Quadratic Equations

**Quadratic Polynomials**

The general form of a quadratic polynomial in x is ax2 + bx + c

where a,b,c, are real numbers and 

**Zeros of quadratic polynomials**

When a quadratic polynomial is equated to zero, we get two values of x which is called zeros of a quadratic polynomial.

**Quadratic Equations**

A quadratic polynomial equated to zero is called a quadratic equation. The general form of a quadratic equation is ax2+ bx + = 0, where a, b, c are real numbers and 

**Quadratic formula**

Let ax2 + bx + c = 0

Multiplying by 4a we get

4a2x2 + 4abx + 4ac = 0

Or, (2ax)2+ 2.2ax.b + b2 = b2 - 4ac

Or, (2ax + b)2 = b2 - 4ac



Let D = b2 - 4ac



Or, 





If  is not real and, therefore, the equation has no real roots.

"D" is called the discriminat of the quadratic equation.

**Case-I**, When 

i. e. then the equation has two distinct real roots  given by



**Case-II,** when D = b2 - 4ac = 0



Both the roots are real and equal.

**Example 1**

 Solve: x2 - 16 = 0

**Solution**

 x2 - 16 = 0

(x + 4) (x - 4) = 0

Either x + 4 = 0 which gives x = -4

Or, x - 4= 0, which gives x = 4

 x = -4, 4

**Example 2.**

Solve: 8x2 - 22x - 21 = 0

S**olution**

8x2 - 22x - 21 = 0

8x2 - (28 - 6) x - 21 = 0

8x2 - 28x + 6x -21 = 0

4x(2x - 7) + 3(2x - 7) = 0

(2x - 7) (4x + 3) = 0

Either 2x - 7 = 0 or, 4x + 3 = 0

Either 2x = 7 or, 4x = -3

Either x = 7/2 or, x = -3/4



**Example 3**

Solve: 

Solution:

$$\frac{1}{x+1 }+ \frac{1}{x+2 }= \frac{1}{x+4 }$$

$$\frac{\left( x+2 \right)+ (x+1) }{\left(x+1 \right)(x+2)}= \frac{1}{x+4 }$$

$$\frac{\left( 2x+3 \right) }{x^{2 }+x+2x+2}= \frac{1}{x+4 }$$

Or, (2x + 3) (x + 4) = x2 + 3x + 2

Or, 2x2 + 8x + 3x + 12 = x2 + 3x + 2

Or, x2 + 8x + 10 = 0

Here a = 1, b = 6, c = 8

D = b2 - 4ac

 = (8)2 - 4x1x10

= 64 - 40

= 24







**Example 4.**

Find the value of p for which the quadratic equation 2x2 + 3x + p = 0 has real roots.

**Solution**

2x2 + 3x +p = 0

a = 2, b = 3, c = p

D = b2 - 4ac

 = 32 - 4x 2 x p

 = 9 – 8p

For real roots, 

or, 

or, 

or, 



**Example5**

Find the value of  such that the quadratic equation  has equal roots.

**Solution:-**



Here****

D = b2 - 4ac







For equal roots, 

i.e. 

Or, 

But for  the quadratic equation does not exist.



 **Application of Quadratic Equations**

**Example 1**

A rectangular field is 16m long and 10m wide, there is a path of uniform width all around it having an area of 120sq.m final the width of the path.

**Solution**

 Let width of the path be x m.



Area of the outer rectangle

= (16 + 2x) (10 + 2x)

= 160 + 32x + 20x + 4x2

= 160 + 52x + 4x2

Area of the inner rectangle

= 16m x 10m

= 160m2

Area of the path

= 160 + 52x + 4x2 - 160 = 120

or, 4x2 + 52x - 120 = 0

or, x2 + 13x - 30 = 0

or, (x + 15) (x - 2) = 0

or, x = -15, 2

As width cannot be negative  x = 2 m

1. e. width of the path = 2 m

**Example 2**

The sum of the reciprocals of Rehman’s ages (in years), 3 years ago and 5 years from now is 1/3. Find his present age.

