Notes on Momentum & Impulse

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1 What is momentum, and why is it important?

1.1 Momentum of a single object

Let us start with a single object with mass m and velocity $\boldsymbol{v}$. The momentum of the object is *defined* to be

$$\boldsymbol{p}_{\text{object}} = m\boldsymbol{v}. \quad (1)$$

It is not really known why $\boldsymbol{p}$ stands for momentum, but this name has stuck and is just something to get used to.

We will build our intuition about momentum a little later via some examples (see §**??**). For the moment, let us concentrate on some of the important properties it has. We start with those we can read straight off from the definition above:

1. *Momentum is a vector*
   Because we obtain momentum by multiplying mass m (a "number") and velocity $\boldsymbol{v}$, the end result is also a vector. i.e. it has a magnitude and direction.

2. *Momentum of an object is parallel to the object's velocity*
   Multiplication of $\boldsymbol{v}$ by m changes the magnitude and units, but as m is a number it has no information about direction. Because masses are always positive, $\boldsymbol{p}$ and $\boldsymbol{v}$ for the same object *always point the same way.*[1]

3. *The SI units of momentum are kg m/s*

[1] If this is not obvious, refer back to the notes on Vectors.

1.2 Momentum of a system

If we have a system that consists of two objects we can construct the momentum of the system:

$$\boldsymbol{p}_{\text{system}} = \boldsymbol{p}_{\text{obj 1}} + \boldsymbol{p}_{\text{obj 2}}, \tag{2}$$

where it is emphasised that the addition is done as *vectors*. Generalising to many different objects is simply a case of adding more momenta.

The reasons that momentum is so important are two-fold:

- Momentum is a state function. More explicitly, to know the momentum of an object we only need to know its mass and velocity **at that instant**.
- The momentum of a *closed momentum system* is conserved.

A closed momentum system means a system that is not being "pushed" or "pulled" overall by outside influences. *This is different from a closed energy system!* It is possible to have an open energy system and a closed momentum system (e.g. an object being heated while at rest), or a closed energy system and an open momentum system. As an exercise see if you can come up with an example of an closed momentum system that is an open energy system (they do exist). The way we determine if a momentum system is open or closed is by studying the pushes and pulls – known in physics as forces – which we will do next. For the time being, we will use our intuition for pushes and pulls.

The above properties of momentum make it very similar to energy. The basic approach to solving problems using momentum conservation will be the same as the approach to energy conservation. We pick an initial and final time, and ask what the momentum has to be in order to be conserved. Because momentum is a state function, we don't have to worry about the messy details between those two times *provided that the system was closed throughout*.

Example #1

A bike with a student on it is initially going 10 m/s to the right. The mass of the student and bike combined is 75 kg. What is the momentum of the student + bike?

	Energy	Momentum
A state function?	Yes	Yes
Conserved in a closed system?	Yes	Yes
Is a number?	Yes	No
Is a vector?	No	Yes
Closed if:	No external heat or work	No external push or pull

Table 1: Comparison of important similarities and differences between energy and momentum

Solution:

This is simply a question that gets us to apply the momentum formula. We know m and $\boldsymbol{v}$, so calculation is trivial:

$$\boldsymbol{p}_{\text{student + bike}} = m\boldsymbol{v} = (75\,\text{kg})(10\,\text{m/s right}) = 750\,\text{kg m/s right}$$

As a vector, it is fairly simple as well:

$$\xrightarrow[\text{750 kg m/s}]{\boldsymbol{p}_{\text{student+bike}}}$$

Example #2

The student from the previous question stops pedalling the bike, and it eventually comes to a stop. What is the final momentum? Is the momentum of the student + bike conserved?

Solution:

- The final velocity is zero, therefore the final momentum is also zero.
- Because $\boldsymbol{p}_i \neq \boldsymbol{p}_f$, we see that the momentum of the bike+student *is not conserved* (duh!).

The only way momentum is not conserved is if you have an open system; i.e. something from outside your system must be pushing or pulling things *inside* your system. In this case it is a combination of pushes on the ground and the air (things that we did not include in our system) which slow the student down. We normally refer to these pushes as friction and air resistance respectively.

Naturally, the *total* momentum is conserved.

Example #3

A 10 kg ice block is pushed across a frozen lake at a speed of 9 m/s. The block is caught by a 50 kg boy, and the two of them slide together. How fast are the two of them going? (You may neglect friction and air resistence).

