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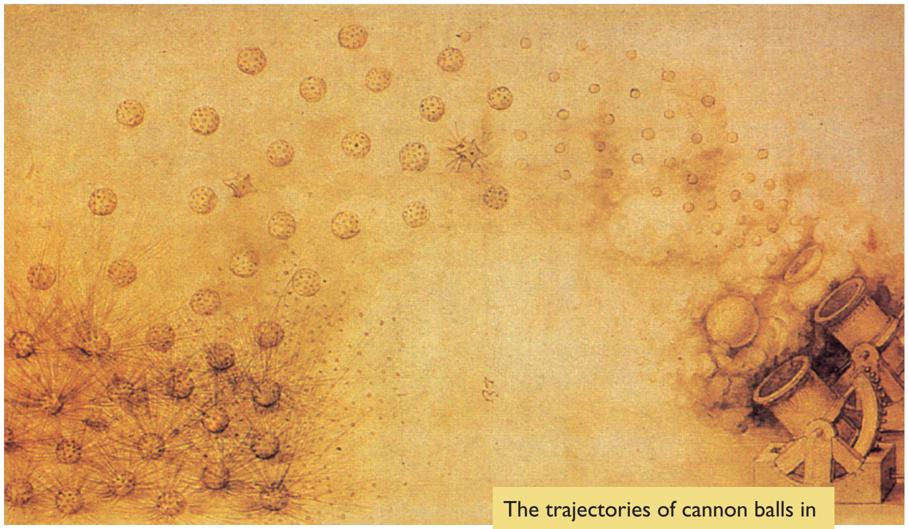
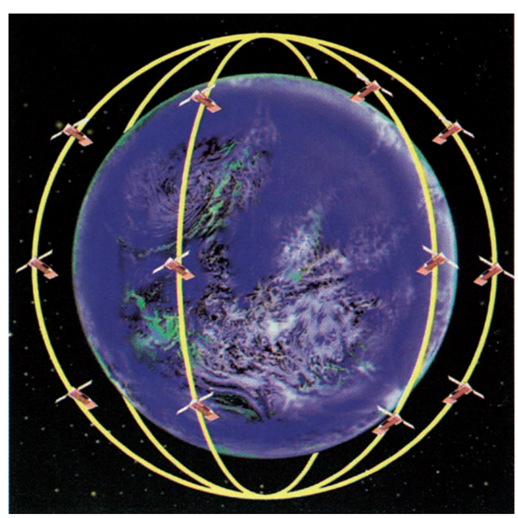
CHAPTER 3 Motion in a Plane



The trajectories of cannon balls in a drawing by Leonardo da Vinci.

In this chapter we shall generalize our discussion of accelerated motion to include nonlinear motion. For simplicity we shall limit our study to motion in a single plane. There are many examples of such motion: the trajectory of a baseball, football, or any other projectile is in a vertical plane, the trajectory of a car rounding a curve is in a horizontal plane, and the trajectory of an earth satellite is in a plane passing through the center of the earth.

Each of the satellites in this proposed new satellite network moves in a plane. The satellites would provide a worldwide cellular telephone system, enabling subscribers to be reached at the same telephone number, no matter where they travel throughout the world.



3–1 Acceleration on a Curved Path

In Chapter 2 acceleration was defined as the rate of change of velocity. For linear motion, which is restricted to one unchanging direction, this definition implies that there is nonzero acceleration only when there is a change of speed. For nonlinear motion, however, the definition implies that there is acceleration even when the speed is constant, for the following reason: as a particle moves along a curved path, its velocity vector constantly changes direction. Since there is a change in the direction of the velocity vector, the particle is accelerated whether its speed changes or not.

In studying linear motion in Chapter 2, we were able to represent any motion as being either along the x-axis or along the y-axis. With motion in two dimensions, we need to be concerned with movement in both the x and y directions—in other words, with movement in the xy plane. We shall obtain an expression for average acceleration in the xy plane by applying the definition of average acceleration (Eq. 2–3):

$$\boxed{\overline{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t}} \qquad \text{(average acceleration)} \qquad (3\text{–}1)$$

In two dimensions, the change in velocity vector $\Delta\mathbf{v}$ in general has components Δv_x and Δv_y. The x and y components of the average acceleration vector are therefore the rates of change of the x and y components of velocity:

$$\overline{a}_x = \frac{\Delta v_x}{\Delta t} \qquad (3\text{–}2)$$

$$\overline{a}_y = \frac{\Delta v_y}{\Delta t} \qquad (3\text{–}3)$$

As we learned in Chapter 2, the instantaneous acceleration $\mathbf{a}$ is the limiting value of the average acceleration for a time interval approaching zero (Eq. 2–4):

$$\boxed{\mathbf{a} = \underset{\Delta t \to 0}{\text{limit}} \frac{\Delta \mathbf{v}}{\Delta t}} \qquad \text{(instantaneous acceleration)} \qquad (3\text{–}4)$$

In two dimensions, the instantaneous acceleration has components a_x and a_y, which are the respective limits of $\overline{a}_x$ and $\overline{a}_y$ for Δt approaching zero:

$$a_x = \underset{\Delta t \to 0}{\text{limit}} \frac{\Delta v_x}{\Delta t} \qquad (3\text{–}5)$$

$$a_y = \underset{\Delta t \to 0}{\text{limit}} \frac{\Delta v_y}{\Delta t} \qquad (3\text{–}6)$$

EXAMPLE 1 Accelerating on a Curve

A car initially is traveling east at a speed of 20.0 m/s. Then, 3.00 s later, having rounded a curve, the car is traveling 30.0° north of east at a speed of 25.0 m/s. Find the car's average acceleration during the 3.00 s interval.

SOLUTION In Fig. 3–1a we have sketched the motion of the car, with initial velocity **v** and final velocity **v′**. The corresponding change in velocity $\Delta\mathbf{v} = \mathbf{v}' - \mathbf{v}$ is indicated in Fig. 3–1b. Our knowns are: $\mathbf{v} = 20.0$ m/s directed east, $\mathbf{v}' = 25.0$ m/s directed 30.0° north of east, and $\Delta t = 3.00$ s. Our task is to find $\overline{\mathbf{a}}$ in terms of **v** and **v′**. We shall first find the x and y components of the car's average acceleration. Applying Eq. 3–2, we express the x component of acceleration in terms of the change in the x component of velocity:

$$\overline{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{v_x' - v_x}{\Delta t}$$

From Fig. 3–1b we find

$$\overline{a}_x = \frac{v' \cos 30.0° - v}{\Delta t}$$

$$= \frac{(25.0 \text{ m/s})(\cos 30.0°) - 20.0 \text{ m/s}}{3.00\text{s}}$$

$$= 0.550 \text{ m/s}^2$$

Next we find the y component of acceleration, applying Eq. 3–3 and again using Fig. 3–1b:

$$\overline{a}_x = \frac{\Delta v_y}{\Delta t} = \frac{v_y' - v_y}{\Delta t}$$

$$= \frac{(25.0 \text{ m/s})(\sin 30.0°) - 0}{3.00\text{s}}$$

$$= 4.17 \text{ m/s}^2$$

The magnitude and direction of $\overline{\mathbf{a}}$ are found from its components in the usual way. First the magnitude:

$$|\overline{\mathbf{a}}| = \sqrt{\overline{a}_x^{\,2} + \overline{a}_y^{\,2}} = \sqrt{(0.550 \text{ m/s}^2)^2 + (4.17 \text{ m/s}^2)^2}$$

$$= 4.21 \text{ m/s}^2$$

As indicated in Fig. 3–1c, the vector $\overline{\mathbf{a}}$ makes an angle θ with the x-axis, where

$$\theta = \arctan\left(\frac{\overline{a}_y}{\overline{a}_x}\right) = \arctan\left(\frac{4.17}{0.550}\right)$$

$$\approx 82°$$

The car experiences an average acceleration of 4.21 m/s^2 in a direction 82° north of east. This considerable acceleration is caused mainly by the change in direction (Δv_y) rather than by the change in speed. The same rate of change of speed on a straight road would produce an acceleration of only (25.0 m/s − 20.0 m/s)/3.00 s = 1.67 m/s^2.

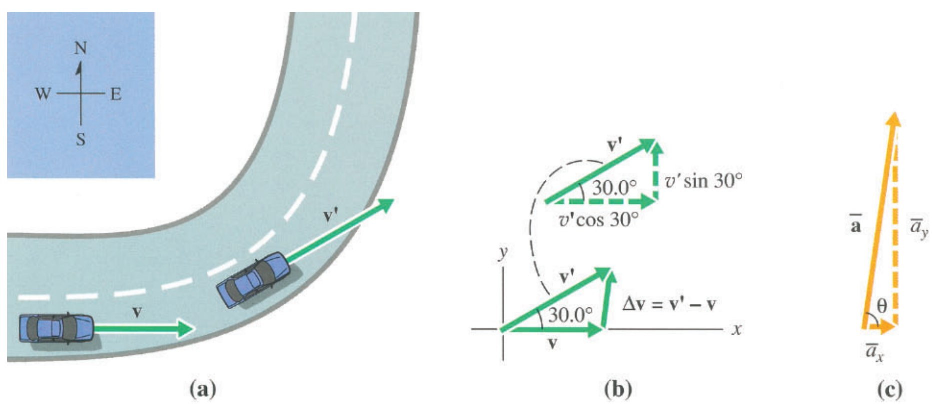


Fig. 3–1

If the x and y components of a particle's acceleration are known functions of time, we can find expressions for both the particle's velocity (v_x and v_y) and its position (x and y) as functions of time. In other words, we can predict the future motion of the particle. We shall see an example of this in the following section, for the particularly simple case of projectile motion.

3–2 Projectile Motion

In Section 2–3 we learned that, in the absence of air resistance, a freely falling object near the surface of the earth has constant acceleration. Whether an object falls from rest or is given some vertical initial velocity, the same free-fall equations apply.

Now suppose we attempt to describe the motion of an object that is thrown into the air with an initial velocity that is not vertically directed. The object is thrown, or *projected,* with an initial velocity vector $\mathbf{v}_0$ that makes some angle θ_0 with the horizontal, as in Fig. 3–2.

