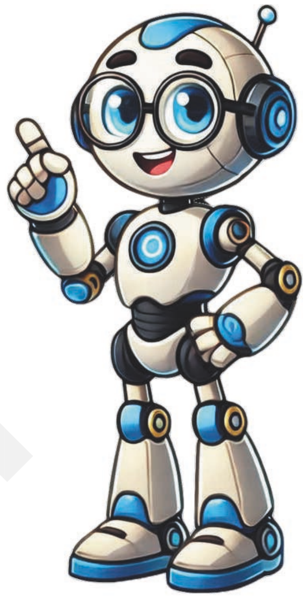
5
Chapter

Squares and Square Roots

We'll cover the following key points:

→ Squares
→ Perfect squares or square numbers
→ Properties of perfect squares
→ Square roots
→ To find the square root of any perfect square number by repeated subtraction
→ To find the square root of any perfect square number by the prime factorization method
→ Square root of a perfect square by the long division method
→ Finding the ones and tens digits of the square root
→ Square root of numbers in decimal form
→ To find the value of square root correct up to certain places of decimal



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Learning Outcomes

By the end of this chapter, students will be able to:

- Understand the concept of square numbers and identify perfect squares.
- Learn and apply the properties of square numbers.
- Calculate squares of numbers up to two digits mentally or using standard methods.
- Identify square roots of perfect squares.
- Use the prime factorization method to find square roots of non-perfect squares.
- Solve real-life problems involving areas of squares and square roots.
- Develop problem-solving skills using patterns in squares and square roots.
- Recognize and use the relationship between squaring and taking square roots.
- Differentiate between perfect and non-perfect squares.
- Apply shortcut techniques like estimating square roots for quick calculations.
- Explore patterns and sequences involving square numbers.
- Solve word problems involving squares and square roots in various contexts.
- Understand and apply the Pythagorean Theorem using square numbers.

Mind Map

SQUARE AND SQUARE ROOTS

Introduction

e.g.,

$a^2 = a \times a$

$5^2 = 5 \times 5 = 25$

$6^2 = 6 \times 6 = 36$

Properties of square numbers

Unit digit in the square of any natural number

$\Rightarrow$ 0, 1, 4, 5, 6, 9

Can never be $\Rightarrow$ 2, 3, 7, 8

e.g.,

$8^2 = 8 \times 8 = 64$

$9^2 = 9 \times 9 = 81$

Some more interesting patterns

i. Adding triangular numbers

1 + 3 = 4

ii. Number between square numbers

There are 2n non perfect square numbers between n^2 and $(n + 1)^2$

iii. Adding odd numbers

$1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

iv. A sum of consecutive natural numbers

$3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

v. Product of two consecutive even or odd natural numbers.

$11 \times 13 = 143 = 12^2 - 1$

$13 \times 15 = 195 = 14^2 - 1$

vi. Some more patterns in square numbers

1^2	=	1
11^2	=	1 2 1
111^2	=	1 2 3 2 1
1111^2	=	1 2 3 4 3 2 1

Find the square of a number

$23^2 = (20 + 3)^2$

$= 20\,(20 + 3) + 3\,(20 + 3)$

$= 20^2 + 20 \times 3 + 3 \times 20 + 3^2$

$= 400 + 60 + 60 + 9$

$= 529$

For any natural number m > 1

2m, $m^2 - 1$, $m^2 + 1$ form triplet e.g., 3, 4, 5

e.g.,

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Square roots

Symbol $\sqrt{\ }$

e.g.,

$\sqrt{9} = 3$

$\sqrt{16} = 4$

- Through repeated subtraction.
- Through prime factorisation
- By division method