Solution:

This example is one that we will refer back to as we develop more techniques. The boy is pushed by the block, and the block is pushed by the boy. Therefore although momentum for each is not conserved, the block and boy are not pushed overall by anything else, so momentum for the boy and block **together** will be conserved.

Let us look at the initial momentum. Let us assume that the block is going to the right initially. Then we have

$$\boldsymbol{p}_{\text{block,i}} = m_{\text{block}}\boldsymbol{v}_{\text{block,i}} = (10\,\text{kg})(9\,\text{m/s right}) = 90\,\text{kg m/s right} \tag{3}$$

$$\boldsymbol{p}_{\text{boy, i}} = m_{\text{student}}\boldsymbol{v}_{\text{boy}} = 0\,\text{kg m/s} \tag{4}$$

$$\boldsymbol{p}_{\text{system,i}} = \boldsymbol{p}_{\text{block,i}} + \boldsymbol{p}_{\text{boy,i}} = 90\,\text{kg m/s, right} \tag{5}$$

Once they collide they are moving together and therefore have the same velocity. We know because momentum is conserved that

$$\boldsymbol{p}_{\text{system,f}} = \boldsymbol{p}_{\text{system,i}} = 90\,\text{kg m/s, right.}$$

To solve for the final velocity, we use the fact that the boy and the block are travelling together; i.e. it is as if they are a single body of mass $m = 50\,\text{kg} + 10\,\text{kg} = 60\,\text{kg}$. Solving for the final velocity:

$$\boldsymbol{p}_{\text{system,f}} = 90\,\text{kg m/s,right} = (60\,\text{kg})\boldsymbol{v}_{\text{system, f}}$$
$$\Rightarrow \boldsymbol{v}_{\text{system, final}} = \frac{90\,\text{kg m/s, right}}{60\,\text{kg}} = 1.5\,\text{m/s, right.}$$

i.e. The final velocity of the block and boy together is 1.5 m/s *in the same direction that the block was initially moving*!

Note: one object or two?

You may be worried with the last step of example #3 where we treated the boy and the block as one object. After all, even though they are travelling together they are still individual objects – so why does it work? To answer this, consider the boy and the block as separate objects, and we know that they are travelling at the same velocity $\boldsymbol{v}_f$. The momenta are

$$\begin{aligned}
\boldsymbol{p}_{\text{block,f}} &= m_{\text{block}}\boldsymbol{v}_f = (10\,\text{kg})\boldsymbol{v}_f \\
\boldsymbol{p}_{\text{boy,f}} &= m_{\text{boy}}\boldsymbol{v}_f = (50\,\text{kg})\boldsymbol{v}_f \\
\text{Combining: } \boldsymbol{p}_{\text{system,f}} &= \boldsymbol{p}_{\text{block,f}} + \boldsymbol{p}_{\text{boy,f}} \\
&= (10\,\text{kg})\boldsymbol{v}_f + (50\,\text{kg})\boldsymbol{v}_f \\
&= (60\,\text{kg})\boldsymbol{v}_f
\end{aligned}$$

Setting this equal to the initial momentum then gets the answer we got above. The whole reason this "trick" works is that the velocities are the same, so it factors out and we can just add the masses – or treat this system as if it were a single object.

It is just as well it works like this: after all, in physics there is no such fundamental object as a "boy". The boy is a system of roughly 10^{24} atoms that are joined![2] Because these atoms are all travelling along at more or less the same velocity we can just lump them all together and talk about the boy as a single object. It is the same "trick" that allows us to incorporate the boy and the block into one system. The good news is that if it does make you uncomfortable, you are still free to treat them as separate systems (and this is the approach taken in this course).

2 Momentum charts

Example #3 is a typical momentum conservation argument. While it seems simple, because the explanations take some room and the calculations are not really organised it allows for mistakes to be made and not caught. Physics 7 uses *momentum charts* to help organise calculations, and allow you to rapidly check if your argument is inconsistent.

[2] Similarly, those atoms are made up of other things. But this detail is unimportant for what follows, which is just as well because otherwise we could not answer this simple question until we knew what the most fundamental building blocks were (we still don't and never will).

2.1 The anatomy of a momentum chart

When presenting an argument based on momentum arguments, it address contain three issues:

1. *The system in question*
   This should be a collection of individual objects in your system. In example #3 the system would consist of the boy and the block.

2. *Is the system open or closed?*
   This is a yes/no question, based on whether or not the system is pushed or pulled from the outside.

