12
Chapter

Factorisation of Algebraic Expressions

We'll cover the following key points:

- → Factors of an algebraic expression
- → Common factors and greatest common factors of monomials
- → Factorisation when a common monomial factor occurs in each term
- → Factorisation when a binomials is common
- → Factorisation by grouping
- → Factorisation when a binomial is the difference of two squares
- → Factorisation when an expression is a perfect square
- → Factorisation of quadratic trinomials
- → Factorisation using identities
- → Linear equations in one variable
- → Rules for solving linear equations in one variable
- → Equations reducible to linear form

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Learning Outcomes

By the end of this chapter, students will be able to:

- Understand the concept of factorization as the process of expressing an algebraic expression as a product of its factors.
- Identify common factors in terms, including numerical and variable components.
- Factorize algebraic expressions using the method of grouping.
- Apply distributive property to factorize expressions.
- Factorize quadratic expressions of the form $ax^2 + bx + c$.
- Recognize and factorize algebraic identities such as $a^2 - b^2 = (a + b)(a - b)$
- Simplify expressions by canceling common factors in rational expressions.
- Solve word problems involving factorization in real-life scenarios.
- Verify factorized forms by expanding and comparing with the original expression.
- Develop problem-solving skills by applying factorization techniques in equations.

Mind Map

FACTORISATION OF ALGEBRAIC EXPRESSIONS

Factors of natural numbers

e.g.,

$30 = 2 \times 15$

$= 3 \times 10 = 5 \times 6$

Method of common factors

e.g.,

$2x + 4$

$= 2 \times x + 2 \times 2$

Factors 2 is common to both the terms

$= 2 \times (x + 2)$

$= 2\,(x + 2)$

Factorisation using Identities

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)\,(a - b) = a^2 - b^2$
- $(x + a)\,(x + b) = x^2 + (a + b)x + ab$

Division of algebraic expressions

- **Monomial by monomial**

$6x^2 \div 2x$

$= \frac{6x^2}{2x} = \frac{2 \times 3 \times x \times x}{2 \times x} = 3x$

- **Polynomial by monomial**

$4y^2 + 5xy + 6y \div 2y$

$= \frac{4y^2 + 5xy + 6y}{2y}$

$= \frac{4y^2}{2y} + \frac{5xy}{2y} + \frac{6y}{2y} = 2y^2 + \frac{5}{2}x + 3$

- **Polynomial by polynomial**

$7x^2 + 14x \div x + 2$

$= \frac{7x^2 + 14x}{x + 2}$

$= \frac{7 \times x \times x + 2 \times 7 \times x}{x + 2}$

$= \frac{7x\,(x+2)}{x + 2} = 7x$

Factors of algebraic expressions

e.g.,

$5xy = 5 \times x \times y$

$12x^2y = 12 \times x^2 \times y$

$= 4 \times 3 \times x \times x \times y$

$= 2 \times 2 \times 3 \times x \times x \times y$

Factorisation by regrouping terms

e.g.,

$2xy + 3x + 2y + 3$

$= 2 \times x \times y + 3 \times x + 2 \times y + 3$

$= x \times (2y + 3) + 1 \times (2y + 3)$

$= (2y + 3)\,(x + 1)$

Based on NCERT*

Introduction

Factors of an Algebraic Expression

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We are familiar with factors and multiples of a given positive integer. We also know that a given positive integer can always be expressed as the product of two or more prime factors. For example, 70 can be written as $70 = 2 \times 5 \times 7$. Each of the numbers, 2, 5 and 7 is called the **factor** of 70 and the process of expressing an integer as the product of two or more integers is called *factorization*.

Let us consider the monomial $3x^2y$.

The monomial $3x^2y$ can be written as $3x^2y = 1 \times 3x^2y$; $3 \times x^2y$; $3x^2 \times y$; $3x \times x \times y$; $3xy \times x$; $3y \times x \times x$; $3y \times x^2$; $3 \times x \times x \times y$.

Thus, the factors of $3x^2y$ are $1, 3, x, 3x, y, 3y, xy, 3xy, x^2, 3x^2, x^2y$ and $3x^2y$.

In the light of above discussion, we can define the term factor as:

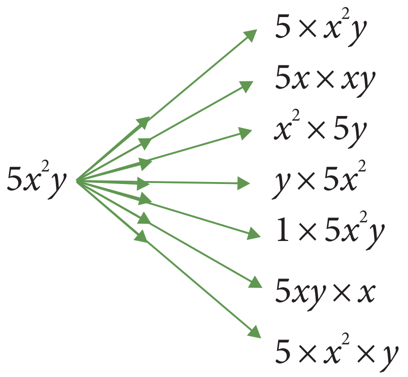
An algebraic expression can be expressed as the product of numbers and algebraic expressions. Each of these numbers and expressions are called the factors of the given algebraic expression and the algebraic expression is called the **product of these expressions**.

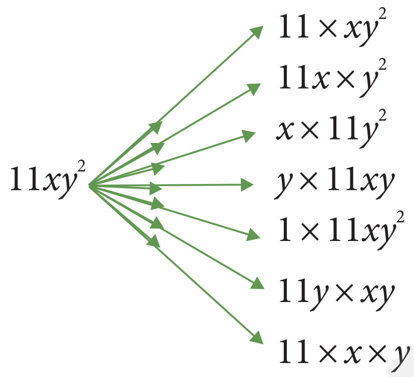
In other words, factorization is the reverse process of multiplication.