Square roots of decimals

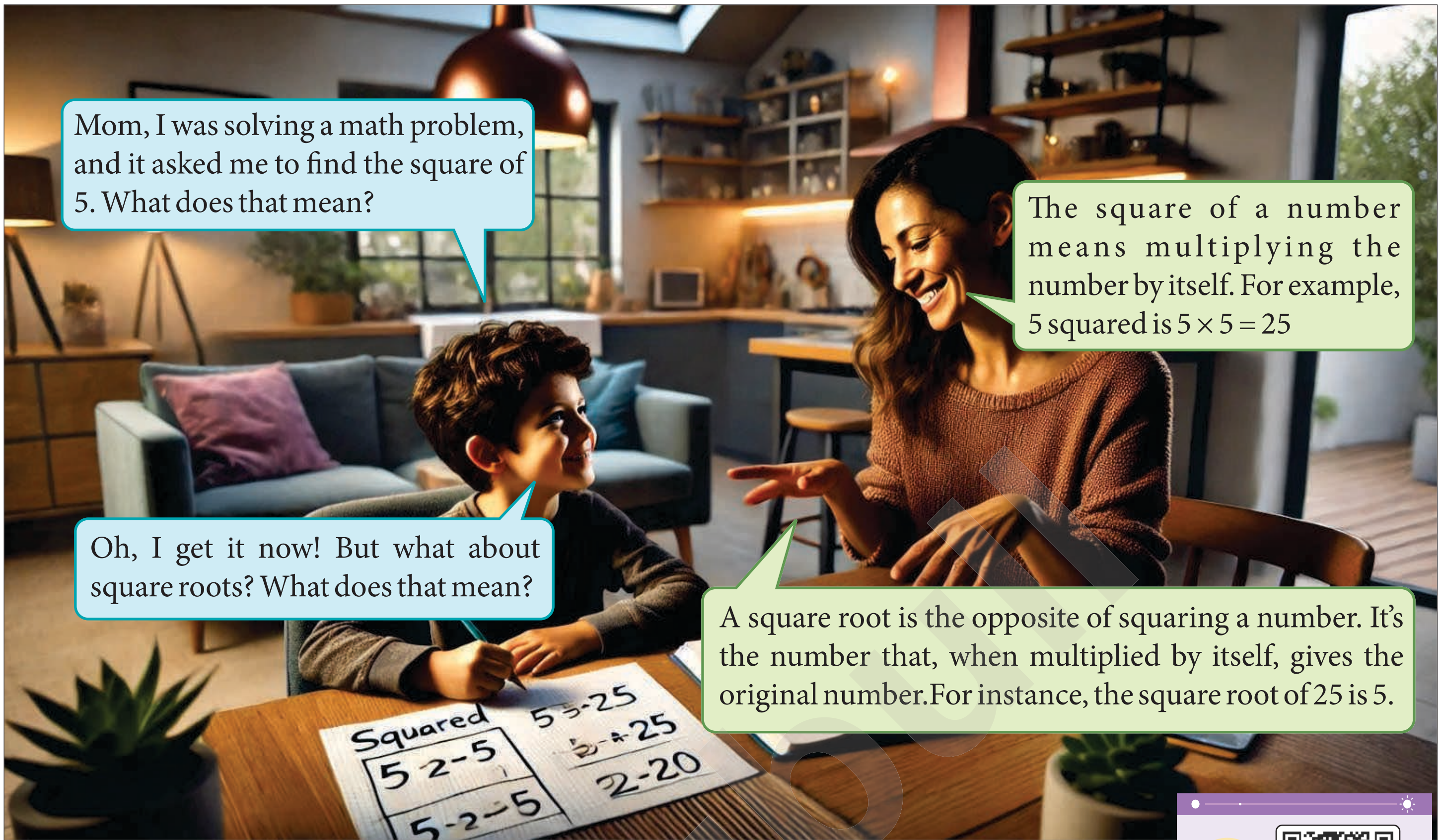
e.g.,

$\sqrt{3.6} = 0.6$

$\sqrt{1.44} = 1.2$

Based on NCERT*

Introduction



Take a Task

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Squares

When a number is multiplied by itself, we say that the number has been squared or the product is the square of that number. In other words, if a is any number, then $a^2 = a \times a$.

Examples : $3^2 = 3 \times 3 = 9$,

$4^2 = 4 \times 4 = 16$,

$6^2 = 6 \times 6 = 36$, etc.

The above examples can be expressed by saying that the *square of 3 is 9*, *the square of 4 is 16* and the square of 6 is 36 and so on, i.e., the square of a number is that number raised to the power 2.

Perfect Squares of Square Numbers

A natural number 'a' is called a perfect square if it is the square of some natural number 'b', *i.e.*, $a = b^2$.

We have $4 = 2^2$; $25 = 5^2$; $64 = 8^2$; $100 = 10^2$

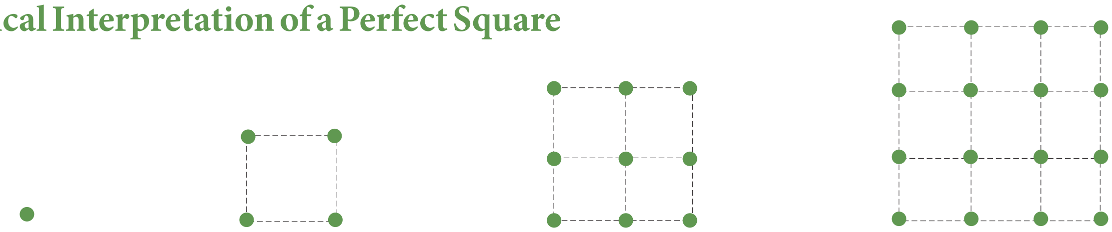
Thus, 4, 25, 64, 100 are the perfect squares of the numbers 2, 5, 8, 10, etc.

To find out whether a given number is a perfect square or not, we write the number as a product of its prime factors. If the number is a perfect square, its factors can be grouped in pairs such that both the factors in each pair are equal.

If a natural number a is a perfect square, then there exist a natural number such that $b^2 = a$.

9 is a perfect square because there is a natural number 3 such that $3^2 = 9$.

Geometrical Interpretation of a Perfect Square



$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$

For every square number, there is a corresponding geometrical figure of a square.

How to Check a Perfect Square ?

Working Rules

1. Write the number as the product of its prime factors.
2. If the number is a perfect square, group the prime factors into pairs.
3. If not, the number is not a perfect square.

Numbers between Consecutive Square Numbers

Let us see some examples of non-square numbers between two consecutive square numbers in the following table.

Squares of Natural Consecutive Numbers	Non-square Numbers between Consecutive Square Numbers	Number of Non-square Numbers between Consecutive Square Numbers
1 and 2; $1^2 = 1, 2^2 = 4$ 2 and 3; $2^2 = 4, 3^2 = 9$ 3 and 4; $32 = 9, 4^2 = 16$ M $m^2, (m+1)^2$	Between 1 and 4 : 2, 3 Between 4 and 9 : 5, 6, 7, 8 Between 9 and 16 : 10, 11, 12, 13, 14, 15	$2 = 2 \times 1$ $4 = 2 \times 2$ $6 = 2 \times 3$ M $2m = 2 \times m$

From the above table, the number of non-square numbers between the squares of two consecutive natural numbers m and $(m + 1)$ is equal to $2m$. We can express the difference between two consecutive square numbers m^2 and $(m+1)^2$ as

$$\{(m+1)^2 - m^2\} = (m+1+m)(m+1-m) = 2m+1$$

Subtracting 1 from both the sides, we get $\{(m+1)^2 - m^2\} - 1 = 2m$

Therefore, the number of non-square numbers (*i.e.*, $2m$) between two consecutive square numbers m^2 and $(m+1)^2$ is 1 less than their difference.

Example 1: **Are 784 and 3,675 perfect squares? If so find the number whose square are 784 and 3,675.**

Solution : Resolving 784 into prime factors, we get

$784 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

Since no factor is left after grouping the factors into doublets, 784 is a perfect square.

2	784
2	392
2	196
2	98
7	49
7	7
	1

Clearly, 784 is square of $2 \times 2 \times 7 = 28$.

Resolving 3,675 into prime factors, we find that

3,675 $= 3 \times 5 \times 5 \times 7 \times 7$

Making pairs of equal factors, we find that 3 is left over.

These, 3,675 is not a perfect square.

