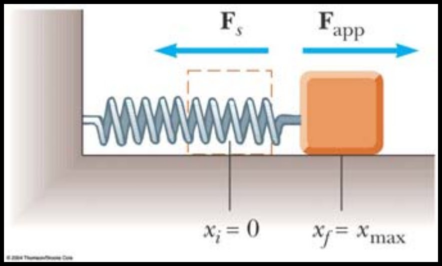
Electric Potential

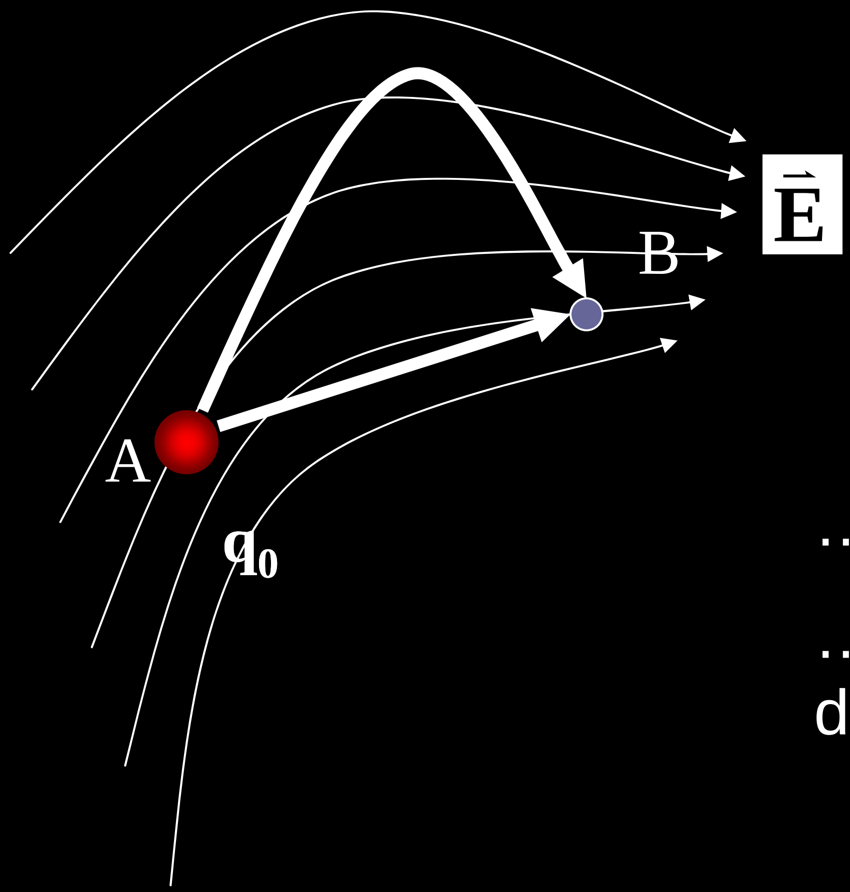
- Flashback to PHY113…
  - Work done by a conservative force is independent of the path of the object.
  - Gravity and elastic forces are examples.
  - This leads to the concept of potential energy and helps us avoid tackling problems using only forces.



$$W = \int_{x_i}^{x_f} F_s dx = -\Delta U$$

- Electrostatic forces are also conservative.

Electric Potential Energy



$$\vec{\mathbf{F}} = q_0\vec{\mathbf{E}}$$

$$\Delta U = -\int_{path} \vec{\mathbf{F}}.d\vec{\mathbf{s}} = -q_0 \int_{path} \vec{\mathbf{E}}.d\vec{\mathbf{s}}$$

…but **F** is a conservative force

…so the path we take does not matter

$$\Delta U = -q_0 \int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{s}}$$

Electric Potential

- Work, ΔU, is dependent on the magnitude of the test charge, q_0.
- We'd like to have a quantity independent of the test charge and only an attribute of the electric field.

$$V = \frac{U}{q_0}$$

Electric Potential
(J/C=V)

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{\mathbf{E}}.d\vec{\mathbf{s}}$$

Potential Difference
between A and B

$$V_P = -\int_\infty^P \mathbf{E}.d\mathbf{s}$$

Electric Potential at P

Unit Pit Stop

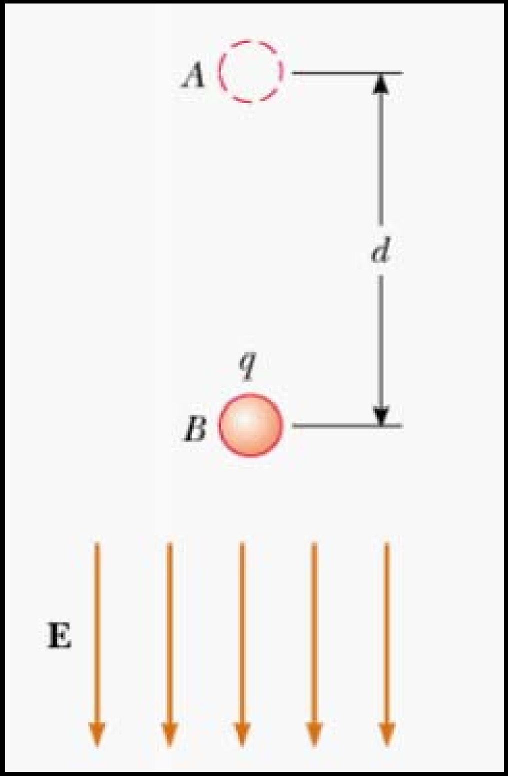
Potential $V = J/C$

Energy $J = N \cdot m$

Electric Field $N/C = \frac{N}{C}\left(\frac{V.C}{J}\right)\left(\frac{J}{N.m}\right) = V/m$

