3

Chapter

Understanding Quadrilaterals

We'll cover the following key points:

→ Quadrilateral

→ Vertices, sides, angles and diagonals of a quadrilateral

→ Interior and exterior of a quadrilateral

→ Convex and concave quadrilaterals

→ Angle sum property of a quadrilateral

→ Interior and exterior angles of a quadrilateral

→ Kinds of quadrilaterals

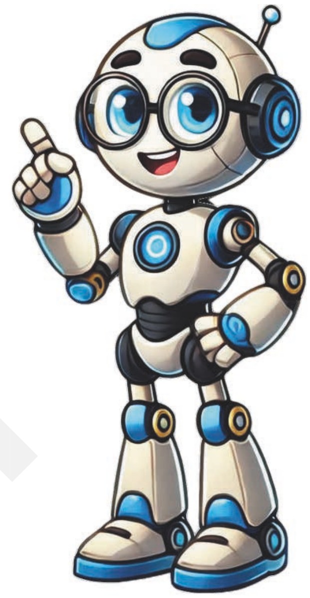
→ Properties of a parallelogram

→ Verification of properties of parallelogram

→ Some special parallelograms

→ Verification of properties of a rectangle

→ Verification of properties of a square



Hi, I'm EeeBee

Do you Remember fundamental concept in previous class.

In class 6th we learnt

→ Curves and Polygon

Still curious? Talk to me by scanning the QR code.

Learning Outcomes

By the end of this chapter, students will be able to:

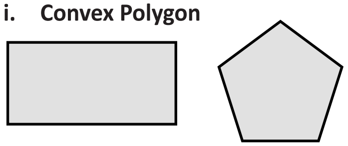
- Identify and classify different types of quadrilaterals based on their properties.
- Understand the properties of special quadrilaterals like squares, rectangles, parallelograms, rhombuses, and trapeziums.
- Apply the properties of quadrilaterals to solve problems related to angles and sides.
- Calculate the sum of interior and exterior angles of various quadrilaterals.
- Derive the relationship between the angles in a quadrilateral and use it in problem-solving.
- Recognize and use symmetry in quadrilaterals.
- Solve problems involving the area and perimeter of different types of quadrilaterals.
- Understand and apply the concept of diagonals in quadrilaterals and how they intersect.
- Use coordinate geometry to locate and represent quadrilaterals on the Cartesian plane.
- Develop reasoning skills by proving basic theorems related to quadrilaterals.

Mind Map

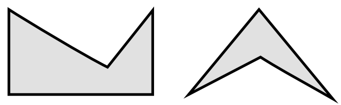
UNDERSTANDING QUADRILATERALS

Convex and concave polygons

i. **Convex Polygon**

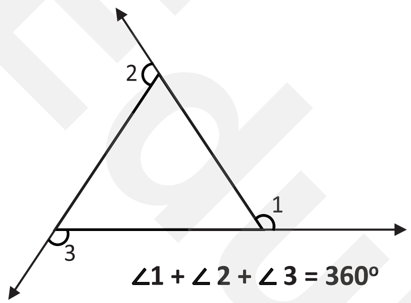


ii. **Concave Polygon**



Sum of exterior angle of a polygon

360°



$\angle 1 + \angle 2 + \angle 3 = 360°$

Angles of Parallelogram

Opposite angles of a parallelogram are of equal measure.

$\angle A = \angle C$, $\angle B = \angle D$

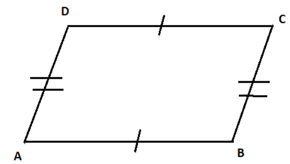
Adjacent angles are supplementary

$\angle A + \angle B = 180°$

$\angle B + \angle C = 180°$

$\angle C + \angle D = 180°$

$\angle D + \angle A = 180°$

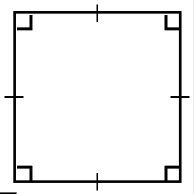


Regular and Irregular Polygons

i. **Regular Polygons**

Both equiangular and equilateral.

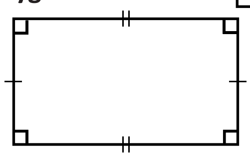
e.g., Square



ii. **Irregular Polygons**

Not regular

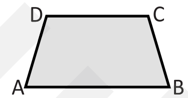
e.g., Rectangle



Kind of Quadrilaterals

i. **Trapezium**

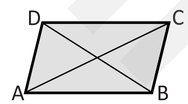
One pair of parallel sides



AB and CD are parallel

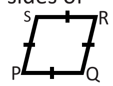
ii. **Parallelogram**

- Opposite sides are parallel
- AB and CD are parallel
- BC and DA are parallel
- AC and BD are diagonals
- AB and BC are adjacent sides
- $\angle A$ and $\angle C$, $\angle B$ and $\angle D$ are opposite angles



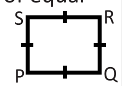
Some special Parallelograms

i. **Rhombus:** A parallelogram with sides of equal length.

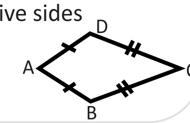


ii. **Rectangle:** A parallelogram with a right angle.

iii. **Square:** A rectangle with sides of equal length.