5	3675
5	735
3	147
7	49
7	7
	1

Properties of Perfect Squares

The squares of natural numbers obey several interesting properties. Some of them are given below.

PROPERTIES	EXAMPLES
1. A number ending with 2, 3, 7 or 8 can never be a perfect square.	1. None of 32, 23, 57 and 68 are perfect squares.
2. A number ending in an odd number of zeros is never a perfect square.	2. None of 10, 34,000, 4,68,00,000 are perfect squares.
3. Squares of even numbers are always even.	3. Square of 8 is 64. (Both 8 and 64 are even.)
4. Squares of odd numbers are always odd.	4. Square of 7 is 49. (Both 7 and 49 are odd.)
5. Square of a natural number n is equal to the sum of the first n odd numbers. In general, n^2 = Sum of the first n odd numbers.	5. $1^2 = 1$ = Sum of the first odd number. $2^2 = 4 = 1 + 3$ = Sum of the first 2 odd numbers. $3^2 = 9 = 1 + 3 + 5$ = Sum of the first 3 odd numbers, etc.
6. For every natural number n, we have: $(n + 1)^2 - n^2 = (n + 1 + n)(n + 1 - n) = (2n + 1)$	6. $3^2 - 2^2 = 3 + 2 = 5$, $15^2 - 14^2 = 15 + 14 = 29$ Observe that obtained number is the sum of the given numbers.
7. Square of a proper fraction is smaller than the fraction.	7. $\left(\frac{3}{5}\right)^2 = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$ and $\frac{9}{25} < \frac{3}{5}$, since $(9 \times 5) < (25 \times 3)$.
8. The square of a natural number (other than 1) is a multiple of 3 or exceeds a multiple of 3 by 1.	8. $4^2 = 16 = (3 \times 5) + 1$, $5^2 = 25 = (3 \times 8) + 1$
9. The square of a natural number (other than 1) is a multiple of 4 or exceeds a multiple of 4 by 1.	9. $4^2 = 16 = (4 \times 4)$, $5^2 = 25 = (4 \times 6) + 1$,
10. **Pythagorean Triplet:** A triplet (a, b, c) of three natural numbers a, b and c is called a Pythagorean triplet, if $a^2 + b^2 = c^2$. For any natural number m, $m > 1$ $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.	10. Taking $m = 4$, we find that (8, 15, 17) is a Pythagorean triplet.

Example 2 : **Find the Pythagorean triplet whose smallest member is 10.**

Solution : For every natural number $m > 1$, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.

Putting $2m = 10$, *i.e.*, $m = 5$, we get the triplet (10, 24, 26).

Example 3 : **Find the smallest number by which 504 should be multiplied to make it a perfect square.**

2	504
2	252
2	126
3	63
3	21
7	7
	1

Solution : Let us find the factors of 504.

$$504 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times 7$$

Making pairs of the factors of 1008, we find that 2 and 7 do not have a pair.

If 504 is multiplied by 14, then all the factors of 504 will become pairs and the number so obtained will be a perfect square.

Therefore, the smallest number by which 504 should be multiplied to make it a perfect square is 14.

Example 4 : **Prove that the following numbers are not perfect squares :**

(*i*) 8,927 (*ii*) 2,058 (*iii*) 43,453 (*iv*) 52,222 (*v*) 260

Solution : (*i*) The number 8,927 ends in 7, so it is not a perfect square.

(*ii*) The number 2,058 ends in 8, so it is not a perfect square.

(*iii*) The number 43,453 ends in 3, so it is not a perfect square.

(*iv*) The number 52,222 ends in 2, so it is not a perfect square.

(*v*) The number 260 has odd number of zeros at the end, so it is not a perfect square.

Example 5 : **Find the least square number which is divisible by 4, 8 and 12.**

Solution : The LCM of 4, 8 and 12 is 24.

24 is the least number divisible by 4, 8 and 12. By prime factorisation, we get

$$24 = \underline{2 \times 2} \times 2 \times 3$$

To make it a perfect square, it must be multiplied by $(2 \times 3 =)$ 6.

i.e., $24 \times 6 = 144$

Therefore, 144 is the smallest perfect square divisible by 4, 8 and 12.

Short Method of Squaring Numbers

For large numbers, multiplication may prove to be laborious and time consuming,. In this section, we will find the square of two or three-digit numbers quickly without actual multiplication.

We use the method for squaring a two digit number with the help of the identity; $(a + b)^2 = a^2 + 2ab + b^2$.

To square a two digit number *ab* (where *a* and *b* are the tens digit and units digit respectively), we write a^2, $2ab$ and b^2 respectively in these columns as follows:

Let us take $(ab)^2 = (45)^2$.

Column I	Column II	Column III
a^2 $(4^2 = 16)$	$2ab$ $(2 \times 4 \times 5)$	b^2 $(5 \times 5) = 25$

Then we go through the following steps:

Working Rules

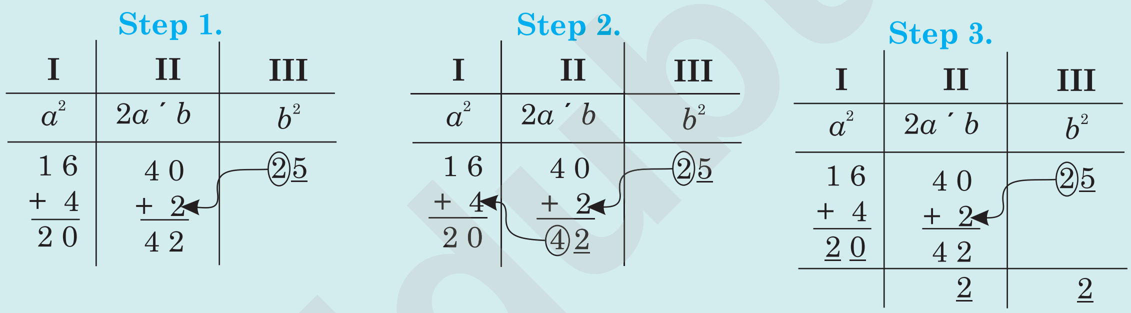
Step 1. Underline the units digit of b^2 (in column III) and take over the tens digit of b^2 and add to $2a \times b$.

