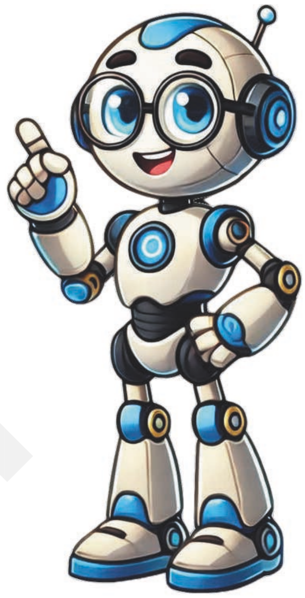
10
Chapter

Exponents and Powers

We'll cover the following key points:

→ Positive integral exponents of a rational number
→ Negative integer exponent
→ Laws of exponents
→ Extension of laws of exponents
→ Use of exponents to express small numbers in standard form



Hi, I'm EeeBee

Do you Remember fundamental concept in previous class.
In class 7th we learnt
→ Exponents
→ Laws of Exponents

Still curious? Talk to me by scanning the QR code.

Learning Outcomes

By the end of this chapter, students will be able to:

- Understand and apply the laws of exponents for multiplication, division, and powers of powers.
- Simplify expressions involving exponents using the laws of exponents.
- Evaluate numerical expressions with exponents and powers.
- Convert numbers written in exponential form into standard form and vice versa.
- Solve problems related to scientific notation using exponents and powers.
- Apply the concepts of exponents to solve real-life problems, such as large numbers in population growth or area calculations.
- Identify and understand the concept of negative exponents and zero exponents.
- Use exponents to represent powers of 10 and solve related problems.
- Compare and order numbers written in exponential form.
- Recognize and solve problems involving fractional exponents.

Mind Map

EXPONENTS AND POWERS

Powers with negative exponents

- $a^{-2} = \frac{1}{a^2}$
- $10^{-2} = \frac{1}{10^2}$

Laws of exponents

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^m \times b^m = (ab)^m$
- $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
- $a^0 = 1$

Use of exponent to express small numbers in standard form

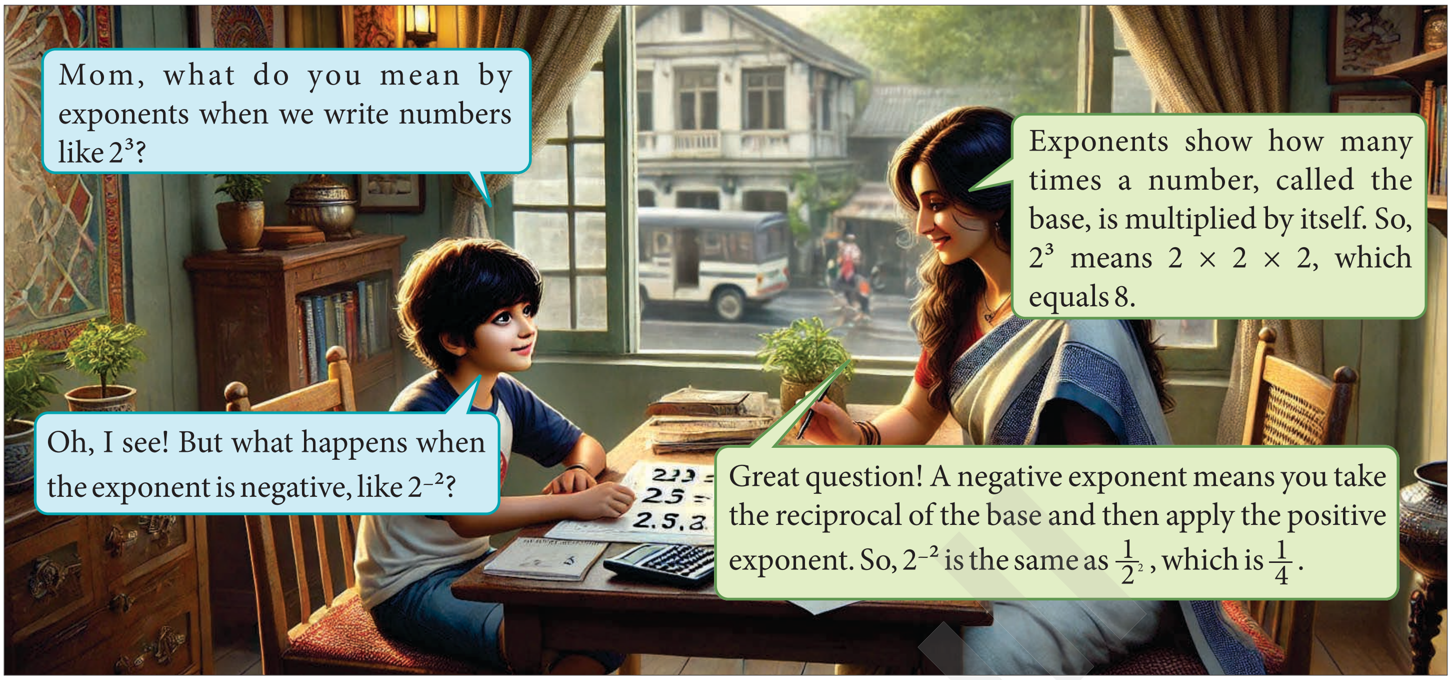
e.g.,

$0.0007 = \frac{7}{10000} = \frac{7}{10^4} = 7 \times 10^{-4}$

$0.000039 = \frac{39}{1000000} = \frac{39}{10^6} = 39 \times 10^{-6}$

Based on NCERT*

Introduction



Positive Integral Exponents of a Rational Number

The rational number multiplied with itself a number of times can be written as the rational number raised to the power a natural number equal to the number of times the rational number is multiplied with itself.

Note: $\frac{2}{9} \times \frac{2}{9} \times \frac{2}{9} \times \frac{2}{9}$ can be written as $\left(\frac{2}{9}\right)^4$ and it is read as $\frac{2}{9}$ raised to the power 4.

Here, $\frac{2}{9}$ is called the **base** and 4 is called the **exponent or index**.

We define the positive integral exponent (power) of a rational number as under:

Let $\frac{a}{b}$ be any positive rational number with n as any integer. Then

$$\left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}, \ldots, \times n \text{ times}$$

$$= \frac{a^n}{b^n}$$

So, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where n is a positive integer.

> **REMEMBER**
> In case of numbers with base as 0, the exponent decreases by 1 and the value becomes $\frac{1}{10}$ of the previous value.

This notation of writing the product of a rational number by itself several times is called the **exponential notation** or **power notation**.

Example 1 : Express each of the following in power notation:

(*i*) $-3 \times -3 \times -3 \times -3 \times -3$ (*ii*) $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}$ (*iii*) $\frac{-4}{9} \times \frac{-4}{9} \times \frac{-4}{9}$

Solution : (*i*) $-3 \times -3 \times -3 \times -3 \times -3 = (-3)^5$ (*ii*) $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} \times \frac{3}{7} = \left(\frac{3}{7}\right)^4$

(*iii*) $\frac{-4}{9} \times \frac{-4}{9} \times \frac{-4}{9} = \left(\frac{-4}{9}\right)^3$

Example 2: **Express: (*i*) $\frac{-8}{27}$ and (*ii*) $\frac{16}{49}$ as the powers of rational numbers.**

Solution: (*i*) We have, $-8 = -2\times-2\times-2 = (-2)^3$ and $27 = 3\times3\times3 = 3^3$

$$\therefore \quad \frac{-8}{27} = \frac{(-2)^3}{3^3} = \left(\frac{-2}{3}\right)^3$$

(*ii*) We have, $16 = 4\times4 = 4^2$ and $49 = 7\times7 = 7^2$

$$\therefore \quad \frac{16}{49} = \frac{4^2}{7^2} = \left(\frac{4}{7}\right)^2.$$

Example 3: **Express each of the following powers of rational numbers as a rational number:**

(*i*) $\left(\frac{-2}{3}\right)^3$ (*ii*) $\left(\frac{-3}{8}\right)^4$

Solution: (*i*) $\left(\frac{-2}{3}\right)^3 = \frac{-2}{3}\times\frac{-2}{3}\times\frac{-2}{3} = \frac{-2\times-2\times-2}{3\times3\times3} = \frac{-8}{27}$

(*ii*) $\left(\frac{-3}{8}\right)^4 = \frac{-3}{8}\times\frac{-3}{8}\times\frac{-3}{8}\times\frac{-3}{8} = \frac{-3\times-3\times-3\times-3}{8\times8\times8\times8} = \frac{81}{4096}$

From the above examples, we arrive at the following general result.

Result: If $\frac{p}{q}$ is any rational number and n is any positive integer, then $\frac{p^n}{q^n} = \left(\frac{p}{q}\right)^n$

The above result can be used to express the power of a rational number as a rational number and vice versa.

Example 4: **Simplify and express the result as a rational number.**

(*i*) $\left(-\frac{3}{2}\right)^2\times\left(-\frac{2}{3}\right)^3$ (*ii*) $(-3)^2\times\left(\frac{-7}{12}\right)^2$

Solution: (*i*) $\left(-\frac{3}{2}\right)^2\times\left(-\frac{2}{3}\right)^3 = (-1)^2\cdot\frac{3^2}{2^2}\times(-1)^3\cdot\frac{2^3}{3^3}$

$$= \frac{3\times3}{2\times2}\times\left(-\frac{2\times2\times2}{3\times3\times3}\right) = -\frac{2}{3} \qquad [\because (-1)^2 = 1]$$

(*ii*) $(-3)^2\times\left(\frac{-7}{12}\right)^2 = (-3)^2\times\frac{(-7)^2}{12^2}$

$$= 9\times\frac{49}{144} = 1\times\frac{49}{16} = \frac{49}{16} = 3\frac{1}{16}.$$

Take a Task

Watch Remedial

Negative Integer Exponent

So far we have considered numbers with base 10. The same is valid for other bases too. Let us see how.

Consider the following:

$$2^5 = 2\times2\times2\times2\times2 = 32$$

i.e., $2^5 \div 2 = 32 \div 2 = 16 = 2^4$

$2^4 \div 2 = 16 \div 2 = 8 = 2^3$

$2^3 \div 2 = 8 \div 2 = 4 = 2^2$

$2^2 \div 2 = 4 \div 2 = 2 = 2^1$

$2^1 \div 2 = 1 = 2^0$

$2^0 \div 2 = 1 \div 2 = \frac{1}{2} = 2^{-1}$

$2^{-1} \div 2 = \frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{2 \times 2} = \frac{1}{2^2} = 2^{-2}$

$2^{-2} \div 2 = \frac{1}{2^2} \div 2 = \frac{1}{2^2} \times \frac{1}{2} = \frac{1}{2^3} = 2^{-3}$

Note

For any non zero integer a, $a^{-m} = \frac{1}{a^m}$ where m is any positive integer and a^{-m} is called the multiplicate inverse of a^m.

Let $\frac{a}{b}$ be any rational and n be any negative integer.

Then, $\left(\frac{a}{b}\right)^{-m}$ can be defined as $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$.

Example 5: Evaluate.

(i) $\left(\frac{1}{3}\right)^{-2}$ (ii) $\left(\frac{-2}{7}\right)^{-3}$

Solution: (i) $\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^{2} = \frac{3 \times 3}{1 \times 1} = \frac{9}{1} = 9$

(ii) $\left(\frac{-2}{7}\right)^{-3} = \left(\frac{7}{-2}\right)^{3} = \frac{7 \times 7 \times 7}{-2 \times -2 \times -2} = \frac{343}{-8} = -42\frac{7}{8}$

0 as an Exponent.

If $\frac{a}{b}$ be any rational number, then $\left(\frac{a}{b}\right)^{0} = 1$

Example 6: Evaluate.

(i) $\left(\frac{-1}{2}\right)^{0}$ (ii) $\left(\frac{11}{13}\right)^{0}$

Solution: (i) $\left(\frac{-1}{2}\right)^{0} = 1$

(ii) $\left(\frac{11}{13}\right)^{0} = 1.$

Note

1. If $x^n = 1$, then $n = 0$ since $x^0 = 1$ for all values of x except, then $x = +1$ or -1
2. If x is $+1$, $x^n = 1^n = +1$ for all values of n.
3. If $x = -1$, $x^n = (-1)^n = +1$, if n is even and $x^n = (-1)^n = -1$, if n is odd.

Example 7: Find the absolute value of the following:

(i) $\left(\frac{4}{-3}\right)^{4}$ (ii) $\left(\frac{3}{7}\right)^{5}$ (iii) $\left(\frac{-2}{-7}\right)^{3}$

Solution: (*i*) Absolute value of $\left(\frac{4}{-3}\right)^4 = \left|\left(\frac{4}{-3}\right)^4\right| = \left|\frac{4^4}{(-3)^4}\right|$

$$= \left|\frac{4\times4\times4\times4}{-3\times-3\times-3\times-3}\right| = \left|\frac{256}{81}\right| = \frac{256}{81}$$

(*ii*) Absolute of $\left(\frac{3}{7}\right)^5 \quad = \left|\left(\frac{3}{7}\right)^5\right| = \left|\frac{3^5}{7^5}\right|$

$$= \left|\frac{3\times3\times3\times3\times3}{7\times7\times7\times7\times7}\right| = \left|\frac{243}{16807}\right| = \frac{243}{16807}$$

(*iii*) Absolute value of $\left(\frac{-2}{-7}\right)^3 = \left|\left(\frac{-2}{-7}\right)^3\right| = \left|\left(\frac{2}{7}\right)^3\right|$

$$= \left|\frac{2^3}{7^3}\right| = \left|\frac{2\times2\times2}{7\times7\times7}\right| = \left|\frac{8}{343}\right| = \frac{8}{343}$$

Example 8 : Simplify: $\left(\frac{2}{3}\right)^{-3} \div \left(\frac{4}{3}\right)^{-3}$

Solution : $\left(\frac{2}{3}\right)^{-3} \div \left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} \div \left(\frac{3}{4}\right)^{3}$

$$= \frac{3\times3\times3}{2\times2\times2} \div \frac{3\times3\times3}{4\times4\times4} = \frac{27}{8} \div \frac{27}{64} = \frac{27}{8}\times\frac{64}{27} = 8.$$

Example 9 : Solve : $\left(\frac{-1}{2}\right)^{4} \times \left(\frac{-1}{2}\right)^{5}$

Solution : $\left(\frac{-1}{2}\right)^{4} \times \left(\frac{-1}{2}\right)^{5} = \left(\frac{-1}{2}\right)^{4+5} = \left(\frac{-1}{2}\right)^{9} = \left(\frac{-2}{1}\right)^{-9} = (-2)^{-9}$ $\quad \left[\because \left(\frac{x}{y}\right)^n = \left(\frac{y}{x}\right)^{-n}\right]$

Example 10 : By what number should $\left(\frac{3}{4}\right)^{-3}$ **be divided so that quotient becomes** $\frac{1}{27}$ **.**

Solution : Let $\left(\frac{3}{4}\right)^{-3}$ be divided by x to get the quotient $\frac{1}{27}$.

Then, $\left(\frac{3}{4}\right)^{-3} \div x = \frac{1}{27}$

$\Rightarrow \left(\frac{3}{4}\right)^{-3} \times \frac{1}{x} = \frac{1}{27}$ $\quad \left[\because \text{Reciprocal of } x \text{ is } \frac{1}{x}.\right]$

$\Rightarrow \left(\frac{4}{3}\right)^{3} \times \frac{1}{x} = \frac{1}{27}$ $\quad \left[\because \left(\frac{p}{q}\right)^{-m} = \left(\frac{q}{p}\right)^{m}\right]$

$\Rightarrow \frac{4\times4\times4\times1}{3\times3\times3\times x} = \frac{1}{27}$

$\Rightarrow 3\times3\times3\times x = 4\times4\times4\times27$ $\quad$ [Using cross multiplication]

$\Rightarrow x = \frac{4\times4\times4\times27}{3\times3\times3} = 64$

Hence, the required number is 64.

Exercise 10.1

1. Express each of the following in power notation :

(i) $\frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11}$ (ii) $\left(\frac{-2}{7}\right) \times \left(\frac{-2}{7}\right) \times \left(\frac{-2}{7}\right)$

(iii) $\frac{(-3)}{8} \times \frac{(-3)}{8} \times \frac{(-3)}{8} \times \frac{(-3)}{8}$ (iv) $\frac{x}{3} \times \frac{x}{3} \times \frac{x}{3} \times \frac{x}{3} \times \frac{x}{3} \times \frac{x}{3} \times \frac{x}{3}$

(v) $\frac{(-7)}{y} \times \frac{(-7)}{y} \times \frac{(-7)}{y} \times \frac{(-7)}{y}$ (vi) $\frac{(-p)}{q} \times \frac{(-p)}{q} \times \frac{(-p)}{q} \times \frac{(-p)}{q} \times \frac{(-p)}{q} \times \frac{(-p)}{q} \times \frac{(-p)}{q}$

2. Express each of the following powers of rational numbers as a rational number :

(i) $\left(\frac{4}{5}\right)^3$ (ii) $\left(\frac{-3}{7}\right)^4$ (iii) $\left(\frac{-2}{5}\right)^2$ (iv) $\left(\frac{2}{3}\right)^4$ (v) $\left(\frac{-1}{3}\right)^5$

3. Express each of the following rational numbers in power notation :

(i) $\frac{16}{25}$ (ii) $\frac{-8}{27}$ (iii) $\frac{-125}{729}$ (iv) $\frac{-216}{343}$ (v) $\frac{1}{64}$

4. Evaluate.

(i) $\left(\frac{2}{3}\right)^{-1}$ (ii) $\left(\frac{3}{7}\right)^{-2}$ (iii) $\left(\frac{4}{9}\right)^{-5}$ (iv) $\left(\frac{5}{9}\right)^{1} \times \left(\frac{5}{9}\right)^{-1}$ (v) $\left(\frac{13}{17}\right)^{2} \times \left(\frac{13}{17}\right)^{-3}$

5. Find the value.

(i) $\left(\frac{2}{3}\right)^0$ (ii) $\left(\frac{8}{13}\right)^0$ (iii) $\left(\frac{7}{13}\right)^0 \times \left(\frac{8}{11}\right)^0$ (iv) $\left(\frac{6}{7}\right)^0 \times (5)^0 \times \left(\frac{1}{7}\right)^0$

6. Simplify the following :

(i) $(-3)^3 \times \left(\frac{1}{3}\right)^3$ (ii) $(-4)^2 \times \left(\frac{-1}{4}\right)^2$ (iii) $5^3 \div \left(\frac{1}{-15}\right)^2$ (iv) $\left(\frac{-9}{7}\right)^3 \div 3^0$

(v) $\left(\frac{3}{4}\right)^4 \times \left(\frac{2}{3}\right)^5$ (vi) $\left(\frac{2}{7}\right)^3 \times \left(\frac{7}{2}\right)^2$ (vii) $\left(\frac{-3}{4}\right)^{-4} \times \left(\frac{2}{-3}\right)^0$ (viii) $\left(\frac{1}{3}\right)^3 \times \left(\frac{3}{2}\right)^{-2}$

7. Find the reciprocals of:

(i) $(-3)^2$ (ii) $(-4)^4$ (iii) $\left(\frac{-2}{5}\right)^2$ (iv) $\left(\frac{-3}{7}\right)^3$

(v) $\left(\frac{5}{9}\right)^3 \times \left(\frac{9}{5}\right)^5$ (vi) $\left(\frac{-1}{8}\right)^8 \div \left(\frac{-1}{8}\right)^2$

8. Find the absolute value of:

(i) $\left(\frac{-10}{13}\right)^2$ (ii) $\left(\frac{-4}{7}\right)^3$ (iii) $\left(\frac{7}{9}\right)^4$ (iv) $\left(\frac{11}{12}\right)^4$

9. Show that:

(i) $\left(\frac{3}{7}\right)^5 = \frac{243}{16807}$ (ii) $\left(\frac{-8}{11}\right)^3 = \frac{-512}{1331}$

10. Find the reciprocals of the following and express in exponential form :

(*i*) $-\dfrac{675}{392}$ (*ii*) $\dfrac{1296}{625}$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

HOTS (Higher Order Thinking Skills)

Experiential Learning

1. Simplify and express as a rational number:

$$\left(\frac{1}{3}\right)^3 \times \left(\frac{-12}{3}\right)^4 \times \left(-\frac{9}{2}\right)^3$$

2. Simplify the following:

(*i*) $\left(\frac{3}{7}\right)^3 \times \left(\frac{7}{3}\right)^5$ (*ii*) $\left(\frac{-1}{5}\right)^8 \div \left(\frac{-1}{5}\right)^2$ (*iii*) $1 \div \frac{(-3)^5}{(-2)^5}$ (*iv*) $\left(\frac{-2}{3}\right)^{10}$

Laws of Exponents

Take a Task

Watch Remedial

We shall learn the laws of exponents which are very useful to do operations of multiplication and division in numbers involving exponents.

Observe the following example :

$$\left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^4 = \left(\frac{1}{4} \times \frac{1}{4}\right) \times \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$$

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \left(\frac{1}{4}\right)^6 = \left(\frac{1}{4}\right)^{2+4}$$

What do we observe ? We observe that when two rational numbers having the same base are multiplied, we get another rational number having the same base with an exponent which is the sum of exponents of the given rational numbers.

Thus, we can deduce the following laws :

Law 1 : *If $\frac{a}{b}$ is any rational number and m and n are two integers, then*

$$\left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}$$

Now observe the following examples.

(*i*) $\left(\frac{6}{13}\right)^6 \div \left(\frac{6}{13}\right)^4 = \left(\frac{6}{13}\right)^6 \times \left(\frac{13}{6}\right)^4$ [$\because$ Reciprocal of $\frac{6}{13}$ is $\frac{13}{6}$.]

$$= \frac{6 \times 6 \times 6 \times 6 \times 6 \times 6}{13 \times 13 \times 13 \times 13 \times 13 \times 13} \times \frac{13 \times 13 \times 13 \times 13}{6 \times 6 \times 6 \times 6} = \frac{6 \times 6}{13 \times 13} = \left(\frac{6}{13}\right)^2 = \left(\frac{6}{13}\right)^{6-4}$$

Law 2 : *If $\frac{a}{b}$ is any rational number and m and n are two integers, then*

$$\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m-n} \text{ if } m > n$$

and $\left(\frac{a}{b}\right)^m \div \left(\frac{a}{b}\right)^n = \frac{1}{\left(\frac{a}{b}\right)^{n-m}}$ if $m < n$.

Example 11: Evaluate $\left(\frac{7}{8}\right)^3 \div \left(\frac{7}{8}\right)^1$.

Solution: $\left(\frac{7}{8}\right)^3 \div \left(\frac{7}{8}\right)^1 = \left(\frac{7}{8}\right)^{3-1} = \left(\frac{7}{8}\right)^2$

Now, evaluate $\left[\left(\frac{3}{4}\right)^3\right]^2$.

We have, $\left[\left(\frac{3}{4}\right)^3\right]^2 = \left(\frac{3}{4}\right)^3 \times \left(\frac{3}{4}\right)^3 = \left(\frac{3}{4}\right)^{3+3} = \left(\frac{3}{4}\right)^6$.

Thus, we deduce the following law.

Law 3 : *If $\frac{a}{b}$ is any rational number such that $\frac{a}{b} \neq 0$, and m and n are two integers, then*

$$\left[\left(\frac{a}{b}\right)^m\right]^n = \left(\frac{a}{b}\right)^{m\times n} = \left(\frac{a}{b}\right)^{mn}$$

Example 12 : Simplify.

(i) $\left(\frac{4}{7}\right)^2 \times \left(\frac{4}{7}\right)^3$ (ii) $\left(\frac{-2}{3}\right)^4 \div \left(\frac{-2}{3}\right)^3$

Solution: (i) $\left(\frac{4}{7}\right)^2 \times \left(\frac{4}{7}\right)^3 = \left(\frac{4}{7}\right)^{2+3}$ [*by applying Law 1*]

$= \left(\frac{4}{7}\right)^5 = \frac{4^5}{7^5} = \frac{1024}{16807}$.

(ii) $\left(\frac{-2}{3}\right)^4 \div \left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right)^{4-3} = \left(\frac{-2}{3}\right)^{4-3} = \left(\frac{-2}{3}\right)^1 = \frac{-2}{3}$ [*by applying Law 2*]

Example 13 : Simplify the following:

(i) $\left(\frac{4}{11}\right)^2 \times \left(\frac{4}{11}\right)^5 \div \left(\frac{4}{11}\right)^4$ (ii) $\left[\left(\frac{3}{7}\right)^4 \div \left(\frac{3}{7}\right)^3\right] \div \left(\frac{3}{7}\right)^0$ (iii) $\left[\left(\frac{4}{5}\right)^3 \times \frac{4}{5}\right] \div \left(\frac{5}{4}\right)^3$

Solution: (i) $\left(\frac{4}{11}\right)^2 \times \left(\frac{4}{11}\right)^5 \div \left(\frac{4}{11}\right)^4 = \left(\frac{4}{11}\right)^{2+5-4} \left(\frac{4}{11}\right)^3 = \frac{4\times4\times4}{11\times11\times11} = \frac{64}{1,331}$

(ii) $\left[\left(\frac{3}{7}\right)^{4-3}\right] \div \left(\frac{3}{7}\right)^0 = \left(\frac{3}{7}\right)^1 \div 1 = \frac{3}{7}$

$$(iii)\quad \left[\left(\frac{4}{5}\right)^3 \times \frac{4}{5}\right] \div \left(\frac{5}{4}\right)^3 = \left(\frac{4}{5}\right)^{3+1} \div \left(\frac{5}{4}\right)^3 = \left(\frac{4}{5}\right)^4 \times \left(\frac{4}{5}\right)^3 = \left(\frac{4}{5}\right)^{4+3} = \left(\frac{4}{5}\right)^7 = \frac{16384}{78125}$$

Example 14: **Express $\left(\frac{-1}{2}\right)^4 \times \left(\frac{-1}{2}\right)^5$ as power of a rational number with negative exponent.**

Solution:

$$\left(\frac{-1}{2}\right)^4 \times \left(\frac{-1}{2}\right)^5 = \left(\frac{-1}{2}\right)^{4+5}$$

$$= \left(\frac{-1}{2}\right)^9 = (-2)^{-9}. \qquad \left[\text{Since } \left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{-n}\right]$$

Example 15: **Find the value of x, if $\left(\frac{2}{7}\right)^3 \times \left(\frac{2}{7}\right)^{-6} = \left(\frac{2}{7}\right)^{2x-1}$.**

Solution: We have: $\left(\frac{2}{7}\right)^3 \times \left(\frac{2}{7}\right)^{-6} = \left(\frac{2}{7}\right)^{2x-1}$

$$\Rightarrow \left(\frac{2}{7}\right)^{3+(-6)} = \left(\frac{2}{7}\right)^{2x-1} \quad \Rightarrow \left(\frac{2}{7}\right)^{-3} = \left(\frac{2}{7}\right)^{2x-1}$$

In an equation, when bases on both sides are equal, their powers must also be equal.

$$\therefore \quad 2x - 1 = -3$$

$$\text{or} \quad 2x = -3 + 1 \quad \Rightarrow \quad 2x = -2$$

$$\Rightarrow \quad x = \frac{-2}{2} = -1.$$

Example 16: **Find the value of m, if $\left(\frac{2}{7}\right)^{2m} \div \left(\frac{2}{7}\right)^{3-m} = \left(\frac{2}{7}\right)^6$.**

Solution: We have: $\left(\frac{2}{7}\right)^{2m} \div \left(\frac{2}{7}\right)^{3-m} = \left(\frac{2}{7}\right)^6$

$$\Rightarrow \left(\frac{2}{7}\right)^{2m-(3-m)} = \left(\frac{2}{7}\right)^6 \quad \Rightarrow \left(\frac{2}{7}\right)^{2m-3+m} = \left(\frac{2}{7}\right)^6$$

$$\Rightarrow 2m - 3 + m = 6 \quad \Rightarrow 3m = 6 + 3$$

$$\Rightarrow 3m = 9 \quad \Rightarrow m = \frac{9}{3}$$

Hence, $m = 3$.

Extension of Laws of Exponents

In an exponential expressions, there are cases when the bases are different but the exponents are the same. In such cases, we follow the following two laws:

(*i*) *If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers and m is any non-zero integer, then*

$$\left(\frac{a}{b} \times \frac{c}{d}\right)^m = \left(\frac{a}{b}\right)^m \times \left(\frac{c}{d}\right)^m$$

(*ii*) *If* $\frac{a}{b}$ *and* $\frac{c}{d}$ *are two rational numbers and m is any non-zero integer, then*

$$\left[\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}\right]^m = \left(\frac{a}{b}\right)^m \div \left(\frac{c}{d}\right)^m$$

Let us illustrate the above rules with the help of examples.

Example 17 : Find the value of $2^4 \times 5^4$.

Solution : $2^4 \times 5^4 = (2 \times 5)^4 = 10^4 = 10{,}000$ [using $\left(\frac{a}{b}\right)^m \times \left(\frac{c}{d}\right)^m = \left(\frac{a}{b} \times \frac{c}{d}\right)^m$]

Example 18 : Simplify.

(*i*) $\left(-\frac{2}{7}\right)^3 \times \left(-\frac{1}{5}\right)^3$ (*ii*) $\left(\frac{3}{5}\right)^3 \div \left(\frac{6}{7}\right)^3$

Solution : (*i*) $\left(-\frac{2}{7}\right)^3 \times \left(-\frac{1}{5}\right)^3 = \left(-\frac{2}{7} \times -\frac{1}{5}\right)^3 = \left(\frac{2}{35}\right)^3 = \frac{8}{42{,}875}$

(*ii*) $\left(\frac{3}{5}\right)^3 \div \left(\frac{6}{7}\right)^3 = \left(\frac{3}{5} \div \frac{6}{7}\right)^3$

$= \left(\frac{3}{5} \times \frac{7}{6}\right)^3 = \left(\frac{7}{10}\right)^3 = \frac{343}{1000}$

Exercise 10.2

1. Evaluate.

(*i*) 2^3 (*ii*) $(-3)^{-2}$ (*iii*) $\left(\frac{1}{5}\right)^4$ (*iv*) $\left(\frac{-1}{7}\right)^{-3}$

(*v*) $\left(\frac{2}{5}\right)^2 \times \left(\frac{5}{2}\right)^{-3}$ (*vi*) $\left(-\frac{1}{6}\right)^2 \times \left(-\frac{1}{6}\right)^3$ (*vii*) $\left(-\frac{1}{2}\right)^2 \times \left(-\frac{1}{2}\right)^{-7}$ (*viii*) $\left(\frac{2}{9}\right)^{-5} \times \left(\frac{2}{9}\right)^3$

2. Simplify and express in exponential form.

(*i*) $\left(\frac{3}{7}\right)^7 \div \left(\frac{3}{7}\right)^6$ (*ii*) $\left(\frac{3}{10}\right)^{11} \div \left(\frac{3}{10}\right)^7$ (*iii*) $\left(-\frac{3}{5}\right)^{17} \div \left(-\frac{3}{5}\right)^4$ (*iv*) $\left(-\frac{4}{9}\right)^8 \div \left(-\frac{4}{9}\right)^4$

3. Simplify and express in positive exponent.

(*i*) $\left(\frac{2}{7}\right)^{17} \div \left(\frac{2}{7}\right)^{20}$ (*ii*) $\left(\frac{3}{7}\right)^7 \div \left(\frac{3}{7}\right)^{11}$ (*iii*) $\left(-\frac{3}{7}\right)^9 \div \left(-\frac{3}{7}\right)^{13}$ (*iv*) $\left(-\frac{4}{5}\right)^3 \div \left(-\frac{4}{5}\right)^6$

4. Find the value of the following :

(*i*) 8^0 (*ii*) 3^{8-8} (*iii*) 19^{-19+19} (*iv*) $(-17)^{2\times 9-10-8}$

(*v*) $(4^0 - 6^0) \times 10^0$ (*vi*) $4^0 \times 5^0 \times 10^0 \times 13^0$ (*vii*) $(1^0 - 3^0) \times (7^0 + 2^0)$ (*viii*) $3^0 + 5^0 + 7^0$

5. Find the value of: $\left(\frac{3}{7}\right)^{-3} \times \left(\frac{-3}{7}\right)^{-3}$

6. By what number should $\left(\frac{1}{4}\right)^{-3}$ be multiplied so that the product will be 20 ?

7. Find the value of x in each of the following :

(*i*) $\left(\frac{2}{3}\right)^{7} \times \left(\frac{2}{3}\right)^{9} = \left(\frac{2}{3}\right)^{x}$ (*ii*) $\left(\frac{125}{27}\right) \times \left(\frac{125}{27}\right)^{x} = \left(\frac{5}{3}\right)^{18}$ (*iii*) $\left(\frac{4}{7}\right)^{-5} \times \left(\frac{4}{7}\right)^{-3} = \left(\frac{4}{7}\right)^{x-2}$

(*iv*) $\left(\frac{-3}{7}\right)^{-3} \div \left(\frac{-3}{7}\right)^{-4} = \left(\frac{-3}{7}\right)^{x}$ (*v*) $\left(\frac{7}{11}\right)^{-4} \times \left(\frac{7}{11}\right)^{-2x} = \left(\frac{7}{11}\right)^{8}$

8. Find the value of the following :

(*i*) 12^{0} (*ii*) $\left(\frac{2}{9}\right)^{10} \times \left(\frac{2}{9}\right)^{2} \div \left(\frac{9}{2}\right)^{12}$ (*iii*) $6^{0} \times 4^{0} \times 3^{0} \times 2^{0} \times 5^{0}$

(*iv*) $\{(3)^{-2} + (3)^{-2}\}^{0}$ (*v*) $\left(\frac{-3}{8}\right)^{6} \div \left(\frac{-8}{3}\right)^{-6}$ (*vi*) $\left(\frac{-13}{6}\right)^{o} + (-10)^{o} - \left(\frac{7}{20}\right)^{o}$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

HOTS (Higher Order Thinking Skills)

Experiential Learning

1. Evaluate the following :

(*i*) $\left(\frac{3}{5}\right)^{-2}$ (*ii*) $\left(\frac{3}{4}\right)^{-1} + \left(\frac{3}{2}\right)^{2}$ (*iii*) $\left(\frac{3}{7}\right)^{6} \times \left(\frac{7}{3}\right)^{5} + \left(\frac{3}{7}\right)^{2}$

(*iv*) $\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{4}\right)^{-4} - (4)^{-3}$ (*v*) $\left[\left(\frac{4}{9}\right)^{10} \times \left(\frac{4}{9}\right)^{2}\right]^{2} \div \left[\left(\frac{4}{9}\right)^{4}\right]^{5}$ (*vi*) $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

2. Find the value of the following :

(*i*) $\left(\frac{3}{4}\right)^{3} \times \left(\frac{4}{9}\right)^{3}$ (*ii*) $\left(\frac{5}{13}\right)^{6} \div \left(\frac{10}{13}\right)^{6}$ (*iii*) $\left(-\frac{2}{3}\right)^{4} \div \left(\frac{7}{3}\right)^{4}$ (*iv*) $\left(-\frac{1}{3}\right)^{4} \div \left(\frac{2}{3}\right)^{4}$

Use of Exponents to Express Small Numbers in Standard Form

We often come across the use of very large or very small numbers in various situations. Given below are some facts that involve very large or very small numbers.

1. The distance of moon from the earth is 38,44,00, 000 m (approx).
2. Average radius of the sun is 6,96,340 km.
3. The thickness of human hair is in the range of 0.005 cm to 0.01 cm.
4. The height of Mount Everest is 8,848 m.
5. The thickness of a piece of paper is 0.0025 cm.
6. The speed of light is 2,99,792 km/sec.

Take a Task

Watch Remedial

Observe that there are few numbers which we can read like 4 cm, 8,848 m, 6,95,000 km. There are some large numbers like 15,00,00,000 km and some very small numbers like 0.000002 m. It is a cumbersome task to read, write and use such large or small number. So, we need to express these numbers in the standard form.

Note : 15,00,00,000 km = 1.5×10^{8} km.

Example 19 : Express 0.000003 m in standard form.

Solution : $0.000003\text{ m} = \dfrac{3}{10,00,000}\text{ m}$

$= \dfrac{3}{10^6}\text{ m} = 3\times10^{-6}\text{ m}$

$\therefore$ $0.000003\text{ m} = 3\times10^{-6}\text{ m}$.

Example 20 : Express the following numbers in standard form :

(*i*) 0.0000091 (*ii*) 14,00,000

Solution : (*i*) $0.0000091 = 9.1\times10^{-6}$

(*ii*) $14,00,000 = 1.4\times10^{6}$.

Example 21 : Express the following numbers in usual form :

(*i*) 8.53×10^4 (*ii*) 2.36×10^{-6} (*iii*) 4×10^{-11}

Solution : (*i*) $8.53\times10^4 = 8.53\times10000 = 85,300$

(*ii*) $2.36\times10^{-6} = \dfrac{2.36}{10,00,000} = 0.00000236$

(*iii*) $4\times10^{-11}\ \dfrac{4}{10^{11}} = \dfrac{4}{1,00,00,00,00,000} = 0.00000000004.$

Example 22 : Write the number 4,268 in expanded form using exponents.

Solution : Writing 4,268 in expanded form, we get

$4,268 = 4\times1000 + 2\times100 + 6\times10 + 8\times1$

Writing in expanded exponential form, we get

$4,268 = 4\times10^3 + 2\times10^2 + 6\times10^1 + 8\times10^0$

Exercise 10.3

1. Express the following numbers in standard form :

   (*i*) 4,60,000 (*ii*) 0.00000000136 (*iii*) 5,08,00,00,00,000

   (*iv*) 1,25,00,00,00,00,000 (v) 0.00000003006839

2. Express the following numbers in usual form :

   (*i*) 3.02×10^{-2} (*ii*) 6.5×10^{6} (*iii*) 5.2×10^{-6}

   (*iv*) 2.0053×10^{8} (*v*) 7.3×10^{8} (*vi*) 6.60013×10^{7}

3. Express the result of the following calculation in the scientific form: $\dfrac{3.24\times0.08666}{5.006}$

4. The mass of a proton is 1.68×10^{-24} g and that of an electron is 9.11×10^{-28} g. Find the ratio of their masses.

5. The mass of the earth is 5.98×10^{27} g and the mass of the moon is 7.36×10^{25} g. What is the sum of the masses of the earth and the moon?

6. Choose the correct options.

(*i*) $\left(\frac{-3}{4}\right)^3$ is equal to

(a) $\frac{-3}{4}$ (b) $\frac{-27}{64}$ (c) $\frac{27}{64}$ (d) $\frac{9}{16}$

(*ii*) $\left[\left(\frac{1}{3}\right)^3\right]^{-2}$

(a) $\left(\frac{1}{3}\right)^1$ (b) $\left(\frac{1}{3}\right)^5$ (c) $\left(\frac{1}{3}\right)^6$ (d) $\left(\frac{1}{3}\right)^{-6}$

(*iii*) The value of $(4^0 - 2^0) \times 3^0$ is :

(a) 3 (b) 0 (c) 6 (d) 2

(*iv*) For the numbers 3126.8×10^{-7} to be in standard form how many places are to be shifted?

(a) 3 (b) 7 (c) 4 (d) 1

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Revision Exercise

Experiential Learning

1. Tick (✓) the correct option:

(*i*) The multiplicative inverse of $\left(\frac{2}{3}\right)^{-3}$ is ....

(*a*) $\frac{8}{27}$ ☐ (*b*) $\frac{27}{8}$ ☐ (*c*) $\frac{-8}{27}$ ☐ (*d*) $\frac{-27}{8}$ ☐

(*ii*) The value of $\left[\left(\frac{4}{-3}\right)^{-3}\right]^0$ is ....

(*a*) $\frac{4}{-3}$ ☐ (*b*) $\frac{-3}{4}$ ☐ (*c*) 1 ☐ (*d*) 0 ☐

(*iii*) The value of $\left(\frac{5}{3}\right)^{-8} \div \left(\frac{5}{3}\right)^{-8}$ is equal to ....

(*a*) 1 ☐ (*b*) $\frac{5}{3}$ ☐ (*c*) $\frac{3}{5}$ ☐ (*d*) none ☐

(*iv*) If $\frac{x}{y} = \left(\frac{2}{3}\right)^{-4} \times \left(\frac{3}{5}\right)^{-4}$ then $\left(\frac{x}{y}\right)^2$ is equal to ....

(a) $\left(\frac{2}{5}\right)^{-8}$ ☐ (*b*) $\left(\frac{2}{5}\right)^{16}$ ☐ (*c*) $\left(\frac{2}{5}\right)^{8}$ ☐ (*d*) none ☐

(*v*) The value of $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{8}\right)^{-1} + \left(\frac{1}{9}\right)^{-1}$ is ....

(*a*) $\left(\frac{53}{72}\right)^{-1}$ ☐ (*b*) $\left(\frac{1}{144}\right)^{-1}$ ☐ (*c*) 19 ☐ (*d*) none ☐

2. Find the value of $(5^0 - 3^0) \times 2^5$.

3. Find the value of $(3^{-1} + 4^{-1})^{-1} + 5^{-1}$.

4. Find the value of x for which $\left(\frac{3}{5}\right)^{-4} \times \left(\frac{3}{5}\right)^{3x} = \left(\frac{3}{5}\right)^{5}$.

5. If $(2^{3x-1} + 10) \div 7 = 6$ then find the value of x.

6. Express 35000000 in standard form.

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Mental Maths

Experiential Learning

Fill in the blanks:

1. $\left(\frac{a}{b}\right)^{m} \times \left(\frac{a}{b}\right)^{n}$ = ...
2. $\left[\left(\frac{5}{4}\right)^{2}\right]^{x}$ = ...
3. $\left[\left(\frac{4}{3}\right)^{0} - \left(\frac{4}{3}\right)\right]^{-1}$ = ...
4. $5^{m} \div 5^{-3}$ = ...
5. Multiplicative inverse of 2^{-4} is ....

Scan to Create Your Own Learning Path

Custom Learning Path

Assertion and Reason

Critical Thinking

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct :

(*a*) Both A and R are true and R is the correct explanation of A.

(*b*) Both A and R are true but R is not the correct explanation of A.

(*c*) A is true but R is false.

(*d*) A is false but R is true.

1. **Assertion (A) :** $\left(\frac{-6}{375}\right)^{0}$ is equal to 1.

   Reason (R) : For any non-zero rational number a, we define $a^0 = 1$.

2. **Assertion (A) :** 3^4 is read as '4 raised to power 3'.

   Reason (R) : The reciprocal of $\left(\frac{a}{b}\right)^{n}$ is written as $\left(\frac{a}{b}\right)^{-n}$.

3. **Assertion (A) :** If $2^5 = 2^x$, then $x = 5$

   Reason (R) : If a and b are two rational number and m, n are integers, then $a^m b^n = (ab)^{mn}$.

4. **Assertion (A) :** The value of $x = \frac{5}{7}$ if $4^x = \left(\frac{5}{7}\right)^{-3} \times \left(\frac{5}{7}\right)^{7}$

   Reason (R) : $(25)^{-17} - (25)^{-18}$ is equal to 25×24^{-18}.

Activity

Negative Integer Exponents

Call three students and distribute the colour paper card to each of them. Ask first student to write a rational number 9 on the card then to the second student ask for its reciprocal on the colour card it is expressed as $\frac{1}{9}$. Teacher will tell them this can also be expressed as $\frac{1}{3 \times 3}$.

Or $\quad = \frac{1}{3\sqrt[2]{\ }}$

Or $\quad = 3^{-2}$

Also, 3^{-2} is read as 3 raised to the power –2.

Call another four students repeat the same procedure with the rational number $\left(\frac{9}{16}\right)$.

Ask to write it's reciprocal to the second student on the card. It is $\frac{16}{9}$. Then to the third student express it in the product of repetition of same number surely the student write it as $\frac{1}{3 \times 3} = \frac{4 \times 4}{3 \times 3}$.

Then to the fourth student ask to write its exponential form on colour card surely student express it as :

$$\frac{1}{\left(\frac{3}{4}\right)^2} \quad \text{or} \quad \left(\frac{1}{\frac{3}{4}}\right)^2 \quad \text{or} \quad \left(\frac{3}{4}\right)^{-2}$$

So it is concluded that the reciprocal of $x^n = \frac{1}{x^n}$ *or* $\left(\frac{1}{x}\right)^n = x^{-n}$ and the reciprocal of $\left(\frac{a}{b}\right)^n$ is written as $\left(\frac{a}{b}\right)^{-n}$.

Students will enjoy performing the activity and the rest of the students will enjoy observing it.

Skills covered: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

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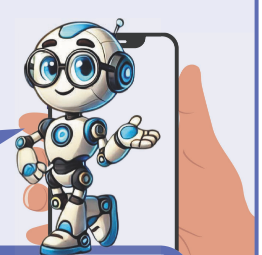

Hi Friend! Use prompts to ask me questions about the chapter we just finished! eeee, lets go!

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