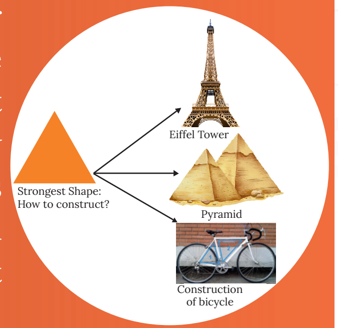
7 Triangle: Three Intersecting Lines

Why This Chapter Matters

Imagine you're an architect designing a new building, or an engineer building a bridge. What's one of the most fundamental shapes you'd use for stability and strength? The triangle! From the pyramids of ancient Egypt to the modern Eiffel Tower, triangles are everywhere. But how do we build a perfect triangle? What rules must its sides and angles follow? In this chapter, we'll unlock the secrets of triangle construction and discover the amazing properties that make them so special. Get ready to draw, measure, and think like a geometrician!



Meet EeeBee.AI

Hello, young mathematicians! I'm EeeBee, your friendly guide through the exciting world of triangles. I love exploring shapes, solving puzzles, and discovering the hidden patterns in mathematics. In this chapter, I'll be right by your side, helping you understand how to build triangles, what makes them special, and how they appear all around us. If you ever get stuck or need a little hint, just look for me! Let's embark on this geometric adventure together!

Learning Outcomes

By the end of this chapter, you will be able to:

- Construct various types of triangles given specific side lengths or angle measures using geometric tools.
- Apply the Triangle Inequality Property to determine if a triangle can be formed from a given set of side lengths.
- Explain the Angle Sum Property of triangles and use it to find unknown angles.
- Identify and construct altitudes of different types of triangles.
- Classify triangles based on their side lengths (equilateral, isosceles, scalene) and angle measures (acute-angled, right-angled, obtuse-angled).
- Solve real-world problems involving triangle properties and construction.

From Last Year's Notebook

- Points, Lines, and Angles: You learned about fundamental geometric shapes like points, lines, line segments, and angles.
- Measurement Tools: You practiced using a protractor to measure and draw angles, and a ruler to draw line segments of specific lengths.
- Types of Angles & Lines: You explored different types of angles (acute, obtuse, right, straight) and lines (parallel, intersecting, perpendicular).
- Compass Skills: You also used a compass to draw circles and arcs.

Connecting to Current Learning:

- These fundamental skills are crucial! We will now build upon this knowledge to understand the more complex world of triangles and their unique characteristics.
- Get ready to put your compass and ruler skills to good use as we learn about constructing triangles!

Real Math, Real Life

- **Architects:** Use triangles for structural integrity in buildings, helping them withstand forces like wind and earthquakes.
- **Engineers:** Rely on triangular frameworks in structures such as bridges, cranes, and even bicycle frames for maximum rigidity and strength.
- **Surveyors:** Utilize triangulation as a method to measure vast distances and accurately map land.
- **Artists and Designers:** Incorporate triangles in their work for visual balance and to create dynamic compositions.
- **Computer Graphics:** Triangles are fundamental in 3D modeling, where countless tiny triangles form complex models.

Quick Prep

1. What is the minimum number of line segments required to form a closed figure?
2. If you have a line segment of 8 cm, how would you accurately mark its midpoint using only a compass and a ruler?
3. Can a triangle have two right angles? Why or why not?
4. Imagine you have three sticks of lengths 3 cm, 4 cm, and 10 cm. Can you form a triangle with these sticks? Why do you think so?
5. What is the difference between a line, a line segment, and a ray?

Introduction

Welcome to the fascinating world of triangles! These three-sided polygons are not just simple shapes; they are the building blocks of geometry and are fundamental to understanding the world around us. From the sturdy framework of a bridge to the precise calculations in surveying, triangles play a crucial role. In this section, we will dive deep into how to construct triangles accurately using various tools, explore the essential rules that govern their existence, and uncover their remarkable properties related to both sides and angles. Get ready to transform abstract concepts into tangible constructions and discover the power of geometric reasoning!

Chapter Overview

- **Introduction to Triangles:** Learn about vertices, sides, angles, and how to name triangles.
- **Constructing Triangles:**
  - Using given side lengths (e.g., equilateral, general triangles) with compass and ruler.
  - Understanding the Triangle Inequality Property (sum of two sides > third side) and its real-world implications.
  - Constructing triangles using given sides and angles (SAS and ASA criteria) with ruler, protractor, and compass.
- **Properties of Triangles:** Explore the Angle Sum Property (sum of angles = 180°) and the Exterior Angle Property.
- **Altitudes of Triangles:** Define and construct altitudes (heights) using a set square.
- **Types of Triangles:** Differentiate triangles based on sides (equilateral, isosceles, scalene) and angles (acute, right, obtuse).

From History's Pages

The study of triangles, originating in ancient civilizations, was formalized by the Greeks. Euclid's "**The Elements**" (c. 300 BCE) established foundational geometry, including triangle properties, congruence, and similarity, logically. Indian mathematicians later advanced trigonometry. Ancient construction methods using compass and straightedge are also vital. This chapter builds on these historical **principles**, exploring triangle properties like the angle sum property through hands-on construction and logical reasoning, bringing timeless geometry to life.

Introduction to Triangles and Construction with Given Side Lengths

A triangle is a fundamental polygon with three sides and three angles. It is the simplest closed figure formed by straight line segments. Every triangle has three corner points called vertices, three sides (line segments connecting the vertices), and three angles (formed by the intersection of the sides at each vertex). We name a triangle using its vertices, for example, triangle ABC (denoted as ΔABC). Understanding how to construct a triangle accurately is the first step to exploring its many properties. We will begin by learning how to construct triangles when the lengths of all three sides are provided. This involves using a compass and a ruler to ensure precision.

Sub-concepts to be covered

1. Definition and components of a triangle (vertices, sides, angles).
2. Naming conventions for triangles.
3. Construction of equilateral triangles (all sides equal).
4. Construction of general triangles (all sides different).

Definition and components of a triangle

A triangle is a closed plane figure formed by three line segments.

Vertices: The three points where the sides meet are called vertices. (e.g., A, B, C in ΔABC). **Fig. 7.1**

Sides: The three line segments forming the triangle are its sides. (e.g., AB, BC, CA in ΔABC). **Fig. 7.2**

Angles: The three angles formed inside the triangle by the intersection of its sides are its interior angles. (e.g., $\angle A$, $\angle B$, $\angle C$ in ΔABC). **Fig. 7.3**

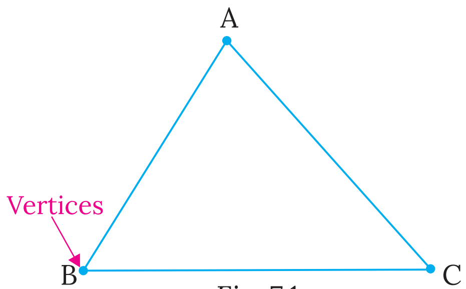


Fig. 7.1

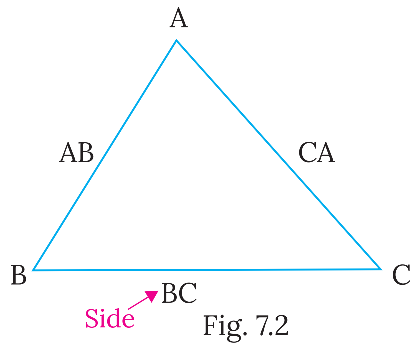


Fig. 7.2

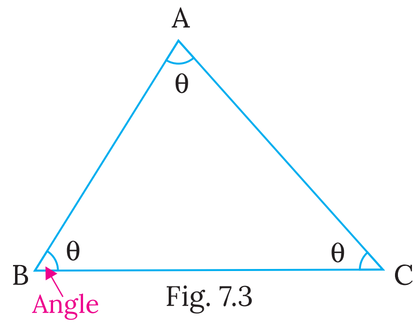


Fig. 7.3

Naming conventions for triangles.

A triangle is named by its vertices, such as ΔABC, ΔBCA, or ΔCAB. The order of vertices does not change the triangle itself.

Construction of equilateral triangles (all sides equal).

An equilateral triangle is a special type of triangle where all three sides are of equal length. Consequently, all three angles are also equal (each 60°). Constructing an equilateral triangle requires precise use of a compass to ensure all sides are identical.

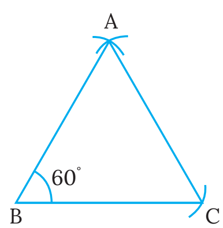


Fig. 7.4

Key points: All sides are equal, all angles are equal (60°).

Common errors: Not using the compass correctly to maintain the same radius, leading to unequal sides.

Construction of general triangles (all sides different).

This refers to constructing any triangle when the lengths of its three sides are given. This method is often called the SSS (Side-Side-Side) criterion for triangle construction. It relies on the principle that if three side lengths are known, a unique triangle can often be formed (provided the triangle inequality holds, which we will discuss later).

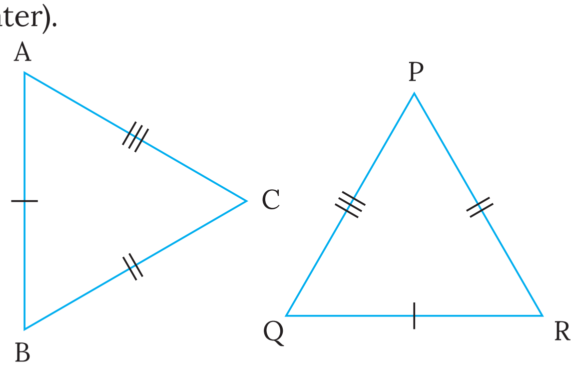


Fig. 7.5

Key points: Choose one side as the base, use arcs from the endpoints to locate the third vertex.

Common errors: Inaccurate measurement of arcs, not ensuring arcs intersect.

Mathematical Explanation

The construction of a triangle when all three side lengths are given (SSS criterion) relies on the fundamental property of circles: all points on a circle are equidistant from its center.

Equilateral Triangle Construction

To construct an equilateral triangle with side length 's' cm:

Step 1: Draw a straight line segment of length 5.5 cm. Label the endpoints A and B. **Fig. 7.6**

Step 2: Using a compass, place the pointer on point A and draw an arc with a radius of 5.5 cm. **Fig. 7.7**

Step 3: Without changing the compass width, place the pointer on point B and draw another arc that intersects the first arc. Label the intersection point as C. **Fig. 7.8**

Step 4: Using a ruler, draw line segments AC and BC to complete the triangle. You now have an equilateral triangle ABC with each side measuring 5.5 cm. **Fig. 7.9**

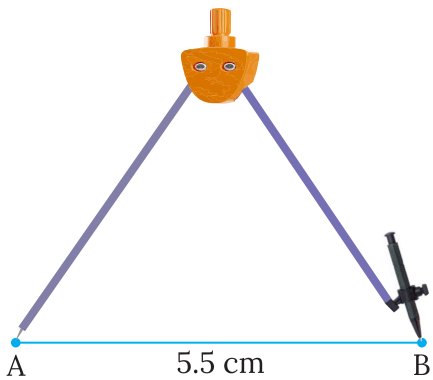


Fig. 7.6

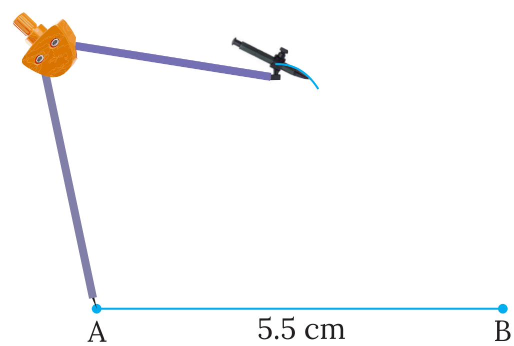


Fig. 7.7

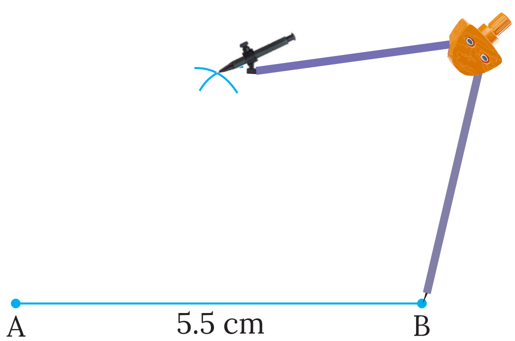


Fig. 7.8

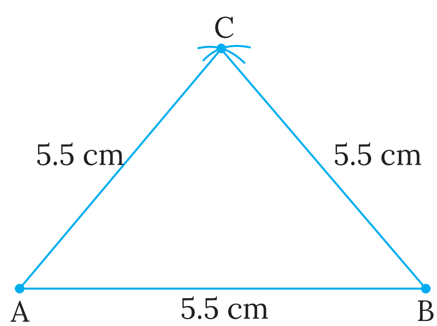


Fig. 7.9

General Triangle Construction (SSS)

To construct a triangle with side lengths 'a', 'b', and 'c' cm:

This method guarantees that the constructed triangle has the specified side lengths, provided such a triangle can exist (which leads to the Triangle Inequality Property).

