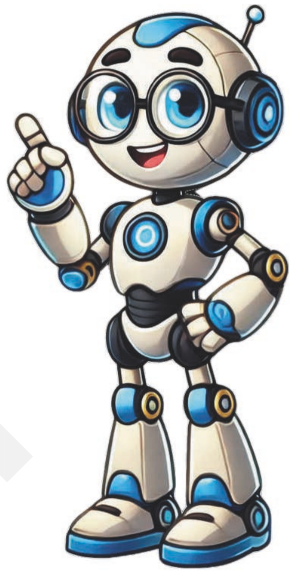
6

Chapter

Cubes and Cube Roots

We'll cover the following key points:

→ Cubes
→ Finding the cube of a two-digit number (alternative method).
→ Properties of cubes of numbers
→ Cube root of a number
→ Cube root of a negative perfect cube
→ Cube root of product of integers
→ Cube root of a rational number



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Learning Outcomes

By the end of this chapter, students will be able to:

- Understand the concept of cube numbers and their properties.
- Learn how to calculate the cube of a number.
- Identify and understand the cube of numbers from 1 to 10.
- Understand the concept of cube roots and their properties.
- Learn how to find the cube root of perfect cubes.
- Identify cube roots of numbers from 1 to 10.
- Solve problems involving the cubes and cube roots of numbers.
- Apply the properties of cubes and cube roots in real-life situations.
- Understand the relationship between cube and cube root.
- Develop skills in simplifying expressions involving cubes and cube roots.
- Use the prime factorization method to find the cube root of a number.
- Compare the cube of a number with its cube root to understand their inverse relationship.
- Solve word problems involving cubes and cube roots, applying appropriate mathematical strategies.

Mind Map

CUBE AND CUBE ROOTS

Cube

$a^3 = a \times a \times a$

$2^3 = 2 \times 2 \times 2 = 8$

$3^3 = 3 \times 3 \times 3 = 27$

Smallest multiple that is a perfect cube

e.g.,

$392 = 2 \times 2 \times 2 \times 7 \times 7$

Need one more 7 to make it perfect cube

Cube root through prime factorisation

e.g.,

$27 = 3 \times 3 \times 3 = 3^3$

Cube root of 27 = $\sqrt[3]{27} = 3$

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$

$\sqrt[3]{3375} = 3 \times 5 = 15$

Some interesting patterns

i. Adding consecutive odd numbers

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

ii. Cube and their prime factors

$4 = 2 \times 2,\ 4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$6 = 2 \times 3,\ 6^3 = 216 = 2 \times 3 \times 2 \times 3 \times 2 \times 3$

