Chapter 8

Algebraic Expressions and Identities

We'll cover the following key points:

- → Algebraic expression
- → Algebraic expressions on the number line
- → Types of algebraic expressions
- → Addition of algebraic expressions
- → Subtraction of algebraic expressions
- → Multiplication of algebraic expressions
- → Multiplication of two monomials
- → Multiplication of three or more monomials
- → Multiplication of a monomial by a polynomial
- → Some special products (special identities)
- → Division of algebraic expressions

Hi, I'm EeeBee

Still curious? Talk to me by scanning the QR code.

Do you Remember fundamental concept in previous class.

In class 7^{th} we learnt

- → Like and unlike terms
- → Monomials, Binomials, Trinomials and Polynomial

Learning Outcomes

By the end of this chapter, students will be able to:

- Define and identify terms, factors, and coefficients in an algebraic expression.
- Classify algebraic expressions as monomials, binomials, trinomials, or polynomials.
- Perform addition, subtraction, and multiplication of algebraic expressions.
- Apply the distributive property to simplify algebraic expressions.
- Factorize algebraic expressions using common factors and grouping methods.
- Derive and verify standard identities such as: =(a+b)(a−b).
  - $(a + b)^2 = a^2 + 2ab + b^2$
  - $(a - b)^2 = a^2 - 2ab + b^2$
  - $a^2 - b^2 = (a + b)(a - b)$
- Use algebraic identities to simplify and solve expressions.
- Solve real-life problems by modeling situations using algebraic expressions and identities.
- Develop logical reasoning and analytical skills through manipulation of algebraic expressions.

Mind Map

ALGEBRIC EXPRESSION AND IDENTITIES

Addition and subtraction of algebraic expressions

e.g.,

i. $(3x + 4y) + (5x + 3y)$

$= (3x + 5x) + (4y + 3y)$

$= 8x + 7y$

ii. $(4x + 5xy) - (y + 2xy)$

$= (4x) + (5xy - 2xy) - y$

$= 4x + 3xy - y$

Multiplication of algebraic expression

Multiplying a monomial by a monomial

e.g.,

i. $4 \times x = 4x$

ii. $3x \times x = 3 \times x^2 = 3x^2$

iii. $-5 \times -2 \times 7x = (-5 \times -2 \times 7) \times x = 70x$

iv. $x^2 \times 3x = 3 \times x^2 \times x = 3x^3$

Multiplying a monomial by a polynomial

e.g.,

i. $3 \times (2x + 5)$

$= 3 \times 2x + 3 \times 5$

$= 6x + 15$

ii. $2x \times (5 + 4x - 3y)$

$= 2x \times 5 + 2x \times 4x + 2x \times -3y$

$= 10x + 8x^2 - 6xy$

Multiplying polynomial by polynomial

e.g.,

i. $(2x + 5) \times (3x + 7)$

$= 2x \times 3x + 2x \times 7 + 5 \times 3x + 5 \times 7$

$= 6x^2 + 14x + 15x + 35$

$= 6x^2 + 29x + 35$

ii. $(5x - 3) \times (2 + 3x - y)$

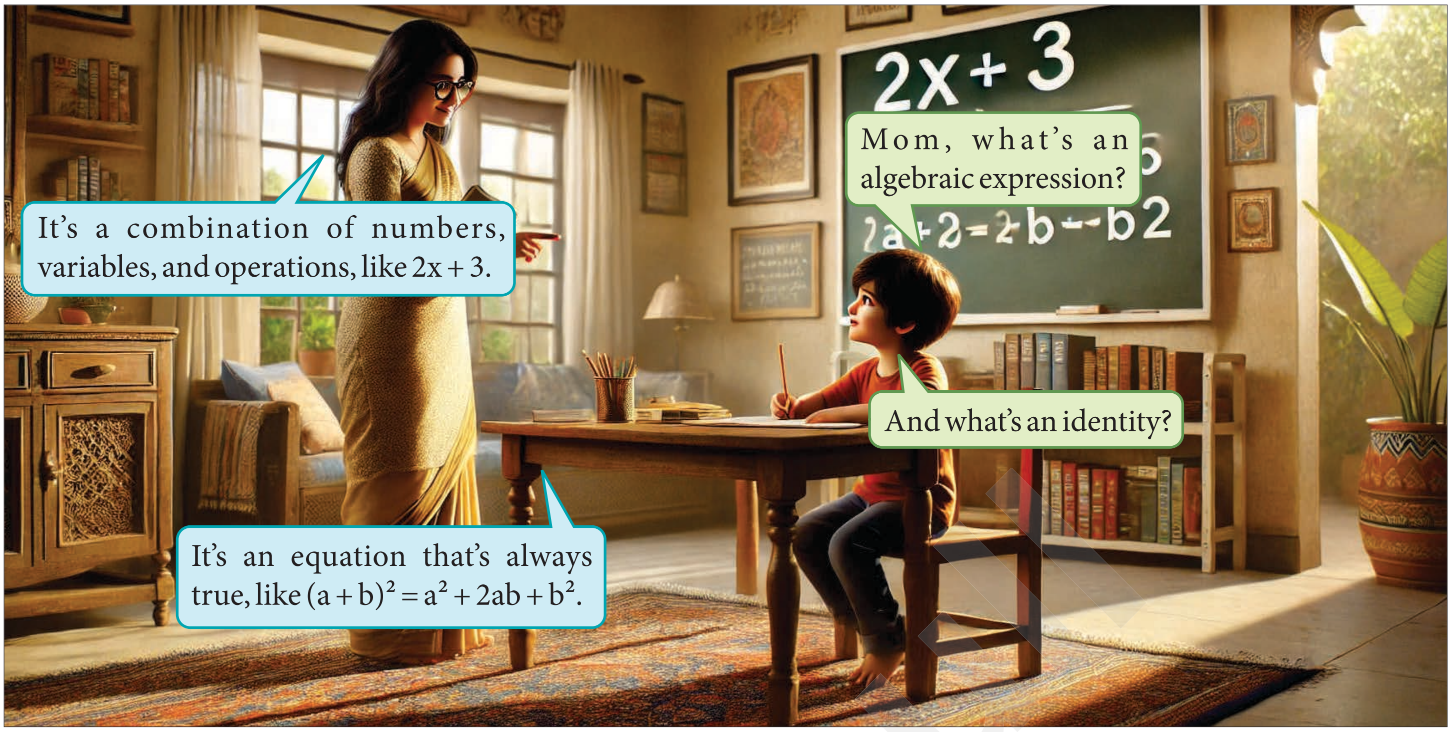
$= 5x \times 2 + 5x \times 3x + 5x \times -y - 3 \times 2 - 3 \times 3x - 3 \times -y$

$= 10x + 15x^2 - 5xy - 6 - 9x + 3y$

$= 15x^2 + x - 5xy + 3y - 6$

Based on NCERT*

Introduction



Algebraic Expression

A combination of constants and variables connected by the signs of fundamental operations – addition, subtraction, multiplication and division is called an **algebraic expression**.

