UNIT II PROPERTIES OF MATER AND THERMAL PHYSICS

2.1 INTRODUCTION:

Elasticity is a branch of Physics which deals with the elastic property of materials. When an external force is applied to a body, there will be some change in its length, shape and volume. When this external force is removed, and if the body regains its original shape and size, then the body is said to be a ***Perfectly Elastic body***. If the body does not regain its original shape or size after removal of the applied force, then it is said to be ***Perfectly Plastic body.***

2.1.1STRESS AND STRAIN:

A body is said to be rigid body, if the distance between any two points in a body is unaltered due to application of the force. In practice no body is perfectly rigid. When a body is subjected to some external forces the body will offer some resistance to the deforming forces, as a result some work is done on the body and this work is stored as the elastic potential energy. Now if the deforming forces re removed the energy stored brings back the body to its original condition.

2.1.1 STRESS

Stress is defined as the restoring force per unit area which brings back the body to its original value from the deformed state. As long as no permanent change is produced in the body, the restoring force is equal to the force applied; Unit of Stress is N/m^2.

TYPES OF STRESS

i. **Normal Stress**

When the force is applied perpendicular to the surface of the body, then the stress applied is ***Normal Stress.***

ii. **Tangential Stress**

When the force is applied along the surface of the body, then the stress applied is called as tangential stress. The tangential stress is also called as ***Shearing Stress***.

2.1.2 STRAIN:

Strain is defined as the change in dimension (fractional deformation) produced by the external force of the body. In other way it can also be defined as the ration of the change in dimension to the original dimension.

Strain = $\frac{Change.in.Dimension}{Original.Dimension}$

2.2 TYPES OF STRAIN

a. Longitudinal or Tensile Strain

Definition: it is defined as the ratio between the changes in length to the original length without any change in its shape, after the removal of the external forces.

Explanation:

When a force is applied along it length of the body of the original length 'L', due to this applied force if the change in length is l then

$$\text{Longitudinal Strain} = \frac{l}{L}$$

b. Shearing Strain

Definition:

It is defined as the angular deformation produced on the body due to the application of external tangential forces on it.

Explanation

Let ABCD be a body with its CD fixed a shown in the figure. A tangential force is applied on the upper surface AB of the body. Therefore the body shears to angle ϕ and it goes to a new position $A'B'CD$. Then this angle ϕ measured in radians is called Shearing Strain.

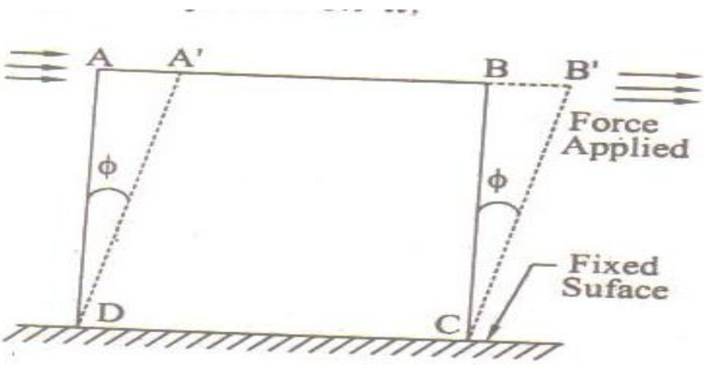


Fig 1.1

c. Volumetric Strain

Definition: it is defined as the ration between the change in volume to the original volume without any change in its shape.

Explanation: When forces are applied normal to the surface of the body of volume 'V', it undergoes a change in volume (v), then

$$\text{Volumetric Strain} = \frac{v}{V}$$

2.3 HOOKE'S LAW

It is found that stress and strain always accompany each other, without stress, strain will not be there and vice versa.

Robert Hooke, proposed a relation between Stress and Strain and I s named as Hooke's Law by his name.

Statement: according to this law, "Stress is directly proportional to the Strain produced, within the elastic limit"

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = \text{E} \times \text{Strain}$$

$$E = \frac{Stress}{Strain} Nm^{-2}$$

Where E is called as modulus of the elasticity. The value of E depends upon the nature of the material.

2.3.1 ELASTIC LIMIT

When the forces are applied to the bodies, each and every body has a tendency to oppose the force and will try to regain its original position after the removal of the force. When the applied force is increased beyond the maximum value, the body does not regain its original position completely, even after the removal of the external force. Hence the maximum stress up to which a body can recover its original shape and size, after removing the external forces is called ***Elastic limit***.

2.3.2 YIELD POINT

Let us consider a wire loaded by some forces. It obeys Hooke's law and the wire extends up to a limit called elastic limit (A) as shown in the figure. If the load is applied beyond the elastic limit, Hooke's law is no longer obeyed and the extension increases to the point 'B' even for a small increase in the load. Now, even if the load is removed the wire will not regain its original position. Therefore, the point at which the body loses its elasticity is called ***Yield Point*** represented by 'B'

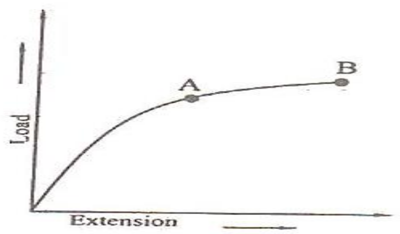


Fig 1.2

2.3.3 ELASTIC FATIGUE

If a body is continuously subjected to Stress or Strain, it gets fatigued (weak), called ***Elastic Fatigue.***

Example: consider two torsional pendulums A and B of same wires. Let the pendulum 'A' be set into oscillation continuously. After some time if 'A' comes to rest, make both pendulums 'A' and 'B' to oscillate simultaneously. It is found that pendulum 'A' comes to rest earlier than pendulum 'B' due to elastic fatigue.

2.3.4 STRESS – STRAIN DIAGRAM

Let us consider a body which is subjected to a uniformly increasing stress. Due to the application of stress, the change in dimension of the body takes place i.e. the strain is developed. If we plot a graph between stress and strain we get a curve as shown in the figure and called ***STRESS – STRAIN diagram***

1. From figure, it is found that the body obeys Hooke's law upon the region OA called as elastic range.
2. As soon as the maximum elastic limits i.e. yield point 'B' is crossed, the strain increases rapidly than the stress.
3. At this stage the body remains partly elastic and partly plastic which is represented by the curve BC.

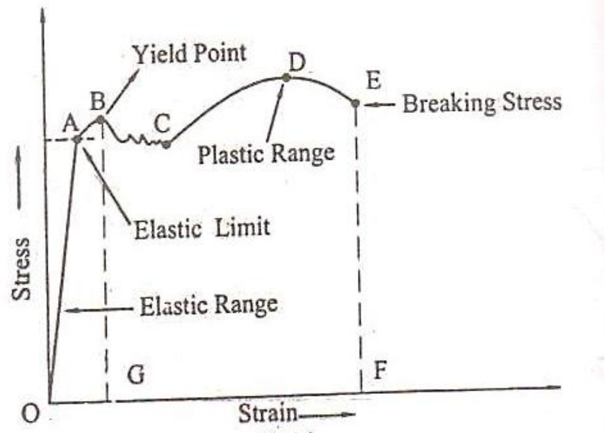


Fig 1.3

4. Now, even if a small external force is applied, the body will take a new path CD and remains as Plastic range, where D is called Ultimate Strength.
5. After this , the body will not come to its original state and the body acquires a permanent residual strain and it breaks down at a point called a breaking stress, indicated by dotted line EF.

2.4 TYPES OF MODULI OF ELASTICITY

Depending upon the three types of Strain, there are three types of elastic moduli, viz.

a) Young's Modulus(Y) or modulus corresponding to Longitudinal Strain
b) Bulk modulus(K) or modulus corresponding to the volume Strain
c) Rigidity modulus (n) or modulus corresponding to the shearing strain.

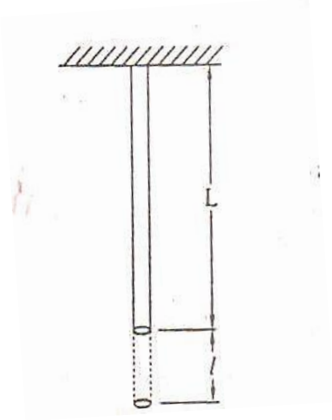
2.4.1 YOUNGS MODULUS (Y)

Definition: it is defined as the ratio between the longitudinal stress to longitudinal strain, within the elastic limits.

I.e. Youngs Modulus (Y) = $\frac{Logitudnal.Stress}{Logitudinal.Strain} Nm^{-2}$ or Pascals

Explanation: Let us consider a wire of length 'L' with an area of cross section 'A'. Let one end of the wire is fixed and the other end is loaded or stretched as shown in the figure.