Step 2. Underline the units digit of $2ab$ and b^2 added in column II and take over the remaining digits of $2ab$, if any, to column I and add with a^2.

Step 3. Underline all the digits in column I.

The underlined digits give the required square, *i.e.*,

$45^2 = 2{,}025$



Exercise 5.1

1. Find the perfect square numbers between (*i*) 10 and 40 (*ii*) 50 and 160.
2. Using prime factorization method, find which of the following numbers are perfect squares.
   49; 121; 332; 729; 1575; 576; 1,225; 1,000; 1,428
3. Which of the following numbers are not perfect squares?
   121; 81; 55; 144; 217; 69; 3,200; 1,600; 4,000; 8,100
4. Which of the following numbers square would end with digit 1?
   1; 77; 82; 123; 109; 161
5. Which of the following perfect squares are squares of even numbers?
   576; 484; 169; 1,296; 676; 900; 625; 1,331
6. Which of the following perfect squares are squares of odd numbers?
   729; 225; 196; 121; 289; 49; 1,089; 784; 1,600; 1,936

7. Find the square of the following numbers :

(*i*) $\frac{2}{3}$ (*ii*) $\frac{13}{25}$ (*iii*) $\frac{101}{220}$ (*iv*) 1.15 (*v*) 12.1

8. Find the smallest number by which each of the given number:

(*i*) is multiplied so that product is a perfect square.

(*a*) 512 (*b*) 1,323

(*ii*) is divided so that the result is a perfect square.

(*a*) 1,575 (*b*) 6,912

9. Fill in the missing numbers in the following patterns :

(*i*)

$7^2 = 49$

$67^2 = 4,489$

$667^2 = ...$

$6,667^2 = ...$

$66,667^2 = ...$

(*ii*) Express the following as sum of squares of two numbers.

(*a*) $25^2 = ... + ...$

(*b*) $13^2 = ... + ...$

(*c*) $41^2 = ... + ...$

(*d*) $26^2 = ... + ...$

(*iii*)

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$4^2 + 5^2 + ...^2 = 21^2$

$6^2 + ...^2 + 42^2 = 43^2$

10. Evaluate.

(*i*) $(15)^2 - (14)^2$ (*ii*) $(34)^2 - (33)^2$ (*iii*) $(732)^2 - (731)^2$ (*iv*) $(112)^2 - (111)^2$

(*v*) $(2,115)^2 - (2,114)^2$ (*vi*) $(1,002)^2 - (1,001)^2$

11. Find the Pythagorean triplets, one of whose members is:

(*i*) 24 (*ii*) 99 (*iii*) 82 (*iv*) 14

12. Find the square of the following numbers using square method :

(*i*) 12 (*ii*) 25 (*iii*) 37 (*iv*) 43

(*v*) 54 (*vi*) 98

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Square Roots

If $a = b^2$, then it can be read as *a* is square of *b*. If we take the reverse direction - form the square of a number to the number itself, we call *b* is a square root of *a*. In other words, the square root of a given number *n* is that natural number which when multiplied by itself gives *n* as the product.

Thus, square root of 9 is 3, since $9 = 3^2$.

Take a Task

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Since, $3^2 = (-3)^2 = 9$. Therefore, both 3 and –3 are the square roots of 9. Similarly, other square numbers also have two square roots, *i.e.*, one positive and the other negative. The positive square root of a number is also called *principal square root*.

Through out the book, we will consider only the positive square root.

The symbol '$\sqrt{\ }$' stands for the positive square root. Thus, $\sqrt{4} = 2$. It is not correct to write $\sqrt{4} = -2$.

If a is a perfect square, then its square root is an integer. Otherwise, a does not have an integral square root. Throughout this section, square stands for perfect square and square root means integral square root.

Remark - We take $\sqrt{36} = 6$ and not $\sqrt{36} = \pm 6$.

First ten perfect squares and their square roots are given below:

Perfect Square (n)	*Square Root* ($\sqrt{n}$)	*Perfect Square* (n)	*Square Root* ($\sqrt{n}$)
1	$\sqrt{1} = 1$	36	$\sqrt{36} = 6$
4	$\sqrt{4} = 2$	49	$\sqrt{49} = 7$
9	$\sqrt{9} = 3$	64	$\sqrt{64} = 8$
16	$\sqrt{16} = 4$	81	$\sqrt{81} = 9$
25	$\sqrt{25} = 5$	100	$\sqrt{100} = 10$

Observe that:

(*i*) 9, 49 and 81 are odd numbers. Their square roots 3, 7 and 9 respectively are also odd numbers.

(*ii*) 16, 64 and 100 are even numbers. Their square roots 4, 8 and 10 respectively are also even numbers.

(*iii*) $\frac{16}{25}$ is a rational number. Its square root $\frac{4}{5}$ is also a rational number. Symbolically, $\sqrt{\frac{p^2}{q^2}} = \frac{p}{q}$.

(*iv*) 0.021 is a decimal number. Its square root 0.014 is also a decimal number.

To Find the Square Root of any Perfect Square Number by Repeated Subtraction

We know that the sum of first n odd numbers is n^2.

i.e., $1 + 3 + 5 .... + (2n - 1) = n^2$

To find the square root of any number, we subtract from it the odd numbers 1, 3, 5, ..., successively. If the given number is a perfect square, we will get zero at some stages.

The number of times we have to perform subtraction to arrive at zero gives the square root of the given number.

This is the simplest method for finding the square root of a perfect square.

Example 6 : **Find the square roots of 121 and 169 by the method of repeated subtraction.**

Solution : (*i*)

$121 - 1 = 120$	$120 - 3 = 117$
$117 - 5 = 112$	$112 - 7 = 105$
$105 - 9 = 96$	$96 - 11 = 85$
$85 - 13 = 72$	$72 - 15 = 57$
$57 - 17 = 40$	$40 - 19 = 21$
$21 - 21 = 0$	

Thus, total number of subtraction is 11.