For example, $x + 5xy - 5$ is an algebraic expression.

Terms of an Algebraic Expression

Various parts of an algebraic expression which are separated by the signs '+' or '–' are called the **terms** of the algebraic expression.

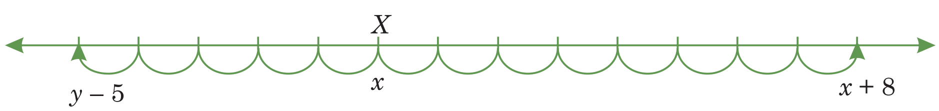
For example, (*i*) $x^2 + 5x - 7$ is an algebraic expression consisting of three terms i.e., x^2, $5x$ and -7.

(*ii*) This expression, $\frac{3a^3}{5} + 6a^2 + 7a + 5$ is an algebraic expression consisting of four terms, i.e., $\frac{3a^3}{5}$, $6a^2$, $7a$ and 5.

Algebraic Expressions on the Number Line

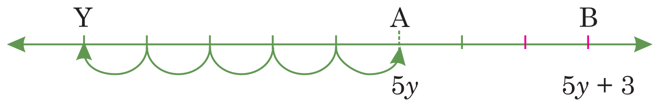
An algebraic expression does not have any particular position on the number line. But, its variable is assigned a particular place on the number line, the position relative to the variable is defined on the number line.

For example, if the variable x of the expression $x + 8$ is represented by the point X on the number line as shown below, then the expression $x + 8$ is at a distance of 8 units to the right of X.



Similarly, $y - 5$ is at a distance of 5 units to the left of X.

If variable y is represented by Y as shown below, then $5y$ is represented by A, which is at a distance equal to 5 times the distance of y from Y. The expression $5y + 3$ is represented by B, which is at a distance of 3 units to the right of $5y$.



Like Terms and Unlike Terms

1. **Like Terms** : The terms having same literal factors are called like terms.
   Example : $5x, 6x; 3xy, 5xy; 8x^2y; 9x^2y$ are like terms.
2. **Unlike Terms** : The terms having different literal factors are called unlike terms.
   Example : $3x, 3y; 4xy$ and $4xy^2$ are unlike terms

Types of Algebraic Expressions

An algebraic expression can have one or more than one term. Based upon the number of terms in an

Algebraic Expressions	Examples
Monomial : *An algebraic expression having only one term is called a* ***monomial****.*	$2x, -5b, 4m, 2m^2$
Binomial : *An algebraic expression having two terms is called a* ***binomial****.*	$3x + 6, -3y - 8, 4m - 5n, 6x^2 - 2y$
Trinomial : *An algebraic expression having three terms is called a* ***trinomial****.*	$7xy + 3x + 4, 1 + 2m - 3n, a^2 + ab + b^2$
Quadrinomial : *An algebraic expression having four terms is known as a* ***quadrinomial****.*	$2xyz + 7xy - 2yz - 6$
Polynomial : *An algebraic expression having several terms is called a* ***polynomial****.*	*binomials, trinomials and quadrinomals are also polynomials.*

Note

$5 + 3x - 6$ is not a trinomial, because $5 + 3x - 6 = 3x - 1$, which is basically a binomial.

Constant Polynomials : A polynomial which have no literal factor is called a constant polynomial, *e.g.*, 6, 4, –3, 2.

Degree of constant polynomial is always 0.

Factors : When two or more numbers or variables are multiplied then each one of them is called a **factor** of the product.

A constant factor is called a *numerical factor* while a variable factor is known as a *literal factor*.

Examples : (*i*) In $6x^2y$, we have 6 as the numerical factor whereas x^2 and y are the literal factors.

(*ii*) In $-2ab$, the numerical factor is -2 while a and b are literal factors.

Coefficients : *In a product of numbers and variables, any of the factors is called the* ***coefficient*** *of the product of the other factors.*

Examples : (*i*) In $5xy$, the coefficient of y is $5x$ and the coefficient of x is $5y$.

(*ii*) In $-b$, the coefficient of b is -1.

(*iii*) In $2x^2y^3z^2$, the coefficient of x^2 is $2y^3z^2$, the coefficient of y^3 is $2x^2z^2$ and the coefficient of z^2 is $2x^2y^3$.

Degree of an Algebraic Expression or Polynomial

An algebraic expression or polynomial which contains only one variable say x, is known as an algebraic expression in one variable.

$4x^2 + 8x + 6$ is an algebraic expression in variable x.

$3p^2 + 5p - 8$ is an algebraic expression in variable p.

The highest exponent of the variable in an algebraic expression is called the *degree of the algebraic expression*.

The degree of $3a^2 + 6$ is 2. The degree of $5x^3 - 2x + 6$ is 3. The degree of $7p^4 - 3p^2 - 1$ is 4.

If an algebraic expression contains more than one variable say x and y, then it is called an algebraic expression in two variables. Its degree is the highest exponent (power) of the term obtained by adding the exponents of the variables. For example, $4xy - 5x^2y + 2y$ is an algebraic expression in x and y. Its degree is 3 which is the highest of all the terms.

Note

A constant like 6 or -10 is known as an algebraic expression of degree 0, because we can think of a number like 6 as $6x^0$.

Addition of Algebraic Expressions

An algebraic expression consists of like and unlike terms. While adding algebraic expressions, we collect and add all the like terms. The sum of several like terms is the like term whose coefficient is the sum of the coefficients of these like terms. We cannot add the coefficients of two unlike terms.