Common Factors and Greatest Common Factors of Monomials

Look at the factorization of the monomials $5x^2y$ and $11xy^2$:

Factorisation of $5x^2y$: Factorisation of $11xy^2$:





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Factors of $5x^2y$ are $1, 5, x, 5x, x^2, 5x^2, y, 5y, xy, 5xy, x^2y, 5x^2y$.

Factors of $11xy^2$ are $1, 11, x, 11x, y, 11y, y^2, 11y^2, xy, 11xy, xy^2, 11xy^2$.

The factors 1, x, y and xy are common to both $5x^2y$ and $11xy^2$. Thus, they are called the *common factors* of $5x^2y$ and $11xy^2$.

Highest Common Factors

The *highest common factors* (HCF) of the two monomials is the greatest factor that exactly divides both the monomials.

To find the HCF of the given monomials, we determine the HCF of their numerical coefficients as well as, of variables.

Working Rules

Step 1. Find the highest common factor of the numerical coefficients.
HCF of 6 and 3 is 3.

Step 2. Find the common variables of both monomials.
Common variables of x^3y and x^2y are x and y.

Step 3. Find the smallest exponent of each common variable.
Smallest exponents of x is 2 and y is 1.

Step 4. Write the common variables with their smallest exponents, *i.e.*, x^2 and y.

Step 5. Multiply the HCF of numerical coefficients with common variables written with their smallest exponents. Here, we have :

For example, to find the HCF of $6x^3y$ and $3x^2y$, $= 3 \times x^2 \times y = 3x^2y$

Thus, HCF of $6x^3y$ and $3x^2y$ is $3x^2y$.

Therefore, *HCF of two monomials can also be defined as product of the HCF of their numerical coefficients and the common variable with the smallest exponents.*

Example 1 : **Find the HCF of $30x^4y$ and $70x^3y^2$.**

Solution : Factorisation of $30x^4y = 2 \times 3 \times 5 \times x \times x \times x \times x \times y$

Factorisation of $70x^3y^2 = 2 \times 5 \times 7 \times x \times x \times x \times y \times y$

Common factors is $2 \times 5 \times x \times x \times x \times y = 10x^3y$.

$\therefore$ The HCF of $30x^4y$ and $70x^3y^2$ is $10x^3y$.

Factorisation when a Common Monomial Factor Occurs in each Term

In this method, find the HCF of the terms and divide each term of the expression by it. Finally, write the given expression as the product of the HCF and the quotient obtained.

Example 2 : **Factorize:**

(*i*) $18x^2 + 6x^3$ (*ii*) $4x^2y - 6xy^2$ (*iii*) $16a^3 - 24x^2 + 44a$

Solution: (*i*) The HCF of $18x^2$ and $6x^3$ is $6x^2$.

$\therefore \quad 18x^2 + 6x^3 = 6x^2(3 + x)$.

(*ii*) The HCF of $4x^2y$ and $6xy^2$ is $2xy$.

$\therefore \quad 4x^2y - 6xy^2 = 2xy(2x - 3y)$.

(*iii*) $16x^3 - 24x^2 + 44x$

$$16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$$
$$24x^2 = 2 \times 2 \times 2 \times 3 \times x \times x$$
$$44x = 2 \times 2 \times 11 \times x$$

$$\therefore \; 16x^3 - 24x^2 + 44x = (2 \times 2 \times 2 \times 2 \times x \times x \times x) - (2 \times 2 \times 2 \times 3 \times x \times x) + (2 \times 2 \times 11 \times x)$$
$$= (4x \times 4x^2) - (4x \times 6x) + (4x \times 11) = 4x\,(4x^2 - 6x + 11).$$

Example 3 : **Factorize: $18a^2b - 9ab^2 + 15abc$.**

Solution : The HCF of the terms of the given expression is $3ab$.

$\therefore \quad 18a^2b - 9ab^2 + 15abc = 3ab(6a - 3b + 5c)$.

Factorisation when a Binomial is Common

When each of the terms of an algebraic expression has a binomial as a factor, write the given expression as the product of the binomial and the quotient obtained on dividing the given expression by the binomial.

Example 4 : **Factorize: $2a(x + 3y) + 3b(x + 3y)$.**

Solution : $2a(x + 3y) + 3b(x + 3y)$

Here, $(x + 3y)$ is common in each term.

$\therefore \quad 2a(x + 3y) + 3b(x + 3y) = (x + 3y)(2a + 3b)$.

Example 5 : **Factorize : $(7a + 2b)(2x + y) - (2a - b)(2x + y)$.**

Solution: $(7a + 2b)(2x + y) - (2a - b)(2x + y)$

$= (2x + y)[(7a + 2b) - (2a - b)]$ [Taking $(2x + y)$ common]

$= (2x + y)\,[(7a + 2b - 2a + b)]$

$= (2x + y)\,[7a - 2a + 2b + b] = (2x + y)(5a + 3b)$.