An experimental fact that was first recognized by Galileo is that any projectile experiences exactly the same acceleration as a freely falling body does: its acceleration vector, denoted by $\mathbf{g}$, is a constant vector of magnitude 9.8 m/s^2 pointing in the downward direction.* Choosing our coordinate system as in Fig. 3–3, we have $a_x = 0$, $a_y = -g$, and $x_0 = y_0 = 0$, where x_0 and y_0 are the coordinates of the original position of the projectile. Both components of acceleration are constant. In Sections 2–2 and 2–3 we derived equations of motion for constant acceleration in either the x or the y direction. So we may apply the two sets of equations: Eqs. 2–7 and 2–11 for the x direction (with a_x and x_0 set equal to zero) and Eqs. 2–13 through 2–16 for the y direction (with y_0 set equal to zero):

$$\begin{aligned} &(a)\ a_x = 0 \\ &(b)\ v_x = v_{x0} \\ &(c)\ x = v_{x0}t \end{aligned} \tag{3–7}$$

$$\begin{aligned} &(a)\ a_y = -g \\ &(b)\ v_y = v_{y0} - gt \\ &(c)\ y = \tfrac{1}{2}(v_{y0} + v_y)t \\ &(d)\ y = v_{y0}t - \tfrac{1}{2}gt^2 \\ &(e)\ v_y^2 = v_{y0}^2 - 2gy \end{aligned} \tag{3–8}$$

The components of the initial velocity vector, v_{x0} and v_{y0}, determine the entire motion of the projectile. In many sports, the skill of an athlete rests on his or her ability to impart the correct initial velocity to a ball, thereby determining where it will go. In basketball, for example, once the ball leaves a player's hand, its value of $\mathbf{v}_0$ is fixed and its motion is thereafter governed by Eqs. 3–7 and 3–8 (as long as its path is unobstructed). Whether the player makes the basket depends on the value of $\mathbf{v}_0$. In football, the crucial problem for the quarterback trying to complete a pass is to release the ball at the right time with the right initial velocity, so that it arrives downfield in the hands of the intended receiver.

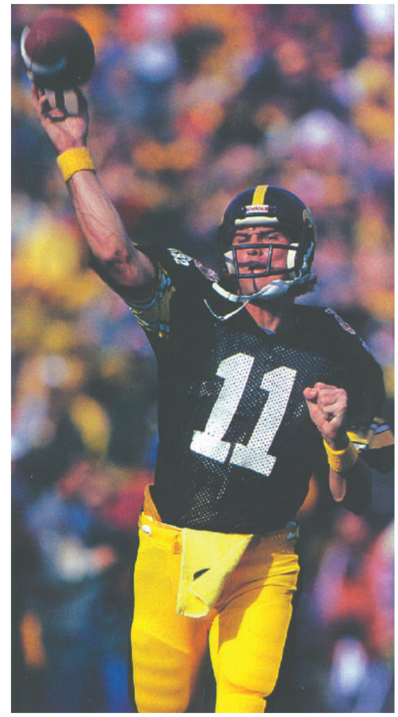


Fig. 3–2 Projectile motion.

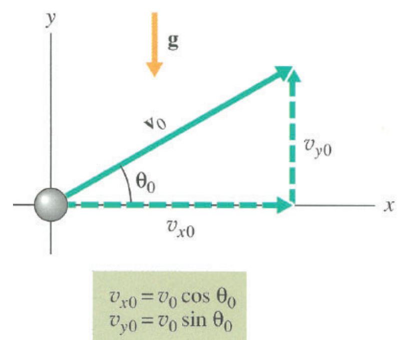


Fig. 3–3 Components of a projectile's initial velocity $\mathbf{v}_0$.

*We assume here that air resistance is negligible and that the trajectory of the particle is very small compared with the radius of the earth.

EXAMPLE 2 Projecting a Marble Horizontally

A marble rolls along a table at a constant speed of 1.00 m/s and then falls off the edge of the table to the floor 1.00 m below. (a) How long does the marble take to reach the floor? (b) At what horizontal distance from the edge of the table does the marble land? (c) What is its velocity as it strikes the floor? (d) Indicate in a diagram the marble's position and velocity at 0.100 s intervals.

SOLUTION Projectile motion of the marble begins as it leaves the table (Fig. 3–4). Since the marble is initially moving horizontally, $v_{y0} = 0$ and $v_{x0} = 1.00$ m/s. In order to use Eqs. 3–7 and 3–8, we must take the origin to be at the edge of the table, so that $x_0 = y_0 = 0$. **(a)** This problem can be stated: Find t when $y = -1.00$ m. Because $v_{y0} = 0$, Eq. 3–8d reduces to

$$y = -\tfrac{1}{2}gt^2$$

Solving for t, we find

$$t = \sqrt{-2y/g} = \sqrt{(-2)(-1.00\text{ m})/(9.80\text{ m/s}^2)} = 0.452\text{ s}$$

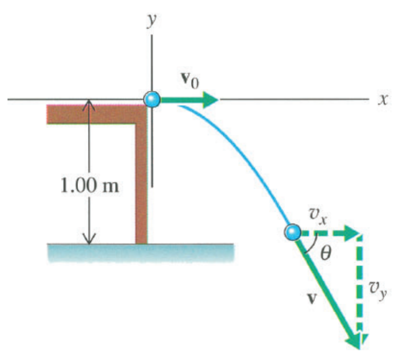


Fig. 3–4

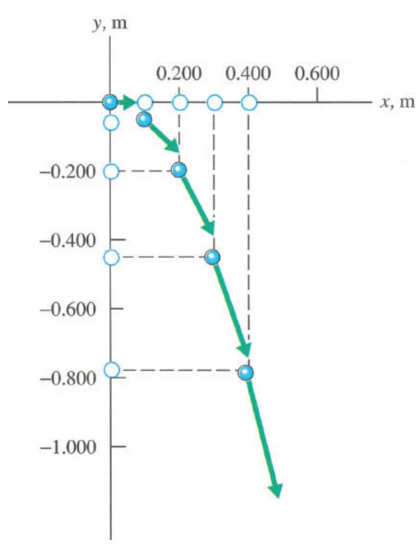


Fig. 3–5

(b) Here we want the marble's x coordinate at the instant it strikes the floor; that is, we wish to find x when $t = 0.452$ s. Eq. 3–7 gives

$$x = v_{x0}t = (1.00\text{ m/s})(0.452\text{ s}) = 0.452\text{ m}$$

(c) Here we must find $\mathbf{v}$ at $t = 0.452$ s. The x component of velocity is constant throughout the motion (Eq. 3–7b):

$$v_x = v_{x0} = 1.00\text{ m/s}$$

We find the y component by using Eq. 3–8b:

$$v_y = v_{y0} - gt = 0 - (9.80\text{ m/s}^2)(0.452\text{ s}) = -4.43\text{ m/s}$$

Thus the velocity vector has magnitude

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.00\text{ m/s})^2 + (-4.43\text{ m/s})^2} = 4.54\text{ m/s}$$

and is directed at an angle θ below the horizontal, where

$$\theta = \arctan\left(\frac{|v_y|}{v_x}\right) = \arctan\left(\frac{4.43}{1.00}\right) = 77.3°$$

As the marble hits the floor, its velocity is 4.54 m/s directed 77.3° below the horizontal.

(d) The marble's coordinates and velocity components at times $t = 0$, 0.100 s, 0.200 s, 0.300 s, and 0.400 s are obtained by direct substitution into Eqs. 3–7b and c, and Eqs. 3–8b and d. The results were used to obtain Fig. 3–5. As illustrated in the figure, the motion in the x direction is that of a body moving at constant velocity, while the motion in the y direction is that of a freely falling body. What we see is the vector sum of these two effects. Fig. 3–6 shows the motion of a horizontally projected object.

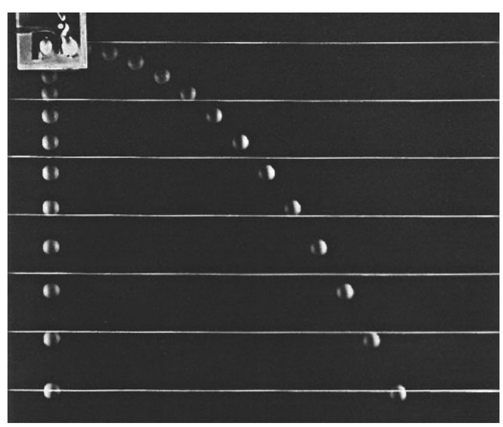


Fig. 3–6 One ball is released from rest, and at the same instant the other is given a horizontal initial velocity. Both balls are at the same elevation at any instant.

EXAMPLE 3 Throwing Too High

A quarterback, standing on his opponents' 35-yard line, throws a football directly downfield, releasing the ball at a height of 2.00 m above the ground with an initial velocity of 20.0 m/s, directed 30.0° above the horizontal. (a) How long does it take for the ball to cross the goal line, 32.0 m (35 yards) from the point of release? (b) The ball is thrown too hard and so passes over the head of the intended receiver at the goal line. What is the ball's height above the ground as it crosses the goal line?

SOLUTION To better visualize the situation described here, we first sketch the trajectory (Fig. 3–7):

(a) The problem here is to find t when $x = 32.0$ m. We can use Eq. 3–7c ($x = v_{x0}t$), if we first find v_{x0}. From Fig. 3–7 we see that

$$v_{x0} = v_0 \cos\theta_0 = (20.0\ \text{m/s})(\cos 30.0°)$$
$$= 17.3\ \text{m/s}$$

Now we apply Eq. 3–7c and solve for t.

$$x = v_{x0}t$$
$$t = \frac{x}{v_{x0}} = \frac{32.0\ \text{m}}{17.3\ \text{m/s}} = 1.85\ \text{s}$$

(b) We want to find y when $x = 32.0$ m, or, since we have already found the time in part (a), we can state this: find y when $t = 1.85$ s. We apply Eq. 3–7d:

$$y = v_{y0}t - \tfrac{1}{2}gt^2$$

where

$$v_{y0} = v_0 \sin\theta_0 = (20.0\ \text{m/s})(\sin 30.0°)$$
$$= 10.0\ \text{m/s}$$

Thus

$$y = (10.0\ \text{m/s})(1.85\ \text{s}) - \tfrac{1}{2}(9.80\ \text{m/s}^2)(1.85\ \text{s})^2$$
$$= 1.73\ \text{m}$$

Since $y = 0$ is 2.00 m above the ground, this means the ball is 3.73 m above the ground as it crosses the goal line—much too high to be caught at that point.