Example 1: **Add $2x^2+5xy+y^2$, $3x^2-2xy-3y^2$ and $3xy+5y^2$.**

Solution : $(2x^2+5xy+y^2)+(3x^2-2xy-3y^2)+3xy+5y^2$

$= (2x^2+3x^2)+(5xy-2xy+3xy)+(y^2-3y^2+5y^2)$

$= (2+3)\,x^2+(5-2+3)\,xy+(1-3+5)\,y^2 = 5x^2+6xy+3y^2.$

Take a Task

Watch Remedial

Example 2: **Add $4x^3+8xy+y^3$, $2y^3-12xy-4x^3$ and $5x^3+2xy-9y^3+5$.**

Solution: **Horizontal Method:**

$(4x^3 + 8xy + y^3) + (2y^3 - 12xy - 4x^3) + (5x^3 + 2xy - 9y^3 + 5)$

$= (4x^3 - 4x^3 + 5x^3) + (y^3 + 2y^3 - 9y^3) + (8xy - 12xy + 2xy) + 5$

$= (4 - 4 + 5)x^3 + (1 + 2 - 9)y^3 + (8 - 12 + 2)xy + 5$

$= (4 + 5 - 4)x^3 + (3 - 9)y^3 + (8 + 2 - 12)xy + 5$

$= (9 - 4)x^3 + (-6)y^3 + (10 - 12)xy + 5 = 5x^3 - 6y^3 - 2xy + 5.$

Column method:

$$\begin{array}{rrrrr}
 & 4x^3 & +8xy & +y^3 & \\
- & 4x^3 & -12xy & +2y^3 & \\
+ & 5x^3 & +2xy & -9y^3 & +5 \\
\hline
 & 5x^3 & -2xy & -6y^3 & +5 \\
\hline
\end{array}$$

Subtraction of Algebraic Expressions

In order to subtract an algebraic expression from another, we change the signs (from '+' to '–' or from '–' to '+') of all the terms of the expression which is to be subtracted and then we add two expressions.

While subtracting two expressions by column method, we indicate the change of sign of every term in the expression to be subtracted below the original sign of each term.

Observe the following examples.

Example 3: **Subtract $3x-y+5z$ from $8x-3y+6z$.**

Solution: In column method, we arrange the expressions so that the like terms are one below the other and change the sign of each term to be subtracted as shown below.

Column Method :

$$\begin{array}{ccccc}
8x & - & 3y & + & 6z \\
3x & - & y & + & 5z \\
- & + & & - & \\
\hline
5x & - & 2y & + & z \\
\hline
\end{array}$$

Horizontal Method :

$8x-3y+6z-(3x-y+5z)$

$= 8x-3y+6z-3x+y-5z$

$= (8x-3x)+(-3y+y)+(6z-5z)$

$= 5x-2y+z.$

Example 4: **Subtract $3x - 2y + 6z$ from $7x - y + 8z$.**

Solution: Firstly, arrange expressions so that the like terms are one below the other. Then, we change the sign of each term to be subtracted as shown below:

$$\begin{array}{l} 7x - y + 8z \\ 3x - 3y + 6z \\ - \;\; + \quad\; - \\ \hline 4x + 2y + 2z \\ \hline \end{array}$$

Example 5: **Subtract $8a^3 + 3a^2b + 5ab$ from $6a^2b - 6a^3 + 12ab - b^3$.**

Solution: $(6a^2b - 6a^3 + 12ab - b^3) - (8a^3 + 3a^2b + 5ab)$

On opening the second brackets, the terms inside the brackets change sign.

$= 6a^2b - 6a^3 + 12ab - b^3 - 8a^3 - 3a^2b - 5ab$

$= (6a^2b - 3a^2b) + (-6a^3 - 8a^3) + (12ab - 5ab) - b^3$

$= 3a^2b - 14a^3 + 7ab - b^3.$

Exercise 8.1

1. Identify the terms, their numerical coefficients for each of the following expressions:

   (i) $7 + x - x^2$ (ii) $2m^2n^2 - 4m^2n^2p^2$ (iii) $\frac{a}{3} + \frac{b}{3} - ab$ (iv) $0.2ab - 0.4a + 0.7b$

2. Classify the following polynomials as monomials, binomials and trinomials. Which polynomials do not fit in any of these categories?

   (i) $x + y$ (ii) $2a^2 + a^3 - 7$ (iii) $7mn + 13np - pq + 7q$

   (iv) $-9x^2$ (v) $8y^3 - 6 + 3y$ (vi) $x^2y - y^2x$

3. Write the degree of each of the following:

   (i) $11y^2$ (ii) $8x - 5x^2 + 5$ (iii) $15x^2 - 7xy + \frac{2}{7}y^2$ (iv) $8x^3 - 3x^2y^2 + 5xy^2$

4. Show on the number line:

   (i) x, $x + 4$ and $x - 3$ (ii) x, $3x$ and $3x + 2$

5. Add the following:

   (i) $4x$, $2x$ and $-5x$ (ii) $p^2 + 5pq$ and $6p^2 - 4pq$

   (iii) $-12xy$, $9xy$ and $-3xy$ (iv) $4x^3 + 3xy + 3$ and $2x^3 - 6xy + 11$

6. Subtract the following:

   (i) $-13a^2b$ from $5a^2b$ (ii) $-3x$ from $-6x$

   (iii) $2x^2 + 3y + 4$ from $14x^2 - 6y - 5$ (iv) $3pq - 4q^2 - 7p^2$ from $p^2 + 3q^2 - 4pq$

   (v) $7x^3 + y^3 + 2z^3$ from $12x^3 - 8y^3 + z^3 + 8$ (vi) $-x^2y^2 + z^2$ from $5x^2y^2 - 3z^2$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Multiplication of Algebraic Expressions

Before going through the multiplication of algebraic expressions, we must remember the following points:

(*i*) *The product of two factors with like signs is positive and the product of two factors with unlike signs is negative.*

i.e., $(+)\times(+) = +$, $(+)\times(-) = -$, $(-)\times(+) = -$ and $(-)\times(-) = +$

(*ii*) *If x is any variable and m, n are positive integers,* then $x^m \times x^n = x^{m+n}$

For example, $x^5 \times x^3 = x^{5+3} = x^8$, $y^3 \times y^6 = y^{3+6} = y^9$, etc.

Take a Task

Watch Remedial

When two algebraic expressions are multiplied, the result obtained is called the *product* and the two expressions which are multiplied are called *factors* or *multiplicands*. The multiplicands in a multiplication operation may be two monomials, one monomial and one binomial, two binomials or two polynomials.

Multiplication of Two Monomials

Let us multiply the monomials $7xy$ and $3x$ that contain two literals x and y.

$$7xy \times 3x = 7 \times x \times y \times 3 \times x$$
$$= (7\times 3)\times x \times x \times y = 21x^2y.$$

Thus, the product of monomials $7xy$ and $3x$ is $21x^2y$.

Clearly, the coefficient 21 of the product $21x^2y$ is equal to the product of the coefficients in $7xy$ and $3x$. Also, the variable x^2y in the product $21x^2y$ is equal to the product of the variable parts xy and x in the monomials $7xy$ and $3x$ respectively.

The following two rules for the multiplication of two monomials are given below:

Working Rules

Rule 1. *The numerical coefficient of the product of two monomials is equal to the product of their numerical coefficients.*

Rule 2. *The variable part in the product of two monomials is equal to the product of the variable parts in the given monomials.*

Example 6: **Multiply.**

(*i*) $5ab$ by $3a^3b$ (*ii*) $6ab^2$ by $-2ab^2$

Solution: We have:

(*i*) $(5ab)\times(3a^3b) = (5\times 3)\times(ab\times a^3b)$
$= 15a^{1+3}b^{1+1} = 15a^4b^2$

(*ii*) $(6ab^2)\times(-2ab^2) = [6\times(-2)]\times(ab^2\times ab^2)$
$= -12a^{1+1}b^{2+2} = -12a^2b^4$.

