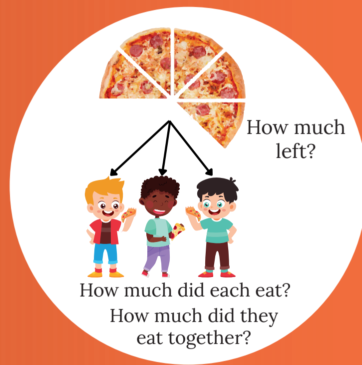
# 8 Fraction

## Why This Chapter Matters

Imagine you're baking a cake, and the recipe calls for "$\frac{3}{4}$ cup of sugar." Or perhaps you're sharing a pizza with friends, and everyone gets "$\frac{1}{8}$ of the whole." Fractions are everywhere in our daily lives!

But what happens when you need to make half of that recipe, or if you want to know how much pizza three friends eat? How do we combine or separate these fractional parts? This chapter will unlock the secrets of multiplying and dividing fractions, showing you how these operations help us solve real-world puzzles.




## Meet EeeBee.AI

Hello, future mathematicians! I'm EeeBee, your friendly guide through the fascinating world of numbers. In this chapter, we're going to dive deep into fractions – those tricky but super useful parts of a whole. Don't worry if they seem a bit confusing at first; I'll be here to help you understand every step, share cool facts, and even point out common mistakes. Let's make fractions fun and easy together!

## Learning Outcomes

**By the end of this chapter, you will be able to:**

- Define multiplication of a fraction by a whole number and by another fraction.
- Calculate the product of fractions, including mixed fractions.
- Explain the concept of reciprocals and their role in fraction division.
- Perform division of a fraction by a whole number and by another fraction.
- Solve real-life word problems involving multiplication and division of fractions.
- Compare the product/quotient of fractions with the original numbers and draw conclusions.
- Appreciate the historical development of fraction arithmetic in different cultures.

## From Last Year's Notebook

- Fractions represent parts of a whole.
- Fractions can be understood as division (e.g., $\frac{3}{4}$ means 3 divided by 4).
- Fractions can be located as numbers on a number line.
- **You know different types of fractions:**
  - Proper fractions (numerator < denominator, e.g., $\frac{1}{2}$)
  - Improper fractions (numerator ≥ denominator, e.g., $\frac{5}{3}$)
  - Mixed fractions (whole number + proper fraction, e.g., $1\frac{2}{3}$)
- You can convert between improper and mixed fractions.
- You practiced adding and subtracting fractions with both like and unlike denominators.
- You learned about equivalent fractions $\left(\text{e.g.,} \frac{1}{2} = \frac{2}{4}\right)$, a skill that will be very useful now!

## Real Math, Real Life

- Fractions are fundamental for understanding quantities and proportions.
- **They're vital in everyday life for:** Cooking (scaling recipes), Shopping (discounts), Telling time
- **Professionally, fractions are indispensable for:** Engineers designing structures, Scientists measuring precisely, Financiers calculating interest, Athletes tracking stats
- Mastering fractions opens doors to diverse careers from architecture to data science, making them crucial for problem-solving and understanding our world.


## Quick Prep

1. Convert $2\frac{1}{4}$ into an improper fraction.
2. Convert $\frac{17}{4}$ into a mixed fraction.
3. If you have 12 apples and give away $\frac{1}{4}$ of them, how many apples do you give away?
4. Draw a rectangle and shade $\frac{2}{3}$ of it.
5. If a recipe calls for $\frac{1}{2}$ cup of flour, and you want to double the recipe, how much flour do you need?

## Introduction

Welcome to the exciting world of multiplying fractions! You've already mastered adding and subtracting fractions, which is like combining or taking away parts of a whole. Now, we're going to explore what it means to multiply fractions. This isn't just about making numbers bigger; sometimes, multiplying fractions can actually make them smaller! Understanding fraction multiplication is crucial for solving problems involving "**parts of parts**," scaling recipes, calculating areas, and much more. Get ready to discover how this powerful operation works!

### Chapter Overview

**Introduction:** Begins with foundational fraction work, leading into multiplication and division.

**Multiplication of Fractions:** Covers multiplying fractions by whole numbers (repeated addition, product rule, real-life examples) and fractions by fractions (visual area models, product rule, simplification, mixed fractions). Includes properties like product comparison based on factors and commutative property.

**Division of Fractions:** Introduces the concept of reciprocals, then details dividing fractions by whole numbers and fractions by fractions (using **Brahmagupta's formula:** multiply by reciprocal). Addresses mixed fractions and quotient properties.

**Problem Solving:** Integrates combined operations, word problems, geometric applications (area, volume), and fractional relations in figures.

**Enrichment & Conclusion:** Features historical context, highlighting ancient Indian contributions, and concludes with a chapter wrap-up including summary, review questions, and application projects.

### From History's Pages

"**Fractions have a long history! Ancient Egyptians** used them for building. But did you know that the way we add, subtract, multiply, and divide fractions today largely came from India. Around **800 BCE**, Indians used complex fractions for precise measurements. Later, in the $7^{th}$ century CE, the Indian mathematician **Brahmagupta** wrote down the rules for fraction operations we still use. This knowledge then traveled to the Arab world and eventually to Europe, showing how mathematical ideas develop globally, with India playing a huge role in understanding fractions!"

## Multiplication of a Fraction by a Whole Number

When we multiply a fraction by a whole number, it's like adding that fraction to itself a certain number of times. For example, if you walk $\frac{1}{4}$ km each day for 3 days, you've walked $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ km. This repeated addition is the same as multiplying $3 \times \frac{1}{4}$. This concept helps us find the total quantity when a fractional amount is repeated multiple times. We will explore how to perform this multiplication and understand its meaning.

**Sub-concepts to be covered**

1. Repeated Addition as Multiplication
2. Rule for Multiplying a Fraction by a Whole Number
3. Multiplying Mixed Fractions by a Whole Number

## Repeated Addition as Multiplication

Multiplication of a fraction by a whole number can be understood as repeated addition of the fraction. For instance, $5 \times \left(\frac{2}{7}\right)$ means adding $\left(\frac{2}{7}\right)$ five times: $\frac{2}{7}+\frac{2}{7}+\frac{2}{7}+\frac{2}{7}+\frac{2}{7}$. This visualizes the process and connects it to prior knowledge of whole number multiplication.

**Example**

If a painter uses $\frac{1}{3}$ of a can of paint for each wall, how much paint does he use for 3 walls? This is $3\times\left(\frac{1}{3}\right)=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{3}{3}$

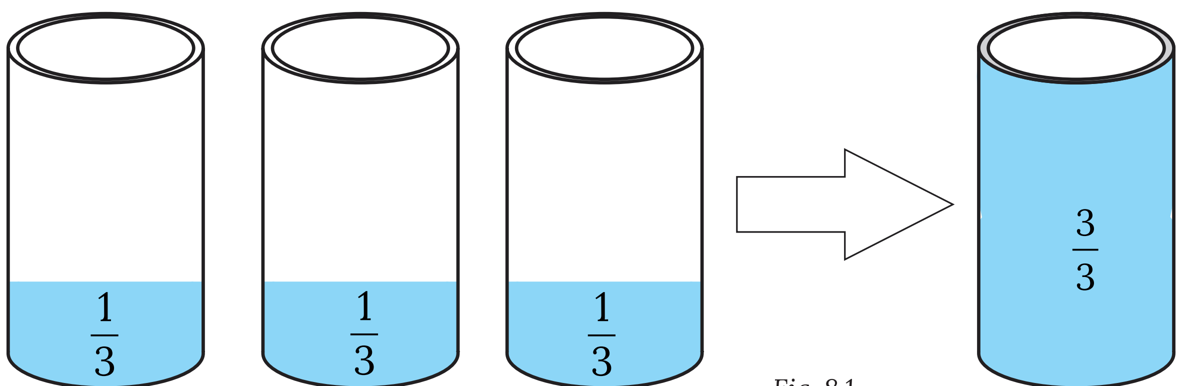



*Fig. 8.1*

## Rule for Multiplying a Fraction by a Whole Number

To multiply a fraction $\frac{a}{b}$ by a whole number (c), we multiply the numerator (a) by the whole number (c) and keep the denominator (b) the same. So, $c\times\left(\frac{a}{b}\right)=\left(\frac{c\times a}{b}\right)$. This rule is a shortcut derived from the repeated addition concept.

**Example:** Calculate $6 \times \frac{3}{8}$.

According to the rule, this is $\frac{6\times 3}{8}=\frac{18}{8}$.

### How this works?

Consider a fraction $\frac{a}{b}$, where 'a' is the numerator and 'b' is the denominator. This means we have 'a' parts out of 'b' equal parts of a whole.

When we multiply this fraction by a whole number c, we are essentially taking c groups of $\frac{a}{b}$.

For example, $3\times\left(\frac{2}{8}\right)$ means we have three groups, and each group contains $\frac{2}{8}$ of something.

Group 1: $\frac{2}{8}$ Group 2: $\frac{2}{8}$ *Fig. 8.2* Group 3: $\frac{2}{8}$

If we add these together, $\frac{2}{8}+\frac{2}{8}+\frac{2}{8}=\frac{(2+2+2)}{8}=\frac{6}{8}$.

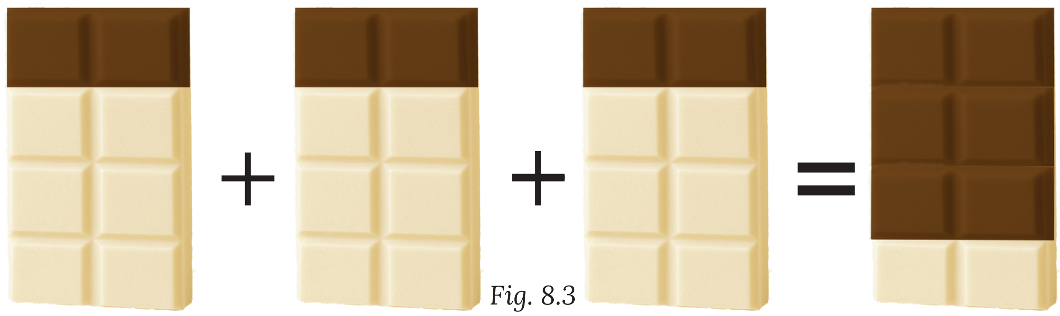


*Fig. 8.3*

Notice that the numerator 2 was added 3 times. This is equivalent to 3 × 2. The denominator, 8, which represents the total number of equal parts the whole is divided into, remains unchanged because we are still talking about eighths.

We are not changing the size of the parts, only how many of those parts we have.

**Example:** A baker divides a packet of 1 kg flour into 7 equal parts and uses only 2 parts out of it for one small cake. What fraction of flour will he need to bake 5 such cakes?