Factorisation by Grouping

Sometimes, there is no common factor among all the terms of an algebraic expression. So, to factorize such expression having four terms, we use the method called *factorization by grouping*.

In this method, we have the following rule :

Working Rules

Rearrange the terms of the given expression in a way that all the terms have a common factor. Rewrite the expression using distributive law by taking out the factor which is common to each group.

Example 6: **Factorize: $xy + 3x + y + 3$.**

Solution: Here, there is no factor common in all the four terms. But we can take terms in two groups which have a common factor as shown below:

$$\begin{aligned} xy + 3x + y + 3 &= (xy + 3x) + (y + 3) \\ &= x(y + 3) + 1(y + 3) \\ &= (x + 1)(y + 3). \end{aligned}$$

Example 7: **Factorize: $6ab - b^2 + 12ac - 2bc$.**

Solution:

$$\begin{aligned} 6ab - b^2 + 12ac - 2bc &= (6ab + 12ac) + (-b^2 - 2bc) \\ &= 6a\,(b + 2c) - b\,(b + 2c) \\ &= (6a - b)(b + 2c). \end{aligned}$$

Example 8: **Factorize: $xy^2 - yz^2 - xy + z^2$.**

Solution:

$$\begin{aligned} xy^2 - yz^2 - xy + z^2 &= (xy^2 - yz^2) - (xy - z^2) \\ &= y\,(xy - z^2) - 1(xy - z^2) \\ &= (xy - z^2)\,(y - 1). \end{aligned}$$

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Exercise 12.1

1. Find all the possible factors of each of the following monomials :
   (i) $6x^2$ (ii) $8x^2y$ (iii) $13pq$ (iv) $5xy^2$
2. Find all the common factors of each of the following :
   (i) $3x^2, 9xy$ (ii) $6mn, 2n$ (iii) $45a, 15a^2b$ (iv) $12x^2y, 36axy$
3. Factorize each of the following :
   (i) $3x^2 + 9$ (ii) $30x - 25y$ (iii) $42x^2y^2 - 36y$ (iv) $5x^3 + 10x^2y$
   (v) $54m^2 - 18mn - 27n^2$ (vi) $5a^2 + 5a + 15ab + 10b$
4. Factorize each of the following :
   (i) $2x\,(x + y) + 4y\,(x + y)$ (ii) $a + 2 + 2ab + a^2b$ (iii) $x(4x + 12y) - 3(2x + 6y)$
   (iv) $6(x + 2y)^3 - 8(x + 2y)^2 + 4(x + 2y)$ (v) $2x(a - b)^3 + 3y(a - b)^2 - 5\,(a - b)$
   (vi) $5a\,(2b - 3c)^2 + 2a(2b - 3c)$

5. Factorize each of the following :

(i) $x^2 + yz + xy + xz$ (ii) $ab - xy + ya - xb$ (iii) $8 - 4x - 2x^2 + x^3$ (iv) $xz^2 + xt^2 + yz^2 + yt^2$

(v) $1 + x + 8y + 8xy$ (vi) $5 - 2a - 6ab + 15b$

(vii) $(x+y)^3 - 3(x+y)^2 + 4(x+y)$ (viii) $a(y+z)^3 + b(y+z)^4 + c(y+z)^2$

(ix) $3a(xy+3) + b(xy+3)$ (x) $14(m-n) - 21(m-n)$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Factorisation when a Binomial is the Difference of Two Squares

We are aware of the standard identity $(a+b)(a-b) = a^2 - b^2$.

Since factorization is the inverse process of multiplication, we can regard this standard identity as a special case of factorization as given below:

$$a^2 - b^2 = (a+b)(a-b).$$

Example 9 : **Factorize each of the following.**

(i) $\mathbf{x^2 - 144}$ **(ii)** $\mathbf{49x^2 - 36y^2}$

Solution : (i) $x^2 - 144 = (x)^2 - (12)^2$

$= (x+12)(x-12)$

(ii) $49x^2 - 36y^2 = (7x)^2 - (6y)^2 = (7x+6y)(7x-6y).$

Factorisation when an Expression is a Perfect Square

For factorizing algebraic expressions expressible as a perfect square, we use the following formulae:

(i) $a^2 + 2ab + b^2 = (a+b)^2 = (a+b)(a+b)$

(ii) $a^2 - 2ab + b^2 = (a-b)^2 = (a-b)(a-b)$

Example 10 : **Factorize: $\mathbf{9x^2 + 30x + 25}$.**

Solution : $9x^2 + 30x + 25 = (3x)^2 + 2 \times 3x \times 5 + (5)^2$

$= (3x+5)^2$

$= (3x+5)(3x+5).$ $[\quad (a+b)^2 = (a+b)(a+b)]$

Example 11 : **Factorize: $\mathbf{1 - 12a + 36a^2}$.**

Solution : $1 - 12a + 36a^2 = 36a^2 - 12a + 1$

$= (6a)^2 - 2 \times 6a \times 1 + (1)^2$

$= (6a-1)^2 = (6a-1)(6a-1).$

Exercise 12.2

1. Factorize:

(i) $x^2 - 16z^2$ (ii) $a^2 - (x+2)^2$ (iii) $81(a-b)^2 - 36(a+b)^2$

(iv) $x^8 - 81$ (v) $a^2 + 2ab + b^2 - z^2$ (vi) $x^8 - y^8$

(vii) $\frac{x^2}{36} - 4$ (viii) $\frac{x^2}{4} - 16z^2$ (ix) $\frac{x^2}{9} - 16$