Therefore, $\sqrt{121} = 11$.

(ii)

$169 - 1 = 168$	$168 - 3 = 165$
$165 - 5 = 160$	$160 - 7 = 153$
$153 - 9 = 144$	$144 - 11 = 133$
$133 - 13 = 120$	$120 - 15 = 105$
$105 - 17 = 88$	$88 - 19 = 69$
$69 - 21 = 48$	$48 - 23 = 25$
$25 - 25 = 0$	

Thus, total number of subtraction is 13. Therefore, $\sqrt{169} = 13$.

Remark : For any two perfect squares *m* and *n*, we have

(*i*) $\sqrt{mn} = \sqrt{m} \times \sqrt{n}$ (*ii*) $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$ $(n \neq 0)$

To Find the Square Root of a Perfect Square By the Prime Factorization Method

Working Rules

Step 1. Find the prime factors of the given number.

Step 2. Make pairs of similar factors.

Step 3. Choose one factor from each pair and find their product.

Example 7 : **Find the square root of 1,764.**

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Solution : By prime factorization method, we get

$$1764 = \underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}$$

$$= 2^2\times3^2\times7^2$$

$$\therefore \sqrt{1764} = 2\times3\times7 = 42.$$

Example 8 : **Find the smallest number by which 1,125 should be multiplied so as to get a perfect square. Also, find the square root of the square number so obtained.**

Solution: Prime factorization of $1,125 = \underline{3\times3}\times\underline{5\times5}\times5$

Here, we find that the prime factor 5 does not occur in a pair.

If we multiply 1125 by 5, then

$1,125\times5 = \underline{3\times3}\times\underline{5\times5}\times\underline{5\times5}$

Now, each prime factor occurs in a pair.

Therefore, $1,125\times5$, *i.e.*, 5,625 is a perfect square.

Thus, the required smallest number = 5.

Also, $\sqrt{5625} = 3\times5\times5 = 75.$

5	1125
5	225
5	45
3	9
3	3
	1

Example 9: **Find the value of :**

(*i*) $\sqrt{147}\times\sqrt{243}$ (*ii*) $\sqrt{980}\div\sqrt{1620}$

Solution: (*i*) $\sqrt{147}\times\sqrt{243} = \sqrt{147\times243}$

$$= \sqrt{3\times7\times7\times3\times3\times3\times3\times3} = 3\times3\times3\times7 = 189$$

(*ii*) $\sqrt{980}\div\sqrt{1620} = \dfrac{\sqrt{980}}{\sqrt{1620}}t$

$$= \sqrt{\frac{980}{1620}} = \sqrt{\frac{2\times2\times5\times7\times7}{2\times2\times5\times3\times3\times3\times3}}$$

$$= \sqrt{\frac{7\times7}{3\times3\times3\times3}} = \frac{7}{3\times3} = \frac{7}{9}$$

Example 10 : **Find the least square number which is exactly divisible by each of the numbers 8, 9 and 10.**

Solution : The least square number divisible by each one of 8, 9 and 10 is their LCM.

LCM of 8, 9 and 10 = $\underline{2\times2}\times2\times\underline{3\times3}\times5 = 360$

Prime factorization of $360 = \underline{2\times2}\times\underline{3\times3}\times2\times5$

So, we need to make pairs of 2 and 5 to make perfect square.

$\therefore$ 360 should be multiplied by 2×5.

Hence, the required square number is $360\times10 = 3,600$.

2	8,	9,	10
2	4,	9,	5
2	2,	9,	5
3	1,	9,	5
3	1,	3,	5
5	1,	1,	5
	1,	1,	1

Exercise 5.2

1. Find the square roots of the following numbers by prime factorization method :

   (*i*) 256 (*ii*) 900 (*iii*) 6,561 (*iv*) 41,616

   (*v*) 8,281 (*vi*) 5,929 (*vii*) 8,464 (*viii*) 10,000

2. Find the square root of the following :

   (*i*) $\frac{676}{625}$ (*ii*) $\frac{961}{25}$ (*iii*) $\frac{64}{169}$ (*iv*) $1\frac{396}{9,604}$

   (*v*) 31.36 (*vi*) 0.0225 (*vii*) 0.000049

3. Find the smallest number by which 3,528 must be multiplied so that the product becomes a perfect square.

4. By which smallest number should 3,528, 1,152 and 7,776 be divided so that the resulting number is a perfect square? Find the square root of the perfect square so obtained.
   (Hint: Divide by the number which is not a pair in the prime factorization.)

5. Find the least square divisible by each one of the digits of the following :

   (*i*) 4, 5 and 8 (*ii*) 16, 15 and 20 (*iii*) 16, 18 and 45

6. Find the smallest number by which 9,126 must be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square so obtained.

7. In a garden there are as many trees in a row as there are number of rows in the garden. If there are 1,024 trees in they garden, find the number of trees in a row.

8. Find the smallest number by which 4,860 must be divided so that it becomes a perfect square. Also, find the square root of the perfect square so obtained.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Square Root of a Perfect Square by the Long Division Method

Finding square roots by the prime factorization method is efficient only when the number has small prime factors. For large and complex numbers, it becomes difficult and time consuming to obtain prime factors. To overcome this difficulty, we use an alternative method called the 'Long Division Method'.

Working Rules

Step 1. Divide the given number into pairs of two digits starting with the digit at the ones place. Each pair and the remaining one digit (if any) is called a *period*.

Step 2. Choose of a whole number whose square is equal to or just less than the first period. This number will become the divisor and the quotient.

Step 3. Write the product of the divisor and the quotient in the first period below the given period. Subtract and bring down the next period to the right of the remainder. This becomes the new dividend.

Step 4. Now, the new divisor is obtained by taking two times the quotient and annexing a suitable digit which is also taken as the next digit of the quotient, chosen in such a way that the product of the new divisor and this digit is equal to or just less than the new dividend.

Step 5. Repeat steps 2, 3 and 4 till all the periods have taken up. Now, the quotient so obtained is the required square root of the given number.