Cube roots

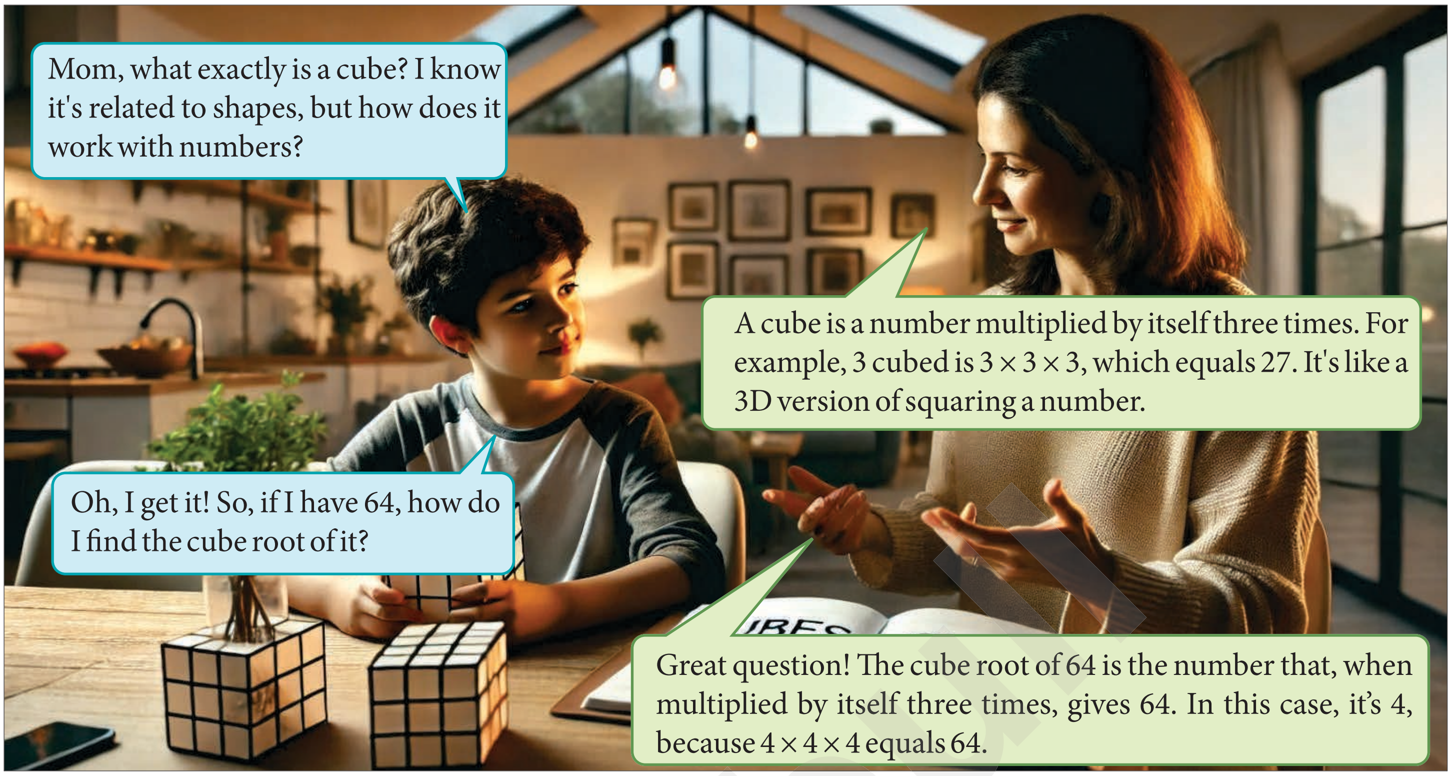
Symbol $\sqrt[3]{\ }$

Cube root of 8 = $\sqrt[3]{8} = 2$

Cube root of 27 = $\sqrt[3]{27} = 3$

Based on NCERT*

Introduction



Cubes

Take a Task

Watch Remedial

Observe the following number statements :

$2^3 = 2 \times 2 \times 2 = 8$; $3^3 = 3 \times 3 \times 3 = 27$; $4^3 = 4 \times 4 \times 4 = 64$; $5^3 = 5 \times 5 \times 5 = 125$, $10^3 = 10 \times 10 \times 10 = 1{,}000$.

The above statements can also be expressed by saying that the cube of 2 is 8, the cube of 3 is 27, the cube of 4 is 64, the cube of 5 is 125,..., the cube of 10 is 1,000. It means the cube of a number is that number raised to the power 3. In general if n is a positive number then

$a^3 = a \times a \times a$ and $(-a)^3 = (-a) \times (-a) \times (-a)$

Perfect Cubes

A natural number is called a perfect cube or a cube number, if it is the cube of some natural number. In short, a natural number n is a perfect cube if $n = m^3$ for some natural number m. Each one of the numbers 1, 8, 27, 64, 125, 216, 343, 512, 729 and 1,000 is a perfect cube. A number a is a perfect cube if there exists a natural number b such that $a = b$ and $-a^3 = (-b)^3$.

To check whether a number is a perfect cube or not

To check whether a given natural number is a perfect cube or not, we factorise the number and try to group together triplets of prime factors. If no factor is left over, we say that it is a perfect cube, otherwise it is not a perfect cube.

Finding the Cube of a Two-Digit Number (Column Method)

To find x^3, we first find x^2 and then multiply it with x. Here, we discuss a method of finding x^3, where x is a two digit number.

Now, let us learn the column method of finding cubes of 2-digit number as given below.

Working Rules

To find the cube of two-digit numbers

Let a = xy be a 2-digit number, where x is the tens digit and y is the ones digit.

Step 1. Use the identify $(x+y)^3 = x^3+3x^2y+3xy^2+y^3$.

Step 2. Make four columns and write x^3, $3x^2y$, $3xy^2$ and y^3 respectively in each column.

Step 3. Remaining process will be same as discussed earlier for finding the square of two-digit number.

Example 1: Find the cube of the number 65 using the column method:

Solution : (*i*) Let $(ab)^3 = (65)^3$,

a^3	$3a^2b$	$3ab^2$	b^3
$a^3 = 216$	$a^2 3b = 540$	$b^2 3a = 450$	$b^3 =$ (12) 5
+ 58	+ 46	+ 12	
27 4	(58) 6	(46) 2	
27 4	6	2	5

Hence, $(65)^5 = 2{,}74{,}625$

Example 2 : Is 288 a perfect cube?

Solution:

2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Resolving 288 into prime factors, we get

$288 = \underline{2 \times 2 \times 2} \times 2 \times 2 \times 3 \times 3$

Observe that the prime factors of 288 cannot be grouped into triplets of equal factors.

Therefore, 288 is not a perfect cube.

Example 3 : What is the least number by which 968 should be multiplied so that the product is a perfect cube?

Solution : Resolving 968 as a product of prime factors, we get

2	968
2	484
2	242
11	121
11	11
	1

$968 = 2 \times 2 \times 2 \times 11 \times 11$

We see that 2 occurs as a prime factor of 968 thrice, but 11 occurs as a prime factor twice.

Thus, if we multiply 968 by 11, the product would be

$2 \times 2 \times 2 \times 11 \times 11 \times 11 = 10,648$, which is a perfect cube.

Hence, the required least number is 11.

Example 4: By what number will you divide 1,125 so that the resultant number is a perfect cube?

Solution : Resolving 1,125 as a product of prime factors, we get

3	1125
3	375
5	125
5	25
5	5
	1

$1,125 = 3 \times 3 \times \underline{5 \times 5 \times 5}$

The factor 5 is a triple but 3 is only a pair. Therefore, if we divide 1,125 by $3 \times 3 = 9$, the number obtained will be a perfect cube.