**Solution :-** Let his present age be x

His age 3 years ago = x – 3 and 5 years hence = x + 5

According to the question

$$ \frac{1}{x-3}+ \frac{1}{x+5}= \frac{1}{3}$$

$$\frac{\left(x+5 \right)+ (x-3)}{\left(x-3\right)(x+5)} = \frac{1}{3}$$

$$\frac{2x+2}{x^{2}-3x+5x-15} = \frac{1}{3}$$

$$or$$

$$\frac{2x+2}{x^{2}-3x+5x-15} = \frac{1}{3}$$

 x2 + 2x – 15 = 6x + 6
 x2 – 4x – 21 = 0
(x + 3)(x – 7) = 0

Either, x + 3 = 0 Or, x – 7 = 0

Thus, x = 7 or – 3; but age cannot be negative.

Hence his age is 7 years.

**Example 3.**

A plane left 30 minutes later than the scheduled time and in order to reach the destination 1500km away in time, it has to increase the speed by 250 km/hr from the usual speed. Find the usual speed.

**Solution**

Let the usual speed of plane be x km/hr.

The increased speed of the plane = (x + 250) km/hr.

Distance = 1500 km

$$\frac{1500}{x}- \frac{1500}{x+250}= \frac{1}{2}$$

or, 

or, x2 + 250x - 750000 = 0

or, x2 + 1000x - 750x - 750000 = 0

or, x(x + 1000) -750 (x + 1000) = 0

or, ( x + 1000) ( x - 750) = 0

or, x = -1000, 750

As speed cannot be negative

 Usual speed = 750 km/hr.

**Example 4.**

If the list price of a book is reduced by Rs. 5, a person can buy 5 more books for Rs.300. Find the original list price of the book.

**Solution**

 Let the original price of the book be Rs. x.

Then the reduced price of the book = Rs. (x - 5)

Total amount = Rs. 300

According to the question



Or, 

Or, 

Or, 5(x2 - 5x) = 1500

Or, x2 - 5x = 300

Or, x2 - 5x - 300 = 0

Or, x2 - 20x + 15x - 300 = 0

Or, x(x - 20) + 15 (x - 20) = 0

Or, (x - 20) (x + 15) = 0

x = -15, 20

As price cannot be negative x = Rs. 20.

**Example 5**

 In a class test, the sum of Shefali’s marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

**Solution**

Let her marks in Mathematics = x, then marks in English = 30 – x.
As per statements, her marks in Mathematics = x + 2

Her marks in English

30 – x – 3 = 27 – x
(x + 2)(27 – x) = 210
Or,

27x – x2 + 54 – 2x = 210
Or,    x2 – 25x + 156 = 0
Or,

(x – 12)(x – 13) = 0
Either,   x – 12 = 0, Or, x – 13 = 0

Thus, x = 12, 13

Hence, either marks in Mathematics = 12 & marks in English = 18,

Or, marks in Mathematics = 13 & marks in English = 17.

**Example 6.**

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m2? If so, find its length and breadth.

**Solution**

Let breadth = x, then length = 2x

According to question, *l* × b = 2x × x = 800

Or, 2x2 = 800

Or, x2 = 400

Or, x = 20s

Hence, it is possible to design of length  = 2×20 = 40 m and breadth = 20 m.

 **Example 7.**

Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

**Solution**

 Let present age of one friend be x and that of the other 20 – x.

4 years ago age of first friend = x – 4, and that of the other = 20 – x – 4 = 16 – x.

According to question,  (x – 4 )(16 – x ) = 48

Or, 16x – x2 – 64 + 4x =48

Or, x2 + 20x – 112 = 0

Or, x2 – 20x + 112 = 0

Here, a = 1, b = – 20, c = 112

D = b2 – 4ac = (– 20 )2 – 4 ×1 × 112 = – 48 < 0.

Hence, the given situation is not possible.

**Example 8.**

A number consists of two digits whose product is 18. when 27 is subtracted from the number, the digits change their places. Find the number.

**Solution**

Let the digit at unit place be x and the digit at ten’s place be y.

 Number 10y + x

Reversed number = 10x + y

According to the question

10y + x - 27 = 10x + y

Or, 9y - 9x = 27

Or, y - x = 3

 y = x + 3 ------------- (i)

xy = 18 ----------------- (ii)

x(x + 3) = 18

Or, x2 + 3x - 18 = 0

Or, x2 + 6x - 3x - 18 = 0

Or, x(x + 6) -3 ( x + 6) = 0

Or, x = -6, 3

As digits ae never negative  x = 3

From (i) y = x + 3

= 3 + 3

= 6

 Number = 10y + x = 10 X 6 + 3 = 63