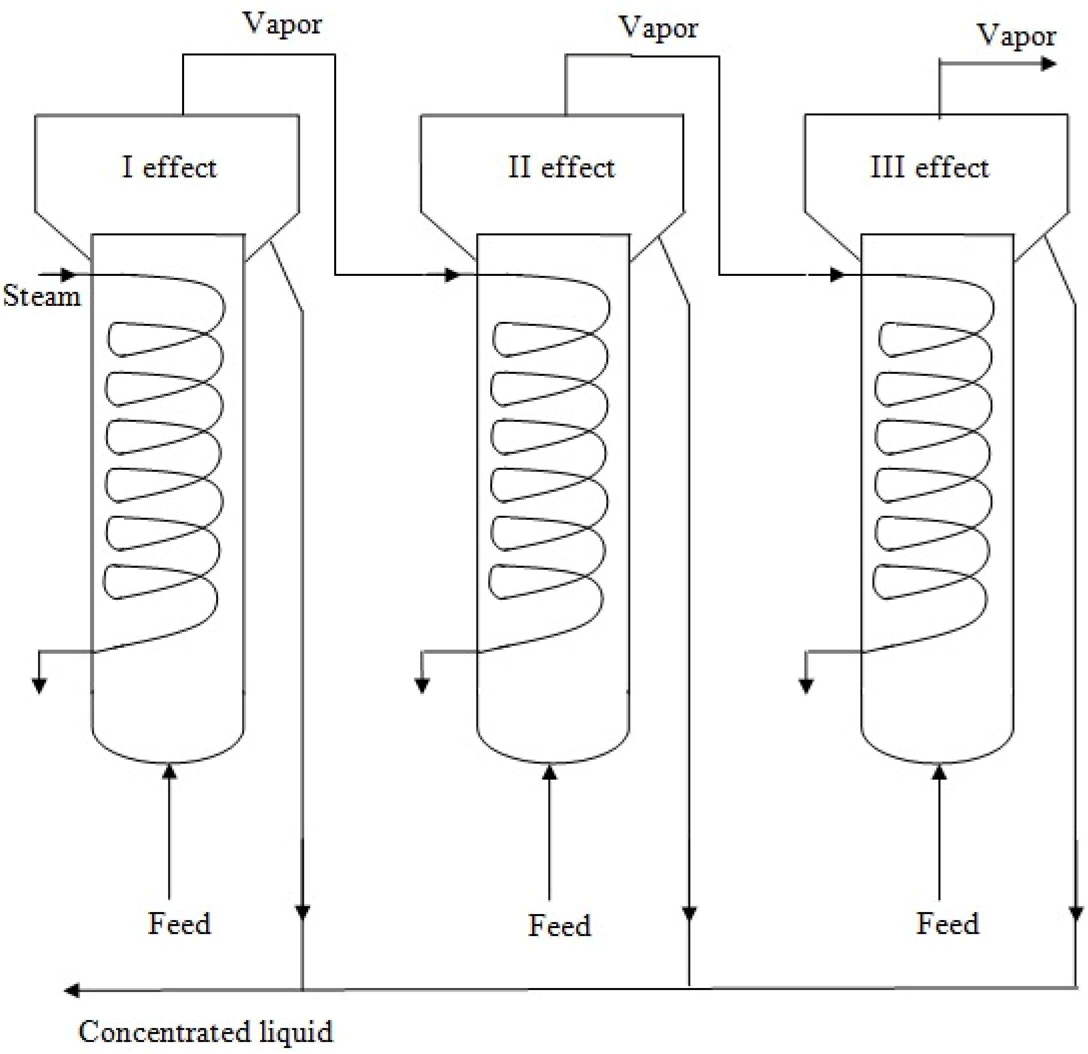

Fig.9.8: Parallel feed arrangement in triple-effect evaporator

9.7 Enthalpy Balance

9.7.1 Single effect evaporator

The latent heat of condensation of the steam is transferred to the boiling solution through the heating surface in order to vaporize the water. Thus, two enthalpy balances are required one for the liquid and another for the steam.