Let 'l' be the change in length due to the action of the force, then



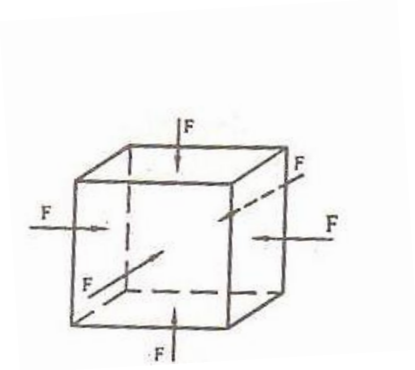
The longitudinal stress = $\frac{F}{A}$

And the Longitudinal Strain = $\frac{l}{L}$

$$\therefore Youngs Modulus Y = \frac{F/A}{l/L}$$

$$Y = \frac{FL}{Al} Nm^{-2}$$

2.4.2 BULK MODULUS



Definition: It is defined as the ration between the volume stress or bulk stress to the volume strain or bulk strain within the elastic limits

Bulk Modulus (K) = $\frac{Bulk Stress}{Bulk Strain} Nm^{-2}$

Explanation: let us consider a body of volume 'V' with an area of cross section 'A'. Let three equal forces act on the body in mutually perpendicular directions as shown in the figure. Let 'v' be the change in volume, due to the action of forces, then

The volume stress or bulk stress = $\frac{F}{A}$

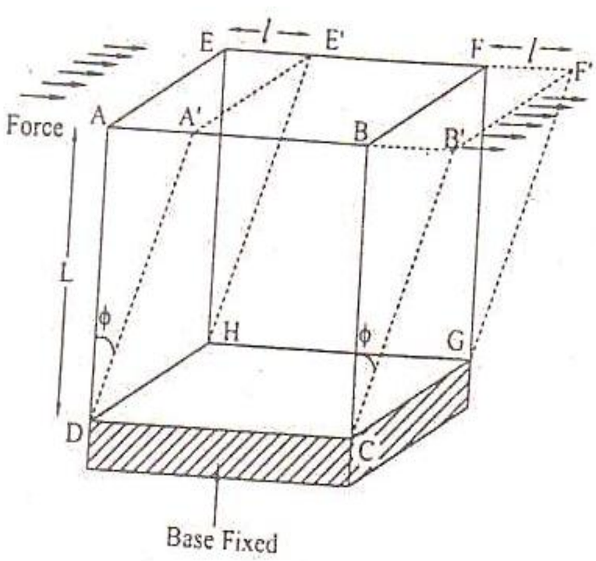
The volume strain or bulk strain = $\frac{v}{V}$

Bulk Modulus (K) = $\frac{F/A}{v/V} = \frac{FV}{vA}$

K = $\frac{PV}{v} Nm^{-2}$ where P = F/A

2.4.3 RIGIDITY MODULUS

Definition: it is defined as the ratio between the tangential stress to the shearing strain, within the elastic limits.



Rigidity Modulus (n) = $\frac{Tangential.Stress}{Tangential.Strain} Nm^{-2}$

Explanation: Let us consider a solid cube ABCDEFGH wherein lower CDHG is fixed as shown in the figure. A tangential force 'F' is applied over the upper face ABEF. The result is that the cube gets deformed into rhombus shape $A'B'CDE'F'GH$ i.e. the lines joining the two faces are shifted to an angle ϕ. If 'L' is the original 'l' is the relative displacement of the upper face of the cube with respect to the lower fixed face, then

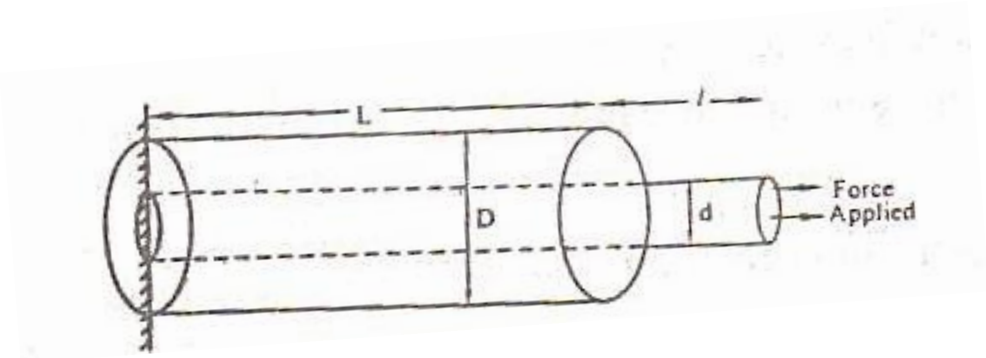
We can write the tangential stress = F/A

The shearing stress ϕ can be defined as the ration of the relative displacement between the two layers in the direction of the stress, to the distance measured perpendicular to the layers.

Rigidity modulus (n) = $\frac{Tangential.Stress}{Shearing.Strain} = \frac{F}{A\phi} Nm^{-2}$

2.4.4 POISSONS RATIO (σ)

Definition: it is defined as the ratio between the lateral strain per unit stress(β) to the longitudinal strain per unit stress(α), within the elastic limits.



i.e. Poisson's Ratio $\sigma = \frac{Lateral.Strain}{Longitudinal.Strain} = \frac{\beta}{\alpha}$

$$\sigma = \frac{\beta}{\alpha} = a \text{ Constant}$$

Explanation: Let us consider a wire, fixed at one end and is stretched along the other end as shown in the figure

Due to force applied the wire becomes longer but it also becomes thinner i.e. although there is an increase in its length, there is a decrease in its diameter as shown in the figure. Therefore the wire elongates freely in the direction of tensile force and contracts laterally in the direction perpendicular to the force. Let 'L' be the original length and 'D' be the original diameter of the wire after the application of force, let the length increase from L to L + l and the diameter decreases from D to d, then

Longitudinal Strain = l/L

And Lateral Strain = (D - d)/D

$$\sigma = -\frac{(D-d)/D}{l/L}$$

(-ve sign indicates the decrease in length)

$$\sigma = -\frac{L(D-d)}{lD}$$

The negative sign indicates that longitudinal strain and lateral strain are opposite to each other.

2.5 FACTORS AFFECTING ELASTICITY

It is found that bodies lose their elastic property even within the elastic limit, due to elastic fatigue. Therefore, the manufacturer should choose the material in such a way that it should regain its elastic property even when it is subjected to large number of cycles of stresses.

For example substances like quartz, Phosphor, Bronze etc. may be employed in manufacturing of galvanometers, electrometers etc, after knowing their elastic properties.

Apart from elastic fatigue some materials will have change in their elastic property because of the following factors.

1. Effect of Stress
2. Effect of annealing
3. Change in temperature
4. Presence of impurities
5. Due to the nature of crystals

a) **Effect of Stress**: we know that when a material is subjected to large number of cycles of stresses, it loses its elastic property even within the elastic limit. Therefore the working stress on the material should be kept lower than the ultimate tensile strengthing and the safety factor.

b) **Effect of Annealing**: Annealing is a process by which the material is heated to a very high temperature and then it is slowly cooled. Usually this process is adopted for the materials to increase the softness and ductility in the materials. But if annealing is made to a material it results in the formation of large crystal grains, which ultimately reduces the elastic property of the material.

c) **Effect of temperature**: The elastic property of the materials changes with the temperature. Normally the elasticity increases with the decrease in temperature and vice-versa.

   Examples:

   1. The elastic property of lead increases when the temperature is decreased.
   2. The carbon filament becomes plastic at higher temperatures.

d) Effect of impurities: The addition of impurities produces variation in the elastic property of the materials. The increase and decrease of elasticity depends upon the type of impurity added to it.

   Examples:

   1. When potassium is added to gold, the elastic property of gold increases.
   2. When carbon is added to molten iron, the elastic property of Iron decreases provided the carbon content should be more than1% in iron.

e) **Effect of nature of crystals:** The elasticity also depends upon the types of the crystals, whether it is a single crystal or poly crystals. For a single crystal the elasticity is more and for a poly crystal the elasticity is less.

2.6 BENDING OF BEAMS

2.6.1 Beam: A beam is defined as a rod or bar. Circular or rectangular of uniform cross-section whose length is very much greater than its other dimensions, such as breadth and thickness. It is commonly used in the construction of bridges to support roofs of the buildings etc. since the length of the beam is much greater than its other dimensions the shearing stresses are very small.

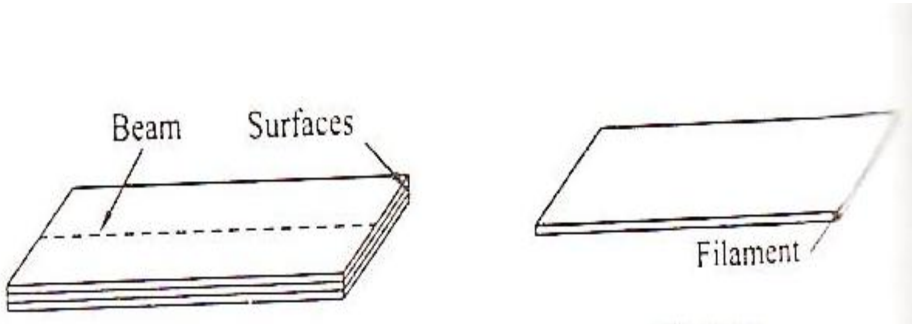
Assumptions:

While studying about the bending of beams, the following assumptions have to be made.