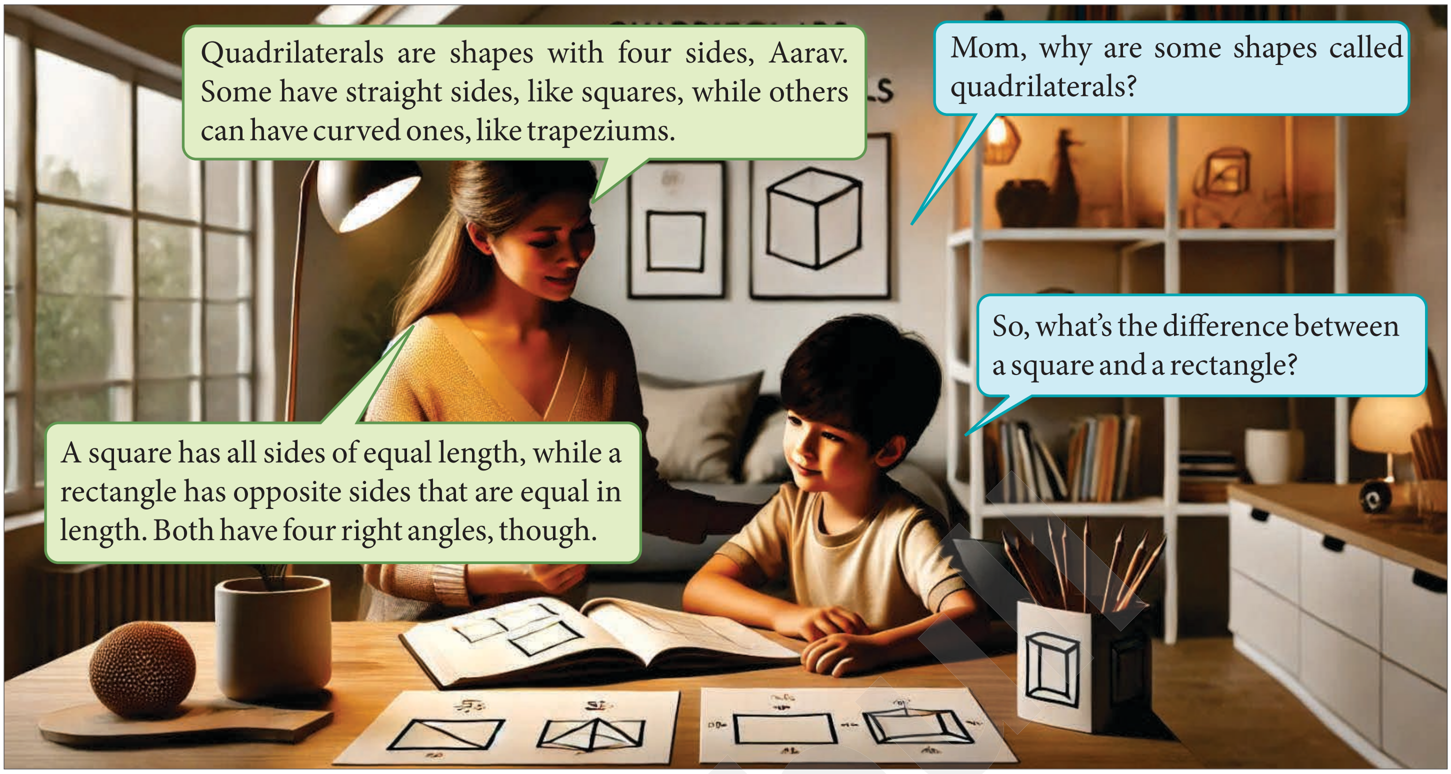


iv. **Kite:** A quadrilateral with exactly two pairs of equal consecutive sides



Based on NCERT*

Introduction



Quadrilateral

If P, Q, R, S are 4 points in a plane such that (i) no three of them are collinear (ii) the line segments PQ, QR, RS and SP do not intersect except at their end points. Then, the figure formed by of these four line segments is called the **quadrilateral PQRS**.

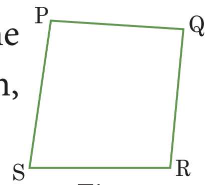
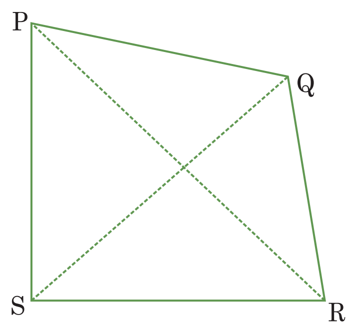


Fig.

Vertices, Sides, Angles and Diagonals of a Quadrilateral

In the quadrilateral PQRS ; we have the following elements:



1. **Vertices :** The four end points are called its *vertices*.
   Example : P, Q, R and S.
2. **Opposite vertices :** A quadrilateral has two pair of opposite vertices.
   Example : P and R, Q and S.
3. **Sides :** The four line segments are called its *sides*.
   Example : PQ, QR, RS and SP.
4. **Adjacent sides :** Two sides of a quadrilateral which have a common endpoint are called its *adjacent sides*.
   Example : QR, RS; RS, SP; SP, PQ and PQ, QR.
5. **Opposite sides :** Two sides of a quadrilateral are called its *opposite sides*, if they do not have a common endpoint.
   Example : PQ, SR and QR, PS.
6. **Angles :** $\angle SPQ$ or $\angle P$, $\angle PQR$ or $\angle Q$, $\angle QRS$ or $\angle R$ and $\angle RSP$ or $\angle S$ are angles of quadrilateral PQRS.

7. **Adjacent angles :** Two angles of a quadrilateral having a common side are called its *adjacent angles*.

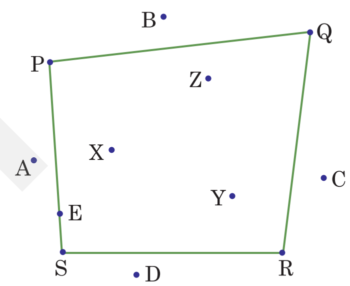
Example : ∠P, ∠Q; ∠Q, ∠R; ∠R, ∠S and ∠S, ∠P.

8. **Opposite angles :** Two angles of a quadrilateral which are not adjacent angles are known as the *opposite angles* of the quadrilateral.

Example : ∠P, ∠R and ∠Q, ∠S.

9. **Diagonals :** The two line segments joining the opposite vertices are called its *diagonals*.
Example : PR and QS.

Interior and Exterior of a Quadrilateral



In the given figure, PQRS is a quadrilateral :

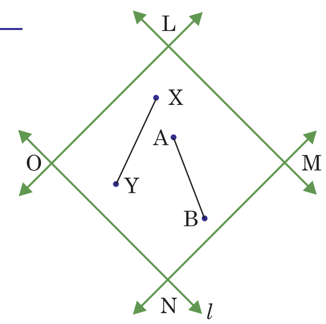
(*i*) X, Y and Z are points in the **interior of the quadrilateral**.

(*ii*) A, B, C and D are points in the **exterior of the quadrilateral**.

(*iii*) P, Q, R, S and E are points **on the quadrilateral**.

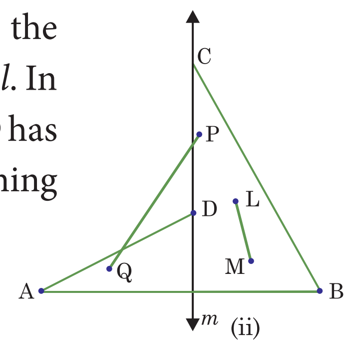
The quadrilateral separates its interior and exterior. The interior and the quadrilateral together from the *quadrilateral region*.

Convex and Concave Quadrilaterals



(i)

A quadrilateral in which a line containing two of its vertices has the remaining two vertices on the same side is called a *convex quadrilateral*. In Fig. (*i*), LMNO is a convex quadrilateral. Line *l* containing vertices N and O has vertices L and M on its same side. In this quadrilateral, if we take any two points, the segment joining, these points lies wholly in its interior.



(ii)

A quadrilateral in which a line containing the two of its vertices has the remaining two vertices on the opposite side is called a *concave quadrilateral*. In Fig. (*ii*), ABCD is a concave quadrilateral. Here, *m* contains vertices C and D has vertices A and B on opposite sides. In this quadrilateral, the line segment joining the points P and Q does not lie wholly in its interior.