Let us construct a triangle ABC, with **sides lengths** AB = 7 cm, BC = 4 cm, and CA = 6 cm. The steps for construction of triangle are:

Step 1: Mark a point A. Measure the length of 7 cm using compass and scale. With the help of Compass mark an arc, placing it at point A. (**Fig. 7.10)**

Step 2: Mark a point B on the arc. Now measure the length of 6 cm. Again using compass mark an arc above point B using the same point (A) **Fig. 7.11**

Step 3: Now measure the length of 4 cm. Using the compass placed at point B cut an arc such that it crosses the previous arc. **Fig. 7.12**

Step 4: Name the point as C where the two arcs cross each other. At the end join the points A, B, and C with a ruler to give the required triangle. Thus, the obtained triangle is the required triangle ABC with the given measurements. **Fig. 7.13**

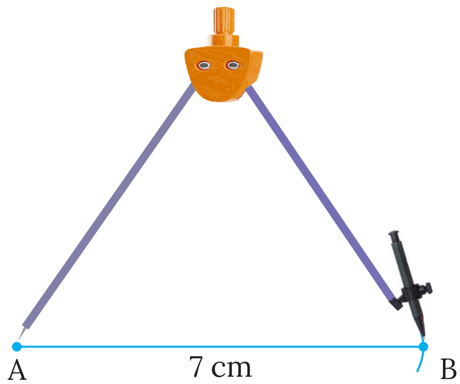


Fig. 7.10

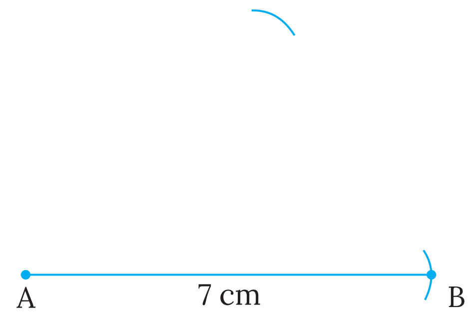


Fig. 7.11

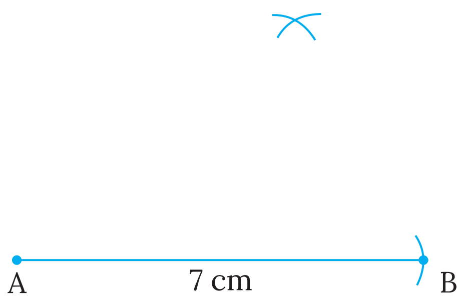


Fig. 7.12

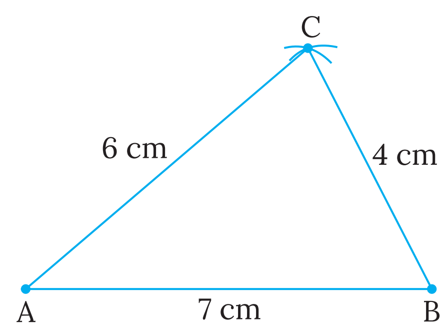


Fig. 7.13

Example: Construct an equilateral triangle with side length 6 cm.

Step 1: Draw a line segment AB of length 6 cm. **Fig. 7.14**

Explanation: This forms the base of our triangle.

Step 2: With A as the center, open the compass to a radius of 6 cm. Draw an arc above the line segment AB. **Fig. 7.15**

Explanation: All points on this arc are 6 cm away from A. The third vertex C must lie on this arc.

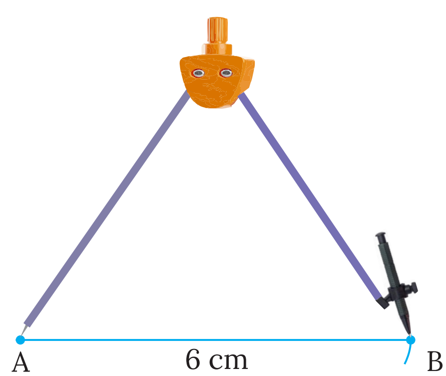


Fig. 7.14

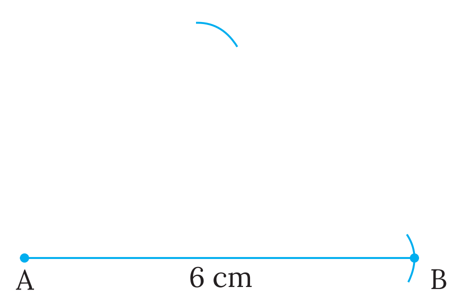


Fig. 7.15

Step 3: With B as the center, and the same radius of 6 cm, draw another arc intersecting the first arc. **Fig. 7.16**

Explanation: All points on this second arc are 6 cm away from B. The intersection point is C, which is 6 cm from both A and B.

Step 4: Label the intersection point as C. Join AC and BC. **Fig. 7.17**

Explanation: This completes the triangle. Since AC = 6 cm and BC = 6 cm (by construction), and AB = 6 cm (by initial drawing), ΔABC is an equilateral triangle.

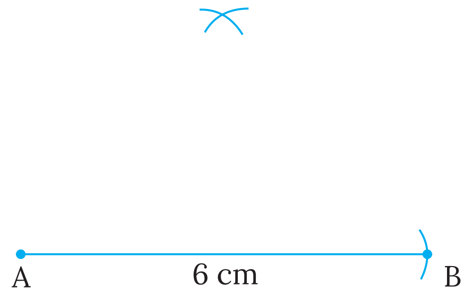


Fig. 7.16

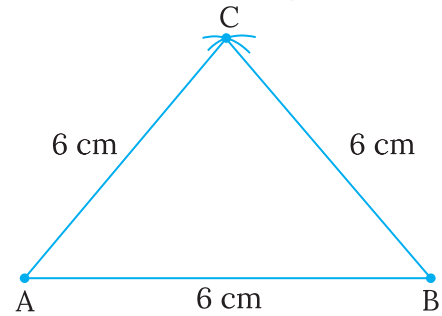


Fig. 7.17

Example: Construct a triangle PQR with sides PQ = 7 cm, QR = 5 cm, and RP = 6 cm.

Step 1: Draw a line segment PQ of length 7 cm. **Fig. 7.18**

Explanation: We choose PQ as the base.

Step 2: With P as the center, open the compass to a radius of 6 cm (length of RP). Draw an arc above PQ. **Fig. 7.19**

Explanation: This arc contains all possible locations for vertex R that are 6 cm from P.

Step 3: With Q as the center, open the compass to a radius of 5 cm (length of QR). Draw another arc intersecting the first arc. **Fig. 7.20**

Explanation: This arc contains all possible locations for vertex R that are 5 cm from Q. The intersection point is R, which satisfies both conditions.

Step 4: Label the intersection point as R. Join PR and QR. **Fig. 7.21**

Explanation: This completes ΔPQR with the given side lengths.

Fig. 7.18

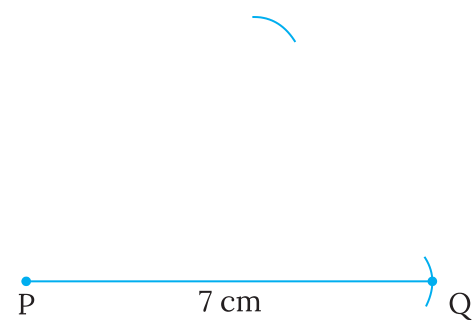


Fig. 7.19

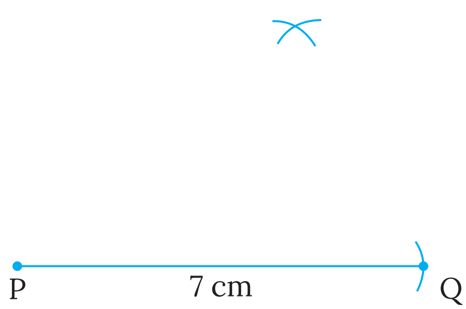


Fig. 7.20

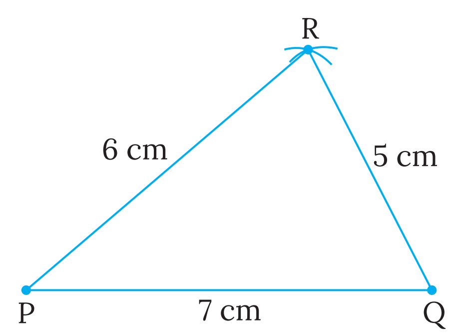


Fig. 7.21

Example: Can you construct a triangle with sides 3 cm, 4 cm, and 8 cm?

Step 1: Draw a line segment AB of length 8 cm.

Step 2: With A as the center, draw an arc with radius 3 cm.

Step 3: With B as the center, draw an arc with radius 4 cm.

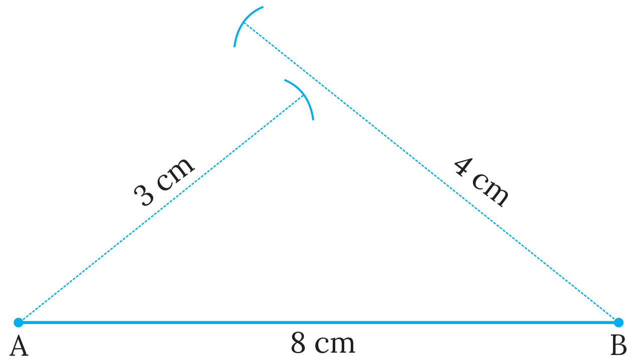


Fig. 7.22

Observation: The two arcs do not intersect.

Conclusion: A triangle cannot be constructed with these side lengths. (This leads to the Triangle Inequality Property, which will be discussed in detail later).

Example: Construct an isosceles triangle XYZ where XY = XZ = 7 cm and YZ = 4 cm.

Step 1: Draw a line segment YZ of length 4 cm. **Fig. 7.23**

Step 2: With Y as the center, open the compass to a radius of 7 cm. Draw an arc above YZ. **Fig. 7.24**

Step 3: With Z as the center, and the same radius of 7 cm, draw another arc intersecting the first arc. **Fig. 7.25**

Step 4: Label the intersection point as X. Join XY and XZ. **Fig. 7.26**

Explanation: This constructs an isosceles triangle because two sides (XY and XZ) are of equal length (7 cm).