2. Factorize :

(i) $4a^2 - 4a + 1$ (ii) $25a^2 + 30ab + 9b^2$ (iii) $16x^2 - 40x + 25$

(iv) $16a^2b^2 - 8ab^2 + b^2$ (v) $9a^2 - 6a + 1$ (vi) $-8xyz + 2x^2 + 8y^2z^2$

(vii) $9x^2 - 24x + 16$ (viii) $a^2 - a + \frac{1}{4}$ (ix) $25x^2 - 40x + 16$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Factorisation of Quadratic Trinomials

Case I. *When the expression is of the form* $x^2 + px + q$.

In this type of expressions, we use the identity,

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

So, in order to factorize $x^2 + px + q$, we find two numbers a and b such that $(a + b) = p$ and $ab = q$.

Now, $x^2 + px + q = x^2 + (a+b)x + ab = (x+a)(x+b)$.

Example 12 : Factorize:

(i) $x^2 + 6x + 9$ (ii) $x^2 - 2x - 35$

Solution : (i) The given expression is $x^2 + 6x + 9$.

First, we split 6 into two parts whose sum is 6 and product is 9.

Clearly, such numbers are 3 and 3.

$\therefore \quad x^2 + 6x + 9 = x^2 + 3x + 3x + 9$

$= x(x+3) + 3(x+3) = (x+3)(x+3)$.

(ii) $x^2 - 2x - 35$

First we split –2 into two parts whose sum is –2 and product is –35.

Clearly, $-2 = -7 + 5$, such that $-7 \times 5 = -35$.

$\therefore \quad x^2 - 2x - 35 = x^2 - 7x + 5x - 35$

$= x(x-7) + 5(x-7) = (x-7)(x+5)$.

Case II. *When the expression is of the form* $ax^2 + bx + c$.

In this type of expressions, we split up b into two parts whose sum is b and product is ac.

Example 13 : Factorize:

(i) $x^2 + 7x + 12$ (ii) $2x^2 - 3bx - 2b^2$

Solution : (i) The given expression is $x^2 + 7x + 12$.

We have, $4 \times 3 = 12$

First, we find two numbers whose sum is 7 and product is 12.

Clearly, the numbers are 4 and 3.

$\therefore \quad x^2 + 7x + 12 = x^2 + 4x + 3x + 12$

$= x(x+4) + 3(x+4) = (x+4)(x+3)$

(ii) The given expression is $2x^2 - 3bx - 2b^2$.

We have, $2 \times (-2) = -4$

First, we find two numbers whose sum is –3 and product is –4.

Clearly, the numbers are –4 and 1.

$$\therefore \quad 2x^2 - 3bx - 2b^2 = 2x^2 - 4bx + bx - 2b^2$$
$$= 2x(x - 2b) + b(x - 2b) \quad = (x - 2b)(2x + b).$$

Exercise 12.3

Factorize.

1. $4x^2 + 8x + 3$
2. $a^2 + 9a + 20$
3. $x^2 + 9x + 18$
4. $3x^2 - 15x + 18$
5. $a^2 - 24a + 128$
6. $x^2 - 40x + 279$
7. $x^2 - 2x - 24$
8. $x^2 - 21x + 54$
9. $z^2 - z - 2$
10. $m^2 - 4m - 12$
11. $-x^2 + 3x + 18$
12. $a^2 - 5a - 176$
13. $3x^2 + 5x + 2$
14. $3x^2 + 15x - 72$
15. $4x^2 - 19x + 15$
16. $3m^2 + 14m + 8$
17. $m^2 + 3m - 4$
18. $2x^2 + x - 6$
19. $6x^2 + x - 7$
20. $-z^2 - 13z - 40$
21. $-(-4x^2 + 4x + 15)$
22. $x^2 - 17x + 60$
23. $2x^2 - 3ax - 2a^2$
24. $3x^2 - 15x + 18$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Factorisation using Identities

We have already learned the following identities :

$$(a + b)^2 = a^2 + 2ab + b^2 = (a + b)(a + b)$$
$$(a - b)^2 = a^2 - 2ab + b^2 = (a - b)(a - b)$$
$$(a + b)(a - b) = a^2 - b^2$$

These identities are very useful in finding the factors of a variety of algebraic expressions which are perfect squares or the product of the sum and difference of two variables.

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Example 14 : Factorize : $x^2 + 4x + 4$.

Solution : If we observe carefully, we can see that this expression is the square of the expression $(x + 2)$. The first term is the square of x, *i.e.*, x^2. The last term is the square of 2, *i.e.*, 4. The middle term is $2 \times x \times 2 = 4x$.

So we can write $x^2 + 4x + 4 = (x + 2)^2 = (x + 2)(x + 2)$

Example 15 : Factorize : $x^2 - 6x + 9$.