3. *An initial and final time*
   You should have an initial and final time clearly in mind when presenting any conservation law argument.

A typical momentum chart looks like this:

Object	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Object #1		+		=	
Object #2		+		=	
$\vdots$		+		=	
Object #n		+		=	
System:		+	Important box here!	=	

We then fill up the chart by putting vectors in various boxes inside the chart. The way the chart is written it should be obvious that the first two columns add to give the third column. Because we find the momentum of a system by adding up all the momenta in the system, we can obtain the last line (for the system) by adding all the previous rows.

In all our examples so far we have at least mentioned an initial and final time, and we have been very explicit about the objects in our system. Where do we answer the question about whether the system is an open or closed system? Recall that the momentum of a system is closed if the momentum of the entire system is conserved. This is the same as saying

$$\text{Closed system} \Rightarrow \boldsymbol{p}_{\text{system, i}} = \boldsymbol{p}_{\text{system, f}}$$

or

$$\text{Closed system} \Rightarrow \boldsymbol{\Delta} p_{\text{system}} = \boldsymbol{p}_{\text{system, f}} - \boldsymbol{p}_{\text{system, i}} = 0$$

A system is closed *only if* Δp_{system} is 0, which means the red box must have a zero in it. If the red box does not have a zero in it, then the system is open.

2.2 Using momentum charts

Now we know how to build a momentum chart, how do we actually go about using one? Naturally the easiest way to get a feel for this is by doing examples yourself, but here we present you with a guide to get you started.

1. *Write in the information already given*
2. *Ask yourself if the system is open or closed. If it is closed, put a '0' in the box for Δp_{system} (i.e. the box in red in previous chart)*
3. *Fill in the rest of the chart using the fact that the rows and columns must add.*
   This step is very similar to filling in latin squares, or (more popular nowadays) doing a Sudoku puzzle.

Example #4

Do example #3 using a momentum chart.

Solution:

The objects in the system are the boy and the block, and as described above it is a *closed* system. So far, we know that our momentum chart looks like (step 2 above):

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Boy		+		=	
Block		+		=	
System:		+	0 kg m/s	=	

Let us go through step 1, and use the initial information. We have a boy at rest, and a moving block. The calculation as before is

$$\boldsymbol{p}_{\text{boy, i}} = m_{\text{student}} \boldsymbol{v}_{\text{boy}} = 0\,\text{kg m/s} \tag{6}$$

$$\boldsymbol{p}_{\text{block,i}} = m_{\text{block}} \boldsymbol{v}_{\text{block,i}} = (10\,\text{kg})(9\,\text{m/s right}) = 90\,\text{kg m/s right} \tag{7}$$

$$\boldsymbol{p}_{\text{system,i}} = \boldsymbol{p}_{\text{block,i}} + \boldsymbol{p}_{\text{boy,i}} = 90\,\text{kg m/s, right} \tag{8}$$

and the resulting momentum chart is

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta}p$	=	$\boldsymbol{p}_f$
Boy	0	+		=	
Block	$\xrightarrow{90\text{ kg m/s}}$	+		=	
System:	$\xrightarrow{90\text{ kg m/s}}$	+	0 kg m/s	=	

We can now use the fact we have almost all the information in the last line. We can find out the final momentum, and we fill in the momentum chart (step 3):

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta}p$	=	$\boldsymbol{p}_f$
Boy	0	+		=	
Block	$\xrightarrow{90\text{ kg m/s}}$	+		=	
System:	$\xrightarrow{90\text{ kg m/s}}$	+	0 kg m/s	=	$\xrightarrow{90\text{ kg m/s}}$

Now we have to use additional information that the final velocities of both objects are the same (as they are travelling together). We have

$$\boldsymbol{p}_{\text{system, f}} = \boldsymbol{p}_{\text{boy,f}} + \boldsymbol{p}_{\text{block,f}} = (50\,\text{kg})\boldsymbol{v}_f + (10\,\text{kg})\boldsymbol{v}_f = (60\,\text{kg})\boldsymbol{v}_f$$

From this, we can tell that the final velocity is 1.5 m/s as before.