Fig. 7.23

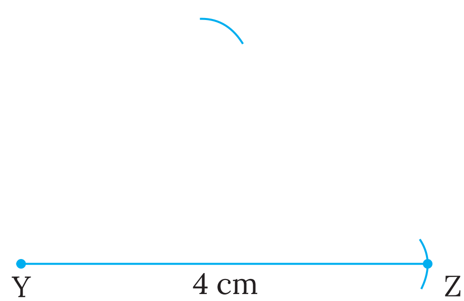


Fig. 7.24

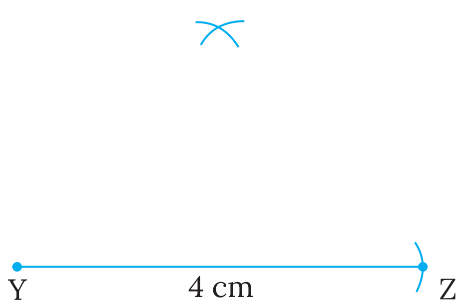


Fig. 7.25

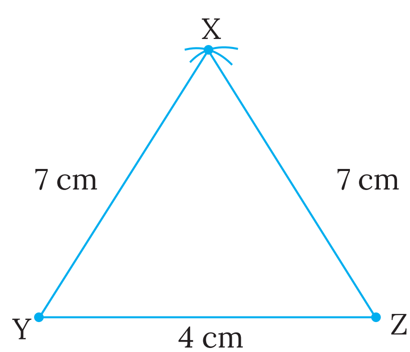


Fig. 7.26

Knowledge Checkpoint

- What are the three main components of a triangle?
- If you are given three side lengths, what tool is most important for accurate construction?
- Can you construct a triangle with sides 2 cm, 2 cm, and 2 cm? What kind of triangle would it be?

Activity

Triangle Challenge Kit

Objectives: To understand the Triangle Inequality Property through hands-on experimentation.

Materials: Strips of cardboard or plastic of various lengths (e.g., 3 cm, 4 cm, 5 cm, 6 cm, 7 cm, 8 cm, 9 cm, 10 cm, 12 cm), fasteners (e.g., paper clips, brads).

Procedure:

1. Students work in pairs or small groups.
2. Each group receives a set of cardboard strips.
3. Task 1: "Can you build it?" – Students are given sets of three lengths (e.g., 5 cm, 7 cm, 9 cm; then 3 cm, 4 cm, 8 cm) and asked to try and form a triangle by joining the strips at their ends with fasteners.
4. Task 2: "Predict and Test" – For each set, before attempting to build, students predict whether a triangle can be formed and record their reasoning. Then they attempt to build and compare with their prediction.
5. Task 3: "Discover the Rule" – Based on their observations, students try to formulate a rule about which sets of lengths can form a triangle and which cannot.

Inquiry Focus: This activity is designed to lead students to discover the Triangle Inequality Property through hands-on exploration and observation, rather than being directly told the rule.

Key Terms

- **Vertex (Vertices):** A corner point of a triangle where two sides meet.
- **Side:** A line segment forming part of the boundary of a triangle.
- **Angle:** The space (measured in degrees) between two intersecting lines or surfaces at or close to the point where they meet.
- **Equilateral Triangle:** A triangle with all three sides of equal length.
- **Isosceles Triangle:** A triangle with two sides of equal length.
- **Scalene Triangle:** A triangle with all three sides of different lengths.
- **Compass:** A geometric tool used for drawing circles or arcs and for measuring distances.
- **Ruler:** A tool used for measuring lengths and drawing straight lines.

Do It Yourself

- If you have three sticks of lengths 5 cm, 5 cm, and 10 cm, can you form a triangle? Why or why not? What happens if you try to make the construction?
- Is it possible to construct two different triangles if you are only given the lengths of their three sides? What does this tell you about the uniqueness of a triangle given its side lengths?

Fact Flash

- The word "**triangle**" comes from the Latin "**triangulus**," meaning "**three-cornered**."
- The Bermuda Triangle is a mythical area in the Atlantic Ocean, famous for unexplained disappearances of ships and planes, named for its triangular shape.
- The strongest shape in nature is often considered to be the triangle due to its rigidity. This is why you see it so often in construction!

Mental Mathematics

- An equilateral triangle has a perimeter of 21 cm. What is the length of each side?
- The sides of a triangle are 5 cm, 12 cm, 13 cm. Without drawing, what type of triangle is this?
- A triangle has sides 7 cm, 24 cm, 25 cm. What is the perimeter? What special triangle is this?

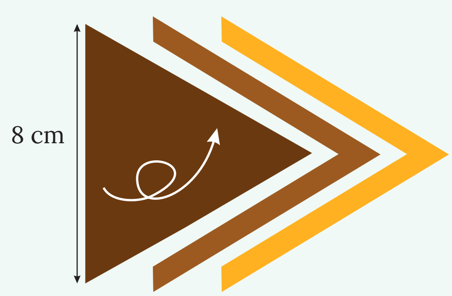
Gap Analyzer™ Homework

Watch Remedial

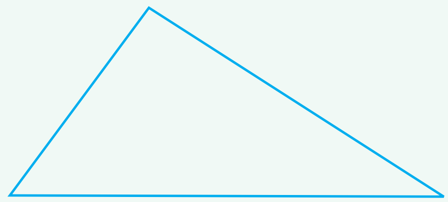
Exercise 7.1

1. **Fill in the Blanks:**
   a) A triangle has ______ vertices, ______ sides, and ______ angles.
   b) An equilateral triangle has all its sides of ______ length.
   c) To construct a triangle given its three side lengths, we use a ______ and a ______.
   d) The symbol for a triangle is ______.
2. A gardener wants to create a triangular flower bed with sides 8 meters, 10 meters, and 12 meters. He wants to first draw a sketch of the same first in a sheet with all dimensions in cm. Could you help him to draw the sketch?

3. Rehman wants to create a logo for his brand. For that he has to create an equilateral triangle first. Sketch the logo for his brand.



4. Draw an isosceles triangle with base 4 cm and other sides 6 cm.
5. Perimeter of an equilateral triangle is 18 cm. What will be the length of each side? Draw the triangle.
6. Draw two triangles PQR and PQS with vertex on the same side and with same base PQ = 5 cm. In ΔPQR, QR = 4 cm and PR = 3 cm whereas in ΔPQS, sides PS = 6 cm and QS = 5cm.
7. Measure the sides of the given triangle and make a duplicate triangle on your own.



The Triangle Inequality Property

We've learned how to construct triangles when given three side lengths. But what if the lengths are 3 cm, 4 cm, and 8 cm? If you try to construct this triangle, you'll find the arcs don't meet! This brings us to a crucial rule in geometry: **Triangle Inequality Property**.

This property states that for any triangle, the sum of the lengths of any two sides must be greater than the length of the third side. It's a fundamental condition for a triangle to exist. We'll explore why this is true using real-world analogies and visualize it with circles.

Sub-concepts to be covered

1. Statement and meaning of the Triangle Inequality Property.
2. Real-world analogy: Shortest path between two points.
3. Visualizing triangle existence through intersecting circles.
4. Cases of circle intersection (intersecting, touching, non-intersecting) and their implications for triangle formation.

Statement and meaning of the Triangle Inequality Property

For any triangle, the sum of the lengths of any two sides is always greater than the length of the third side. If the sides of a triangle are **a**, **b**, and **c**, then:

- $a + b > c$
- $b + c > a$
- $a + c > b$

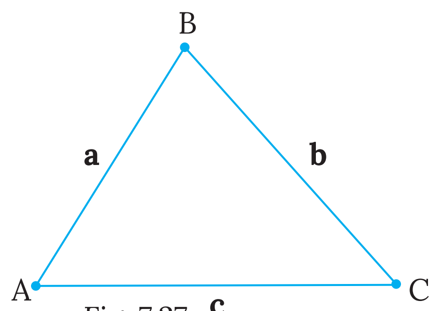


Fig. 7.27

Key points: All three conditions must be satisfied for a triangle to exist. If even one condition is not met, a triangle cannot be formed.

Common errors: Only checking one or two conditions, or confusing "greater than" with "greater than or equal to."

Real-world analogy: Shortest path between two points

Imagine walking from point A to point C. The shortest path is always a straight line directly from A to C. If you go from A to B and then from B to C, the total distance (AB + BC) will always be greater than or equal to the direct distance AC. In a triangle, if A, B, C are vertices, then AB + BC > AC. This intuitive understanding forms the basis of the triangle inequality.

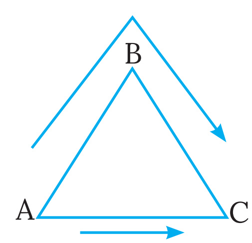


Fig. 7.28

Visualizing triangle existence through intersecting circles

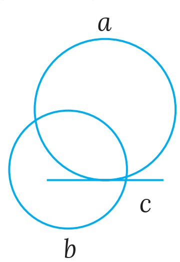
Let's imagine we have three sticks of lengths **a**, **b**, and **c**. We want to see if we can join them to make a triangle.

Set the Base: First, lay down the longest stick, **c**, as the base of your potential triangle.

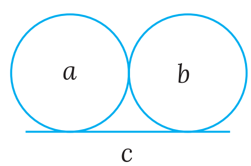
Reach Out! Now, imagine holding stick **a** at one end of the base **c**, and stick **b** at the other end of the base **c**.

Cases of circle intersection

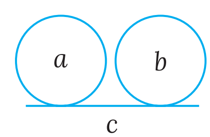
- **Intersecting Circles:** If stick **a** and stick **b** are long enough to reach across and cross each other in the air, you've found the third corner! You can form a triangle. This happens when the combined length of **a** and **b** is more than what's needed to just stretch across **c** (i.e., $a + b > c$). They overlap, giving you that meeting point. **Fig. 7.29**
- **Circles Touching Each Other:** What if stick **a** and stick **b** are just long enough to meet exactly at a single point on the line **c** itself? They don't cross above it; they just touch it. In this case, your 'triangle' will be completely flat, just a straight line. This happens when their combined length is exactly equal to **c** (i.e., $a + b = c$). It's like they've collapsed onto the base. **Fig. 7.30**
- **Circles Not Intersecting:** What if stick **a** and stick **b** are too short to even touch each other when held from the ends of **c**? They just dangle, unable to connect. In this situation, you cannot form a triangle. This happens when their combined length is less than 'c' (i.e., $a + b < c$). They simply can't bridge the gap! **Fig. 7.31**



Intersecting Circles
Fig. 7.29



Circle Touching Each Other
Fig. 7.30



Circle Not Intersecting
Fig. 7.31

When constructing a triangle using a compass and ruler:

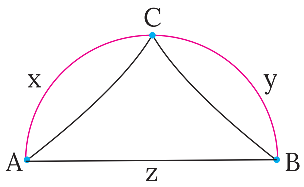
Let the given side lengths be x, y, and z.

1. Draw the longest side, say z, as the base AB.
2. Draw an arc with radius x centered at A.
3. Draw an arc with radius y centered at B.
   (i) If $x + y > z$: The arcs will intersect at two distinct points (above and below the base AB). Either intersection point can be chosen as the third vertex, forming a valid triangle. This confirms the triangle inequality. **(Fig. 7.32)**

   (ii) If $x + y = z$: The arcs will touch at exactly one point on the line segment AB. This means the **triangle** collapses into a straight line, and no true triangle is formed. **(Fig. 7.33)**

   (iii) If $x + y < z$: The arcs will not intersect at all. This means no third vertex can be found that satisfies the given side lengths, and thus no triangle can be formed. **(Fig. 7.34)**

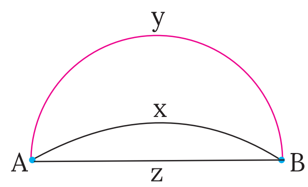
(i)



x + y > z

Fig. 7.32

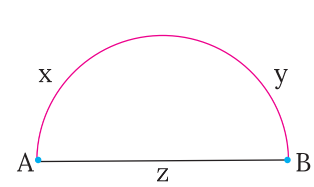
(ii)



x + y = z

Fig. 7.33

(iii)



x + y < z

Fig. 7.34

This visual and conceptual understanding reinforces why the triangle inequality is a necessary condition for triangle existence.