Solution: Baker divides one packet of flour into 7 equal parts and uses 2 parts out of it

So, fraction of flour used for one cake = $\frac{2}{7}$ kg

Number of cakes = 5

Total flour needed = $5\times\left(\frac{2}{7}\right)$ kg

Using the rule: $\frac{(5\times2)}{7}=\frac{10}{7}$ kg


*Fig. 8.4*

### Multiplying Mixed Fractions by a Whole Number

To multiply a mixed fraction by a whole number, first convert the mixed fraction into an improper fraction. Then, apply the rule for multiplying a fraction by a whole number.

**Example 1 :** Find the product of 4 and $2\frac{1}{5}$.

**Solution:** Convert $2\frac{1}{5}$ to an improper fraction: $\frac{2\times5+1}{5}=\frac{11}{5}$.

Now multiply: $4\times\frac{11}{5}=\frac{4\times11}{5}=\frac{44}{5}$

Convert back to a mixed fraction: $\frac{44}{5}=8\frac{4}{5}$.

### How this works?

When dealing with mixed fractions, say $c \times d\frac{e}{f}$, the first step is to convert the mixed fraction $d\frac{e}{f}$ into an improper fraction.

A mixed fraction $d\frac{e}{f}$ means d whole units plus $\frac{e}{f}$ of another unit.

To convert $d\frac{e}{f}$ to an improper fraction, we recognize that each whole unit d can be expressed as $d\times\frac{f}{f}$.

So, $d\frac{e}{f} = \frac{(d \times f)}{f} + \frac{e}{f} = \left(\frac{d \times f + e}{f}\right)$.

Once converted to an improper fraction, say $\frac{P}{Q}$, the multiplication proceeds as $c \times \left(\frac{P}{Q}\right) = \left(\frac{c \times P}{Q}\right)$.

- Always remember to simplify your final answer to its lowest terms and convert improper fractions back to mixed fractions if the context requires it.

## Multiplying Two Fractions

Now that we've mastered multiplying a fraction by a whole number, let's explore what happens when we multiply a fraction by another fraction. This is like finding a **part of a part**.

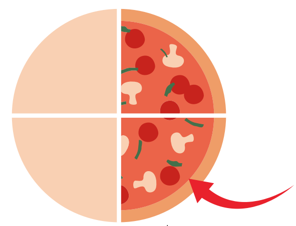


*Fig. 8.5*

For example, if you have $\frac{1}{2}$ of a pizza left, and you eat $\frac{1}{4}$ of that remaining half, how much of the original whole pizza did you eat? This is where multiplying two fractions comes in handy. We'll use visual models to make this concept clear and then discover the simple rule that governs it.

### Sub-concepts to be covered

1. Visualizing Fraction Multiplication (Area Model)
2. Rule for Multiplying Two Fractions
3. Simplifying to Lowest Form (Cancelling Common Factors)
4. Multiplying Mixed Fractions

### Visualizing Fraction Multiplication (Area Model)

The area model uses a unit square (a square with side length 1 unit) to represent the whole. Fractions are represented by dividing the square into rows and columns. The product of two fractions is shown by the overlapping shaded area. This provides a concrete, intuitive understanding of "**fraction of a fraction**."

### Example

To visualize $\frac{1}{2} \times \frac{1}{4}$: Draw a unit square. Divide it horizontally into 2 parts and shade $\frac{1}{2}$.

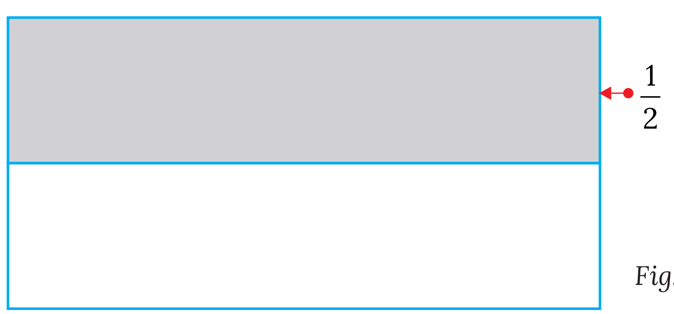


*Fig. 8.6*

Then, divide the shaded part vertically into 4 parts and shade $\frac{1}{4}$. The area where both shadings overlap represents the product.

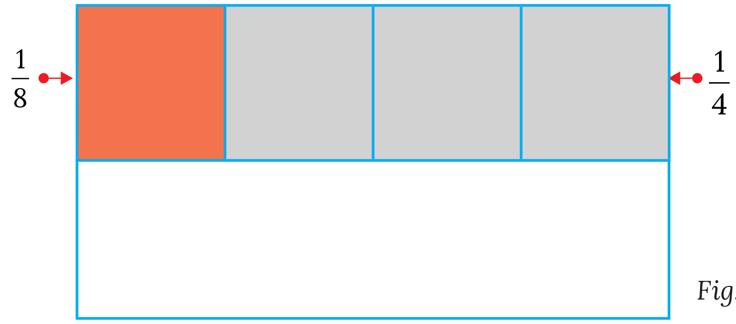



*Fig. 8.7*

Here, Shaded orange part represents $\frac{1}{4}$ part of $\frac{1}{2}$ part of the whole rectangle.

**Rule for Multiplying Two Fractions**

To multiply two fractions $\left(\frac{a}{b}\right)$ and $\left(\frac{c}{d}\right)$, multiply the numerators together and multiply the denominators together. So, $\left(\frac{a}{b}\right)\times\left(\frac{c}{d}\right)=\left(\frac{a\times c}{b\times d}\right)$. This rule is a direct consequence of the area model.

**Example:**

- Calculate $\frac{3}{5}\times\frac{2}{7}$.
- Using the rule: $\frac{3\times 2}{5\times 7}=\frac{6}{35}$.

**Example 2 :** Shade the area obtained after multiplying $\frac{3}{4}\times\frac{2}{3}$

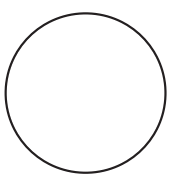


*Fig. 8.8*

**Solution:** First, consider the fraction $\frac{3}{4}$ and divide the circle into 4 equal parts and shade 3 out of them.

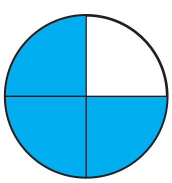


*Fig. 8.9*

Now, consider the fraction $\frac{2}{3}$ and divide the shaded part into 3 equal parts and shade 2 out of them.

Here, the shaded part is already divided into 3 equal parts, so we choose 2 parts out of them and shade them yellow

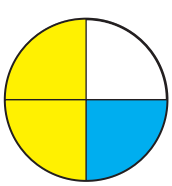


*Fig. 8.10*

The yellow part is representing the product of $\frac{3}{4}\times\frac{2}{3}=\frac{6}{12}=\frac{1}{2}$

### Simplifying to Lowest Form (Cancelling Common Factors)

Before multiplying, if a numerator and a denominator share a common factor, they can be divided by that factor. This "**cancelling**" simplifies the numbers, making the multiplication easier and often resulting in a product that is already in its lowest terms. This is based on the property that $\left(\frac{a \times c}{b \times c}\right) = \frac{a}{b}$.

**Example:** Multiply $\frac{4}{9} \times \frac{3}{8}$.

- Notice 4 and 8 share a common factor of 4. (4 ÷ 4 = 1, 8 ÷ 4 = 2)
- Notice 3 and 9 share a common factor of 3. (3 ÷ 3 = 1, 9 ÷ 3 = 3)
- So, $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

### Multiplying Mixed Fractions

To multiply mixed fractions, first convert all mixed fractions into improper fractions. Then, apply the rule for multiplying two fractions (multiplying numerators and denominators), simplifying by cancelling common factors if possible.

Finally, convert the improper fraction product back to a mixed fraction if required.

**Example:** Calculate $2\frac{1}{3} \times 1\frac{1}{2}$.

Convert into improper fractions: $2\frac{1}{3} = \frac{7}{3}; 1\frac{1}{2} = \frac{3}{2}$.

Multiply numerators and denominators: $\frac{7}{3} \times \frac{3}{2}$.

Cancel common factors, which is 3 here: $\frac{7}{1} \times \frac{1}{2} = \frac{7}{2}$.

Convert back to mixed fraction: $\frac{7}{2} = 3\frac{1}{2}$.

**Example 3 :** Calculate $\left(\frac{5}{12}\right) \times \left(\frac{8}{15}\right)$.

**Solution:** $\left(\frac{5}{12}\right) \times \left(\frac{8}{15}\right)$

**Look for common factors:**

5 and 15 share a common factor of 5. (5 ÷ 5 = 1, 15 ÷ 5 = 3)

8 and 12 share a common factor of 4. (8 ÷ 4 = 2, 12 ÷ 4 = 3)

Rewrite with cancelled terms: $\left(\frac{1}{3}\right) \times \left(\frac{2}{3}\right)$

Multiply new numerators: 1 × 2 = 2

Multiply new denominators: 3 × 3 = 9

Product = $\frac{2}{9}$

**Example 4 :** What is the area of the rectangular plot?

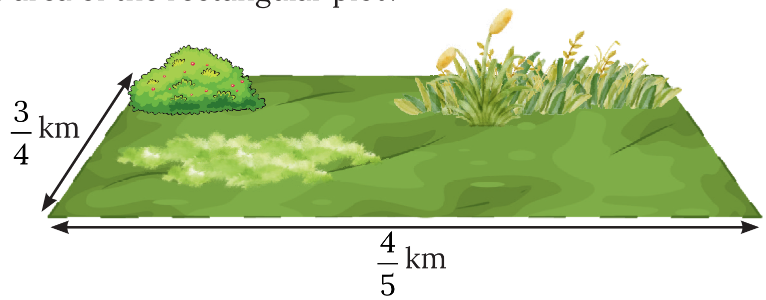



*Fig. 8.11*

**Solution:** Length = $\frac{4}{5}$ km

Breadth = $\frac{3}{4}$ km

Area of rectangular plot = Length × Breadth

$$= \left(\frac{4}{5} \times \frac{3}{4}\right) \text{ km}$$

Cancel common factor 4: $\left(\frac{1}{5}\right) \times \left(\frac{3}{1}\right) = \frac{3}{5}$ km²

**Example 5 :** Multiply $3\frac{1}{5}$ by $2\frac{1}{4}$.

**Solution:** Convert mixed fractions to improper fractions;

$$3\frac{1}{5} = \frac{3 \times 5 + 1}{5} = \frac{16}{5}$$

$$2\frac{1}{4} = \frac{2 \times 4 + 1}{4} = \frac{9}{4}$$

Multiply the improper fractions: $\left(\frac{16}{5}\right) \times \left(\frac{9}{4}\right)$

Look for common factors: 16 and 4 share a common factor of 4. (16 ÷ 4 = 4, 4 ÷ 4 = 1)

Rewrite with cancelled terms: $\left(\frac{4}{5}\right) \times \left(\frac{9}{1}\right)$

**Multiply new numerators:** 4 × 9 = 36

**Multiply new denominators:** 5 × 1 = 5

Product = $\frac{36}{5}$

Convert back to mixed fraction: $\frac{36}{5} = 7\frac{1}{5}$

## Knowledge Checkpoint

- What is $7 \times \frac{3}{10}$?
- A recipe calls for $\frac{1}{2}$ cup of sugar. If you triple the recipe, how much sugar do you need?
- Convert $1\frac{3}{4}$ to an improper fraction and then multiply it by 5.