Solution : The first term is the square of x. The last term is the square of 3. The second term $(-6x)$ is equal to $2 \times x \times (-3)$ or $2\,x\,(-x) \times 3$.

So, $x^2 - 6x + 9) = (x - 3)^2$ or $(3 - x)^2$.

Example 16 : Factorize : $x^2 - 16$.

Solution: This expression is the difference of the squares of x and 4. Hence, they are the product of the sum and difference of the square root of the terms

$$x^2 - 16 = (x)^2 - (4)^2 = (x + 4)\,(x - 4)$$

Let us verify :

$$(x+4)(x-4) = x(x-4)+4(x-4)$$
$$= x^2-4x+4x-16$$
$$= x^2-16$$

Example 17: Factorize : $p^2+25q^2+10pq-36$.

Solution: $p^2+25q^2+10pq-36$

$$= (p^2+25q^2+10pq)-36$$
$$= (p^2+10pq+25q^2)-36$$
$$= (p+5q)^2-36 \quad = (p+5q)^2-6^2$$
$$= (p+5q+6)(p+5q-6)$$

Example 18: Evaluate : 703×697.

Solution: $703 = (700+3)$ and $697 = (700-3)$

So, $703\times 697 = (700+3)(700-3)$

$$= 700^2-3^2 = 490000-9 = 489991$$

Example 19: Evaluate : 40.7×39.3.

Solution: $40.7 = (40+0.7)$ and $39.3 = (40-0.7)$

Hence, $40.7\times 39.3 = (40+0.7)(40-0.7)$

$$= (40)^2-(0.7)^2$$
$$= 1600-0.49 = 1599.51$$

Example 20: Evaluate : $(888)^2-(112)^2$.

Solution: $888^2-112^2 = (888+112)(888-112)$

$$= 1000\times 776 = 776000$$

Example 21: Factorize : $11p^5-44p$.

Solution: $11p^5-44p = 11p(p^4-4)$

$$= 11p[(p^2)^2-2^2]$$
$$= 11p(p^2-2)(p^2+2)$$

Example 22: Factorize : x^8-y^8.

Solution: $x^8-y^8 = (x^4+y^4)(x^4-y^2)$

$$= (x^4+y^4)[(x^2)^2-(y^2)^2]$$
$$= (x^4+y^4)(x^2+y^2)(x^2-y^2)$$
$$= (x^4+y^4)(x^2+y^2)(x+y)$$

Example 23: Factorize : $p^4q^{12}-p^{12}q^4$.

Solution: $p^4q^{12}-p^{12}q^4 = p^4q^4(q^8-p^8)$

$$= p^4q^4(q^4+p4)(q^4-p^4)$$
$$= p^4q^4(q^4+p^4)(q^2+p^2)(q^2-p^2)$$
$$= p^4q^4(q^4+q^4)(q^2+p^2)(q+p)(q-p)$$

Example 24: Factorize: p^4+p^2+1.

$$p^4+p^2+1 = p^4+p^2+1+p^2-p^2$$
$$= p^4+2p^2+1-p^2 \qquad = (p^4+2p^2+1)-p^2$$
$$= (p^2+1)^2-(p)^2 \qquad = (p^2+1+p)(p^2+1-p)$$
$$= (p^2+p+1)(p^2-p+1)$$

Exercise 12.4

1. Regroup and factorize:

(a) $x^2+xy+9x+9y$ (b) $15xy+6x+10y+4$ (c) $4x^2-16xy-3x+12y$

(b) $12ab-8b-6+9a$ (e) $3xy-4y-3x+4$ (f) $ab+4a-3b-12$

(g) $9x^2+3xy-12x-4y$ (h) $12x-8+3xy-2y$

2. Factorize the following using appropriate identity.

(a) a^2+6a+9 (b) $4x^2+4xy+y^2$ (c) $4a^2+20ab+25b^2$

(d) $25x^2-20xy+4y^2$ (e) $4x^2-12x+9$ (f) $121+22x+x^2$

(g) x^2-y^2 (h) $16a^2+88a+121$ (i) $4p^2-16q^2$

(j) $25x^2-49y^2$ (k) $36x^2-84xy+49y^2$ (l) x^4-81

(m) m^6-25 (n) $64x^2+16x+1$ (o) $144-24x+x^2$

(p) $25a^2b^2-9p^2q^2$ (q) $36a^2-36ab+9b^2$ (r) $64-9t^2$

3. Factorize:

(a) $x^2+12x+36$ (b) $4x^2+12xy+9y^2$ (c) $100p^2-20p+1$

(d) $9m^2-66mn+121n^2$ (e) $c^2-(x-y)^2$ (f) $9a^2-4(b-c)^2$

4. Evaluate:

(a) 307×293 (b) 4.15×3.85 (c) $(72)^2-(68)^2$ (d) 67×73

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Linear Equations in one Variable

A way of describing an equation is in terms of its degree. The degree of an equation is equal to the highest power of the variable in it. If an equation has only one variable and the highest power of the variable is one, then it is called a **linear equation in one variable.**

The following equations are linear equations in one variable, with their variables being *x, a* and *b* respectively.,

$$4x+7=13,\ 10a-4=22,\ 8n=40$$

But the following algebraic expressions are not linear expressions since the highest power of their variables is 2, *i.e.*, more than 1.

$$3x^2+4y^2,\ 5x^2+5,\ 3x^3-15$$

In this chapter, you will study about linear equations in one variable, or about equations which can be reduced to linear equations.