Example 1 : Check if a triangle can be formed with side lengths 7 cm, 10 cm, and 15 cm.

Solution: **Condition 1:** $7 + 10 > 15 \rightarrow 17 > 15 \rightarrow$ True

Condition 2: $10 + 15 > 7 \rightarrow 25 > 7 \rightarrow$ True

Condition 3: $7 + 15 > 10 \rightarrow 22 > 10 \rightarrow$ True

Conclusion: Since all three conditions are satisfied, a triangle can be formed with these side lengths.

Example 2 : Determine if a triangle can be formed with side lengths 5 cm, 8 cm, and 14 cm.

Solution: **Condition 1:** $5 + 8 > 14 \rightarrow 13 > 14 \rightarrow$ False

Conclusion: Since one condition is not satisfied (13 is not greater than 14), a triangle cannot be formed with these side lengths. We don't even need to check the other two conditions once one fails. But let's check once, if those are true or not.

Condition 2: $5 + 14 > 8 \rightarrow 19 > 8 \rightarrow$ True

Condition 3: $8 + 14 > 5 \rightarrow 22 > 5 \rightarrow$ True

Only one false is enough to disqualify triangle formation.

Conclusion: No triangle can be formed.

Example 3 : A student claims they constructed a triangle with sides 6 cm, 6 cm, and 12 cm. Is this possible?

Solution: **Condition 1:** $6 + 6 > 12 \rightarrow 12 > 12 \rightarrow$ False

Conclusion: No, it is not possible. The sum of the two shorter sides is equal to the third side, which means the "triangle" would collapse into a straight line.

Condition 2: $6 + 12 > 6 \rightarrow 18 > 6 \rightarrow$ True

Condition 3: $6 + 12 > 6 \rightarrow 18 > 6 \rightarrow$ True

One condition failed again (sum equals the third side). This creates a degenerate triangle (a straight line), not a true triangle.

Conclusion: No triangle can be formed.

Example 4 : Which of the following sets of lengths can form a triangle?

a) 4, 4, 6 b) 3, 4, 5 c) 1, 5, 5 d) 4, 6, 8 e) 3.5, 3.5, 3.5

Solution: **a) 4, 4, 6:**

$4 + 4 > 6 \rightarrow 8 > 6 \rightarrow$ True

$4 + 6 > 4 \rightarrow 10 > 4 \rightarrow$ True

Conclusion: Yes, a triangle can be formed. (It would be an isosceles triangle).

b) 3, 4, 5:

$3 + 4 > 5 \rightarrow 7 > 5 \rightarrow$ True

$3 + 5 > 4 \rightarrow 8 > 4 \rightarrow$ True

$4 + 5 > 3 \rightarrow 9 > 3 \rightarrow$ True

Conclusion: Yes, a triangle can be formed. (It would be a right-angled triangle, as $3^2 + 4^2 = 5^2$).

c) 1, 5, 5:

$1 + 5 > 5 \rightarrow 6 > 5 \rightarrow$ True

$5 + 5 > 1 \rightarrow 10 > 1 \rightarrow$ True

Conclusion: Yes, a triangle can be formed. (It would be an isosceles triangle).

d) 4, 6, 8:

$4 + 6 > 8 \rightarrow 10 > 8 \rightarrow$ True

$4 + 8 > 6 \rightarrow 12 > 6 \rightarrow$ True

$6 + 8 > 4 \rightarrow 14 > 4 \rightarrow$ True

Conclusion: Yes, a triangle can be formed.

e) 3.5, 3.5, 3.5:

$3.5 + 3.5 > 3.5 \rightarrow 7 > 3.5 \rightarrow$ True

Conclusion: Yes, a triangle can be formed. (It would be an equilateral triangle).

Knowledge Checkpoint

- If the sides of a triangle are 'x', 'y', and 'z', write down the three inequalities that must be true for the triangle to exist.
- Can a triangle have side lengths 1 cm, 2 cm, and 3 cm? Explain why.
- What happens during construction if the triangle inequality is not satisfied?

Activity

Triangle or Not?" Card Sort

Objectives: To develop reasoning skills by checking whether a given set of three side lengths can form a triangle.

Materials: Index cards with sets of three numbers written on them (e.g., (3, 4, 5), (2, 2, 5), (10, 15, 20), (7, 7 ,7), (6, 8, 10), (1, 2, 3), etc.).

Procedure:

1. Students work individually or in pairs.
2. Each student/pair receives a stack of cards.
3. Students sort the cards into two piles: "Can form a triangle" and "Cannot form a triangle."
4. For each card, they must write down the sum of the two shorter sides and compare it to the longest side to justify their sorting.
5. Discuss results as a class, focusing on the reasoning for each card.

Inquiry Focus: Reinforces the application of the Triangle Inequality Property and encourages systematic checking of conditions.

Key Terms

- **Triangle Inequality Property:** A fundamental theorem stating that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.
- **Degenerate Triangle:** A "**triangle**" where the three vertices lie on a single straight line, meaning the sum of two sides equals the third side. It does not form a true enclosed area.

Fact Flash

- The Triangle Inequality is not just for geometry! It's a fundamental concept in many areas of mathematics, including analysis and metric spaces, where it defines what a "**distance**" is.
- In some non-Euclidean geometries (like spherical geometry, on the surface of a sphere), the triangle inequality can be different. For example, on a sphere, the sum of two sides can be less than the third side if you consider great circle paths! But for Grade 7, we stick to flat Euclidean geometry.

Do It Yourself

- If you have two sides of a triangle, say 8 cm and 15 cm, what is the smallest possible integer length for the third side? What is the largest possible integer length?
- Can you think of a scenario in real life where ignoring the triangle inequality could lead to a struc tural failure or an impossible design?

Mental Mathematics

- Sides are 5, 7, 11. Is it a triangle?
- Sides are 2, 2, 4. Is it a triangle?
- If two sides are 6 and 9, what is the smallest whole number the third side can be?
- If two sides are 6 and 9, what is the largest whole number the third side can be?

Gap Analyzer™ Homework

Watch Remedial

Exercise 7.2

1. **Which of the following sets of lengths can form a triangle (given in cm)?**

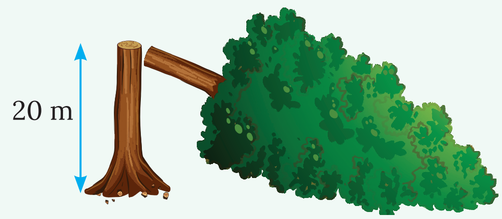
a) 2, 3, 5	b) 5, 7, 10	c) 1, 2, 4	d) 6, 6, 12	e) 8, 9, 15
f) 7, 7, 7	g) 3, 6, 9	h) 4, 5, 6	i) 10, 10, 1	j) 2, 9, 10

2. **Which of the following statements about forming a triangle with given side lengths is true?**
   a) A triangle can always be formed if one side is very small.
   b) If the sum of the two smaller sides is equal to the largest side, a triangle can be formed.
   c) The sum of any two sides must be greater than the third side for a triangle to be formed.
   d) A triangle can only be formed if all three sides are equal.
   e) The difference between any two sides must be greater than the third side.

3. **Fill in the Blanks:**
   a) According to the Triangle Inequality Property, the sum of any two sides of a triangle must be ______ than the third side.
   b) If the sum of two sides is equal to the third side, the points are ______.
   c) If the arcs drawn during construction do not intersect, it means the triangle ______ exist.

4. Rubeena wants to know if the triangle is possible when one of the two equal sides of an isosceles triangle is 4 cm and the third side is 9 cm.

5. In Ram's garden, a tree of total height 65 m broke in such a way that its top half fell to the ground while still attached at the break point. The height of the standing vertical portion is 20 m. Find the possible values of the horizontal distance between the base of the tree and the point where the top touches the ground.



6. The sides of a triangle are in the ratio 2:3:5. If the sum of all three sides is 50 cm, then is the triangle possible?

7. Fill in the below table and determine if the triangle is possible or not:

S.No.	Perimeter	Side AB	Side BC	Triangle possible or not
i.	43	13	13	
ii.	65	12	19	
iii.	87	42	11	
iv.	98	29	45	

Construction with Given Sides and Angles (SAS & ASA Criteria)

So far, we've constructed triangles using only side lengths. But what if we know some side lengths AND some angle measures? This opens up new ways to construct triangles. We'll explore two powerful criteria: SAS (Side-Angle-Side), where two sides and the angle between them are known, and ASA (Angle-Side-Angle), where two angles and the side between them are known. These methods are crucial for constructing specific triangles and understanding their uniqueness.

Sub-concepts to be covered

1. Construction using SAS (Side-Angle-Side) criterion.
2. Construction using ASA (Angle-Side-Angle) criterion.
3. Conditions for existence of triangles based on angle sums.

Construction using SAS (Side-Angle-Side) Criterion

If two sides and the included angle (the angle formed by those two sides) are given, a unique triangle can be constructed.

Key points: The angle must be the one between the two given sides.

Tools: Ruler, protractor, pencil.

Common errors: Using a non-included angle, inaccurate angle measurement with protractor.

Construction using ASA (Angle-Side-Angle) Criterion

If two angles and the included side (the side common to both angles) are given, a unique triangle can be constructed.

Key points: The side must be the one connecting the vertices of the two given angles.

Tools: Ruler, protractor, pencil.

Common errors: Using a non-included side, inaccurate angle measurement.

Conditions for Existence of Triangles based on Angle Sums

While SAS and ASA generally lead to unique triangles, there's a critical condition related to angles: the sum of the angles in any triangle must be 180°.

For ASA construction: If the sum of the two given angles is 180° or more, the lines forming the third vertex will never intersect, and thus no triangle can be formed. For example, if angles are 90° and 90°, they form parallel lines.

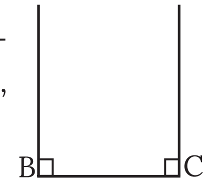


Fig. 7.35

Key points: The sum of the two given angles must be less than 180°.
Common errors: Attempting to construct a triangle with angle sums ≥ 180°.

Mathematical Explanation

The SAS and ASA criteria are fundamental for proving triangle congruence (which you'll learn in higher grades) and for constructing unique triangles.

SAS Construction

To construct a triangle ABC where AB = **c** cm, AC = **b** cm, and ∠A = **x** degrees:
Step 1: Draw a **line segment** AB of length 'c' cm. This forms one side of the triangle.
Step 2: At point A, use a **protractor** to draw a ray AX such that ∠BAX = **x** degrees. This ray represents the direction of the second side.
Step 3: On ray **AX**, measure and mark a point **C** such that AC = **b** cm. This fixes the length of the second side.
Step 4: Join points **B** and **C**. This completes the third side, **BC**, and forms **ΔABC**.

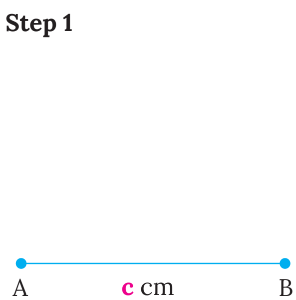


Fig. 7.36

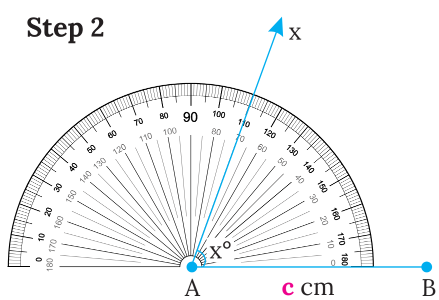


Fig. 7.37

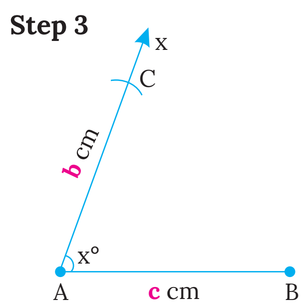


Fig. 7.38

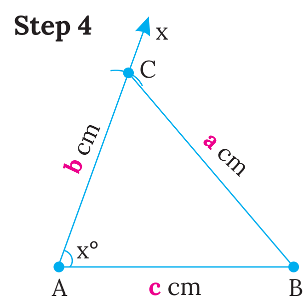


Fig. 7.39

This method works because the two sides and the angle between them uniquely define the shape and size of the triangle.