## Activity

**Fraction Jumps on a Number Line**

**Objective:** To visually understand multiplication of a fraction by a whole number as repeated jumps on a number line.

**Materials:** Long strip of paper (or chalk on the floor), ruler, markers.

**Procedure:**

1. Draw a number line from 0 to 10 on the paper strip. Mark whole numbers clearly.
2. **Part 1:** Choose a unit fraction, e.g., $\frac{1}{3}$. Ask students to show $4 \times \frac{1}{3}$.

- Students start at 0 and make 4 jumps, each jump being $\frac{1}{3}$ unit long.
- Mark where each jump lands $\left(\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}\right)$.
- Discuss the final position $\left(\frac{4}{3} \text{ or } 1\frac{1}{3}\right)$.

3. **Part 2:** Choose a non-unit fraction, e.g., $\frac{2}{5}$. Ask students to show $3 \times \frac{2}{5}$.

- Students start at 0 and make 3 jumps, each jump being $\frac{2}{5}$ units long.
- Mark where each jump lands $\left(\frac{2}{5}, \frac{4}{5}, \frac{6}{5}\right)$.
- Discuss the final position $\left(\frac{6}{5} \text{ or } 1\frac{1}{5}\right)$.

4. **Part 3 (Challenge):** Use a mixed fraction, e.g., $2 \times 1\frac{1}{4}$.

- Convert $1\frac{1}{4}$ to $\frac{5}{4}$. Make 2 jumps of $\frac{5}{4}$.
- Discuss the result $\left(\frac{10}{4} \text{ or } 2\frac{1}{2}\right)$.

**Inquiry Questions:**

- How does this activity relate to repeated addition?
- What happens to the denominator when you multiply a fraction by a whole number?
- Can you predict the landing spot before making the jumps?

## Fact Flash

- Did you know that the word "**fraction**" comes from the Latin word "**fractio**," meaning "**to break**"?
- Did you know that the word "**cancel**" in mathematics means to remove common factors from the numerator and denominator of a fraction or expression?

## Do It Yourself

If you multiply a whole number by a proper fraction, will the product always be smaller than the whole number? Why or why not? Give examples.

## Mental Mathematics

1. $10 \times \frac{3}{5}$ 2. $18 \times \frac{1}{3}$ 3. $24 \times \frac{5}{6}$ 4. $7 \times \frac{1}{7}$ 5. $16 \times \frac{3}{6}$ 6. $30 \times \frac{8}{5}$

## Key Terms

- **Numerator:** The top number in a fraction, indicating how many parts are being considered.
- **Denominator:** The bottom number in a fraction, indicating the total number of equal parts a whole is divided into.
- **Area Model:** A visual representation used to demonstrate multiplication, especially of fractions, by showing the area of a rectangle.
- **Unit Square:** A square with side lengths of 1 unit, used as a visual representation of a whole in the area model.
- **Common Factor:** A number that divides two or more numbers without leaving a remainder.
- **Cancelling**: The process of dividing a numerator and a denominator by a common factor before multiplying, to simplify calculations.

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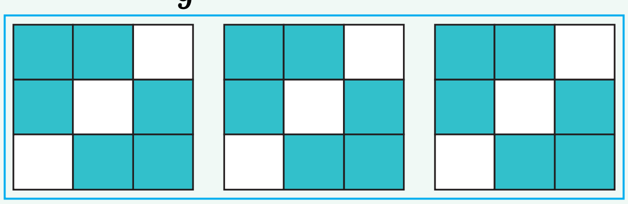
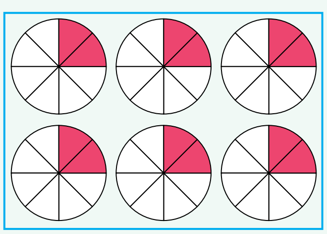
## Exercise 8.1

**1. How many $\frac{2}{9}$ parts are shaded in the below image?**

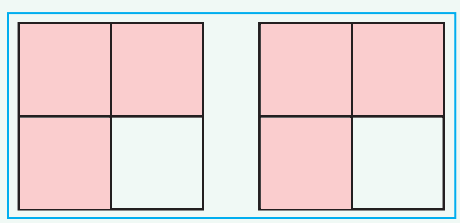


**2. Write the multiplicative sentence of the given models**

a)



b)



**3. Convert the following Repeated Addition into Multiplication Sentences.**

a) $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$

b) $\frac{2}{7}+\frac{2}{7}+\frac{2}{7}$

c) $\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}$

d) $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$

e) $\frac{3}{10}+\frac{3}{10}+\frac{3}{10}+\frac{3}{10}+\frac{3}{10}+\frac{3}{10}+\frac{3}{10}$

f) $\frac{4}{11}+\frac{4}{11}+\frac{4}{11}+\frac{4}{11}+\frac{4}{11}$

g) $\frac{7}{15}+\frac{7}{15}+\frac{7}{15}$

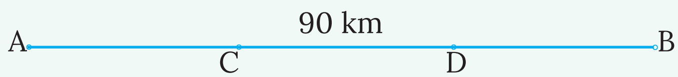
4. **For each problem, calculate the product. Remember to express your final answer as a mixed number in its lowest terms.**

a) $2\frac{1}{2}\times1\frac{3}{5}$ b) $4\frac{2}{3}\times1\frac{1}{4}$ c) $3\frac{3}{4}\times2\frac{2}{5}$ d) $1\frac{5}{6}\times2\frac{1}{3}$

e) $5\frac{1}{2}\times2\frac{3}{4}$ f) $2\frac{7}{8}\times1\frac{1}{3}$ g) $4\frac{1}{5}\times3\frac{1}{7}$ h) $1\frac{3}{10}\times2\frac{5}{13}$

5. A book has 240 pages. Rohan has read $\frac{3}{4}$ of the book. Of the pages he has read, $\frac{2}{3}$ are from the first section. How many pages are in the first section that Rohan has read?

6. Ram is travelling in a car which is moving at a constant speed. Points A, B, C and D are at equal distances. Ram has travelled from A to C in one hour.




a) How many kilometers has Ram covered in one hour?

b) In how much time will Ram reach the point B?

7. A bottle contains 2 liters of juice. If you drink $\frac{1}{4}$ of the juice from the bottle every day, how much juice (in liters) do you drink in 3 days? (This requires two steps: finding the daily amount, then multiplying by the number of days).

## Product Relationship (Greater/Less Than)

When we multiply whole numbers (other than 1), the product is usually larger than the numbers being multiplied (e.g., 3 × 5 = 15, where 15 > 3 and 15 > 5). But with fractions, things can be different! Sometimes the product is smaller than the original numbers, sometimes larger, and sometimes in between. Understanding these relationships helps us estimate answers and check if our calculations make sense.

**Sub-concepts to be covered**

1. Product when both numbers are greater than 1
2. Product when both numbers are between 0 and 1 (proper fractions)
3. Product when one number is between 0 and 1 and the other is greater than 1

### Product when both numbers are greater than 1

When you multiply two numbers, both of which are greater than 1 (this includes whole numbers greater than 1, or improper fractions), the product will always be greater than both of the original numbers. This is consistent with whole number multiplication.

**Examples:** a) 2 × 3 = 6 (6 > 2 and 6 > 3)


b) $\left(\frac{5}{2}\times\frac{7}{3}\right)=\frac{35}{6}=5\frac{5}{6}\left(\frac{5}{2}\left(=2\frac{1}{2}\right)<5\frac{5}{6}\text{ and }\frac{7}{3}\left(=2\frac{1}{3}\right)<5\frac{5}{6}\right)$

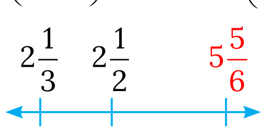




*Fig. 8.12*

### Product when both numbers are between 0 and 1 (proper fractions)

When you multiply two proper fractions (numbers between 0 and 1), the product will always be smaller than both of the original fractions. This is because you are taking a "**part of a part,**" which naturally results in a smaller part of the original whole.

**Example:** a) $\left(\frac{1}{2}\right)\times\left(\frac{1}{4}\right)=\frac{1}{8}\left(\frac{1}{8}<\frac{1}{2} \text{ and } \frac{1}{8}<\frac{1}{4}\right)$

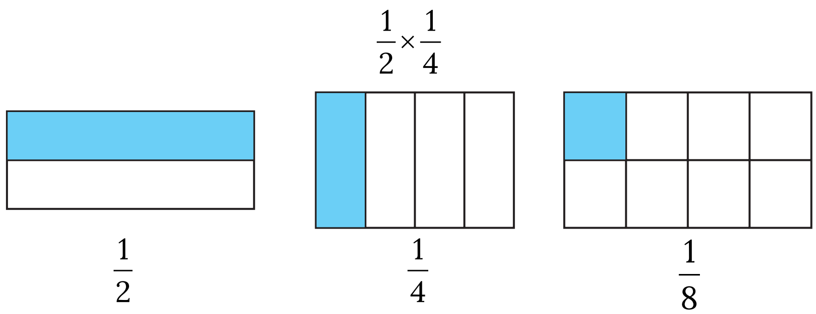



*Fig. 8.13*

b) $\left(\frac{2}{3}\right)\times\left(\frac{3}{5}\right)=\frac{6}{15}=\frac{2}{5}\left(\frac{2}{5}<\frac{2}{3} \text{ and } \frac{2}{5}<\frac{3}{5}\right)$

### Product when one number is between 0 and 1 and the other is greater than 1

When you multiply a proper fraction (between 0 and 1) by a number greater than 1 (a whole number or an improper fraction), the product will be greater than the proper fraction but smaller than the number greater than 1. It will fall between the two original numbers.

**Example:** a) $\left(\frac{1}{2}\right)\times 6=3\left(3>\frac{1}{2} \text{ but } 3<6\right)$

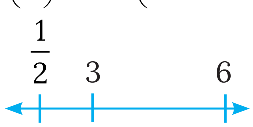



*Fig. 8.14*

b) $\left(\frac{3}{4}\right)\times\left(\frac{8}{3}\right)=2\left(2>\frac{3}{4} \text{ but } 2<\frac{8}{3}\left(2\frac{2}{3}\right)\right)$

**Example 6 :** Without calculating, state whether the product of $\frac{3}{4}\times\frac{5}{6}$ will be greater than, less than, or equal to $\frac{3}{4}$.