Note

Above rules are also applicable for the product of three or more monomials.

Multiplication of Three or More Monomials

To multiply three or more monomials, we multiply the first two monomials and then multiply the resulting monomial by the third monomial.

This method can be extended to the product of any number of monomials.

Example 7 : **Find the product of $\frac{3}{7}x^2y^3z^0$ and $\frac{-14}{9}x^4yz^4$.**

Solution :

$$\frac{3}{7}x^2y^3z^0 \times \frac{-14}{9}x^4yz^4 = \left(\frac{3}{7} \times \frac{-14}{9}\right) \times (x^2 \times x^4) \times (y^3 \times y) \times (z^0 \times z^4)$$

$$= \left(\frac{1}{1} \times \frac{-2}{3}\right) \times x^{2+4} \times y^{3+1} \times z^{0+4} \quad = \frac{-2}{3}x^6y^4z^4.$$

Example 8: **Multiply $\frac{9}{4}a^3b^2$ by $\frac{-8}{9}a^2b^3$.**

Solution:

$$\frac{9}{4}a^3b^2 \times \frac{-8}{9}a^2b^3 = \left(\frac{\cancel{9}^1}{\cancel{4}_1} \times \frac{-\cancel{8}^2}{\cancel{9}_1}\right) \times (a^3b^2 \times a^2b^3)$$

$$= -2a^5b^5.$$

Check Your Progress

Experiential Learning

Multiply the following monomials :

(*i*) $2x^2y$ and $3x^2y^2$ (*ii*) $7x^2$ and $-5y^3$ (*iii*) $3xy$ and $-5y^2$

(*iv*) $\frac{14}{27}pq$ and $\frac{4}{7}p^2$ (*v*) $\frac{-2}{5}a^2b^2c$ and $\frac{5}{3}ab^2c$ (*vi*) $\frac{-9}{11}x^3y$ and $\frac{-3}{7}x^2y$

Mental Maths

Experential Learning

Complete the table of products.

First Monomial ® Second Monomial –	$6x$	$-10xy$	$3a^2b^2$	$2x^2y^2$	$-3mn^2$
$4x$	$24x^2$				
$6xy$		$-60x^2y^2$			
$5a^2b^2$			$15a^4b^4$		
$-9x^3y^2$					
$7mn^2$				$14mn^2x^2$	

Multiplication of a Monomial by a Polynomial

Multiplication of a Monomial by a Binomial

When a binomial is multiplied by a monomial, each term of the binomial is multiplied by the monomial using distributive law.

$$a \times (b + c) = a \times b + a \times c$$

Also, $$a \times (b - c) = a \times b - a \times c$$

Example 9 : Multiply $(3x + 7)$ by $9x$.

Solution :

Step 1. Multiply first $3x$ by $9x$.

$$3x \times 9x = 3 \times 9 \times x^{(1+1)} = 27x^2$$

Step 2. Multiply 7 by $9x$.

$$7 \times 9x = 63x$$

Step 3. Add the results of step 1 and step 2.

Therefore, $(3x + 7) \times 9x = 27x^2 + 63x$.

Example 10 : If the length and breadth of a rectangle are $(2x + 7)$ units and $2y$ units respectively, find the area of the rectangle.

Solution : Area of the rectangle $= l \times b = (2x+7) \times 2y$

$= 2x \times 2y + 7 \times 2y = 4xy + 14y$

Hence, the area of the rectangle is $4xy + 14y$ sq. units.

Multiplication of Two Binomials

To multiply two binomials, each term of one binomial has to be multiplied by each term of the other binomial. Then all the like terms are grouped together through addition or subtraction.

In general,

$$(a + b) \times (c + d) = a \times (c + d) + b \times (c + d)$$
$$= (a \times c + a \times d) + (b \times c + b \times d)$$
$$= ac + ad + bc + bd$$

Thus, to *multiply any two binomials, we multiply each term of one binomial by each term of the other and add the products*.

Example 11 : Multiply: $(8x + 2y^2)$ and $(6x - 4y)$

Solution : $(8x + 2y^2) \times (6x - 4y) = 8x(6x - 4y) + 2y^2(6x - 4y)$

$= 8x \times 6x - 8x \times 4y + 2y^2 \times 6x - 2y^2 \times 4y$

$= 48x^2 - 32xy + 12xy^2 - 8y^3$.

There is another way for multiplying each term of one binomial with each term of another binomial and arranging these products so that like terms are combined columnwise. This method is known as the *Column Method* of multiplication of two binomials.

Example 12 : **Multiply $(4a + 2b)$ and $(3a + 3b)$ by column method.**

Solution : We have,

$$\begin{array}{rrrl}
 & 4a & + \; 2b & \\
\times & 3a & + \; 3b & \\
\hline
 & 12ab & + \; 6b^2 & \leftarrow (4a + 2b) \times 3b \\
12a^2 + & 6ab & & \leftarrow (4a + 2b) \times 3a \\
\hline
12a^2 + & 18ab & + \; 6b^2 &
\end{array}$$

Multiplication of Trinomials by Binomials and Trinomials

We know that a trinomial is an algebraic expression containing three terms. The distributive law of multiplication can be extended to the product of algebraic expressions containing any number of terms. Thus, we have

$$\begin{aligned}(a + b)(x + y + z) &= a(x + y + z) + b(x + y + z) \\ &= ax + ay + az + bx + by + bz\end{aligned}$$

Rule : Multiply each term of the binomial with every term of the trinomial and add all the products so obtained.

