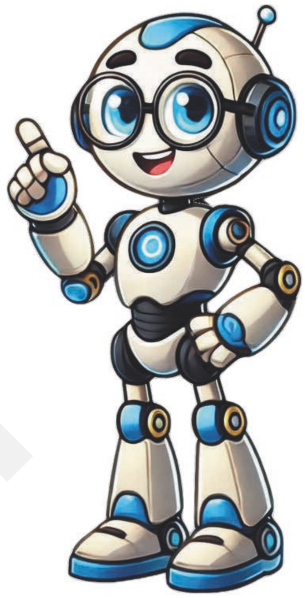
9
Chapter

Mensuration

We'll cover the following key points:

- → Area of a trapezium
- → Area of general quadrilateral
- → Area of special quadrilateral-rhombus
- → Area of a polygon
- → Volume and surface area of solids
- → Volume of cuboid and cube
- → Surface area of cuboid and cube
- → Diagonal of cuboid and cube
- → Right circular cylinder
- → Volume of a right circular cylinder
- → Total surface area of a right circular cylinder



Hi, I'm EeeBee

Do you Remember fundamental concept in previous class.
In class 5th we learnt
- → Concept of Area
- → Volume

Still curious? Talk to me by scanning the QR code.

Learning Outcomes

By the end of this chapter, students will be able to:

- Understand and differentiate between 2D and 3D shapes based on their properties.
- Calculate the perimeter and area of plane figures such as rectangles, squares, triangles, circles, and parallelograms.
- Apply formulas to find the surface area of 3D objects like cubes, cuboids, and cylinders.
- Derive and use formulas to calculate the volume of solids such as cubes, cuboids, cylinders, and cones.
- Solve real-life problems involving the perimeter, area, surface area, and volume of shapes.
- Convert units of measurement (e.g., cm^2 to m^2, cm^3 to m^3) effectively in calculations.
- Explore the concept of lateral surface area and total surface area for 3D objects.
- Analyze and interpret problems related to combining or dividing shapes to find unknown measures.
- Enhance problem-solving skills by applying mensuration concepts in diverse practical contexts.

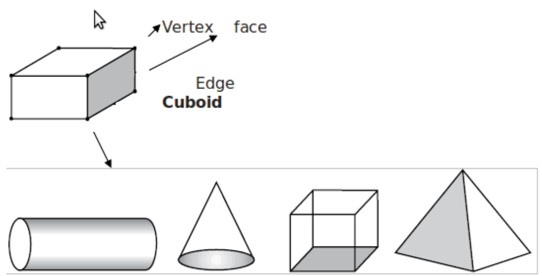
Mind Map

MENSURATION

Area of polygon

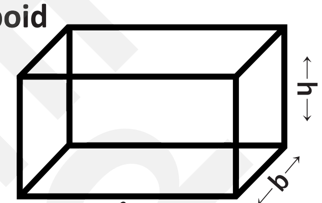
- Area of trapezium = $\frac{1}{2}$ h (a + b)
  a and b are parallel sides h is height.
- Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$
  d_1 and d_2 are diagonals

Solid shapes



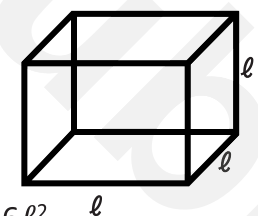
Surface area

- **Cuboid**



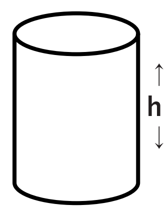
TSA = 2(ℓ b + bh + hℓ)

- **Cube**



TSA = $6\ell^2$

- **Cylinder**



CSA = $2\pi rh$

TSA = $2\pi r(r + h)$

Volume

i. Cube

Volume = ℓ^3

i. Cuboid

Volume = ℓ × b × h

i. Cylinder

Volume = $\pi r^2 h$

Volume and Capacity

i. Amount of space occupied by an object – volume.

ii. Quantity that container holds – capacity.

$1m^3$ = 1000 litres

1000 cm^3 = 1 litre

Based on NCERT*

Introduction

Let's have a quick glance on these concept :

(i) Area of a square = side × side = a^2 sq units

Perimeter of a square = $4 \times a$ unit

(ii) Area of a rectangle = length × breadth = $l \times b$ sq units

Perimeter of a rectangle = 2 × (Length + Breadth)

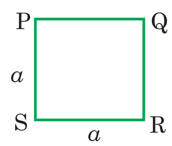
$= 2 \times (l + b)$

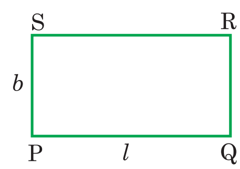
Area of a parallelogram = Base × height

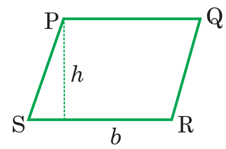
$= b \times h$

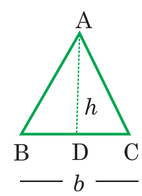
Area of a triangle = $\frac{1}{2}$ × base × height

$= \frac{1}{2} \times b \times h$









In this chapter, we shall learn how to find the area of a trapezium and a polygon.

Area of a Trapezium

Trapezium is a quadrilateral in which one pair of opposite sides are parallel to each other.

Let DEFG be a trapezium in which DE || GF as shown in the given figure :

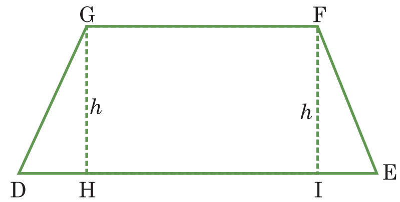
Now, draw FI ⊥ DE and GH ⊥ DE. Let FI = GH = h

∴ Area of trapezium DEFG

$$= \text{Area of DDHG} + \text{Area of rectangle FGHI} + \text{Area of } \Delta\text{EFI}$$

$$= \frac{1}{2} \times \text{DH} \times \text{HG} + \text{HI} \times \text{HG} + \frac{1}{2} \text{IE} \times \text{FI}$$

$$= \frac{1}{2} \times DH \times h + HI \times h + \frac{1}{2} IE \times h$$



$$= \frac{1}{2} \times (DH + 2HI + IE)h$$

$$= \frac{1}{2} \times (DH + HI + IE + HI)h$$

$$= \frac{1}{2} \times (DE + HI)h \qquad (\because DE = DH + HI + IE)$$

$$= \frac{1}{2} \times (DE + GF)h \qquad (\because HI = FG)$$

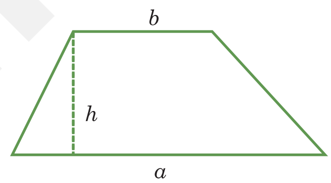
$$= \frac{1}{2} \times \text{(Sum of parallel sides)} \times \text{height}$$

$\therefore$ Area of a trapezium $= \frac{1}{2}$ (Sum of parallel sides) $\times$ altitude

Thus, *the area of a trapezium equals half the sum of parallel sides multiplied by the altitude.*

Example 1 : Two parallel sides of a trapezium are 22 m and 18 m long. The distance between these sides is 12 m. Find the area of the trapezium.