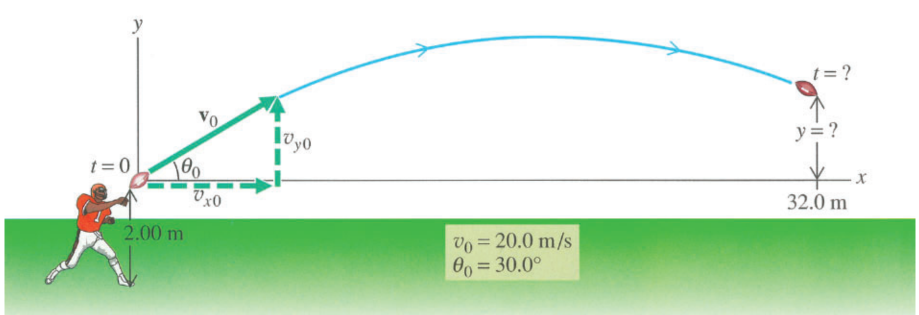


Fig. 3–7

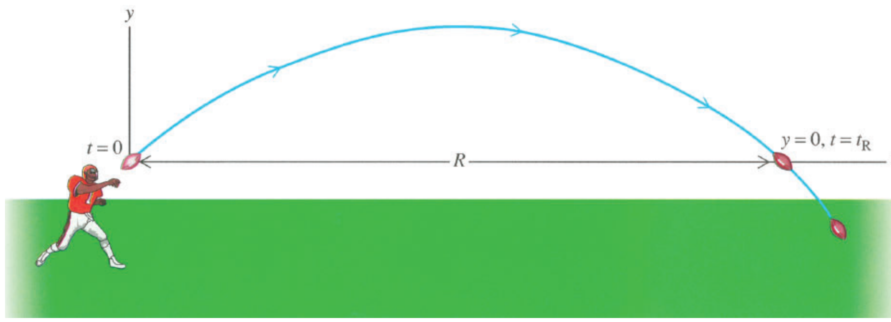


Fig. 3–8 The range of a projectile.

Although Eqs. 3–7 and 3–8 are sufficient to solve any problem in projectile motion, it is sometimes convenient to have other formulas indicating various aspects of the projectile's path. We shall find an expression for a projectile's horizontal range R, the horizontal distance traveled by the projectile before returning to its initial elevation ($y = 0$), illustrated in Fig. 3–8. And we shall find an expression for the time t_R the projectile takes to travel the distance R. We can find the time t_R at which the projectile returns to its initial elevation using Eq. 3–8d, setting $y = 0$.

$$y = 0 = v_{y0}t - \tfrac{1}{2}gt^2$$

The nonzero solution to this equation is the time t_R:

$$t_R = \frac{2v_{y0}}{g} \tag{3–9}$$

We can obtain an expression for the horizontal range by applying Eq. 3–7c.

$$x = v_{x0}t$$

Setting $x = R$ when $t = t_R$ and using Eq. 3–9 for t_R, we find

$$R = v_{x0}\left(\frac{2v_{y0}}{g}\right)$$

Next we substitute

$$v_{x0} = v_0 \cos \theta_0 \quad \text{and} \quad v_{y0} = v_0 \sin \theta_0$$

to obtain

$$R = \frac{2v_0^2 \sin \theta_0 \cos \theta_0}{g}$$

We can use the trigonometric identity $\sin 2\theta_0 = 2 \sin \theta_0 \cos \theta_0$ to express this result more concisely:

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \tag{3–10}$$

The dependence of R on v_0^2 shows that a doubling of v_0 quadruples the range. For example, if a ball is thrown a distance of 20 m, doubling the initial speed will increase the range to 80 m. For a fixed value of v_0, R will be maximum at an angle $\theta_0 = 45°$, for which $\sin 2\theta_0 = \sin 90° = 1$.

Fig. 3–9 Water is projected from two tubes at the same speed—from one at an angle of 30° and from the other at 60°. Why are the ranges equal?

EXAMPLE 4 Snowball Strategy

A clever strategy in a snowball fight is to throw two snowballs at your opponent in quick succession, the first one with a high trajectory and the second one with a lower trajectory and shorter time of flight, so that they both reach the target at the same instant. Suppose your opponent is 20.0 m away. You throw both snowballs with the same initial speed v_0, but θ_0 is 60.0° for the first snowball and 30.0° for the second. If they are both to reach their target at the same instant, how much time must elapse between the release of the two snowballs?

SOLUTION We need to find the time of flight for each snowball. The time t_R is determined by v_{y0}, the vertical component of initial velocity, according to Eq. 3–9:

$$t_R = \frac{2v_{y0}}{g} = \frac{2v_0 \sin \theta_0}{g}$$

To find t_R, we need to know, in addition to the initial angle θ_0 (a given), the initial speed v_0, which is not given. We can find v_0 by applying the range equation (Eq. 3–10):

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

Solving for v_0, we obtain

$$v_0 = \sqrt{\frac{Rg}{\sin 2\theta_0}}$$

We obtain the same value for v_0 whether we use $\theta_0 = 30.0°$ or $\theta_0 = 60.0°$, since $\sin 2(30.0°) = \sin 2(60.0°)$:

$$v_0 = \sqrt{\frac{(20.0\text{ m})(9.80\text{ m/s}^2)}{\sin 60.0°}} = 15.0\text{ m/s}$$

Now we can apply Eq. 3–9 and find t_R for each snowball.

$$t_R = \frac{2v_{y0}}{g} = \frac{2v_0 \sin \theta_0}{g}$$

For the first snowball,

$$t_R = \frac{2(15.0\text{ m/s})(\sin 60.0°)}{9.80\text{ m/s}^2} = 2.65\text{ s}$$

For the second snowball,

$$t'_R = \frac{2(15.0\text{ m/s})(\sin 30.0°)}{9.80\text{ m/s}^2} = 1.53\text{ s}$$

Thus you should wait a time Δt before making your second throw, where Δt is the difference in the times of flight:

$$\Delta t = t_R - t'_R = 2.65\text{ s} - 1.53\text{ s} = 1.12\text{ s}$$

The trajectory equation describes the path of a projectile by relating its x and y coordinates. We derive this equation by solving Eq. 3–7c for t to obtain $t = x/v_{x0}$ and then substituting this expression for t into Eq. 3–8d:

$$\begin{aligned} y &= v_{y0}t - \tfrac{1}{2}gt^2 \\ &= v_{y0}\left(\frac{x}{v_{x0}}\right) - \tfrac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2 \end{aligned}$$

From Fig. 3–3 we see that $v_{y0}/v_{x0} = \tan \theta_0$. Substituting into the equation above, we obtain

$$y = (\tan \theta_0)x - \left(\frac{g}{2v_{x0}^2}\right)x^2 \qquad (3\text{–}11)$$

This equation has the form $y = ax + bx^2$, the general equation for a parabola. Thus a projectile has a parabolic trajectory (Fig. 3–10).

Fig. 3–10 Parabolic paths of projectiles.

EXAMPLE 5 Shooting With the Right Velocity

A basketball player shoots the ball at a hoop 3.00 m above the floor from a horizontal distance of 6.00 m from the center of the hoop. The ball leaves the player's hand 2.00 m above the floor at an angle of 45.0° with the horizontal. With what initial speed must the ball be shot in order to hit the center of the basket, without hitting the rim or backboard?

SOLUTION First we sketch the motion (Fig. 3–11). Once again we must take the origin to be at the initial position of the ball because the equations describing projectile motion were derived on the assumption $x_0 = y_0 = 0$. Our problem is to find v_0 such that $y = 1.00$ m when $x = 6.00$ m. We use the trajectory equation (Eq. 3–11) because it relates v_0 to quantities that are given:

$$y = (\tan\theta_0)x - \frac{gx^2}{2v_{x0}^2}$$

Substituting $v_{x0} = v_0\cos\theta_0$, we have

$$1.00\text{ m} = (\tan 45.0°)(6.00\text{ m}) - \frac{(9.80\text{ m/s}^2)(6.00\text{ m})^2}{2v_0^2\cos^2 45.0°}$$

Solving for v_0, we find

$$v_0 = 8.40\text{ m/s}$$

Through experience a good shooter knows how to give the ball just this initial speed.

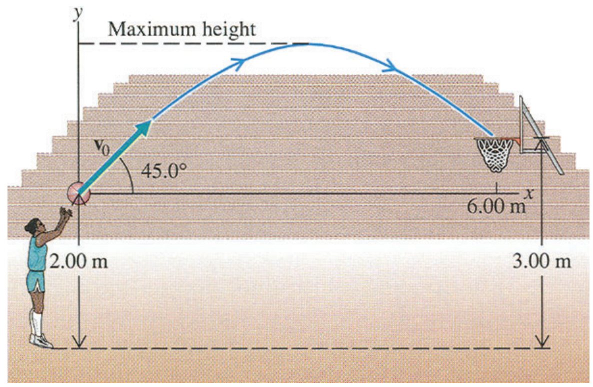


Fig. 3–11

3–3 Circular Motion

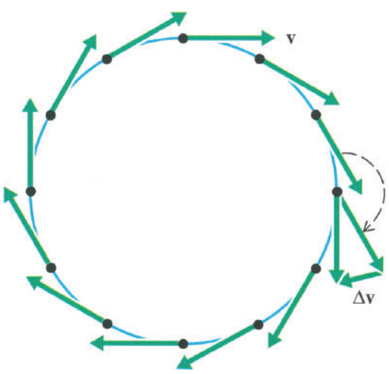


Fig. 3–12 A particle moving along a circular path at constant speed has a velocity **v** that changes direction. The velocity change $\Delta\mathbf{v}$ from one position to the next is constructed in the figure.

A particle moving along a circular path at constant speed is said to undergo uniform circular motion. Fig. 3–12 shows the position and the velocity vector at selected points for a particle moving in this way. The dots are equally spaced and the velocity vectors are of uniform length, indicating that the speed is constant. However, the fact that **v** changes direction as the particle moves along the circle implies that there is nonzero acceleration. The change in **v** from one point to the next is a vector, $\Delta\mathbf{v}$, pointing toward the inside of the curve, as indicated in Fig. 3–12. The particle's path is bending inward, and therefore its acceleration is inward.