Example 13 : **Find the product of $(a^2 + 2b)$ and $(a^2 - 2ab + 2b^2)$ by horizontal method.**

Solution : $(a^2 + 2b) \times (a^2 - 2ab + 2b^2)$

$$\begin{aligned}
&= a^2 \times (a^2 - 2ab + 2b^2) + 2b \times (a^2 - 2ab + 2b^2) \\
&= a^2 \times a^2 - a^2 \times 2ab + a^2 \times 2b^2 + 2b \times a^2 - 2b \times 2ab + 2b \times 2b^2 \\
&= a^4 - 2a^3b + 2a^2b^2 + 2a^2b - 4ab^2 + 4b^3
\end{aligned}$$

Example 14 : **Multiply $6p^2 + 2q^2 - q$ and $2p - q + 6$ by column method.**

Solution :

$$\begin{array}{l}
6p^2 + \; 2q^2 \; - \; q \\
\times \; 2p \; - \; q \; + \; 6 \\
\hline
12p^3 + 4pq^2 - 2pq \\
\qquad\quad - 6p^2q - 2q^3 + q^2 \\
\qquad\qquad\qquad 36p^2 + 12q^2 - 6q \\
\hline
12p^3 + 4pq^2 - 2pq - 6p^2q - 2q^3 + 13q^2 + 36p^2 - 6q \\
\hline
\end{array}$$

Exercise 8.2

1. Express each of the following products as a monomial:

(i) $ab \times bc \times ca$ (ii) $5xy \times (-3x^2) \times 8xy^3$ (iii) $-12p^2q \times 3pq^2 \times 0$

(iv) $\frac{-2}{7} p^2q^2 \times \frac{7}{8} pq^2 \times \frac{3}{7} p^2q$ (v) $\frac{-3}{15} x^2y \times \frac{-15}{8} y^2z \times \frac{-1}{5} z^2x$

2. Find the product of $3xy$, $\frac{-1}{9}x^2y^2z^2$ and $-8xy$ and verify the result for $x = -2, y = 2$ and $z = -3$.

3. Find the product of $12xy$ and $\frac{-1}{3}x^2y^2$ and verify the result by replacing variables by some numbers.

4. Multiply.

(*i*) $(5x)\times(7x+5)$ (*ii*) $(6a)\times(3a-3)$ (*iii*) $2pq\times(3ab-9y)$

5. Multiply the following :

(*i*) (x^2-6y) and $(2x^2-3y)$ (*ii*) $(3a^2+4b)$ and $(5a^2+3b)$

(*iii*) $\left(\frac{3}{4}p-\frac{5}{3}q\right)$ and $(3p-6q)$ (*iv*) $\left(\frac{1}{2}ab^2-3b\right)$ and $(7ab^2+4b)$

6. Find the product of:

(*i*) $(x+5y)$ and (x^2-xy+y^2) (*ii*) $(3x+y)$ and $(4x+5y+7)$

(*iii*) (a^2+2) and (a^2-5a+7) (*iv*) $(5a+9b)$ and $\left(3a+\frac{2}{3}b+5\right)$

(*v*) $\left(\frac{1}{5}p^2+\frac{2}{5}\right)$ and $\left(2p^2-3pq+\frac{5}{3}\right)$ (*vi*) (a^2-b^2) and $(a^2+ab+3b^2)$

7. Multiply :

(*i*) $(3x-2)$ and $(5x^3-3x^2+4x-7)$ (*ii*) $(2a+b)$ and $(3a^2+5ab+4b^2-5)$

(*iii*) $(4z+5y+2)$ and $(x^2-xy+y+1)$ (*iv*) (x^2+x+1) and $(8x^3-6x^2+6x-1)$

8. Find products :

(*i*) $(a+2b+3)\times(a-3b+7)$ (*ii*) $(x^2+2x-7)\times(3x^2-4x+3)$

(*iii*) $(x^2+xy-y^2)\times(6x^2+5y-2)$ (*iv*) $(2m^2+5m+3)\times(m^2+3m+7)$

9. Perimeter of a triangle is $17a+11b+13c$ and two of its sides are $6a+3b-4c$ and $7a+8b+3c$. Find the third side.

10. Subtract the sum of $18a + 13b + 15c$ and $16a - 20b + 11c$ from the sum of $18a + 12b - 10c$ and $9a + 13b - 15c$.

11. If A = $5x + 16y - 15z$, B = $12x - 13y + 12z$, C = $8x - 6y + 21z$, find :

(*a*) A + B + C (*b*) A – B – C (*c*) A + B – C (*d*) 2A + 3B + C

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Some Special Products (Special Identities)

We are familiar with linear equations in one variable. We know that an equation contains two algebraic expressions on either side of the equal to (=) sign. Consider the equation $5x + 6 = 16$.
For $x = 2$, we have, LHS = $5 \times 2 + 6 = 16$ and RHS = 16.
Thus, LHS = RHS, when $x = 2$. In other words, the equation $5x + 6 = 16$ is true only when $x = 2$.
On the otherhand, the equality $3m + 5m = 8m$ is true for all values of x. This kind of equalities are known as *identities*.
Thus, *an identity is an equality, which is true for all values of the variable(s).*
In this section, we shall study about three important identities.

Identity I. $(a+b)^2 = a^2+2ab+b^2$ or, $(a+b)^2 = a^2+b^2+2ab$

Square of the sum of binomial is equal to the square of the first term plus twice the product of the unlike terms plus square of the second term.

Proof: We have, $(a+b)^2 = (a+b)\times(a+b)$

$= a\times(a+b)+b\times(a+b)$ [Using distributive law]

$= (a^2+ab)+(ba+b^2)$ [Using law of exponents]

$= a^2+(ab+ab)+b^2$ (Since $ab=ba$)

$= a^2+2ab+b^2$ [Combining and adding like terms]

Thus, $(a+b)^2 = a^2+2ab+b^2$

Identity II. $(a-b)^2 = a^2-2ab+b^2$ or $(a-b)^2 = a^2+b^2-2ab$

Square of the difference of two unlike terms is equal to the square of the first term minus twice the product of the unlike terms plus square of the second term.

Proof: We have, $(a-b)^2 = (a-b)\times(a-b)$

$= a\times(a-b)-b\times(a-b)$ [Using distributive law]

$= (a^2-ab)-(ba-b^2)$ [Using law of exponents]

$= a^2-ab-ab+b^2$ (Since $ab=ba$)

$= a^2-2ab+b^2$ [Adding like terms]

Thus, $(a-b)^2 = a^2-2ab+b^2$

Identity III. $(a+b)(a-b) = a^2-b^2$

The product of sum and difference of two binomials is equal to the square of first term minus the square of second term.

Proof: We have, $(a+b)(a-b) = (a+b)\times(a-b)$

$= a\times(a-b)+b\times(a-b)$ [Using distributive law]

$= (a^2-ab)+(ba-b^2)$ [Using law of exponents]

$= a^2-ab+ab-b^2$ (Since $ab=ba$)

$= a^2-b^2$ [Adding like terms]

Thus, $(a+b)(a-b) = a^2-b^2$.