Solution : The area of a trapezium is given by



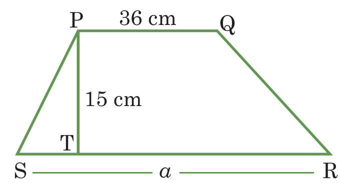
$$A = \frac{1}{2} \times (a + b) \times h$$

Here, $a = 22$ m, $b = 18$ m and $h = 12$ m

$$\therefore \quad A = \frac{1}{2} \times (22 + 18) \times 12 \text{ m}^2 = 40 \times 6 \text{ m}^2 = 240 \text{ m}^2.$$

Example 2 : The area of a trapezium is 900 cm^2. If the distance between parallel sides is 15 cm and one of the parallel side is 36 cm, find the length of the other side.

Solution : Area of trapezium $= \frac{1}{2} \times (a + b) \times h$



$$900 = \frac{1}{2} \times (a + 36) \times 15$$

$$\Rightarrow \quad 900 \times 2 = 15a + 540$$

$$\Rightarrow \quad 1800 - 540 = 15a$$

$$\Rightarrow \quad 1260 = 15a$$

$$\therefore \quad a = \frac{1260}{15} = 84 \text{ cm}$$

Hence, the length of other parallel side is 84 cm.

Example 3 : The parallel sides of a trapezium are 50 cm and 80 cm. Its non-parallel sides are equal, each equal to 25 cm. Find the area of the trapezium?

Solution : Let ABCD be the given trapezium in which AB = 80 cm, CD = 50 cm and non-parallel sides AD and BC each equal to 25 cm.

Draw CM $\perp$ AB and draw CT || AD meeting AB in T.

Now, in trapezium ABCD.

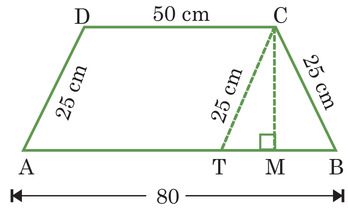
$$TB = AB - AT = AB - CD = 80\text{ cm} - 50\text{ cm} = 30\text{ cm}$$

Also $CT = DA = 25$ cm [Opposite sides of the parallelogram ABCD]

And $AD = BC = 25$ cm [given]

$\therefore$ $CT = CB = 25$ cm

$\therefore$ ΔCTB is an isosceles triangle.



Since $CM \perp AB$ or $CM \perp TB$,

$\therefore$ M is the mid-point of TB

$\therefore$ $TM = MB = \frac{1}{2} \times 30\text{ cm} = 15\text{ cm}.$

Now, in DCMB

$$CM^2 = BC^2 - BM^2$$

$$= (25)^2 - (15)^2 = 625 - 225 = 400\text{ cm}^2$$

$\therefore$ $CM = \sqrt{400} = 20$ cm

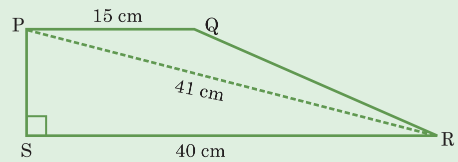
$\therefore$ Area of the trapezium ABCD

$$= \frac{1}{2}\text{(sum of the parallel sides)} \times \text{distance between them}$$

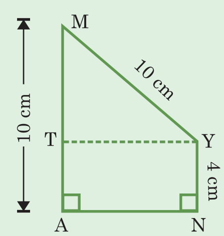
$$= \frac{1}{2}(80 + 50) \times 20\text{ cm}^2 = 1300\text{ cm}^2.$$

Exercise 9.1

1. Two parallel sides of a trapezium ABCD are of lengths 10 cm and 19 cm respectively, and the distance between them is 12 cm. Find the area of the trapezium?
2. The parallel sides of trapezium are 85 cm and 63 cm and its area is 2664 cm^2. Find the height of the trapezium?
3. PQRS is a trapezium with area 69 cm^2 and PQ and SR are its parallel sides and PQ = 2 (SR) –1. If SR = 8 cm, find the height of the trapezium.
4. The ratio of the lengths of the parallel sides of a trapezium is 3 : 2. The shortest distance between them is 15 cm. If the area of the trapezium is 450 cm^2. Find the length of the parallel sides.
5. The parallel sides of a trapezium are 50 cm and 80 cm. The non-parallel sides are equal each being 17 cm. Find the area of the trapezium.
6. In the given figure PQ and SR are parallel sides of a trapezium PQRS and $\angle PSR = 90°$. Given that PQ = 15 cm, SR = 40 cm and the diagonal PR = 41 cm. Calculate the area of the trapezium.



7. If the parallel sides of a trapezium are 24 m and 20 m long. The distance between these sides is 6 m. Find the area of the trapezium.
8. Find the area of the trapezium MANY shown in the given figure.



9. The area of a trapezium is 960 cm^2. If the parallel sides are 34 cm and 46 cm long. Calculate the distance between them.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

HOTS (Higher Order Thinking Skills)

Experiential Learning

1. The area of trapezium is 78 cm^2 and its height is 13 cm. If one of the parallel sides is longer than the other by 2 cm, find the lengths of the two parallel sides.
2. The distance between parallel sides of a trapezium is 3 cm, one of its parallel sides is 8 cm and its area is 37.5 cm^2. Find the length of the other parallel side.
3. Find the area of a trapezium ABCD in which AB || DC, AB = 78 cm, CD = 52 cm, AD = 28 cm and BC = 30 cm.