Example 11 : **Find the square root of 49,37,284 and 6,43,204.**

Solution: Making periods and using division method, we have

	2 2 2 2
2	$\overline{4}\ \overline{93}\ \overline{72}\ \overline{84}$
	4
42	93
	84
442	9 72
	8 84
4442	88 84
	88 84
	0

	8 0 2
8	$\overline{64}\ \ \overline{32}\overline{04}$
	64
1602	32 04
	32 04
	0

1. In 49,37,284, we pair the digits from the right end till 4 is left. We find a number whose square is equal to or less than 4 and write its square root, divisor and quotient.
2. In 6,43,204, at the second division step, 1602 is greater than 32, so we bring down 04 and placed 0 at both dividend and quotient.

Thus, $\sqrt{4937284} = 2{,}222$ and $\sqrt{643204} = 802$

Note

A quicker way to determine the number of digits in the square root of a square number is to place a bar over every pair of digits starting from the units digit. If the number of digits in the given number is odd, then the leftmost single digit too has a bar. The number of bars is the number of digits in the square root of given number.

For example, if $n = 441$ then $\sqrt{n}$ has two digits as there are two bars in $\overline{4}\,\overline{41}$.

Example 12 : **What least number should be subtracted from 2,361, so that the resulting number becomes a perfect square ?**

Solution : Let us try to find the square root of 2,361.

	4 8
4	$\overline{23}\ \overline{61}$
	16
88	7 61
	704
	57

This shows that $(48)^2$ is less than 2,361 by 57. So, 57 must be subtracted from 2,361 to get a perfect square.

$\therefore$ The required least number is 57.

Example 13 : **What least number must be added to 6,710 to make the sum a perfect square?**

Solution : Let us find the square root of 6,710.

	81
8	$\overline{67}\,\overline{10}$
	64
161	310
	161
	149

From the solution, it is clear that 6,710 is greater than $(81)^2$ but less than $(82)^2$.

Thus, the required number to be added

$= (82)^2 - 6710$

$= 6724 - 6710 = 14.$

Example 14 : **Find the greatest 6-digit number which is a perfect square.**

Solution: The greatest 6-digit number is 9,99,999. Now we try to find whether 9,99,999 is a perfect square.

	9 9 9
9	99 99 99
	81
189	18 99
	17 01
1989	1 98 99
	1 79 01
	19 98

We get remainder 1,998 showing that 9,99,999 is not a perfect square.

Subtracting 1,998 from 9,99,999 we can make it a perfect square.

We get 999999 – 1998 = 9,98,001.

Finding the Ones and Tens Digits of the Square Root

We find the ones and tens digits of the square root of a given number and then the actual square root.

Working Rules

Step 1. Guess the possible units digits of the square root of the given number.

Step 2. Find the greatest number whose square is equal to or less than the number obtained after striking off the ones and tens digits of the given number. This is the tens digit of the squared number.

Step 3. After combining the ones and tens digits, there are two possible numbers as the square root. Square one of them to get the right answer.

Example 15 : **Find the square root of 256.**

Solution :

Step 1. The ones digit of the square root may be 4 or 6.

Step 2. Deleting the ones and tens digits of 256, we are left with the number 2. The tens digit of the square root of 256 is 2. (The largest square < 2 is 1.)

Step 3. The square root of 256 is probably 14 or 16. By multiplying, we confirm that

$$\sqrt{256} = 16.$$

Example 16 : **Find the square root of 5625.**

Solution :

Step 1. Ones digit = 5

Step 2. Tens digit = 7 [Striking off 2 and 5 from 5,625, we are left with 56, and 7 is the number whose square 49 < 56.]

Step 3. The square root of 5,625 is 75.

$$\sqrt{5625} = 75$$

Exercise 5.3

1. Using the division method, find the square root of the following numbers :
   (*i*) 841 (*ii*) 4,624 (*iii*) 1,024 (*iv*) 8,649
   (*v*) 7,921 (*vi*) 15,129 (*vii*) 55,225 (*viii*) 9,74,169
2. Find the least number which must be subtracted from the following to make them a perfect square :
   (*i*) 1,989 (*ii*) 1,19,766 (*iii*) 6,249 (*iv*) 1,525 (*v*) 2,73,682
3. Find the least number which must be added to the following to make them a perfect square :
   (*i*) 6,708 (*ii*) 45,15,600 (*iii*) 7,912 (*iv*) 3,90,615
4. Find the missing digit in 1,102* so that the number becomes a perfect square.
5. The area of a square field is 7,20,801 m^2. A man cycles along its boundary at 18 km/hr. In how much time will he return to the starting point?
6. Find the greatest number of 4 and 5 digits which is a perfect square. Find also the square root of these numbers.
7. Find the least numbers of 4 and 5 digits which are perfect squares. Also find the square root of these numbers.
8. Find the square root of the following by finding the ones and the tens digits :
   (i) 121 (ii) 625 (iii) 841 (iv) 676
9. 1525 students are to be arranged in rows to form a square. Find the number of students in each row and the number that are left out.
10. The area of a square field is 4,761 m^2. A rectangular field whose length is five times its breadth had its perimeter equal that of a square field. Find the area of a rectangular field.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

HOTS (Higher Order Thinking Skills)

Critical Thinking

1. Find the smallest number by which 4,410 must be multiplied to make it a perfect square. Also find the square root of the perfect square so obtained.
2. Find the smallest number by which 5,776 must be divided to make it a perfect square. Also find the square root of the perfect square so obtained.
3. A garden has many trees in each row as there are rows in the garden. If the total number of trees in the garden is 7,921, find the number of rows of trees.

Take a Task

Watch Remedial

Square Root of Numbers in Decimal Form

In order to find the square root of a number in decimal form, we take the following steps.

Working Rules

Step 1. Consider the decimal number is made of 2 parts : integral and decimal part.

Step 2. Place bars on the periods as usual in the integral part.

Step 3. Make the number of decimal places even by affixing single or more number of zeros at the end.

Step 4. Similarly mark off the periods in the decimal part beginning with the first decimal place.