**Solution:** When multiplying two proper fractions, the product is always smaller than both original fractions. Since $\frac{5}{6}$ is a proper fraction (less than 1), multiplying $\frac{3}{4}$ by $\frac{5}{6}$ will result in a product smaller than $\frac{3}{4}$.

**Example 7 :** Without calculating, state whether the product of $2\frac{1}{3}\times 4$ will be greater than, less than, or equal to 4.

**Solution:** Both numbers ($2\frac{1}{3}$ and $4$) are greater than 1. When multiplying two numbers greater than 1, the product is always greater than both original numbers. Therefore, the product will be greater than 4.

**Example 8 :** Without calculating, state whether the product of $\frac{7}{8}\times1\frac{1}{2}$ will be greater than, less than, or between $\frac{7}{8}$ and $1\frac{1}{2}$.

**Solution:** When multiplying a proper fraction by a number greater than 1, the product will be greater than the proper fraction but less than the number greater than 1. So, the product will be between $\frac{7}{8}$ and $1\frac{1}{2}$.

**Example 9 :** A student claims that multiplying any two fractions always results in a smaller number. Is this true? Justify your answer with an example.

**Solution:** No, this statement is false. While it's true for proper fractions, it's not true for all fractions. For example, if we multiply two improper fractions, the product will be larger.

Consider $\left(\frac{5}{3}\right)\times\left(\frac{7}{4}\right)$. Both are improper fractions (greater than 1).

Product = $\left(\frac{5}{7}\right)\times\left(\frac{3}{4}\right)=\frac{35}{12}=2\frac{11}{12}$

Here, $2\frac{11}{12}$ is greater than $\frac{5}{3}\left(1\frac{2}{3}\right)$ and $\frac{7}{4}\left(1\frac{3}{4}\right)$.

## Activity

**Fraction Product Exploration**

**Objective:** To empirically discover the relationship between the product of fractions and the original factors.

**Materials:** Index cards, calculator (optional).

**Procedure:**

1. Divide students into small groups.
2. Provide each group with a set of index cards. On each card, write a multiplication problem involving fractions $\left(\text{e.g.,}\frac{1}{2}\times\frac{1}{4},\ \frac{5}{3}\times\frac{2}{1},\ \frac{2}{5}\times\frac{7}{2},\ 1\frac{1}{2}\times2\frac{1}{3},\text{etc.}\right)$. Ensure a mix of all three cases (proper × proper, improper × improper/whole, proper × improper/whole).
3. For each card, students first predict the relationship: Will the product be greater than both, smaller than both, or between the two factors? They should write down their prediction.
4. Then, they calculate the actual product.
5. Finally, they verify their prediction by comparing the product with the original factors.
6. After completing all cards, groups discuss their findings and summarize the rules they observed for each case.

**Inquiry Questions:**

- Were your predictions always correct? If not, what did you learn?
- Can you formulate a general rule for each type of multiplication (proper × proper, improper × improper, proper × improper)?
- Why do you think these relationships occur?

## Knowledge Checkpoint

- Will the product of $\frac{1}{5} \times \frac{1}{6}$ be greater than or less than $\frac{1}{5}$?
- Will the product of $3 \times \frac{4}{3}$ be greater than or less than 3?
- Will the product of $\frac{2}{3} \times \frac{5}{2}$ be greater than, less than, or between $\frac{2}{3}$ and $\frac{5}{2}$?
- Will the product of $3 \times \frac{4}{6}$ be greater than or less than 3?
- Will the product of $\frac{4}{6} \times \frac{5}{5}$ be greater than, less than, or between $\frac{4}{6}$ and $\frac{6}{6}$ $\frac{2}{3}$ ?

## Key Terms

- **Proper Fraction:** A fraction where the numerator is smaller than the denominator (value between 0 and 1).
- **Improper Fraction:** A fraction where the numerator is greater than or equal to the denominator (value greater than or equal to 1).
- **Scaling:** The effect of multiplication on a number, making it larger or smaller.

## Fact Flash

- This concept is related to "**scaling factors**" used in geometry and computer graphics to enlarge or shrink images.

## Do It Yourself

- Can you find two fractions (not equal to 1) whose product is exactly 1? What is special about these fractions?
- If you multiply a number by itself, and the product is smaller than the original number, what kind of number must it be?

## Mental Mathematics

- If you multiply 10 by 2, will the answer be greater than 10, less than 10, or equal to 10?
- If you multiply 10 by $\frac{1}{2}$, will the answer be greater than 10, less than 10, or equal to 10?
- If you multiply 5 by 1, will the answer be greater than 5, less than 5, or equal to 5?
- If you multiply $\frac{1}{4}$ by 2, will the answer be greater than $\frac{1}{4}$, less than $\frac{1}{4}$, or equal to $\frac{1}{4}$?

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## Exercise 8.2

**1. Fill in the blank with "greater than", "less than", or "equal to".**

a) When you multiply 7 by $\frac{1}{2}$, the product will be ______________ 7.

b) The product of $\frac{1}{3}$ and $\frac{1}{2}$ will be ______________ $\frac{1}{3}$.

c) If you multiply 25 by $\frac{9}{8}$, the result will be ______________ 25.

d) For any number n greater than 1, if you multiply n by $2\frac{1}{5}$, the product will always be _______ n.

e) When a number is multiplied by a proper fraction (a fraction between 0 and 1), the product will always be ______________ the original number.

**2. Consider the product of 15 and $\frac{3}{5}$.**

a) Will the product be greater than 15, less than 15, or equal to 15?

b) Will the product be greater than $\frac{3}{5}$, less than $\frac{3}{5}$, or equal to $\frac{3}{5}$?

**3. A car is travelling at the highest speed limit on the road. If it travels for $2\frac{1}{2}$ hours, will the total distance covered be greater than, less than, or equal to 70 km?**


4. You have a full glass of water. If you drink $\frac{1}{3}$ of the glass, and then someone else drinks $\frac{1}{2}$ of what's left, will the total amount of water consumed by both of you be more or less than the initial $\frac{1}{3}$ you drank?

**(Hint:** Think about what fraction of the original glass is left after your drink**).**

**5.** A baker uses $2\frac{1}{5}$ cups of sugar for a large cake recipe. If they decide to make a smaller cake that equires only $\frac{3}{5}$ times the amount of sugar, will they use more or less than $2\frac{1}{5}$ cups of sugar? Justify your answer.

**6. A local restaurant sells a popular sandwich. Last month, they sold 350 sandwiches. This month, due to a special promotion, they expect to sell $\frac{6}{5}$ times the number of sandwiches sold last month. Without calculating the exact number, answer the following questions:**

a) Will the restaurant sell more or fewer than 350 sandwiches this month?

b) If the sales decrease by $\frac{2}{3}$ times the number of sandwiches sold last month, then will they sell more or fewer than 350 sandwiches now?

## Division of Fractions

Division is about splitting a quantity into equal groups or finding out how many times one quantity fits into another. When we divide fractions, it's no different! Imagine you have a certain amount of fabric, and you want to make smaller pieces of a specific fractional size. How many pieces can you make? Or, if you know a fractional part of a whole, how do you find the whole?

Division of fractions helps us answer these kinds of questions.

Get ready to learn the powerful concept of reciprocals, which will make fraction division surprisingly simple!

### Reciprocal of a Fraction

Before we dive into dividing fractions, we need to understand a very important concept called the reciprocal (also known as the multiplicative inverse).

The reciprocal of a number is what you multiply that number by to get a product of 1. For example, what do you multiply 2 by to get 1? The answer is $\frac{1}{2}$. So, $\frac{1}{2}$ is the reciprocal of 2.

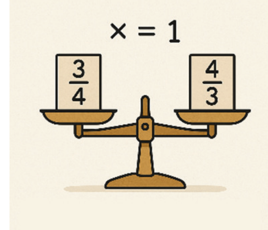


*Fig. 8.15*

**Sub-concepts to be covered**

1. Definition of Reciprocal
2. Finding the Reciprocal of a Fraction
3. Finding the Reciprocal of a Whole Number
4. Finding the Reciprocal of a Mixed Fraction

### Definition of Reciprocal

The reciprocal of a non-zero number is the number that, when multiplied by the original number, results in a product of 1. If a number is **x**, its reciprocal is $\frac{1}{x}$ .

**Example:** The reciprocal of 5 is $\frac{1}{5}$, because $5 \times \left(\frac{1}{5}\right) = 1$.

- The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, because $\left(\frac{3}{4}\right) \times \left(\frac{4}{3}\right) = 1$.

### Finding the Reciprocal of a Fraction

To find the reciprocal of a fraction $\left(\frac{a}{b}\right)$, simply swap its numerator and denominator. The reciprocal is $\left(\frac{b}{a}\right)$.

**Example:**
- Reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$ as $\frac{2}{7} \times \frac{7}{2} = 1$.
- Reciprocal of $\frac{9}{11}$ is $\frac{11}{9}$ as $\frac{9}{11} \times \frac{11}{9} = 1$.

### Finding the Reciprocal of a Whole Number

To find the reciprocal of a whole number (c), first express it as a fraction $\left(\frac{c}{1}\right)$. Then, swap the numerator and denominator. The reciprocal is $\left(\frac{1}{c}\right)$.

**Example:** Reciprocal of 8 is $\frac{1}{8}\left(\text{since } 8 = \frac{8}{1}\right)$.

- Reciprocal of 1 is 1 $\left(\text{since } 1 = \frac{1}{1}\right)$.

### Finding the Reciprocal of a Mixed Fraction

To find the reciprocal of a mixed fraction, first convert the mixed fraction into an improper fraction. Then, find the reciprocal of that improper fraction by swapping its numerator and denominator.

**Example:** Reciprocal of $2\frac{1}{3}$:

- Convert to improper: $2\frac{1}{3} = \frac{2 \times 3 + 1}{3} = \frac{7}{3}$.
- Reciprocal of $\frac{7}{3}$ is $\frac{3}{7}$.

**Some more examples:**

**Example 10 :** Find the reciprocal of $\frac{5}{9}$.

**Solution:** Swap the numerator and denominator.

Reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$.

**Example 11 :** What is the reciprocal of 12?

**Solution:** Write 12 as a fraction: $\frac{12}{1}$.

Swap the numerator and denominator: $\frac{1}{12}$.

**Example 12 :** Find the reciprocal of $3\frac{2}{5}$.

**Solution:** Convert $3\frac{2}{5}$ to an improper fraction: $\frac{3 \times 5 + 2}{5} = \frac{17}{5}$.

Swap the numerator and denominator of $\frac{17}{5} : \frac{5}{17}$.