Properties of Cubes of Numbers

Properties	Examples
1. Cube of the numbers 0, 1, 4, 5, 6 and 9 or the numbers which have these numbers at its units place end with the same digit.	$11^3 = 1,331$; $14^3 = 2,744$; $16^3 = 4,096$.
2. Cube of 2 and numbers having 2 at its unit place end in 8. Also cube of 8 and numbers ending in 8 end in 2.	$12^3 = 1,728$; $18^3 = 5,832$.

3. Cube of 3 and the numbers having 3 at its unit place end in 7. Also cube of 7 and numbers ending in 7 end in 3.	$13^3 = 2,197; 27^3 = 19,683$.
4. Cubes of all even natural numbers are even.	$4^3 = 64,\ 8^3 = 512, 12^3 = 1728$.
5. Cubes of all odd natural numbers are odd.	$3^3 = 27,\ 7^3 = 343, 9^3 = 729$.
6. The cube of any multiple of 2 is divisible by 8.	$4^3 = 64$, which is divisible by 8. $6^3 = 216$, which is divisible by 8.
7. The cube of any multiple of 3 is divisible by 27.	$6^3 = 6 \times 6 \times 6 = 3 \times 3 \times 3 \times 2 \times 2 \times 2 = 27 \times 8$ is divisible by 27. $9^3 = 9 \times 9 \times 9 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$ $= 27 \times 27$ is divisible by 27.
8. Cubes of negative integers are negative.	$(-1)^3 = (-1) \times (-1) \times (-1) = -1;$ $(-2)^3 = (-2) \times (-2) \times (-2) = -8;$
9. *For any rational number* $\frac{a}{b}$, *we have* $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$.	$\left(\frac{3}{7}\right)^3 = \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \frac{3 \times 3 \times 3}{7 \times 7 \times 7} = \frac{3^3}{7^3};$ $\left(\frac{-5}{6}\right)^3 = \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right) \times \left(\frac{-5}{6}\right)$ $= \frac{(-5) \times (-5) \times (-5)}{6 \times 6 \times 6} = \frac{(-5)^3}{6^3}$

Example 5 : **Find the one's digit of the cube of each of the following numbers :**

(*i*) 149 (*ii*) 1,024

Solution : (*i*) In 149 , one's digit is 9.

Since, the cube of 9 = $9 \times 9 \times 9 = 729$

∴ One's digit of the cube of this number is 9.

(*ii*) In 1,024, one's digit is 4.

Since, the cube of 4 = $4 \times 4 \times 4 = 64$

$\therefore$ One's digit of the cube of the number 1,024 is 4.

Example 6 : **Express 6^3 as the sum of odd numbers.**

Solution : As 6^3 is cube of 6, so there are 6 odd numbers in the pattern.

1st odd number = $n(n - 1) + 1 = 6(6 - 1) + 1 = 31$

2nd odd number = $n(n - 1) + 3 = 6(6 - 1) + 3 = 33$ and so on

$\therefore$ Sum of 6 odd numbers = $31 + 33 + 35 + 37 + 39 + 41 = 216 = 6^3$.

Example 7 : **Find the value of $20^3 - 19^3$ using the pattern :**

$$n^3 - (n-1)^3 = 1 + n(n-1) \times 3$$

Solution : $20^3 - 19^3 = 20^3 - (20 - 1)^3 = 1 + 20(20 - 1) \times 3$

$= 1 + 20 \times 19 \times 3 = 1{,}141.$

Exercise 6.1

1. Find the cubes of the following numbers:

   (*i*) 1 (*ii*) 8 (*iii*) 2.7 (*iv*) 50 (*v*) 3.8

   (*vi*) −7 (*vii*) $\frac{4}{5}$ (*viii*) $\frac{-8}{13}$ (*ix*) $2\frac{1}{6}$

2. Which of the following numbers are perfect cubes ?

   (*i*) 49 (*ii*) 2,700 (*iii*) 243 (*iv*) 343

   (*v*) 900 (*vi*) 64,000 (*vii*) 13,824 (*viii*) 1,25,000

3. Find the cube root of the following by prime factorization method :

   (*i*) 512 (*ii*) 13,824 (*iii*) 15,625 (*iv*) 91,125

4. Find the smallest number by which 1,372 must be multiplied so that the product is a perfect cube.

5. What is the smallest number by which 15,552 must be divided so that the quotient is a perfect cube?

6. What is the smallest number by which 2,304 must be multiplied so that the product is a perfect cube?

7. Which of the following are the cubes of odd numbers?

   (*i*) 512 (*ii*) 15,625 (*iii*) 1,331 (*iv*) 7,29,000

   (*v*) 46,656 (*vi*) 2,744

8. Which of the following are the cubes of even natural numbers?

   (*i*) 216 (*ii*) 13,824 (*iii*) 343 (*iv*) 1,728

   (*v*) 3,375 (*vi*) 5,832

9. Which of the following are the cubes of negative integers?

(*i*) 64 (*ii*) –8,000 (*iii*) –2,197 (iv) 3,658

(*v*) –3,375 (*vi*) 729

10. Give two examples to show that the cube of an odd natural number is always odd.

11. Express the following numbers as the sum of odd numbers :

(*i*) 7^3 (*ii*) 12^3 (*iii*) 28^3

12. Find the cube of the following by column method :

(*i*) 36 (*ii*) 42 (*iii*) 79 (*iv*) 84 (*v*) 105

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Mental Maths

Experiential Learning

State whether the following are true or false:

(*i*) 800 is a perfect cube.