Let us say, in a triangle ABC, the lengths of the sides are AB = 4 cm. AC = 7 cm and ∠CAB = 60°. The steps for its construction are:

Step 1: Draw a straight line and mark its left endpoint as A. Set the compass to a width of 4 cm. Place the pointer head of the compass at A and cut an arc on the line. Mark the point as B where the arc crosses the line. **Fig. 7.40**

Step 2: Construct a 60° angle with line AB at point A. Set the compass to a width at 7 cm. **Fig. 7.41**

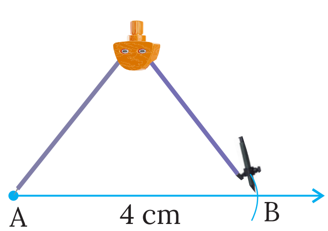


Fig. 7.40

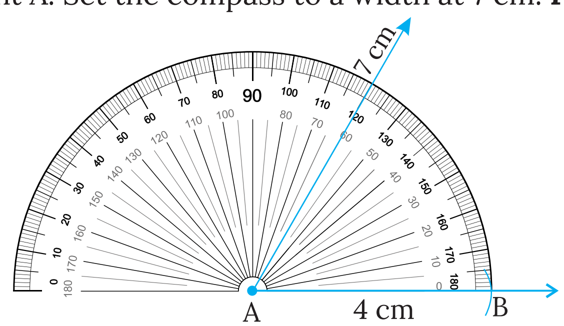


Fig. 7.41

Step 3: Place the pointer head of the compass at A and cut an arc on the 60° line. **Fig. 7.42**

Step 4: Mark the point as C where the arc crosses the line. Join points B and C using a ruler. **Fig. 7.43**

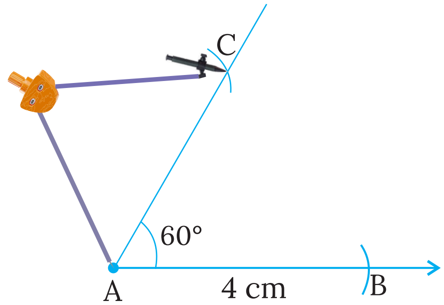


Fig. 7.42

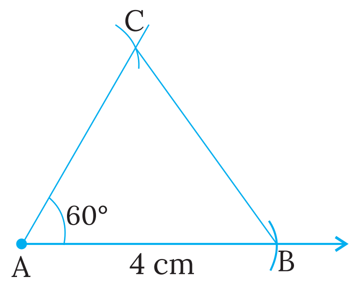


Fig. 7.43

ASA Construction

To construct a triangle ABC where AB = **c** cm, ∠A = **x** degrees, and ∠B = **y** degrees: image

Step 1: Draw a line segment AB of length 'c' cm. This forms the included side.

Step 2: At point A, use a protractor to draw a ray AX such that ∠BAX = **x** degrees.

Step 3: At point B, use a protractor to draw a ray BY such that ∠ABY = **y** degrees.

Step 4: The point where rays AX and BY intersect is the third vertex, C.

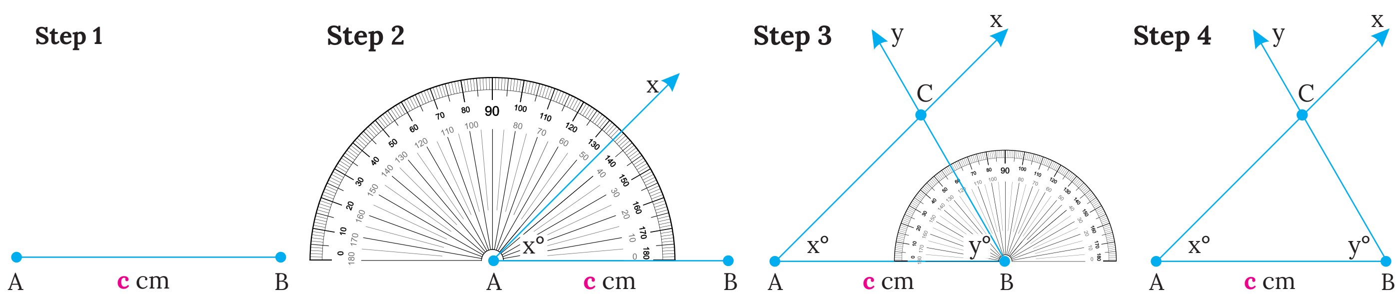


Fig. 7.44

This intersection point C is unique, provided the rays are not parallel or diverging. This condition is directly related to the Angle Sum Property: for the rays to intersect and form a triangle, the sum of ∠A and ∠B must be less than 180°. If ∠A + ∠B ≥ 180°, the rays will either be parallel or diverge, and no triangle will be formed.

For the ASA criterion to be satisfied, the given side must necessarily be the one that is enclosed between the known angles.

Now coming back to constructing triangles with the ASA criterion, you will require a ruler and a protractor. Let us say, in a triangle ABC, the measure of the angles are ∠CAB = 75° and ∠ABC = 60°. The length of side AB = 3 cm. The steps for its construction are as follows:

Step 1: Using the ruler, construct a line segment AB of length 3 cm. **Fig. 7.45**

Step 2: Using the protractor, draw a ray at point B making 60° with the line BA. **Fig. 7.46**

Fig. 7.45

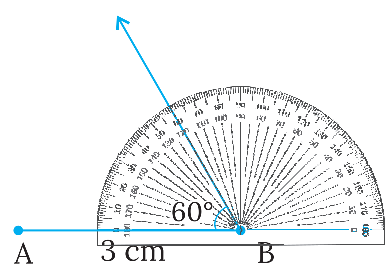


Fig. 7.46

Step 3: Similarly draw a ray at point A making 75° with the line AB using the protractor. **Fig. 7.47**

Step 4: Mark the point where the two rays meet as C. **Fig. 7.48**

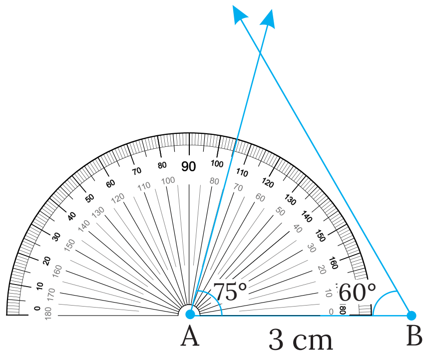


Fig. 7.47

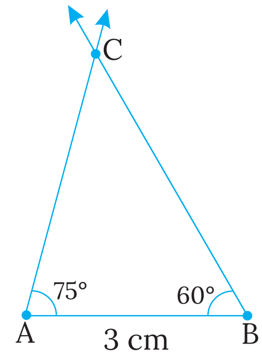


Fig. 7.48

Example (SAS): Construct a triangle PQR with PQ = 6 cm, QR = 5 cm, and ∠Q = 70°.

Step 1: Draw a line segment PQ of length 6 cm. **Fig. 7.49**

Explanation: This is one of the given sides.

Step 2: At point Q, place the protractor and draw a ray QX making an angle of 70° with PQ. **Fig. 7.50**

Explanation: This sets the direction for the second side, QR, at the given included angle.

Fig. 7.49

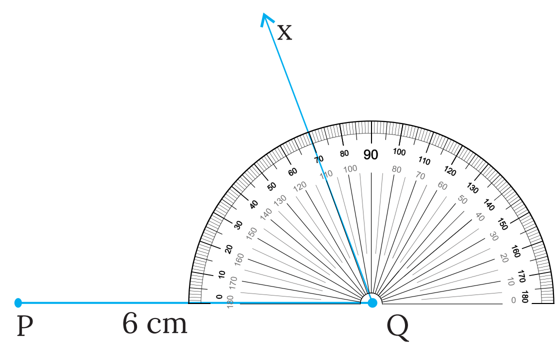


Fig. 7.50

Step 3: With Q as the center, open the compass to a radius of 5 cm (length of QR). Mark a point R on the ray QX. **Fig. 7.51**

Explanation: This fixes the length of the second side.

Step 4: Join P and R. **Fig. 7.52**

Explanation: This completes ΔPQR.

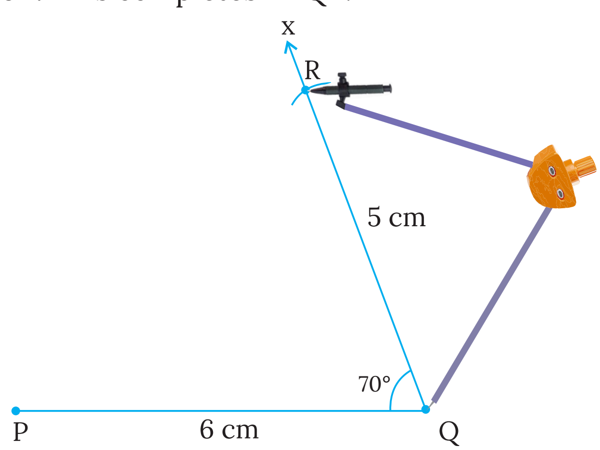


Fig. 7.51

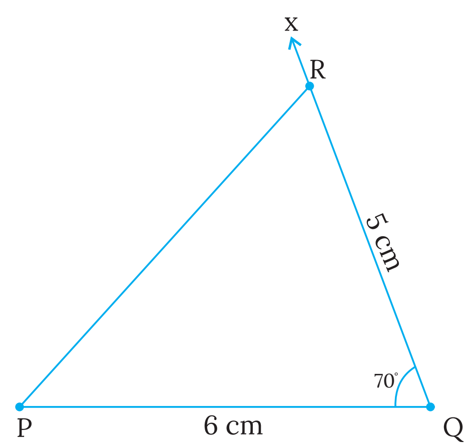


Fig. 7.52

Example (ASA): Construct a triangle XYZ with YZ = 7 cm, ∠Y = 40°, and ∠Z = 60°.

Step 1: Draw a line segment YZ of length 7 cm. **Fig. 7.53**

Explanation: This is the included side.

Step 2: At point Y, place the protractor and draw a ray YA making an angle of 40° with YZ. **Fig. 7.54**

Explanation: This sets the direction for side XY.

Fig. 7.53

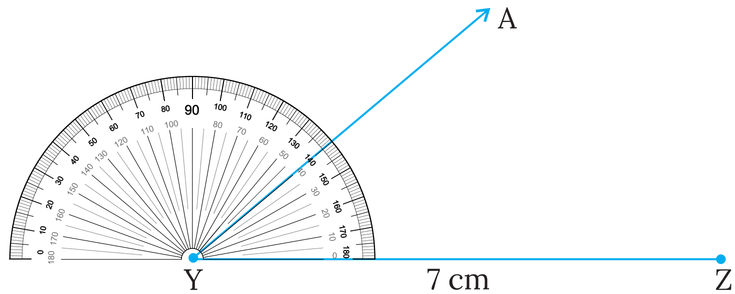


Fig. 7.54

Step 3: At point Z, place the protractor and draw a ray ZB making an angle of 60° with ZY (on the same side as ray YA). **Fig. 7.55**

Explanation: This sets the direction for side XZ.

Step 4: The point where rays YA and ZB intersect is X. **Fig. 7.56**

Explanation: This completes ΔXYZ.