Next we shall determine the direction and magnitude of the instantaneous acceleration of a particle in uniform circular motion, using Fig. 3–13. Fig. 3–13a shows the particle's velocity vector drawn tangent to the trajectory at two points P and P′. The particle moves from P to P′ along an arc of length Δs over a time interval Δt, and the velocity vector changes from **v** to $\mathbf{v}' = \mathbf{v} + \Delta\mathbf{v}$ during this time. The triangle formed by vectors **v**, **v**′, and $\Delta\mathbf{v}$ is constructed in the figure. We see that vector $\Delta\mathbf{v}$ is in the same direction as a line from the center of the arc Δs to point O at the center of the circle. The particle's average acceleration $\bar{\mathbf{a}} = \Delta\mathbf{v}/\Delta t$ is a vector proportional to $\Delta\mathbf{v}$, and so it points in the same direction as $\Delta\mathbf{v}$.

In Fig. 3–13b $\Delta\mathbf{v}$ is constructed for a shorter time interval Δt. Again vectors $\Delta\mathbf{v}$ and $\bar{\mathbf{a}}$ are directed toward the center of the circle. Because Δt is shorter, this second figure shows a smaller change in velocity, $\Delta\mathbf{v}$. However, the vector $\bar{\mathbf{a}} = \Delta\mathbf{v}/\Delta t$ has slightly greater magnitude than in the first figure. As the time interval Δt approaches zero, $\Delta\mathbf{v}$ also approaches zero, but the ratio $\Delta\mathbf{v}/\Delta t$ approaches as a limit the instantaneous acceleration $\mathbf{a}$, shown in Fig. 3–13c. Notice that **a is directed toward the center of the circle,** perpendicular to $\mathbf{v}$.

Next we wish to find an expression for the magnitude of $\mathbf{a}$. In Fig. 3–13a the angle θ between the equal length vectors $\mathbf{v}$ and $\mathbf{v}'$ is the same as the angle θ between the two equal length lines OP and OP′, since $\mathbf{v}$ is perpendicular to OP and $\mathbf{v}'$ is perpendicular to OP′. (Remember that the velocity vector is always tangent to the trajectory and therefore is always perpendicular to the radius of the circle.) This means that the triangle formed by the velocity vectors is geometrically similar to the triangle OPP′. Therefore ratios of corresponding sides of these two triangles are equal. For very short time intervals, the length of line PP′ is approximately equal to the arc length Δs. Thus

$$\frac{|\Delta\mathbf{v}|}{v} \approx \frac{\Delta s}{r}$$

Multiplying both sides of this equation by $v/\Delta t$, we obtain

$$\frac{|\Delta\mathbf{v}|}{v} \approx \frac{v}{r}\,\frac{\Delta s}{\Delta t}$$

As Δt approaches 0, the equation becomes exact and $\Delta s/\Delta t$ approaches v, the instantaneous speed. Thus the limiting value of $|\Delta\mathbf{v}|/\Delta t$, the magnitude of the instantaneous acceleration $\mathbf{a}$, is given by

$$\boxed{a = \frac{v^2}{r}} \qquad \text{(uniform circular motion)} \qquad (3\text{–}12)$$

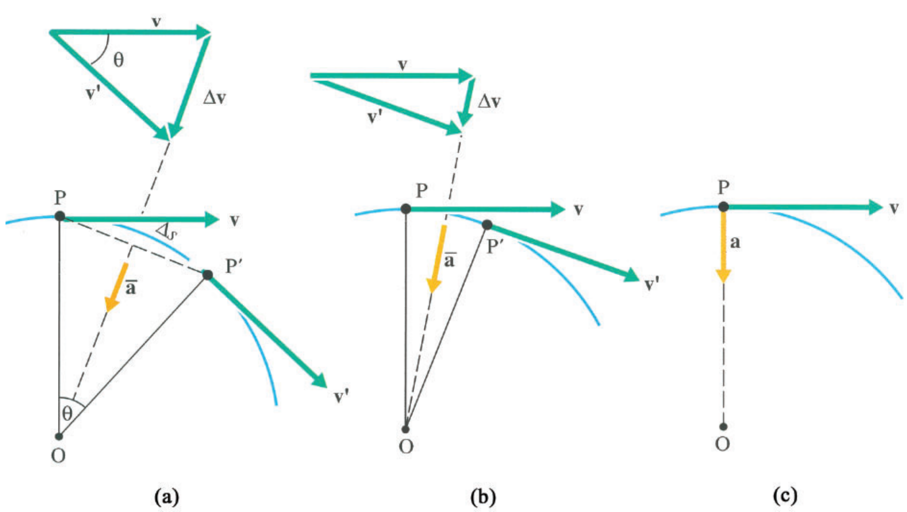


Fig. 3–13 **(a)** A particle's velocity change $\Delta\mathbf{v}$ and average acceleration $\bar{\mathbf{a}}$ are constructed for some time interval Δt. **(b)** $\Delta\mathbf{v}$ and $\bar{\mathbf{a}}$ are constructed for a shorter time interval. **(c)** Instantaneous acceleration, $\mathbf{a}$.

In Fig. 3–14 the illustration of Fig. 3–12 is repeated, this time with the acceleration vectors drawn in. The acceleration vector always points from the instantaneous position of the particle to the center of the circle. The term **centripetal acceleration** is used to describe **a**. "Centripetal" means 'directed toward the center'.

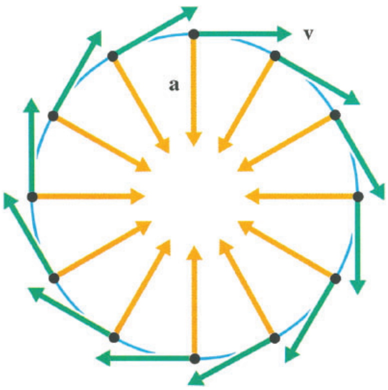


Fig. 3–14 Centripetal acceleration **a** is at any instant directed toward the center of the circular path, perpendicular to velocity **v**.

Note that although the magnitude of **a** is constant for uniform circular motion, the acceleration vector is not constant; it continuously changes direction—that direction always being toward the center of the circle.

As a physical example of uniform circular motion, suppose a rock is attached to a string and swung horizontally overhead along a circular path at a constant rate. The rock experiences centripetal acceleration; its acceleration vector points inward along the string (Fig. 3–15). We shall see in Chapter 4 that a force is required to produce an acceleration. The string maintains the circular path and its associated inward acceleration by exerting an inward pull (a force) on the rock. The greater the speed of the rock, the greater the acceleration and the greater the force provided by the string. A person swinging a rock in this way must pull harder on the string as the speed is increased. If at some instant the string is released, the rock will fly off tangent to the circle, in the direction of its instantaneous velocity.

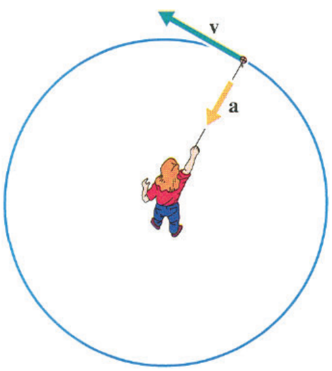
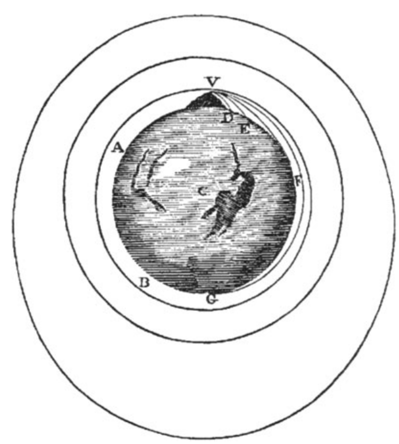


Fig. 3–15 A rock swung overhead from a string moves along a circular path.

The effect of acceleration is felt by passengers in a car making a sharp turn. Suppose that a car moves at a speed of 10 m/s around a curve of radius 20 m. The car's centripetal acceleration $v^2/r = (10 \text{ m/s})^2/20 \text{ m} = 5 \text{ m/s}^2$, or approximately 0.5 g. Such a turn would cause a passenger to move relative to the car until something (perhaps the door) can provide a strong enough force to compel the passenger to follow the circular path. This force is directed inward, toward the center of the circle, that is, in the direction of the centripetal acceleration.

It is possible for a projectile to have a circular trajectory. If the initial velocity of a projectile is very large, so that its trajectory is not very small relative to the size of the earth, the description of projectile motion in Section 3.2 is no longer valid. If the initial velocity is great enough (on the order of thousands of meters per second), the projectile may become a satellite orbiting the earth (Fig. 3–16). For the special case of a circular orbit (achieved by giving the satellite just the right initial velocity), the satellite's speed is constant and its motion is uniform and circular. In this case the centripetal acceleration is equal to gravitational acceleration. This means that if a satellite could orbit the earth at an elevation of only a few kilometers above the surface, its centripetal acceleration would be 9.8 m/s^2, directed toward the center of the earth. Air resistance precludes such a low orbit, however. At a more realistic elevation of 200 km, a satellite would experience a centripetal acceleration of 9.2 m/s^2, the same acceleration that would be experienced by a body released from rest at this altitude (see Problem 26).

Fig. 3–16 In this drawing from the *Principia,* Newton showed how a particle projected horizontally from a high elevation might be given an initial velocity great enough to orbit the earth. For smaller values of v_0 the paths (D and E) are approximately parabolic, but if v_0 is sufficiently great, the particle *falls around the earth,* rather than into it. Thus Newton anticipated artificial satellites over 300 years ago.



EXAMPLE 6 Swinging an Arm

Extend your arm straight up overhead and then swing it in a vertical plane, so that your hand follows a circular path. If you rotate your arm as fast as possible, you should be able to achieve a rate of 2.0 revolutions per second. Compute the acceleration of the blood in your fingertips at this rate, assuming your arm is 65 cm long.

SOLUTION As indicated in Fig. 3–17, the acceleration vector is directed toward the center of the circle, which is at your shoulder joint. The magnitude of the acceleration is given by Eq. 3–12.

$$a = \frac{v^2}{r}$$

To use this equation, we must know the speed v. We compute v by dividing the path length for one revolution ($2\pi r$, the circumference of the circle) by the time for one revolution, called the **period** and denoted by T:

$$v = \frac{2\pi r}{T}$$

Since the rate of rotation is 2.0 rev/s, the period $T = 0.50$ s. Combining the two preceding expressions, we obtain

$$a = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$= \frac{4\pi^2(0.65\text{ m})}{(0.50\text{ s})^2} = 100\text{ m/s}^2$$

This is approximately 10 g. Such an acceleration is quite painful.