Area of General Quadrilateral

A general quadrilateral can be split into triangles by drawing one of its diagonals. This "triangulation" helps us to find a formula for any general quadrilateral. Observe the figure:

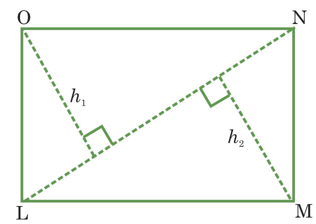
Area of quadrilateral LMNO

$$= (\text{area of } \Delta LON) + (\text{area of } \Delta LMN)$$

$$= (\frac{1}{2} LN \times h_1) + (\frac{1}{2} LN \times h_2)$$

$$= \frac{1}{2} \times LN \times (h_1 + h_2)$$

$$= \frac{1}{2} \times d\,(h_1 + h_2)$$, where d denotes the length of diagonal LN.



Area of Special Quadrilateral–Rhombus

We can use the method of splitting into triangles (which we called "triangulation") to find the formula for the area of a rhombus. Look at Fig. 15.10. It represents a rhombus. Therefore, its diagonals are perpendicular bisectors of each other.

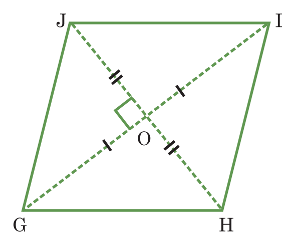
Area of rhombus GHIJ $= (\text{area of } \Delta GJI) + (\text{area of } \Delta GHI)$

$$= (\frac{1}{2} \times GI \times JO) + (\frac{1}{2} \times GI \times OH)$$

$$= \frac{1}{2} \times GI \times (JO + OH)$$

$$= \frac{1}{2} \times GI \times JH$$

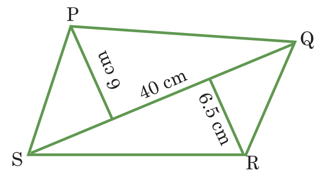
$$= \frac{1}{2} \times d_1 \times d_2$$, where $GI = d_1$ and $JH = d_2$



Thus, the area of a rhombus is half the product of its diagonals.

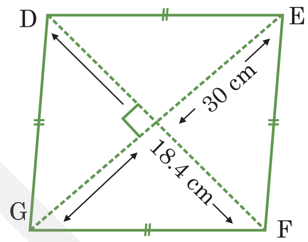
Example 4: **Find the area of the quadrilateral, if the length of the diagonal of a quadrilateral is 40 cm and the perpendiculars drawn to it from the opposite vertices are 6 cm and 6.5 cm.**

Solution: Area of quadrilateral



$$= \text{Area of } \Delta PQS + \text{Area of } \Delta QRS$$
$$= \left(\frac{1}{2}\times 40\times 6+\frac{1}{2}\times 40\times 6.5\right)\text{ cm}^2$$
$$= (120+130)\text{ cm}^2 = 250\text{ cm}^2.$$

Example 5: **Find the area of a rhombus whose diagonals are of lengths 18.4 cm and 30 cm.**



Solution: Area of rhombus $= \frac{1}{2}\times d_1 d_2$,

where d_1 and d_2 are lengths of diagonals.

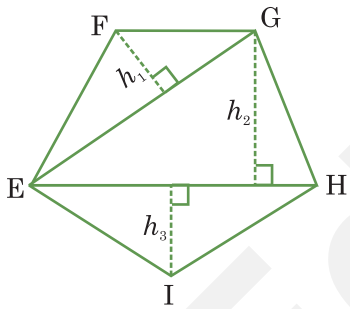
$$= \left(\frac{1}{2}\times 18.4\times 30\right)\text{ cm}^2$$
$$= 276\text{ cm}^2.$$

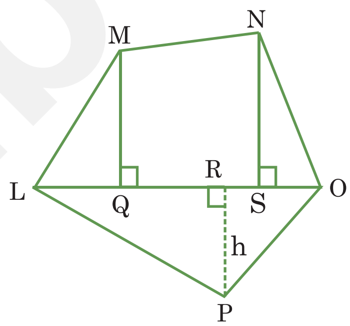
Area of a Polygon

A polygon is made up of four or more straight line segments. To find the area of a polygon, we split it into triangle and rectangle or trapezium.

Take a Task

Watch Remedial

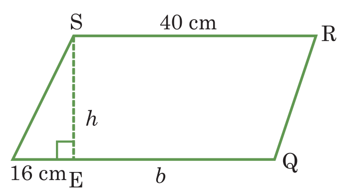




By constructing two diagonals EG and EH, the pentagon EFGHI is divided into three parts. So, area of EFGHI = area of ΔEFG + area of ΔEGH + area of ΔEIH.

By constructing one diagonal LO and two perpendiculars MQ and NS on it, pentagon LMNOP is divided into four parts.

So, area of LMNOP = area of right-angled ΔLQM + area of trapezium MQSN + area of right-angled ΔNSO + area of ΔLPO. (Identify the parallel sides of trapezium MQSN.)



Example 6: **In the given figure, PQRS is a parallelogram in which SR = 40 cm, SE ⊥ PQ and PE = 16 cm. If the area of the parallelogram is 480 cm², find SE and QR.**

Solution: Here, PQ = SR = 40 cm

Also, area of parallelogram = Base × height

$$480 = PQ\times SE$$
$$\Rightarrow \quad 480 = 40\times SE$$
$$\Rightarrow \quad SE = \frac{480}{40} = 12\text{ cm}$$

Again in right-angled DPES,

$$PS^2 = PE^2 + SE^2 \quad \text{(by Pythagorus theorem)}$$

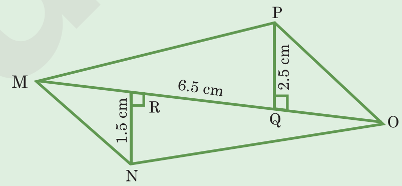
$$\Rightarrow \quad PS^2 = 16^2 + 12^2 \quad \Rightarrow \quad PS^2 = 256 + 144$$

$$\Rightarrow \quad PS^2 = 400 \quad \Rightarrow \quad PS = \sqrt{400} = 20 \text{ cm}$$

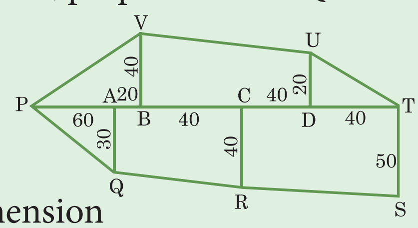
Hence, PS = QR = 20 cm.