Notice that we did not need the entire momentum chart – four of the blocks have no values. Even though we have the answer, let us finish off the momentum chart. We know that

$$\boldsymbol{p}_{\text{boy, f}} = m_{\text{boy}}\boldsymbol{v}_f = (50\,\text{kg})(1.5\,\text{m/s}) = 75\,\text{kg m/s} \tag{9}$$

$$\boldsymbol{p}_{\text{block, f}} = m_{\text{block}}\boldsymbol{v}_f = (10\,\text{kg})(1.5\,\text{m/s}) = 15\,\text{kg m/s} \tag{10}$$

Note that these two add to give 90 kg m/s, as required for momentum conservation. Placing this into the momentum chart we have

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta}p$	=	$\boldsymbol{p}_f$
Boy	0	+		=	$\xrightarrow{75\text{ kg m/s}}$
Block	$\xrightarrow{90\text{ kg m/s}}$	+		=	$\xrightarrow{15\text{ kg m/s}}$
System:	$\xrightarrow{90\text{ kg m/s}}$	+	0 kg m/s	=	$\xrightarrow{90\text{ kg m/s}}$

I have shifted the final momentum vector of the block to one side to make it easier to show that the two rows add to give the final momentum of the system.

There are lots of possible ways of getting the last two entries. By looking at the first row, we see that Δp_{boy} is 75 kg m/s to the right. By using the middle column, we see that Δp_{block} has to be 75 kg m/s *to the left*. The final momentum chart is

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Boy	0	+	$\xrightarrow{75 \text{ kg m/s}}$	=	$\xrightarrow{75 \text{ kg m/s}}$
Block	$\xrightarrow{90 \text{ kg m/s}}$	+	$\xleftarrow{75 \text{ kg m/s}}$	=	$\xrightarrow{15 \text{ kg m/s}}$
System:	$\xrightarrow{90 \text{ kg m/s}}$	+	0 kg m/s	=	$\xrightarrow{90 \text{ kg m/s}}$

The final step is to check that the momentum chart makes sense. In this case, we would check the "block" row of the momentum chart and make sure it adds up.

2.3 Pros and Cons

Notice that the momentum chart did not allow any shortcuts – every calculation we did in example #3 we had to repeat in example #4. In fact, to complete the momentum chart I needed to do *more* calculations than just finding the velocity! Provided we were only asked to find the velocity, it would be perfectly acceptable to not finish the momentum chart.

The main advantage of momentum charts are that they organise the calculation for us – it is clear what has been calculated, what is left to calculate and what relationships must exist between them. The other advantage that they have is that once the problem is complete, you can check the relationships between the rows and columns and *check* that the system is consistent.

You MUST use momentum charts in physics 7B. If you find them useful, then you should continue using them when doing physics in the future.

2.4 Impulse

Impulse is simply another name for $\boldsymbol{\Delta} p$

At the moment there is no new physics involved in impulse; it is just a name! I will always use the symbol $\boldsymbol{\Delta} p$ to mean impulse as I see no point in unnecessarily proliferating symbols. Be warned that my opinion on the economy of symbols is not universal, and so you will often see others referring to impulse as $\boldsymbol{J}$.[3]

Recall that the momentum of an object is conserved if nothing *exterior* pushes or pulls it. i.e.

$$\boldsymbol{\Delta} p_{\text{obj}} = 0 \Rightarrow \text{no net external push or pull.} \tag{11}$$

Thus if there *is* an impulse on an object, we know that it has been pushed or pulled:

$$\boldsymbol{\Delta} p_{\text{obj}} \neq 0 \Rightarrow \text{an external push or pull.} \tag{12}$$

This is known as Newton's first law of motion, although this is not the language in which it is usually expressed.

The other point which is important to stress is that the impulse and the average net push or pull always point in the same direction. More will be made of this point when forces are discussed.

3 Collisions

Let us think about the case where two objects collide, and they push each other around but are not pushed or pulled overall by anything else. Then the two objects form a closed system and so the system's momentum is conserved.

One might be tempted to use conservation of energy and set the energy before the collision and after the collision to be the same. After all, this is what we learnt in physics 7A! However, we are not guaranteed that these are closed *energy* systems, only that they are closed *momentum* systems. As explained in table 1 these are not necessarily the same! This system could give off heat or noise as a result of the collision. Only if we included transfers to the environment as well would we find that the energy is conserved.

Even if the system *is* a closed energy system, some energy may go into raising the temperature of the object or melting it. That way all the energy stays in the system, but the *kinetic* energy of the system is not conserved. The special case where the kinetic energy of the objects before and after the collision is the same is so special that it receives its own name: an elastic collision. Any other collision is inelastic.