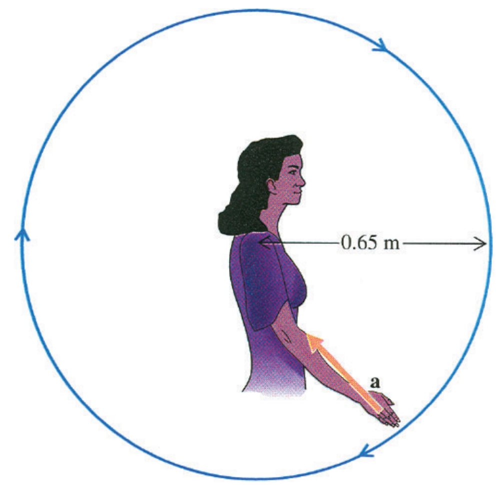


Fig. 3–17

Fig. 3–18 A car moves relative to the earth, and the earth moves relative to the car.

3–4 Reference Frames and Relative Motion

We shall sometimes refer to a coordinate system used in the description of motion as a "reference frame." It is not hard to imagine yourself standing at the origin of a reference frame and viewing a moving body.

Since we are earthbound, it is always tempting to think of the earth as stationary and to describe any motion from the earth's reference frame, that is, from a set of coordinate axes fixed with respect to the surface of the earth.* However, there is no fundamental reason to give this reference frame a uniquely privileged status. The earth moves relative to other bodies. Suppose, for example, you are in a moving car or train. As you look out the window, you see passing trees and buildings. Relative to you, these objects are moving. Indeed, the entire earth is moving relative to you (Fig. 3–18).

We shall often find it convenient to describe the motion of a body from a reference frame that is moving relative to the earth. For example, a moving plane or train serves as a natural reference frame for describing the motion of passengers inside.

It is sometimes useful to be able to go from one reference frame to another, that is, to have transformation equations that allow us to relate descriptions of motion in two different reference frames. Such relationships are of practical importance to an airplane pilot, for example. The pilot is obviously interested in the plane's motion relative to the earth. But the plane is moving relative to the *air,* and so, when determining the plane's position, the pilot must know how to compensate for the velocity of the air relative to the earth. We shall now derive the equations relating the relative motion of three bodies (earth, air, and plane in our example). Each of the three arbitrary moving bodies, labeled A, B, and C, are treated on equal footing. Each body moves relative to the other two.

Relative displacement is a displacement vector directed from one body to another. Let $\mathbf{D}_{AB}$ denote the displacement of A relative to B, that is, the displacement vector required to go from the position of B to the position of A (Fig. 3–19a). Using this notation, $\mathbf{D}_{AC}$ denotes the displacement of A relative to C, and $\mathbf{D}_{CB}$ denotes the displacement of C relative to B. These relative displacement vectors are related to each other. From Fig. 3–19b we see that vectors $\mathbf{D}_{AB}$, $\mathbf{D}_{AC}$, and $\mathbf{D}_{CB}$ form a vector addition triangle, with $\mathbf{D}_{AB}$ equal to the vector sum of the other two vectors:

$$\mathbf{D}_{AB} = \mathbf{D}_{AC} + \mathbf{D}_{CB} \tag{3–13}$$

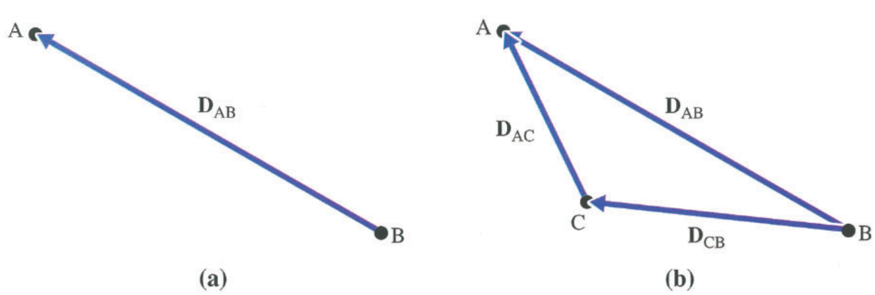


Fig. 3–19 **(a)** The displacement $\mathbf{D}_{AB}$ of A relative to B is a vector directed from B to A. **(b)** The relative displacements of any three bodies are related by a vector addition triangle.

*Historically, this led to the view that the earth was the center of the universe—the geocentric theory advocated by Aristotle.

As the bodies move, these relative displacement vectors change. **The change in relative displacement per unit time is called relative velocity.** The changes in relative displacement vectors are related in the same way as the relative displacements at any instant:

$$\Delta\mathbf{D}_{\mathrm{AB}} = \Delta\mathbf{D}_{\mathrm{AC}} + \Delta\mathbf{D}_{\mathrm{CB}}$$

Dividing by the time interval Δt during which the displacement occurs, we obtain

$$\frac{\Delta\mathbf{D}_{\mathrm{AB}}}{\Delta t} = \frac{\Delta\mathbf{D}_{\mathrm{AC}}}{\Delta t} + \frac{\Delta\mathbf{D}_{\mathrm{CB}}}{\Delta t}$$

Taking the limit of this expression as Δt approaches zero, we obtain a relationship between the relative velocity vectors $\mathbf{v}_{\mathrm{AB}}$, $\mathbf{v}_{\mathrm{AC}}$, $\mathbf{v}_{\mathrm{CB}}$, each of which describes the velocity of one body relative to another:

$$\mathbf{v}_{\mathrm{AB}} = \mathbf{v}_{\mathrm{AC}} + \mathbf{v}_{\mathrm{CB}} \tag{3–14}$$

This expression tells us that the velocity of A relative to B is the vector sum of the velocity of A relative to C and the velocity of C relative to B. The significant point to note about this equation is the relationship between the subscripts. (We may want to use subscripts other than A, B, and C to denote the bodies in relative motion, and we can, just as long as we always maintain the same relationship between subscripts.) The first subscript on the left side of the equation (A) is the same as the first subscript on the right side. The last subscript on the left side (B) is the same as the last subscript on the right side. The second and third subscripts on the right side are the same (C). Following this rule, we could write, for example, $\mathbf{v}_{\mathrm{XZ}} = \mathbf{v}_{\mathrm{XY}} + \mathbf{v}_{\mathrm{YZ}}$.

Sometimes it is useful to relate $\mathbf{v}_{\mathrm{AB}}$, the velocity of A relative to B, to $\mathbf{v}_{\mathrm{BA}}$, the velocity of B relative to A. From Fig. 3–20 we see that

$$\mathbf{D}_{\mathrm{BA}} = -\mathbf{D}_{\mathrm{AB}}$$

and the change in relative displacement is given by

$$\Delta\mathbf{D}_{\mathrm{BA}} = -\Delta\mathbf{D}_{\mathrm{AB}}$$

Dividing by Δt, we have

$$\frac{\Delta\mathbf{D}_{\mathrm{BA}}}{\Delta t} = \frac{-\Delta\mathbf{D}_{\mathrm{AB}}}{\Delta t}$$

Taking the limit as Δt approaches zero, we obtain a second useful relationship between relative velocities:

$$\mathbf{v}_{\mathrm{BA}} = -\mathbf{v}_{\mathrm{AB}} \tag{3–15}$$

Thus each vector is the negative of the other; the relative velocity vectors have equal magnitudes but are oppositely directed. For example, if A is moving east at 20 m/s relative to B, then B is moving west at 20 m/s relative to A.

Application of the equations of relative motion requires only a careful labeling of the subscripts corresponding to the bodies and simple vector addition.

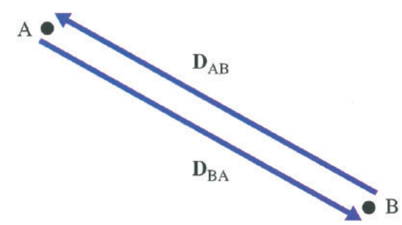


Fig. 3–20 The displacement of B relative to A ($\mathbf{D}_{\mathrm{BA}}$) is a vector equal in magnitude but opposite in direction to the displacement of A relative to B ($\mathbf{D}_{\mathrm{AB}}$).

EXAMPLE 7 Navigating a Plane in a Crosswind

A pilot flying with an airspeed of 325 km/h wishes to fly due north in a 70.0 km/h wind blowing from east to west. In what direction should she head, and what is her speed relative to the earth?

SOLUTION First we assign a letter to each body: E, earth; P, plane; and A, air. Next we express the given information in terms of the relative velocity vectors:

$$\mathbf{v}_{AE} = 70.0\ \text{km/h, west}$$
$$|\mathbf{v}_{PA}| = 325\ \text{km/h}$$
$$\mathbf{v}_{PE}\ \text{is directed north}$$

In this problem, we have been given one vector completely, only the magnitude of the second vector, and only the direction of the third vector. Our task is to find the direction of the second vector and the magnitude of the third. Using the relationship described in Eq. 3–14, we have

$$\mathbf{v}_{PE} = \mathbf{v}_{PA} + \mathbf{v}_{AE}$$

We can now use the vector triangle corresponding to this equation (Fig. 3–21) to obtain the desired information:

$$\theta = \arcsin\left(\frac{v_{AE}}{v_{PA}}\right) = \arcsin\left(\frac{70.0}{325}\right) = 12.4^\circ$$

$$v_{PE} = \sqrt{v_{PA}^2 - v_{AE}^2} = \sqrt{(325\ \text{km/h})^2 - (70.0\ \text{km/h})^2} = 317\ \text{km/h}$$

To have her velocity relative to the earth directed north, the pilot must point the plane 12.4° east of north. The resulting speed relative to the ground has been reduced to 317 km/h. Fig. 3–22 indicates the position of the plane as seen from the earth at three successive instants.