Electron-Volt $1eV = e(1V) = \left(1.6\times10^{-19}C\right)\left(1J/C\right) = 1.6\times10^{-19}J$

Electric Potential in a Uniform Field



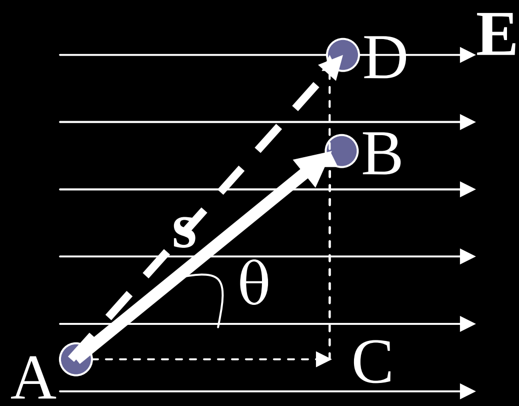
$$\Delta V = -\int_A^B \mathbf{E}.d\mathbf{s} = -\int_A^B E\cos 0^\circ ds = -E\int_A^B ds$$

$$\Delta V = -Ed \qquad \Delta U = -q_0 Ed$$

Electric field lines point to decreasing potential.

A positive charge will lose potential energy and gain kinetic energy when moving in the direction of the field.

Equipotential Surfaces



$$\Delta V = -\int_A^B \mathbf{E}.d\mathbf{s} = -\mathbf{E}.\int_A^B d\mathbf{s} = -\mathbf{E}.\mathbf{s}$$

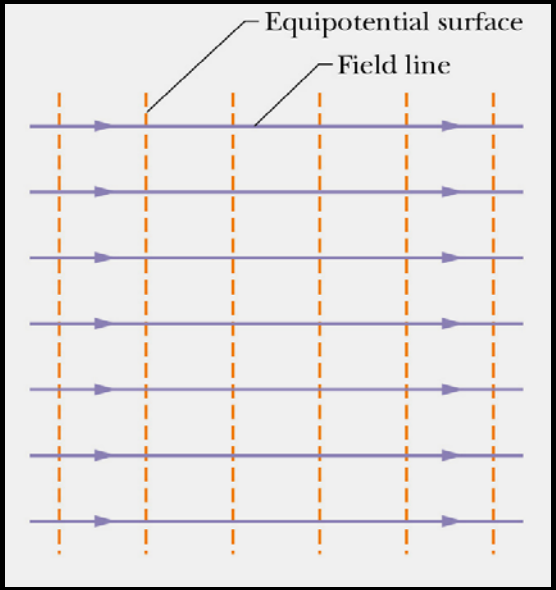
$$\Delta V = -(\mathbf{E}.\mathbf{s})_{AC} - (\mathbf{E}.\mathbf{s})_{CB}$$
$$\Delta V = -Es_{AC}\cos 0° - Es_{CB}\cos 90°$$
$$\Delta V = -Es\cos\theta$$

$$\Delta U = -q_0(\mathbf{E}.\mathbf{s})_{AC} - q_0(\mathbf{E}.\mathbf{s})_{CB}$$
$$\Delta U = -q_0 Es_{AC}\cos 0°$$
$$\Delta U = -q_0 Es\cos\theta$$

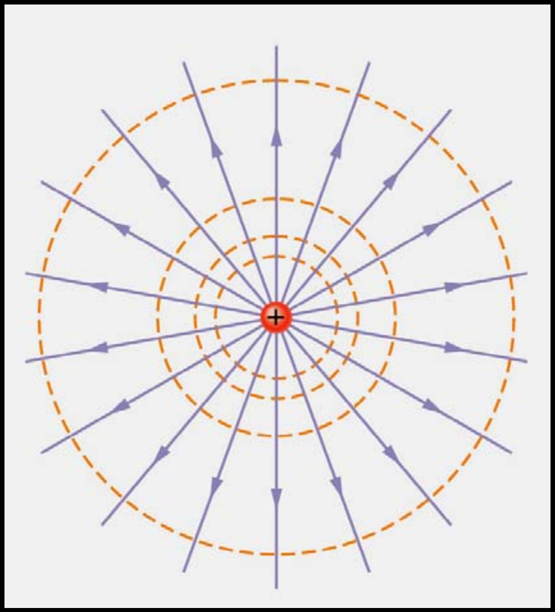
$$\Delta V_{AD} = -\int_A^D \mathbf{E}.d\mathbf{s} = -\mathbf{E}.\int_A^D d\mathbf{s} = -\mathbf{E}.\mathbf{s} = -(\mathbf{E}.\mathbf{s})_{AC} - (\mathbf{E}.\mathbf{s})_{CD} = -Es\cos\theta$$

No work is done moving a charged particle perpendicular to a field (along equipotential surfaces)