Rules for Solving Linear Equations in one Variable

A linear equation is an equation having the variable with power 1. The basic principle used in solving any linear equation is that any operation performed should be done on both sides of the equation to hold it true.

The rules which are important in solving a linear equations in one variable are:

1. The same quantity can be added or subtracted to (from) both sides of an equation without changing its equality.
2. Both sides of an equation may be multiplied or divided by the same non-zero number without changing its equality.
3. If $\frac{ax+b}{cx+d} = \frac{r}{s}$, then $s(ax+b) = r(cx+d)$. This process is called *cross multiplication*.
4. Any term of an equation may be taken from one side to the other with a change in its sign. This does not affect the equality of the statement. This process is called *transposition*. The sign is changed from

   (i) $+$ to $-$ (ii) $-$ to $+$ (iii) $\times$ to $\div$ (iv) $\div$ to $\times$

Illustrative Examples :

Example 25: **Find the solution for $3x + 7 = 37$.**

Solution : $3x + 7 = 37$

Step 1. Subtract 7 from both sides.

$$3x + 7 - 7 = 37 - 7$$

$$\Rightarrow \quad 3x = 30$$

Step 2. Divide both sides by 3.

$$\frac{3x}{3} = \frac{30}{3}$$

$\Rightarrow$ $x = 10$ is the required solution.

Example 26 : **Solve $\frac{4x}{5} - 8 = 24$ and verify the solution**

Solution : $\frac{4x}{5} + 8 = 24$

Transposing -8 to the RHS, we get

$$\frac{4x}{5} = 24 + 8 = 32$$

Multiplying both sides by 5, we get

$$5 \times \frac{4x}{5} = 5 \times 32$$

$$\Rightarrow \quad 4x = 160$$

Dividing both sides by 4, we have :

$$x = \frac{160}{4} = 40$$

Now, LHS $= \frac{4}{5} \times 40 - 8 = \frac{160}{5} - 8 = 32 - 8 = 24$

Example 27: **Solve the equation $\frac{4}{7}x - \frac{1}{4} = 8$ and check the result.**

Solution : We have, $\frac{4}{7}x - \frac{1}{4} = 8.$

Adding $\frac{1}{4}$ to both sides of the equation, we get :

$$\Rightarrow \frac{4}{7}x - \frac{1}{4} + \frac{1}{4} = 8 + \frac{1}{4}$$

$$\Rightarrow \frac{4}{7}x - 0 = \frac{8}{1} + \frac{1}{4}$$

$$\Rightarrow \frac{4}{7}x = \frac{32+1}{4}$$

$$\Rightarrow \frac{4}{7}x = \frac{33}{4} \quad \text{.... }(i)$$

Multiplying both sides of eqn. (*i*) by 7, we get :

$$\frac{4}{7}x \times 7 = \frac{33}{4} \times 7$$

$$\Rightarrow \quad 4x = \frac{231}{4} \quad \text{... (ii)}$$

Now, dividing both sides of eqn. (*ii*) by 4, we get :

$$\Rightarrow \quad \frac{4x}{4} = \frac{231}{4} \times \frac{1}{4}$$

$$\Rightarrow \quad x = \frac{231}{16} = 14\frac{7}{16}$$

Thus, $x = 14\frac{7}{16}$ is the solution of the given equation.

Check: Substituting $x = 14\frac{7}{16}$ in the given equation, we get :

$$\text{LHS} = \frac{4}{7} \times 14\frac{7}{16} - \frac{1}{4}$$

$$= \frac{4}{7} \times \frac{231}{16} - \frac{1}{4}$$

$$= \frac{33}{4} - \frac{1}{4} = \frac{33-1}{4} = \frac{32}{4} = 8 \text{ (RHS)}$$

i.e., for $x = 14\frac{7}{16}$, we have, LHS = RHS

∴ Our solution is correct.

Note

From examples 2 and 3, it is obvious that the solution of an equation can be a rational number. The coefficient of the variable can also be a rational number.

Example 28: **Solve $8x - 3(x - 5) = 5 + 3x$ and check the result.**

Solution: We have,

$$8x - 3(x - 5) = 5 + 3x$$

or $8x - 3x + 15 = 5 + 3x$ (By removing brackets)

or $5x - 3x = 5 - 15$ (By transposition)

or $2x = -10.$

$\therefore$ $x = -5.$

Check: LHS $= 8x - 3(x - 5) = 8 \times (-5) - 3[(-5) - 5]$

$$= 8 \times (-5) - 3[-5 - 5]$$

$$= 8 \times (-5) - 3 \times (-10)$$

$$= -40 + 30 = -10$$

RHS $= 5 + 3x = 5 + 3 \times (-5)$

$$= 5 - 15 = -10$$

$\therefore$ LHS = RHS. Hence, our solution is correct.