(*ii*) –3,375 is the cube of 15.

(*iii*) If 'x' divides 'xy', then x^3 divides $(xy)^3$.

(*iv*) A perfect cube does not end with two zeroes.

(*v*) The cube of a number having units digit as 8 ends with 2 as units digit.

(*vi*) The cube of a 2-digit number may have 7 or more digits.

(*vii*) There is no perfect cube which ends with 8.

Cube Root of a Number

Cube root : A number a is the cube root of a number b, if $b = a^3$.

The cube root of a number b is denoted by $\sqrt[3]{b}$. $\sqrt[3]{b}$ is also called a **radical**; b is called the **radicand** and 3 is called the **index** of the radical.

Observe these:

(*a*) $1 = 1^3 \quad \therefore \quad \sqrt[3]{1} = 1$

(*b*) $8 = 2^3 \quad \therefore \quad \sqrt[3]{8} = 2$

(*c*) $27 = 3^3 \quad \therefore \quad \sqrt[3]{27} = 3$

(*d*) $64 = 4^3 \quad \therefore \quad \sqrt[3]{64} = 4$

(*e*) $-125 = (-5)^3 \quad \therefore \quad \sqrt[3]{-125} = -5$

Take a Task

Watch Remedial

Methods of Finding Cube Roots

1. Using pattern or repeated subtraction of consecutive odd numbers :

$1^3 - 0^3 = 1$ or, $1^3 = 1$

$2^3 - 1^3 = 7$ or $2^3 = 1 + 7$

$3^3 - 2^3 = 19$ or $3^3 = 1 + 7 + 19$

$4^3 - 3^3 = 37$ or $4^3 = 1 + 7 + 19 + 37$

$5^3 - 4^3 = 61$ or $5^3 = 1 + 7 + 19 + 37 + 61$

$6^3 - 5^3 = 91$ ....

What do we conclude from the above pattern ?

From the above pattern, we conclude that any cube number can be written as a sum of 1, 7, 19, 37, 61 and so on. We can also obtain these numbers by putting n = 1, 2, 3, 4, 5, ... in the expression $1 + n \times (n - 1) \times 3$.

For example : $1 + 1 \times (1 - 1) \times 3 = 1$; $1 + 2 \times (2 - 1) \times 3 = 7$; $1 + 3 \times (3 - 1) \times 3 = 19$.

Thus, the numbers in this sequence are: 1, 7, 19, 37, 61, 91, 127, etc. However, to find the cube root of a perfect cube, we perform the reverse operation, i.e., subtraction. We subtract 1, 7, 19, 37, 61, 91, ... successively to get 0. The number of times we subtract is the cube root of the given number.

Observe the following examples to make the concept more clear.

Example 8 : Find the cube root of (*i*) 216 and (*ii*) 512 using repeated subtraction of consecutive odd numbers.

Solution: (*i*)

$216 - 1 = 215$ (subtract 1)

$215 - 7 = 208$ (subtract 7)

$208 - 19 = 189$ (subtract 19)

$189 - 37 = 152$ (subtract 37)

$152 - 61 = 91$ (subtract 61)

$91 - 91 = 0$ (subtract 91)

Since the subtraction is carried out six times to get 0, thus $\sqrt[3]{216} = 6$.

(*ii*) We subtract 1, 7, 19, ... and so on successively from 512, till we get 0.

$512 - 1 = 511$; $511 - 7 = 504$;

$504 - 19 = 485$; $485 - 37 = 448$;

$448 - 61 = 387$; $387 - 91 = 296$;

$296 - 127 = 169$; $169 - 169 = 0$.

We subtracted numbers 1, 7, 19, ... 8 times.

Thus, cube root of 512 is 8, i.e., $\sqrt[3]{512} = 8$.

2. Using prime factorization method

To find the cube root of a perfect cube number with the help of prime factorization method, follow the steps given below :

Working Rules

- Resolve the given number into prime factors.
- Make triples of similar factors, or arrange them in groups taking three equal factors at a time.
- Take the product of prime factors choosing one out of every triple.
- This product is the required cube root of the given number.

Example 9: **Find the cube root of 5,832.**

Solution: By prime factorization, we have

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$5832 = \underline{2\times2\times2}\times\underline{3\times3\times3}\times\underline{3\times3\times3}$$

$$= 2^3\times3^3\times3^3$$

$$\therefore \sqrt[3]{5832} = \sqrt[3]{2^3 \times 3^3 \times 3^3}$$

$$= 2\times3\times3 = 18.$$

3. Cube root using units digit

This method can be used to find cube roots of perfect cubes having at most six digits. The cube root of a number having at most six digits is a 2-digit number, because the least seven digit number is 1000000 whose cube root is 100, the least 3-digit number.