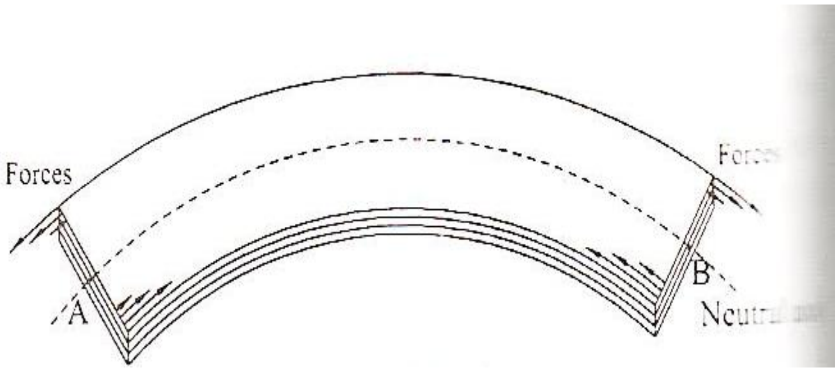
1. The length of the beam should be large compared to other dimensions.
2. The load(forces) applied should be large compared to the weight of the beam
3. The cross-section of the beam remains constant and hence the geometrical moment of inertia I_g also remains constant
4. The shearing stresses are negligible
5. The curvature of the beam is very small

2.6.2 Bending of a Beam and neutral axis

Let us consider a beam of uniform rectangular cross-section in the figure. A beam may be assumed to consist of a number of parallel longitudinal metallic fibers placed one over the other and are called as filaments as shown in the figure.

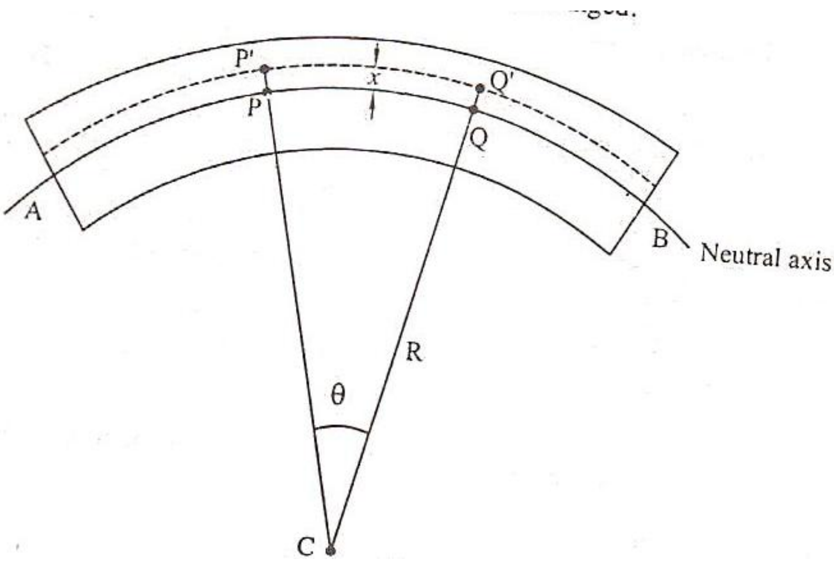


Let the beam be subjected to deforming forces at its ends as shown in the figure. Due to the deforming force the beam bends. We know the beam consists of many filaments. Let us consider a filament AB at the center of the beam. It is found that the filaments (layers) lying above AB gets elongated, while the filaments lying below AB gets compressed. Therefore the filament i.e. layer AB which remains unaltered is taken ass the reference axis called Neutral Axis and the plane is called Neutral Plane. Further, the deformation of any filament can be measured with reference to the neutral axis.



2.6.3 EXPRESSION FOR BENDING MOMENT

Let us consider a beam under the action of deforming forces. The beam bends into a circular arc as shown in the figure. Let AB be the neutral axis of the beam. Here the filaments above AB are elongated and the filaments below AB are compressed. The filament AB remains unchanged.



Let PQ be the arc chosen from the neutral axis. If R is the radius of curvature of the neutral axis and θ is the angle subtended by it at its center of curvature 'C'.

Then we can write original length

$$PQ = R\theta \longrightarrow \quad 1$$

Let us consider a filament $P'Q'$ at a distance 'x' from the neutral axis.

$\therefore$ We can write the extended length

$$P'Q' = (R + x)\theta \longrightarrow \quad 2$$

From equations 1 and 2 we have,

Increase in length = $P'Q' - PQ$

Or increase in its length = $(R+x)\theta - R\theta$

$\therefore$ Increase in length = $x\theta$ → 3

We know Linear Strain = Increase in Length / Original length

Linear Strain = $\frac{x\theta}{R\theta} = \frac{x}{R}$ → 4

We know, the Youngs Modulus of the Material

$$Y = \frac{Stress}{Linear\,strain}$$

Or stress = Y × Linear Strain → 5

Substituting 4 in 5, we have

$$Stress = \frac{Yx}{R}$$

If δA is the area of cross-section of the filament $P'Q'$, then,

The tensile force on the area δA = Stress × Area

Ie. Tensile Force = $\frac{Yx}{R}.\delta A$

We know the Moment of Force = Force × Perpendicular Distance

$\therefore$ Moment of the tensile force about the neutral axis AB or PQ = $\frac{Yx}{R}.\delta A.x$

PQ = $\frac{Y}{R}.\delta A.x^2$

The Moment of force acting on both the upper and lower halves of the neutral axis can be got by summing all the moments of tensile and compressive forces about the neutral axis.

$\therefore$ The moment of all the forces about the neutral axis = $\frac{Y}{R}..\sum x^2\delta A$

Here $I_g = \sum x^2\delta A = AK^2$ is called as the geometrical moment of inertia.

Where, A is the total area of the beam and K is the radius of Gyration.

∴ Total Moment of all the forces Or Internal bending Moment = $\frac{YI_g}{R}$ → 6

SPECIAL CASES

a) Rectangular Cross Section

If 'b' is the breadth and'd' is the thickness of the beam, then

Area A = bd and $K = \frac{d^2}{12}$

$$\therefore I_g = AK^2 = \frac{bd.d^2}{12} = \frac{bd^3}{12}$$

Substituting the value of I_g in equation 6, we can write

Bending moment for rectangular across section = $= \frac{bd^3}{12R}$ → 7

b) Circular Cross section

For a circular cross section if 'r' is the radius, then Area A = πr^2 and $K^2 = \frac{r^2}{4}$

$$I_g = AK^2 = \frac{\pi r^2 \times r^2}{4} = \frac{\pi r^4}{4}$$

Substituting the value of I_g in equation 6 we can write

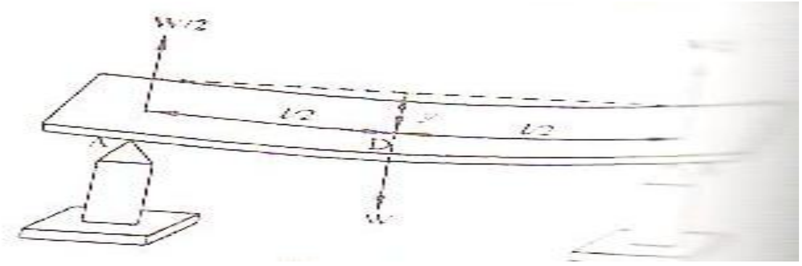
The Bending moment of a circular cross section = $\frac{\pi Y r^4}{4R}$ → 8

2.6.4 NON – UNIFORM BENDING – DEPRESSION OF THE MID POINT OF A BEAM LAODED AT THE MIDDLE

Theory:

Let us consider a beam of length 'l' (distance between the two knife edges) supported on the two knife edges A and B as shown in the figure. The load of weight 'W' is suspended at the Centre 'C'. it is found that the beam bends and the maximum displacement is at the point 'D' where the load is given.

Due to the load (W) applied, at the middle of the beam the reaction W/2 is acted vertically upwards at each knife edges. The bending is called Non- Uniform Bending



The beam may be considered as two cantilevers, whose free end carries a load of W/2 each of length l/2 and fixed at the point 'D'.