Example 29: **Solve $\frac{2x - 18}{5} - x + \frac{x - 1}{2} = 14$ and check the result.**

Solution: We have,

$$\frac{2x - 18}{5} - x + \frac{x - 1}{2} = \frac{14}{1}$$

$$\Rightarrow \frac{2x - 18}{5} - \frac{x}{1} + \frac{x - 1}{2} = \frac{14}{1}$$

$$\Rightarrow \frac{2(2x - 18) - 10x + 5(x - 1)}{10} = \frac{140}{10}$$

$$\Rightarrow \frac{4x - 36 - 10x + 5x - 5}{10} = \frac{140}{10}$$

$$\Rightarrow -41 - x = 140$$

$$\Rightarrow -x = 140 + 41$$

$$\Rightarrow x = -181$$

Check: Substituting $x = -181$ in the given equation, we have

$$\text{LHS} = \frac{2 \times (-181) - 18}{5} - (-181) + \frac{-181 - 1}{2} = \frac{-362 - 18}{5} + 181 + \frac{-182}{2}$$

$$= \frac{-380}{5} + 181 + (-91)$$

$$= -76 + 181 - 91 = 181 - 167 = 14 = \text{RHS}$$

Since, LHS = RHS, our answer is correct.

Equations Reducible to Linear Form

To reduce a given equation to linear form, cross-multiply its terms on both sides of equality and after cross-multiplication we get an equation involving variables having power more than one. on simplifying the terms with power more than one, like x^2 or y^2, cancel out the terms and you are left with a linear equation which you can solve in the usual manner.

Example 30: **Solve:** $\frac{5x+2}{7x-5} = \frac{5x+1}{7x+2}$ **and verify your result.**

Solution: We have, $\frac{5x+2}{7x-5} = \frac{5x+1}{7x+2}$

$\Rightarrow \quad (5x+2)(7x+2) = (7x-5)(5x+1)$ (By cross multiplication)

$\Rightarrow \quad 5x(7x+2)+2(7x+2) = 7x(5x+1)-5(5x+1)$

$\Rightarrow \quad 35x^2+10x+14x+4 = 35x^2+7x-25x-5$

$\Rightarrow \quad 35x^2+24x+4 = 35x^2-18x-5$

$\Rightarrow \quad 35x^2+24x-35x^2+18x = -5-4$ (By transposition)

$\Rightarrow \quad 42x = -9$

$\Rightarrow \quad x = \frac{-3}{14}$

Check: Substituting $x = \frac{-3}{14}$, in the given equation, we have,

$$\text{LHS} = \frac{5\times\frac{-3}{14}+2}{7\times\frac{-3}{14}-5} = \frac{\frac{-15}{14}+2}{\frac{-3}{2}-5}$$

$$= \frac{\frac{-15+28}{14}}{\frac{-3-10}{2}}$$

$$= \frac{\frac{13}{14}}{\frac{-13}{2}} = \frac{13}{14}\times\frac{2}{-13} = \frac{-1}{7}$$

$$\text{RHS} = \frac{5\times\frac{-3}{14}+1}{7\times\frac{-3}{14}+2} = \frac{\frac{-15}{14}+1}{\frac{-3}{2}+2}$$

$$= \frac{\frac{-15+14}{14}}{\frac{-3+4}{2}}$$

$$= \frac{\frac{-1}{14}}{\frac{1}{2}}$$

$$= \frac{-1}{14}\times\frac{2}{1} = \frac{-1}{7}$$

We have, L HS = RHS. Hence, our answer is correct.

Example 31 : **Solve the equation given below :**

$$\frac{3}{8}(7x-1)-\left(4x-\frac{1-2x}{2}\right)=x+\frac{1}{4}$$

Solution : We have, $\frac{3}{8}(7x-1)-\left(4x-\frac{1-2x}{2}\right)=x+\frac{1}{4}$

Removing brackets; we get :

$$\frac{21x}{8}-\frac{3}{8}-\frac{4x}{1}+\frac{1-2x}{2}=\frac{x}{1}+\frac{1}{4}$$

or, $\frac{21x}{8}\times 8-\frac{3}{8}\times 8-\frac{4x}{1}\times 8+\frac{1-2x}{2}\times 8=\frac{x}{1}\times 8+\frac{1}{2}\times 8$

[Multiply throughout by LCM of denominators 8]

or, $21x-3-32x+4(1-2x)=8x+4$

or, $21x-3-32x+4-8x=8x+4$

or, $21x-32x-8x-8x=3-4+4$ [By transposition]

or, $21x-48x=3$

or, $-27x=3$

or, $x=\frac{-3}{27}=\frac{-1}{9}$

Hence $x=\frac{-1}{9}$ is the solution of the given equation.