**Example 13 :** Is 0.25 the reciprocal of 4? Justify your answer.

**Solution:** Convert 0.25 to a fraction: $0.25 = \frac{25}{100} = \frac{1}{4}$.

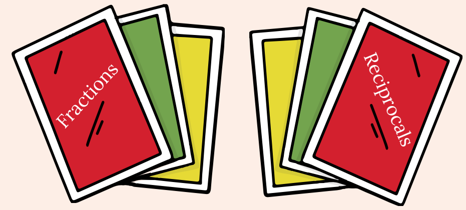
Now, check if $\left(\frac{1}{4}\right) \times 4 = 1$. Yes, $\left(\frac{1}{4}\right) \times 4 = \frac{4}{4} = 1$.

## Activity

**Reciprocal Card Match**

**Objective:** To practice identifying reciprocals quickly and accurately.

**Materials:** Index cards. On half the cards, write various numbers (whole numbers, proper fractions, improper fractions, mixed fractions). On the other half, write their corresponding reciprocals.




**Procedure:**

1. Shuffle all the cards and spread them face down on a table.
2. Students take turns flipping over two cards.
3. If the two cards form a reciprocal pair $\left(\text{e.g.,} \frac{3}{5} \text{ and } \frac{5}{3}\right)$, the student keeps the pair and takes another turn.
4. If they are not a pair, the cards are flipped back over.
5. The game continues until all pairs are found.

**Inquiry Questions:**

- What strategy did you use to find pairs quickly?
- Did you notice any patterns between the type of number and its reciprocal (e.g., proper fraction reciprocal is improper)?
- Can you create a new pair of reciprocal cards for a friend?

## Knowledge Checkpoint

- What is the reciprocal of $\frac{7}{10}$?
- Find the reciprocal of 6.
- What is the reciprocal of $1\frac{3}{4}$?

## Key Terms

- **Reciprocal:** A number that, when multiplied by a given number, results in a product of 1. Also known as multiplicative inverse.
- **Multiplicative Inverse:** Another term for reciprocal.
- **Unit Fraction:** A fraction with a numerator of 1 $\left(\text{e.g.,} \frac{1}{2}, \frac{1}{7}\right)$.

## Do It Yourself

- If a number is greater than 1, what can you say about its reciprocal?
- If a number is between 0 and 1, what can you say about its reciprocal?
- Can you think of a real-life situation where finding the reciprocal of a number would be useful?

## Fact Flash

- The only numbers that are their own reciprocals are 1 and -1.
- The concept of reciprocals is very old, but the term "**reciprocal**" itself became common in mathematics around the $17^{th}$ century.
- In some ancient cultures, division was often thought of as multiplication by a reciprocal, even if they didn't use the exact term "**reciprocal**."

## Mental Mathematics

- What is the reciprocal of three-fifths?
- If you multiply a fraction by its reciprocal, what is the product?
- What is the reciprocal of the whole number 7?
- What is the reciprocal of one and a half $1\frac{1}{2}$?
- If a fraction has a reciprocal of $\frac{2}{3}$, what was the original fraction?

Gap Analyzer™ Homework

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## Exercise 8.3

**1. Fill in the blank with the correct word or number.**

a) The product of a number and its reciprocal is always ____________.

b) The reciprocal of a proper fraction is always an ____________ fraction.

c) The number ____________ does not have a reciprocal.

d) The reciprocal of the whole number 7 is _________.

e) Before finding the reciprocal of a mixed fraction like $3\frac{2}{11}$, you must first convert it into an _________ fraction.

**2. Find the reciprocal of each given fraction.**

a) 23  b) $\frac{5}{9}$  c) $\frac{13}{7}$  d) $9\frac{2}{3}$  e) $\frac{14}{11}$

f) $\frac{15}{2}$  g) $1\frac{7}{8}$  h) $2\frac{4}{7}$  i) $\frac{1}{20}$  j) 15

3. What is the reciprocal of the fraction representing one slice of the given pizza?

4. What will be the reciprocal of the product obtained by multiplying $\frac{2}{7} \times \frac{3}{8}$?

5. Anthony has 28 candies with him. He is eating 3 candies after each hour. What will be the reciprocal of the fraction of candies left after 4 hours?

6. A tailor has a piece of fabric that is $\frac{7}{10}$ of a meter long. She needs $\frac{1}{2}$ of the cloth piece to make a decorative piece. What is the reciprocal of the fraction of cloth she used for making decorative piece?

## Division of a Fraction

Fig. 8.16

Imagine you have $\frac{3}{4}$ of a cake, and you want to share it equally among 3 friends. How much cake does each friend get?

This is a division problem: $\frac{3}{4} \div 3$.

**Sub-concepts to be covered**

1. Relating Division to Multiplication by Reciprocal
2. Rule for Dividing a Fraction by a Whole Number
3. Dividing Mixed Fractions by a Whole Number

### Relating Division to Multiplication by Reciprocal

Division is the inverse operation of multiplication. If a ÷ b = c, then c × b = a. Using this, we can rewrite $\left(\frac{a}{b} \div c\right)$ as $\left(\frac{a}{b} \times \frac{1}{c}\right)$. This is because dividing by c is the same as multiplying by its reciprocal, $\left(\frac{1}{c}\right)$.

**Example:** $\left(\frac{1}{2} \div 4\right)$: This means dividing $\frac{1}{2}$ into 4 equal parts. Visually, if you have half a pizza and cut it into 4 pieces, each piece is $\frac{1}{8}$ of the whole pizza. Mathematically, $\left(\frac{1}{2}\right) \times \left(\frac{1}{4}\right) = \frac{1}{8}$.

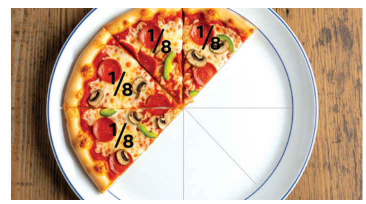



Fig. 8.17

### Rule for Dividing a Fraction by a Whole Number

To divide a fraction $\left(\frac{a}{b}\right)$ by a whole number (c), multiply the fraction by the reciprocal of the whole number. So, $\left(\frac{a}{b}\right) \div c = \left(\frac{a}{b}\right) \times \left(\frac{1}{c}\right) = \frac{a}{b \times c}$.

**Example:** Calculate $\frac{5}{6} \div 2$.

- Reciprocal of 2 is $\frac{1}{2}$.
- $\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$.

### Dividing Mixed Fractions by a Whole Number

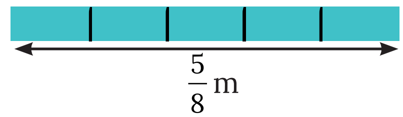
To divide a mixed fraction by a whole number, first convert the mixed fraction into an improper fraction. Then, apply the rule for dividing a fraction by a whole number (multiply by the reciprocal of the whole number).

**Example:** Find the quotient of $4\frac{1}{2} \div 3$.

- Convert $4\frac{1}{2}$ to an improper fraction: $\frac{4 \times 2 + 1}{2} = \frac{9}{2}$.
- Reciprocal of 3 is $\frac{1}{3}$.

- $\frac{9}{2} \times \frac{1}{3} = \frac{9 \times 1}{2 \times 3} = \frac{9}{6}$.
- Simplify: $\frac{9}{6} = \frac{3}{2} = 1\frac{1}{2}$.

*Fig. 8.18*




**Example 14 :** What is the length of each piece of the given ribbon?

**Solution:** Total ribbon length = $\frac{5}{8}$ meter

Number of pieces = 5

Length of each piece = $\left(\frac{5}{8}\right) \div 5$

Reciprocal of 5 is $\frac{1}{5}$.

$\left(\frac{5}{8}\right) \times \left(\frac{1}{5}\right) = \left(\frac{5}{1} \times \frac{8}{5}\right)$

Cancel common factor 5: $\left(\frac{1 \times 1}{8 \times 1}\right) = \frac{1}{8}$ meter

**Example 15 :** Calculate $\frac{12}{15} \div 4$.

**Solution:** $\frac{12}{15} \div 4$

Reciprocal of 4 is $\frac{1}{4}$.

$\left(\frac{12}{15}\right) \times \left(\frac{1}{4}\right)$

Cancel common factor 4 (between 12 and 4): $\left(\frac{3}{15}\right) \times \left(\frac{1}{1}\right) = \frac{3}{15}$

Simplify $\frac{3}{15}$ by dividing by 3: $\frac{1}{5}$

**Example 16 :** A baker has $2\frac{1}{4}$ kg of dough. He wants to make 3 identical loaves of bread. How much dough will be in each loaf?

**Solution:** Total dough = $2\frac{1}{4}$ kg

Number of loaves = 3

*Fig. 8.19*

Convert $2\frac{1}{4}$ to an improper fraction: $\frac{2 \times 4 + 1}{4} = \frac{9}{4}$ kg.

Dough per loaf = $\left(\frac{9}{4}\right) \div 3$

Reciprocal of 3 is $\frac{1}{3}$.

$\left(\frac{9}{4}\right)\times\left(\frac{1}{3}\right)$

Cancel common factor 3 (between 9 and 3): $\left(\frac{3}{4}\right)\times\left(\frac{1}{1}\right)=\frac{3}{4}$ kg

## Division of a Fraction by Another Fraction

What if you have $\frac{5}{6}$ of a large cake, and you want to know how many $\frac{1}{12}$ sized slices you can cut from it? This is a division problem: $\left(\frac{5}{6}\right)\div\left(\frac{1}{12}\right)$ or, if you know that $\frac{2}{3}$ of a journey is 10 km, how long is the whole journey? This also involves dividing a fraction by a fraction. The "invert and multiply" rule, using reciprocals, is the universal method for solving these problems.

### Sub-concepts to be covered

1. Rule for Dividing a Fraction by a Fraction
2. Dividing Mixed Fractions by Fractions
3. Quotient Relationship (Greater/Less Than Dividend)

### Rule for Dividing a Fraction by a Fraction

To divide a fraction $\left(\frac{a}{b}\right)$ by another fraction $\left(\frac{c}{d}\right)$, multiply the first fraction (the dividend) by the reciprocal of the second fraction (the divisor). So, $\left(\frac{a}{b}\right)\div\left(\frac{c}{d}\right)=\left(\frac{a}{b}\right)\times\left(\frac{d}{c}\right)=\left(\frac{a\times d}{d\times c}\right)$.

**Example:** Calculate $\left(\frac{3}{4}\right)\div\left(\frac{1}{8}\right)$.

Reciprocal of $\left(\frac{1}{8}\right)$ is $\left(\frac{8}{1}\right)$.

$\left(\frac{3}{4}\right)\times\left(\frac{8}{1}\right)=\left(\frac{3\times 8}{4\times 1}\right)=\frac{24}{4}=6$.

- This means there are 6 one-eighths in three-quarters.

### Dividing Mixed Fractions by Fractions

To divide mixed fractions by other fractions (or mixed fractions), first convert all mixed fractions into improper fractions. Then, apply the rule for dividing a fraction by a fraction (multiply the first improper fraction by the reciprocal of the second improper fraction). Simplify the result.