We can find the cube root as follows:

(*i*) Look at the digit at the units place of the perfect cube and determine the digit at the units place using the table given below:

Units digit of the given number	0	1	2	3	4	5	6	7	8	9
Units digit of the cube root	0	1	8	7	4	5	6	3	2	9

(*ii*) Strike out from the right, last three (*i.e.*, units, tens and hundreds) digits of the number. If nothing is left, we do not proceed. The digit in step (*i*) is the cube root.

(*iii*) Take the number left in step (*ii*). Find the largest single digit number, whose cube is less than or equal to this left over number. This will be the tens digit of the cube root.

Observe the following example.

Example 10: **Find the cube roots of the following numbers by finding their units and tens digits:**

(*i*) 1,728 (*ii*) 1,75,616

Solution: (*i*) In 1,728, the units digit is 8. Therefore, the digit at the units place in the cube root is 2.

After striking out the last three digits from the right, we are left with the number 1. Now, 1 is the only number whose cube is less than 1.

Therefore, the tens digit is 1.

$$\therefore \sqrt[3]{1728} = 12$$

(*ii*) In 175616, the units digit is 6. Therefore, units digit of the cube root is 6.

After striking out the last three digits from the right, the number left is 175.

Also, $5^3 < 175$ and $6^3 > 175$.

Hence, the digit at tens place of the cube root is 5.

$$\therefore \sqrt[3]{175616} = 56.$$

Cube Root of a Negative Perfect Cube

We cannot find the square root of a negative number, but we can find the cube root of a negative number.

The cube of a negative number is always a negative number.

In general, if a is any number, then $(-a) \times (-a) \times (-a) = (-a)^3$ or $\sqrt[3]{(-a)^3} = (-a)$.

Alternatively, $\sqrt[3]{(-a)^3} = \sqrt[3]{(-1)^3} \times \sqrt[3]{a^3}$

$= \sqrt[3]{a^3} \times (-1)$

$= -1 \times a = -a$

Hence, $\sqrt[3]{-125} = -5$ since $\sqrt[3]{125} = 5$

$\sqrt[3]{-216} = -6$ since $\sqrt[3]{216} = 6$, etc.

Example 11: Find the cube root of –2,917.

Solution: We have: $\sqrt[3]{-2197} = -\sqrt[3]{2197}$

13	2197
13	169
13	13
	1

Now, resolving 2197 into prime factors, we get

$2{,}197 = \underline{13 \times 13 \times 13}$

$\therefore \quad \sqrt[3]{-2197} = \sqrt[3]{-13 \times -13 \times -13}$

Therefore, $\sqrt[3]{-2197} = -13$.

Cube Root of Product of Integers

In order to find the cube root of the product of two integers, we use the following result.

For any two integers a and b, we have $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.

The above result is true for all values of a and b.

Example 12 : Show that: $\sqrt[3]{-216 \times 1728} = \sqrt[3]{-216} \times \sqrt[3]{1728}$

Solution : LHS $= \sqrt[3]{-216 \times 1728} = \sqrt[3]{-[2\times2\times2\times3\times3\times3]\times2\times2\times2\times2\times2\times2\times3\times3\times3}$

$= -(2\times3)\times2\times2\times3$

$= -6 \times 12 = -72$

RHS $= \sqrt[3]{-216} \times \sqrt[3]{1728}$

$= \sqrt[3]{2\times2\times2\times3\times3\times3} \times \sqrt[3]{2\times2\times2\times2\times2\times2\times3\times3\times3}$

$= -(2 \times 3) \times (2 \times 2 \times 3)$

$= -6 \times 12 = -72$

We have LHS = RHS,

So $\sqrt[3]{-216 \times 1728} = \sqrt[3]{-216} \times \sqrt[3]{1728}$

Cube Root of a Rational Number

The cube root of a rational number is equal to the quotient of cube roots of its numerator and denominator.

In general, let $\frac{a}{b}$ be any rational number, then $\frac{a}{b}$ can be written as $a \times \frac{1}{b}$.

$$\therefore \quad \sqrt[3]{\frac{a}{b}} = \sqrt[3]{a \times \frac{1}{b}} = \sqrt[3]{a} \times \sqrt[3]{\frac{1}{b}}$$

$$= \sqrt[3]{a} \times \frac{1}{\sqrt[3]{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

So, $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$.

Example 13 : Find the cube root of $\frac{1,331}{9,261}$.

Solution: $\sqrt[3]{\frac{1331}{9261}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{9261}}$

$$= \frac{\sqrt[3]{11 \times 11 \times 11}}{\sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}} = \frac{11}{3 \times 7} = \frac{11}{21}$$

To find rational number whose cube is nearly equal to a given number. We find the approximate value of the cube root of the number.