Let ABCD be a quadrilateral. Join one of its diagonals, say AC.

REMEMBER

(i) In a concave quadrilateral, one angle is greater than 180° but in convex quadrilateral all angles are less than 180°. In Fig. (ii), ∠ADC is greater than 180°.

(ii) In a concave quadrilateral, one diagonal lies completely inside the quadrilateral and one completely outside the quadrilateral, whereas in a convex quadrilateral, both diagonals lie in the interior of the quadrilateral.

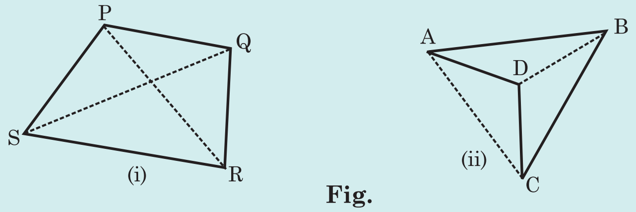
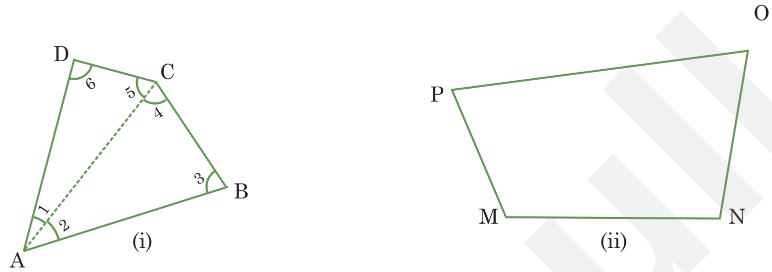


Fig.

Take a Task

Watch Remedial

Angle Sum Property of a Quadrilateral



In ΔACD, $\angle 1 + \angle 6 + \angle 5 = 180°$ (Sum of the angles of a triangle is 180°.)

and in ΔABC, $\angle 2 + \angle 3 + \angle 4 = 180°$ (Sum of the angles of a triangle is 180°.)

Adding the above equations, we get

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360° \qquad (\because \angle A = \angle 1 + \angle 2)$$

or $$\angle A + \angle B + \angle C + \angle D = 360° \qquad (\angle C = \angle 4 + \angle 5)$$

We will find that the sum of all the angles of a quadrilateral is 360°. If we draw another quadrilateral MNOP, we see that

$$\angle M + \angle N + \angle O + \angle P = 360°.$$

We conclude that *the sum of the angles of a quadrilateral is 360°.*

Draw one of the diagonals, say BD.

The quadrilateral is divided into two triangles.

Sum of the angles of each triangle = 2 right angles or 180°

Sum of the angles of 2 triangles = 2×2 right angles

= $2 \times 180°$ or 360°.

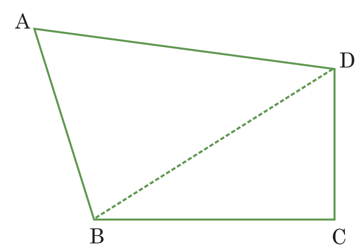


Fig.

$\therefore$ Sum of the angles of the quadrilateral = 4 right angle or 360°.

The angles of a quadrilateral are called its interior angles.

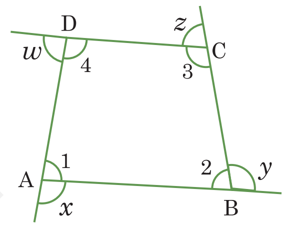
Interior and Exterior Angles of a Quadrilateral

In Fig., $\angle 1, \angle 2, \angle 3$ and $\angle 4$ are interior angles.

If the sides of a quadrilateral are produced in an order (as shown in Fig.), then $\angle x, \angle y, \angle z, \angle w$ are called the **exterior angles** of the quadrilateral.

Exterior Sum Property: *If the sides of a quadrilateral are produced in an order, the sum of the four exterior angles so formed is 360°.*

We know that the sum of the angles of a linear pair is 180°.



$\therefore \quad \angle x + \angle 1 = 180°.$

Similarly, $\quad \angle y + \angle 2 = 180°,$

$\angle z + \angle 3 = 180°$

and $\quad \angle w + \angle 4 = 180°.$

Adding the angles on either side, we get

$$(\angle x + \angle y + \angle z + \angle w) + (\angle 1 + \angle 2 + \angle 3 + \angle 4) = 720°$$

But $\quad (\angle 1 + \angle 2 + \angle 3 + \angle 4) = 360°$

$\Rightarrow \quad \angle x + \angle y + \angle z + \angle w + 360° = 720°$

$\therefore \quad \angle x + \angle y + \angle z + \angle w = 360°.$

Take a Task

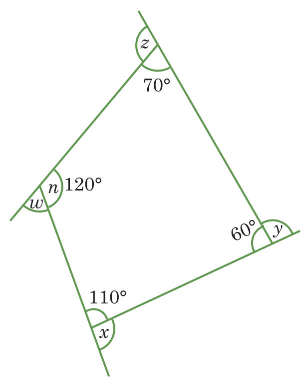
Watch Remedial

The above result is true for all polygons, *e.g.*, triangle, pentagon, hexagon, etc.

Hence, the sum of the measures of the external angles of any polygon is 360°.

Example 1 : From the given figure, find $x + y + z + w$.

Solution : Sum of the angles of a quadrilateral = 360°



$70° + 60° + 110° + x = 360°$

$\Rightarrow \quad 240° + x = 360°$

$\Rightarrow \quad x = 360° - 240° = 120°.$

Since the sum of the linear pair is 180°, we have

$w + 120° = 180° \quad \text{... (1)}$

$\Rightarrow \quad x + 110° = 180° \quad \text{... (2)}$

$\Rightarrow \quad y + 60° = 180° \quad \text{... (3)}$

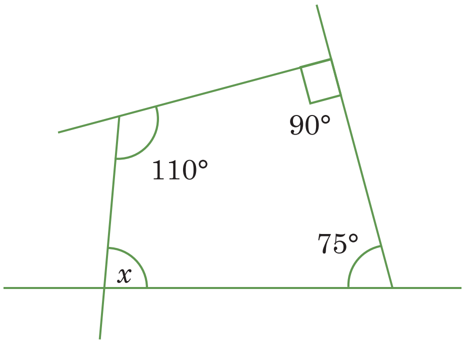
$\Rightarrow \quad z + 70° = 180° \quad \text{... (4)}$

Adding the equations (1), (2), (3) and (4), we get

$$\Rightarrow \quad x + y + z + w + 360° = 720°$$

$$\Rightarrow \quad x + y + z + w = 720° - 360° \quad \Rightarrow \quad x + y + z + w = 360°.$$

Example 2 : **In the given figure, find the value of $\angle x$.**



Solution :

$$x + 90° + 75° + 110° = 360°$$

$$\Rightarrow \quad x + 275° = 360°$$

$$\Rightarrow \quad x = 85°.$$

Example 3 : **Find the number of sides of a regular polygon whose each exterior angle has a measure of 120°.**

Solution : Total measure of all exterior angles = 360°

Measure of each exterior angle = 120°

Therefore, the number of exterior angles $= \dfrac{360°}{120°} = 3$

Hence, the polygon has 3 sides.