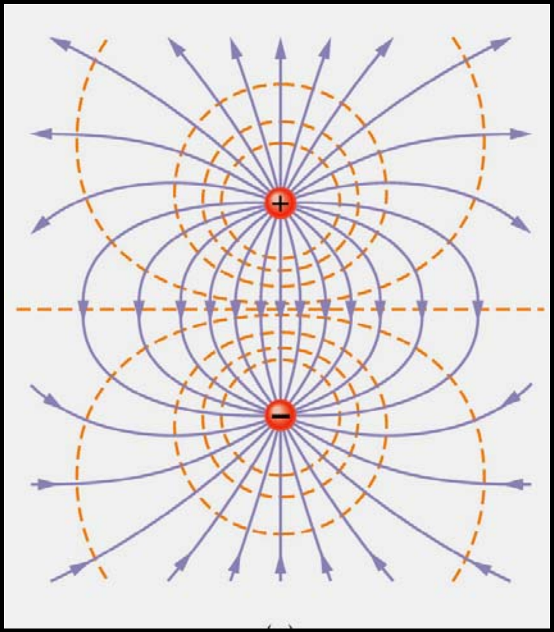
Equipotential Surfaces



Uniform Field

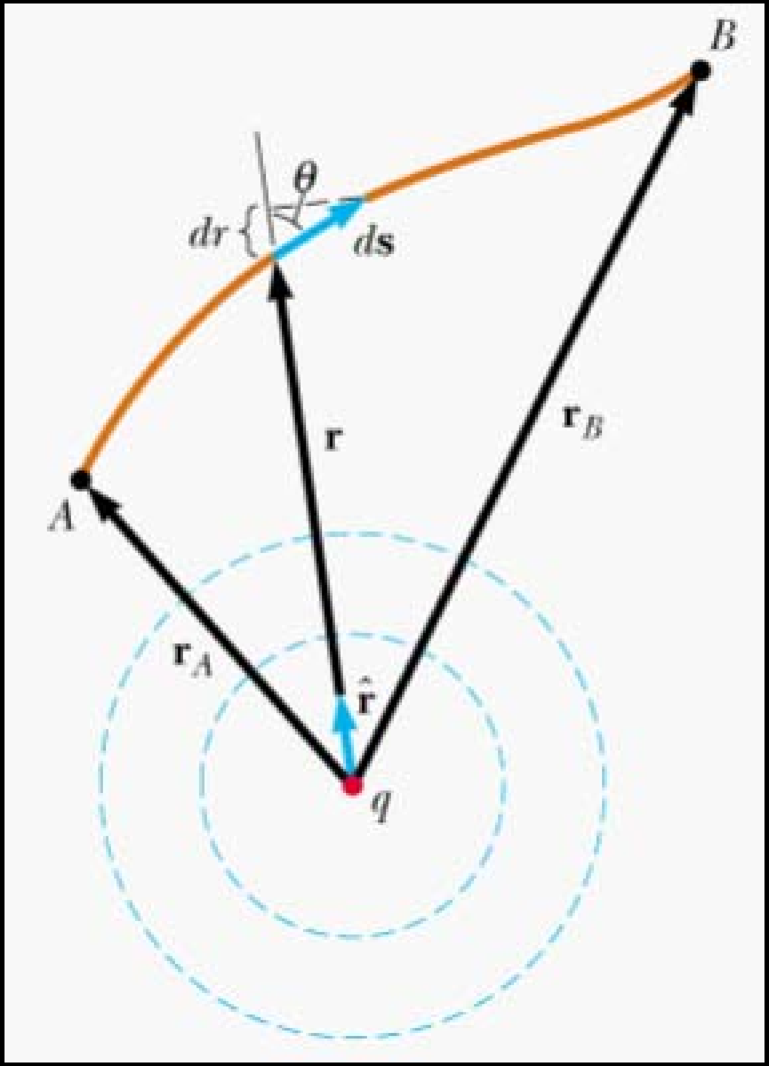


Point Charge



Electric Dipole

Electric Potential of a Point Charge



$$V_B - V_A = -\int_A^B \mathbf{E}.d\mathbf{s}$$

$$\mathbf{E}.d\mathbf{s} = k_e \frac{q}{r^2}\hat{\mathbf{r}}.d\mathbf{s} = k_e \frac{q}{r^2} ds\cos\theta = k_e \frac{q}{r^2} dr$$

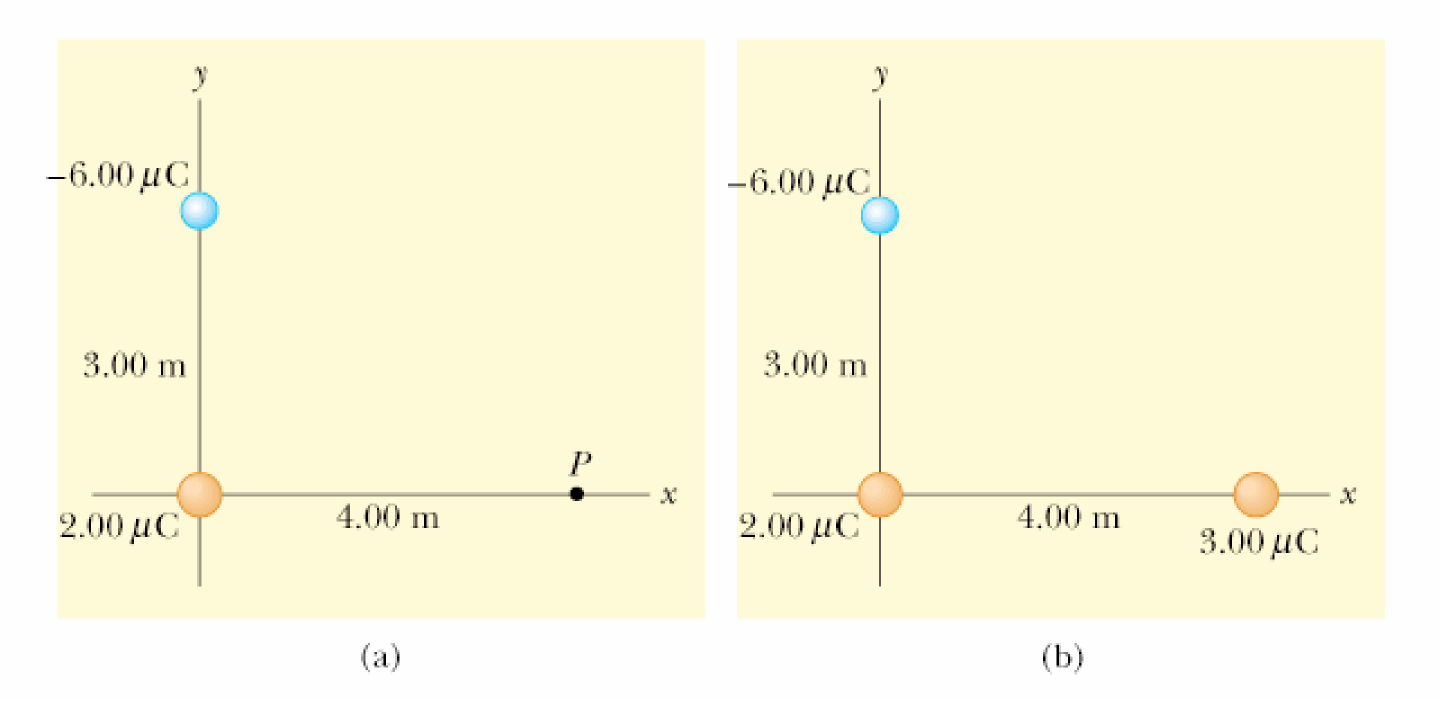
$$V_B - V_A = -\int E_r dr$$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_{A=\infty} = 0$$