[3] I think the only reason for using $\boldsymbol{J}$ for impulse is that people are too lazy to type "Δ"!

Types of collisions:

- *Elastic collision*
  A collision in which KE is conserved.

- *Inelastic collision*
  A collision in which KE is **not** conserved.

- *Completely inelastic collision*
  A collision where objects stick together. These collisions are those in which the most kinetic energy is lost.

Note that in *all* of these collisions momentum is still conserved.

Example #5

Cart A is heading right at a speed of 5 m/s, and cart B is heading left at a speed of 5 m/s on a horizontal frictionless surface. Both carts have identical masses. The two carts collide. Below some possibilities for the final velocities of cart A and B. State for each set if the combination are possible, if the combination conserves momentum, and if the collision is elastic.

a) Cart A comes to rest, Cart B comes to rest.

b) Cart A goes left at 1 m/s, Cart B goes right at 1 m/s.

c) Cart A goes left at 1 m/s, Cart B goes right at 7 m/s.

d) Cart A goes into the page at 5 m/s, Cart B comes out of the page at 5 m/s.

e) Cart A goes left at 8 m/s, Cart B goes right at 8 m/s

Solution

Nothing is pushing the system overall from outside, so we are going to consider the two Carts to form a closed momentum system. This tells us that we should put $\boldsymbol{\Delta} p_{\text{system}} = 0$ in the lower box on our momentum chart (step 2). To fill out the momentum chart let us call the mass of a cart m, fill in the additional information for the initial conditions and then complete what we can:

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Cart A	$\xrightarrow{(5\text{ m/s})m}$	+	??	=	??
Cart B	$\xleftarrow{(5\text{ m/s})m}$	+	??	=	??
System:	0 kg m/s	+	0 kg m/s	=	0 kg m/s

This is as far as we can get on momentum conservation alone. To go any further requires that we say something about how the blocks pushed one another *or* make a statement about initial and final conditions. I will do cases a), c) and e) and let you work b) and d) out for yourselves.

Although kinetic energy does not need to be conserved, we need to know it to say if the collision is elastic or not. First we compute the initial kinetic energy of the carts:

$$\begin{aligned} KE_{\text{Cart A, i}} &= \frac{1}{2}m(5\,\text{m/s})^2 = (12.5\,\text{J/kg})m \\ KE_{\text{Cart B,i}} &= \frac{1}{2}m(5\,\text{m/s})^2 = (12.5\,\text{J/kg})m \\ &\Rightarrow KE_{\text{system,i}} = (25\,\text{J/kg})m \end{aligned}$$

Scenario a)

Here we are told that the final velocities are both zero, so the momentum chart looks like this:

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Cart A	$\xrightarrow{(5\text{ m/s})m}$	+		=	0 kg m/s
Cart B	$\xleftarrow{(5\text{ m/s})m}$	+		=	0 kg m/s
System:	0 kg m/s	+	0 kg m/s	=	0 kg m/s

We could easily fill in the middle column (but we don't need to – we have all the information we need already). It is probably a good exercise to fill it in anyway, but I will leave you to do that. The point is that momentum *is* conserved, so this collision is allowed.

Let us look at the kinetic energy now. Obviously the final KE is 0 J, whereas the initial KE was (25 J/kg)m. We have *lost* kinetic energy! This is okay, as that kinetic energy has simply been converted into other forms of energy – conservation of energy is still okay. But because the *kinetic* energy is not conserved, this is an inelastic collision (in fact, it is a completely inelastic collision).

Conclusion: allowed, conserves momentum, (completely) inelastic.

Scenario c)

Let us put the final momenta given in c) into our momentum chart:

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Cart A	$\xrightarrow{(5\text{ m/s})m}$	+		=	$\xleftarrow{(1\text{ m/s})m}$
Cart B	$\xleftarrow{(5\text{ m/s})m}$	+		=	$\xrightarrow{(7\text{ m/s})m}$
System:	0 kg m/s	+	0 kg m/s	=	0 kg m/s

By looking at the final column we see that there is no way that these vectors add to zero. Therefore:

$\boldsymbol{p}$ *not conserved* $\Rightarrow$ *this processes cannot happen!*

If you had calculated the final kinetic energy you would have seen that it *was* conserved. However, momentum conservation rules this out as an allowed process.