Example 15 : **Simplify.**

(*i*) $(3x+5y)^2$ (*ii*) $(2a-3b)^2$ (*iii*) $(3x+5)(3x-5)$

Solution: (*i*) $(3x+5y)^2 = (3x)^2+2\times(3x)\times(5y)+(5y)^2$ [Using $(a+b)^2=a^2+2ab+b^2$]

$= 9x^2+30xy+25y^2$

(*ii*) $(2a-3b)^2 = (2a)^2-2\times(2a)\times(3b)+(3b)^2$ [Using $(a-b)^2=a^2-2ab-b^2$]

$= 4a^2-12ab+9b^2$

(*iii*) $(3x+5)(3x-5) = (3x)^2-5^2$ [Using a^2-b^2]

$= 9x^2-25.$

Example 16 : **Write down the squares of:**

(*i*) $\left(\frac{3}{2}x+\frac{1}{3}y\right)$ (*ii*) $\frac{2}{5}x-\frac{2}{5}y$

Solution: (*i*) $\left(\frac{3}{2}x+\frac{1}{3}y\right)^2 = \left(\frac{3}{2}x\right)^2 + 2\times\frac{3}{2}x\times\frac{1}{3}y + \left(\frac{1}{3}y\right)^2$

[Using $(a+b)^2 = a^2 + 2ab + b^2$]

$$= \frac{9}{4}x^2 + xy + \frac{1}{9}y^2$$

(*ii*) $\left(\frac{2}{5}x-\frac{2}{5}y\right)^2$

$$= \left(\frac{2}{5}x\right)^2 - 2\times\frac{2}{5}x\times\frac{2}{5}y + \left(\frac{2}{5}y\right)^2$$

[Using $(a-b)^2 = a^2 - 2ab - b^2$]

$$= \frac{4}{25}x^2 - \frac{8}{25}xy + \frac{4}{25}y^2.$$

Example 17 : **Evaluate the following using identities :**

(*i*) $(63)^2$ (*ii*) $(94)^2$

Solution : (*i*) We have, $(63)^2 = (60+3)^2$

$$= (60)^2 + 2\times(60)\times(3) + (3)^2$$

$$= 3600 + 360 + 9 = 3969$$

Thus, $(63)^2 = 3969$.

(*ii*) We have, $(94)^2 = (100-6)^2$

$$= (100)^2 - 2\times(100)\times(6) + (6)^2$$

$$= 10000 - 1200 + 36 = 8836$$

Thus, $(94)^2 = 8836$.

Example 18 : **If $\left(x+\frac{1}{x}\right) = 5$, find the value of $x^2+\frac{1}{x^2}$.**

Solution : We have, $\left(x+\frac{1}{x}\right) = 5$

On squaring both the sides, we get

$$\left(x+\frac{1}{x}\right)^2 = (5)^2$$

or, $x^2 + 2 + \frac{1}{x^2} = 25$ [Using $(a+b)^2 = a^2 + 2ab + b^2$]

or, $x^2 + \frac{1}{x^2} = 25 - 2$

Thus, $x^2 + \frac{1}{x^2} = 23$.

Example 19: **If $(a+b)=9$ and $ab=14$, find the value of a^2-b^2.**

Solution : To find $a-b$, we use the formulae

$$(a+b)^2-(a-b)^2 = (a^2+2ab+b^2)-(a^2-2ab+b^2)$$
$$= a^2+2ab+b^2-a^2+2ab-b^2 = 4ab$$
$$\Rightarrow (a+b)^2-(a-b)^2 = 4ab$$

Substituting the values of $(a+b)$ and ab, we get

$$(9)^2-(a-b)^2 = 4\times 14$$
$$\Rightarrow 81-(a-b)^2 = 56$$
$$\Rightarrow (a-b)^2 = 81-56 = 25$$
$$\Rightarrow (a-b) = 5$$

Now $a^2-b^2 = (a+b)(a-b) = 9\times 5 = 45$

$\therefore$ $a^2-b^2 = 45$

Exercise 8.3

1. Simplify:

(i) $(3a+b)^2$ (ii) $(4x+4y)^2$ (iii) $(2x^2+3y)^2$ (iv) $(5x+12)^2$

(v) $(2p-3q)^2$ (vi) $\left(\frac{2}{5}x-4\right)^2$ (vii) $\left(\frac{9}{4}x-3y\right)^2$ (viii) $\left(\frac{1}{7}x+\frac{1}{2}y\right)^2$

2. Find the square of each of the following :

(i) $(x+5)$ (ii) $(3+5b)$ (iii) $(4a-6b)$ (iv) $\left(1-\frac{3}{5}x\right)$

(v) $\left(p+\frac{1}{2}q\right)$ (vi) $\left(\frac{a}{3}-\frac{3}{a}\right)$

3. Find each of the following products :

(i) $(7x+y)(7x-y)$ (ii) $(3a+5y)(3a-5y)$ (iii) $(7x+9y)(7x-9y)$

(iv) $\left(\frac{3}{5}x+4\right)\left(\frac{3}{5}x-4\right)$ (v) $\left(6-\frac{1}{5}x\right)\left(6+\frac{1}{5}x\right)$

4. Using identities find the value of each of the following :

(i) $(102)^2$ (ii) $(67)^2$ (iii) $(997)^2$ (iv) $(496)^2$ (v) 106×94

5. If $x+\frac{1}{x}=3$, find the value of $\left(x^2+\frac{1}{x^2}\right)$ and $x^4+\frac{1}{x^4}$.

6. If $a-\frac{1}{a}=4$, find the value of $\left(a-\frac{1}{a}\right)^2$ and $a^2+\frac{1}{a^2}$.

7. Find the value of a if:

(i) $6a=29^2-23^2$ (ii) $48a=25\times 25-23\times 23$ (iii) $38^2-32^2=4a$

8. Find the value of each of the following expressions :

(*i*) $49x^2 + 70x + 25$, when $x = 1$

(*ii*) $\frac{1}{9}x^2 + 6xy + 81y^2$, when $x = -1$ and $y = 1$

(*iii*) $16x^2 - \frac{16}{3}xy + \frac{4}{9}y^2$, when $x = \frac{1}{3}$ and $y = \frac{1}{2}$

9. Simplify:

(*i*) $\frac{36^2 - 35^2}{71}$

(*ii*) $\frac{(a+b)^2 - (a-b)^2}{2a}$

(*iii*) $\frac{a^2 - b^2}{a - b}$

10. Find the product of the following:

(*i*) $(x+y)(x-y)(x^2+y^2)$

(*ii*) $(x+2y)(x-2y)(x^2+4y^2)$

(*iii*) $(16x^2+25y^2)(4x-5y)(4x+5y)$

(*iv*) $\left(\frac{1}{9}a^2+9b^2\right)\left(\frac{1}{3}a+3b\right)\left(\frac{1}{3}a-3b\right)$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Division of Algebraic Expressions

Division of a Monomial by a Monomial

The division of a monomial by a monomial is alike to the division of integers. In fact, dividing a monomial say a another by another monomial say *b* means finding a monomial *c* such that $a = bc$ and we write

$$a \div b = c \text{ or } \frac{a}{b} = c.$$

Here, *a* is called the *dividend*, *b* is called the *divisor* and *c* is called the *quotient*.