The following assumptions are required, in order to make the enthalpy balance,

1. Flow of non-condensable is negligible
2. The superheat and sub-cooling (see fig. 9.4) of the condensable steam is negligible
3. No solid precipitates out from the concentrating solution

The enthalpy balance for the steam side is,

$$\dot{q}_s = \dot{m}_s(h_s - h_c) = \dot{m}_s\lambda_s$$

Where,

$\dot{q}_s =$ rate of heat transfer through heating surface from steam

$\dot{m}_s =$ flow rate of steam

λ_s = latent heat of condensation of steam

h_s = specific enthalpy of steam

h_c = specific enthalpy of condensate

Enthalpy balance for the liquid side is (eq.9.3),

$$\dot{q}_l = (\dot{m}_{fl} - \dot{m}_{cl})h_v - \dot{m}_{fl}h_{fl} + \dot{m}_{cl}h_{cl} \tag{9.3}$$

Where,

$\dot{q}_l =$ rate of heat transfer from heating surface to the liquid

h_v = specific enthalpy of vapour

h_{cl} = specific enthalpy of concentrated liquid

h_{fl} = specific enthalpy of liquid feed

$\dot{m}_{fl} =$ flow rate of liquid feed

$\dot{m}_{cl} =$ flow rate of concentrated liquid

The enthalpy balance at steam side and liquid side will be same in the absence of any heat loss (eq.9.4). Thus,

$$(\dot{m}_{fl} - \dot{m}_{cl})h_v - \dot{m}_{fl}h_{fl} + \dot{m}_{cl}h_{cl} = \dot{m}_s\lambda_s \tag{9.4}$$

The area of heat transfer A can be calculated from

$$\dot{q}_l = \dot{q}_s = U_D A \Delta T$$

When $\Delta T = (T_b - T_c)$;
T_b = Saturated temperature of steam in the shell

T_s = Boiling point of the solution at the prevailing pressure

U_D = Overall coefficient (dirty)

9.7.2 Effect of heat of dilution

Most of the solutions when mixed or diluted at constant temperature do not give significant heat effect. It is generally true for organic solutions (like sugar). However, many of the inorganic solutions (like sulfuric acid, potassium hydroxide, calcium carbonate etc.) evolve significant heat on dilution. Therefore, an equivalent amount of heat is required (in addition to the latent heat of vaporization), when dilute solutions of these inorganic chemicals are concentrated. Enthalpy-concentration diagram are helpful in order to find the enthalpy of the solution at different concentration of these chemicals in the solution.

9.7.3 Multiple effect evaporators

The steam goes into I-effect and heat the solution by the latent heat of condensation. If the heat required to boil the feed is negligible, it follows that practically all this heat

$$\dot{q}_s = A_1 U_1 \Delta T_1$$

must appear as latent heat in the vapor that leaves the I-effect and enter into II-effect as steam. The temperature of the condensate leaving the II-effect will be very near the temperature T_1 of the vapors from the boiling liquid in the I-effect. Thus, in steady state operation all the heat that was expanded in creating vapor in the I-effect must be given by when this same vapor condenses in the II-effect and so on.

The heat delivered into the II-effect will be,

$$\dot{q}_{v1} = A_2 U_2 \Delta T_2$$

$$\dot{q}_{v1} = \dot{q}_s$$

The
Similarly, for III-effect

$$A_1 U_1 \Delta T_1 = A_2 U_2 \Delta T_2 = A_3 U_3 \Delta T_3 = \dot{q} \qquad (9.5)$$

It can be seen (eq. 9.5) that the temperature drops in a multiple effect evaporator is approximately inversely proportional to the heat-transfer coefficient.

The total available temperature drop will be given by eq.9.6,

$$\sum \Delta T = \Delta T_1 + \Delta T_2 + \Delta T_3 \tag{9.6}$$

$$= (T_s + T_{v3}) - \sum BPE$$

where,
T_s : Steam temp. (I-effect); T_{v3} : Vapor temperature leaving III-effect

BPE : boiling point elevation in the solution in various effects

Illustration

A triple effect forward feed evaporator is used to concentrate a liquid which has marginal elevation in boiling point. The temperature of the stream to the first effect is 105°C, and the boiling point of the solution within third effect is 45°C. The overall heat transfer coefficients are,

2,200 W/m2: in the I-effect,
1,800 W/m2: in the II-effect,
1,500 W/m2: in the III-effect.

Find out at what temperatures the fluid boils in the I and II effects.

Solution:

Assumptions
1. We may assume that there is no elevation in boiling point in the evaporators.
2. Area of all the three evaporators are same ($A_I = A_{II} = A_{III} = A$)

Total temperature drop = (105-45) °C = 60 °C
Using eq. 9.5, the temperature drop across I-effect,

$$\Delta T_I = \frac{\frac{1}{2200}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 15.2\ °C$$

Similarly, the temperature drop across II-effect,

$$\Delta T_{II} = \frac{\frac{1}{1800}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 18.6\ °C$$

And the temperature drop across III-effect,

$$\Delta T_{III} = \frac{\frac{1}{1500}}{\frac{1}{2200} + \frac{1}{1800} + \frac{1}{1500}} \times 56 = 22.3\ °C$$

Therefore, the boiling point in the first effect will be = (105 – 15.2) °C = 89.8 °C
Similarly, the boiling point in the second effect will be = (89.8 – 18.6)°C = 71.2 °C.