Example 4 : **The angles of a quadrilateral are in the ratio of 2 : 4 : 5 : 1. Find the measure of each angle.**

Solution : Let the angles be $2x$, $4x$, $5x$ and $1x$.

By angle sum property of the quadrilateral, we have

$$2x + 4x + 5x + x = 360°$$

$$\Rightarrow \quad 12x = 360°$$

$$\Rightarrow \quad x = \frac{360°}{12} = 30°$$

Now, $2x = 2 \times 30° = 60°$;

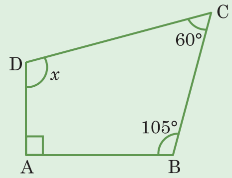
$4x = 4 \times 30° = 120°$;

$5x = 5 \times 30° = 150°$;

Thus, the angles of the quadrilateral are 60°, 120°, 150° and 30°.

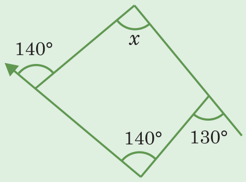
Exercise 3.1

1. In the given figure, ABCD is a quadrilateral. Find x.

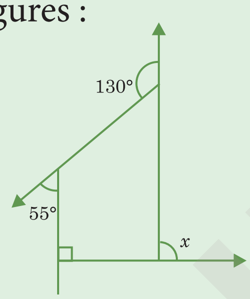


2. Find the value of x in each of the following figures :

(*i*)

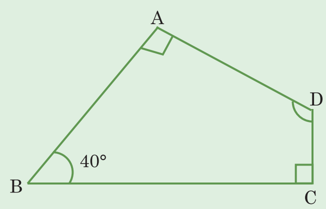


(*ii*)



3. Name the polygon whose each exterior angle is 72°.
4. Two angles of a quadrilateral are 53° and 67° and the remaining two angles are in the ratio 2 : 3. Find the measure of each of the two angles.
5. Two angles of a quadrilateral are 67° and 115°, and the remaining two angles are equal. Find the measure of each of the equal angles.
6. Is it possible to have a quadrilateral whose angles are of measures 105°, 155°, 55° and 65° ? Give reason.
7. The four angles of a quadrilateral are in the ratio 2 : 2 : 3 : 3. Find the measure of each of the angles.
8. How many sides does a regular polygon has if each of its interior angle is 160° ?

   [**Hint:** Interior angle of a polygon $= \dfrac{(n-2)\times 180°}{n}$ where n = number of sides.]
9. Three angles of a quadrilateral are in the ratio 1 : 2 : 3. The sum of the least and the greatest of these angles is equal to 180°. Find all the angles of this quadrilateral.
10. Three angles of a quadrilateral are equal. Fourth angle is 150°. Find the measure of other three angles.
11. In Fig., find ∠ADC.



12. Fill in the blanks :

   (*i*) The line segment joining the opposite vertices is called ____________ of the quadrilateral.

   (*ii*) A quadrilateral has ____________ pairs of opposite angles.

   (*iii*) The sum of the angles of a quadrilateral is ____________ .

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills (*iv*) Sum of the four exterior angles formed by producing the four sides of a quadrilateral is equal to ____________ .

(*v*) A quadrilateral has ____________ pairs of opposite vertices.

13. The angles A, B, C and D of a generated ABCD are in the ratio 2 : 3 : 7 : 8.

(*i*) Find the measure of each angle. (*ii*) Is ABCD a trapezium? Why ?

(*iii*) Is ABCD a parallelogram? Why ?

14. PQRS is a parallelogram. What special name will be given to the parallelogram, if the following additional facts are known ?

(*i*) PQ = PS (*ii*) ∠PQR = 90° (*iii*) PQ = PS and ∠PSR = 90°

Check Your Progress

Experiential Learning

Choose the correct option:

1. The number of pairs of adjacent angles after of a quadrilateral are :

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

2. How many right angles are there in the sum of the angles of a quadrilateral ?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

3. A quadrilateral is said to be convex, if for each line drawn the lies on the same side of the line.

(*a*) sides (*b*) vertices (*c*) angles (*d*) diagonals

4. How many diagonals at the least can lie inside a concave quadrilateral?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 9

Take a Task

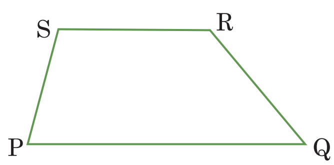
Watch Remedial

Kinds of Quadrilaterals

Based on the nature of the sides or angles of a quadrilateral, it gets special names :

Trapezium:

A trapezium is a quadrilateral in which at least one pair of opposite sides are parallel.



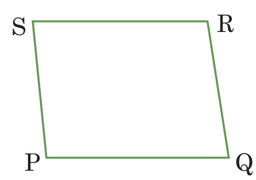
In the figure, PQRS is a quadrilateral with PQ P SR.

Note

If the non-parallel sides of a trapezium are of equal length, we call it an *isosceles trapezium*.

Parallelogram :

A parallelogram is a quadrilateral in which each pair of opposite sides are parallel.

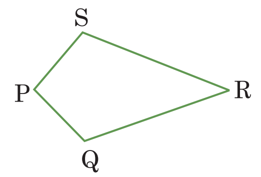


In the figure, PQ P SR, PS P QR, therefore, PQRS is a parallelogram.

Note

We can easily observe that every parallelogram is a trapezium but the converse is not true.

Kite :



A kite is a quadrilateral which has two pairs of equal adjacent sides and unequal opposite sides.

Thus, a quadrilateral PQRS is a kite, if PQ = PS, QR = RS but PS ≠ QR and PQ ≠ SR.

Properties of a Parallelogram

We know, that a parallelogram is a quadrilateral in which each pair of opposite sides are parallel.

Now, let us study the relationship between its sides, angles and diagonals with the help of an activity.

Activity : Draw a pair of parallel lines l and m. Draw another pair of parallel lines say e and f such that e intersect l and m at P and S and f intersect l and m at Q and R, respectively. A parallelogram PQRS is formed. Join the opposite vertices, *i.e.*, PR and SQ and name the point of intersection as O, as shown in Fig.