Approximate cube roots of numbers can be found by using the cube root on Table 4.1

Table 4.1

x	$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$	x	$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$
1	1.000	2.154	4.642	21	2.759	5.944	12.81
2	1.260	2.714	5.848	22	2.802	6.037	13.01
3	1.442	3.107	6.694	23	2.844	6.127	13.20
4	1.587	3.420	7.368	24	2.884	6.214	13.39
5	1.710	3.684	7.937	25	2.924	6.300	13.57
6	1.817	3.915	8.434	26	2.962	6.383	13.75
7	1.913	4.121	8.879	27	3.000	6.463	13.92
8	2.000	4.309	9.283	28	3.037	6.542	14.09
9	2.080	4.481	9.655	29	3.072	6.619	14.26
10	2.154	4.642	10.00	30	3.107	6.694	14.42
11	2.224	4.791	10.32	31	3.141	6.768	1458
12	2.289	4.932	10.63	32	3.175	6.840	14.74
13	2.351	5.006	10.91	33	3.208	6.910	14.89
14	2.410	5.192	11.19	34	3.240	6.980	15.04
15	2.466	5.313	11.45	35	3.271	7.047	15.18

16	2.520	5.429	11.70	36	3.302	7.114	15.33
17	2.571	5.540	11.93	37	3.332	7.179	15.47
18	2.621	5.646	12.16	38	3.362	7.243	15.60
19	2.668	5.749	12.39	39	3.391	7.306	15.74
20	2.714	5.848	12.60	40	3.420	7.368	15.87
41	3.448	7.429	16.01	71	4.141	8.921	19.22
42	3.476	7.489	16.13	72	4.160	8.963	19.31
43	3.503	7.548	16.26	73	4.179	9.004	19.40
44	3.530	7.606	16.39	74	4.198	9.045	19.49
45	3.557	7.663	16.51	75	4.217	9.086	19.57
46	3.583	7.719	16.63	76	4.236	9.126	19.66
47	3.609	7.775	16.75	77	4.254	9.166	19.75
48	3.634	7.830	16.87	78	4.273	9.205	19.83
49	3.659	7.884	16.98	79	4.291	9.244	19.92
50	3.684	7.937	17.10	80	4.309	9.283	20.00
51	3.708	7.990	17.21	81	4.327	9.322	20.08
52	3.733	8.041	17.32	82	4.344	9.360	20.17
53	3.756	8.093	17.44	83	4.362	9.398	20.25
54	3.780	8.143	17.54	84	4.380	9.435	20.33
55	3.803	8.193	17.65	85	4.397	9.473	20.41
56	3.826	8.243	17.76	86	4.414	9.510	20.49
57	3.849	8.291	17.86	87	4.431	9.546	20.57
58	3.871	8.340	17.97	88	4.448	9.583	20.65
59	3.893	8.387	18.07	89	4.465	9.619	20.72
60	3.915	8.434	18.17	90	4.481	9.655	20.80
61	3.936	8.481	18.27	91	4.498	9.691	20.88
62	3.958	8.527	18.37	92	4.514	9.726	20.95
63	3.979	8.573	18.47	93	4.531	9.761	21.03
64	4.000	8.618	18.57	94	4.547	9.796	21.10
65	4.021	8.662	18.66	95	4.563	9.830	21.18
66	4.041	8.707	18.76	96	4.579	9.865	21.25
67	4.062	8.750	18.85	97	4.595	9.899	21.33
68	4.082	8.794	18.95	98	4.610	9.933	21.40
69	4.102	8.837	19.04	99	4.626	9.967	21.47
70	4.121	8.879	19.18				

Example 14 : **By using the cube root table, find the value of:**

(*i*) $\sqrt[3]{3}$ (*ii*) $\sqrt[3]{30}$ (*iii*) $\sqrt[3]{300}$

Solution : Let the given value be x

(*i*) For $x = 3$, $\sqrt[3]{3} = 1.442$ (*ii*) $\sqrt[3]{30} = 3.107$

(*iii*) $\sqrt[3]{300} = \sqrt[3]{10 \times 30} = \sqrt[3]{10} \times \sqrt[3]{30}$

$= 2.154 \times 3.107 = 6.692$

or, $\sqrt[3]{300} = \sqrt[3]{100 \times 3} = \sqrt[3]{100} \times \sqrt[3]{3}$

$= 4.642 \times 12.442 = 6.694$

Example 15 : **Find the cube root of 1,080.**

Solution : Writing 1,080 as the product of its prime factors, we have

$$1{,}080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$$

$$\therefore \sqrt[3]{1080} = \sqrt[3]{2^3} \times \sqrt[3]{3^3} \times \sqrt[3]{5}$$

$$= 2 \times 3 \times \sqrt[3]{5} = 6 \times 1.710 = 10.26$$

Example 16 : **Find the cube root of $\frac{28}{343}$.**

Solution :

$$\sqrt[3]{\frac{28}{343}} = \frac{\sqrt[3]{28}}{\sqrt[3]{343}}$$

$$= \frac{\sqrt[3]{28}}{7} = \frac{3.037}{7} = 0.434$$

Example 17 : **Show that:** $\sqrt[3]{-216 \times 1728} = \sqrt[3]{-216} \times \sqrt[3]{1728}$

Solution : L.H.S. $= \sqrt[3]{-216 \times 1728}$

$= \sqrt[3]{[-(2 \times 2 \times 2 \times 3 \times 3 \times 3)] \times [2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3]}$

$= [-(2 \times 3)] \times [2 \times 2 \times 3] = -6 \times 12 = -72$

R.H.S. $= \sqrt[3]{-216} \times \sqrt[3]{1728}$

$= -\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} \times \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= -(2 \times 3) \times (2 \times 2 \times 3) = -6 \times 12 = -72$

Hence, L.H.S. = R.H.S.