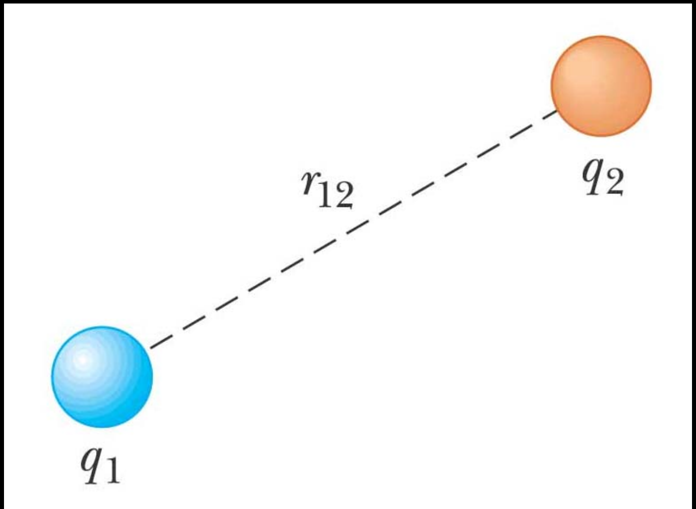
$$V = k_e \frac{q}{r}$$

Two Point Charges

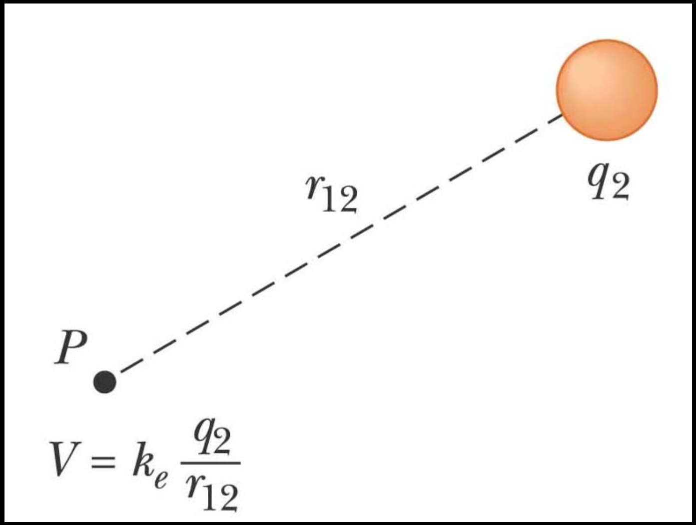


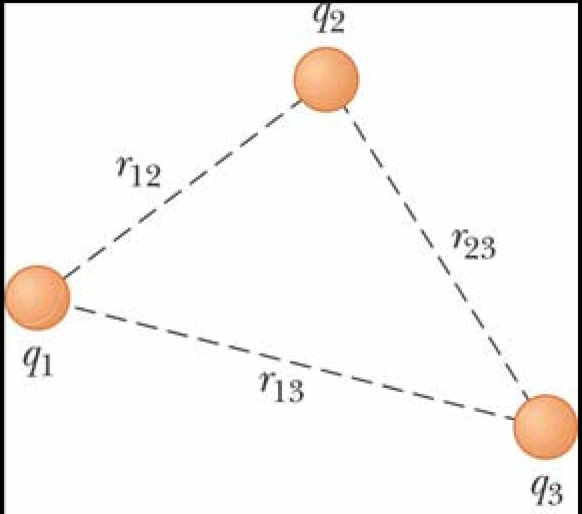
$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

A System of Point Charges



$$U = k_e \frac{q_1 q_2}{r_{12}}$$

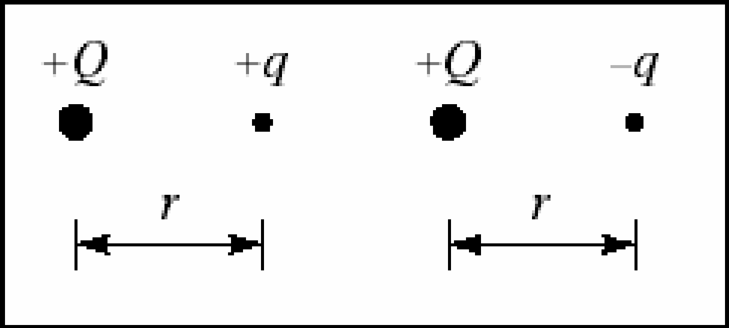




$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}} \right)$$

Concept Question

Two test charges are brought separately into the vicinity of a charge $+Q$. First, test charge $+q$ is brought to a point a distance r from $+Q$. Then this charge is removed and test charge $-q$ is brought to the same point. The electrostatic potential energy of which test charge is greater:



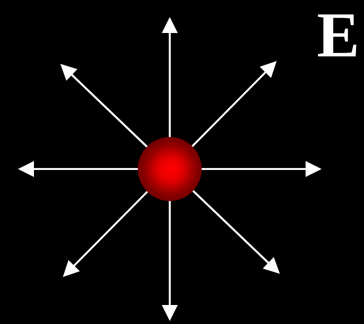
1. $+q$
2. $-q$
3. It is the same for both.