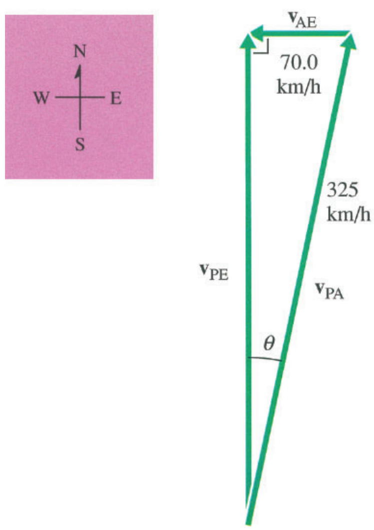


Fig. 3–21 Velocity $\mathbf{v}_{PE}$ is the vector sum of vectors $\mathbf{v}_{PA}$ and $\mathbf{v}_{AE}$.

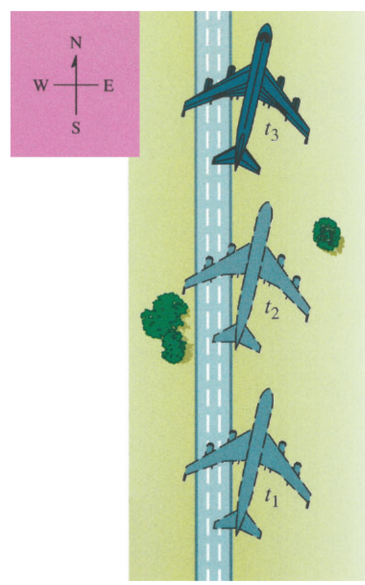


Fig. 3–22

EXAMPLE 8 Walking on a Moving Sidewalk

An airline passenger late for a flight walks on an airport "moving sidewalk" at a speed of 5.00 km/h relative to the sidewalk, in the direction of its motion. The sidewalk is moving at 3.00 km/h relative to the ground and has a total length of 135 m. (a) What is the passenger's speed relative to the ground? (b) How long does it take him to reach the end of the sidewalk? (c) How much of the sidewalk has he covered by the time he reaches the end?

SOLUTION The situation is sketched in Fig. 3–23a. We assign a letter to each body in relative motion: P, passenger; S, sidewalk; G, ground. The relative velocities $\mathbf{v}_{PS}$ and $\mathbf{v}_{SG}$ are given:

$$\mathbf{v}_{PS} = 5.00 \text{ km/h, to the right}$$

$$\mathbf{v}_{SG} = 3.00 \text{ km/h, to the right}$$

(a) Here we must find the magnitude of the vector $\mathbf{v}_{PG}$, given the magnitude and direction of two other vectors. We find the velocity $\mathbf{v}_{PG}$ by using Eq. 3–14:

$$\mathbf{v}_{PG} = \mathbf{v}_{PS} + \mathbf{v}_{SG}$$

Here the vectors are parallel, and so the vector addition is quite simple (Fig. 3–23b). We add vectors by adding magnitudes:

$$\begin{aligned} v_{PG} &= v_{PS} + v_{SG} \\ &= 5.00 \text{ km/h} + 3.00 \text{ km/h} \\ &= 8.00 \text{ km/h} \end{aligned}$$

(b) The length of the sidewalk is 135 m, and so this is the distance Δx_G the passenger travels relative to the ground. So our problem is to find Δt when $\Delta x_G = 135$ m. The rate at which this distance along the ground is covered by the passenger is v_{PG}, where

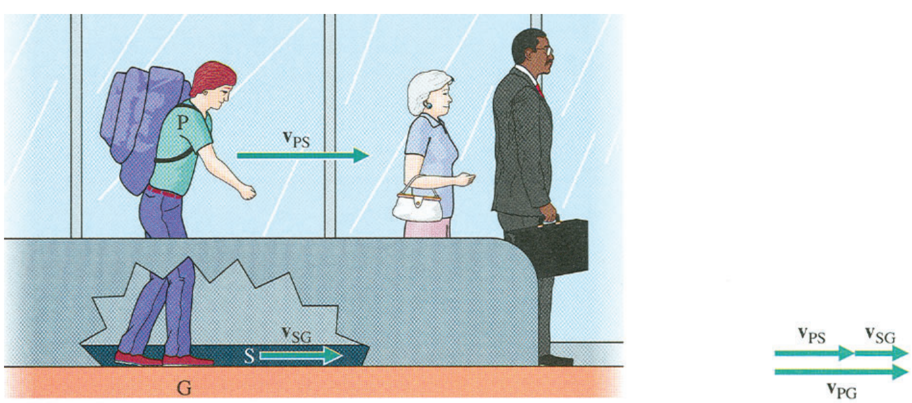
$$v_{PG} = \frac{\Delta x_G}{\Delta t}$$

Therefore

$$\begin{aligned} \Delta t = \frac{\Delta x_G}{v_{PG}} &= \frac{135 \text{ m}}{8.00 \text{ km/h}\left(\dfrac{1.00 \text{ m/s}}{3.60 \text{ km/h}}\right)} \\ &= 60.8 \text{ s} \end{aligned}$$

(c) The problem here is to determine how much of the sidewalk's surface the passenger moves over. If he were standing still and not walking along the surface, he would cover none of it. Because he is moving relative to the surface at velocity $\mathbf{v}_{PS}$, he does move some distance Δx_S relative to the surface. The problem is to find Δx_S when $\Delta t = 60.8$ s, since we found in part (b) that this is the time interval during which he is on the moving sidewalk. His velocity relative to the sidewalk is $v_{PS} = \Delta x_S/\Delta t$, and so

$$\begin{aligned} \Delta x_S &= v_{PS}\,\Delta t \\ &= (5.00 \text{ km/h})\left(\frac{1.00 \text{ m/s}}{3.60 \text{ km/h}}\right)(60.8 \text{ s}) \\ &= 84.4 \text{ m} \end{aligned}$$



(a) (b)

Fig. 3–23

EXAMPLE 9 Running in the Rain

Rain is falling vertically at a speed of 20.0 m/s. A woman runs through the rain at a speed of 5.00 m/s. (a) What is the velocity of the rain relative to the woman? (b) How far in front of her would an umbrella have to extend to keep the rain off if she holds the umbrella 1.50 m above her feet?

SOLUTION **(a)** We assign the following letters: W, woman; R, rain; and E, earth. We are given the relative velocities $\mathbf{v}_{RE}$ and $\mathbf{v}_{WE}$, which are drawn in Fig. 3–24a. We want to find the relative velocity $\mathbf{v}_{RW}$. Following the usual rule, we obtain an expression for $\mathbf{v}_{RW}$:

$$\mathbf{v}_{RW} = \mathbf{v}_{RE} + \mathbf{v}_{EW}$$

To perform the vector addition, we must first find the velocity $\mathbf{v}_{EW}$, which is not given. But $\mathbf{v}_{EW}$ is the negative of $\mathbf{v}_{WE}$, which is given. Performing the vector addition, we obtain the diagram shown in Fig. 3–24b. From the figure we find

$$v_{RW} = \sqrt{v_{RE}{}^2 + v_{EW}{}^2} = \sqrt{(20.0\ \text{m/s})^2 + (5.00\ \text{m/s})^2}$$
$$= 20.6\ \text{m/s}$$

and

$$\theta = \arctan\left(\frac{v_{EW}}{v_{RE}}\right) = \arctan\left(\frac{5.00\ \text{m/s}}{20.0\ \text{m/s}}\right)$$
$$= 14.0°$$

(b) The rain as seen by the running woman is shown in Fig. 3–24c. From the figure we see that the distance d the umbrella must extend is

$$d = (1.50\ \text{m})(\tan 14.0°)$$
$$= 0.375\ \text{m}$$
$$= 37.5\ \text{cm}$$

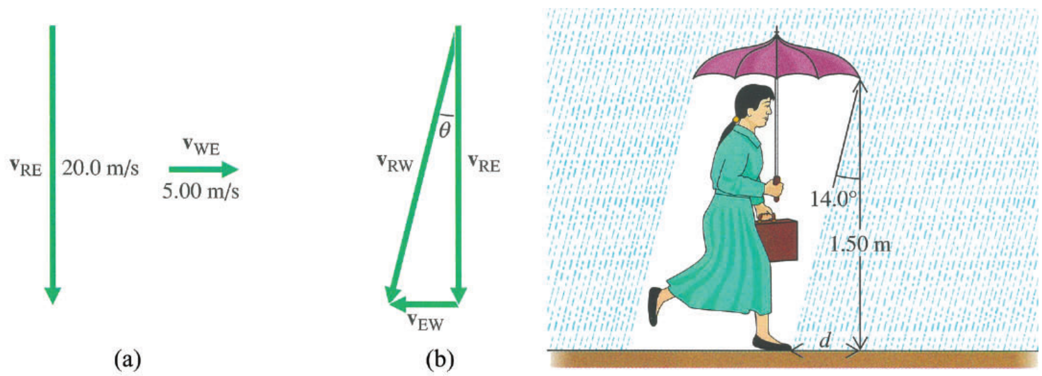


Fig. 3–24

CHAPTER 3 SUMMARY

For motion in a plane, average acceleration $\overline{\mathbf{a}}$, defined as $\Delta\mathbf{v}/\Delta t$, has Cartesian components

$$\overline{a}_x = \frac{\Delta v_x}{\Delta t} \quad \text{and} \quad \overline{a}_y = \frac{\Delta v_y}{\Delta t}$$

whereas instantaneous acceleration $\mathbf{a}$, defined as

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

has components

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \quad \text{and} \quad a_y = \lim_{\Delta t \to 0} \frac{\Delta v_y}{\Delta t}$$

In the absence of air resistance, the motion of a projectile over a limited region near the earth's surface is described by the following equations for the projectile's x and y coordinates, and its velocity components v_x and v_y:

$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_{x0} \qquad v_y = v_{y0} - gt$$

$$x = v_{x0}t \qquad y = \tfrac{1}{2}(v_{y0} + v_y)t$$

$$y = v_{y0}\,t - \tfrac{1}{2}gt^2$$

$$v_y{}^2 = v_{y0}{}^2 - 2gy$$

The x and y coordinates of a projectile are related at any instant by the equation

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_{x0}{}^2}\right)x^2$$

The range R of a projectile is the horizontal distance it travels before falling to its original elevation and is given by

$$R = \frac{v_0{}^2 \sin 2\theta_0}{g}$$

A projectile travels the distance R in time

$$t_R = \frac{2v_{y0}}{g}$$

When a particle moves along a circular path of radius r at constant speed v, its acceleration, called centripetal acceleration, is directed toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$

Relative velocity vectors are related by the equations

$$\mathbf{v}_{AB} = \mathbf{v}_{AC} + \mathbf{v}_{CB}$$

$$\mathbf{v}_{BA} = -\mathbf{v}_{AB}$$

Questions

1 Is an object moving along a curved path necessarily accelerated?

2 If an object moves along a linear path, is its acceleration necessarily zero?

3 If your speedometer reading is constant, does this necessarily mean your car is not accelerating?

4 The initial speed with which a ball is thrown is doubled, with the angle of projection fixed. Is the maximum height to which the ball rises doubled?