Example 18 : **Find the value of :**

(*i*) $\sqrt[3]{0.064}$ (*ii*) $\sqrt[3]{0.000216}$

2	64
2	32
2	16
2	8
2	4
2	2
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

Solution : (*i*) $\sqrt[3]{0.064} = \sqrt[3]{\frac{64}{1000}} = \frac{\sqrt[3]{64}}{\sqrt[3]{1000}}$

$$= \frac{\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}}{\sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}} = \frac{2 \times 2}{2 \times 5} = \frac{2}{5} = 0.4$$

(ii) $\sqrt[3]{0.000216} = \sqrt[3]{\dfrac{216}{1000000}}$

$= \dfrac{\sqrt[3]{216}}{\sqrt[3]{1000000}}$

$= \dfrac{\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}}{\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}}$

$= \dfrac{2 \times 3}{2 \times 2 \times 5 \times 5} = \dfrac{6}{100} = 0.06$

2	216
2	108
2	54
3	27
3	9
3	3
	1

2	1000000
2	500000
2	250000
2	125000
2	62500
2	31250
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

Example 19 : **What is the smallest number by which 5,400 must be multiplied to make it a perfect cube?**

Solution : The prime factorisation of 5,400 is

$$5{,}400 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 5 \times 5$$

The factors 5×5 do not appear in group of three.

If we multiply 5,400 by 5, the number obtained will be 27,000 whose all prime factors can be grouped as triplets.

$$27{,}000 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$$

Therefore, 27,000 is a perfect cube.

Exercise 6.2

1. Find the cube roots of the following numbers by successive subtraction :

   (i) 64 (ii) 125 (iii) 512 (iv) 216

2. Find the cube root of each of the following numbers by prime factorization method :

   (i) 27 (ii) 27,000 (iii) 1,331 (iv) 15,625

   (v) 17,576 (vi) –6,14,125 (vii) –85,184 (viii) –19,683

3. Find the units digit of the cube roots of the following numbers :

   (i) 74,088 (ii) 18,60,867 (iii) 8,000 (iv) 8,57,375 (v) 13,824

4. Find the cube roots of the following numbers by finding their units and tens digits :

   (i) 12,167 (ii) 15,625 (iii) 1,03,823 (iv) 35,937 (v) 27,000

5. Find the cube roots of the following :

(*i*) $\frac{125}{1,728}$ (*ii*) $\frac{-64}{1,331}$ (*iii*) $\frac{343}{216}$ (*iv*) $\frac{3,375}{512}$

(*v*) $2^6 \times 3^6 \times 5^3$ (*vi*) $(-6)^3 \times (-3)^3$ (*vii*) $11^3 \times 7^3 \times 5^3$

6. Evaluate :

(*i*) $\sqrt[3]{27 \times 216}$ (*ii*) $\sqrt[3]{1331 \times 1728}$ (*iii*) $\sqrt[3]{5^3 \times 6^3}$ (*iv*) $\sqrt[3]{64 \times 125}$

7. Show that:

(*i*) $\sqrt[3]{27 \times 216} = \sqrt[3]{27} \times \sqrt[3]{216}$ (*ii*) $\frac{\sqrt[3]{-64}}{\sqrt[3]{512}} = \sqrt[3]{\frac{-64}{512}}$

8. Estimate the cube roots of the following numbers up to three places of decimal by using . Table 4.1:

(*i*) 9 (*ii*) 14 (*iii*) 25 (*iv*) 38

(*v*) 82 (*vi*) 36 (*vii*) 6,112 (*viii*) 8,042

9. Three numbers are in the ratio 1 : 2 : 3. The sum of their cubes is 62,208. Find the numbers.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

HOTS (Higher Order Thinking Skills)

Experiential Learning

1. The sum of the cubes of three numbers which are in the ratio 2 : 3 : 4 is 33,957. Find the numbers.
2. Find the length of a cube whose volume is 820.584 cubic metres.
3. Find the volume of a cube, each of whose edge measures 4.3 cm.
4. What is the smallest number by which we divide 6,912 so that the quotient becomes a perfect cube. Find the cube root of the quotient.