Getting From V to E

$$V = -\int_A^B \mathbf{E}.d\mathbf{s}$$

$$dV = -\mathbf{E}.d\mathbf{s}$$

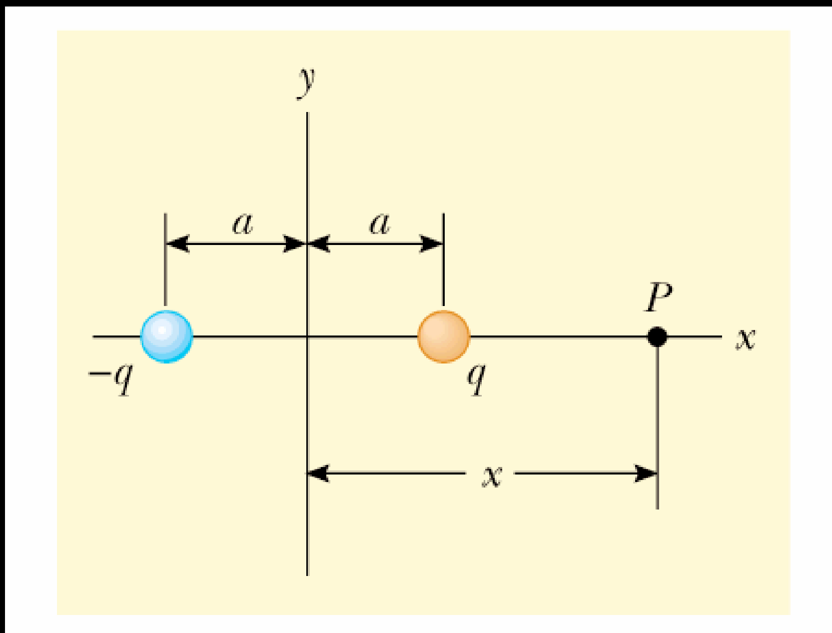
$$E_x = -\frac{dV}{dx}$$



$$E_r = -\frac{dV}{dr}$$

In general: $$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

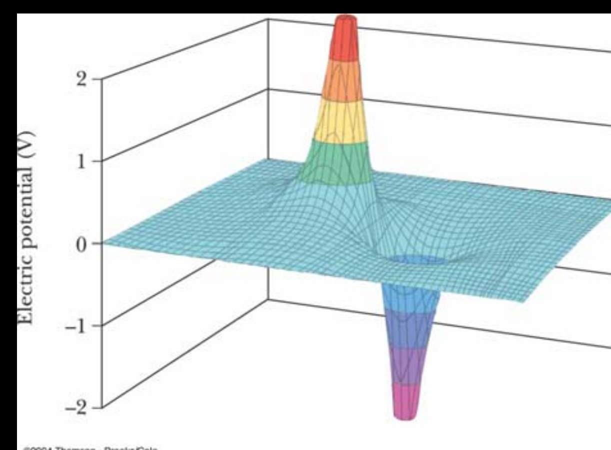
Electric Potential of a Dipole



$$V_P = \frac{2k_e q a}{x^2 - a^2}$$

$$V \approx \frac{2k_e q a}{x^2}$$ (x >> a)