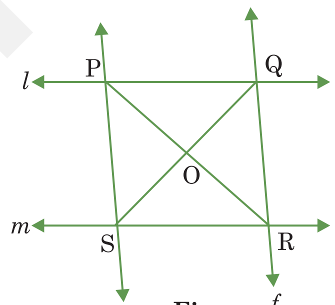


Fig.

Now, measure the sides PQ, QR, RS and SP. Measure ∠P, ∠Q, ∠R, and ∠S. Measure OP, OQ, OR and OS.

Repeat the above activity with other pair of parallel lines. You will find that:

PQ = QR and RS = QR; ∠P = ∠R and ∠Q = ∠S; OP = OR and OQ = OS.

Thus, in a parallelogram :

(*i*) *opposite sides are equal.* (*ii*) *opposite angles are equal.*

(*iii*) *diagonals bisect each other.*

Verification of Properties of Parallelogram

We know that ***a parallelogram is a quadrilateral*** in which ***opposite sides are parallel.*** We will prove with the help of experiment that, in a parallelogram :

(*i*) *Opposite sides are equal.*

(*ii*) *Opposite angles are equal.*

(*iii*) *Diagonals bisect each other.*

In ΔSPQ and ΔSRQ, we have ∠1 = ∠3. (when two parallel lines are cut by a transversal, then alternate angles are equal.)

Also, ∠2 = ∠4

and, $SQ = SQ$ (Common)

So, by ASA congruence condition

$$\Delta SPQ \cong \Delta SRQ$$

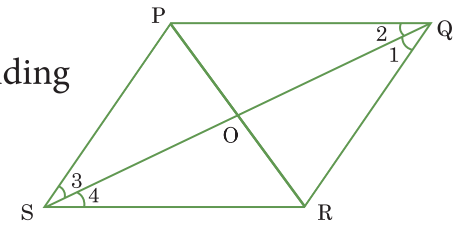


Fig.

This implies that PQ =SR, PQ = SR and $\angle P = \angle R$, because corresponding sides and angles of congruent triangles are equal.

Similarly, for diagonal PR.

$$\Delta PSR \cong \Delta PQR$$

$\Rightarrow$ $PS = QR$ and $\angle S = \angle Q$

Thus, in a parallelogram PQRS, we find opposite sides are equal, i.e.,

$$PQ = SR \text{ and } PS = QR$$

and opposite angles are equal

i.e., $\angle P = \angle R$ and $\angle S = \angle Q$

Hence, we have proved the properties (*i*) and (*ii*).

Now, to prove the result (*iii*) i.e., the diagonals bisect each other at point O.

In ΔPOQ and ΔSOR, we have

$PQ = SR$ (From above)

$\angle POQ = \angle SOR$ (Vertically opposite angles)

and $\Rightarrow 4 = \angle 2$ (Alternate angles)

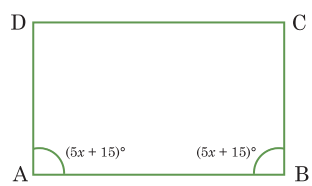
$\Rightarrow$ $\Delta POQ \cong DSOR$ (By AAS criterion)

$\Rightarrow$ $OP = OR$ and $OS = OQ$ (cpct)

Hence, proved.

Example 5 : ABCD is a parallelogram in which $\angle A = (5x + 15°)$ and ÐB = $(5x + 15°)$. Find the measure of all angles of the parallelogram.

Solution : In the parallelogram ABCD, $\angle A = 5x + 15°$ and $\angle B = 5x + 15°$.



$\therefore$ $5x + 15° + 5x + 15° = 180°$

$\Rightarrow$ $10x + 30° = 180°$

$\Rightarrow$ $10x = 180° - 30°$

$\Rightarrow$ $10x = 150°$

$\Rightarrow$ $x = 15°$

$\therefore$ $\angle A = 5x + 15 = 5 \times 15 + 15 = 75 + 15 = 90°$

$\angle B = 5x + 15 = 5 \times 15 + 15 = 75 + 15 = 90°$.

But, $\angle A = \angle C$ [Opposite angles of a P gm are equal.]

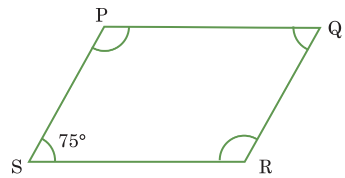
$\angle B = \angle D$ [Opposite angles of a || gm are equal.]

$\therefore$ $\angle C = \angle 90°$ and $\angle D = \angle 90°$

$\therefore$ The angles of the parallelogram are 90°, 90°, 90°, 90° and is a rectangle.

Example 6 : **In a parallelogram PQRS figure, if $m\angle S = 75°$. What is the measure of the other angles?**

Solution : Let PQRS be the parallelogram with $\angle S = 75°$.



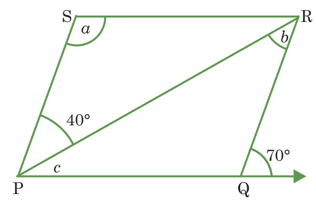
Then, $\angle Q = 75°$ (Opposite angles are equal.)

$\angle R = 180° - 75°$ (Supplementary angle to $\angle S$.)

$= 105°$

$\therefore$ $\angle P = 105°$ (Opposite angle to $\angle R$.)

Example 7 : **In figure, PQRS is a parallelogram. Find the angles *a*, *b* and *c*. State the properties used to find them.**



Solution : $\angle PQR + 70° = 180°$ (Angles of linear pair.)

$\Rightarrow$ $\angle PQR = 180° - 70° = 110°$

$\angle S = \angle PQR$ (Opposite angles of a P gm are equal)

$\Rightarrow$ $a = 110°$ (Alternate angles)

$\therefore$ $b = 40°$ (Corresponding angles)

$\therefore$ $\angle SPQ = \angle Q = 70°$ (Corresponding angles)

$\Rightarrow$ $40° + c = 70°$

$\Rightarrow$ $c = 70° - 40° = 30°$

Hence, $a = 110°$, $b = 40°$ and $c = 30°$.

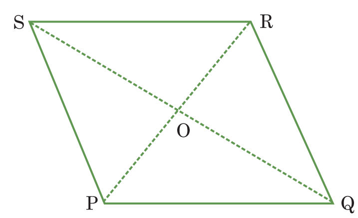
Take a Task

Watch Remedial

Some Special Parallelograms

1. Rhombus

A rhombus has all the properties of a parallelogram in which any pair of adjacent sides and opposite angles are equal.



Thus, a quadrilateral PQRS is a rhombus if PQ P SR, PS P QR and PQ = QR = RS = SP.

Since, a rhombus is a parallelogram, all the properties of parallelogram holds true for rhombus also. Therefore, (*i*) Opposite sides of a rhombus are equal, *i.e.*, PQ = SR and SP = QR.