While dividing a monomial by a monomial, follow the given procedure:

- The coefficient of the quotient of two monomials is equal to the quotient of their coefficients.
- The variable part in the quotient of two monomials is equal to the quotient of the variables in the given monomials.

Example 20 : **Divide (*i*) $64x^3y^2$ by $16xy$ (*ii*) $-56ab^2c$ by $8ab$**

Solution : (*i*) $\frac{64x^3y^2}{16xy}$

$$= \frac{64}{16} \times \frac{x \times x \times x \times y \times y}{x \times y} = 4x^2y$$

(*ii*) $\frac{-56ab^2c}{8ab} = \frac{-56}{8} \times \frac{a \times b \times b \times c}{a \times b} = -7bc.$

Division of a Polynomial by a Monomial

For dividing a polynomial in one variable by a monomial in the same variable, we divide each term of the polynomial by the monomial.

Example 21 : **Divide: $20x^3y + 12x^2y - 16xy$ by $4xy$.**

Solution: We have, $(20x^3y + 12x^2y - 16xy) \div 4xy$

$$= \frac{20x^3y + 12x^2y - 16xy}{4xy} = \frac{20x^3y}{4xy} + \frac{12x^2y}{4xy} - \frac{16xy}{4xy}$$

$$= 5x^2 + 3x - 4.$$

Division of a Polynomial by a Binomial or a Polynomial

Working Rules

1 Arrange the terms of dividend and divisor in descending order of their degrees.

2 Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

3 Multiply the divisor by the first term of the quotient and subtract the result from the dividend to obtain the remainder.

4 Consider the remainder (if any) as dividend and repeat step 2 to obtain the second term of the quotient.

5 Repeat the above process till we obtain a remainder which is either zero or a polynomial of degree less than that of the divisor.

Example 22 : **Divide $x^2 + 7x + 8$ by $x + 2$.**

Solution: **Explanation:**

1. Divide x^2 of the dividend by x of the divisor.
   The first term of the quotient. is $x^2 \div x = x$.
2. Now multiply each term of $x + 2$ by x, and write the product in the respective column.
3. On subtracting x^2 get cancelled and $5x - 2x = 3x$ will remain.
4. Divide $3x$ by x. The result $3x$, $x = 3$ is the second term of the quotient.
5. Again multiply $(x + 2)$ by 3 and subtract the remainder is 0.
   Hence, the required quotient is $x + 3$.

Quotient $= x + 3$

Remainder $= 0$

$$\begin{array}{r|l|l} x+2 & x^2 + 5x + 6 & x+3 \\ & x^2 + 2x & \\ & - \quad - & \\ \hline & 3x + 6 & \\ & 3x + 6 & \\ & - \quad - & \\ \hline & 0 & \\ \hline \end{array}$$

Example 23 : Divide $3x^3 + 2x^2 - 4x + 3$ by $3x - 1$.

Solution: Arranging terms of dividend and divisor in descending order and then dividing:

$$
\begin{array}{r|l}
3x-1 & 3x^3 + 2x^2 - 4x + 3 \;(\; x^2 + x - 1 \\
 & \underline{3x^3 - x^2} \\
 & \quad - \quad + \\
 & \qquad 3x^2 - 4x \\
 & \qquad \underline{3x^2 - x} \\
 & \qquad - \quad + \\
 & \qquad\qquad -3x + 3 \\
 & \qquad\qquad \underline{-3x + 1} \\
 & \qquad\qquad + \quad - \\
 & \qquad\qquad\qquad 2
\end{array}
$$

$\therefore$ Quotient $= x^2 + x - 1$ and remainder $= 2$.

Word problems

In word problems on algebraic expressions, we would not be given an algebraic solution. But we have to convert the given statement into mathematical statement.

Example 24 : Find the length of a rectangle having breadth 5 metres and area 355 sq. metres.

Solution: Let the length of rectangle be x metre. Breadth given is 5 metres.

We know that, Length $\times$ Breadth = Area

$\Rightarrow \quad x \times 5 = 355$

$\Rightarrow \quad x = \frac{355}{5} = 71$

Thus, length of the rectangle is 71 metres.

Example 25 : Kavir throught of a number, doubled it and subtracted 15 from it. If the result was 49, find the number.

Solution: Let the number throught by Kavir be x. After doubling, it becomes $2x$.

According to given condition, we have

$2x - 15 = 49 \quad \Rightarrow \quad 2x = 49 + 15$

$\Rightarrow \quad 2x = 64 \quad \Rightarrow \quad x = \frac{64}{2} = 32$

Thus, the number through by Kavir is 32.

Example 26 : Two numbers are in the ratio 2: 3. If sum of the two numbers is 100, find the numbers.

Solution: Let the required numbers be $2x$ and $3x$ respectively then $2x + 3x = 100$

$\Rightarrow \quad 5x = 100 \quad \Rightarrow \quad x = \frac{100}{5} = 20$

Thus, one number $= 2x = 2 \times 20 = 40$

and other number $= 3x = 3 \times 20 = 60$

Exercise 8.4

1. Divide the following :

   (*i*) $4a^2b$ by ab (*ii*) $-90a^3b$ by $5ab$ (*iii*) $35x^2y^3z^4$ by $5xyz^2$

   (*iv*) $-144x^3y^2z$ by $12xy$ (*v*) $125m^6n^3p^2$ by $-5mnp^2$ (*vi*) $-160a^4b^2c^2$ by $-20a^2b^2c$

2. Divide the following :

   (*i*) $6x^2 + 10xy + 8x$ by $2x$ (*ii*) $21x^2y^3 - 48xy^2 + 36x^3y^3$ by $3xy$

   (*iii*) $20a^2 - 15a^3b^2 + 20a^4$ by $-5a$ (*iv*) $15x^3 - 3x^2 + 9x$ by $3x$

   (*v*) $18m^5n^3p^3 - 63m^2np + 27m^2n^2p^2$ by $9m^2np$ (*vi*) $56x^2y^2 - 32x^4y + 48x^2y$ by $-8x^2y$

3. Find the quotient in each of the following :

   (*i*) $(x^2 + 11x - 26)$ by $(x - 2)$ (*ii*) $(2a^3 - 6a^2b - 13ab^2 - 35b^3)$ by $(a - 5b)$

   (*iii*) $(x^5 + 1)$ by $(x + 1)$ (*iv*) $15x^2 + x - 6$ by $3x + 2$

4. Find the quotient and remainder in each of the following :

   (*i*) $(4a^3 + a - 5) \div (a - 1)$ (*ii*) $(8y^3 - 6y^2 + 10y + 15) \div (4y + 1)$