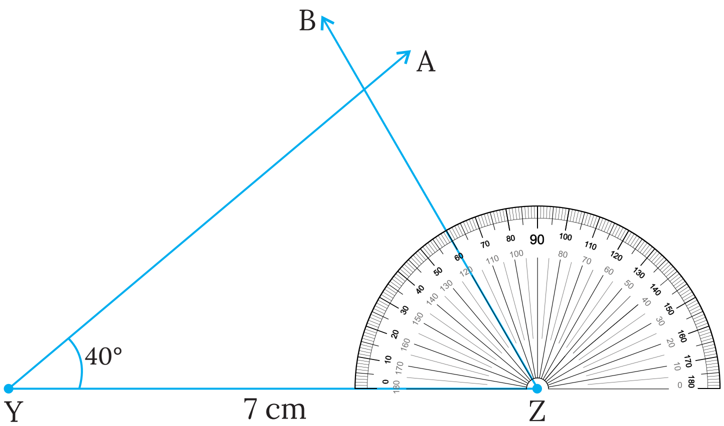


Fig. 7.55

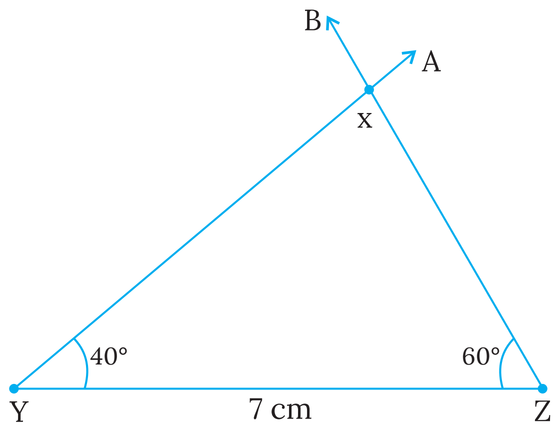


Fig. 7.56

Example: Can you construct a triangle ABC with AB = 8 cm, ∠A = 100°, and ∠B = 90°?

Step 1: Draw AB = 8 cm. **Fig. 7.57**

Step 2: Draw a ray from A at 100° to AB. **Fig. 7.58**

Fig. 7.57

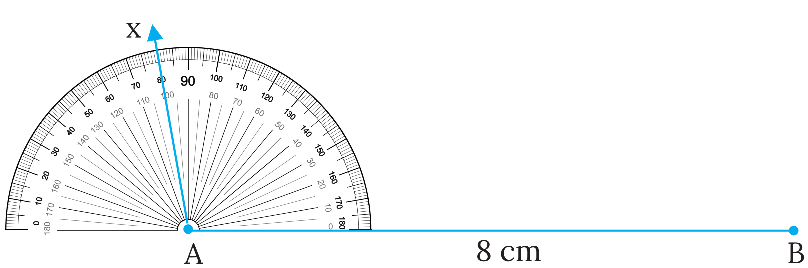


Fig. 7.58

Step 3: Draw a ray from B at 90° to AB. **Fig. 7.59**

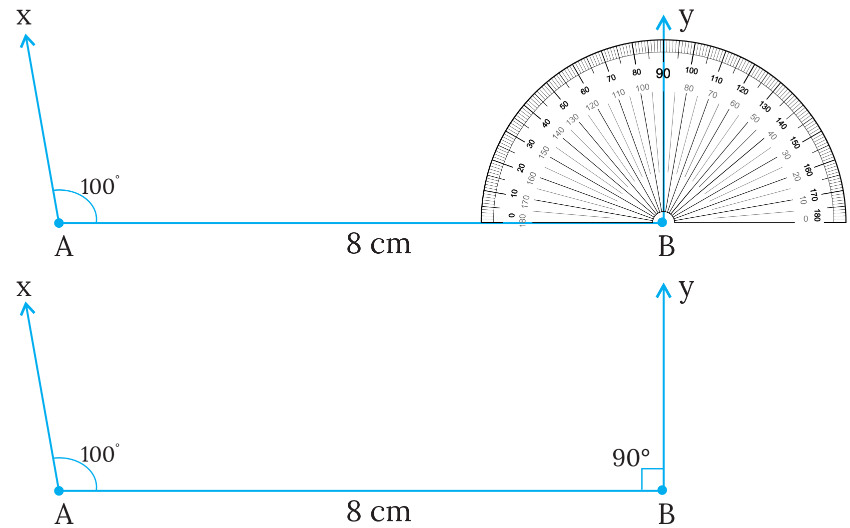


Fig. 7.59

Observation: The two rays diverge and do not intersect.

Reasoning: The sum of the two given angles is 100° + 90° = 190°. Since 190° is greater than 180°, the lines will not intersect to form a triangle.

Conclusion: No, a triangle cannot be constructed with these measurements.

Example (SAS): Construct a triangle DEF with DE = 4 cm, EF = 6 cm, and ∠E = 110°.

Step 1: Draw a line segment DE of length 4 cm. **Fig. 7.60**

Step 2: At point E, draw a ray EX making an angle of 110° with DE. **Fig. 7.61**

Fig. 7.60

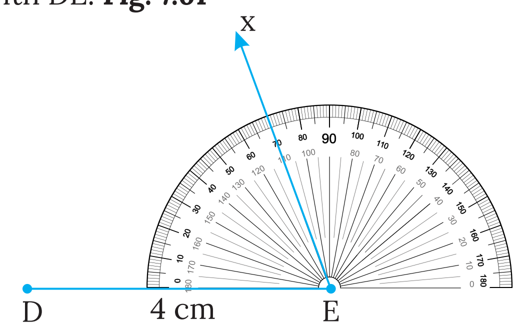


Fig. 7.61

Step 3: With E as the center, open the compass to a radius of 6 cm. Mark a point F on the ray EX. **Fig. 7.62**

Step 4: Join D and F. This completes ΔDEF. **Fig. 7.63**

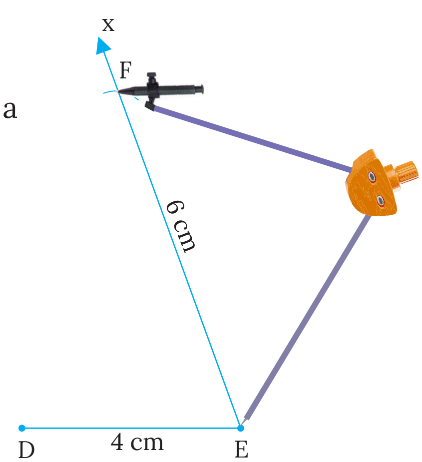


Fig. 7.62

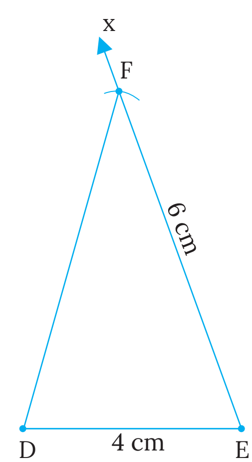


Fig. 7.63

Activity

Triangular Wall Hanging

Objectives: To develop accuracy and precision in constructing triangles using the ASA and SAS methods.

Materials: Cardboard, rulers, protractors, pencils, scissors, decorative material like colorful sheet, mirrors, adhesive,etc.

Procedure:

1. **Prepare the Base:**
   - Cut out a rectangular piece of cardboard to work on
2. **Choose Construction Type:**
   - Teacher assigns half the students ASA and the other half SAS (or each student makes both).
3. **Construct Triangle (ASA method):**
   - Draw a base line of given length (e.g., 6 cm).
   - At one end of the base, use the protractor to construct the given angle (say 45°).
   - At the other end of the base, construct the second given angle (say 60°).
   - Extend the rays; they meet at the third vertex.
   - Triangle is formed using ASA.
4. **Construct Triangle (SAS method):**
   - Draw one side of given length (e.g., 7 cm).
   - At one end of this side, draw the given angle (say 50°) using protractor.
   - On this angle's arm, measure and mark the second side (say 5 cm).
   - Join the marked point to the other endpoint of the base.
   - Triangle is formed using SAS.
5. **Cut Out the Triangle:**
   - Carefully cut out the triangle from cardboard along the drawn lines.
6. **Decorate:**
   - Paste colorful sheet, mirrors, or fabric on the triangle.
   - Add borders, glitter, or patterns to make it attractive.
7. **Make It a Wall Hanging:**
   - Punch a small hole at the top vertex.
   - Tie a ribbon/string through the hole.
   - Your ASA/SAS triangular wall hanging is ready!

Inquiry Focus: Develops precision in construction, reinforces the conditions for triangle existence, and encourages measurement and verification.

Knowledge Checkpoint

- What does SAS stand for in triangle construction?
- What does ASA stand for in triangle construction?
- If you are given two angles of 70° and 120° and an included side, can you construct a triangle? Why or why not?

Key Terms

- **Included Angle:** The angle formed by two specific sides of a triangle.
- **Included Side:** The side that is common to two specific angles of a triangle.
- **SAS Criterion:** A rule for constructing a unique triangle when two sides and the included angle are known.
- **ASA Criterion:** A rule for constructing a unique triangle when two angles and the included side are known.
- **Protractor:** A geometric tool used for measuring and drawing angles.

Fact Flash

- The SAS and ASA criteria are not just for construction; they are also two of the main criteria used to prove that two triangles are congruent (identical in shape and size).
- The concept of "**included**" is very important. If the angle is not included (e.g., SSA - Side-Side-Angle), it might be possible to construct two different triangles, or no triangle at all! This is known as the "**ambiguous case**" in trigonometry.

Do It Yourself

- If you are given two sides and a non-included angle (e.g., sides 5 cm, 7 cm, and an angle of 30° not between them), do you think you can always construct a unique triangle? Try it!
- Why is it impossible to construct a triangle if the sum of two angles is 180°? What kind of lines would be formed?

Mental Mathematics

- A triangle has sides 4 cm, 5 cm, and an included angle of 90°. What type of triangle is it?
- A triangle has angles 30° and 60°, and the included side is 10 cm. What is the third angle?
- If you are given two sides and an included angle, how many measurements are you given in total?

Gap Analyzer™ Homework

Watch Remedial

Exercise 7.3

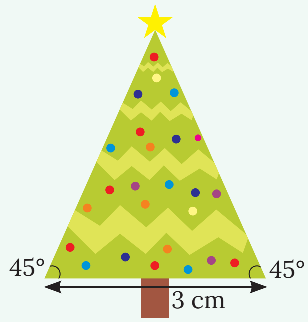
1. **Fill in the Blanks:**

   a) In SAS criterion, the angle must be ______ the two given sides.

   b) In ASA criterion, the side must be ______ to both given angles.

   c) For a triangle to be constructed using ASA criterion, the sum of the two given angles must be ______ 180°.

2. **For each of the following sets of measurements, state whether a unique triangle can be constructed using the SAS or ASA criterion. If yes, name the criterion. If no, explain why not.**
   a) Triangle PQR: PQ = 6 cm, QR = 8 cm, ∠Q = 70°
   b) Triangle ABC: AB = 5 cm, BC = 7 cm, ∠A = 40°
   c) Triangle XYZ: ∠X = 60°, ∠Y = 80°, XY = 7 cm
   d) Triangle LMN: ∠L = 50°, ∠M = 70°, MN = 4 cm
3. **Jaiswal wants to make a Christmas tree shape bookmark. He wants to take the base of the triangle as 3 cm and angles on each side to be 45°. Make a sketch for him to cut it out.**



4. Construct a triangle with sides 8 cm and 5 cm. The angle between them is 65°. Measure the other two angles formed in the triangle.
5. A drone is programmed to fly horizontally and at point A it turns 45°, fly 100 meters to point B, then turn 70° and fly to point C. If the distance AB is 100 meters, describe how you would represent this flight path on a diagram.
6. A carpenter is building a triangular shelf. He has two pieces of wood, 30 cm and 40 cm long. Keeping base as 40 cm, he wants the angle between them to be 95°. Can he make this shelf? If so, how would he determine the length of the third side?
7. You are designing a kite. One part of the kite is a triangle with two sides measuring 25 cm and 35 cm, and the angle between them is 130°. How would you construct this part of the kite?
8. **A student was given the task to construct a triangle GHI with GH = 8 cm, ∠G = 60°, and ∠H = 90°.**
   (a) Can such a triangle be constructed?
   (b) If yes, then draw the triangle and the steps of construction.
9. **A student wants to construct a triangle ABC. They are given two side lengths: AB = 7 cm and BC = 9 cm.**
   (a) What additional information (one piece only) is essential to construct a unique triangle using the SAS criterion?
   (b) If they wanted to use the ASA criterion, what information would ideally be given instead? Explain why.