$$E_x = -\frac{dV}{dx} = \frac{4k_e q a}{x^3}$$

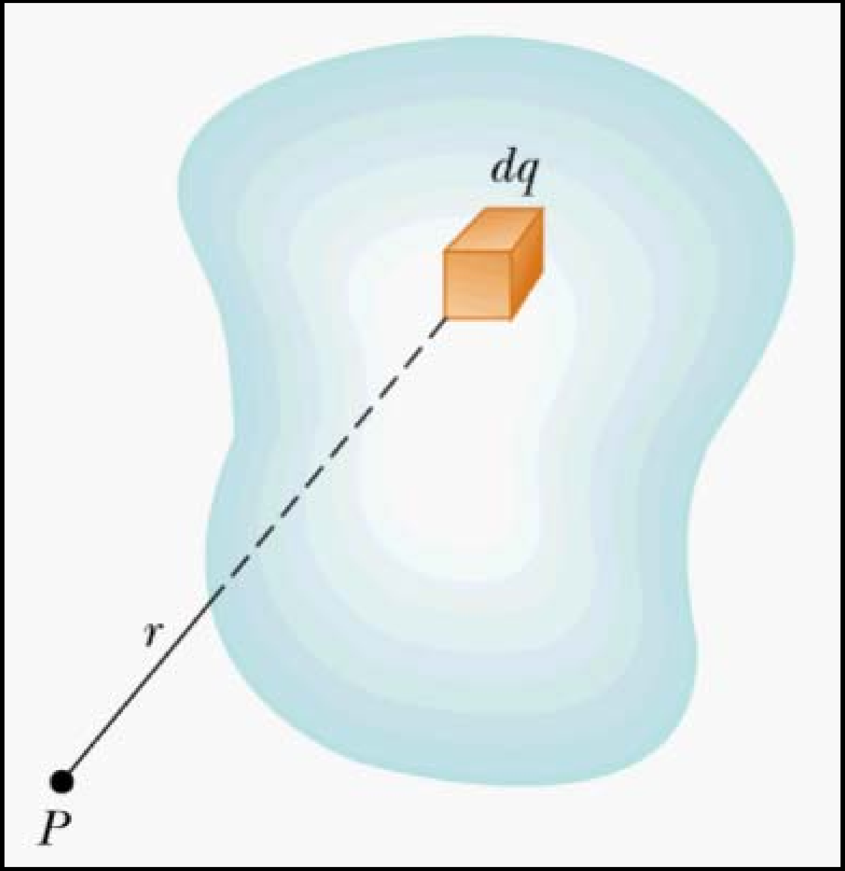


Between the Charges

$$V_P = \frac{2k_e q x}{a^2 - x^2}$$

$$E_x = -\frac{dV}{dx} = -2k_e q\left(\frac{a^2 + x^2}{\left(a^2 - x^2\right)^2}\right)$$

Electric Potential Due to Continuous Charge Distributions



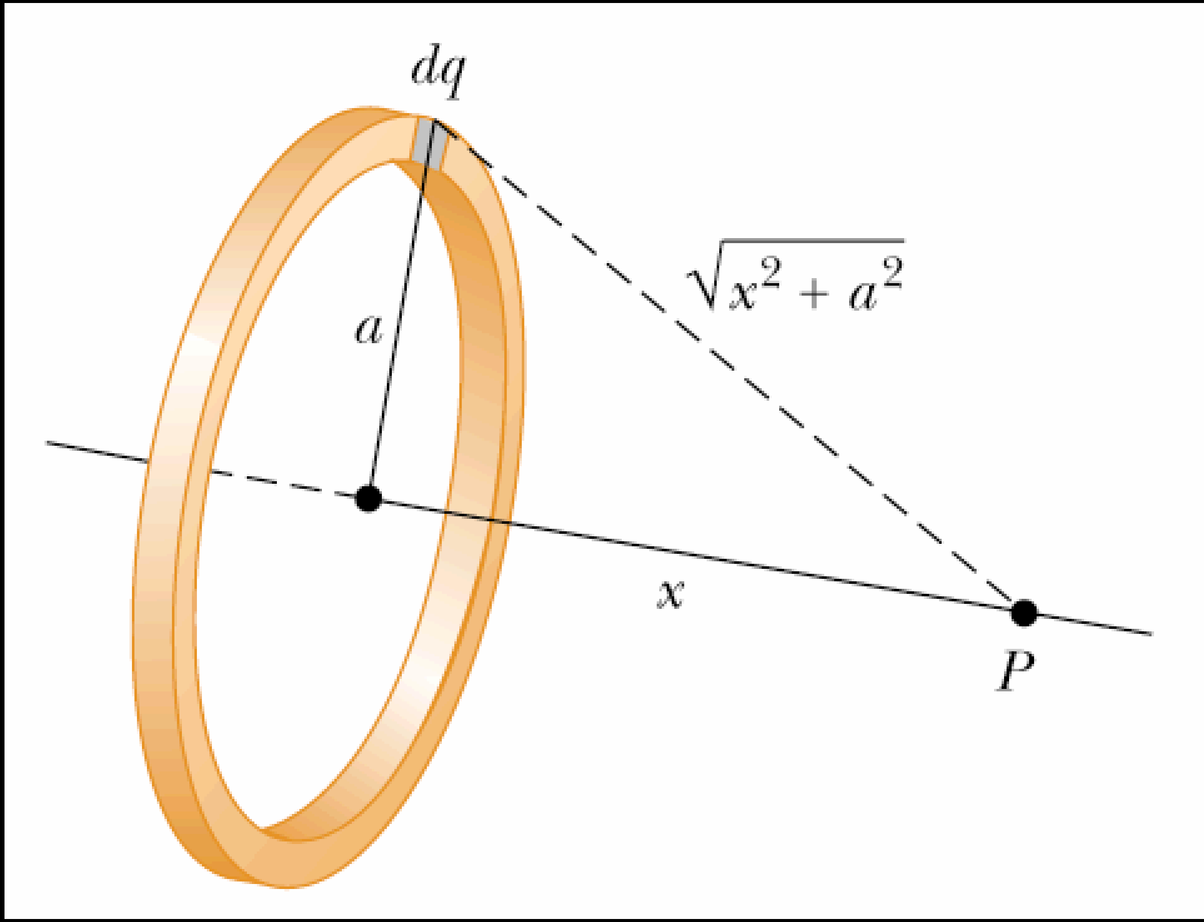
Start with an infinitesimal charge, *dq*.