(*ii*) Opposite angles of a rhombus are equal, *i.e.*, $\angle P = \angle R$ and $\angle Q = \angle S$.

(*iii*) Diagonals of a rhombus bisect each other, *i.e.*, OP = OR and OS = OQ.

Measure $\angle POQ$ and $\angle SOR$.

You will find that both the angles measure 90°. Similarly, on measuring $\angle POS$ and $\angle QOR$, you will find that both the angles are 90°.

Therefore, $\angle POQ = \angle SOR = \angle POS = \angle QOR = 90°$.

Verification of Diagonal Property of Rhombus

Consider Δs POQ and POB and AOD in order to prove that the diagonals PR and QS are perpendicular to each other.

In ΔPOQ and ΔPOS,

Because,	QO = OS	(O is mid-point of QS)
	OP = OP	(Common side)
	PQ = PS	(Sides of a rhombus are equal)
So,	$\Delta POQ = \Delta POS$	(By SSS congruence condition)
$\Rightarrow$	$\angle POQ = \angle POS$	
But	$\angle POQ + \angle POS = 180°$	(Linear pair)
$\therefore$	$\angle POQ = \angle POS = 90°$	

This, the diagonals of a rhombus bisect each other at right angles.

Example 8: In figure PQRS is a rhombus whose diagonals PR and QS intersect at a point O. If side PQ = 10 cm and diagonal QS = 16 cm. Find the length of diagonal PR.

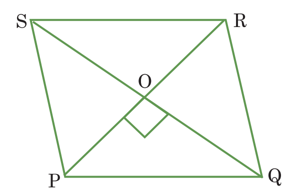


Fig.

Solution: We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore$ $QO = \frac{1}{2} QS = \frac{1}{2} \times 16 \text{ cm} = 8 \text{ cm}$

$PQ = 10 \text{ cm}$ and $\angle POQ = 90°$. (given)

From right ΔOPQ, we have :

$PQ^2 = PO^2 + QO^2$

$\Rightarrow$ $PO^2 = PQ^2 - QO^2$

$\Rightarrow$ $PO^2 = 10^2 - 8^2$

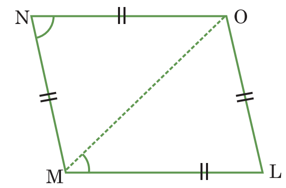
$\Rightarrow$ $PO^2 = 100 - 64$

$\Rightarrow \quad PO^2 = 36$

$\Rightarrow \quad PO^2 = 6^2$

$\Rightarrow \quad PO = 6$ cm

$\therefore \quad PR = 2 \times PO = 2 \times 6 = 12$ cm.



Example 9: **Diagonal MO of a rhombus LMNO is equal to one of its sides LM figure. Find all the angles of the rhombus.**

Solution: In ΔLMO,

$LO = LM$ [All the sides of a rhombus are equal.]

and $LM = MO$ [given]

$\therefore \quad LO = LM = MO$

Thus, DLMO is an equilateral triangle.

Hence, each angle of $\Delta LMO = 60°$.

Now, $NO = MN = LM$ [All the sides of a rhombus are equal.]

and $LM = MO$ [given]

$\therefore \quad NO = MN = MO$

Thus, ΔMNO is also an equilateral triangle.

Hence, each angle of $\Delta MNO = 60°$.

Thus, angles of the rhombus LMNO:

$\angle L = 60°$

$\angle M = \angle LMO + \angle NMO = 60° + 60° = 120°$,

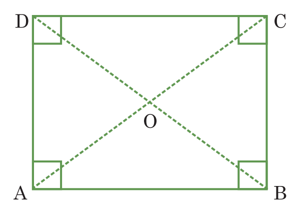
$\angle N = 60°$,

and $\angle O = \angle NOM + \angle MOL = 60° + 60° = 120°$.

2. Rectangle

A rectangle is a quadrilateral in which opposite sides are equal and one angle measures 90°.

In a rectangle ABCD (Fig.):



(*i*) opposite sides are equal, *i.e.*, $AB = DC$ and $AD = BC$;

(*ii*) each angle is a right angle;

(*iii*) diagonals are equal, *i.e.*, $AC = BD$;

(*iv*) diagonals bisect each other but not necessarily at right angles.

Verification of Properties of a Rectangle

We know that a rectangle is a parallelogram one of whose angles is 90°. In Fig., ABCD is a rectangle, *i.e.*, a parallelogram with $\angle A = 90°$.

Since, ABCD is a parallelogram, therefore

$$AB = DC; AD = BC;$$

$$\angle A = \angle C \text{ and } \angle B = \angle D.$$

Now, AB || DC and AD is a transversal to these lines.

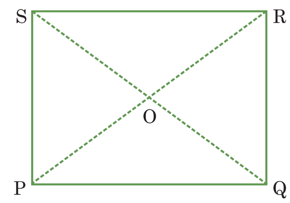
Therefore, $\angle A + \angle D = 180°$ [Interior angles on the same side of the transversal]

But, $\angle A = 90°$

Therefore, $\angle D = 180° - \angle A = 180° - 90° = 90°$

Hence, $\angle B = 90°$

and $\angle C = 90°$ [$\angle B = \angle D$ and $\angle A = \angle C$]



Thus, we can say that *each angle of a rectangle is a right angle.*

3. Square

A square is a quadrilateral in which all sides are equal and one angle measures 90°.

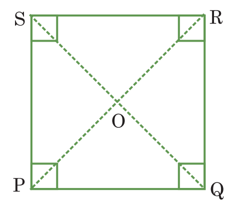
In a square PQRS figure:

(*i*) all the sides are equal, *i.e.*, PQ = QR = RS = SP;

(*ii*) each angle is of 90°;

(*iii*) diagonals are equal, *i.e.*, PR = QS ;

(*iv*) diagonals bisect each other at right angles.



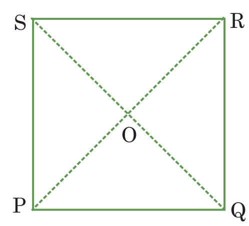
Verification of Properties of Square

We know that a parallelogram with a pair of adjacent sides equal and one angle a right angle is called a square.

In figure , PQRS is a square, *i.e.*, it is a parallelogram with PQ = PS and $\angle P = 90°$.

Now, PQRS is a parallelogram with PQ = PS. Therefore, PQRS is a rhombus.

Hence, PQ = QR = RS = SP, PR ⊥ QS, OP = OR and OQ = OS (Fig.).



Also, PQRS is a parallelogram with $\angle P = 90°$.

Therefore, it is a rectangle.

Hence, $\angle P = \angle Q = \angle R = \angle S = 90°$ and PR = QS.

What do we conclude from the above discussion?