Exercise 9.2

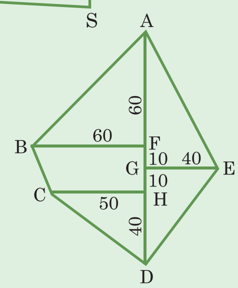
1. Find the area of rhombus whose diagonals are of lengths 14 cm and 10 cm.
2. The diagonal of a quadrilateral is 28.4 cm long. The perpendicular distances of the diagonal from opposite vertices are of lengths 7.20 cm and 8.80 cm. Find the area of the quadrilateral.
3. Find the area of a field which is in the form of a quadrilateral with one diagonal of length 60 cm and length of the perpendiculars drawn from the opposite vertices on this diagonal are 16 cm and 14 cm respectively.
4. Find the area of field if the diagonal of a quadrilateral shaped field is 24 m and the perpendiculars drawn on it from the remaining opposite vertices are 13 m and 8 m.
5. Find the area of quadrilateral MNOP shown in the figure.



6. Find the area of the quadrilateral PQRS in which PR = 36 cm, perpendiculars QE and SF from Q and S to PR are 16 cm and 18 cm respectively.



7. Find the area of the polygon shown in the figure. (All dimension are in cm.)
8. Find the area of the field as shown in the figure. (All dimension in metres.)



9. How many cubic metres of earth must be dug out to sink a well which is 30 m deep and has a diameter of 8 metres ?
10. How much area of metal sheet is required to make an open cylindrical tank of radius 2.8 m and height 120 m ?
11. 40 circular discs each of radius 7 cm and thickness 0.5 cm are placed one above the other to form a solid cylinder. Find the volume of the cylinder so formed.
12. A cylindrical tube open at one end is made of thick iron which is 3 cm. If the external radius is 21 cm and the length of the tube is 140 cm. Find the capacity of tube.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Volume and Surface Area of Solids

We have studied about the concepts of perimeter and area of some two-dimensional figures like rectangle, parallelogram and square in the previous chapter. In this chapter, we will study about the surface area and volume of some solid figures like cube, cuboid and cylinder. Such figures are called *three-dimensional* figures.

Volume of Cuboid and Cube

The space occupied by a solid is called volume. The volume is indicated by the symbol 'V'.

Standard Units of Volume

The space occupied by a unit cube, each of whose edges is 1 cm long, is 1 *cubic centimetre*. It is written as 1 cu cm or 1 cm^3. *Cubic centimetre is the standard unit of volume.*

Look at some other standard units of volume and their relations:

$$1 \text{ cm} = 10 \text{ mm} \Rightarrow 1 \text{ cm}^3 = 1{,}000 \text{ mm}^3$$

$$1 \text{ m} = 100 \text{ cm} \Rightarrow 1 \text{ m}^3 = 10^6 \text{ cm}^3$$

$$1 \text{ dm} = 10 \text{ cm} \Rightarrow 1 \text{ dm}^3 = 1{,}000 \text{ cm}^3$$

$$1 \text{ m} = 10 \text{ dm} \Rightarrow 1 \text{ m}^3 = 1{,}000 \text{ dm}^3$$

$$1 \text{ km} = 1{,}000 \text{ m} \Rightarrow 1 \text{ km}^3 = 10^9 \text{ m}^3$$

Take a Task

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Capacity, *i.e.*, volume of a vessel is expressed in *litres*.

$$1 \text{ cm}^3 = 1 \text{ m}l$$

$$1{,}000 \text{ cm}^3 = 1{,}000 \text{ m}l = 1 \text{ litre}$$

$$1 \text{ m}^3 = 1000 \text{ litres} = 1 \text{ kilolitre.}$$

The volume V of a cuboid of length l, breadth b and height h is given as

$$V = (l \times b \times h) \text{ cubic units.}$$

From the above formula, we observe that:

(*i*) $l = \dfrac{V}{b \times h}$ (*ii*) $b = \dfrac{V}{l \times h}$ (*iii*) $h = \dfrac{V}{l \times b}$

where V = volume, l = length, b = breadth and h = height.

where V = volume, l = length, b = breadth and h = height.

In case of a cube, $l = b = h$.

Therefore, volume of a cube

$$V = (l \times l \times l) \text{ cubic units}$$

or, $$V = l^3 \text{ cubic units.}$$

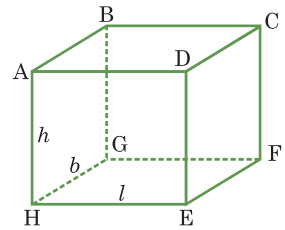
As a cuboid has six faces and each face has some area. We can define surface area as: *The surface* *area of a cuboid is the sum of the areas of all its faces.*

Let us consider a cuboid whose length, breadth and height are l, b and h respectively (Fig.).

(i) Area of EFGH = Area of ABCD = length × breadth

(ii) Area of EFCD = Area of ABGH = breadth × height

(iii) Area of ADHE = Area of BCFG = length × height



∴ By adding areas of six surfaces,

Surface area of a cuboid = 2 (length × breadth + breadth × height + length × height)

$$= \mathbf{2\,(l \times b + b \times h + l \times h)}$$

Since measure of each edge of a cube is same.

∴ The surface area of a cube = $\mathbf{6\,(side)^2 = 6\,l^2}$.

Lateral surface area of a cuboid = Area of (EFCD + ABGH + ADHE + BCFG)

= 2 area of (ADHE + EFCD)

= 2 (length × height + breadth × height)

= 2 (length + breadth) × height = $\mathbf{2\,(l + b) \times h}$

Lateral surface area of a cube = 2 (side + side) × side

= $2\,(l + l) \times l$ = $\mathbf{4\,l^2 = 4\,(side)^2}$.

Take a Task

Watch Remedial

REMEMBER

(i) To obtain the surface area of a cuboid, its length, breadth and height must be expressed in the same unit.

(ii) We can also express surface area as total surface area or whole surface area.

Area of four walls means the area of four sides of the cuboid (not the area of the base and the top.)

Area of four sides of a cuboid or four walls of a room can be written as

$= (l \times h + l \times h) + (b \times h + b \times h)$

$= 2\,(l \times h) + 2(b \times h) \quad = \quad 2\,(lh + bh) \quad = 2\,(l + b) \times h.$

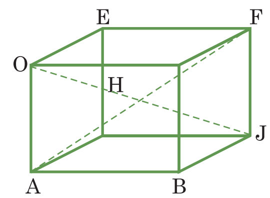
= (sum of four walls of the room) × height

= (perimeter of the room) × height

If the walls of the room has doors and windows, the area of doors and window is to be subtracted from the area of four walls of the room to obtain the exact area of four walls.