$$dV = k_e \frac{dq}{r}$$

Then integrate over the whole distribution

$$V = k_e \int \frac{dq}{r}$$

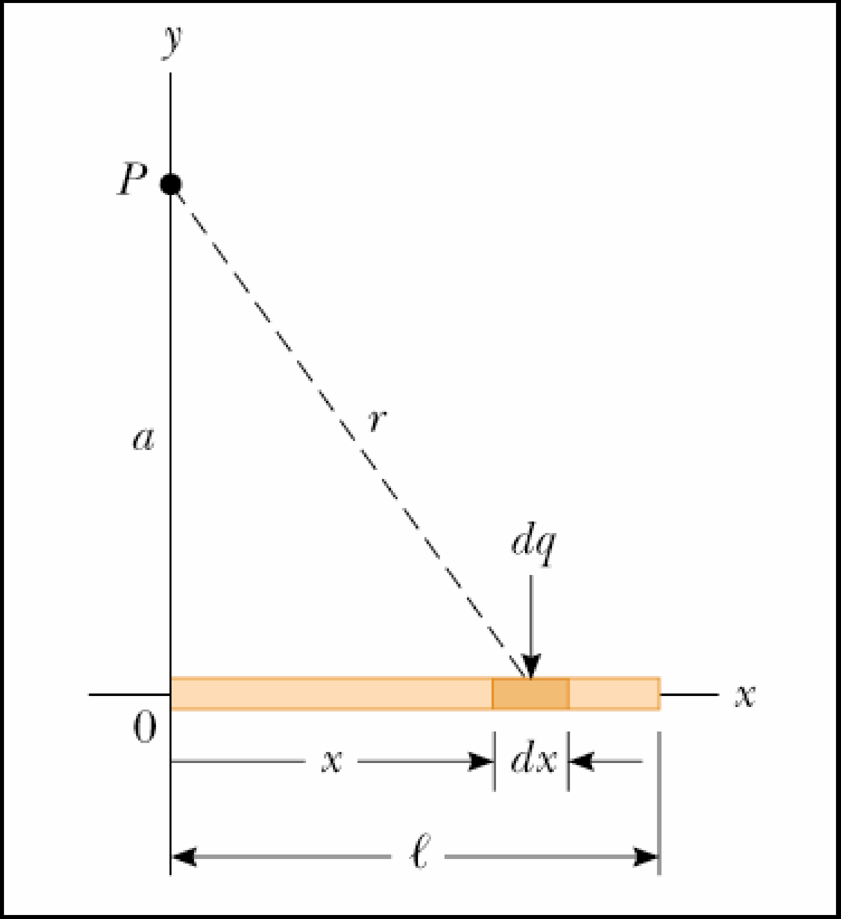
Electric Potential Due to a Uniformly Charged Ring



$$V = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$

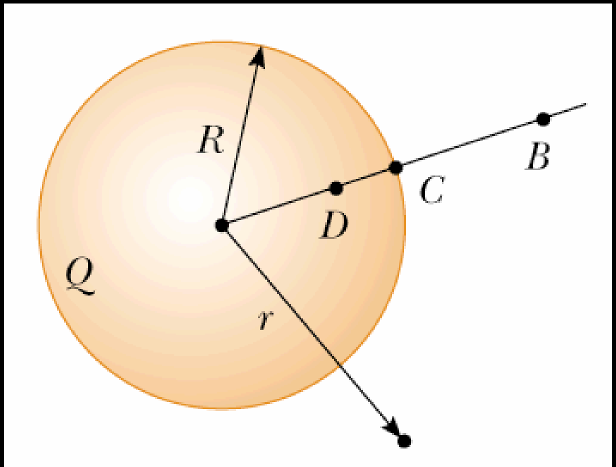
$$E = \frac{k_e Q x}{\left(x^2 + a^2\right)^{3/2}}$$

Electric Potential Due to a Finite Line of Charge



$$V = \frac{k_e Q}{l} \ln\left(\frac{l + \sqrt{l^2 + a^2}}{a}\right)$$

Electric Potential Due to a Uniformly Charged Sphere



r > R

$$E_r = k_e \frac{Q}{r^2}$$

$$V_B = -\int_{\infty}^{r} E_r dr = -k_e Q \int_{\infty}^{r} \frac{dr}{r^2}$$

$$V_B = k_e \frac{Q}{r}$$

$$V_C = k_e \frac{Q}{R}$$

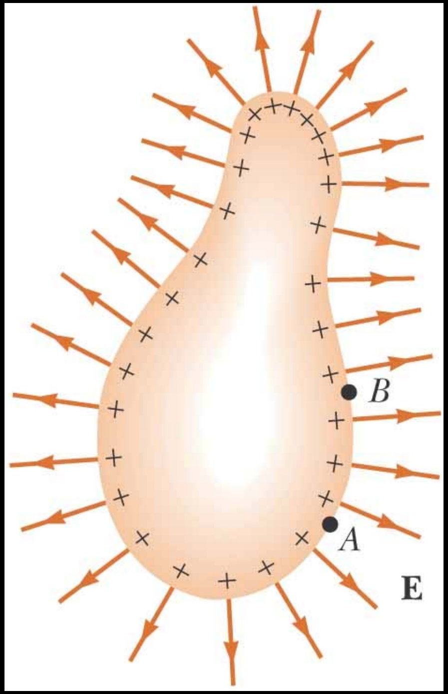
r < R

$$E_r = \frac{k_e Q}{R^3} r$$

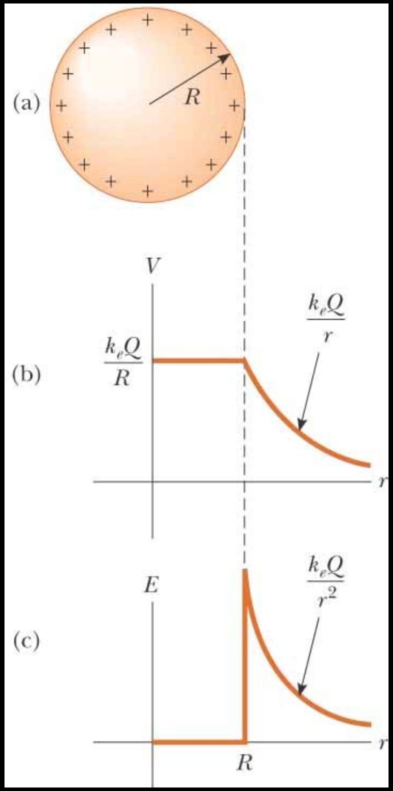
$$V_D - V_C = -\int_{R}^{r} E_r dr = \frac{k_e Q}{2R^3}\left(R^2 - r^2\right)$$