We conclude that in a square :

(*i*) *All the sides are of equal length.*

(*ii*) *Each of the angles is a right angle.*

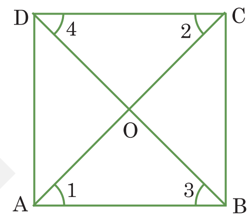
(*iii*) *The diagonals bisect each other at right angles.*

(*iv*) *The diagonals are of equal length.*

(*v*) *Every square is a rectangle but every rectangle is not a square.*

(*vi*) *Every square is a rhombus but every rhombus is not a square.*

Example 10 : In a square ABCD, AC and BD are its two diagonals figure. AC and BD intersect at O. Prove that:



(*i*) AC = BD

(*ii*) AO = CO and BO = DO

Solution : (*i*) In ΔABC and ΔABD,

AB = AB [Common]

BC = AD [Sides of the square]

Included ∠ABC = Included ∠BAD [SAS property of congruency]

∴ ΔABC ≅ ΔABD

∴ AC = BD

(*ii*) In ΔAOB and ΔCOD,

AB = CD [Sides of the square]

∠1 = ∠2 [Alt. angles, AB P CD and AC intercepts them]

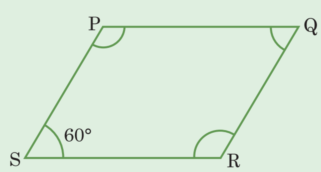
∠3 = ∠4 [Alt. angles, AB P CD and BD intercepts them]

∴ ΔAOB ≅ ΔCOD [ASA property of congruency]

∴ AO = CO and BO = DO.

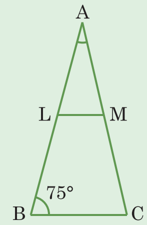
Exercise 3.2

1. In a parallelogram PQRS, figure. If $m\angle S = 60°$. What is the measure of the other angles?



2. Two adjacent angles of a parallelogram are in the ratio of 4 : 5. Find the measure of each of its angles.

3. Two adjacent sides of a parallelogram are 4 cm and 7 cm long. Find its perimeter.



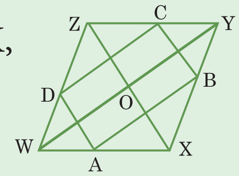
4. In figure, LM || BC. What type of quadrilateral is BLMC ? If ∠B = 75°, ∠A = 30°. Find the measures of ∠BLM and ∠CML.

5. Two adjacent angles of a parallelogram are in the ratio 7 : 11. Find the measure of each of its angles.

6. The perimeter of a parallelogram is 20 cm. If the ratio of the length to breadth is 3 : 2. What are the dimensions of the parallelogram ?

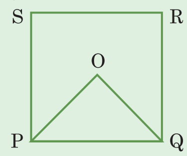
7. In a parallelogram ABCD, if ∠DAB = 75° and ∠ABC = 105°. Find ∠BCD and ∠ADC.

8. Two sides of a parallelogram are such that one of the sides is 10 cm longer than the other side. If perimeter of the parallelogram is 140 cm. Find the length of each of its sides.

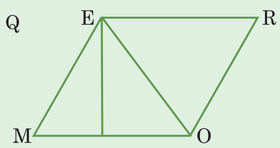


9. In figure, WXYZ is a rhombus. A, B, C and D are the midpoints of sides WX, XY, YZ and WZ respectively. Show that ABCD is a rectangle.

10. PQRS is a rhombus with PQ = 10 cm. If OQ = 6 cm, then what is the length of the diagonal PR ?



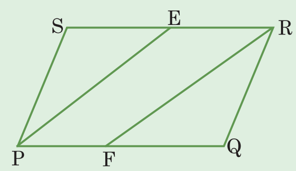
11. In the adjoining figure PQRS is a parallelogram, PO and QO are the bisectors of ∠P and ÐQ respectively. Prove that POQ = 90°.



12. In figure, MORE is a rhombus in which the altitude from E to the side MO bisects it. Find the measures of the angles of the rhombus.

13. Prove that any adjacent angles of a parallelogram are supplementary.

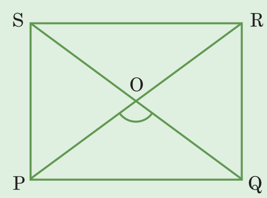
14. The sides of a rectangle are in the ratio 7 : 5 and its perimeter is 72 cm. Find the length and breadth of the rectangle.



15. In the figure, PQRS is a parallelogram in which line segments PE and RF bisect the angles ∠P and ∠R respectively. Show that PE P RF.

16. The breadth of a rectangle is 6 cm and each of its diagonals measures 10 cm. Find its length.

17. PQRS is a square. Its diagonals are PR and QS. Find the measures of ∠RPS and ∠SQR ?



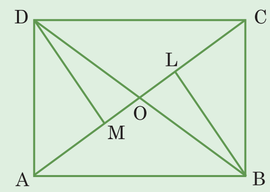
18. In the figure, PQRS is a rectangle and its diagonals PR and QS intersect at O. If ∠POQ = 110°. Find the measures of:

(*i*) ∠PQO (*ii*) ∠PSQ (*iii*) ∠ORS

19. The diagonals of a rectangle DEFG intersect at O. If ∠FOE = 60°, find ∠OGD.

20. PQRS is a rectangle. The perpendicular SN from S on PR divides ∠S in the ratio 2 : 3. Find ∠NSQ.

21. In figure, ABCD is a rectangle, LB ⊥ AC and MD ⊥ PC. Prove that:



(*i*) ΔAMD ≅ ΔCLB (*ii*) LB = MD

22. Two adjacent angles of a parallelogram are $(3x + 25)°$ and $(2x - 5)°$. Find the value of x.

23. Find the length of diagonal of the square if the perimeter is 60 cm.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Revision Exercise

Conceptual Learning

Tick (✓) the correct option:

1. A closed plane curve that does not intersect itself is a:

   (*a*) simple closed curve (*b*) open curve

   (*c*) closed curve (*d*) non-simple curve

2. A regular quadrilateral is a:

   (*a*) square (*b*) rectangle

   (*c*) parallelogram (*d*) rhombus

3. The sum of the exterior angles of an octagon is:

   (*a*) 180° (*b*) 360° (*c*) 540° (*d*) 720°

4. The line segment joining the two non-adjacent vertices of a polygon is called:

   (*a*) side (*b*) diagonal (*c*) adjacent sides (*d*) none of these

5. The interior angle of a polygon of '*n*' sides is given by:

   (*a*) $\frac{n(n-2)}{180°}$ (*b*) $\frac{n}{(n+2)} \times 180°$ (*c*) $\frac{(n-2)}{n} \times 180°$ (*d*) $\frac{(n-2)}{180°}$

6. The angles of a quadrilateral are in the ratio 2 : 3 : 5 : 8. The smallest angle is:

   (*a*) 40° (*b*) 50° (*c*) 20° (*d*) 60°

7. The number of pairs of opposite sides in a quadrilateral is:

   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

8. The sum of an exterior angle and an adjacent interior angle of a quadrilateral is:

   (*a*) 90° (*b*) 180°

   (*c*) 360° (*d*) none of these

Mental Maths

Experiential Learning

1. If the angles of a quadrilateral are in the ratio of 1 : 2 : 3 : 4. Find the smallest angle.
2. The length of a rectangle is 8 cm and each of its diagonal measures 10 cm. Find it breadth.
3. Find the fourth angle, if the three acute angles of a quadrilateral, each measuring 75°.
4. Two sides of a parallelogram are in the ratio of 5 : 3. If the perimeter is 64 cm. Find the length of it sides.
5. The four angles of a quadrilateral are in the ratio 3 : 5 : 7 : 9. Find the angles of the quadrilateral.