Diagonal of Cuboid and Cube

If l, b and h be the length, breadth and height of a cuboid, then



$$\textbf{diagonal} = \sqrt{l^2 + b^2 + h^2} \textbf{ units.}$$

In case of a cube $l = b = h$.

$\therefore$ Diagonal of the cube $= \sqrt{l^2 + l^2 + l^2} = \sqrt{3l^2} = \sqrt{3}\, l$ **units.**

Example 7 : How many bricks will be required for a wall of dimensions 9 m × 5 m × 0.8 m, if each brick measures 12 cm × 15 cm × 10 cm?

Solution : Here, length of the wall $= 9 \text{ m} = 900 \text{ cm}$

breadth of the wall $= 5 \text{ m} = 500 \text{ cm}$

height of the wall $= 0.8 \text{ m} = 80 \text{ cm}$

Volume of the wall $= l \times b \times h = (900 \times 500 \times 80) \text{ cm}^3$

Volume of the bricks $= (12 \times 15 \times 10) \text{ cm}^3$

Number of the bricks that can be placed in the cuboid

$$= \frac{900 \times 500 \times 80}{12 \times 15 \times 10} = 20{,}000$$

Hence, the required bricks are 20,000.

Example 8 : Find the surface area of a cube whose volume is 1,331 cm³.

Solution : Let the length of each edge of the cube be a cm.

Then, its volume $= (a^3) \text{ cm}^3$

According to the question, $a^3 = 1{,}331 \text{ cm}^3$

or, $a^3 = 11^3 \text{ cm}^3$

$\therefore$ $a = 11 \text{ cm}$

Total surface area of the cube $= 6a^2 \text{ cm}^2$

$= 6 \times 11 \times 11 \text{ cm}^2 = 726 \text{ cm}^2$.

Example 9 : A rectangular field is 70 m long and 50 m broad. In one corner of the field, a pit which is 10 m long, 7.5 m broad and 8 m deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of field.

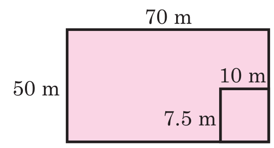


Fig.

Solution : Area of the field $= (70 \times 50) \text{ m}^2 = 3{,}500 \text{ m}^2$

Area of the pit $= (10 \times 7.5) \text{ m}^2 = 75 \text{ m}^2$

Area over which the earth is spread out

$= (3500 - 75) \text{ m}^2 = 3{,}425 \text{ m}^2$

$$\text{Volume of earth dug out} = (10 \times 7.5 \times 8)\ m^3 = 600\ m^3$$

$$\text{Rise in level} = \frac{\text{Volume of earth dug out}}{\text{Area over which the earth is spread out}}$$

$$= \left(\frac{600}{3425}\right) m = \left(\frac{600 \times 100}{3425}\right) cm = 17.5\ cm.$$

Example 10 : The dimension of a room – 10 m long, 10 m wide and 5 m height is to be measured with a long rope. Find the length of the rope.

Solution : Here, $l = 10$ m, $b = 10$ m and $h = 5$ m.

Length of the longest rope = length of the diagonal

$$= \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{10^2 + 10^2 + 5^2}$$

$$= \sqrt{100 + 100 + 25} = \sqrt{225} = 15\ m.$$

Example 11 : Whose diagonal is larger – a cube of edge 12 cm or a cuboid of dimensions 11 cm × 9 cm × 8 cm ?

Solution : First, we find the diagonals.

For the cube, $l = 12$ cm

$\therefore$ Diagonal of the cube $= \sqrt{3}\ l$

$$= \sqrt{3} \times 12\ cm$$

$$= 1.73 \times 12\ cm = 20.76\ cm.$$

For cuboid, $l = 11$ cm, $b = 9$ cm and $h = 8$ cm

$\therefore$ Diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$

$$= \sqrt{11^2 + 9^2 + 8^2}\ cm$$

$$= \sqrt{121 + 81 + 64}\ cm$$

$$= \sqrt{266}\ cm = 16.31\ cm.$$

Clearly, $20.76 > 16.31$.

Hence, cube's diagonal is larger.

Example 12 : Find the surface area of the cube whose side is 7 cm.

Solution: Here, side $(l) = 7$ cm.

$\therefore$ Surface area of the cube $= 6 \times (l)^2$

$$= 6 \times (7)^2\ cm^2$$

$$= 6 \times 49\ cm^2 = 294\ cm^2.$$

Example 13 : A hall is 22 m in length 18 m in breadth and 5 m in height. Find the cost of cementing its floor and walls at the rate of ₹ 25 per square.

Solution: We have,

l = length of the hall = 22 m

b = breadth of the hall = 18 m

h = height of the hall = 5 m

Therefore, area of the four walls of the hall

$= 2h(l+b) = 2 \times 5(22+18) = 10 \times 40 = 400 \text{ m}^2$

Area of the floor of the hall $= l \times b = 22 \times 18 \text{ m}^2 = 396 \text{ m}^2$

Therefore, total area to be cemented $= (400 + 396) \text{ m}^2 = 796 \text{ m}^2$

Let the cost of cementing of 1 square metre surface = ₹ 25

Cost of cementing the floor and the walls = ₹ (25×796) = ₹ 19,900

Therefore, the cost of cementing the floor and the walls = ₹ 19,900.

Exercise 9.3

1. Find the volume and total surface area of the cuboids whose dimensions are:
   (*i*) length = 10 cm, breadth = 6 cm, height = 3 cm
   (*ii*) length = 12 cm, breadth = 4 cm, height = 2 cm
   (*iii*) length = 11 cm, breadth = 7 cm, height = 10 cm
   (*iv*) length = 9 cm, breadth = 7.5 cm, height = 50 cm
2. Find the volume and total surface area of the cubes whose edges are:
   (*i*) 10 cm (*ii*) 4.5 dm (*iii*) 3.6 cm (*iv*) 8 cm
3. Whose diagonal is larger – a cuboid of dimensions 9 cm × 3 cm × 8 cm or a cube of edge 11 cm ?
4. Find the surface area of a cuboidal box whose length, breadth and height are 12.5 cm, 4 cm and 8.2 cm respectively. Find its lateral surface area also.
5. The edges of a cuboid are in the ratio 1 : 2 : 3 and its surface area is 88 cm^2. Find the volume of the cuboid.
6. If the surface area of cube is 1,176 cm^2. Find the volume of cube.
7. If the volume of a cube is 4,913 cm^3. Find it surface area.
8. Total surface area of a cube is 600 sq. cm. Find the length of its each edge.
9. Find the diagonal of the cube whose each edge is 12 cm.
10. A cuboidal solid wooden box contains 72 cm^3 of wood. If the length and breadth are 9 cm and 4 cm respectively. Find its height.
11. The length, breadth and height of a room is 6 m, 4.5 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of ₹ 8 per sq. m.