Hence we can say the elevation of A above D as the depression below 'A'. We know the depression of a cantilever

$$y = \frac{Wl^3}{3YI_g} \longrightarrow \quad 1$$

Therefore substituting the value of l and l/2 and was W/2 in the expression for the depression of the cantilever we have

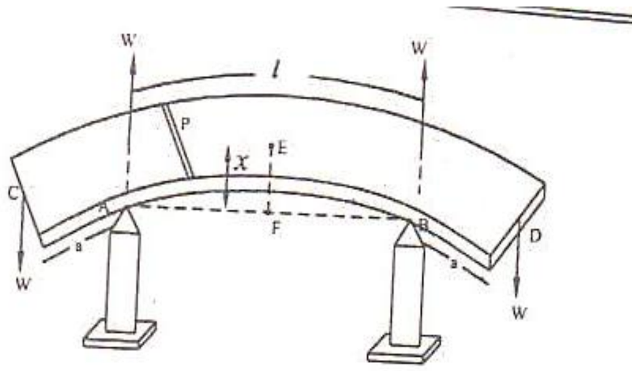
$$y = \frac{(W/2)(l/2)^3}{3YI_g} \longrightarrow \quad 2$$

Or $$y = \frac{Wl^3}{48YI_g} \longrightarrow \quad 3$$

2.6.5 UNIFORM BENDING – ELEVATION AT THE CENTER OF THE BEAM LOADED AT BOTH THE ENDS

Theory:

Let us consider a beam of negligible mass, supported symmetrically on the two knife edges A and B as shown. Let the length between A and B is 'l'. Let equal weights W; be added to either end of the beam C or D.

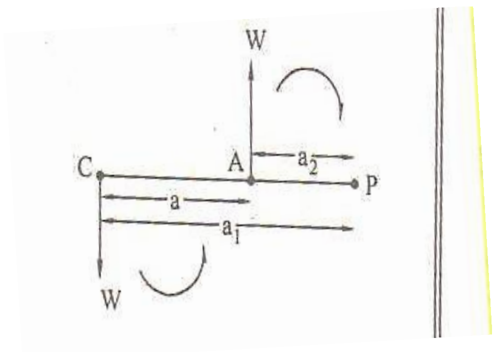


Let CA = BD

Due to load applied the beam bends from position F to E into an arc of a circle and produces as elevation 'x' from position F to E. let 'W' be the reaction produced at the points A and B acts vertically upwards as shown in the figure.

Consider a point 'P' on the cross section of the beam. Then the forces acting on the part PC of the beam are

a. Force W at 'C' and
b. Reaction W at A as shown in the figure



Let he distance PC = a_1 and PA = a_2, then

The external bending moment about 'P' is

$$M_p = W \times a_1 - W \times a_2$$

Here the clockwise moment is taken as negative and anticlockwise moment is taken as positive.

External bending moment about P can be written as

$$M_p = W \times (a_1 - a_2)$$

$$M_p = Wa \longrightarrow \quad 1$$

We know the internal bending moment = $\frac{YI_g}{R} \longrightarrow \quad 2$

Under equilibrium condition

External bending moment = Internal bending moment

∴ We can write Equation 1 = Equation 2

$$Wa = \frac{YI_g}{R} \longrightarrow 3$$

Since for a given load (W) Y, I_g a nd R are constant the bending is called Uniform Bending Here it is found that the elevation 'x' forms an arc of the circle of radius 'R', as shown in the figure.

From the ΔAFO we can write

$$OA^2 = AF^2 + FO^2$$

Since OF = FE, therefore we can write

$$OA^2 = AF^2 + FE^2$$

$$AF^2 = OA^2 - FE^2$$

Rearranging we can write

$$AF^2 = FE\left[\frac{OA^2}{FE} - FE\right] \longrightarrow 4$$

Here AF = l/2 ; FE = x= R/2; OA = R

∴ Equation 4 can be written as

$$\left(\frac{l}{2}\right)^2 = x\left[\frac{R^2}{(R/2)} - x\right]$$

$$\frac{l^2}{4} = x[2R - x]$$

$$\frac{l^2}{4} = 2xR - x^2$$

If the elevation is very small, then the term x^2 can be neglected.

∴ We can write $\frac{l^2}{4} = 2xR$

Or $x = \frac{l^2}{8R}$

∴ Radius of the curvature $R = \frac{l^2}{8x}$ $\longrightarrow$ 5

Substituting the value of 'R' value in equation 3, we have

$$W.a = \frac{YI_g}{\left(l^2/8x\right)}$$

$$W.a = \frac{8YI_g x}{l^2} \longrightarrow \quad 6$$

Rearranging equation 6 we have

The elevation of Point 'E' above 'A' is $x = \frac{Wal^2}{8YI_g}$

2.6.6 DEPRESSION OF A CANTILEVER WHEN LOADED AT ITS END

CANTILEVER:

A cantilever is a beam fixed horizontally at one end loaded to the other end.

THEORY:

Let us consider a beam fixed at one –end and loaded at its other end as shown in the figure

Due to load applied at the free end, a couple is created between the two forces

a. Force (load 'W') applied at the free end towards downward direction and
b. Reaction(R) acting in the upward direction at the supporting end

The external bending couple tends to bend in the clockwise direction. But since one end of the beam is fixed, the beam cannot rotate. Therefore external bending couple must be balanced by another equal and opposite couple, created due to elastic nature of the body i.e. called as internal bending moment.

$\therefore$ Under equilibrium condition

External bending moment = Internal bending Moment

2.6.7 DEPRESSION OF A CANTILEVER – LOADED AT ITS ENDS

THEORY:

Let 'l' be the length of the cantilever OA fixed at 'O'. Let 'W' be the weight suspended (loaded) at the free end of the cantilever. Due to the load applied the cantilever moves to a new position OA' as shown in the figure.

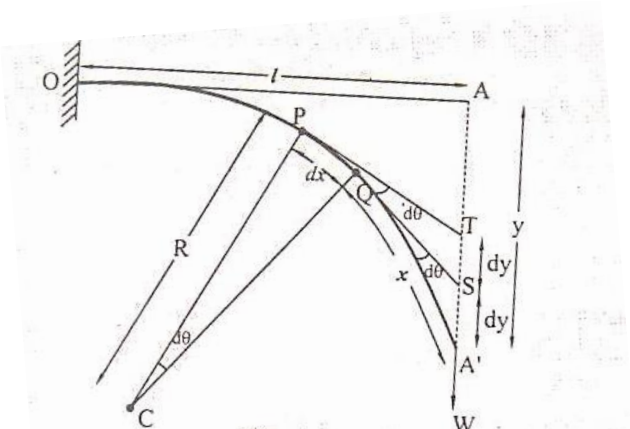


Fig.A3

Let us consider an element PQ of the beam of length dx, at a distance OP = x from the fixed end. Let 'C' be the center of curvature of the element PQ and let 'R' be the radius of the curvature.

Due to the load applied at the free end of the Cantilever, an external couple is created between the load W t 'A' and the force of reaction at 'Q'. Here, the arm of couple (Distance between the two equal and opposite forces) is (l-x).

∴ The External Bending Moment = W* (l-x) ⟶ 1

We know the internal bending moment = $\frac{YI_g}{R}$ ⟶ 2

We know under thermal equilibrium

External bending moment = Internal bending Moment

Therefore, we can write Eqn 1 = Eqn 2

$$W^*(l-x) = \frac{YI_g}{R}$$

Or $R = \frac{YI_g}{W(l-x)}$ ⟶ 3

Two tangents are drawn at points P and Q, which meet the vertical line A A' at T and S respectively

Let the smallest depression produced from T to S = dy and

Let the angle between the two tangents $= d\theta$

Then we can write

The angle between CP and CQ is also dθ i.e. $\angle PCQ = d\theta$

$\therefore$ We can write the arc length PQ = R. dθ=dx

$$d\theta = \frac{dx}{R} \longrightarrow \quad 4$$

Substituting equation 3 in equation 4, we have

$$d\theta = \frac{dx}{\left[YI_g / W(l-x)\right]}$$

$$d\theta = \frac{W}{YI_g}(l-x)dx \longrightarrow \quad 5$$

From the $\Delta QA'S$ we can write Sin dθ = $\frac{dy}{(l-x)}$

If dθ is very small then we can write

$$dy = (l-x)\,d\theta \longrightarrow \quad 6$$

Substituting equation 5 in equation 6 we have

$$dy = \frac{W}{YI_g}(l-x)^2.dx \longrightarrow \quad 7$$

$\therefore$ Total depression at the free end of the cantilever can be derived by integrating the equation 7 within the limits 0 to 'l'

Therefore $$y = \frac{W}{YI_g}\int_0^l (l-x)^2 .dx$$

$$y = \frac{W}{YI_g}\int_0^l \left(l^2 - 2lx + x^2\right)dx$$

$$y = \frac{W}{YI_g}\left[l^2x - \frac{2lx^2}{2}. + \frac{x^3}{3}\right]_0^l$$

$$y = \frac{W}{YI_g}\left[l^3 - l^3. + \frac{l^3}{3}\right]$$

$$y = \frac{W}{YI_g}\left[\frac{l^3}{3}\right]$$

$\therefore$ Depression of the Cantilever at free end $y = \frac{Wl^3}{3YI_g} \longrightarrow \quad 8$

SPECIAL CASES:

a. RECTANGULAR CROSS SECTION

If 'b' is the breadth and'd' is the thickness of the beam then we know

$$I_g = \frac{bd^3}{12}$$

Substituting the value of I_g in equation 8 we can write

The depression produced at the free end for a rectangular cross section

$$y = \frac{Wl^3}{3Y\left(bd^3/12\right)} = \frac{4Wl^3}{Ybd^3}$$

b. CIRCULAR CROSS SECTION

If 'r' is the radius of the circular cross section, then

We know $I_g = \frac{\pi r^4}{4}$

Substituting the value of I_g in equation 8 we can write

The depression produced $y = \frac{Wl^3}{3Y\,\pi r^4/4} = \frac{4Wl^3}{3\pi r^4 Y}$

2.7 MODES OF HEAT TRANSFER

Heat is one of the forms of energy. It is transmitted from one place to another by three different ways.

They are

Conduction

Convection

Radiation

2.7.1 Thermal conduction

It is a well-known fact that heat is conducted through the material of the body.

In conduction, heat transfer takes place from one point to another through a material medium without the actual movement of the particles in that medium.

The heat is transmitted from a body of higher temperature to that of lower temperature.

As an example, when a metal rod is heated at one end, the heat gradually flows along the length of the rod and the other end of the rod also becomes hot after some time. This shows that heat has travelled through the molecules of the rod from one end to other. The molecules in the rod remain fixed in their mean positions.

On heating the energy molecules increases and they start vibrating about their mean positions. They collide with the neighboring molecules. Because of this collision, the neighboring molecules are set into vibration.

Each molecule thus transfers some of the heat it receives from its predecessor to its successor. Thus the transmission of heat takes place by molecular vibrations in case of conduction.

DEFINITION

It is the process of transmission of heat from one point to another through substance (or some medium) without the actual motion of the particles.

Conduction always requires some material medium. The material medium must be solid. As it requires medium, the conduction process takes place over vacuum. In fluids (liquid and gas), heat transmission is through the process of convection.

2.7.2 Thermal conductivity

The ability of a substance to conduct heat energy is measured by thermal conductivity

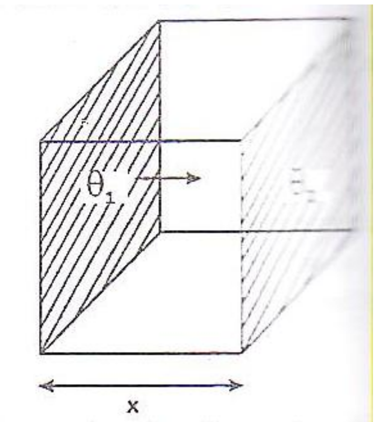
EXPRESSION FOR THERMAL CONDUCTIVITY

Consider a slab of material of length x meter and area of cross section A as shown in the figure.

One end of the slab is maintained at a higher temperature θ_1 and the other end at a lower temperature is θ_2. Heat flows from the hot end to the cold end. It is found that the amount of heat (Q) conducted from one end to another end is

- Directly proportional to the area of cross-section(A)
- Directly proportional to the temperature difference between the ends $\theta_1 - \theta_2$

- Directly proportional to the time of conduction(t)
- Inversely proportional to the length of (x).



$$Q \propto A$$
$$Q \propto \theta_1 - \theta_2$$
$$Q \propto t$$
$$Q \propto \frac{1}{x}$$

Combining all these factors, we have

$$Q \propto \frac{A(\theta_1 - \theta_2)t}{x} \quad \longrightarrow 1$$

$$Q = \frac{KA(\theta_1 - \theta_2)t}{x} \quad \longrightarrow 2$$

Where K is the proportionality constant. It is known as coefficient of thermal conductivity or simple thermal conductivity. Its value depends upon the nature of the material.

If $A = 1m^2$ $(\theta_1 - \theta_2) = 1$ Kelvin ; x= 1 m; t = 1 sec

Then K = Q

This condition defines the coefficient of Thermal conductivity

Definition:

It is defined as the amount of heat conducted per second normally across unit area of cross-section of the material per unit temperature difference per unit length.

The quantity $(\theta_1 - \theta_2)/x$ denote the rate of fall of temperature with respect to distance. It is known as temperature gradient.

For smaller values, $(\theta_1 - \theta_2)/2$ is written as $\frac{d\theta}{dx}$

Rewriting the expression 2 we get

$$Q = -KA\frac{d\theta}{dx}t$$

The negative sign indicates the fall of temperature with distance.

Unit :

We know that $K = \frac{Qx}{A(\theta_1 - \theta_2)t}$

Substituting the units, we have

$\frac{Joule \times metre}{metre^2 .Kelvin.Second}$ Watt/Meter/Kelvin = W/M/K = $Wm^{-1}K^{-1}$

2.7.3 Newton's Law of Cooling

Statement:

It states that the rate at which a body loses heat is directly proportional to the temperature difference between the body and that of the surrounding.

The amount of heat radiated depends upon the area and nature of the radiating surface.

If 'θ' is the temperature of the body at any instant and 'θ_0' the temperature of the surroundings, then according to Newton's law of cooling, heat lost is proportional to the difference of temperature between the body and surroundings i.e. $(\theta_1 - \theta_0)$

If dQ is the quantity of heat lost in a small time dt , then

$$-\frac{dQ}{dt} \propto (\theta_1 - \theta_0)$$

$$-\frac{dQ}{dt} = k(\theta_1 - \theta_0)$$

Where k is the constant depending upon the area and the nature of the radiating surface. The negative sign indicates that there is decrease of heat with time.

Expression when a heat body cools from $\theta_1{}^0C$ to $\theta_2{}^0C$ in time t

Consider a body of mass m, specific heat capacity S and at temperature θ. Suppose , the temperature falls by a small amount $d\theta$ in time dt.

Thus , the amount of heat lost

$$dQ = mSd\theta$$

Rate of loss of heat

$$\frac{dQ}{dt} = mS\frac{d\theta}{dt} \longrightarrow \quad 1$$

From Newton's law of cooling,

$$-\frac{dQ}{dt} = k(\theta - \theta_0) \longrightarrow \quad 2$$

From equations 1 and 2

$$-mS\frac{d\theta}{dt} = k(\theta - \theta_0) \longrightarrow \quad 3$$

Rearranging the equation 3

$$\frac{d\theta}{(\theta - \theta_0)} = -\frac{k}{mS}dt$$

$$\frac{d\theta}{(\theta - \theta_0)} = -Kdt \longrightarrow \quad 4$$

Where K is another constant and is independent of time t

$$K = \frac{k}{mS}$$

Integrating Equation 4 on both sides

$$\int \frac{d\theta}{(\theta - \theta_0)} = -K\int dt \longrightarrow \quad 5$$

$$\log_e(\theta - \theta_0) = -Kt + c \longrightarrow \quad 6$$

Where c is the constant of integration .this equation is of the form

$y = mx + c$ and it represents a Straight line .

If the cooling takes place from $\theta_1{}^0C$ **to** $\theta_2{}^0C$ in time t then taking the limits, we have

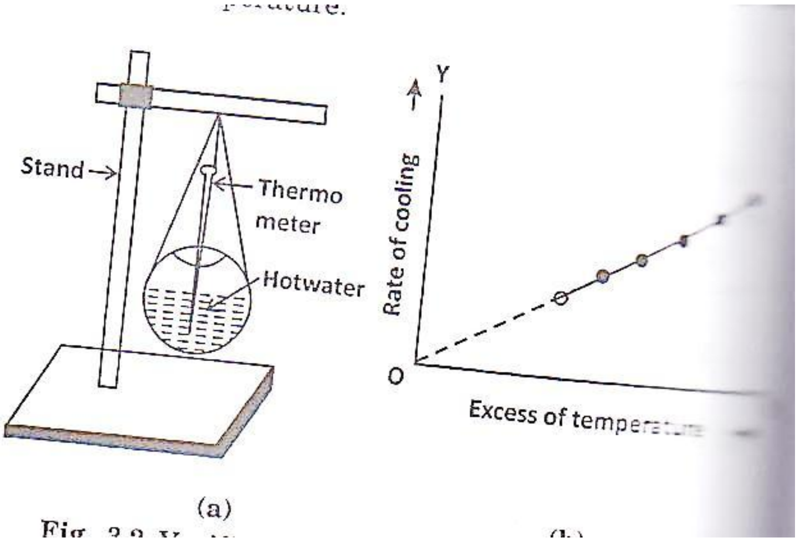
$$[\log_e(\theta - \theta_0)]_{\theta_1}^{\theta_2} = -Kt$$

$$\log\left(\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}\right) = -Kt \longrightarrow \quad 7$$

2.7.4 VERIFICATION OF NEWTON'S LAW OF COOLING

The given empty spherical calorimeter is filled with boiling water and a thermometer is kept in the orifice as shown in the figure. When the temperature reaches 800C, a stop clock is started. The time taken for every 2^{0}C fall in temperature is noted, till the temperature reaches 60 ^{0}C.

The rate of cooling at various temperatures is determined. A graph is drawn with rate of cooling along y-axis and the excess of temperature of the calorimeter over the surrounding along the x- axis.



The graph is found to be a straight line, thereby, showing that the rate of cooling is proportional to the excess of temperature.

Limitations

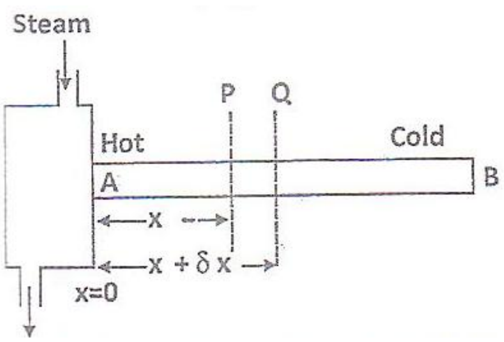
- The temperature difference between the hot body and surrounding should be low.
- The heat loss is only by radiation and convection.
- The temperature of hot body should be Uniform throughout.

Applications

The specific heat capacity of the liquid is determined by using this law.

2.7.5 RECTILINEAR FLOW OF HEAT THROUGH A ROD

Consider a long rod AB of Uniform cross section heated at one end A as shown in the figure



Then there is flow of heat along the length of the bar and heat is also radiated from its surface. B is the cold end.

Consider the flow of heat between the sections P and Q at distance x and x + δx from the hot end.

Excess temperature at Section P above the surroundings = θ

Temperature gradient at section P = $\frac{d\theta}{dx}$

Excess temperature at Section $Q = \theta + \frac{d\theta}{dx}\delta x$

$\therefore$ Temperature Gradient at $Q = \frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\delta x\right)$

$$\frac{d\theta}{dx} + \frac{d^2\theta}{dx^2}\delta x$$

Heat flowing (entering) through P in one second

$$Q_1 = -KA\frac{d\theta}{dx} \longrightarrow \quad 1$$

Heat flowing (leaving) through Q in one second

$$Q_1 = -KA\left(\frac{d\theta}{dx} + \frac{d^2\theta}{dx^2}\delta x\right)$$

$$Q_1 = -KA\frac{d\theta}{dx} - K\frac{d^2\theta}{dx^2}\delta x \longrightarrow \quad 2$$

$\therefore$ Net gain of heat by element δx in one second

$$Q = Q_1 - Q_2$$

$$Q = -KA\frac{d\theta}{dx} - \left(-KA\frac{d\theta}{dx} - KA\frac{d^2\theta}{dx^2}\delta x\right)$$

$$Q = -KA\frac{d\theta}{dx} + KA\frac{d\theta}{dx} + KA\frac{d^2\theta}{dx^2}\delta x$$

$$Q = KA\frac{d^2\theta}{dx^2}\delta x \longrightarrow \quad 3$$

2.7.6 BEFORE THE STEADY STATE IS REACHED

Before the steady state is reached, the amount of heat Q is used in two ways. A part of the heat is used in raising the temperature of the rod and the remaining heat is lost by radiation from the surface of the element.

Heat absorbed per second to raise the temperature of the rod

= mass $\times$ Specific heat capacity $\times \frac{d\theta}{dt}$

$$= (A \times \delta x)\rho \times S \times \frac{d\theta}{dt} \longrightarrow \quad 4$$

Where A = area of cross section of the rod

ρ = density of the rod

S = specific heat capacity of the bar

$\frac{d\theta}{dt}$ = rate of rise of temperature

Heat loss per second due to radiation

$$Ep\delta x\theta \longrightarrow 5$$

Where

E = emissive power of the surface

P = Perimeter of the bar

p δx = surface area of the element

θ = average excess of temperature of the element over that of the surroundings

Amount of heat (Q)

= amount of heat absorbed + amount of heat lost

$$Q = (A \times \delta x)\rho \times S \times \frac{d\theta}{dt} + Ep\delta x\theta \longrightarrow 6$$

Substituting the value of Q from equation (3) in equation (6) we have

$$KA\frac{d^2\theta}{dx^2}\delta x = (A \times \delta x)\rho \times S \times \frac{d\theta}{dt} + Ep\delta x\theta \longrightarrow 7$$

Dividing both L.H.S and R.H.S of the equation 7 by KA δx, we have,

$$\frac{d^2\theta}{dx^2} = \frac{\rho S}{K}\frac{d\theta}{dt} + \frac{Ep}{KA}\theta \longrightarrow 8$$

This equation 8 is the standard differential equation for the flow of heat through the rod.

SPECIAL CASES:

The thermal conductivity of any material is determined using equation 8 by considering the actual condition of the material

Case 1: When heat is lost by radiation is negligible.

If the rod is completely covered by some insulating materials, then there is no loss of heat due to radiation. Hence, the heat lost by radiation $Ep\delta x\theta$ is zero.

In that case, the total heat gained by the rod is completely used to raise the temperature of the rod.

From equation 8

$$\frac{d^2\theta}{dx^2} = \frac{\rho S}{K}\frac{d\theta}{dt} = \frac{1}{h}\frac{d\theta}{dt} \longrightarrow \quad 9$$

Here $\frac{K}{\rho S} = h$, the thermal diffusivity of the rod

Case 2: After the steady state is reached

After the steady state is reached, there is no raise of temperature.

Hence $\frac{d\theta}{dt} = 0$

From equation 8

$$\frac{d^2\theta}{dx^2} = \frac{Ep}{KA}\theta$$

Taking $\frac{Ep}{KA} = \mu^2$

$$\frac{d^2\theta}{dx^2} = \mu^2\theta \longrightarrow \quad 10$$

This is second order differential equation

The general solution of this equation is

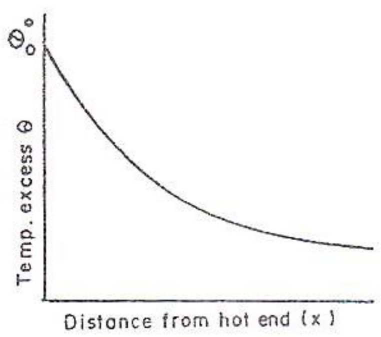
$$\theta = Ae^{+\mu x} + Be^{-\mu x} \longrightarrow \quad 11$$

Here A and B are two unknown constants which can be determined from the boundary conditions of the problem.

Suppose the bar is of infinite length

Excess of temperature above the surroundings of the hot end = θ_0

Temperature of the other end (cold end) = 0



First boundary condition is at x = 0; $\theta = \theta_0$

From equation 11

$$\theta_0 = A + B$$

Secondary boundary condition is at x = $\infty, \theta = 0$

From equation 11

$$0 = Ae^{\infty}$$

But e^{∞} cannot be zero, therefore A should be Zero

i.e. A = 0

Then $\theta_0 = B$

Substituting the values of A and B in equation 11, we have

$$\theta = \theta_0 e^{-\mu x} \longrightarrow \quad 12$$

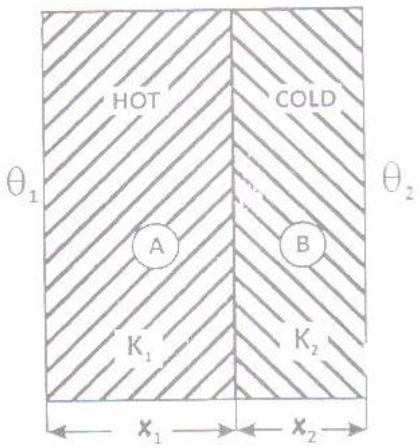
This equation 12 represents the excess of temperature of a point at distance x from the hot end after the steady state is reached and it represents an exponential curve.

The temperature falls exponentially from the hot end as shown in the figure.

2.7.7 HEAT CONDUCTION THROUGH A COMPOUND MEDIA OF TWO LAYERS

BODIES IN SERIES

Let us consider a composite slab (or compound wall) of two different materials A and B with thermal conductivities K_1 and K_2 and of thickness x_1 and x_2



The temperatures of the outer faces of A and B are $\theta_1 and \theta_2$

The temperature of the surfaces in contact is θ. When the steady state is reached, the amount of heat flowing per second (Q) through every layer is same.

Amount of heat flowing through the material (A) per second

$$Q = \frac{K_1 A(\theta_1 - \theta)}{x_1} \longrightarrow \quad 1$$

Amount of heat flowing through the material (B) per second

$$Q = \frac{K_2 A(\theta - \theta_2)}{x_2} \quad 2$$

Here , the equations 1 and 2 are equal $\longrightarrow$

$$\frac{K_1 A(\theta_1 - \theta_2)}{x_1} = \frac{K_2 A(\theta_1 - \theta_2)}{x_2} \longrightarrow \quad 3$$

$$K_1 A(\theta_1 - \theta)x_2 = K_2 A(\theta - \theta_2)x_1$$

$$K_1 \theta_1 x_2 - K_1 \theta x_2 = K_2 \theta x_1 - K_2 \theta_2 x_1$$

$$K_1 \theta_1 x_2 + K_2 \theta_2 x_1 = K_2 \theta x_1 - K_1 \theta x_2$$

$$K_1 \theta_1 x_2 + K_2 \theta_2 x_1 = \theta(K_2 x_1 + K_1 x_2)$$

$$\theta = \frac{K_1\theta_1 x_2 + K_2\theta_2 x_1}{K_2 x_1 + K_1 x_2} \longrightarrow \quad 4$$

This is the expression for interface temperature of two composite slabs

Substituting the value of θ inn equation 1 we have,

$$Q = \frac{K_1 A}{x_1}\left[\theta_1 - \left(\frac{K_1\theta_1 x_2 + K_2\theta_2 x_1}{K_2 x_1 + K_1 x_2}\right)\right]$$

$$Q = \frac{K_1 A}{x_1}\left[\left(\frac{K_2\theta_1 x_1 + K_1\theta_1 x_2 - K_1\theta_1 x_2 - K_2\theta_2 x_1}{K_2 x_1 + K_1 x_2}\right)\right]$$

$$Q = \frac{K_1 A}{x_1}\left[\left(\frac{K_2\theta_1 x_1 - K_2\theta_2 x_1}{K_2 x_1 + K_1 x_2}\right)\right]$$

$$Q = \frac{K_1 K_2 A}{x_1}\left[\left(\frac{\theta_1 x_1 - \theta_2 x_1}{K_2 x_1 + K_1 x_2}\right)\right]$$

$$Q = \frac{K_1 K_2 A}{x_1}\left[\left(\frac{x_1(\theta_1 - \theta_2)}{K_2 x_1 + K_1 x_2}\right)\right]$$

$$Q = K_1 K_2 A\left[\left(\frac{(\theta_1 - \theta_2)}{K_2 x_1 + K_1 x_2}\right)\right]$$

$$\frac{K_1 K_2 A(\theta_1 - \theta_2)}{K_2 x_1 + K_1 x_2}$$

$$\frac{A(\theta_1 - \theta_2)}{\frac{K_2 x_1}{K_1 K_2} + \frac{K_1 x_2}{K_1 K_2}}$$

$$\frac{A(\theta_1 - \theta_2)}{\frac{x_1}{K_1} + \frac{x_2}{K_2}} \longrightarrow \quad 5$$

'Q' is the value of heat flowing through the compound wall of the two materials

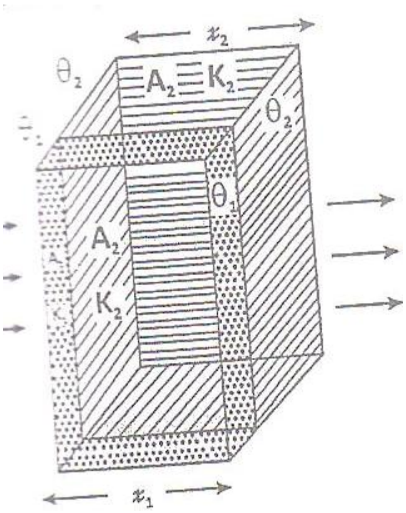
The method can be extended to composite slab with more than two slabs.

In general for any number of walls or slabs, the amount of heat conducted is

$$\frac{A(\theta_1 - \theta_2)}{\sum \frac{x}{K}}$$

Bodies in Parallel

Let us consider a composite slab (or Compound wall) of two different materials A and B with thermal conductivities $K_1 and K_2$ and of thickness $x_1 and x_2$. They are arranged in parallel as shown in the figure.



Let the faces of the material be at temperature θ_1 and the respective other end faces be at θ_2 temperature. A_1 and A_2 be the area of cross- section of the materials

Amount of heat flowing through the first material (A) in one second

$$Q_1 = \frac{K_1 A_1(\theta_1 - \theta_2)}{x_1} \longrightarrow$$

1

Similarly the amount of heat flowing through the second material (B) in one second

$$Q_1 = \frac{K_2 A_2(\theta_1 - \theta_2)}{x_2}$$

The total heat flowing through these materials per second is the sum of these two heats Q_1 and Q_2

$$Q = Q_1 + Q_2$$

$$Q = \frac{K_1 A_1(\theta_1 - \theta_2)}{x_1} + \frac{K_2 A_2(\theta_1 - \theta_2)}{x_2}$$

$\therefore$ The amount of heat flowing per second

$$Q = (\theta_1 - \theta_2)\left(\frac{K_1 A_1}{x_1} + \frac{K_2 A_2}{x_2}\right)$$

In general

$$Q = (\theta_1 - \theta_2)\sum \frac{KA}{x}$$

2.7.8 Methods to determine Thermal conductivity

The thermal conductivity of a material is determined by different methods.

1. Searle's Method – for good conductor like Metallic rod
2. Forbe's Method – for determining the absolute conductivity of metals
3. Lee's Disc method – for bad conductors
4. Radial Flow method – for Bad conductors.

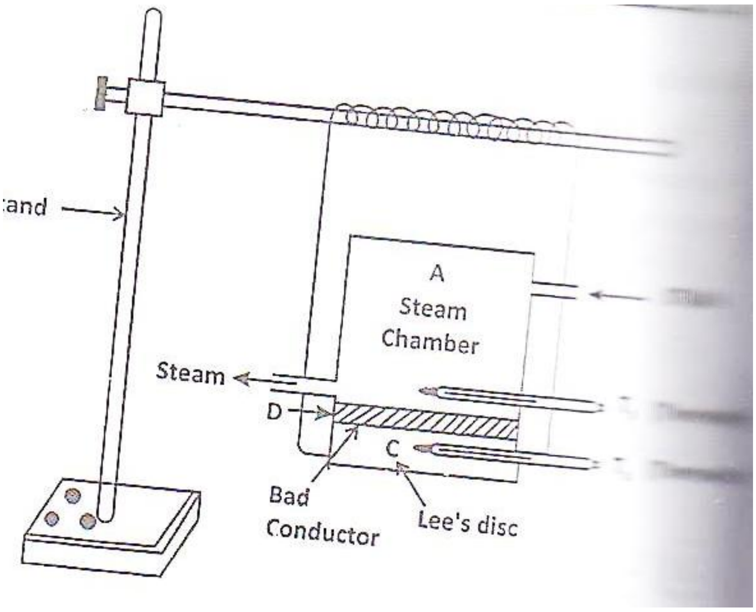
2.7.9 LEE'S DISC METHOD FOR BAD CONDUCTORS

The thermal conductivity of bad conductors like ebonite or card board is determined by this method.

Description:

The apparatus consists of circular metal disc or slab C (Lee's Disc) by strings from as stand The given bad conductor (such as glass, ebonite) is taken in the form of the disc (D). This disc has the same diameter as that of the slab and is placed over it.

A cylindrical hollow steam chamber A having the same diameter as that of the slab is placed over the bad conductor. There are holes in the steam chamber and the slab through which thermometers T_1 and T_2 are inserted to record the respective temperatures.



Working:

Steam is passed through the steam chamber until the temperatures of the chamber and the slab are ready. When the thermometers show steady temperatures, there readings θ_1 and θ_2 are noted. The radius (r) of the disc D and its thickness (d) are also noted.

Observation and Calculation

Thickness of the bad conductor = d meter

Radius of the bad conductor = r meter

Mass of the Slab (c) = M kg

Steady temperature in the slab = θ_1

Steady temperature in the steam chamber = θ_2

Thermal conductivity of the bad conductor = K

Rate of cooling at θ_2 = R

Specific heat capacity of the slab =S

Area of cross-section A = πr^2

Amount of heat conducted through the specimen per second

$$Q = \frac{KA(\theta_1 - \theta_2)}{d} = \frac{K(\pi r^2)(\theta_1 - \theta_2)}{d} \longrightarrow \quad 1$$

At this stage, all the heat conducted through the bad conductor is completely radiated by the bottom flat surface and the curved surface of the Slab C.