Revision Exercise

Conceptual Learning

1. Choose the correct option :

(*i*) If the length of the side of cube is 24 cm, its volume is :

(*a*) 13824 m^3 (*b*) 13824 cm^3 (*c*) 138.24 cm (*d*) 138.24 cm^3

(*ii*) The cube of 81 is :

(*a*) 513441 (*b*) 513414 (*c*) 513144 (*d*) 531441

(*iii*) The cube root of 4.096 is :

(*a*) 1.6 (*b*) 16 (*c*) 0.16 (*d*) 0.016

(*iv*) The value of $\sqrt[3]{648} \times \sqrt[3]{576}$ is :

(*a*) 18 (*b*) 72 (*c*) 36 (*d*) 154

2. Find the cube in each of the following :

(*a*) 12 (*b*) 24 (*c*) –49

3. Find the cube root in each of the following using prime factorisation :

(*a*) 15625 (*b*) 0.125 (*c*) $\left(\frac{-27}{343}\right)$ (*d*) 13824

(*e*) 250047 (*f*) 0.008

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4. Find the cube root of each of the following perfect cubes by estimation :

(*a*) 255 (*b*) 698 (*c*) 35 (*d*) 400

5. Find the cube root of each of the following:

(*a*) – 125 (*b*) 2744 (*c*) – 2700 (d) 474552 (e) – 1331

6. Estimate the cube root of the following upto one decimal place:

(*a*) 35 (*b*) 950 (*c*) 850 (d) 7 (e) 80

Mental Maths

Experiential Learning

1. The cube root of 1 is ....
2. Find the value of $\sqrt[3]{\frac{125}{343}}$
3. Find the value of $\sqrt[3]{-125 \times -1331}$.
4. Find the length of each side of a cubical box the volume of which is 61 cm^3.
5. Find the volume of a cube whose surface area is 216 m^2.

Scan to Create Your Own Learning Path

Custom Learning Path

HOTS (Higher Order Thinking Skills)

Experiential Learning

1. What is the smallest number by which 1372 may be multiplied so that the product is a perfect cube?
2. What is the smallest number by which 8640 may be divided so that the quotient is a perfect cube?

Assertion and Reason

Critical Thinking

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false. (*d*) A is false but R is true.

1. **Assertion (A) :** Cube of a natural number is a natural number.

   Reason (R) : Cube of a natural number which is a multiple of 3 is a multiple of 27.

2. **Assertion (A) :** $2^3 = 8$, $4^3 = 64$, $6^3 = 216$.

   Reason (R) : Cube of all even natural numbers are even.

3. **Assertion (A) :** Cubes of all negative numbers are negative.

   Reason (R) : The cube root of 4.913 is 2.7.

4. **Assertion (A) :** $\sqrt[3]{-721} + \sqrt{(8^2)}$ is equal to -7.

   Reason (R) : Cube of a number less than 1 is also less than 1.

5. **Assertion (A) :** The unit digit of the cube root of a perfect cube ending in 7 is 3.

   Reason (R) : Cube of a natural number of the form $3x$ is divisible by 27.

Activity

To find cubes

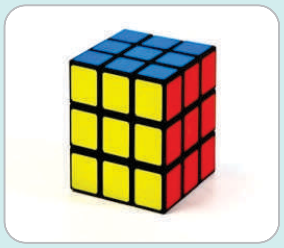
- Make a group of four students, distribute the colour sheets.
- Ask one student to write a number 6 on blue sheet another student write it in cube form like 6^3 or 6 raised to power 3 on green sheet.
- Another student write its expanded form and write the product of $6 \times 6 \times 6 = 216$
- So teacher will explain the cube of a natural number is number raised to the power 3, explain here its prime factors $6 \times 6 \times 6 = 6^3$.
- Make pairs of same factors, write it at once. So we have write 6 one time also explain this is called perfect cube, repeat this for different numbers like 8, 11, 13 a perfect cube if any pair left, then the number would be not a perfect cube.
- Repeat the for some not perfect cube number.
- To make teaching- learning process a success, teacher will make sure that almost all students in the class will participate in this activity.

Skills Developed: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Thinking Skills

The volume of a cube is 729 cubic centimeters.

1. Find the side length of the cube.
2. If the side length of the cube were increased by 2 cm, what would the new volume of the cube be?
3. How does the change in the side length affect the volume? Explain your reasoning.



Skills covered: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Competency based Questions

A hotel plans to build a cubic-shaped storage tank for its water supply. The volume of the tank is $1728 cm^3$. Due to increased demand, the hotel wants to increase the tank size so that the new volume is 8 times the original volume. What will be the new side length of the tank?

A) 24 cm B) 32 cm C) 48 cm D) 64 cm

Skills covered: Interpersonal skills, Observation, Application and Decision making skills

Case Study

A marble factory produces blocks of marble with different sizes, and the factory workers use a specific process to cut the blocks into smaller pieces. The factory has been given a special task to cut a large cube-shaped marble block into smaller cube-shaped pieces.

The factory's manager has the following information:

- The side of the large cube-shaped block is 64 cm.
- The goal is to cut the large cube into smaller cubes, each with a side length of 4 cm.

Based on this information answer the following questions

1. Calculate the volume of the large cube-shaped marble block.
2. How many smaller cubes can be made from the large block?
3. What is the cube root of the volume of the large marble block?
4. If the factory decides to make cubes with a side length of 2 cm instead of 4 cm, how many cubes will be produced now?

Skills covered: Research, Logical Reasoning, Problem-Solving, Practical Application