12. A box is 150 cm long and 25 cm wide. If its height is 6 cm. Find the area of four walls of the box.
13. A water tank is 25 m long, 20 m wide and 10 m deep. How many litres of water does it contain?
14. A milk container in the form of a cuboid is 60 cm long and 24 cm wide. How high should it be made to hold 9 litres of milk?

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

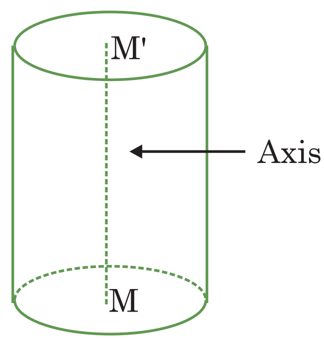
HOTS (Higher Order Thinking Skills)

Experiential Learning

1. A field is 78 m long and 27 m broad. A rectangular shaped pit of length 13 m. and breadth 12 m. is dug inside the field and the mud taken out is spread evenly over the remaining part of the field to a thickness of 25 cm. Find the depth of the pit.
2. The cost of white washing the total surface area of a cube at the rate of 15 paise per sq cm is ₹ 396.90. Find the volume of the cube.
3. Find the side of the new cube if three cubes of iron whose edges are 6 cm, 8 cm and 10 cm respectively are melted and formed into a single cube.

Right Circular Cylinder

In our every day life, we come across objects such as a drum, an electric tube, a circular pencil etc. These objects are right circular cylinders as they have a curved (lateral) surface with congruent plane ends. Each plane end is circular in shape, and the two plane ends are parallel. Each of the plane ends is called a base of the cylinder.



In the given figure, MM' is the axis of the cylinder.

The line segment which joins the centres of two bases is called the axis of the cylinder. The axis is perpendicular to the circular ends.

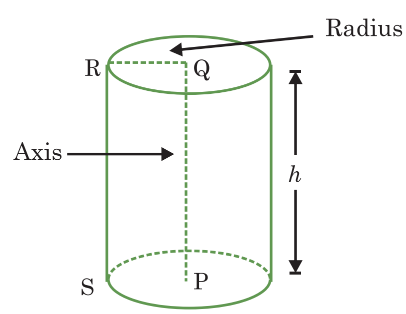
Volume of a Right Circular Cylinder

Let r be the radius and h be the height of the cylinder.

Volume of a cylinder = Area of the base × height

= $(\pi r^2 \times h)$ cubic units

= $(\pi r^2 h)$ cubic units.



Total Surface Area of a Right Circular Cylinder

Consider a right circular cylinder of radius r and height h as shown in figure.

Each of the bases is a circle of radius r.

Therefore, length of each circular edge is $2\pi r$.

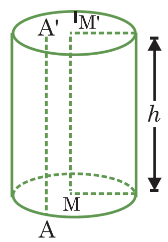
Cut figure. (i) along a line on the surface parallel to axis of the cylinder. We obtain a rectangle whose length is equal to the circumference of the base of the right circular cylinder and the breadth is equal to the height of the cylinder (figure. (ii))

Therefore, area of the curved surface of the right circular cylinder

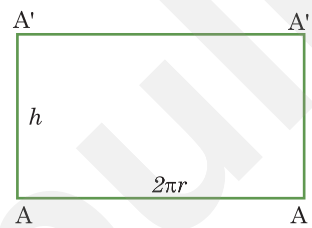
= area of the rectangle

= circumference of the base of the cylinder × height

$= 2\pi r \times h \qquad = 2\pi rh$ sq unit.



(i)



(ii)

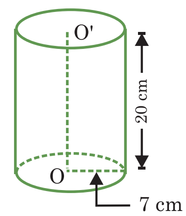
Area of the curved surface of a right circular cylinder = 2πrh

If we add the area of the two circular ends, we obtain the total surface area of the right circular cylinder.

Therefore, total surface area $= 2\pi rh + 2\pi r^2 \quad = 2\pi r(h + r)$ sq units

Total surface area of a right circular cylinder = $2\pi r(h + r)$ sq units.

Example 14 : Find the volume, area of the curved surface and the total surface area of right circular cylinder, whose height and radius of the base are 20 cm and 7 cm respectively.



Solution : Here, $r = 7$ cm, $h = 20$ cm

$$\text{Volume} = \pi r^2 h$$
$$= \frac{22}{7} \times 7 \times 7 \times 20\ \text{cm}^3 = 3{,}080\ \text{cm}^3$$

$$\text{Area of the curved surface} = 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 7 \times 20\ \text{cm}^2 = 880\ \text{cm}^2$$

Total surface area of the cylinder

= Curved surface area + area of two bases

$$= 880\ \text{cm}^2 + 2\pi r^2$$
$$= 880\ \text{cm}^2 + 2 \times \frac{22}{7} \times 7 \times 7\ \text{cm}^2$$
$$= (880 + 308)\ \text{cm}^2 = 1{,}188\ \text{cm}^2.$$

Example 15 : The radius and height of a cylinder are in the ratio 7 : 2. If the volume of the cylinder is 8,316 cm^3. Find the total surface area of the cylinder.

Solution : Let the radius and height of cylinder be $7x$ and $2x$.

Volume of the cylinder = 8,316 cm^3

$$V = \pi r^2 h$$

$$\Rightarrow \quad 8316 = \frac{22}{7} \times 7x \times 7x \times 2x$$

$$\Rightarrow \quad 8316 = 308x^3 \qquad \Rightarrow \quad x^3 = \frac{8316}{308}$$

$$\Rightarrow \quad x^3 = 27$$

$$\therefore \quad x = 3$$

So, $r = 7 \times 3 = 21$ cm and $h = 2 \times 3 = 6$ cm.