$$V_D = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

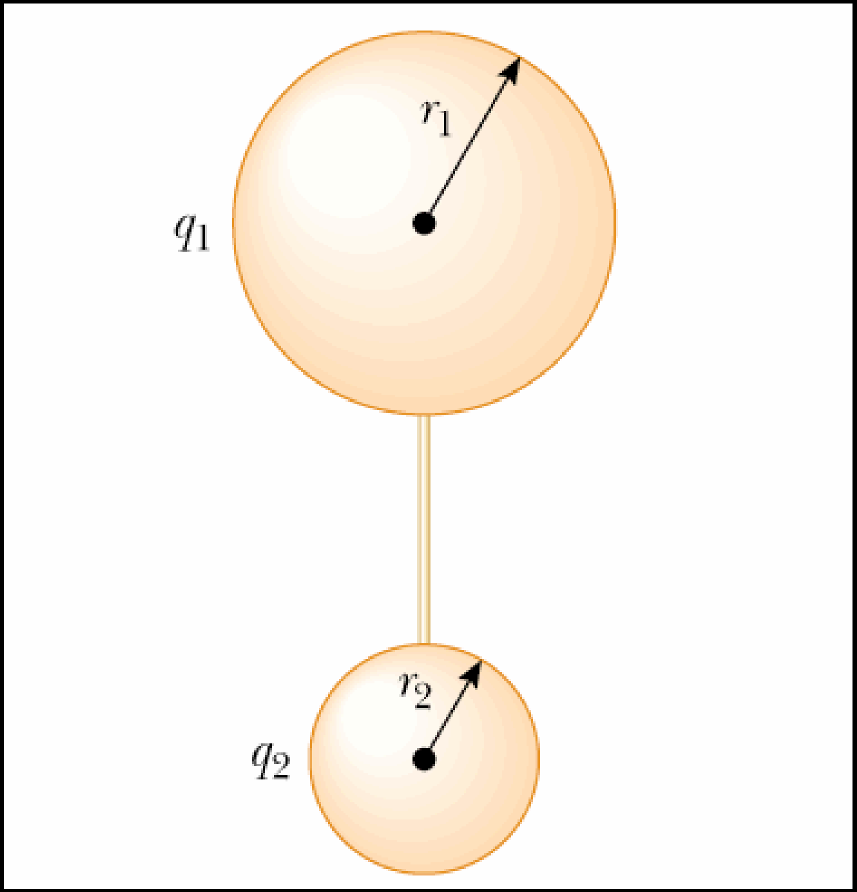
Potential Due to a Charged Conductor



- Charges always reside at the outer surface of the conductor.
- The field lines are always perpendicular to surface.
- Then $\mathbf{E}.d\mathbf{s}=0$ on the surface at any point.
- Which means, $V_B\text{-}V_A=0$ along the surface.
- The surface is an equipotential surface.
- Finally, since $\mathbf{E}=0$ inside the conductor, the potential V is constant and equal to the surface value.



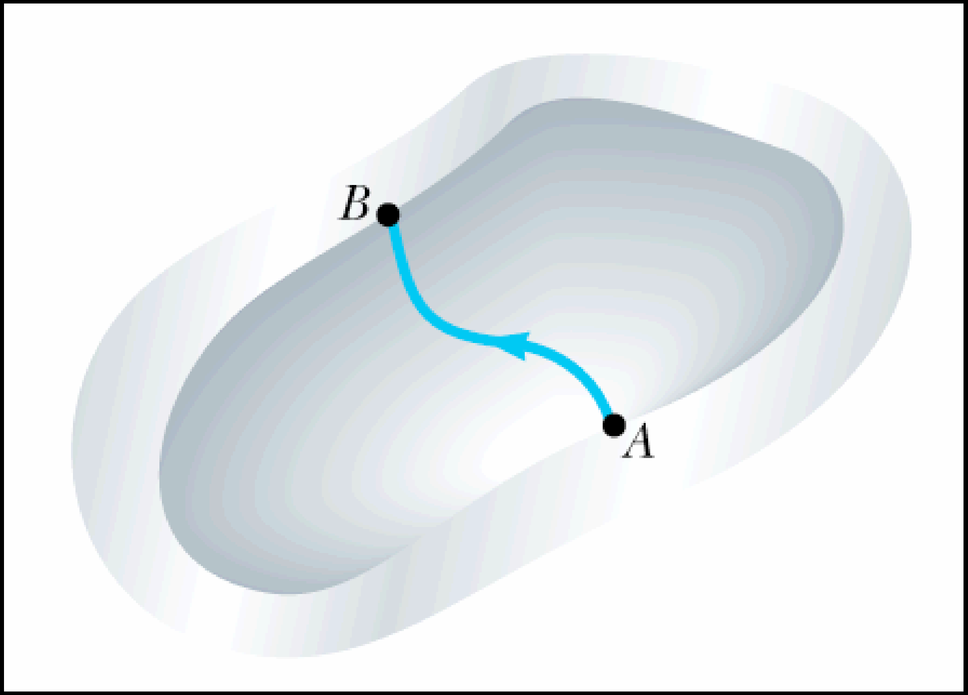
Connected Charged Conducting Spheres



$$\frac{q_1}{q_2} = \frac{r_1}{r_2}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

Cavity Within a Conductor



$$V_B - V_A = -\int_A^B \mathbf{E}.d\mathbf{s} = 0$$

We can always find a path where E.*d*s is non-zero.

But, since $\Delta V=0$ for all paths, E must be zero everywhere in the cavity.

A cavity without any charges enclosed by a conducting wall is field free.

Summary

- Charges can have different electric potential energy at different points in an electric field.
- Electric potential is the electric potential energy per unit charge.
- All points inside a conductor are at the same potential.