Angle Sum Property and Exterior Angle Property

Triangles have some amazing properties that are true for all triangles, regardless of their size or shape. Two of the most important are the Angle Sum Property and the Exterior Angle Property. The Angle Sum Property states that the sum of the interior angles of any triangle is always 180°. The Exterior Angle Property relates an exterior angle to the two opposite interior angles. These properties are incredibly useful for finding unknown angles without having to measure them.

Sub-concepts to be covered

1. Angle Sum Property of a triangle (sum of interior angles is 180°).
2. Proof of Angle Sum Property using parallel lines.
3. Verification of Angle Sum Property using paper folding.
4. Exterior Angle of a triangle.
5. Exterior Angle Property (relation to interior opposite angles).

Angle Sum Property of a triangle

The sum of the measures of the three interior angles of any triangle is always 180°.

For ΔABC, $\angle A + \angle B + \angle C = 180°$.

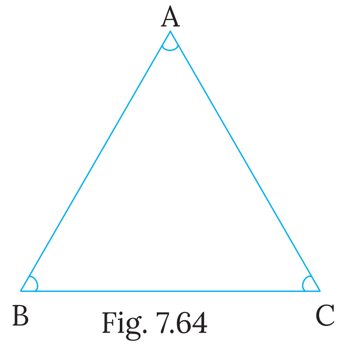


Fig. 7.64

Key points: This property holds true for all types of triangles (equilateral, isosceles, scalene, acute, right, obtuse).

Common errors: Assuming the sum is 360° (like a quadrilateral), or miscalculating when finding a missing angle.

A diagram of a triangle ABC with a line XY drawn through A parallel to BC, clearly showing alternate interior angles ($\angle XAB = \angle B$, $\angle YAC = \angle C$) and the straight angle at A.

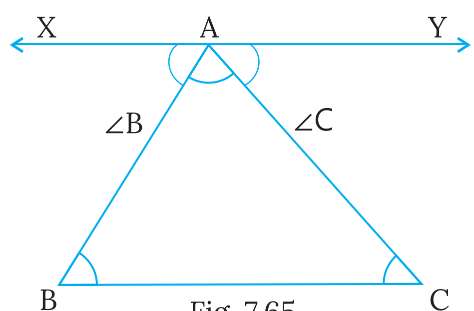


Fig. 7.65

Proof of Angle Sum Property using parallel lines

This is a classic geometric proof.

1. Draw a triangle ABC.
2. Draw a line XY passing through vertex A and parallel to the base BC.

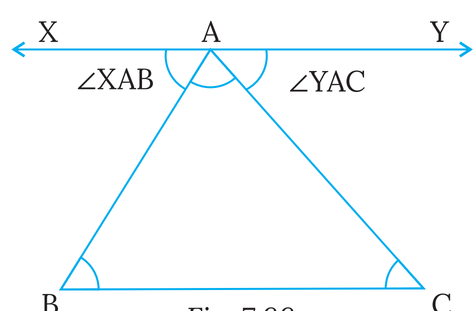


Fig. 7.66

3. Using properties of parallel lines and transversals:
   - ∠XAB = ∠ABC (Alternate interior angles, with AB as transversal).
   - ∠YAC = ∠ACB (Alternate interior angles, with AC as transversal).
4. Since XAY is a straight line, ∠XAB + ∠BAC + ∠CAY = 180°.
5. Substituting the alternate interior angles: ∠ABC + ∠BAC + ∠ACB = 180°.

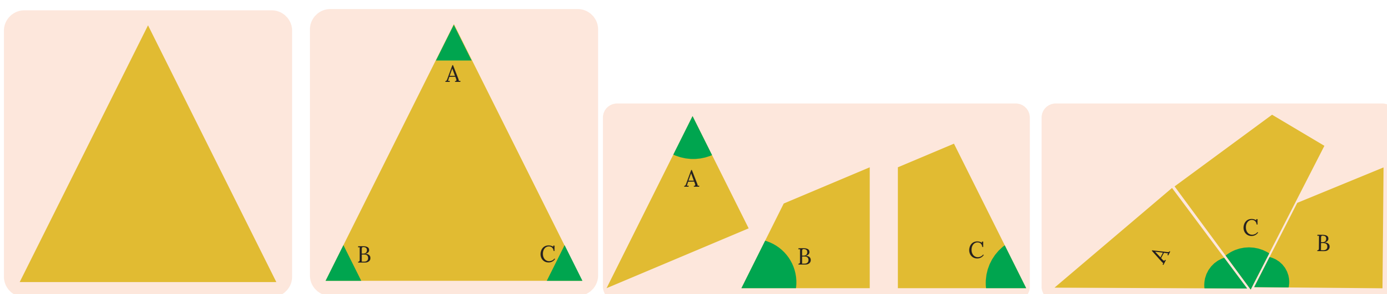
Key points: This proof elegantly connects the properties of parallel lines to the properties of triangles.

Verification of Angle Sum Property using paper folding

Steps:

1. **Cut a Triangle:** Take a piece of paper and cut out any triangle (scalene, isosceles, or equilateral).
2. **Label the Angles:** Mark the three corners (vertices) of the triangle as A, B, and C.
3. **Fold the Corners:** Carefully fold each corner (angle) towards the opposite side so that the tip touches the center of the triangle. This helps identify each angle separately. Tear or cut the three corners along the fold lines, separating the three angles from the triangle.
4. **Align on a Straight Line:** Place the three angle pieces adjacent to each other on a straight edge (like the edge of a ruler or paper strip), making sure the vertices touch and fit without gaps.

- **Observe the Angle Formation:** You will notice that the three angles together form a straight angle (a straight line), which measures 180°.



Step 1 Step 2 Fig. 7.67 Step 3 Step 4

Conclusion: This demonstrates that the sum of the three interior angles of any triangle is 180°, which is the Angle Sum Property of a Triangle.

Exterior Angle of a triangle

When one side of a triangle is extended, the angle formed outside the triangle is called an exterior angle. Each vertex has two exterior angles, which are vertically opposite and thus equal.

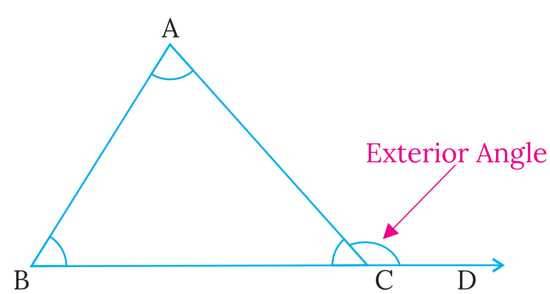


Fig. 7.68

For ΔABC, if side BC is extended to D, then ∠ACD is an exterior angle.

Key points: An exterior angle and its adjacent interior angle form a linear pair (sum to 180°).

Exterior Angle Property

An exterior angle of a triangle is equal to the sum of its two interior opposite angles.
For ΔABC with exterior angle ∠ACD: ∠ACD = ∠BAC + ∠ABC (or ∠A + ∠B).

Key points: This property is a direct consequence of the Angle Sum Property and linear pairs. It provides a shortcut for finding exterior angles or interior angles.

Common errors: Confusing interior opposite angles with the adjacent interior angle.
A diagram of a triangle ABC with side BC extended to D, clearly showing the exterior angle ∠ACD and the interior opposite angles ∠A and ∠B.

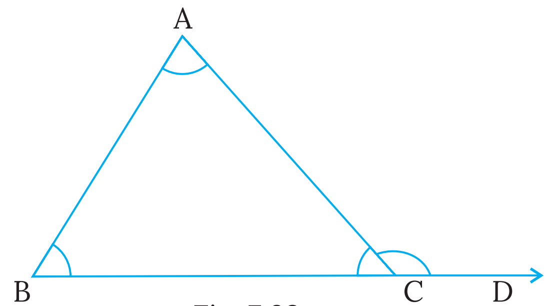


Fig. 7.69

Mathematical Explanation

Angle Sum Property (Proof): Let's consider ΔABC.
Draw a line XY parallel to BC passing through A.

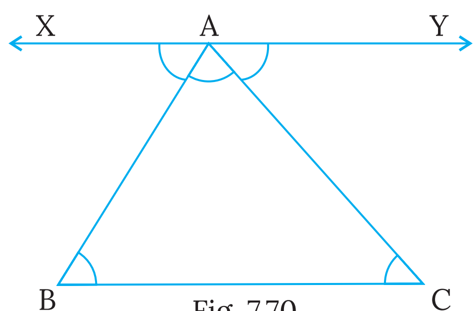


Fig. 7.70

- Since XY || BC and AB is a transversal, ∠XAB = ∠ABC (alternate interior angles).
- Since XY || BC and AC is a transversal, ∠YAC = ∠ACB (alternate interior angles).
- Angles on a straight line sum to 180°. So, ∠XAB + ∠BAC + ∠CAY = 180°.
- Substitute the alternate interior angles: ∠ABC + ∠BAC + ∠ACB = 180°.

Therefore, the sum of the angles of any triangle is 180°.

Example 5 : Find the missing angle in a triangle if two angles are 55° and 75°.

Solution: Let the angles be A, B, and C. We know A = 55°, B = 75°.
By Angle Sum Property: A + B + C = 180°
55° + 75° + C = 180°
130° + C = 180°
C = 180° - 130°
C = 50°

Conclusion: The missing angle is 50°.

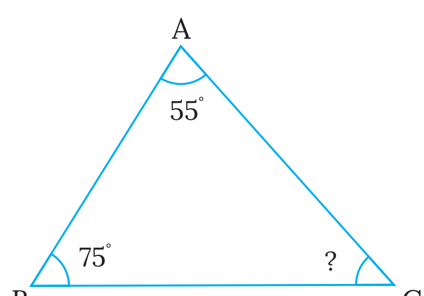


Fig. 7.71

Example 6 : In ΔPQR, if ∠P = 90° and ∠Q = 45°, find ∠R.

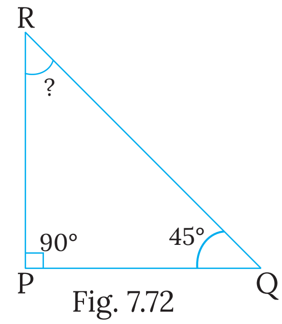


Fig. 7.72

Solution: P + Q + R = 180°

90° + 45° + R = 180°

135° + R = 180°

R = 180° - 135°

R = 45°

Conclusion: ∠R = 45°. (This is an isosceles right-angled triangle).

Exterior Angle Property (Proof): Consider ΔABC. Extend side BC to point D, forming exterior angle ∠ACD.