Putting the value of x in total surface area of cylinder, we have:

Total surface area = $2\pi r(h + r)$

$$= 2 \times \frac{22}{7} \times 21 \times (6 + 21) \qquad = 2 \times 22 \times 3 \times 27 \qquad = 3{,}564 \text{ sq units.}$$

Example 16 : The circumference of the base of a cylinder is 88 cm and its height is 30 cm. Find the volume of the cylinder and its lateral surface area.

Solution: Let the radius of the base of the cylinder be r cm.

We know that, circumference = $2\pi r$

$$\Rightarrow \quad 88 = 2 \times \frac{22}{7} \times r$$

$$\Rightarrow \quad r = \frac{88 \times 7}{22 \times 2} = \frac{4 \times 7}{2} = 2 \times 7 = 14 \text{ cm}$$

Now, r = 14 cm and h = 30 cm

$\therefore$ Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times 30 \text{ cm}^3 = 18{,}480 \text{ cm}^3$$

Lateral surface area of the cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 30 \text{ cm}^2 = 2{,}640 \text{ cm}^2.$$

Example 17 : How many full bags of wheat can be emptied into a circular drum of height 2 m and base radius 4.2 m. If the space required by each bag of wheat is 2.1 cu. m.?

Solution : Here, $r = 4.2$ m and $h = 2$ m.

Volume of the circular drum = $\pi r^2 h = \left(\frac{22}{7} \times 4.2 \times 4.2 \times 2\right) m^3$

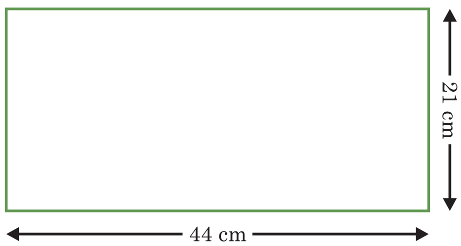
$$\therefore \quad \text{Number of bags} = \frac{\text{Volume of the drum}}{\text{Volume of a bag of wheat}}$$

$$= \frac{22}{7} \times \frac{4.2 \times 4.2 \times 2}{2.1}$$

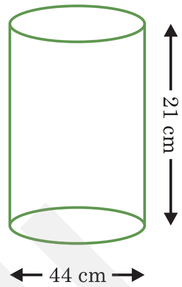
$$= 52.8 \cong 53 \text{ (full bags).}$$

Example 18 : A rectangular piece of paper is 44 cm long and 21 cm broad. It is rolled along its length to form a cylinder. Find the volume of the cylinder so formed.

Solution:



(a)



(b)

Height of the cylinder = 21 cm

Circumference of the cylinder = 44 cm

$$2\pi r = 44 \text{ cm}$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44 \text{ cm}$$

$$\Rightarrow \quad r = \frac{44 \times 7}{44} = 7 \text{ cm}$$

$$\therefore \text{ Volume (V)} = \pi r^2 h$$

$$= \frac{22}{7} \times (7)^2 \times 21$$

$$= \frac{22}{7} \times 7 \times 7 \times 21 = 3{,}234 \text{ cm}^3.$$

Example 19 : The diameters of two cylinders are in the ratio of 3 : 5. Find the ratio of their heights if their volumes is same.

Solution: Let r_1 and r_2 are radii of the bases of two cylinders respectively.

Ratio of their diameters $= \frac{d_1}{d_2} = \frac{2r_1}{2r_2} = \frac{r_1}{r_2} = \frac{3}{5}$.

Volume of two cylinders is same, say V.

Let h_1 and h_2 be their heights respectively,

According to question, $\pi r_1^2 h_1 = \pi r_2^2 h_2$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{h_2}{h_1} \quad \Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 = \frac{h_2}{h_1}$$

$$\Rightarrow \quad \left(\frac{3}{5}\right)^2 = \frac{h_2}{h_1} \quad \Rightarrow \quad \frac{h_1}{h_2} = \frac{9}{25}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{25}{9}$$

Hence, the ratio of their heights $h_1 : h_2 = 25 : 9$.

Exercise 9.4

1. Find the volume, lateral surface area and total surface area of each of the cylinders whose dimensions are:
   (*i*) radius of the base = 0.21 m, height = 7 cm
   (*ii*) radius of the base = 3.5 m, height = 12 m
   (*iii*) radius of the base = 21 cm, height = 4 cm
   (*iv*) radius of the base=10.5 dm, height=12 dm
2. A cylindrical water tank has inner radius 3.5 m and depth 21 m. Find the capacity of the tank.
3. If the capacity of a cylindrical tank is 1848 m^3 and the diameter of its base is 28 m. Find the depth of the tank.
4. A cylindrical tank open at the top has a base of radius 10.5 cm and height 14 cm. Find the cost of painting the inner part of the vessel at the rate of ₹ 10 per sq. m.
5. The diameters of two cylinders are in the ratio 3 : 4. Find the ratio of their heights, if their volume is same.
6. The radius and height of a cylinder are in the ratio 3 : 4 and its volume is 5,346 cm^3. Find its radius.
7. The circumference of the base of a cylinder is 66 cm and its height is 60 cm. Find the volume of the cylinder and its lateral surface area.
8. How many cubic metres of earth must be dug out to sink a well which is 30 m deep and has a diameter of 8 metres ?
9. How much area of metal sheet is required to make an open cylindrical tank of radius 2.8 m and height 120 m ?
10. 40 circular discs each of radius 7 cm and thickness 0.5 cm are placed one above the other to form a solid cylinder. Find the volume of the cylinder so formed.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Revision Exercise

Conceptual Learning

A. Tick (✓) the correct option:

(*i*) A cube has a volume of 27000 cm^2. Its edge length is :

(*a*) 30 cm (*b*) 60 cm (*c*) 54 cm (*d*) 90 cm

(*ii*) The volume of a cylinder whose radius of the base is 7 cm and height 20 cm is :

(*a*) 1540 cm^3 (*b*) 3080 cm^3 (*c*) 2560 cm^3 (*d*) 1920 cm^3

(*iii*) The maximum length of a pencil that can be kept in a rectangular box of length 12 cm, breadth 9 cm, and height 8 cm is :

(*a*) 13 cm (*b*) 17 cm (*c*) 18 cm (*d*) 19 cm

(*iv*) The volume of a cylinder whose diameter is equal to its height is :

(a) $\pi r^3 h$ (*b*) $\frac{\pi h^3}{8}$ (*c*) $\frac{\pi r^3}{8}$ (*d*) $2\pi r^3$

(*v*) The number of cubes of edge 10 cm that can be placed in a cube of edge 1 m is :

(*a*) 10 (*b*) 100 (*c*) 1000 (*d*) none

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Mental Maths

Experiential Learning

1. The diagonals of a rhombus are 18 cm and 12 cm. Find its area?
2. Two parallel sides of a trapezium are 9 cm and 7 cm, its area is 48 cm^2. Find the distance between the parallel sides.
3. Find the area of trapezium' if the parallel sides are of length 9 cm and 7 cm and height is 5 cm.
4. Find the area of a trapezium if the parallel sides are 25 cm and 13 cm and its non-parallel sides are equal to 10 cm.
5. The area of the rhombus is 216 cm^2 and one of it diagonals is 18 cm. Find the other diagonal.
6. If the side of a cube is '*a*' cm. Find its total surface.
7. If the volume of a cube is 512 cm^3. Find its total surface.