**Example:** Find the quotient of $2\frac{1}{2}\div 1\frac{1}{4}$.

- Convert: $2\frac{1}{2}=\frac{5}{2};\ \ 1\frac{1}{4}=\frac{5}{4}$.
- $\frac{5}{2}\div\frac{5}{4}$
- Reciprocal of $\frac{5}{4}$ is $\frac{4}{5}$.
- $\left(\frac{5}{2}\right)\times\left(\frac{4}{5}\right)$

- Cancel 5s and 2s/4s: $\left(\frac{1}{1}\right)\times\left(\frac{2}{1}\right)=2$.

### Quotient Relationship (Greater/Less Than Dividend)

Unlike whole number division where the quotient is usually smaller than the dividend, with fractions, the quotient can be larger or smaller than the dividend, depending on the divisor.

If the divisor is a proper fraction (between 0 and 1): The quotient will be greater than the dividend. (e.g., $6 \div \frac{1}{2} = 12$; $12 > 6$. You're asking how many halves are in 6 wholes.)

$$6 \div \frac{1}{2} = 12$$

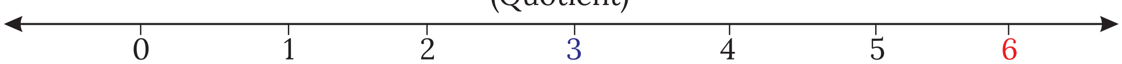
*Fig. 8.20*


If the divisor is an improper fraction (greater than 1) or a whole number greater than 1: The quotient will be less than the dividend. (e.g., $6 \div 2 = 3$; $3 < 6$. You're splitting 6 into 2 groups.)

$$6 \div 2 = 3$$

(Quotient)

*Fig. 8.21*




**Examples:** a) $\left(\frac{1}{2}\right)\div\left(\frac{1}{4}\right)=2\left(2>\frac{1}{2}\right)$

b) $\left(\frac{3}{4}\right)\div\left(\frac{3}{2}\right)=\frac{1}{2}\left(\frac{1}{2}<\frac{3}{4}\right)$

**Example 17 :** How many pieces of wire, each $\frac{1}{5}$ meter long, can be cut from the given wire?

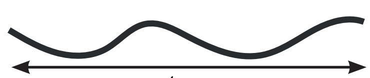


*Fig. 8.22*

$\frac{4}{5}$ m

**Solution:** Total length = $\frac{4}{5}$ meter

Length of each piece = $\frac{1}{5}$ meter

Number of pieces = $\left(\frac{4}{5}\right)\div\left(\frac{1}{5}\right)$

Reciprocal of $\frac{1}{5}$ is $\frac{5}{1}$.

$\left(\frac{4}{5}\right)\times\left(\frac{5}{1}\right)$

Cancel common factor 5: $\left(\frac{4}{1}\right)\times\left(\frac{1}{1}\right)=4$

**Example 18 :** Calculate $\left(\frac{7}{9}\right)\div\left(\frac{14}{3}\right)$.

**Solution:** $\left(\frac{7}{9}\right) \div \left(\frac{14}{3}\right)$

Reciprocal of $\frac{14}{3}$ is $\frac{3}{14}$.

$\left(\frac{7}{9}\right) \times \left(\frac{3}{14}\right)$

Cancel common factor 7 (between 7 and 14): $\left(\frac{1}{9}\right) \times \left(\frac{3}{2}\right)$

Cancel common factor 3 (between 3 and 9): $\left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right) = \frac{1}{6}$

**Example 19 :** A baker uses $1\frac{1}{2}$ cups of sugar for a batch of cookies. If he has $4\frac{1}{2}$ cups of sugar, how many batches of cookies can he make?

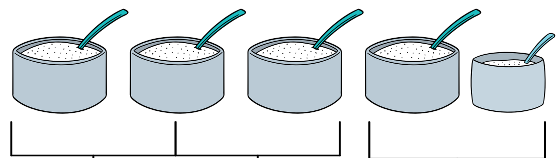


*Fig. 8.23*

**Solution:** Total sugar = $4\frac{1}{2}$ cups

Sugar per batch = $1\frac{1}{2}$ cups

**Convert to improper fractions:**

$4\frac{1}{2} = \frac{4 \times 2 + 1}{2} = \frac{9}{2}$

$1\frac{1}{2} = \frac{1 \times 2 + 1}{2} = \frac{3}{2}$

Number of batches = $\left(\frac{9}{2}\right) \div \left(\frac{3}{2}\right)$

Reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$.

$\left(\frac{9}{2}\right) \times \left(\frac{2}{3}\right)$

Cancel common factor 2 and 3: $\left(\frac{3}{1}\right) \times \left(\frac{1}{1}\right) = 3$

**Example 20 :** If $\frac{3}{5}$ of a tank is filled with water, and this amount is 120 litres, what is the total capacity of the tank?

**Solution:** Let the total capacity be 'x' litres.

$\frac{3}{5}$ of x = 120

$\frac{3}{5} \times x = 120$

$x = 120 \div \frac{3}{5}$

Reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

$x = 120 \times \frac{5}{3}$

**Cancel common factor 3 (between 120 and 3):** 40 × 5 = 200 litres

**Example 21 :** Will the quotient of $\frac{1}{4} \div \frac{1}{8}$ be greater than or less than $\frac{1}{4}$? Justify.

**Solution:** The divisor is $\frac{1}{8}$, which is a proper fraction (less than 1). When the divisor is less than 1, the quotient is greater than the dividend.

**Calculation:** $\left(\frac{1}{4}\right) \div \left(\frac{1}{8}\right) = \left(\frac{1}{4}\right) \times \left(\frac{1}{8}\right) = \frac{8}{4} = 2$

Here, 2 is indeed greater than $\frac{1}{4}$.

## Activity

**"How Many Fit In?" Game**

**Objective:** To understand fraction division as finding how many times one fractional quantity fits into another.

**Materials:** Sets of fraction circles or fraction tiles (representing wholes, halves, thirds, quarters, etc.).

**Procedure:**

1. Give each group a set of fraction circles/tiles.
2. Pose division problems, e.g., "How many $\frac{1}{4}$s are in $\frac{1}{2}$?"
3. Students take a $\frac{1}{2}$ circle and try to cover it completely with $\frac{1}{4}$ pieces. They count how many $\frac{1}{4}$ pieces are needed (2). So, $\frac{1}{2} \div \frac{1}{4} = 2$.
4. Repeat with other examples:

- "How many $\frac{1}{6}$s are in $\frac{2}{3}$?" (Take $\frac{2}{3}$, cover with $\frac{1}{6}$s -> 4 pieces. So, $\frac{2}{3} \div \frac{1}{6} = 4$.)
- "How many $\frac{1}{2}$s are in $1\frac{1}{2}$?" (Take $1\frac{1}{2}$, cover with $\frac{1}{2}$s -> 3 pieces. So, $1\frac{1}{2} \div \frac{1}{2} = 3$.)
- "How many $\frac{3}{4}$s are in $1\frac{1}{2}$?" (Take $1\frac{1}{2}$, cover with $\frac{3}{4}$s -> 2 pieces. So, $1\frac{1}{2} \div \frac{3}{4} = 2$.)

**Inquiry Questions:**

- When did you get a whole number as an answer? When did you get a fraction?
- How does the size of the "fitting" piece (divisor) affect the number of times it fits (quotient)?
- Can you explain the "invert and multiply" rule using this physical model?

## Knowledge Checkpoint

- Calculate $\left(\frac{5}{9}\right) \div 5$.
- A $\frac{7}{8}$ meter long wire is cut into 7 equal pieces. How long is each piece?
- Divide $1\frac{1}{3}$ by 4.

## Key Terms

- **Dividend:** The number being divided.
- **Divisor:** The number by which another number is divided.
- **Quotient:** The result of division.

## Fact Flash

- The division symbol (÷) is called an **obelus**. It was first used for division by Swiss mathematician Johann Rahn in 1659.
- The "**invert and multiply**" rule is a powerful shortcut that simplifies what could otherwise be a complicated process.

## Do It Yourself

- If you divide a proper fraction by a whole number greater than 1, will the quotient always be smaller than the original fraction? Why?
- If you divide an improper fraction by a whole number greater than 1, will the quotient always be smaller than the original improper fraction? Why?

## Mental Mathematics

- What is one-half divided by two?
- If you divide three-fifths by three, what do you get?
- **Calculate:** two-thirds divided by four.
- What is seven-eighths divided by seven?

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## Exercise 8.4

**1. Fill in the blanks:**

a) To divide a fraction by a whole number, you multiply the fraction by the ____________ of the whole number.

b) Dividing by 5 is the same as multiplying by ____________.

c) Before dividing a mixed fraction by a whole number, you must first convert the mixed fraction into an ___________ fraction.

d) The reciprocal of 7 is ___________.

e) When you divide $\frac{2}{3}$ by 2, the answer is ___________.

**2. Solve the following:**

a) $\frac{3}{7} \div 2$ b) $\frac{4}{9} \div 5$ c) $\frac{7}{10} \div 3$ d) $\frac{1}{6} \div 8$ e) $\frac{9}{4} \div 3$ f) $\frac{11}{2} \div 6$ g) $\frac{5}{12} \div 10$ h) $\frac{15}{8} \div 5$

**3. Answer with: Greater than / Less than / Equal to:**

a) Is the quotient of $\frac{3}{4} \div \frac{1}{2}$ greater than or less than $\frac{3}{4}$?

b) Is the quotient of $\frac{5}{6} \div \frac{7}{8}$ greater than or less than $\frac{5}{6}$ ?

c) Is the quotient of $\frac{2}{5} \div \frac{1}{10}$ greater than or less than $\frac{2}{5}$?

d) Is the quotient of $\frac{3}{8} \div \frac{1}{2}$ greater than or less than $\frac{3}{8}$?

e) Is the quotient of $\frac{4}{7} \div \frac{5}{7}$ greater than or less than $\frac{4}{7}$?

**4. Which of the following expressions has a greater value? Show your work.**

a) $\frac{4}{5} \div 2$ b) $\frac{4}{5} \div 4$

5. A student has $\frac{5}{6}$ of a pizza. If she shares it equally with her other 4 friends, what fraction of the whole pizza does each person get?

6. If $\frac{y}{x} \div z = \frac{1}{21}$ and z = 3, what could be the original fraction $\frac{y}{x}$ ?

**7. A teacher showed students a strip of paper divided into half and asked the students some questions:**

a) Without unfolding, if this strip is folded into 3 equal parts and then unfolded completely, what fraction does each part represent?

b) Write the division statement for the situation described in (a) part.

c) Without unfolding, if this strip is folded into 4 equal parts and then unfolded completely, what fraction does each part represent?

d) Write the division statement for the situation described in (c) part.

e) Into how many parts the folded half strip must be folded further so that each part represents $\frac{1}{8}$ part, after unfolding completely.