5 A person running away from you at a speed of 3 m/s throws a ball vertically upward (as seen by him), the ball leaving his hand with an initial velocity of 4 m/s (relative to him). What is the speed of the ball relative to you?

6 When riding a bicycle, is the force of the air resistance greater when riding with the wind or against the wind? Explain.

7 Would you expect a typical trajectory of a Ping-Pong ball to be a parabola? Explain.

8 While driving a car around a curve, an observer sees a road sign. Is the road sign moving with respect to the car? Is it accelerated with respect to the car?

Answers to Odd-Numbered Questions

1 yes; **3** no; **5** 5 m/s; **7** no

Problems (listed by section)

3–1 Acceleration on a Curved Path

Average acceleration

1 A cyclist is initially traveling north, then turns 90.0° and moves west at the same speed. Find the direction of the cyclist's average acceleration.

2 A halfback is initially running south at a speed of 10.0 m/s, quickly cuts to his right, and 0.500 s later is running 60.08 west of south at 10.0 m/s. Find his average acceleration.

3 A pilot claims to have seen a UFO moving initially at a speed of about 440 m/s in an easterly direction and then, in a time interval of only 1.0 s, turning 45° and moving southeast at 440 m/s. Compute the UFO's average acceleration during the turn.

4 A particle is initially moving along the positive x-axis at a speed of 10.0 m/s. After 2.00 s, the particle is moving along the negative y-axis at a speed of 5.00 m/s. Find the x and y components of the particle's acceleration.

5 The initial and final velocities of a particle are shown in Fig. 3–25. Find the particle's average acceleration if the change in velocity takes place in a 10.0 s interval.

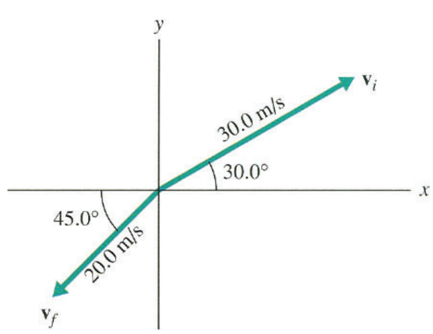


Fig. 3–25

Instantaneous acceleration

6 An aircraft initially flying north at 225 m/s turns toward the east and 0.100 s later is flying 0.0400° east of north at the same speed. Estimate the aircraft's instantaneous acceleration.

7 A baseball has a velocity of 44.0 m/s (98.4 mi/h), directed horizontally, as it is released by a pitcher. The ball's velocity 0.0100 s before it is released is 42.0 m/s, directed 3.00° above the horizontal. Estimate the ball's instantaneous acceleration just before it is released.

8 A child on a Ferris wheel is moving vertically upward at 5.00 m/s at one instant, and 0.100 s later is moving at 5.00 m/s at an angle of 86.0° above the horizontal. Estimate the child's instantaneous acceleration.

3–2 Projectile Motion

9 A water pistol aimed horizontally projects a stream of water with an initial speed of 5.00 m/s.
(a) How far does the water drop in moving 1.00 m horizontally?
(b) How far does it travel before dropping a vertical distance of 1.00 cm?

10 A beam of electrons in a television tube moves horizontally with a velocity of 1.00×10^7 m/s. How far will the electrons drop as they travel a horizontal distance of 20.0 cm?

11 Standing on a balcony, you throw your keys to a friend standing on the ground below. One second after you release the keys, they have an instantaneous velocity of 13.9 m/s, directed 45° below the horizontal. What initial velocity did you give them?

12 A baseball pitcher throws a pitch with an initial velocity of 44.0 m/s, directed horizontally. How far does the ball drop vertically by the time it crosses the plate 18.0 m away?

13 An archer wishes to shoot an arrow at a target at eye level a distance of 50.0 m away. If the initial speed imparted to the arrow is 70.0 m/s, what angle should the arrow make with the horizontal as it is being shot?

14 A fox fleeing from a hunter encounters a 0.800 m tall fence and attempts to jump it. The fox jumps with an initial velocity of 7.00 m/s at an angle of 45.0°, beginning the jump 2.00 m from the fence. By how much does the fox clear the fence? Treat the fox as a particle.

*15 The object of the long jump is to launch oneself as a projectile and attain the maximum horizontal range (Fig. 3–26). Here we shall treat the long jumper as a particle even though the human body is fairly large compared to the size of the trajectory. Actually there is one point within the athlete's body, called the "center of mass" (to be studied in Chapter 5), that behaves as a projected particle. Our analysis of projectile motion implies that the long jumper should try to maximize v_0 and take off at an angle as close to 45.0° as possible. However, it is easier to get a large value of v_{x0} (by a running start) than it is to get a large value of v_{y0}; consequently θ_0 is usually much less than 45.0°. Suppose the jumper takes off with $v_{x0} = 9.00$ m/s and jumps with a value of v_{y0} sufficient to reach a vertical height of 1.00 m. Find v_0, θ_0, and the horizontal range. The world record, as of 1994, is 8.95 m.

Fig. 3–26

16 Suppose that a world-class long jumper jumped on the moon with the same initial velocity as that which produced a world record of 8.95 m on earth. What would be the lunar record for the long jump? Gravitational acceleration on the moon is 1.67 m/s^2.

17 In an article on the use of the sling as a weapon (Korfmann M: *Sci Am* 229:34, Oct. 1973), the author states that a skilled slinger can sling a rock a distance of about 400 m. What is the minimum speed the rock must have, when it leaves the sling, to travel exactly 400 m?

*18 Prove the following:
(a) The maximum height of a projectile equals $v_{y0}^2/2g$.
(b) The time it takes a projectile to reach its maximum height equals v_{y0}/g.
(c) The time it takes a projectile to descend from its maximum height to its original elevation is the same as the time to ascend, v_{y0}/g.
(d) The y component of velocity is reversed when a projectile descends to its original elevation: $v_y = -v_{y0}$.

19 A football is kicked 60.0 meters. If the ball is in the air 5.00 s, with what initial velocity was it kicked?

20 A baseball player hits a home run over the left-field fence, which is 104 m from home plate. The ball is hit at a point 1.00 m directly above home plate, with an initial velocity directed 30.0° above the horizontal. By what distance does the baseball clear the 3.00 m high fence, if it passes over it 3.00 s after being hit?

3–3 Circular Motion

21 A runner moving at a constant speed of 10.0 m/s rounds a curve of radius 5.00 m. Compute the acceleration of the runner. Are these numbers realistic?

22 A large merry-go-round completes one revolution every 10.0 s. Compute the acceleration of a child seated on it, a distance of 6.00 m from its center.

23 In Problem 17, the initial velocity of the rock is produced by rotating the sling in a circle. What rate of rotation, in rev/s, is necessary to give the rock the required speed? Take the radius of the circle to be 1.50 m.

24 A certain centrifuge produces a centripetal acceleration of magnitude exactly 1000g at a point 10.0 cm from the axis of rotation. Find the number of revolutions per second.

25 Find the speed of a lunar orbiter in a circular orbit that is just above the surface of the moon, given that the orbiter's acceleration is equal to the moon's gravitational acceleration of 1.67 m/s^2. The radius of the moon is 1.74×10^6 m.

26 An artificial earth satellite has a circular orbit of radius 6.50×10^6 m (which means it is orbiting approximately 130 km above the surface of the earth) in an equatorial plane. The period T (the time required for one complete orbit) is 5.22×10^3 s (about 1.5 h).
(a) Compute the (constant) speed of the satellite.
(b) If the satellite is directly above the equator and traveling east at time t, find the average acceleration during the time interval from t to $t + T/40.0$.
(c) Find the satellite's instantaneous acceleration at time t.

3–4 Reference Frames and Relative Motion

27 Two cars, A and B, travel in the same direction on a straight section of highway. A has a speed of 70.0 km/h, and B a speed of 90.0 km/h (both relative to the earth).
(a) What is the speed of B relative to A?
(b) If A is initially 400 m in front of B, how long will it take for B to reach A?

28 A plane is headed due west with an air speed of 225 m/s. The wind blows south at 20.0 m/s. Find the velocity of the plane relative to the earth.

29 A plane is headed east with an air speed of 250.0 m/s. The wind blows southeast at 40.0 m/s. Find the velocity of the plane relative to the earth.

30 A man observes snow falling vertically when he is at rest, but when he runs through the falling snow at a speed of 6.00 m/s, it appears to be falling at an angle of 30.0° relative to the vertical. Find the speed of the snow relative to the earth.

***31** You are driving your car with a velocity of 20.0 m/s, north, approaching an intersection. Another car approaches the intersection with a velocity of 25.0 m/s, west (Fig. 3–27).
(a) Find the velocity of the other car relative to you.
(b) The cars are initially each 100.0 m from the intersection. Sketch the path of the other car as you see it.

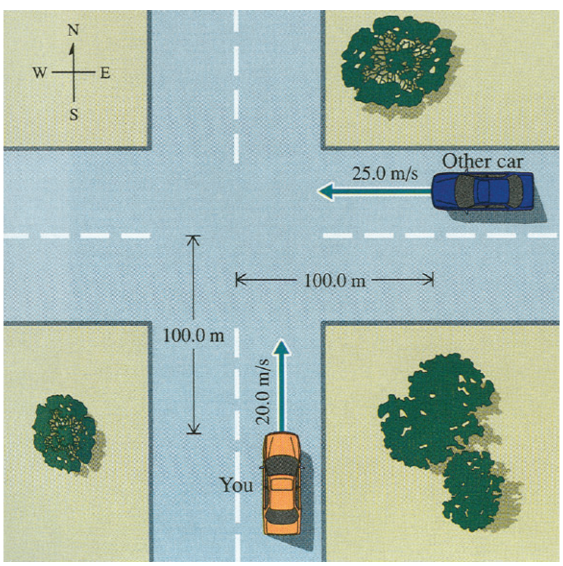


Fig. 3–27

32 A man can row a boat at a speed of 6.00 km/h in still water. If he is crossing a river where the current is 3.00 km/h, in what direction should his boat be headed if he wants to reach a point directly opposite his starting point?