Assertion and Reason

Critical Thinking

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct.

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

1. **Assertion (A) :** Each diagonal of a square bisects the square into two congruent triangles :

   Reason (B): All the angles and sides of a rhombus are equal.

2. **Assertion (A) :** If three angles of a quadrilateral are 90°, 50° and 90°, then the fourth angles is 80°.

   Reason (B) : The sum of four angles of a quadrilateral is 360°.

3. **Assertion (A) :** A rectangle is a square, if its diagonals bisect each other at right angles.

   Reason (B): All rectangles are parallelogram.

4. **Assertion (A) :** If the adjacent sides of a parallelogram are q cm and 6 cm, its perimeter is 30cm.

   Reason (R) : A rhombus has two pairs of parallel sides.

5. **Assertion (A) :** The diagonals of a square bisect each other at right angles.

   Reason (B) : The diagonals of a rectangle bisect each other at right angles.

Thinking Skills

1. The sum of the interior angles of a polygon is 1260°. How many sides does the polygon have?
2. The ratio of the exterior angle to the interior angle of a polygon is 1: 7. How many sides does the polygon have?
3. The ratio of the number of sides of two regular polygons is 6:7, and the ratio of the measures of their exterior angles is 3:2. Find the number of sides of each polygon.
4. A rectangular stadium has a width of 50 meters and a length of 70 meters. Neel walks diagonally across the stadium, while Simran walks along the length and breadth of the stadium. How much distance does Neel travel diagonally? Also, find how much Simran walks along the length and breadth.
5. The sum of the interior angles of a polygon is five times the sum of its exterior angles. Find the number of sides of the polygon.

Skills Developed: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Competency based Questions

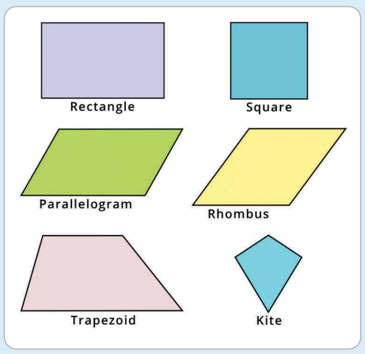
1. Maya and her friends are discussing the different types of quadrilaterals in their geometry class(Shown in the given picture). They are trying to identify which quadrilaterals have all sides equal, which have opposite sides equal, and which have all angles equal.

   A) Square, Rectangle, Rhombus, Parallelogram

   B) Square, Rhombus, Kite

   C) Square, Rectangle, Rhombus, Parallelogram, Kite

   D) Square, Rectangle, Rhombus, Trapezoid



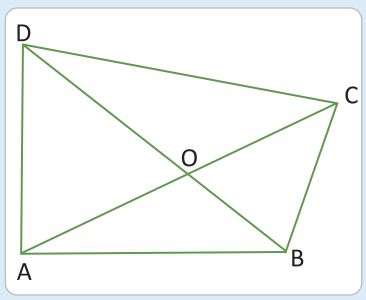
2. In quadrilateral ABCD, diagonals AC and BD intersect at point O. If ∠AOB = 40°, ∠BOC = 100°, and ∠COD = 120°, what is the measure of ∠DOA?

   A) 120°

   B) 140°

   C) 80°

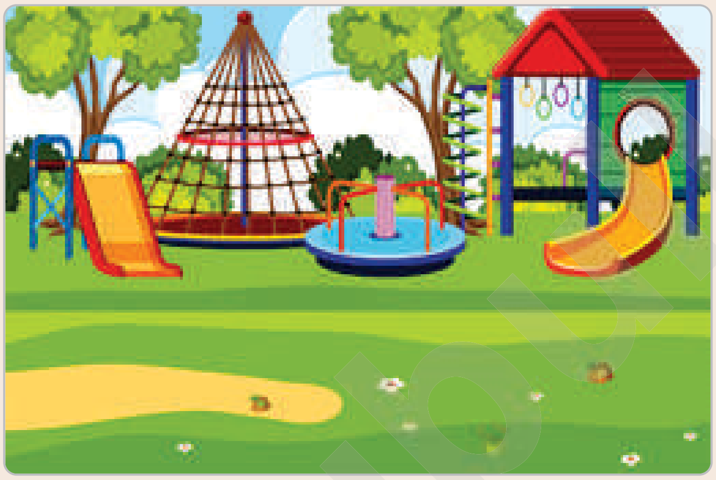
   D) 100°



Skills Developed: Interpersonal skills, Observation, Application and Decision making skills

Case Study

A group of students are designing a playground. They need to build a fence around a rectangular section of the playground and a quadrilateral-shaped garden. The rectangular section of the playground has lengths of 30 meters and 20 meters. The quadrilateral-shaped garden has four sides, and they are given the following measurements: 12 meters, 14 meters, 15 meters, and 18 meters. To ensure the safety of the students, the students want to calculate and analyze the properties of both shapes to verify their designs. They need to find the perimeter and classify the quadrilateral garden.



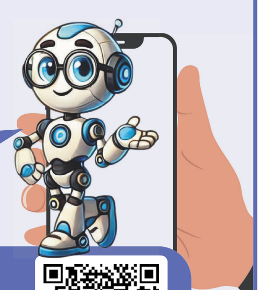
Based on this information answer the following questions:

1. Calculate the perimeter of the rectangular section of the playground.
2. Find the perimeter of the quadrilateral garden.
3. Using the formula for the sum of interior angles of a quadrilateral, calculate the total sum of the interior angles of the garden.

Skills covered: Research, Logical Reasoning, Problem-Solving, Practical Application

EeeBee: Your AI Buddy

Explore! Understanding Quadrilaterals with EeeBee AI Buddy.



Hi Friend! Use prompts to ask me questions about the chapter we just finished! eeee, lets go!

Start by Scanning this QR Code: