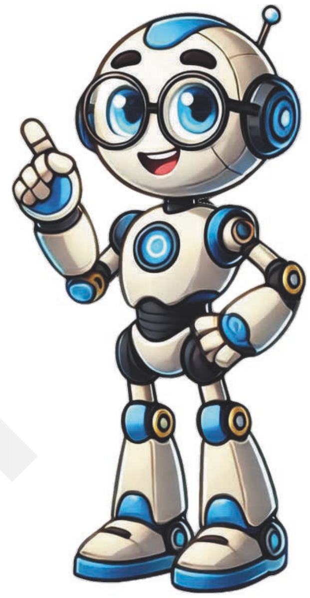
1
Chapter

Rational Numbers

We'll cover the following key points:

- → Recapitulation of rational numbers
- → Comparison of rational numbers
- → Addition of rational numbers
- → Properties of addition of rational numbers
- → Subtraction of rational numbers
- → Properties of subtraction of rational numbers
- → Multiplication of rational numbers
- → Properties of multiplication of rational numbers
- → Division of rational numbers
- → Properties of division of rational numbers
- → Insertion of rational numbers between two given rational numbers



Hi, I'm EeeBee

Do you Remember fundamental concept in previous class.

In class 7th we learnt

- → Positive and Negative Rational Numbers
- → Equivalent rational Numbers
- → Rational number on a number line

Still curious? Talk to me by scanning the QR code.

Learning Outcomes

By the end of this chapter, students will be able to:

- Define and identify rational numbers and distinguish them from other number types.
- Represent rational numbers on a number line accurately.
- Express a rational number in its standard form and simplify it.
- Perform operations (addition, subtraction, multiplication and division) on rational numbers.
- Understand the properties of rational numbers, including closure, commutativity, associativity, distributive property, and identity elements.
- Compare and order rational numbers.
- Identify and find the reciprocal of a rational number.
- Solve problems involving rational numbers in real-life contexts.
- Apply the concept of rational numbers in solving equations and inequalities.
- Relate rational numbers to decimals and percentages.

Mind Map

RATIONAL NUMBERS

PROPERTIES OF RATIONAL NUMBERS

Closure

(Whole numbers, Integers and Rational Numbers)

i. Closed under addition.

$a + b = c$

ii. Closed under subtraction (not whole numbers).

$a - b = c$

iii. Closed under multiplication.

$a \times b = c$

iv. Closed under Division (not whole numbers and integers).

$a \div b = c, b \neq 0$

except when denominators is 0.

Commutativity

(Whole numbers, Integers and Rational Numbers)

i. Addition is commutative.

$a + b = b + a$

ii. Subtraction is not commutative.

$a - b \neq b - a$

iii. Multiplication is commutative.

$a \times b = b \times a$

iv. Division is not commutative.

$a \div b \neq b \div a$

Associativity

(Whole numbers, Integers and Rational Numbers)

i. Addition is associative.

$a + (b + c) = (a + b) + c$

ii. Subtraction is not associative.

$a - (b - c) \neq (a - b) - c$

iii. Multiplication is associative.

$a \times (b \times c) = (a \times b) \times c$

iv. Division is not associative.

$a \div (b \div c) \neq (a \div b) \div c$

The role of zero (0) and One (1)

- 0 is additive identity

$a + 0 = a$

- 1 is the multiplicative identity

$a \times 1 = a$

Distributivity of multiplication over addition and subtraction

➢ **For all Rational Numbers**

a, b and c

$a(b + c) = ab + ac$

$a(b - c) = ab - ac$

Based on NCERT*

Introduction

Recapitulation of Rational Numbers

A rational number is a number of the form $\frac{p}{q}$ or a number which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

For example, each of the numbers $\frac{3}{5}, \frac{-1}{7}, \frac{2}{-9}, \frac{-4}{7}, \frac{0}{12}$ is a rational number.

The integer p in the rational number $\frac{p}{q}$ is called its *numerator* and q is called its *denominator.*

Positive Rational Number. *A rational number is said to be positive, if its numerator and denominator are either both positive integers or both negative integers.*

$\frac{1}{4}, \frac{2}{6}, \frac{-2}{-3}, \frac{-3}{-11}$ is a positive rational number.

Negative Rational Number. *A rational number is said to be negative, if its numerator and denominator are such that one of them is positive integer and the other one is a negative integer.*

$\frac{-5}{2}, \frac{11}{-19}, \frac{-17}{29}$ is a negative rational number.

REMEMBER

1. Each positive integer, *i.e.*, 1, 2, 3, 4, ... is a positive rational number.
2. Each negative integer, *i.e.*, –1, –2, –3, –4, ... is a negative rational number.
3. 0 is neither positive nor negative rational number.

Absolute Value of a Rational Number

In case of integers, absolute value of an integer is obtained by choosing only the numerical value of that integer. For example, the absolute value of $8 = |8| = 8$ and the absolute value of $-3 = |-3| = 3$.

The same definition can be applied to rational numbers.

Absolute value of $\frac{1}{5} = \left|\frac{1}{5}\right| = \frac{1}{5}$

Absolute value of $\frac{-1}{7} = \left|\frac{-1}{7}\right| = \frac{1}{7}$

Thus, the absolute value of a rational number is its numerical value regardless of its sign.

Three Properties of Rational Numbers

Property 1. *If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then* $\frac{p}{q} = \frac{p \times m}{q \times m}$.

Example : $\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$, $\frac{5}{6} = \frac{5 \times 12}{6 \times 12} = \frac{60}{72}$ etc.,

Similarly, $\frac{-6}{11} = \frac{-6 \times (-1)}{11 \times (-1)} = \frac{6}{-11}$, $\frac{1}{-4} = \frac{1 \times (-2)}{-4 \times (-2)} = \frac{-2}{8}$.

Property 2. *If $\frac{p}{q}$ is a rational number and m is a common divisor of p and q, then* $\frac{p}{q} = \frac{p \div m}{q \div m}$.

Examples : (*i*) $\frac{18}{-24} = \frac{18 \div (-1)}{-24 \div (-1)} = \frac{-18}{24} = \frac{-18 \div 6}{24 \div 6} = \frac{-3}{4}$

(*ii*) $\frac{-36}{-72} = \frac{-36 \div (-6)}{-72 \div (-6)} = \frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$.

Equivalent Rational Numbers

If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then each of $\frac{p \times m}{q \times m}$ and $\frac{p \div m}{q \div m}$ is a rational number equivalent to $\frac{p}{q}$.

If the denominator of a rational number is a negative integer, then by using the above property, we can make it positive by multiplying its numerator and denominator by –1.

For example, $\frac{3}{-17} = \frac{3 \times (-1)}{-17 \times (-1)} = \frac{-3}{17}$.

Property 3. If $\frac{p}{q}$ and $\frac{r}{s}$ be two equivalent rational numbers, then $\frac{p}{q} = \frac{r}{s}$, *i.e.,* $p \times s = q \times r$

[Numerator of the first rational number × Denominator of the second rational number] = [Numerator of the second rational number × Denominator of the first rational number]

Standard form of a Rational Number

A rational number $\frac{p}{q}$ is said to be in standard form if q is positive and the integers p and q have no common divisor other than 1.

Example 1: **Express $\frac{-35}{-60}$ in the standard form.**

Solution : Multiplying the numerator and denominator of $\frac{-35}{-60}$ by – 1, we have

$$\frac{-35}{-60} = \frac{-35 \times (-1)}{-60 \times (-1)} = \frac{35}{60}$$

Dividing the numerator and denominator of $\frac{35}{60}$ by 5, we get

$$\frac{35}{60} = \frac{35 \div 5}{60 \div 5} = \frac{7}{12}$$

Thus, the standard form of $\frac{-35}{-60}$ is $\frac{7}{12}$.

Example 2: **Write $\frac{11}{-25}$ with positive denominator.**

Solution : In order to express a rational number with positive denominator, we multiply its numerator and denominator by –1. Therefore,

$$\frac{11}{-25} = \frac{11 \times (-1)}{(-25) \times (-1)} = \frac{-11}{25}$$

Thus, the required number is $\frac{-11}{25}$.

Example 3: **Express $\frac{-4}{7}$ as rational number with numerator 24.**

Solution : In order to express $\frac{-4}{7}$ as a rational number with numerator 24, we first find a number which when multiplied by – 4 gives 24.

Clearly, such a number is 24 ÷ (–4) = (–6).

Multiplying the numerator and denominator of $\frac{-4}{7}$ by (–6), we have

$$\frac{-4}{7} = \frac{(-4) \times (-6)}{7 \times (-6)} = \frac{24}{-42}$$

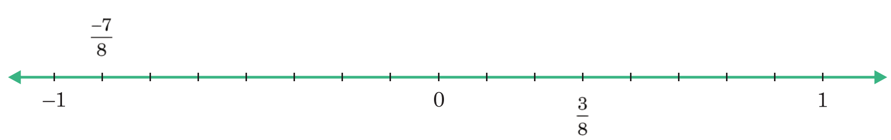
Thus, the required number is $\frac{24}{-42}$.

Representation of Rational Numbers on the Number Line

Every rational number can be represented on a number line.

On the number line, all negative rational numbers, such as $\frac{-p}{q}$, $\frac{p}{-q}$ are shown to the left of 0 and all positive rational numbers to the right of 0.

Observe the following examples.



> In case of rational numbers, every point on the number line can be represented by rational numbers.

Example 4 : **Represent (i) $\frac{2}{5}$ and (ii) $\frac{-1}{5}$ on the number line.**

Solution : (i) Draw a number line. Let O represents 0 and A represents 1.

Since the denominator of $\frac{2}{5}$ is 5, divide OA into five equal parts.

Label the points as P, Q, R and S. Point Q represents $\frac{2}{5}$.

(ii) Draw a number line. Let O represents 0 and B represents –1.

Since the denominator of $\frac{-1}{5}$ is 5, divide OB into five equal parts.

Label the points as J, K, L and M. Point J represents $\frac{-1}{5}$.

Example 5 : **Represent $\frac{13}{4}$ and $-\frac{13}{4}$ on the number line.**

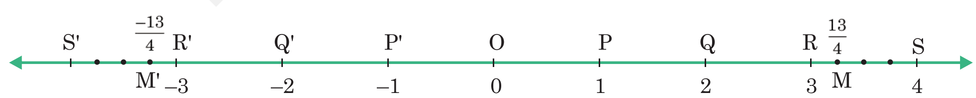
Solution : Draw a number line and mark a point O on it to represent zero.

Now, $\frac{13}{4} = 3\frac{1}{4} = 3 + \frac{1}{4}$.

Mark points P, Q, R and S on the line to the right of 0 to represent integres 1, 2, 3, 4 respectively. Now, take 3 units OP, PQ and QR and divide the fourth unit RS into 4 equal parts. Take one part out of these 4 parts to reach at point M.

Then the point M represents the rational number $\frac{13}{4}$.

$$(\because \text{ OM} = \text{OR} + \text{RM} = 3 + \frac{1}{4} = \frac{13}{4}.)$$



Again, mark points P', Q', R' and S' on the line to the left of zero, to represent integers –1, –2,–3, –4 respectively.

Now, take 3 units OP', P'Q' and Q'R' and divide the fourth unit R'S' into 4 equal parts. Take one part out of these 4 parts to reach at a point M'.

The point M' represents the rational number $-\frac{13}{4}$.

Comparison of Rational Numbers

Comparison of Rational Numbers

Important points for comparison of rational numbers are as follows:

1. Every positive rational number is greater than 0.
2. Every negative rational number is less than 0 and is less than every positive rational number.

Working Rules

In order to compare any two rational numbers, we use the following steps.

Step 1 : Express the denominators as positive integers.

Step 2 : Find the LCM of the positive denominators of the rational numbers obtained in step 1.

Step 3 : Express each rational number (obtained in step 1) with the LCM (obtained in step 2) as common denominator.

Step 4 : Compare the numerators of rational numbers. The number with greater numerator is the greater rational number.

Example 6 : **Which one of the two rational numbers $\frac{-8}{11}$ and $\frac{3}{-7}$ is greater ?**

Solution : Let us write each one of the given rational numbers with positive denominator.

Clearly, denominator of $\frac{-8}{11}$ is positive and the denominator of $\frac{3}{-7}$ is negative. So, we express it with positive denominator as given below.

$$\frac{3}{-7} = \frac{3 \times (-1)}{-7 \times (-1)} = \frac{-3}{7}$$

Now, LCM of the denominators 11 and 7 is 77.

We write each rational number with 77 as common denominator as follows:

$$\frac{-8}{11} = \frac{-8 \times 7}{11 \times 7} = \frac{-56}{77} \quad \text{and} \quad \frac{-3}{7} = \frac{-3 \times 11}{7 \times 11} = \frac{-33}{77}$$

Comparing the numerators, we find that $\frac{-33}{77} > \frac{-56}{77}$ or $\frac{3}{-7} > \frac{-8}{11}$.

Example 7 : **Arrange $\frac{5}{-2}, \frac{5}{6}, \frac{3}{7}$ and $\frac{3}{14}$ in ascending and descending order.**

Solution : We have, $\frac{5}{-2}, \frac{5}{6}, \frac{3}{7}, \frac{3}{14} = \frac{-5}{2}, \frac{5}{6}, \frac{3}{7}, \frac{3}{14}$ [Making the denominators positive]

LCM of 2, 6, 7 and 14 is 42.

$$\therefore \frac{-5}{2} = \frac{-5 \times 21}{2 \times 21} = \frac{-105}{42}, \quad \frac{5}{6} = \frac{5 \times 7}{6 \times 7} = \frac{35}{42}$$

$$\frac{3}{7} = \frac{3 \times 6}{7 \times 6} = \frac{18}{42}, \quad \frac{3}{14} = \frac{3 \times 3}{14 \times 3} = \frac{9}{42}$$

Since, $\frac{-105}{42} < \frac{9}{42} < \frac{18}{42} < \frac{35}{42}$

Hence, the given numbers can be arranged in ascending order as follows:

$$\frac{5}{-2} < \frac{3}{14} < \frac{3}{7} < \frac{5}{6}$$

Since, $\frac{35}{42} > \frac{18}{42} > \frac{9}{42} > \frac{-105}{42}$

Hence, the given numbers can be arranged in descending order as follows:

$$\frac{5}{6} > \frac{3}{7} > \frac{3}{14} > \frac{5}{-2}.$$

Exercise 1.1

1. Choose the correct alternative from the given four alternative options.

   (*i*) Which is the smallest rational number ?

   (*a*) 0 ☐ (*b*) $\frac{-1}{3}$ ☐ (*c*) 1 ☐ (*d*) $\frac{3}{4}$ ☐

   (*ii*) Which of the following is true for rational numbers ?

   (*a*) Every natural number is a rational number. ☐
   (*b*) Zero is a rational number. ☐
   (*c*) Every integer is a rational number. ☐
   (*d*) All the above ☐

   (*iii*) Which of the following rational number is in standard form ?

   (*a*) $\frac{1}{-3}$ ☐ (*b*) $\frac{-2}{-3}$ ☐ (*c*) $\frac{1}{7}$ ☐ (*d*) -3 ☐

   (*iv*) The absolute value of $\frac{-3}{4}$ is

   (*a*) -3 ☐ (*b*) 4 ☐ (*c*) $\frac{3}{-4}$ ☐ (*d*) $\frac{3}{4}$ ☐

2. Fill in the boxes.

   (*i*) $\frac{3}{4} = \frac{\square}{8} = \frac{12}{\square} = \frac{21}{\square}$ (*ii*) $\frac{9}{11} = \frac{27}{\square} = \frac{\square}{55} = \frac{54}{\square}$

3. Express each of the following as a rational number with positive denominator.

   (*i*) $\frac{5}{-7}$ (*ii*) $\frac{-12}{-13}$ (*iii*) $\frac{2}{-9}$ (*iv*) $\frac{13}{-15}$ (*v*) $\frac{-7}{-11}$

4. Represent the following rational numbers on the number line.

   (*i*) $\frac{2}{3}$ (*ii*) $\frac{-7}{8}$ (*iii*) $\frac{-5}{6}$ (*iv*) $\frac{-2}{13}$ (*v*) $\frac{-1}{-3}$

5. Express each of the following rational numbers in the standard form.

   (*i*) $\frac{-10}{22}$ (*ii*) $\frac{14}{-35}$ (*iii*) $\frac{81}{108}$ (*iv*) $\frac{121}{143}$

6. Fill in the boxes with the symbol >, < or = .

(i) $\frac{-7}{8}\square\frac{35}{-40}$ (ii) $\frac{-4}{57}\square\frac{5}{117}$ (iii) $\frac{4}{6}\square\frac{-14}{23}$ (iv) $\frac{-15}{17}\square\frac{3}{-7}$

7. Arrange the following rational numbers in ascending order.

(i) $\frac{1}{3}, \frac{4}{9}, \frac{-4}{11}, \frac{7}{12}$ (ii) $\frac{3}{-2}, \frac{-1}{6}, \frac{1}{3}, \frac{-1}{2}$ (iii) $\frac{7}{10}, \frac{1}{5}, \frac{1}{35}, \frac{3}{7}$ (iv) $\frac{15}{-20}, \frac{3}{4}, \frac{15}{32}, \frac{-2}{6}$

8. Arrange the following rational numbers in descending order.

(i) $\frac{-2}{3}, \frac{3}{4}, \frac{-5}{12}, \frac{2}{9}$ (ii) $\frac{3}{-14}, \frac{-5}{21}, \frac{4}{7}, \frac{-1}{7}$ (iii) $\frac{3}{4}, \frac{-1}{2}, \frac{-2}{5}, -1, \frac{2}{5}$ (iv) $\frac{11}{12}, \frac{-7}{18}, \frac{4}{21}, \frac{-2}{15}, \frac{-1}{3}$

9. Find the rational numbers whose absolute value is :

(i) $\frac{8}{13}$ (ii) 2 (iii) $\frac{2}{11}$ (iv) $\frac{1}{7}$

10. Simplify:

(i) $\left|\frac{9}{10}+\frac{1}{3}\right|$ (ii) $\left|\frac{1}{8}-\frac{5}{16}\right|$ (iii) $\left|\frac{2}{7}\times\frac{-14}{8}\right|$ (iv) $\left|-3\div\frac{1}{9}\right|$

11. Verify the following :

(i) $\left|\frac{5}{8}+\left(\frac{-3}{8}\right)\right| < \left|\frac{5}{8}\right|+\left|\frac{-3}{8}\right|$ (ii) $\left|\frac{11}{15}+\frac{3}{10}\right| = \left|\frac{11}{15}\right|+\left|\frac{3}{10}\right|$

(iii) $\left|\frac{-1}{6}+\frac{2}{5}\right| < \left|\frac{-1}{6}\right|+\left|\frac{2}{5}\right|$ (iv) $\left|\frac{-5}{9}+\frac{18}{25}\right| < \left|\frac{-5}{9}\right|+\left|\frac{18}{25}\right|$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Addition of Rational Numbers

In order to add two rational numbers, rewrite each one of them with positive denominator. For addition purpose, the rational numbers are categorised into two levels.

Case I. Addition of Rational Numbers with Same Denominators

If $\frac{p}{q}$ and $\frac{r}{q}$ are two rational numbers with the same denominator, then $\frac{p}{q}+\frac{r}{q}=\frac{p+r}{q}$.

Example 8 : **Add the following :**

(i) $\frac{1}{7}$ and $\frac{3}{7}$ (ii) $\frac{6}{-13}$ and $\frac{7}{13}$

Solution : (i) $\frac{1}{7}+\frac{3}{7}=\frac{1+3}{7}=\frac{4}{7}$

(ii) We first express $\frac{6}{-13}$ as a rational number with a positive denominator.

We have, $\frac{6}{-13}=\frac{6\times(-1)}{-13\times(-1)}=\frac{-6}{13}$

Now, $\frac{6}{-13}+\frac{7}{13}=\frac{-6}{13}+\frac{7}{13}=\frac{-6+7}{13}=\frac{1}{13}$.

Case II. Addition of Rational Numbers with Unequal Denominators

In order to find the sum of two rational numbers which have unequal denominators, we proceed as per the following steps:

Working Rules

Step 1 : Make the denominators of the rational numbers positive.

Step 2 : Find the LCM of the denominators obtained in step 1.

Step 3 : Express each one of the rational numbers in step 2 so that the LCM obtained in step 2 becomes their common denominator.

Step 4 : Write a rational number whose numerator is equal to the sum of the numerators of rational numbers obtained in step 3 and denominator as the LCM obtained in step 2.

Step 5 : The rational number obtained in step 4 is the required sum.

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers such that q and $s \neq 0$, then $\frac{p}{q} + \frac{r}{s} = \frac{ps + rq}{qs}$

Example 9: **Add $\frac{5}{8} + \frac{11}{12}$.**

Solution: LCM of 8 and 12 is 24.

Rewriting given rational numbers as :

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24} \quad \text{and} \quad \frac{11}{12} = \frac{11 \times 2}{12 \times 2} = \frac{22}{24}$$

Now, $\frac{15}{24} + \frac{22}{24} = \frac{15 + 22}{24} = \frac{37}{24} = 1\frac{13}{24}$

Take a Task

Watch Remedial

Properties of Addition of Rational Numbers

The properties of addition of rational numbers are similar to those of integers. Let us examine them one by one.

Property 1 (Closure Property). *The sum of any two rational numbers is always a rational number, i.e., if* $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left[\frac{a}{b} + \frac{c}{d}\right]$ is also a rational number.

Verification :

Consider the rational numbers $\frac{-6}{7}$ and $\frac{-11}{14}$.

$$\frac{-6}{7} + \frac{-11}{14} = \frac{-12}{14} + \frac{-11}{14} = \frac{-12 - 11}{14} = \frac{-23}{14}$$ which is a rational number.

Property 2 (Commutative Property). *The addition of rational numbers is commutative, i.e., if* $\frac{a}{b}$ *and* $\frac{c}{d}$ *are any two rational numbers, then*

$$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}.$$

Verification : Consider the expressions $\frac{5}{8} + \frac{-4}{7}$ and $\frac{-4}{7} + \frac{5}{8}$.

We have: $\frac{-4}{7} + \frac{5}{8} = \frac{-32}{56} + \frac{35}{56}$

$= \frac{-32+35}{56} = \frac{3}{56}$

and $\frac{5}{8} + \frac{-4}{7} = \frac{35}{56} + \frac{-32}{56} = \frac{35+(-32)}{56} = \frac{3}{56}$

$\therefore \quad \frac{5}{8} + \frac{-4}{7} = \frac{-4}{7} + \frac{5}{8}.$

Take a Task

Watch Remedial

Property 3 (Associative Property). *The addition of rational numbers is associative, i.e.,* $\frac{a}{b}, \frac{c}{d}$ *and* $\frac{e}{f}$ *are any three rational numbers, then* $\frac{a}{b} + \left[\frac{c}{d} + \frac{e}{f}\right] = \left[\frac{a}{b} + \frac{c}{d}\right] + \frac{e}{f}.$

Verification : Consider the following expressions $\left[\frac{2}{3} + \frac{-3}{5}\right] + \frac{-3}{7}$ and $\frac{2}{3} + \left[\frac{-3}{5} + \frac{-3}{7}\right]$.

$\left[\frac{2}{3} + \frac{-3}{5}\right] + \frac{-3}{7} = \left[\frac{10}{15} + \frac{-9}{15}\right] + \frac{-3}{7}$

$= \left[\frac{10+(-9)}{15}\right] + \frac{-3}{7} = \frac{1}{15} + \frac{-3}{7}$

$= \frac{7}{105} + \frac{-45}{105} = \frac{7+(-45)}{105} = \frac{-38}{105}.$

and $\frac{2}{3} + \left[\frac{-3}{5} + \frac{-3}{7}\right] = \frac{2}{3} + \left[\frac{-21}{35} + \frac{-15}{35}\right]$

$= \frac{2}{3} + \left[\frac{-21+(-15)}{35}\right] = \frac{2}{3} + \frac{-36}{35}$

$= \frac{70}{105} + \frac{-108}{105} = \frac{70+(-108)}{105} = \frac{-38}{105}.$

$\therefore \quad \left[\frac{2}{3} + \frac{-3}{5}\right] + \frac{-3}{7} = \frac{2}{3} + \left[\frac{-3}{5} + \frac{-3}{7}\right].$

Property 4 (Additive Property of Zero). *The sum of any rational number and zero (0) is the rational number itself. In other words, if* $\frac{a}{b}$ *is any rational number, then*

$$\frac{a}{b} + 0 = \frac{a}{b} = 0 + \frac{a}{b}.$$

Verification: We have, $\frac{1}{8} + 0 = \frac{1}{8} + \frac{0}{8} = \frac{1+0}{8} = \frac{1}{8}$

[0 can be written as $\frac{0}{8}$. Since it is a rational number.]

and $\quad 0 + \frac{1}{8} = \frac{0}{8} + \frac{1}{8} = \frac{0+1}{8} = \frac{1}{8}$

Property 5 (Additive Inverse or Negative Property). *For every rational number $\frac{a}{b}$ there exists a rational number $\frac{-a}{b}$ such that $\frac{a}{b} + \frac{-a}{b} = 0 = \frac{-a}{b} + \frac{a}{b}$.*

The rational numbers $\frac{a}{b}$ and $\frac{-a}{b}$ satisfying the above property are called *additive inverse* or *negative* of each other.

Examples: (*i*) The additive inverse of $\frac{-13}{17}$ is $-\left[\frac{-13}{17}\right] = -\left[-\frac{13}{17}\right] = \frac{13}{17}$.

(*ii*) The additive inverse of $\frac{-9}{-14}$ is $-\left[\frac{-9}{-14}\right] = -\frac{9}{14}$.

Subtraction of Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then subtracting $\frac{c}{d}$ from $\frac{a}{b}$ means adding additive inverse (negative) of $\frac{c}{d}$ to $\frac{a}{b}$.

Thus, $\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \left[\frac{-c}{d}\right]$. [Since, additive inverse of $\frac{c}{d}$ is $\frac{-c}{d}$.]

Example 10: Subtract $\frac{2}{5}$ from $\frac{-7}{10}$.

Solution: The additive inverse of $\frac{2}{5}$ is $\frac{-2}{5}$.

$$\therefore \quad \frac{-7}{10} - \frac{2}{5} = \frac{-7}{10} + \frac{-2}{5} = \frac{-7}{10} + \frac{-4}{10} = \frac{-7+(-4)}{10} = \frac{-11}{10}.$$

Example 11: Find the value of $\frac{-3}{7} - \frac{-5}{12}$.

Solution: The additive inverse of $\frac{-5}{12}$ is $\frac{5}{12}$.

$$\therefore \quad \frac{-3}{7} - \frac{-5}{12} = \frac{-3}{7} + \frac{5}{12}$$

$$= \frac{-3 \times 12 + 5 \times 7}{7 \times 12} = \frac{-36+35}{84} = \frac{-1}{84}.$$

Properties of Subtraction of Rational Numbers

Property 1 (Closure Property). *If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d}$ is a rational number.*

Verification: (*i*) Consider the rational numbers $\frac{4}{5}$ and $\frac{3}{8}$.

$\frac{4}{5} - \frac{3}{8} = \frac{32-15}{40} = \frac{17}{40}$, which is a rational number.

(*ii*) Consider the rational numbers $\frac{5}{6}$ and $\frac{1}{9}$.

$\frac{5}{6} - \frac{1}{9} = \frac{15-2}{18} = \frac{13}{18}$, which is a rational number.

Property 2 (Commutative Property). *The subtraction of rational numbers is not always commutative, i.e., for any two rational numbers* $\frac{a}{b}$ *and* $\frac{c}{d}$, *we have* $\frac{a}{b} - \frac{c}{d} \neq \frac{c}{d} - \frac{a}{b}$

Verification: Consider the rational numbers $\frac{7}{18}$ and $\frac{1}{6}$.

We have, $\frac{7}{18} - \frac{1}{6} = \frac{7}{18} + \frac{-1}{6} = \frac{7}{18} + \frac{-3}{18}$

$= \frac{7 + (-3)}{18} = \frac{4}{18} = \frac{2}{9}$.

and, $\frac{1}{6} - \frac{7}{18} = \frac{1}{6} + \frac{-7}{18} = \frac{3}{18} + \frac{-7}{18}$

$= \frac{3 + (-7)}{18} = \frac{-4}{18} = \frac{-2}{9}$.

Thus, $\frac{7}{18} - \frac{1}{6} \neq \frac{1}{6} - \frac{7}{18}$.

Property 3 (Associative Property). *The subtraction of rational numbers is not associative, i.e., for any three rational numbers* $\frac{a}{b}$, $\frac{c}{d}$ *and* $\frac{e}{f}$, *we have*

$$\left[\frac{a}{b} - \frac{c}{d}\right] - \frac{e}{f} \neq \frac{a}{b} - \left[\frac{c}{d} - \frac{e}{f}\right].$$

Example: Consider the rational numbers $\frac{3}{8}$, $\frac{2}{7}$ and $\frac{3}{4}$.

We have: $\left[\frac{3}{8} - \frac{2}{7}\right] - \frac{3}{4} = \left[\frac{21-16}{56}\right] - \frac{3}{4} = \frac{5}{56} - \frac{3}{4} = \frac{-37}{56}$

$\frac{3}{8} - \left[\frac{2}{7} - \frac{3}{4}\right] = \frac{3}{8} - \left[\frac{8-2}{28}\right] = \frac{3}{8} - \frac{-13}{28} = \frac{3}{8} + \frac{13}{28} = \frac{47}{56}$

Clearly, $\left[\frac{3}{8} - \frac{2}{7}\right] - \frac{3}{4} \neq \frac{3}{8} - \left[\frac{2}{7} - \frac{3}{4}\right]$.

Property 5 (Existence of Right Identity). *The rational number 0 is the right identity, i.e., for any rational number* $\frac{a}{b}$, *we have* $\frac{a}{b} - 0 = \frac{a}{b}$.

Example: $\frac{3}{7} - 0 = \frac{3}{7}$. Similarly, $\frac{9}{11} - 0 = \frac{9}{11}$.

Example 12: The sum of two rational numbers is $\frac{-6}{7}$. If one of them is $\frac{-15}{7}$, find the other number.

Solution: Let the other number be x.

Then, $x + \left[\frac{-15}{7}\right] = \frac{-6}{7}$

$\Rightarrow \quad x = \frac{-6}{7} - \frac{-15}{7}$

$= \frac{-6}{7} + \frac{15}{7} = \frac{-6 + 15}{7} = \frac{9}{7} = 1\frac{2}{7}$

Hence, the other number is $1\frac{2}{7}$.

Example 13 : **Simplify:** $\frac{3}{2} + \frac{-8}{11} + \frac{-4}{11} + \frac{5}{22}$

Solution : $\frac{3}{2} + \frac{-8}{11} + \frac{-4}{11} + \frac{5}{22}$

$= \frac{33}{22} + \frac{-16}{22} + \frac{-8}{22} + \frac{5}{22}$ ($\therefore$ LCM of 2, 11, 11, 22 is 22.)

$= \frac{33 + (-16) + (-8) + 5}{22} = \frac{14}{22} = \frac{7}{11}$.

Exercise 1.2

1. Verify the following :

(*i*) $\frac{5}{7} + \frac{1}{9} = \frac{1}{9} + \frac{5}{7}$ (*ii*) $\frac{-4}{13} + 0 = 0 + \frac{-4}{13}$

(*iii*) $\frac{1}{9} + \left[\frac{3}{4} + \frac{-4}{5}\right] = \left[\frac{1}{9} + \frac{3}{4}\right] + \frac{-4}{5}$ (*iv*) $\left[\frac{2}{5} + \frac{-7}{10}\right] + \frac{1}{15} = \frac{2}{5} + \left[\frac{-7}{10} + \frac{1}{15}\right]$

(*v*) $\frac{-5}{81} + \frac{5}{81} = 0$ (*vi*) $\left[\frac{2}{9} + \frac{-7}{10}\right] + \frac{1}{15} = \frac{2}{9} + \left[\frac{-7}{10} + \frac{1}{15}\right]$

2. Add the following rational numbers :

(*i*) $\frac{3}{7}$ and $\frac{2}{7}$ (*ii*) $\frac{-5}{13}$ and $\frac{9}{13}$ (*iii*) $\frac{1}{11}$ and $\frac{3}{7}$ (*iv*) 3 and $\frac{7}{12}$

(*v*) $\frac{21}{-32}$ and -5 (*vi*) $\frac{13}{-31}$ and $\frac{-5}{9}$ (*vii*) $\frac{-15}{42}$ and $\frac{-13}{17}$

[**Hint:** (*iv*) and (*v*) write 3 and –5 as $\frac{3}{1}$ and $\frac{-5}{1}$ respectively and then solve.]

3. Find the additive inverse of each of the following :

(*i*) 14 (*ii*) $\frac{8}{15}$ (*iii*) $\frac{-1}{9}$ (*iv*) $\frac{13}{-23}$ (*v*) $\frac{-3}{-14}$ (*vi*) $\frac{4}{-7}$

4. Simplify.

(*i*) $\frac{6}{5} + \frac{19}{-45} + \frac{-5}{3} + \frac{-7}{15}$ (*ii*) $\frac{-2}{3} + \frac{-11}{15} + \frac{8}{3}$ (*iii*) $-3\frac{2}{5} + 3\frac{1}{2} + 3\frac{1}{4}$

5. Find.

(*i*) $\frac{4}{19} - \frac{3}{19}$ (*ii*) $\frac{7}{10} - \frac{-3}{10}$ (*iii*) $\frac{-2}{26} - \frac{-1}{26}$ (*iv*) $\frac{5}{11} - \frac{1}{11}$

(*v*) $\frac{-6}{21} - \frac{3}{7}$ (*vi*) $\frac{3}{18} - \frac{-5}{27}$ (*vii*) $\frac{-21}{11} - \frac{-25}{33}$ (*viii*) $3 - \frac{-13}{15}$

6. Verify the following :

(*i*) $\frac{1}{5} - \frac{3}{8} \neq \frac{3}{8} - \frac{1}{5}$ (*ii*) $-3 - \frac{5}{11} \neq \frac{5}{11} - (-3)$

(*iii*) $\left[\frac{5}{9} - \frac{2}{6}\right] - \frac{2}{7} \neq \frac{5}{9} - \left[\frac{2}{6} - \frac{2}{7}\right]$ (*iv*) $\frac{-6}{11} - 0 = \frac{-6}{11}$

7. Simplify the following expressions :

(i) $\frac{5}{14}+\frac{3}{7}+\frac{1}{21}$ (ii) $3-\left[\frac{5}{9}-\frac{3}{15}\right]$ (iii) $\frac{-2}{6}+\frac{5}{9}-\frac{1}{3}$

(iv) $1+\frac{14}{35}+\frac{-5}{21}+\frac{24}{15}$ (v) $\frac{1}{6}+\frac{-18}{12}+\frac{-4}{3}+\frac{7}{2}$

8. The sum of two rational numbers is –7. If one of them is $\frac{-11}{3}$, find the other.

9. What number should be added to $-\frac{5}{6}$ to get –3 ?

10. Subtract the sum of $-\frac{6}{7}$ and $-\frac{4}{14}$ from the sum of $\frac{1}{2}$ and $-\frac{2}{7}$.

11. From the sum of $\frac{-8}{7}$ and $\frac{-11}{14}$, subtract the difference of $\frac{1}{9}$ and $\frac{-12}{13}$.

12. The weight of a carton full of juice packs is $30\frac{1}{7}$ kg. Seema took $15\frac{3}{4}$ kg of juice packs from it. Find the weight of the carton now.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Mental Maths

Experiential Learning

State true and false.

1. 1 is the additive identity.
2. Additive inverse of $\frac{a}{b}$ is $\frac{b}{a}$.
3. For adding two rational numbers, first you need to find LCM of their denominators.
4. Subtraction is basically addition, where one of the addends has negative sign.

HOTS (Higher Order Thinking Skills)

Experiential Learning

1. Ricky bought two fish bowls. He pours $4\frac{3}{4}$ litres of water in the first bowl and $3\frac{1}{2}$ litres in the second bowl. How much water did Ricky put in two bowls altogether?
2. Gautam and Sumati went for jogging in the morning. Gautam ran $\frac{2}{3}$ of the track and Sumati ran $\frac{5}{6}$ of the same track. Who ran more and by how much?

Multiplication of Rational Numbers

When two or more rational numbers are multiplied, the product is a rational number whose numerator equals to the product of the numerators of the given rational numbers and denominator equals to the product of the denominators of the given rational numbers.

$$\text{Product of two rational numbers} = \frac{\text{Product of their numerators}}{\text{Product of their denominators}}$$

In general, we can say that if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$.

Example 14 : **Multiply.**

(*i*) $\frac{-3}{17}$ by 3 (*ii*) $\frac{-5}{9}$ by $\frac{72}{25}$

Solution : (*i*) $\frac{-3}{17} \times 3 = \frac{-3}{17} \times \frac{3}{1} = \frac{-3 \times 3}{17 \times 1} = \frac{-9}{17}$

(*ii*) $\frac{-5}{9} \times \frac{72}{25} = \frac{-5 \times 72}{9 \times 25} = \frac{-1 \times 8}{1 \times 5} = \frac{-8}{5} = -1\frac{3}{5}$

Properties of Multiplication of Rational Numbers

The multiplication of rational numbers possesses following properties.

Property 1(Closure Property). *The product of any two rational numbers is always a rational number, i.e.,* if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then their product $\frac{a}{b} \times \frac{c}{d}$ is also a rational number.

Verification : (*i*) Consider two rational numbers $\frac{2}{5}$ and $\frac{9}{5}$. Then, $\frac{2}{5} \times \frac{9}{5} = \frac{18}{25}$, which is a rational number.

(*ii*) Now consider the rational numbers $\frac{2}{11}$ and $\frac{-12}{5}$.

Then, $\frac{2}{11} \times \frac{-12}{5} = \frac{-24}{55}$, which is also a rational number.

Property 2 (Commutative Property). *The product of two rational numbers remains the same even if we change their order, i.e.,* if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$.

Verification : Consider two rational numbers $\frac{-3}{4}$ and $\frac{1}{7}$.

Then, $\frac{-3}{4} \times \frac{1}{7} = \frac{(-3) \times 1}{4 \times 7} = \frac{-3}{28}$

and $\frac{1}{7} \times \frac{-3}{4} = \frac{1 \times (-3)}{7 \times 4} = \frac{-3}{28}$

$\therefore \quad \frac{-3}{4} \times \frac{1}{7} = \frac{1}{7} \times \frac{-3}{4}$.

Property 3 (Associative Property). *The product of three or more rational numbers remains the same even if the order in which they are grouped is changed.*

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then $\left[\frac{a}{b} \times \frac{c}{d}\right] \times \frac{e}{f} = \frac{a}{b} \times \left[\frac{c}{d} \times \frac{e}{f}\right]$.

Verification : Let us consider three rational numbers $\frac{2}{7}, \frac{1}{-2}$ and $\frac{4}{5}$. Then

$$\left[\frac{2}{7} \times \frac{1}{-2}\right] \times \frac{4}{5} = \frac{2 \times (-1)}{7 \times 2} \times \frac{4}{5} = \frac{-2}{14} \times \frac{4}{5}$$

$$= \frac{-2 \times 4}{14 \times 5} = \frac{-8}{70} = \frac{-4}{35}$$

By changing order, we get

$$\frac{2}{7} \times \frac{1}{-2} \times \frac{4}{5} = \frac{2}{7} \times \frac{(-1) \times 4}{2 \times 5} = \frac{2}{7} \times \frac{-4}{10}$$

$$= \frac{2 \times (-4)}{7 \times 10} = \frac{-8}{70} = \frac{-4}{35}$$

$$\therefore \left[\frac{2}{7} \times \frac{1}{-2}\right] \times \frac{4}{5} = \frac{2}{7} \times \left[\frac{1}{-2} \times \frac{4}{5}\right].$$

Take a Task

Watch Remedial

Property 4 (Property of Multiplicative Identity or '1'). *If we multiply a rational number by 1, then the product is the number itself.*

In other words, if $\frac{a}{b}$ is any rational number, then $\frac{a}{b} \times 1 = \frac{a}{b} = 1 \times \frac{a}{b}$.

Example :

$$\frac{-4}{9} \times 1 = \frac{-4}{9} \times \frac{1}{1} = \frac{-4 \times 1}{9 \times 1} = \frac{-4}{9}$$

and $$1 \times \frac{-4}{9} = \frac{1}{1} \times \frac{-4}{9} = \frac{1 \times (-4)}{1 \times 9} = \frac{-4}{9}$$

Thus, $$\frac{-4}{9} \times 1 = 1 \times \left(\frac{-4}{9}\right).$$

Similarly, $1 \times \left(\frac{5}{-11}\right) = \frac{5}{-11} \times 1.$

Property 5 (Property of Additive Identity or '0'). *Any rational number when multiplied with zero, gives product as 0.*

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b} \times 0 = 0 = 0 \times \frac{a}{b}$.

Example :

$$\frac{2}{3} \times 0 = \frac{2}{3} \times \frac{0}{1} = \frac{2 \times 0}{3 \times 1} = \frac{0}{3} = 0$$

and $$0 \times \frac{2}{3} = \frac{0}{1} \times \frac{2}{3} = \frac{0 \times 2}{1 \times 3} = \frac{0}{3} = 0$$

$$\therefore \quad \frac{2}{3} \times 0 = 0 \times \frac{2}{3}.$$

Similarly, $\frac{-7}{5} \times 0 = 0 = 0 \times \frac{-7}{5}.$

Property 6 (Distributive Property of Multiplication over Addition). *The multiplication of rational numbers is distributive over addition. In other words,*

if $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \times \left[\frac{c}{d} + \frac{e}{f}\right] = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$.

Verification: Consider three rational numbers, say, $\frac{-1}{4}$, $\frac{2}{3}$ and $\frac{4}{5}$.

$$\text{We have, } \frac{-1}{4} \times \left[\frac{2}{3} + \frac{4}{5}\right] = \frac{-1}{4} \times \left[\frac{10+12}{15}\right]$$

$$= \frac{-1}{4} \times \frac{22}{15} = \frac{-1 \times 22}{4 \times 15} = \frac{-22}{60} = \frac{-11}{30}$$

$$\text{and } \frac{-1}{4} \times \frac{2}{3} + \frac{-1}{4} \times \frac{4}{5} = \frac{-1 \times 2}{4 \times 3} + \frac{-1 \times 4}{4 \times 5}$$

$$= \frac{-2}{12} + \frac{-4}{20} = \frac{-10 + (-12)}{60} = \frac{-22}{60} = \frac{-11}{30}$$

$$\therefore \quad \frac{-1}{4} \times \left[\frac{2}{3} + \frac{4}{5}\right] = \frac{-1}{4} \times \frac{2}{3} + \frac{-1}{4} \times \frac{4}{5}.$$

Property 7 (Reciprocal or Multiplicative Inverse Property).

For every non-zero rational number $\frac{a}{b}$, *there exists a rational number* $\frac{b}{a}$ *such that*

$$\frac{a}{b} \times \frac{b}{a} = 1$$

Here, $\frac{a}{b}$ and $\frac{b}{a}$ are said to be *multiplicative inverse* or *reciprocal* of each other.

Verification: (*i*) Reciprocal of $\frac{3}{11}$ is $\frac{11}{3}$. (Since, $\frac{3}{11} \times \frac{11}{3} = 1$.)

(*ii*) Reciprocal of $\frac{-7}{15}$ is $\frac{-15}{7}$. (Since, $\frac{-7}{15} \times \frac{-15}{7} = 1$.)

Exercise 1.3

1. Solve the following:

(*i*) $\frac{3}{10} \times \frac{2}{9}$ (*ii*) $\frac{2}{3} \times \frac{4}{5}$ (*iii*) $\frac{5}{18} \times \frac{-90}{109}$ (*iv*) $33 \times \frac{5}{-11}$

(*v*) $28 \times \frac{-3}{7}$ (*vi*) $\frac{10}{-16} \times \frac{-32}{-5}$ (*vii*) $\frac{15}{21} \times \frac{-3}{5}$

2. Multiply.

(*i*) $\frac{1}{4}$ by $\frac{5}{2}$ (*ii*) $\frac{-2}{7}$ by $\frac{3}{11}$ (*iii*) $\frac{6}{13}$ by $\frac{-65}{6}$ (*iv*) $\frac{-8}{15}$ by $\frac{3}{16}$

(*v*) $\frac{-8}{7}$ by $\frac{49}{64}$ (*vi*) $\frac{-3}{5}$ by $\frac{-25}{15}$

3. Fill in the blanks.

(*i*) $\frac{1}{3} \times \left[\frac{5}{7} \times \frac{-5}{8}\right] = \left[\frac{1}{3} \times \frac{5}{7}\right] \times ......$ (*ii*) $\left[\frac{-7}{12} \times \frac{3}{-8}\right] \times \frac{11}{5} = ... \times \left[\frac{3}{-8} \times ......\right]$

(*iii*) $\frac{-30}{95} \times 1 = 1 \times ...... = ...$ (*iv*) $\frac{-24}{55} \times 0 = ... \times \frac{-24}{55} = ...$

(*v*) $\frac{-10}{23} \times \left[\frac{-8}{9} + \frac{-2}{5}\right] = \left[\frac{-10}{23} \times ......\right] + \left[...... \times \frac{-2}{5}\right]$

4. Find the reciprocal of each of the following :

(*i*) $\frac{3}{8}$ (*ii*) $\frac{5}{-1}$ (*iii*) 24 (*iv*) –27 (*v*) $\frac{-30}{-83}$ (*vi*) $\frac{-1}{49}$

5. Verify the property, $x \times (y \times z) = (x \times y) \times z$ by taking.

(*i*) $x = \frac{1}{2}, y = \frac{1}{-4}$ and $z = \frac{1}{3}$ (*ii*) $x = \frac{-4}{3}, y = \frac{1}{5}$ and $z = \frac{3}{4}$

6. Verify the property, $x \times (y + z) = (x \times y) + (x \times z)$ by taking.

(*i*) $x = \frac{-3}{4}, y = \frac{5}{2}$ and $z = \frac{1}{5}$ (*ii*) $x = 0, y = \frac{5}{6}$ and $z = \frac{-5}{12}$

7. Simplify.

(*i*) $\frac{2}{7} + \frac{2}{3} \times \frac{1}{2}$ (*ii*) $\frac{-5}{14} \times \frac{14}{3} + \frac{7}{8} \times \frac{-16}{21}$

(*iii*) $\frac{7}{15} \times \frac{-15}{21} + \frac{-3}{5} \times \frac{4}{9}$ (*iv*) $\frac{-2}{5} \times \frac{-3}{10} - \frac{2}{3} \times \frac{-3}{5} + \frac{3}{4} \times \frac{2}{3}$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Division of Rational Numbers

We know that division is the inverse of multiplication.

To divide a rational number by another non-zero rational number, we find the product of first rational number with the reciprocal of the second number.

In general, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \text{reciprocal of } \frac{c}{d}$$

$$= \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}.$$

where $\frac{a}{b}$ is dividend, $\frac{c}{d}$ is divisor and $\frac{a \times d}{b \times c}$ is the quotient.

Take a Task

Watch Remedial

Example 15 : Divide the following :

(*i*) $\frac{2}{7}$ by $\frac{3}{7}$ (*ii*) $\frac{4}{18}$ by $\frac{4}{-9}$

Solution : (*i*) $\frac{2}{7} \div \frac{3}{7} = \frac{2}{7} \times \frac{7}{3} = \frac{2 \times 7}{7 \times 3} = \frac{2}{3}$

(*ii*) $\frac{4}{18} \div \frac{4}{-9} = \frac{4}{18} \times \frac{-9}{4}$ $\left[\because \frac{4}{-9} = \frac{-4}{9}\right]$

$$= \frac{4 \times (-9)}{18 \times 4} = \frac{-1}{2}.$$

Example 16 : The product of two numbers is $\frac{-7}{8}$. If one of the numbers is $\frac{3}{8}$, find the other number.

Solution : Let the other number be x.

Then $\frac{3}{8} \times x = \frac{-7}{8}$ $\Rightarrow x = \frac{-7}{8} \div \frac{3}{8}$

$\Rightarrow x = \frac{-7}{8} \times \frac{8}{3}$ $\Rightarrow x = \frac{-7}{3} = -2\frac{1}{3}$

Hence, the other number is $-2\frac{1}{3}$.

Properties of Division of Rational Numbers

The division of rational numbers possesses following properties.

Property 1 (Closure Property). If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is always a rational number, i.e., the set of all non-zero rational numbers is closed under division.

Verification : Consider the rational numbers $\frac{1}{4}$ and $\frac{4}{5}$.

$$\frac{1}{4} \div \frac{4}{5} = \frac{1}{4} \times \frac{5}{4}$$

$$= \frac{1 \times 5}{4 \times 4} = \frac{5}{16},$$ which is a rational number.

Note

Division is closed for all rational numbers except zero.

Property 2 (Identity Property). For any rational number $\frac{a}{b}$, we have : $\frac{a}{b} \div 1 = \frac{a}{b}$ and $\frac{a}{b} \div (-1) = -\frac{a}{b}$.

Verification : Consider the rational numbers $\frac{7}{6}$.

We have, $\frac{7}{6} \div 1 = \frac{7}{6} \times \frac{1}{1} = \frac{7}{6}$ and $\frac{7}{6} \div (-1) = \frac{7}{6} \times \frac{1}{-1} = \frac{-7}{6}$

Property 3 For every non-zero rational number $\frac{a}{b}$, we have :

(*i*) $\frac{a}{b} \div \frac{a}{b} = 1$ (*ii*) $\frac{a}{b} \div \left(-\frac{a}{b}\right) = -1$ (*iii*) $\left(\frac{-a}{b}\right) \div \frac{a}{b} = -1$

Verification : Consider the rational number $\frac{5}{7}$.

(*i*) $\frac{5}{7} \div \frac{5}{7} = \frac{5}{7} \times \frac{7}{5} = 1$ (*ii*) $\frac{5}{7} \div \left(\frac{-5}{7}\right) = \frac{5}{7} \times \frac{-7}{5} = -1$

(*iii*) $\left(\frac{-5}{7}\right) \div \frac{5}{7} = \frac{-5}{7} \times \frac{7}{5} = -1$

Property 4 For every non-zero rational number $\frac{a}{b}$, we have:

(*i*) $0 \div \frac{a}{b} = 0$ (*ii*) $0 \div \left(\frac{-a}{b}\right) = 0$

Note

The division of rational numbers is neither commutative nor associative.

Verification : Consider the rational number $\frac{3}{5}$.

(*i*) $0 \div \frac{3}{5} = \frac{0}{1} \times \frac{5}{3}$

$= \frac{0 \times 5}{3} = \frac{0}{3} = 0$

(*ii*) $0 \div \frac{-3}{5} = \frac{0}{1} \times \left(\frac{-5}{3}\right)$

$= \frac{0 \times (-5)}{1 \times 3} = \frac{0}{3} = 0.$

Exercise 1.4

1. Verify whether the given number statements are True or False.

(i) $\frac{5}{13} \div 1 = \frac{5}{13}$ (ii) $\frac{3}{10} \div \frac{-4}{7} \neq \frac{-4}{7} \div \frac{3}{10}$ (iii) $\frac{3}{13} \div \frac{-4}{7} = \frac{-4}{7} \div \frac{3}{13}$

(iv) $\frac{3}{5} \div \frac{2}{9} = \frac{2}{9} \div \frac{3}{5}$ (v) $\frac{5}{27} \div \frac{5}{27} = -1$ (vi) $\left(\frac{2}{5} \div \frac{3}{8}\right) \div \frac{-3}{5} \neq \frac{2}{5} \div \left(\frac{3}{8} \div \frac{-3}{5}\right)$

2. Divide.

(i) $\frac{8}{7}$ by $\frac{6}{21}$ (ii) $\frac{3}{10}$ by $\frac{6}{20}$ (iii) $\frac{5}{12}$ by $\frac{-5}{12}$ (iv) 6 by $\frac{1}{4}$

(v) 0 by $\frac{9}{17}$ (vi) $\frac{-6}{7}$ by (-8) (vii) $\frac{12}{56}$ by (-7)

3. What number should be multiplied by $\frac{7}{3}$ to get the product $\frac{-35}{81}$?

4. What number should be multiplied by $\frac{-7}{26}$ to get the product $\frac{56}{78}$?

5. The product of two rational numbers is $\frac{-16}{9}$. If one of the numbers is $\frac{-4}{3}$, find the other.

6. Fill in the blanks.

(i) $\frac{8}{9} \div 1 =$ ______ (ii) ______ $\div \frac{8}{35} = 0$ (iii) $\frac{11}{13} \div$ ______ $= 1$

(iv) ______ $\div \frac{15}{29} = -1$ (v) $1 \div \frac{3}{5} =$ ______ (vi) ______ $\div \frac{13}{31} = 0$

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Insertion of Rational Numbers Between Two Given Rational Number

Between any two rational numbers we can insert infinitely many rational numbers.

If x and y are two rational numbers such that $x < y$, then

$$x < \frac{x+y}{2} < y$$

or $$x < \frac{1}{2}(x+y) < y.$$

The mean of two numbers x and y is $\frac{x+y}{2}$.

In other words, the rational number $\frac{x+y}{2} = (x+y) \div 2$ lies between x and y.

To find the rational numbers between two given rational numbers, we proceed as per the following steps:

Working Rules

Step 1. Find the sum of the two numbers.

Step 2. Divide the sum by 2. The result so obtained is the rational number between the two given numbers.

Step 3. Take one of the given rational numbers and add it to the result obtained in the previous step. Now divide the sum by 2. This gives the another rational number.

Step 4. Repeat step 3 for finding some more rational numbers.

Example 17 : **Find three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.**

Solution : The rational number $\dfrac{\frac{1}{3}+\frac{1}{2}}{2} = \frac{5}{12}$ lies between $\frac{1}{3}$ and $\frac{1}{2}$.

Therefore, we get $\frac{1}{3} < \frac{5}{12} < \frac{1}{2}$.

Now, the rational number $\dfrac{\frac{1}{3}+\frac{5}{12}}{2} = \frac{9}{24}$ lies between $\frac{1}{3}$ and $\frac{5}{12}$,

and the rational number $\dfrac{\frac{5}{12}+\frac{1}{2}}{2} = \frac{11}{24}$ lies between $\frac{5}{12}$ and $\frac{1}{2}$.

This gives us that $\frac{1}{3} < \frac{11}{24} < \frac{5}{12}$ and $\frac{5}{12} < \frac{9}{24} < \frac{1}{2}$.

So, we have found three rational numbers, namely $\frac{11}{24}$, $\frac{5}{12}$ and $\frac{9}{24}$, between the two given rational numbers.

REMEMBER

To find large number of rational numbers between two given rational numbers, we write the equivalent rational number having a sufficiently large common denominator.

Example 18 : **Find twelve rational numbers between $\frac{1}{2}$ and $\frac{4}{3}$.**

Solution : Here, $\frac{1}{2} = \frac{3}{6}$ and $\frac{4}{3} = \frac{8}{6}$.

Between integers 3 and 8, we get only 8 – 3 – 1 = 4 integers. So, it is not easy to locate twelve rational numbers between $\frac{1}{2}$ and $\frac{4}{3}$.

Let us write $\frac{1}{2} = \frac{15}{30}$ and $\frac{4}{3} = \frac{40}{30}$.

Now, between 15 and 40, we have (40 – 15 – 1) = 24 integers.

Out of these, we can choose any twelve. Thus,

$$\frac{1}{2} < \frac{15}{30} < \frac{16}{30} < \frac{17}{30} < \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30} < \frac{25}{30} < \frac{26}{30} < \frac{27}{30} < \frac{40}{30} < \frac{4}{3}.$$

Exercise 1.5

1. Choose the correct alternative from the given four alternative options.

   (*i*) The position of rational number $\frac{-8}{11}$ on the number line is :

   (*a*) to the left of 0 (*b*) to the right of 0 (*c*) at the point of 0 (*d*) None of these (*ii*) If x and y are two rational numbers such that

(*a*) $x + y$ is also a rational number (*b*) $x - y$ is also a rational number

(*c*) $x \times y$ is also a rational number (*d*) All the above

(*iii*) Which of the following property is satisfied for commutative property of rational numbers ?

(*a*) Addition and multiplication (*b*) Subtraction and multiplication

(*c*) Multiplication and division (*d*) Addition and division

2. Find three rational numbers between:

(*i*) $\frac{1}{2}$ and $\frac{1}{5}$ (*ii*) -1 and $\frac{3}{2}$ (*iii*) $\frac{1}{10}$ and $\frac{8}{5}$ (*iv*) $\frac{-7}{15}$ and $\frac{-6}{25}$

3. Find ten rational numbers between:

(*i*) $\frac{4}{25}$ and $\frac{-3}{10}$ (*ii*) $\frac{-6}{27}$ and $\frac{5}{6}$ (*iii*) $\frac{4}{7}$ and $\frac{1}{2}$ (*iv*) $\frac{10}{13}$ and $\frac{12}{13}$

4. Check whether the following is a rational number between $\frac{-3}{8}$ and $\frac{3}{8}$.

(*i*) $\frac{1}{7}$ (*ii*) $\frac{-5}{8}$ (*iii*) $\frac{7}{23}$ (*iv*) $\frac{1}{8}$

5. Find any five rational numbers between $\frac{1}{6}$ and $\frac{1}{5}$.

6. Insert 100 rational numbers between $\frac{-4}{17}$ and $\frac{11}{17}$.

Skills covered: Evaluation skills, analytical skills, problem solving skills, numeracy skills

Revision Exercise

Conceptual Learning

1. **Tick (✓) the correct option:**

(*i*) The sum of the additive inverse and multiplicative inverse of 3 is:

(*a*) $\frac{8}{3}$ ☐ (*b*) -3 ☐ (*c*) $\frac{-8}{3}$ ☐ (*d*) $\frac{-1}{3}$ ☐

(*ii*) The rational number equivalent to $\frac{-24}{45}$ is:

(*a*) $\frac{-8}{15}$ ☐ (*b*) $\frac{-6}{9}$ ☐ (*c*) $\frac{4}{11}$ ☐ (*d*) $\frac{12}{20}$ ☐

(*iii*) What should be subtracted from $\frac{-5}{3}$ to get $\frac{5}{6}$?

(*a*) $\frac{3}{2}$ ☐ (*b*) $\frac{5}{4}$ ☐ (*c*) $\frac{5}{2}$ ☐ (*d*) $\frac{-5}{2}$ ☐

(*iv*) The value of $3 + \frac{5}{-7}$ is:

(*a*) $\frac{-16}{7}$ ☐ (*b*) $\frac{-26}{7}$ ☐ (*c*) $\frac{16}{7}$ ☐ (*d*) $\frac{-8}{7}$ ☐

2. Find the product of $-3\frac{2}{3} \times 1\frac{1}{4}$.

3. Expressed $\frac{-28}{84}$ as a rational number with numerator 4.

4. Find the value of $\left|\frac{-4}{5}+\frac{9}{20}\right|$.

5. The product of two numbers is $\frac{-16}{25}$. If one of the numbers is $\frac{4}{25}$, find the other number.

6. What is the value of $\frac{4}{5} \div \frac{5}{25}$.

7. What is additive inverse of $\frac{-14}{19}$.

8. What number should be added to $\frac{-5}{16}$ to get $\frac{7}{24}$.

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Mental Maths

Experiential Learning

1. Find the standard form of $\frac{25}{-10}$.
2. What should be added to $\left(\frac{3}{6}+\frac{1}{2}-\frac{2}{4}\right)$ to get 1.
3. Multiply $-3\frac{3}{8}$ by $-9\frac{1}{11}$.

Scan to Create Your Own Learning Path

Custom Learning Path

Assertion and Reason

Experiential Learning

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.
Study both the statements and state which of the following is correct.

(*a*) Both A and R are true and R is the correct explanation of A.
(*b*) Both A and R are true but R is not the correct explanation of A.
(*c*) A is true but R is false.
(*d*) A is false but R is true.

1. **Assertion (A) :** The product of two rational numbers is always a positive integer.

   Reason (R) : The product of zero and a rational number is 1.

2. **Assertion (A) :** The denominator of a rational number cannot be zero.

   Reason (R) : Zero is not the reciprocal of any number.

3. **Assertion (A) :** $\frac{-3}{5}\times\left[\left(\frac{2}{3}\right)+\left(\frac{-2}{7}\right)\right]=\frac{-8}{35}$

   Reason (R) : $\frac{-2}{5}\div\frac{1}{2}$ *is eqeual to* $\frac{-5}{4}$

4. **Assertion (A) :** The additive inverse of $\frac{-6}{-17}$ is $\frac{-17}{6}$.

   Reason (R) : If $\frac{-2}{5}=\frac{13}{x}$, then x is equal to $\frac{-65}{2}$.

5. Assertion (A) : $\frac{-2}{3}$ lies on the right of 0 on the number line.

Reason (R) : 0 is the only rational number whose additive inverse is the number itself.

Activity

Finding a rational number between two given numbers.

Ask the class to move for play ground then call student to hold the rope end by touching the rop terminal on ground surface. Ask two students stand on position–2/3 and 1/2 both students observe that the numbers are mixed fraction or simple fraction.

And stand on the position. Now ask another two students make their denominators equal or same.

$$\frac{-2}{3} = \frac{-2 \times 2}{3 \times 2} = \frac{-4}{6} \text{ and } \frac{1}{2} = \frac{1}{2} \times \frac{3}{3} \times \frac{3}{6}$$

Now ask them to insert the numbers between–4 and 6 then to choose –2, 3, so $\frac{-2}{6}, \frac{3}{6}$ are two rational numbers similarly they can find number of rational numbers between two rational numbers.

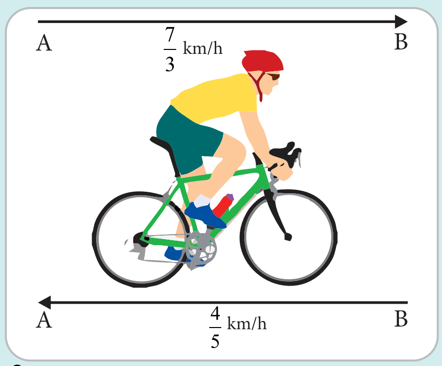
Repeat this with $\frac{7}{6}$ and $\frac{5}{3}$ and ask for 3 rational numbers to insert.

To make teaching-learning process a success, teacher will make sure that almost all the students in the class will participate in this activity.

Skills Developed: Interpersonal skills, Observation, Application and Decision making skills

Thinking Skills

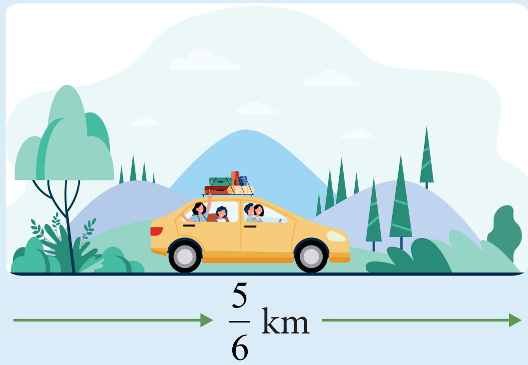
1. A cyclist travels from point A to point B at a speed of $\frac{7}{3}$ km/h and returns from point B to point A at a speed of $\frac{4}{5}$ km/h. If the total distance covered for the round trip is 60 km, find the time taken for the entire journey. Express your answer as a rational number.



2. The prices of five different books in a store are $\frac{18}{5}$, $\frac{3}{7}$, $\frac{11}{6}$, $\frac{5}{4}$, and $\frac{3}{2}$. Find the average price of a book in the store. Express your answer as a rational number.
3. A person travels to a town located 120 km away. He first covers $\frac{3}{5}$ of the distance at a speed of $\frac{8}{3}$ km/h and the remaining distance at a speed of $\frac{7}{2}$ km/h. Calculate the average speed of the person for the entire journey, and express the answer as a rational number.

Skills Developed: Creativity, Observation, Critical Thinking, Logical Reasoning, Reflective Thinking

Competency based Questions



A car travels a distance of $\frac{5}{6}$ km in $\frac{2}{3}$ hours. Based on this information, answer the following:

(a) Calculate the average speed of the car in km per hour.

(b) If the car's speed were to double, what would the new speed be? Express the new speed as a rational number.

(c) The area of a square field is given by the square of its side length. If the side length of the square is $\frac{3}{4}$ meters, calculate the area of the square in square meters.

(d) If you add the average speed from part (a) and the area of the square from part (c), what is the result? Simplify the sum.

Skills covered: Interpersonal skills, Observation, Application and Decision making skills

Integrated Learning

A company is tracking its profit and loss over the first three quarters of the year. The profit or loss in each quarter is recorded as a rational number in fraction form.

Quarter	Profit/Loss (in fractions)
Q1 (January - March)	$-\frac{5}{12}$
Q2 (April - June)	$\frac{7}{8}$
Q3 (July - September)	$-\frac{3}{4}$

Based on this information answer the following questions:

1. Calculate the total profit or loss over the first three quarters by adding the values for each quarter.
2. Find the average profit or loss for the first three quarters by dividing the total by 3.
3. If the company decides to reduce its total loss by $\frac{14}{3}$, calculate the new total profit/loss and average.
4. Are the total profit/loss and average profit/loss rational numbers? Justify your answer.

Skills covered: Research, Logical Reasoning, Problem-Solving, Practical Application