Step 5. Start finding square root by the long division method and place the decimal point in the square root as soon as the integral part is exhausted.

Step 6. Complete the process as usual.

Example 17 : Find the square root of 0.0 24025.

Solution: In the given number, the number of decimal places is already even. So, mark off periods and find the square root as shown along side:

	0 . 1 5 5
1	$0\,.\,\overline{02}\,\overline{40}\,\overline{25}$
	1
25	1 40
	1 25
305	15 25
	15 25
	0

$\therefore \sqrt{0.024025} = 0.155.$

Example 18: Find the square root of 12.4609.

Solution: In the given number, the number of decimal places is already even. So, mark off the periods and proceed as follows to find the square root.

	3. 5 3
3	$\overline{12}.\overline{46}\,\overline{09}$
	9
65	3 46
	3 25
703	21 09
	21 09
	0

$\therefore \sqrt{12.4609} = 3.53.$

To Find the Value of Square Root Correct up to Certain Places of Decimal

Rule : If the square root is required correct up to two places of decimal, we find it up to 3 places of decimal and then round it off up to two places of decimal.

Similarly, if the square root is required correct up to two places of decimal, we find it up to 4 places of decimal and then round it off up to three places of decimal, and so on.

Example 19 : Find the square root of 2 correct to two places of decimal.

Solution: Since we have to find the square root of 2 correct to two places of decimal, we shall first find the square root of 2 up to three places of decimal. For this purpose, we affix 6 zeros to the right of the decimal point. So, we write 2 as 2.000000.

Now, mark off periods and start finding square root as shown below:

	1. 4 1 4
1	$2.\overline{00}\ \overline{00}\ \overline{00}$
	1
24	1 00
	96
281	4 00
	2 81
2824	1 19 00
	1 12 96
	6 04

$\therefore \quad \sqrt{2} = 1.414$ up to three places of decimal.

Hence, $\sqrt{2} = 1.41$.

Estimating Square Root

Consider the following situations :

1. Rahul has a square field of area 150 m^2. He wants to know whether the field is of side 12 m each or not.
2. If we want to find the square root of 250, we first estimate that the square of which number approximates to 250.

   We know that $100 < 250 < 400$ and $\sqrt{100} = 10$ and $\sqrt{400} = 20$.

   So, $10 < \sqrt{250} < 20$

   But still we are not very close to the square number.

   We know that, $15^2 = 225$ and $16^2 = 256$.

   Therefore, $15 < \sqrt{250} < 16$ and 256 is much closer to 250 than 225.

   So, $\sqrt{250}$ is approximately 16.

Example 20 : Find the square root of 238 to the nearest integer.

Solution : $15^2 = 225$ and $16^2 = 256$

Since, $\quad 225 < 238 < 256$

$\therefore \quad 15 < \sqrt{238} < 16$

But 238 is closer to 225 than 256,

So, $\sqrt{238}$ is approximately 15.

Estimating Square Roots

x	$\sqrt{x}$
1	1.000
2	1.414
3	1.732
4	2.000
5	2.236
6	2.449
7	2.646
8	2.828
9	3.000
10	3.162
11	3.317
12	3.464
13	3.606
14	3.742
15	3.873
16	4.000
17	4.123
18	4.243
19	4.359
20	4.472
21	4.583
22	4.690
23	4.796
24	4.899
25	5.000
26	5099
27	5.196
28	5.292
29	5.385
30	5.477
31	5.568
32	5.657

x	$\sqrt{x}$
33	5.745
34	5.831
35	5.916
36	6.000
37	6.083
38	6.164
39	6.245
40	6.325
41	6.403
42	6.481
43	6.557
44	6.633
45	6.708
46	6.782
47	6.855
48	6.928
49	7.000
50	7.071
51	7.141
52	7.211
53	7.280
54	7.384
55	7.416
56	7.483
57	7.550
58	7.616
59	7.681
60	7.746
61	7.810
62	7.874
63	7.937

x	$\sqrt{x}$
64	8.000
65	8.062
66	8.124
67	8.185
68	8.246
69	8.307
70	8.367
71	8.426
72	8.485
73	8.544
74	8.602
75	8.660
76	8.718
77	8.775
78	8.832
79	8.888
80	8.944
81	9.000
82	9.055
83	9.110
84	9.165
85	9.220
86	9.274
87	9.327
88	9.381
89	9.434
90	9.487
91	9.539
92	9.592
93	9.644
94	9.695

Exercise 5.4

1. Find the value of each of the following:

(*i*) $\sqrt{0.9025}$ (*ii*) $\sqrt{12.25}$ (*iii*) $\sqrt{2.56}$ (*iv*) $\sqrt{0.002304}$

(*v*) $\sqrt{37.0881}$ (*vi*) $\sqrt{7260.7441}$ (*vii*) $\sqrt{7.29}$ (*viii*) $\sqrt{0.00053361}$

2. Find the square roots of the following numbers correct up to two places of decimal:

(*i*) 3 (*ii*) 7 (*iii*) 20 (*iv*) 23.1

3. Find the value of each of the following correct up to three places of decimal:

(*i*) $\sqrt{8}$ (*ii*) $\sqrt{473.56}$ (*iii*) $\sqrt{0.000001}$ (*iv*) $\sqrt{0.675}$

4. A decimal fraction is multiplied by itself. If the product is 51.84, find the fraction.
5. The area of a square paper is 331.24 cm^2. Find the length of boundary to be drawn.
6. The area of a square field is 1 m^2. Find the length of one side of the field.
7. A decimal fraction is multiplied by itself. If the product is 61.7796, find the fraction.
8. Estimate the value of the following to the nearest whole numbers :

(*i*) $\sqrt{97}$ (*ii*) $\sqrt{520}$ (*iii*) $\sqrt{1,100}$ (*iv*) $\sqrt{6,800}$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Revision Exercise

Conceptual Learning

1. Tick (✓) the correct option:

(*i*) Which of the following is not a perfect square number?

(*a*) 4 ☐ (*b*) 81 ☐ (*c*) 163 ☐ (*d*) 225 ☐

(*ii*) The square of which of the following would be even number?