8. A container holds $2\frac{2}{3}$ liters of juice. If each bottle holds $\frac{2}{5}$ liter, how many bottles are required to fill all the juice from the container?

## Some Problems Involving Fractions

There are some problems which are not solved directly in single step and requires multi-steps to get the answer.

If your mother gave you and your brother 24 candies each and you have $\frac{1}{2}$ of the candies left while your brother has $\frac{1}{6}$ part of candies left. How many candies do you both have altogether? This problem is not a single step problem and require multi-steps to get the answer.

### Let us see how to calculate this:

Total number of candies = 24

Fraction of candies left with you = $\frac{1}{2}$

Fig. 8.24

Number of candies left with you = $\frac{1}{2} \times 24 = 12$ ⟶ **Involves multiplication of fraction**

Similarly, fraction of candies left with your brother = $\frac{1}{6}$

Number of candies left with him = $\frac{1}{6} \times 24 = 4$ ⟶ **Again, involves multiplication of fraction**

Total number of candies left = 12 + 4 = 16 ⟶ **Involves addition**

**Let us see some more examples:**

**Example 22 :** Seema ate $\frac{2}{3}$ part of a pizza and Rihana ate $\frac{1}{6}$ part of it. If the pizza was divided into 12 equal parts. How many slices are left?

**Solution:** Fraction of pizza eaten by both Seema and Rihana = $\frac{2}{3} + \frac{1}{6}$

$$= \frac{(4+1)}{6} \quad \text{(LCM of 3 and 6 is 6)}$$

$$= \frac{5}{6}$$

Number of slices of pizza = 12

Number of slices eaten = $\frac{5}{6} \times 12 = 10$

Number of slices left = 12 − 10 = 2

**Alternative Method:**

Number of slices eaten by Seema = $\frac{2}{3} \times 12 = 8$

Number of slices eaten by Rihana = $\frac{1}{6} \times 12 = 2$

Total number of slices eaten by both of them = 8 + 2 = 10

Number of slices left = 12 − 10 = 2

**Example 23 :** A water tank was $\frac{5}{6}$ full. After some water is used, it is only $\frac{2}{3}$ full. If each bucket holds $\frac{1}{12}$ of the tank, how many buckets of water were removed?

**Solution:** First, calculate the fraction of water removed:

Fraction of water removed = $\frac{5}{6} - \frac{2}{3}$

$$= \frac{(5-4)}{6} \quad \text{(LCM of 3 and 6 is 6)}$$

$$= \frac{1}{6}$$

Each bucket holds = $\frac{1}{12}$ of the tank

So, $\frac{1}{6} \div \frac{1}{12} = \frac{1}{6} \times \frac{12}{1} = 2$ buckets

This means 2 buckets of water were removed.

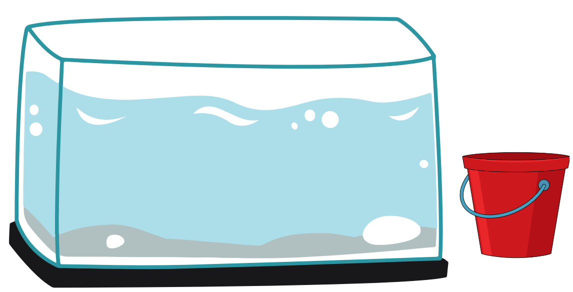

Fig. 8.25

**Example 24 :** Rohan spent $\frac{1}{3}$ of an hour with each of his 3 clients respectively, instead of $\frac{1}{4}$ of an hour. How much more time did he spend meeting the clients?

*Fig. 8.26*

**Solution:** Time Rohan spent with each client = $\frac{1}{3}$

Number of clients = 3

Total time spent = $\frac{1}{3} \times 3 = 1$ hour = 60 minutes

Time Rohan had to spent with these three clients = $\frac{1}{4} \times 3 = \frac{3}{4}$ of an hour

$= \frac{3}{4} \times 60$ minutes = 45 minutes

Time spent more = 60 − 45 = 15 minutes

## Activity

**"Fraction Story Problems" Creation**

**Objective:** To deepen understanding of fraction operations by creating and solving real-world problems.

**Materials:** Blank index cards, pens.

**Procedure:**

1. Divide students into pairs or small groups.
2. Each group creates 3-4 word problems involving multiplication and/or division of fractions.
   - Encourage them to use real-life scenarios (cooking, sharing, travel, building, etc.).
   - They should write the problem clearly on one side of the card.
   - On the back, they write the step-by-step solution.
3. Once problems are created, groups exchange cards with another group.
4. The new group solves the problems they received, then checks their solutions against the answers on the back of the cards.
5. Groups discuss any discrepancies or interesting problems.

**Inquiry Questions:**

- What makes a good fraction word problem?
- Was it easier to create problems or to solve them? Why?
- Did you find any common themes or types of problems that frequently use fraction multiplication/division?

## Knowledge Checkpoint

- A bag had $\frac{7}{8}$ kg of flour. After making some chapatis, only $\frac{1}{4}$ kg of flour is left. How much flour is used?
- If each small packet holds $\frac{1}{16}$ kg, how many packets of flour were used?
- A cookie recipe needs $\frac{3}{4}$ cup of flour for one batch. If each cookie uses $\frac{1}{10}$ of the flour from the batch, how much flour is there in 2 batches worth of cookies?
- A cake weighs $\frac{3}{4}$ kg. Riya eats $\frac{2}{3}$ of it. She then shares the remaining portion equally, such that each plate contains $\frac{1}{6}$ kg of the original cake. How many plates did she use?

## Key Terms

- **Word Problem:** A mathematical problem presented in narrative form, requiring translation into mathematical expressions.
- **Scenario:** A description of a real-life situation.
- **Context:** The circumstances or setting in which an event occurs.

## Fact Flash

- The oldest known mathematical text, the Rhind Papyrus from ancient Egypt (around 1650 BCE), is full of practical problems involving fractions!
- Many ancient civilizations used fractions to solve problems related to taxation and inheritance.

## Do It Yourself

- Can you create a word problem that requires both multiplication and division of fractions to solve?
- Think of a real-life situation where you might need to find "a fraction of a fraction of a whole number."

## Mental Mathematics

- $\frac{1}{3}$ of $\frac{1}{4}$ of 20.
- If $\frac{1}{3}$ of a number is 7, what is the number?
- $\frac{1}{3}$ of $\frac{3}{4}$ of 18 slices means how many slices.
- How many $\frac{1}{2}$s in 5?
- $\frac{1}{2}$ of $\frac{1}{2}$ of 100.

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## Exercise 8.5

1. After interviewing the candidates, Babita decided to hire 3 candidates and reject 2 candidates. If she spent $\frac{1}{20}$ of an hour calling each rejected candidate and $\frac{1}{8}$ of an hour calling each hired candidates. How much time did she spent in total in minutes? How much time did she spend in total in hours?

**2. Find the value of the following:**

a) $\frac{2}{3}$ of $\frac{1}{4}$ of 12  b) $\frac{9}{14}$ of $\frac{3}{16}$ of $\frac{7}{8}$ of 100  c) $\frac{2}{3}+\frac{1}{2}\times\frac{4}{5}$

3. **There are 24 students in a class. Out of which, 17 are boys. 18 students are ready to go to the picnic.**

a) What fraction of students are ready to go to the picnic?

b) What fraction of girls are there in the class?

c) If $\frac{1}{3}$ of the students going to the picnic are boys, then how many girls are going to the picnic?


4. **Arun spends his day doing different activities. The fraction of time he spends on each activity is given below:**

- Studying → $\frac{1}{8}$ of the day
- Sleeping → $\frac{1}{3}$ of the day
- In school → $\frac{1}{3}$ of the day
- Eating dinner → $\frac{1}{24}$ of the day
- Watching TV → $\frac{1}{12}$ of the day
- Playing → $\frac{1}{12}$ of the day

**Answer the following questions based on the data given:**

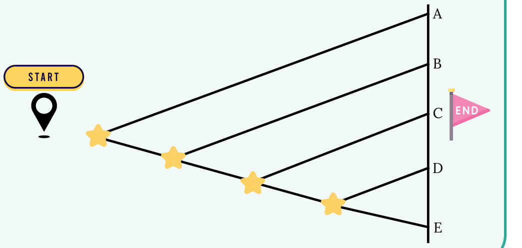
a) How much time does Arun spend on watching TV and playing together?

b) For how many hours does Arun stay in school?

c) Arun spends $\frac{1}{2}$ hour more in studying. What fraction of the day does he now spend on studying?

d) Who takes up more time in Arun's day — playing or eating dinner? By how much?

5. A group of friends were playing a game, in which they had to reach from starting point to the ending point. They had a checkpoint after each meter. At each checkpoint they had to divide into half to go on each path. If there were 4 such checkpoints, then what fraction of original group reached at the ending points?




## Common Misconceptions

**Misconception:** Multiplication always makes numbers bigger, and division always makes numbers smaller.

**Correction:** This is true for whole numbers greater than 1, but not always for fractions.

**Multiplication:** When multiplying by a proper fraction (less than 1), the product is smaller than the original number $\left(\text{e.g.,} 10 \times \frac{1}{2} = 5;\ 5 < 10\right)$. When multiplying two proper fractions, the product is smaller than both $\left(\text{e.g.,} \frac{1}{2} \times \frac{1}{4} = \frac{1}{8};\ \frac{1}{8} < \frac{1}{2} \text{ and } \frac{1}{8} < \frac{1}{4}\right)$.

**Division:** When dividing by a proper fraction (less than 1), the quotient is larger than the dividend $\left(\text{e.g.,} 10 \div \frac{1}{2} = 20;\ 20 > 10\right)$. This is because you are asking how many small pieces fit into a larger quantity.

**Misconception:** To divide fractions, you just flip the second fraction.

**Correction:** This is incomplete. You must also change the operation from division to multiplication. The rule is "Keep, Change, Flip" (KCF). You keep the first fraction, change the division sign to multiplication, and then flip (take the reciprocal of) the second fraction.

**Misconception:** When multiplying or dividing mixed fractions, you can operate on the whole number parts and fractional parts separately.

**Correction:** This is incorrect. Mixed fractions must always be converted to improper fractions before performing multiplication or division.

**Example:** $2\frac{1}{2} \times 3\frac{1}{3}$ is NOT $(2 \times 3) + \left(\frac{1}{2} \times \frac{1}{3}\right)$.

It is $\left(\frac{5}{2}\right) \times \left(\frac{10}{3}\right) = \frac{50}{6} = 8\frac{1}{3}$.