Amount of heat lost per second by the Slab C

Q = Mass X Specific Heat Capacity × Rate of Cooling

$$Q = MSR \longrightarrow \quad 2$$

At steady rate,

Heat conducted through bad conductor per second = heat lost [per second by the slab

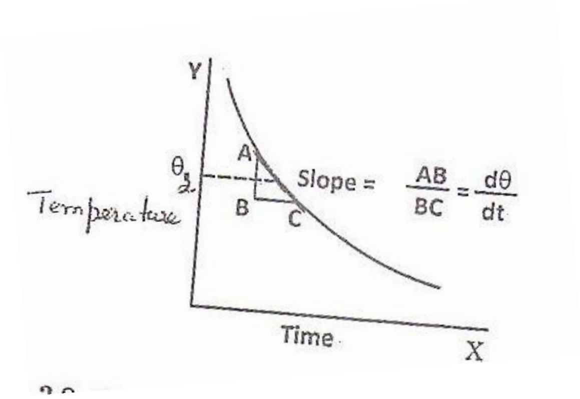
Hence the equations (1) and (2) are equal

$$\frac{K(\pi r^2)(\theta_1 - \theta_2)}{d} = MSR$$

$$\therefore K = \frac{MSRd}{\pi r^2(\theta_1 - \theta_2)} Wm^{-1}K^{-1} \longrightarrow \quad 3$$

2.7.10 Determination of Rate of Cooling

The bad conductor is removed and the steam chamber is placed directly on the slab. The slab is heated to a temperature of about 5^{0}C higher than θ_2. The steam chamber is removed and the slab alone is allowed to cool



As the slab cools, the temperatures of the slab are noted at regular intervals of half a minute until the temperature of the slab falls to about 5^{0}C below θ_2. The time – temperature graph is drawn as shown in the figure and the rate of cooling $\frac{d\theta}{dt}$ at the steady temperature θ_2 is determined.

During the first part of the experiment, the top surface of the slab is covered by the bad conductor. Radiation is taking place only from the bottom surface area and curved surface area.

Total area of = $\pi r^2 + 2\pi rh = \pi r(r+2h)$ where h is the height of C

In the second part of the experiment, heat is radiated from the top surface area, the bottom surface area and the curved sides i.e. over an area

$$2\pi r^2 + 2\pi rh = 2\pi r(r+h)$$

As the rate of cooling is directly proportional to the surfaces are exposed

$$\frac{R}{\frac{d\theta}{dt}} = \frac{\pi r(r+2h)}{2\pi r(r+h)} = \frac{r+2h}{2(r+h)}$$

$$R = \frac{d\theta}{dt} \cdot \frac{(r+2h)}{2(r+h)} \longrightarrow \quad 4$$

Substituting this value in equation 3 , we have

$$K = \frac{MS\frac{d\theta}{dt}d}{\pi r^2(\theta_1-\theta_2)}\times\frac{(r+2h)}{(r+h)} \longrightarrow \quad 5$$

From which K is determined.

2.8 Radial flow of heat

In this method, heat flows from the inner side towards the other side along the radius of the cylindrical shell.

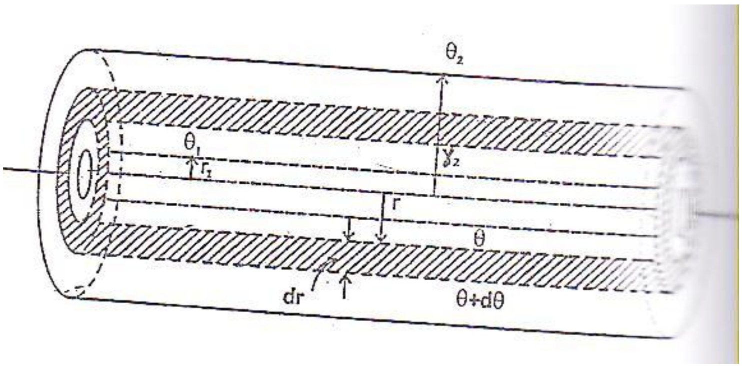
This method is interesting because there is no loss of heat as in the other methods.

2.8.1 Cylindrical Shell method

Consider a cylindrical tube of length l , inner radius r_1 and outer radius r_2 as shown in the figure. The tube carries steam or some hot liquid.

Heat is conducted radially across the walls of the tube. After the steady state is reached, the temperature of the inner surface θ_1 and on the outer surface θ_2. This thick pipe is imagined to consist of a large number of thin coaxial cylinders of increasing radius. Any such thin imaginary cylinder of the material of thickness 'dr' at a distance r from the axis of the pipe is taken.

Amount of heat flowing per second through this elementary cylinder



$$Q = -KA\frac{d\theta}{dr} \longrightarrow \quad 1$$

Now, surface area of the imaginary cylinder

$$A = 2\pi r\times l$$

$$\therefore Q = -2\pi rlK\frac{d\theta}{dr} \longrightarrow \quad 2$$

After steady state is reached, the amount of heat flowing (Q) through all imaginary cylinders is same.

Rearranging, the equation 2, we get

$$\frac{dr}{r} = \left(\frac{-2\pi lK}{Q}\right) d\theta$$

Integrating both sides between their proper limits we have,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{-2\pi lK}{Q} \int_{\theta_1}^{\theta} d\theta$$

$$[\log_e r]_{r_1}^{r_2} = \frac{-2\pi lK}{Q} [\theta]_{\theta_1}^{\theta_2}$$

$$\frac{-2\pi lK}{Q} [\theta_2 - \theta_1]$$

$$[\log_e r_2 - \log_e r_1]_{r_1}^{r_2} = \frac{2\pi lK}{Q} [\theta_1 - \theta_2]$$

$$\log_e \left(\frac{r_2}{r_1}\right) = \frac{2\pi lK}{Q} (\theta_1 - \theta_2)$$

$$Q = \frac{K 2\pi l(\theta_1 - \theta_2)}{\log_e \frac{r_2}{r_1}}$$

$$K = \frac{Q \log_e \frac{r_2}{r_1}}{2\pi l(\theta_1 - \theta_2)}$$

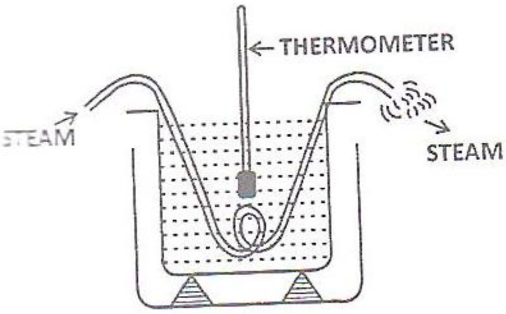
2.8.2 THERMAL CONDUCTIVITY OF RUBBER

Principle:

It is based on the principle of radial flow of heat through a cylindrical shell.

Procedure:

A big empty calorimeter with stirrer is weighted (W_1). It is then filled with two thirds of water and again weighed (W_2). A known length (l) of a rubber tube is immersed in water in the calorimeter as shown in the figure.



The calorimeter is stirred dwell and the initial temperature $\theta_1{}^0C$ is noted. Now one end of the rubber tube is connected to a steam generator and steam is passed through it. The steam is passed continuously till there is rise of 10^0 C in temperature. The time taken (t second) for this rise in temperature is noted. The final temperature of the water $\theta_2{}^0C$ in the calorimeter is also noted.

Observation

Mass of the empty calorimeter with stirrer = W_1 kg

Mass of the calorimeter with water = W_2 Kg

Mass of the water = $(W_2 - W_1)$Kg

Initial temperature of the water = $\theta_1{}^0C$

Final temperature of the water = $\theta_2{}^0C$

Rise in temperature of the water = $(\theta_1 - \theta_2)$

Time for which steam is passed = t seconds

Length of the rubber tube immersed in water= l

Inner radius of the rubber tube = r_1

Outer radius of the rubber tube = r_2

Specific heat capacity of Calorimeter = S_1

Specific heat capacity of Water = S_2

Heat gained by the calorimeter = Mass × Specific heat capacity × change in temperature

$$= W \times S_1 \times (\theta_1 - \theta_2) \longrightarrow 1$$

Heat gained by water = $(W_2 - W_1) \times S_2 \times (\theta_2 - \theta_1) \longrightarrow$ 2

Total heat gained by the calorimeter and water in t second

$$Q = \frac{W_1 S_1(\theta_2 - \theta_1) + (W_2 - W_1) S_2 (\theta_2 - \theta_1)}{t} \longrightarrow 3$$

The expression for the thermal conductivity (K) in the case of cylindrical shell method is given by

$$K = \frac{Q \log_e \frac{r_2}{r_1}}{2\pi l(\theta_1 - \theta_2)} \longrightarrow 4$$

Substituting equation 3 in equation 4, we have

$$K = \frac{(W_1 S_1 + (W_2 - W_1) S_2)(\theta_2 - \theta_1) \log_e \frac{r_2}{r_1}}{2\pi l t\left[\theta_s - \frac{\theta + \theta_2}{2}\right]}$$

Where θ_s = temperature of the steam

$\frac{\theta + \theta_2}{2}$ = average temperature inside of rubber tube