(*a*) 131 ☐ (*b*) 7779 ☐ (*c*) 1057 ☐ (*d*) 2826 ☐

(*iii*) What will be the 'unit's digit' in the square of 1234?

(*a*) 2 ☐ (*b*) 6 ☐ (*c*) 8 ☐ (*d*) 9 ☐

(*iv*) The square of 345 is :

(*a*) 119205 ☐ (*b*) 119025 ☐ (*c*) 191025 ☐ (*d*) 191205 ☐

(*v*) What will be the value of 'x' in Pythagorean triplet (6, 8, x)?

(*a*) 5 ☐ (*b*) 7 ☐ (*c*) 10 ☐ (*d*) 11 ☐

(*vi*) Which is the greatest 4 digit perfect square?

(*a*) 9801 ☐ (*b*) 9800 ☐ (*c*) 9960 ☐ (*d*) 9999 ☐

2. Which least number should be subtracted from 629 to make it a perfect square? :

3. Find the smallest 4-digit number which is a perfect square :

4. Find the greatest 5-digit number which is a perfect square :

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Mental Maths

Experiential Learning

1. What least number must be subtracted from 984 to make it a perfect square?
2. What least number must be added to 6,072 to make it a perfect square?
3. In an auditorium $(1482 + x)$ chairs are placed such that the number of rows is equal to the number columns. Find the least value of x.

Custom Learning Path

Scan to Create Your Own Learning Path

Assertion and Reason

Experiential Learning

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct.

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

1. **Assertion (A) :** The square of a number is that number raised to the power 2.
   Reason (R) : A natural number is called a perfect square, if it is square of a number.
2. **Assertion (A) :** $2^2 = 4$, $8^2 = 64$, $12^2 = 144$, $14^2 = 196$, etc.
   Reason (R) : Squares of even numbers are always even.
3. **Assertion (A) :** A number ending in an odd number of zeroes is never a perfect square.
   Reason (R) : The square root of 1949 is 43 .
4. **Assertion (A) :** If we divide 252 by 7, the quotient is also a perfect square.
   Reason (R) : The square of a positive or negative number is always a positive number.
5. **Assertion (A) :** The square root of $6\frac{19}{25}$ is $3\frac{3}{5}$.
   Reason (R) : The greatest perfect square of six-digit number is 198001.

Activity

Square Roots:

Make a group of five students distribute colour sheet to each student.

Ask one student to write the number 81.

Now teacher explain the step to perform square root of a natural number. Ask another student to resolve the 81 into its prime factor on blue sheet.

Then student show the sheet

$81 = 3 \times 3 \times 3 \times 3$

Ask another student to make pairs of same factors with the help of red sketch.

Ask another student to write the product of prime factors choosing one out of each pair.

Then student answer

$3 \times 3 = 9$

Teacher can explain the class that square root of 81 is 9.

Similarly, ask another eight student to calculate square root by same steps of number 525.

To make teaching-learning process a success, teacher will make sure that almost all students in the class will participate in this activity.

Skills covered: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Thinking Skills

1. The area of a square is 3 times the area of another square. If the side length of the smaller square is 5 cm, what is the side length of the larger square?
2. The height of a triangle is equal to the side length of a square. The area of the triangle is half the area of the square. If the area of the square is 81 cm^2, what is the height of the triangle?
3. Consider a sequence of numbers where each number is the square of a natural number. The sequence starts as follows: 1, 4, 9, 16, 25, 36…

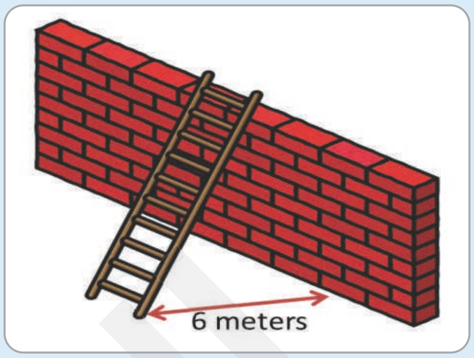
Based on this information answer the following questions:

1. Find the 15th number in this sequence.
2. The 10th number in this sequence is subtracted from the 20th number in the sequence. What is the result?

Skills covered: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Competency based Questions

A ladder is leaning against a wall. The foot of the ladder is 6 meters away from the base of the wall (shown in the picture), and the ladder reaches a height of 8 meters on the wall. The ladder forms a right-angled triangle with the ground and the wall. Calculate the length of the ladder and select the correct option.



A) 6 meters

B) 8 meters

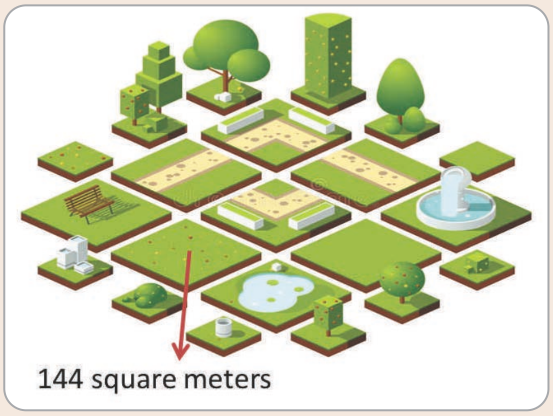
C) 10 meters

D) 12 meters

Skills covered: Interpersonal skills, Observation, Application and Decision making skills

Case Study

A local garden project was underway, where several rectangular plots were being designed. Each plot had to have the same area, but the gardeners wanted the area to be a perfect square number. They decided to design a plot with an area of 144 square meters shown in the picture. The design was meant to showcase the relationship between square numbers and square roots.



The students in the math club were given the following tasks:

1. Calculate the side length of a square plot whose area is 144 square meters.
2. Is the side length of the garden plot a perfect square? Explain your reason.
3. The gardener found that the area of a neighboring plot is 196 square meters. What is the side length of this plot, and is the area a perfect square?

Skills covered: Research, Logical Reasoning, Problem-Solving, Practical Application