Scenario e)

Putting the final momenta into the momentum chart gives

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Cart A	$\xrightarrow{(5\text{ m/s})m}$	+		=	$\xleftarrow{(8\text{ m/s})m}$
Cart B	$\xleftarrow{(5\text{ m/s})m}$	+		=	$\xrightarrow{(8\text{ m/s})m}$
System:	0 kg m/s	+	0 kg m/s	=	0 kg m/s

We see momentum conservation works.

We also see that each of these carts is going faster than it had initially. This implies that $KE_{\text{system,final}} > KE_{\text{system,initial}}$! While we can generally *lose* kinetic energy to the environment or thermal energy, it requires special circumstances for us to suddenly gain large amounts of kinetic energy. So I would rule this scenario *impossible*, as it violates total energy conservation.

(Note: You could think of a rather artifical case, such as a bomb being on the side of cart A to make this work. When the carts touch, the bomb blows up and some of the energy of the explosion speeds the two sides up. Even in this example, momentum would still have to be conserved but we could get “extra” KE. See the next example for a slightly more commonplace example).

Example #6

Bill and Ted are sitting on rolling office chairs on a flat surface. They are both initially at rest. Note that Ted has roughly twice the mass of Bill. Bill pushes off Ted’s chair and goes flying to the right. Neither Bill or Ted put their feet on the floor, they only push off each other.

a) Which way does Ted go?

b) Whose magnitude of momentum is greater: Bill or Ted’s?

c) Whose (magnitude of) final velocity is greater: Bill or Ted’s?

Solution:

Again we have a closed system, and we have Bill and Ted initially at rest. This information (as well as adding the first column and the final row) gives us

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Bill	0 kg m/s	+		=	
Ted	0 kg m/s	+		=	
System:	0 kg m/s	+	0 kg m/s	=	0 kg m/s

So far this is a pretty boring system! The other piece of information that we have is that after pushing off Ted’s chair Bill goes flying to the right. We can put Bill’s final momentum into the chart, and then it tells us what Ted’s final momentum must be:

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Bill	0 kg m/s	+		=	$\longrightarrow$
Ted	0 kg m/s	+		=	$\longleftarrow$
System:	0 kg m/s	+	0 kg m/s	=	0 kg m/s

i.e. Ted must go to the left to conserve momentum, and as they sum to zero the magnitudes of Bill and Ted’s momenta must be the same.

Then there is the question of velocities. Even though the (magnitudes of) the final momentum of Bill and Ted are the same, Ted has a greater mass than Bill. Therefore *Bill must go faster than Ted* (recall $\boldsymbol{p} = m\boldsymbol{v}$).

The other comment to be made is that the initial KE was zero. But the final KE is not, as things are moving! That is okay, because the energy comes from somewhere: namely Bill doing work by pushing on Ted's chair. In the cart example #5e) there was no source to supply additional energy (short of rather creative solutions, such as bombs).

Example #7

(Continued from example #6)
Bill is still going to the right after pushing off Ted's chair. Bill crashes into the wall of the office he was fooling around in and comes to a complete stop. Is the system (Bill + wall) an open or closed momentum system?

Solution:

If this is a normal office wall, we know it won't be moving before or after the collision. We know that Bill was going to the right initially and then stopped. Putting this information into the momentum chart:

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Bill	$\longrightarrow$	+		=	0
Wall	0	+		=	0
System:		+		=	

We now have enough information to fill it in completely:

	$\boldsymbol{p}_i$	+	$\boldsymbol{\Delta} p$	=	$\boldsymbol{p}_f$
Bill	$\longrightarrow$	+	$\longleftarrow$	=	0
Wall	0	+	0	=	0
System:	$\longrightarrow$	+	$\longleftarrow$	=	0

We see $\boldsymbol{\Delta} p_{\text{system}} \neq 0$, so it is an open momentum system.

This means that something outside the system was pushing or pulling things inside the system. In this case it is easy to see the culprit: the reason the wall does not move is because it is dug into the ground. When Bill slammed into the wall, he pushed the wall but *the ground pushed back*. Because the entire Earth was not part of our system, this "external" push caused the momentum to be conserved. If we included the momentum of the entire Earth, everything would have balanced out.

4 Summary

- Momentum is a vector, and points in the direction of motion.
- Momentum of a system is conserved if nothing is pushing or pulling it from outside the system.
- An elastic collision is a collision that conserves kinetic energy.
- An inelastic collision is a collision that does not conserve kinetic energy.
- All collisions (in a closed momentum system) conserve momentum.
- Impulse is another name for "change in momentum", and is sometimes denoted $\boldsymbol{J}$.