Exercise 12.5

Solve each of the following equations and check your results :

1. $8x+4=3(x-1)+7$
2. $x-\frac{(x-1)}{2}=1-\frac{x-2}{3}$
3. $9x-4(x-4)=3x$
4. $9y-5(2y-3)=1-2y$
5. $0.5(2x-5)=1.2(2-5x)$
6. $3x=5x-\frac{8}{5}$
7. $5x-\frac{8}{5}=2x$
8. $\frac{x}{5}-\frac{5}{2}=8$
9. $\frac{3x-5}{2x+15}=\frac{8}{9}$
10. $\frac{1}{2x+1}=\frac{1}{3x+1}$
11. $\frac{7x}{2x-5}=\frac{14}{5}$
12. $\frac{x-5}{6}=\frac{x-6}{3}$
13. $\frac{3x}{5}-\frac{x}{4}=\frac{1}{8}$
14. $2+\frac{5}{3y}=\frac{26}{3y}-1$
15. $\frac{2x-11}{3}+x+\frac{x-1}{3}=12$
16. $\frac{x+5}{3x-7}=\frac{2}{7}$
17. $\frac{3}{2b-5}=\frac{2}{b+3}$
18. $\frac{0.21+0.7a}{a}=0.56$
19. $\frac{11(2+x)-5(x+12)}{3-7x}=2$
20. $\frac{0.3x-5}{1.5x+11}=\frac{-3}{5}$
21. $\frac{3x+4}{6x+7}-\frac{x+1}{2x+3}=0$
22. $9(x-1)-20(x-4)=\frac{96}{5}(x-6)+5$
23. $1.4\,(2-x)=0.5\,(2x-4)$
24. $\frac{x+4}{3}+\frac{2x-3}{35}=\frac{5x-32}{4}-\frac{x+9}{28}$
25. $0.02x-0.13x+0.11=0.101-0.2x$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Revision Exercise

Conceptual Learning

1. Tick (✓) the correct option:

(i) The factorization of $(x-1)^2 - 1$ is —

(*a*) $(x-1)(x-3)$ (*b*) $x(x-3)$ (*c*) $(x-1)(x-2)$ (*d*) $x(x-2)$

(ii) Which of these is not a polynomial?

(*a*) $x^2 + 3x + 7$ (*b*) $9x^3 + \frac{3}{7}x^2 + 5$ (*c*) $\frac{3}{x} + \frac{5}{x^2}$ (*d*) none

(*iii*) The factorization of $x^2 - 9$ is —

(*a*) $x(x+3)$ (*b*) $x(x-3)$ (*c*) $(x+3)(x-3)$ (*d*) $(x+1)^2$

(*iv*) The coefficient of x in $-8xy^2$ is —

(*a*) 8 (*b*) –8 (*c*) $-8y^2$ (*d*) $8y^2$

(*v*) The value of $61^2 - 59^2$ is —

(*a*) 120 (*b*) 240 (*c*) 360 (*d*) none

(*vi*) One factor of $12a^2b^2 - 18ab$ is —

(*a*) $6ab^2$ (*b*) $6a^2b^2$ (*c*) $6ab$ (*d*) none

2. Factorize: $x^2 - 4x - 12$.
3. Factorize: $x^2 + 12x + 36$
4. Find the remainder obtained on dividing $x^3 + 3x^2 - 5x + 4$ by $(x - 1)$.
5. If $ab = 6$ and $(a + b) = 5$, then find the value of $a^2 + b^2$.
6. Factorize: of $x^2 + 12x + 32$
7. Factorize: $p^2 + qr + pq + pr$
8. Find the value of $(x^2 - 4) - (x - 2)(x + 2)$.

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Mental Maths

Experiential Learning

1. Simplify: $a^2 + bc + ab + ac$.
2. Write the factorization of $9x^2 - 16$.
3. Factorize: $x^2 + 11x - 60$.
4. Factorize: $x^2 + 22x - 48$.
5. Find the value of $(502)^2 - (498)^2$.

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HOTS (Higher Order Thinking Skills)

Experiential Learning

1. Find the greatest common factor of the following monomials:
   (*i*) x^2y^3 and $2x^3y^2$ (*ii*) a^3 and $-ba^2$ (*iii*) $2ax^2$ and $-6a^2x$
2. Find the product of $(x + y)$, $(x - y)$ and $(x^2 + y^2)$.
3. If $pq = 6$ and $p + q = 5$, then find the value of $p^2 + q^2$.
4. Find the factorization of $(x + y)^2 + 2(x + y) + 1$.
5. Find the factorization of $16x^2 - 40xy + 25y^2$.

Assertion and Reason

Critical Thinking

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

1. **Assertion (A) :** The factorization of $(x-1)^2-1$ is $x(x+2)$.
   Reason (R) : Factors of $36x^5y^4 + 72x^4y^5$ are $36x^4y^4$ and $(x+2y)$.
2. **Assertion (A) :** $p^2-4q^2 = (p+2q)(p-2q)$
   Reason (R) : Finding factors is opposite of finding product.
3. **Assertion (A) :** $a(c+d) + b(c+d) = (c+d)(a+b)$
   Reason (R) : $ac + bc + ad + bd = (a+b)(c-d)$
4. **Assertion (A) :** $x^2 + 12x + 32 = (x+4)(x+8)$
   Reason (R) : $x^2 + 5x - 6 = (x+6)(x-1)$
5. **Assertion (A) :** $9y^2 + 81y + y + 9 = (y+9)(9y+1)$
   Reason (R) : $x^2 + 11x - 60 = (x-15)(x+4)$

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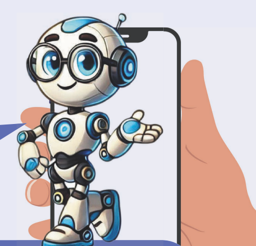

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