## Real-Life (Fraction) : Mathematical Applications

Fractions are essential for representing parts of a whole, dividing quantities, and understanding proportions in everyday life. These applications make fractions relevant, fostering accuracy, fair distribution, and proportional reasoning, aligning with NEP 2020's focus on experiential learning:

1. **Cooking & Recipes:** Adjusting ingredient quantities, e.g., halving a recipe ($\frac{1}{2}$ cup of flour) or doubling it (2 and $\frac{1}{4}$ cups sugar). (Ensures successful culinary outcomes).
2. **Time Management:** Dividing hours, minutes, and seconds, e.g., quarter past three (3 : 15 or 3 and $\frac{1}{4}$ hours) or half an hour. (Crucial for scheduling and punctuality).
3. **Measurements & Sharing:** Dividing objects or quantities fairly, such as cutting a cake into equal slices ($\frac{1}{8}$ of the cake) or sharing a pizza. (Fundamental for equitable distribution).
4. **Discounts & Sales:** Understanding percentage discounts, which are often expressed as fractions (e.g., 25% off is $\frac{1}{4}$ off the price). (Enables smart purchasing decisions).
5. **Construction & DIY:** Measuring lengths and dimensions for projects, often involving fractional parts of an inch or foot, e.g., 3 and $\frac{1}{2}$ inches. (Ensures precision in building and crafting).

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Complete Chapter Test

# EXERCISE

## A. Choose the correct answer.

1. What is the product of $\frac{3}{5}$ and 10?

   a) $\frac{30}{50}$ b) 6 c) $\frac{10}{50}$ d) $\frac{3}{50}$

2. The area of a rectangular garden is $\frac{7}{8}$ sq km. If its length is $\frac{7}{4}$ km, what is its width?

   a) $\frac{1}{2}$ km b) 2 km c) $\frac{7}{32}$ km d) $\frac{14}{32}$ km

3. Which of the following expressions will result in a product smaller than $\frac{5}{6}$?

   a) $\frac{5}{6}\times1\frac{1}{2}$ b) $\frac{5}{6}\times\frac{7}{6}$ c) $\frac{5}{6}\times\frac{3}{4}$ d) $\frac{5}{6}\times2$

4. What is the reciprocal of $2\frac{3}{7}$?

   a) $\frac{7}{17}$ b) $2\frac{7}{3}$ c) $\frac{17}{7}$ d) $\frac{3}{7}$

5. How many pieces of ribbon, each $\frac{1}{6}$ meter long, can be cut from a ribbon that is $2\frac{1}{2}$ meters long?

   a) 15 b) 5 c) 10 d) 25

## Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given. Study both the statements and state which of the following is correct:

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

1. **Assertion (A):** The product of $\frac{2}{3}$ and $\frac{4}{5}$ is $\frac{8}{15}$.

   **Reason (R):** To multiply fractions, you multiply the numerators and multiply the denominators.

2. **Assertion (A):** The reciprocal of $3\frac{1}{4}$ is $\frac{4}{13}$.

   **Reason (R)**: To find the reciprocal of a mixed fraction, convert it to an improper fraction and then flip it.

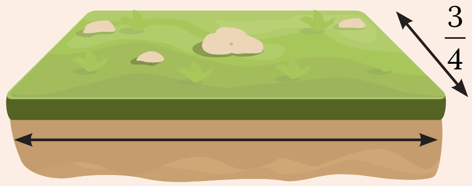
3. **Assertion (A):** When you divide 5 by $\frac{1}{2}$, the quotient is 10.

   **Reason (R)**: Dividing by a proper fraction always results in a quotient smaller than the dividend.

## Case Study

**The Community Garden Project**

A local community decides to start a garden project on a rectangular plot of land that measures $\frac{5}{6}$ km in length and $\frac{3}{4}$ km in width.




- **Part 1:** What is the total area of the garden plot?
- **Part 2:** The community decides to allocate $\frac{2}{5}$ of the total garden area for growing vegetables. What fraction of the entire plot is dedicated to vegetables? What is this area in square kilometers?
- **Part 3:** The vegetable section is to be divided into smaller individual plots, each measuring $\frac{1}{20}$ square kilometers. How many individual vegetable plots can be created?
- **Part 4:** If $\frac{1}{3}$ of the individual vegetable plots are assigned to senior citizens, how many plots do senior citizens receive?

## Project

**The Great Recipe Scaling Challenge**

**Objective:** To apply knowledge of fraction multiplication and division to scale a recipe up and down for different numbers of servings, and to calculate ingredient costs.

**Materials:** Access to internet for recipe and ingredient prices, paper, pen, calculator (optional).

1. **Choose a Recipe:** Find a simple recipe online (e.g., cookies, pancakes, a simple curry) that uses at least 5 ingredients, and whose original serving size is clearly stated (e.g., "serves 4").
2. **Original Recipe Analysis:**
   - Write down the recipe name and its original serving size.
   - List all ingredients and their quantities. Ensure all quantities are in fractional or mixed number form $\left(\text{e.g.,} \frac{1}{2} \text{ cup}, 2\frac{1}{4} \text{ tsp}, \frac{3}{4} \text{ kg}\right)$. If a quantity is a whole number, convert it to a fraction $\left(\text{e.g.,} 2 \text{ eggs,} = \frac{2}{1} \text{ eggs}\right)$.
3. **Scale Up:**
   - Choose a new serving size that is $1\frac{1}{2}$ times the original serving size (e.g., if original serves 4, new serves 6).
   - Calculate the new quantity for each ingredient using fraction multiplication. Show your calculations clearly.

4. **Scale Down:**
   - Choose another new serving size that is $\frac{1}{2}$ of the original serving size (e.g., if original serves 4, new serves 2).
   - Calculate the new quantity for each ingredient using fraction multiplication. Show your calculations clearly.
5. **Ingredient Cost Analysis:**
   - Research the approximate cost of each ingredient (per unit, e.g., cost per kg of flour, cost per dozen eggs, cost per cup of milk). You can use online grocery store websites.
   - Calculate the total cost of ingredients for the original recipe.
   - Calculate the total cost of ingredients for the scaled-up recipe.
   - Calculate the total cost of ingredients for the scaled-down recipe.
6. **Presentation:**
   - Create a clear and organized presentation of your findings. This can be a poster, a digital presentation (e.g., Google Slides), or a detailed report.
   - Include:
   - Recipe name and original serving size.
   - Table showing original, scaled-up, and scaled-down ingredient quantities.
   - All calculations for scaling.
   - Ingredient cost breakdown for each version of the recipe.
   - A brief reflection on what you learned about fraction operations and their real-world utility.

## Source-Based Question

The Vande Bharat Express, a symbol of modern rail travel in India, shows a mixed picture of occupancy across its routes. Currently, 17 out of the 59 operational Vande Bharat trains are running at full capacity, reflecting the growing popularity of this high-speed service among travelers.

*Adapted from reports published in India Today (2024) and Indian Railways occupancy analysis (2025).*

However, the occupancy rates reveal a contrasting scenario. In 13 of the 59 Vande Bharat trains, approximately half of the seats remain unoccupied, highlighting the varied interest in different routes. The unattractive fares are contributing to this and causing the numbers to rise.

Adding to the operational diversity, three Vande Bharat trains are run with 20 coaches. Among them, the Nagpur-Secunderabad route is experiencing lower occupancy, with 1,118 out of 1,328 seats typically unoccupied.

This statistic raises important questions about passenger preferences and the factors influencing travel choices on this particular route.

Railway officials are keenly observing these trends to optimise services and enhance the overall travel experience. Strategies may include targeted marketing campaigns to promote underutilised routes, as well as adjustments in scheduling to align with passenger demand.

As the Indian Railways continues to innovate and expand its services, the Vande Bharat Express stands at the forefront of this transformation.

With full-capacity trains to attract considerable attention, there remains a pressing need to address the occupancy challenges on certain routes, ensuring that the Vande Bharat trains can fully realise their potential as a modern mode of transportation for millions across the country.

**Directions:**

The Vande Bharat Express, a modern semi-high-speed train in India, shows varying occupancy levels across its routes. Railway reports highlight that while many trains run at full capacity, others face challenges with low passenger numbers. The table below summarizes some recent data.

| Category | Details | Numbers / Fractions |
|---|---|---|
| **Total operational Vande Bharat trains** | Currently running across India | 59 trains |
| **Trains running at full capacity** | All seats occupied | 17 out of 59 (≈ 29%) |
| **Trains with ~50% seats unoccupied** | Around half seats empty | 13 out of 59 (≈ 22%) |
| **Trains with 20 coaches** | Extra-long trains in operation | 3 trains |
| **Nagpur–Secunderabad 20-coach train** | Low occupancy: 1,118 seats empty out of 1,328 | Only 210 seats filled (≈ 16% occupancy) |

**Questions on the Data**

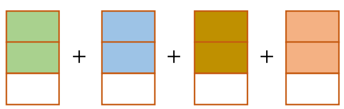
1. What fraction of the trains are full?
2. What fraction of the trains have half their seats empty?
3. What fraction of the Nagpur–Secunderabad train's seats are unoccupied?
4. If in Nagpur-Secunderabad train, out of filled seats $\frac{2}{3}$ are occupied by females and $\frac{1}{6}$ by transgenders, then how many seats are occupied by males?

Mind Map

# Fractions

## Multiplication of a Fraction by a Whole Number

- ❖ Repeated Addition as Multiplication
- ❖ Rule for Multiplying a Fraction by a Whole Number
- ❖ Multiplying Mixed Fractions by a Whole Number

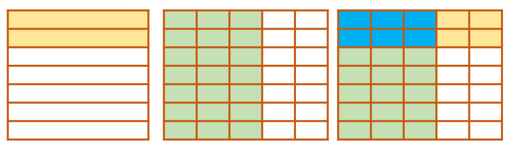


$\frac{2}{3}$ + $\frac{2}{3}$ + $\frac{2}{3}$ + $\frac{2}{3}$

$4 \times \frac{2}{3} = \frac{8}{3}$

## Multiplying Two Fractions

- ❖ Visualizing Fraction Multiplication (Area Model)
- ❖ Rule for Multiplying Two Fractions
- ❖ Simplifying to Lowest Form (Cancelling Common Factors)
- ❖ Multiplying Mixed Fractions



$\frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$

## Reciprocal of a Fractions

- ❖ Definition of Reciprocal
- ❖ Finding the Reciprocal of a Fraction
- ❖ Finding the Reciprocal of a Whole Number
- ❖ Finding the Reciprocal of a Mixed Fraction

$\frac{3}{4}$ Reciprocal → $\frac{4}{3}$

## Product Relationship

- ❖ Product when both numbers are greater than 1
- ❖ Product when both numbers are between 0 and 1 (proper fractions)
- ❖ Product when one number is between 0 and 1 and the other is greater than 1

## Division of a Fraction

- ❖ Relating Division to Multiplication by Reciprocal
- ❖ Rule for Dividing a Fraction by a Whole Number
- ❖ Dividing Mixed Fractions by a Whole Number.
- ❖ Some problems including fractions.

$\frac{1}{2} \div \frac{1}{4}$

Keep Change Flip

$\frac{1}{2} \times \frac{4}{1}$