   (*iii*) $(3x^4 - 3x^3 - 4x^2 - 4x) \div (x^2 - 2x)$ (*iv*) $(x^6 + 3x^2 + 10) \div (x^3 + 1)$

5. Two numbers differ by 5. If their sum is 9, then find the two numbers.

6. The length of a rectangle is twice the breadth. If its perimetre is 60, then find its length.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Revision Exercise

1. Tick (✓) the correct option: Conceptual Learning

(*i*) If $xy = 8$ and $x + y = 6$, then the value of $(x^2 + y^2)$ is:

(*a*) 20 ☐ (*b*) 19 ☐ (*c*) 18 ☐ (*d*) 49 ☐

(*ii*) $6a^2b^3 \div (-2ab) = ?$

(*a*) $3ab^2$ ☐ (*b*) $-3ab^2$ ☐ (*c*) $12a^2b^3$ ☐ (*d*) $-12a^3b^4$ ☐

(*iii*) $95 \times 103 = ?$

(*a*) 9600 ☐ (*b*) 9785 ☐ (*c*) 9700 ☐ (*d*) 9000 ☐

(*iv*) $(x + p)(x + p) = ?$

(*a*) $(x + p)^2$ ☐ (*b*) $x - p^2$ ☐ (*c*) $x + p^2$ ☐ (*d*) $x^2 + p^2$ ☐

(*v*) Subtract $3x(x - 4y + 5z)$ from $4x(2x - 3y + 10z)$:

(*a*) $25xz$ ☐ (*b*) $5x^2$ ☐ (*c*) $5x^2 + 25xy$ ☐ (*d*) $5x^2 - 25xz$ ☐

2. Add $5x^3 + 8xy + y^3$, $2y^3 - 10xy - 7x^3$ and $3x^3 + 2xy - 4y^3$.

3. Add $4x^2 - 5xy + 3y^2$, $-6x^2 - 4xy + 2y^2$ and $-2xy - 4y^2 - 3x^2$.

4. What must be added to $3x^3 - 2x^2 + 5x + 1$, so that the sum may be $x^3 - 2x^2 + 4x - 1$?

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5. Simply the following :

(*i*) $(x+y)(x+yz)$ (*ii*) $(12x + 9)(7x + 3)$ (*iii*) $(a^2 + b^2)(3a + 2b)$

(*iv*) $\left(\frac{1}{2}x - y\right)\left(\frac{1}{2}x + y\right)$ (*v*) $5(x - 2) + 4(x - 3) + 3(x - 4)$

6. Divide:

(*i*) $5x^3 - 15x^2 + 25x$ by $5x$ (*ii*) $4z^3 + 6z^2 - z$ by $-\frac{1}{2}z$

(*iii*) $3x^3y^2 + 2x^2y + 15xy$ by $3xy$

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7. Find the breadth of a rectangle with length 15 metres and area 165 sq. metres.

8. Rajesh thought of a number, tripled it and added 15 to it, the result was 60. Find the number.

Mental Maths

Experiential Learning

1. Find the value of $\frac{5pq(p^2 - q^2)}{3q(p+q)}$.

2. Find the value of $9a^3 \div 3a^3$.

3. Find the value of p, if $9p = (76)^2 - (67)^2$.

4. Evaluate: 97×103.

5. If $\left(x - \frac{1}{x}\right) = 3$ then, find the value of $\left(x^2 + \frac{1}{x^2}\right)$.

Assertion and Reason

Critical Thinking

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given : Study both the statements and state which of the following is correct :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

1. **Assertion (A) :** $x + 3$, $5 - 3x$, $a^2 - 2abc$, are binomials.

Reason (R) : An algebraic expression containing two terms is called a binomial.

2. **Assertion (A) :** The degree of $3x^2y^3y^4$ is 9.

Reason (R) : The highest power of a particular term is the degree of the polynomial.

3. **Assertion (A) :** $(x^3 - 2x^2 + 4x - 1) - (3x^3 - 2x^2 + 5x + 1) = 2x^3 + x - 2$

Reason (R) : Subtraction is the reverse process of addition.

4. **Assertion (A) :** Multiply $(3x^2 + 7y)$ by $-4xy^3 = -12x^3y^3 - 28xy^4$.

Reason (R) : If $a = 1$ and $b = -2$, then value of $-5ab \times 3a^2b^2 = 120$

5. **Assertion (A) :** When the dividend and divisor have the same signs, the quotient has plus sign.

Reason (R) : Divide $24x^6y^8$ by $-6x^3y^2 = -4x^2y^6$

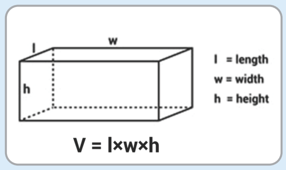
Thinking Skills

1. The area of a rectangular field is $3a^2 + 5ab + 2b^2$. One of its sides is $(a + b)$. Find the length of the fence around the field.
2. A shirt costs ₹ $(a^2 - ab - b^2)$ and a pair of trousers cost ₹$(2a^2 + 8ab - 2b^2)$ and a pair of shoes ₹ $(a^2 - 3ab + 4b^2)$. After collecting these three items from the store, Ranjan paid ₹ $(2a + b)^2$. What balance will Ranjan receive from the cashier?

Skills covered: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Competency based Questions

The volume of a cuboid is given by the expression l   w   h where l = 2x + 1, w = x – 2, and h = x + 3. Find the expression for the volume of the cuboid.



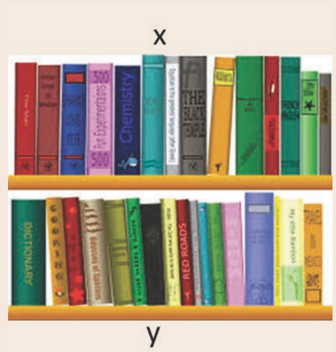
A) 2x3 + 3x2 – 11x – 6 B) 2x4 + 3x2 – 11x – 6

C) 2x3 + 3x3 – 11x – 6 D) 2x2 + 3x2 – 11x – 6

Skills covered: Interpersonal skills, Observation, Application and Decision making skills

Integrated Learning

A library has two shelves where books are arranged. The ratio of books on Shelf 1 to Shelf 2 is initially 3:5. The number of books on Shelf 1 is represented by 3x, and the number of books on Shelf 2 is represented by 5x, where x is a positive integer.



Based on this information answer the following question

1. Find the value of x when the ratio of the number of books on Shelf 1 to Shelf 2 is exactly 3:5.
2. If x=4, calculate the number of books on Shelf 1 and Shelf 2. Then, find the new ratio of books on Shelf 1 to Shelf 2.
3. If the total number of books on both shelves is 120, find the value of x.

Skills covered: Research, Logical Reasoning, Problem-Solving, Practical Application