***33** A river 100.0 m wide flows toward the south at 33.3 m/min. A girl on the west bank wishes to reach the east bank in the least possible time. She can swim 100.0 m in still water in 1.00 min.
(a) How long does it take her to cross the river?
(b) How far downstream does she travel?
(c) What is her velocity relative to land?
(d) What is the total distance she travels?
(e) In what direction must she swim if she wishes to travel straight across the river?

Additional Problems

***34** In the game of darts, the player stands with feet behind a line 2.36 m from a dartboard, with the bull's-eye at eye level. Suppose you lean across the line, release a dart at eye level 1.80 m from the board, and hit the bull's-eye (Fig. 3–28). Find the initial velocity of the dart, if the maximum height of its trajectory is 1.00 cm above eye level.

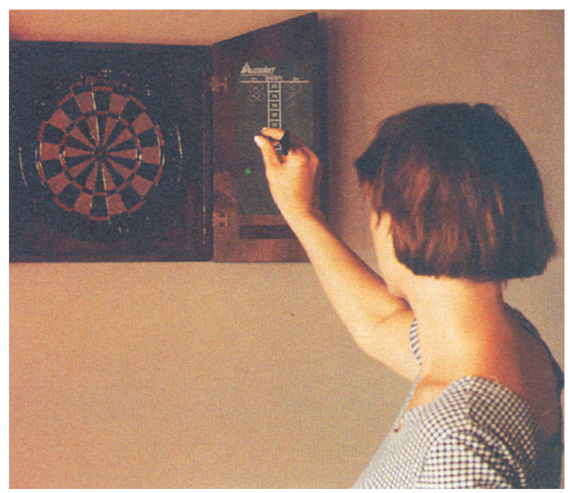


Fig. 3–28

**35 A football is to be thrown by a quarterback to a receiver who is running at a constant velocity of 10.0 m/s directly away from the quarterback, who intends for the ball to be caught a distance of 40.0 m away. At what distance should the receiver be from the quarterback when the ball is released? Assume the football is thrown at an initial angle of 45.0° and that it is caught at the same height at which it is released.

36 A particle requires 4.00 s to complete a circular path of radius 20.0 m. At $t = 0$, the particle is moving east and has instantaneous acceleration directed south. Find the particle's average acceleration from $t = 0$ to (a) $t = 4.00$ s; (b) $t = 2.00$ s; (c) $t = 1.00$ s. (d) Find the magnitude of its instantaneous acceleration at $t = 0$.

*37 In the shot put, a heavy lead weight—the "shot"—is given an initial velocity, starting from an initial elevation approximately equal to the shot putter's height, say, 1.90 m. If $v_0 = 8.00$ m/s, find the horizontal distance traveled by the shot for (a) $\theta_0 = 0°$; (b) $\theta_0 = 40.0°$; (c) $\theta_0 = 45.0°$.

*38 A tennis ball is struck at the base line of the court, 12.0 m from the net. The ball is given an initial velocity with a horizontal component equal to 24.0 m/s at an initial elevation of 1.00 m.

(a) What vertical component of initial velocity must be given to the ball, such that it barely clears the 1.00 m high net?

(b) How far beyond the net will the ball hit the ground?

39 The sun travels about the center of our galaxy in a nearly circular orbit of radius 2.5×10^{17} km in a period of about 2.0×10^{8} years. Compute the magnitudes of the velocity and acceleration of the sun relative to the center of the galaxy.

*40 The moon travels about the earth at uniform speed in a nearly circular orbit of radius 3.8×10^{5} km in a period of about 27 days. The earth travels about the sun at uniform speed in a circular orbit of radius 1.5×10^{8} km in a period of about 365 days. Compute the magnitudes of the velocity and acceleration of (a) the moon relative to the earth and (b) the earth relative to the sun. (c) What is the maximum acceleration of the moon relative to the sun, and during what phase of the moon does this occur? The orbits of the moon and earth are very nearly coplanar.

*41 The earth is approximately a sphere of radius 6.37×10^{3} km. A particle P at rest on the surface of the earth at 40.0° north latitude moves in a circular path as the earth rotates on its axis (Fig. 3–29). Compute the magnitudes of the particle's velocity and acceleration relative to the center of the earth.

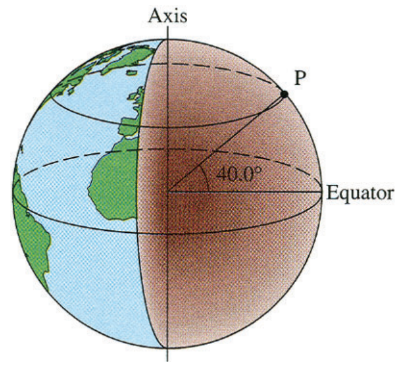


Fig. 3–29

*42 A rocket is fired at 40.0° north latitude with an initial velocity in an eastward direction. Its velocity relative to the center of the earth is of magnitude 11.0 km/s, just enough to escape the earth. Find the velocity of the rocket relative to the ground. What would be the advantage in firing the rocket from a site closer to the equator?

*43 A golfer must hit an approach shot to the green over a tree.

(a) What initial velocity must be imparted to the ball so that it will follow the trajectory indicated in Fig. 3–30?

(b) Find the horizontal distance d that the ball travels after it clears the tree before hitting the ground.

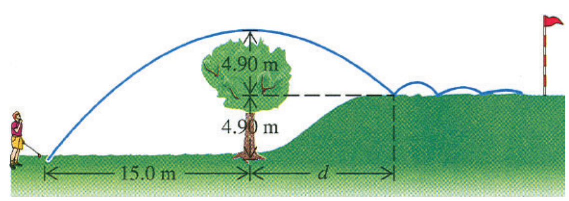


Fig. 3–30

***44** A point that is instantaneously on the top edge of an automobile tire moves in the forward direction at a speed of 40.0 m/s, as the car moves at a constant speed of 20.0 m/s.

(a) Find the velocity of the point relative to a passenger in the car.

(b) Find the centripetal acceleration of the point relative to the earth. The tire's radius is 35.0 cm.

***45** When you walk, your upper leg rotates about the hip joint, the knee describing an approximately circular arc relative to the hip. During its forward motion, the maximum speed of your knee relative to the ground is approximately twice the speed of your hip relative to the ground. What is the maximum centripetal acceleration of your knee when you are walking at a speed of 2 m/s, if the length of your upper leg is 0.5 m?

***46** A tennis ball is served at a height of 3.00 m with an initial horizontal component of velocity equal to 25.0 m/s.

(a) What should the vertical component of initial velocity be if the ball clears the 1.00 m high net, 12.0 m away?

(b) At what angle is the initial velocity vector below the horizontal?

(c) At what angle below the horizontal would the initial velocity vector be directed if there were no gravitational acceleration?

****47** Airplane A is flying at a constant velocity of 1.20×10^2 m/s north. Airplane B is flying at a constant velocity of 1.60×10^2 m/s west.

(a) What is the velocity of B relative to A?

(b) If, at a certain instant, the pilot of A observes the pilot of B 75.0 m directly north, what will be the smallest distance between the pilots as they pass? (HINT: Consider the path of B as seen by A.)

****48** Water from a garden hose has a maximum horizontal range of 10.0 m. The hose is used to put out a fire on a rooftop. Find the initial angle θ_0 that will allow the water to reach the greatest possible horizontal distance d from the edge of the roof, as shown in Fig. 3–31. (HINT: Consider first the angle θ_1.)

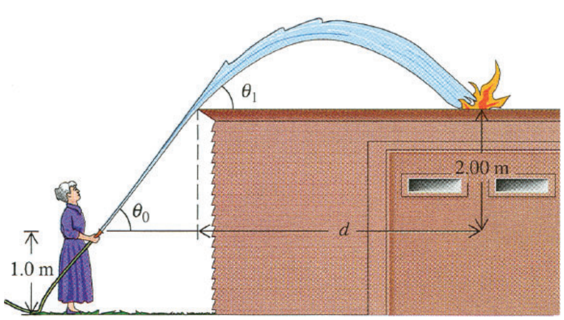


Fig. 3–31

****49** Suppose you are standing on a 30.0° slope and kick a ball on the ground up the slope, giving the ball an initial velocity of 10.0 m/s, directed at an angle of 45.0° above the horizontal.

(a) At what distance from your feet will the ball strike the ground?

(b) Repeat your calculation with the initial velocity directed 50.0° above the horizontal.

****50** A rock is thrown from the edge of a cliff to the ground 20.0 m below. The rock has an initial velocity of 15.0 m/s, directed 30.0° above the horizontal.

(a) How long does it take the rock to reach the ground?

(b) How far from the base of the cliff does the rock strike the ground?

(c) Find the velocity of the rock just before it strikes the ground.

***51** A car is found in swampy ground 100.0 m from the base of a cliff 40.0 m high. The car is headed directly away from the cliff. Find the car's initial velocity, assuming that it left the edge of the cliff with a horizontal initial velocity and that it did not roll after hitting the ground.

***52** A passenger in a car moving at 50.0 km/h looks out her side window and observes rain falling vertically. A man standing outside in the rain observes the rain falling at an angle of 30.0° with the vertical. Find the speed of the rain relative to the earth.

53 Suppose you want to leap from the top of a building to the top of an adjacent building of the same height, across a gap of 3.00 m. With what minimum initial velocity would you have to jump?

***54** A dart player stands 2.40 m from a dartboard and throws a dart, releasing it at eye level, which is also the level of the bull's-eye. The dart strikes the board 0.200 s after it is released, at a point 2.00 cm directly below the center of the bull's-eye.

(a) Find the x and y components of the dart's initial velocity.

(b) Find the angle that the dart's velocity vector makes with the horizontal as it strikes the board.

***55** A car traveling along a straight section of road at a speed of 100.0 km/h approaches a curve of radius 20.0 m. The driver must apply the brakes to round the curve. The car decelerates at the rate of 3.00 m/s^2 while the brakes are applied.

(a) If the centripetal acceleration of the car is not to exceed 4.00 m/s^2 as it rounds the curve, at what distance from the beginning of the curve must the brakes be applied?

(b) If the driver continues to apply the brakes as she begins to round the curve, what is the instantaneous acceleration of the car?