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Custom Learning Path

Assertion and Reason

Critical Thinking

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct.

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

1. **Assertion (A) :** A square is a special type of rhombus.
   Reason (R) : The volume of a water tank is measured in square units.
2. **Assertion (A) :** Area of a rhombus is half the product of its diagonals.
   Reason (R) : If each side of a cuboid is doubled, its volume becomes 8 times.
3. **Assertion (A) :** A cylinder has only one circular end.
   Reason (R) : The structures that have a definite shape are called solids.
4. **Assertion (A) :** A cuboid whose length, breadth and height are equal, is called a cube.
   Reason (R) : The curved surface which joins the two bases of a right circular cylinder is called its total surface.
5. **Assertion (A) :** If the radius and height of a cylinder are respectively *r* and h, then its total surface area is 2prh.
   Reason (R) : The space or region occupied by a body is called its volume.

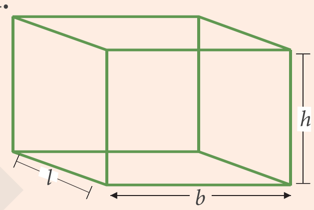
Activity

Surface Area and Volume of Cube and Cuboid

- Divide the class into equal groups. Call one group at front side. Ask two students to pick up cube and cuboid respectively.
- Now teacher explain that the amount of space occupied by a solid is called its volume and the area of the surface forming is known as the surface area.
- Now ask the student to show the length, breadth and height of cuboid.
- Now student learn the formula of surface area and volume of cube, cuboid.
- Volume of cuboid = length × breadth × height
- Volume of cube = edge × edge × edge
- $V = 1^3$
- Surface area of cuboid means there are 6 faces of cuboid and each face has rectangular shape.

  So, total surface area of cuboid = 2 (lb+ bh + hl).

  Total surface area of a cube = $6l^{2}$.



Skills covered: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Thinking Skills

1. A village having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For, how many days, the water of the tank will be sufficient for the village?
2. A cubic tank with 4.6 m edges is filled with water. How much water will be left in the tank if some is drained off to fill a cylindrical tank with a radius of 2.2 m and a height of 4.6 m?
3. A birthday gift is 55 cm long, 40 cm wide and 5 cm high. The sheet of paper you want to use to wrap it measures 75 cm by 100 cm. Is the paper large enough to wrap the gift? Explain.
4. A rectangular piece of paper of width 30 cm and length 77 cm is rolled along its width to form a cylinder. Find the volume and total surface area of the cylinder so formed.
5. A cylindrical water tank has a radius of 7 meters and a height of 10 meters. A smaller cylindrical tank, with a radius of 3 meters and the same height, is placed inside the larger tank, completely submerged in the water. The smaller tank is filled with water, and the water is transferred to the larger tank. What is the volume of the water in the smaller tank?

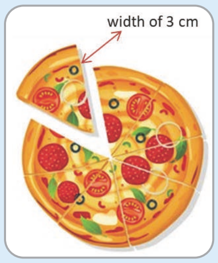
Skills covered: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Competency based Questions

1. A rectangular park has a length of 80 meters and a width of 60 meters. A jogging track of uniform width is constructed along the boundary of the park. If the area of the jogging track is 2400 m^2 (Shown in the given picture), what is the width of the jogging track?

(a) 5 m (b) 6 m (c) 4 m (d) 7 m

2. A circular pizza has a diameter of 30 cm. If the crust of the pizza forms a ring with a uniform width of 3 cm (Shown in the given picture), what is the area of the crust? (Use $\pi = 3.14$)

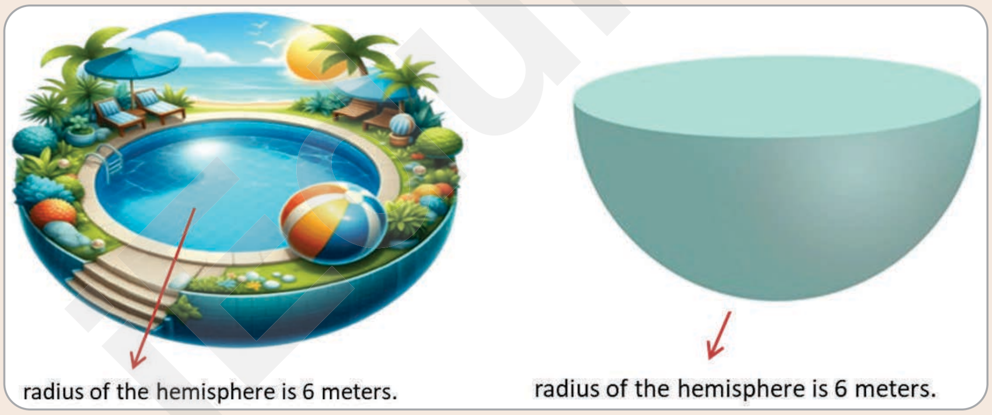


(a) 174.26 cm^2 (b) 178.46 cm^2
(c) 185.04 cm^2 (d) 254.34 cm^2

Skills covered: Interpersonal skills, Observation, Application and Decision making skills

Case Study

A construction company is planning to build a circular swimming pool in a park. The pool is to be shaped like a hemisphere (Shown in the given picture). The radius of the hemisphere is 6 meters.



Based on this information answer the following questions:

a) Calculate the volume of water the pool can hold (in cubic meters).

b) The pool will be surrounded by a circular deck of uniform width. If the total diameter of the deck is 16 meters, find the width of the deck.

c) If the deck is made of tiles, and each tile covers an area of 1 square meter, how many tiles will be required to cover the deck?

Skills covered: Research, Logical Reasoning, Problem-Solving, Practical Application