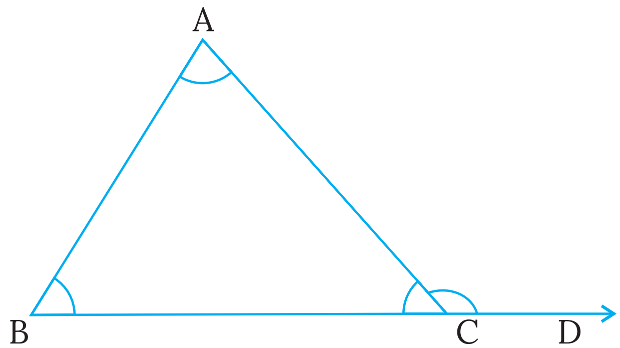


Fig. 7.73

- By Angle Sum Property: ∠BAC + ∠ABC + ∠ACB = 180° (Equation 1)
- Angles on a straight line: ∠ACB + ∠ACD = 180° (Equation 2)
- From Equation 1, ∠BAC + ∠ABC = 180° - ∠ACB.
- From Equation 2, ∠ACD = 180° - ∠ACB.
- Comparing the two results, we get: ∠ACD = ∠BAC + ∠ABC.

This means the exterior angle is equal to the sum of the two interior opposite angles.

Example 7 : In ΔXYZ, side YZ is extended to W. If ∠X = 60° and ∠Y = 70°, find the exterior angle ∠XZW.

Solution: By Exterior Angle Property: ∠XZW = ∠X + ∠Y

∠XZW = 60° + 70°

∠XZW = 130°

Verification (using Angle Sum Property):

First find ∠Z (interior): ∠X + ∠Y + ∠Z = 180°

60° + 70° + ∠Z = 180°

130° + ∠Z = 180°

∠Z = 50°.

Then,

∠XZW + ∠Z = 180° (linear pair)

∠XZW + 50° = 180°

∠XZW = 130°.

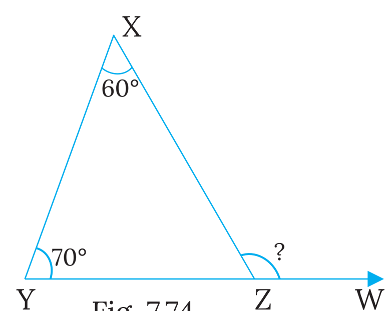


Fig. 7.74

Conclusion: The exterior angle ∠XZW is 130°.

Example 8 : Side PR of ΔPQR is extended to S and side QP is extended to T. Find the value of $\angle PRQ$.

Solution: Since, side QP is extended to T, so QPT is a straight line.

So, $\angle TPS + \angle QPS = 180°$ (linear pair)

$65° + \angle QPS = 180°$

$\angle QPS = 180° - 65°$

$\angle QPS = 115°$

Now, one of the interior opposite angle is $\angle Q = 70°$ and opposite exterior angle is 65°.

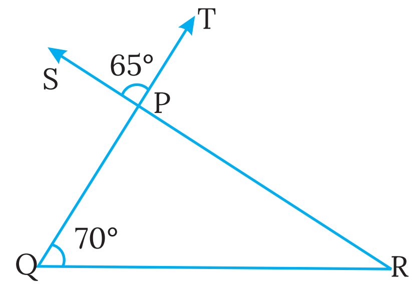


Fig. 7.75

By Exterior Angle Property: $\angle QPS = \angle PQR + \angle PRQ$

$115° = 70° + \angle PRQ$

$\angle PRQ = 115° - 70°$

$\angle PRQ = 45°$

Conclusion: The value of $\angle PRQ$ is 45°.

Activity

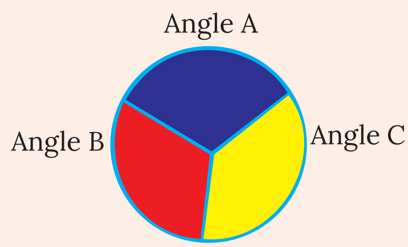
Angle Sum Discovery

Objectives: To help students discover and verify that the sum of the angles of a triangle is always 180°.

Materials: Paper triangles (pre-cut, various shapes: acute, obtuse, right-angled), scissors, rulers, protractors.

Procedure:

1. Each student receives a paper triangle.
2. Method 1 (Measurement): Students measure each angle of their triangle using a protractor and record the measurements. Then, they sum the three angles. They repeat this for 2-3 different triagles.
3. Method 2 (Folding/Tearing): Students tear off the three corners of one of their triangles and arrange them adjacent to each other on a straight line drawn on their notebook. They observe the result.
4. Discussion: Compare results from Method 1 (should be close to 180°) and Method 2 (should form a straight line). Discuss why Method 2 is a more general proof.



Inquiry Focus: Provides empirical evidence for the Angle Sum Property and introduces a visual proof.

Knowledge Checkpoint

- What is the sum of angles in any triangle?
- If an exterior angle of a triangle is 100°, and one interior opposite angle is 40°, what is the other interior opposite angle?
- Can a triangle have angles 30°, 50°, and 100°? Why or why not?

Key Terms

- **Angle Sum Property:** The theorem stating that the sum of the interior angles of any triangle is 180°.
- **Exterior Angle:** An angle formed by one side of a triangle and the extension of an adjacent side.
- **Interior Opposite Angles:** The two interior angles of a triangle that are not adjacent to a given exterior angle.
- **Linear Pair:** Two adjacent angles that form a straight line, summing to 180°.
- **Alternate Interior Angles:** A pair of angles on opposite sides of the transversal and between the two parallel lines.

Do It Yourself

- If you have a triangle with one angle greater than 90° (an obtuse angle), what can you say about the other two angles? Can they also be obtuse?
- If all three angles of a triangle are acute (less than 90°), what is the maximum possible value for any one of those angles?

Fact Flash

- The Angle Sum Property (that angles sum to 180°) is a defining characteristic of Euclidean geometry (the geometry on a flat surface). In other geometries (like on the surface of a sphere or a saddle), the sum of angles in a triangle can be more or less than 180°!
- The exterior angle property is very useful in proving other geometric theorems, making it a powerful tool in geometry.

Mental Mathematics

- Two angles of a triangle are 40° and 60°. What is the third angle?
- An exterior angle is 110°. One interior opposite angle is 50°. What is the other?
- If a triangle has angles x, 2x, and 3x, what is the value of x?

Gap Analyzer™ Homework

Watch Remedial

Exercise 7.4

1. **Fill in the Blanks:**
   a) The sum of the three angles of any triangle is always _______ degrees.
   b) An exterior angle of a triangle is equal to the sum of its two _______ angles.
   c) An exterior angle and its adjacent interior angle form a _______ pair.
2. A ramp is built in the shape of a right-angled triangle. If one acute angle is 35°, what is the measure of the other acute angle?
3. A kite is designed with a triangular section. If two angles of this section are 45° and 100°, what is the third angle?

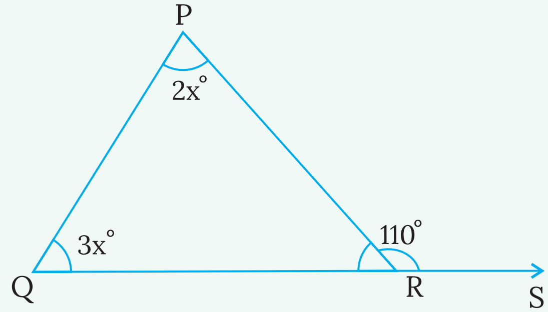
4. A surveyor measures an exterior angle of a triangular plot to be 115°. If one of the interior opposite angles is 60°, what is the other interior opposite angle?
5. A triangle has one angle six less than double of second angle and third angle is 10 less than the first angle. Determine the value of all three angles.

6 In triangle PQR, the measure of angle P is 2x°, angle Q is 3x°, and the exterior angle at R is 110°.

a) Find the measure of the interior angle R of triangle PQR.

b) Using the exterior angle property, set up an equation and solve for the value of x.

c) Calculate the measures of angle P and angle Q.



7. The angles of a triangle are in the ratio 2:3:4.

a) Find the value of common ratio y.

b) Calculate the measure of each angle in the triangle.

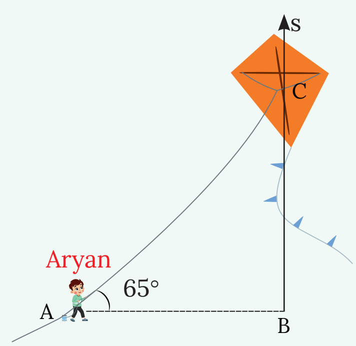
c) If the largest angle in this triangle is an interior angle, what would be the measure of its corresponding exterior angle?

8. Aryan was flying a kite. His kite was making an angle of 65° with the ground.

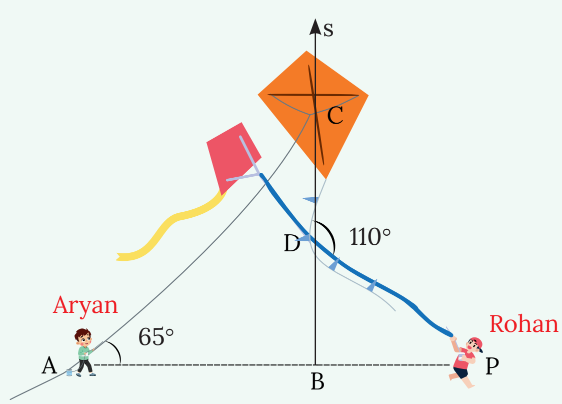
a) What will be the value of ∠ABC.

b) What will be the value of ∠ACB.

c) If side BC is extended to S, then what will be the value of ∠ACS.



If his friend Rohan also joins him and their kites intersect at point D making an angle of 110°. Determine the value of ∠BPD.



Altitudes of Triangles and Types of Triangles

Beyond sides and angles, triangles have other important features. One such feature is an altitude, which represents the "height" of a triangle from a specific vertex to its opposite side. Understanding altitudes is crucial for calculating the area of a triangle and for classifying triangles based on their angles. We will also revisit and formalize the classification of triangles based on both their side lengths and angle measures, connecting these classifications to the properties we've learned.

Sub-concepts to be covered

1. Altitude of a triangle
2. Construction of altitudes using a set square and ruler.

Altitude of a Triangle

An altitude of a triangle is a perpendicular line segment drawn from a vertex to its opposite side (or to the extension of the opposite side). The length of the altitude is the height of the triangle with respect to that base. Every triangle has three altitudes.

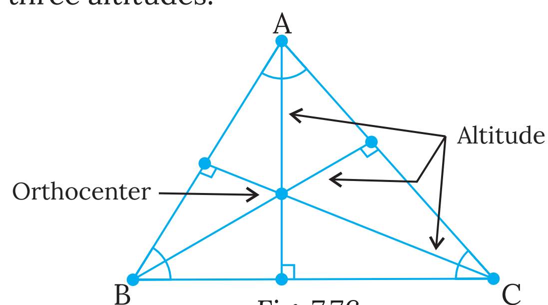


Fig. 7.76

Key points: An altitude must be perpendicular (form a 90° angle) to the base.

Common errors: Drawing a median (line to midpoint) or angle bisector instead of a perpendicular.

Construction of altitudes using a set square and ruler.

Altitudes are constructed using a set square and a ruler to ensure the perpendicularity.

Align the ruler with the base, slide the set square along the ruler until its right-angle edge touches the opposite vertex, then draw the perpendicular line.

Types of triangles

Imagine a simple shape made by connecting three straight lines. That's a triangle! It's one of the most fundamental and strong shapes in geometry. Every triangle always has:

- **Three sides:** These are the straight lines that form the boundaries of the triangle.
- **Three vertices:** These are the "corners" or points where two sides meet.
- **Three interior angles:** These are the angles formed inside the triangle at each vertex.

A very important rule about triangles is that the sum of all three interior angles in any triangle is always 180 degrees. No matter how big or small, or what shape it is, this rule always holds true!

Mathematical Explanation

We can sort triangles into different groups based on two main characteristics

Classification by Sides

This way of classifying triangles looks at the lengths of their three sides.

1. **Scalene Triangle:** A triangle where all three sides have different lengths. This means no two sides are equal.

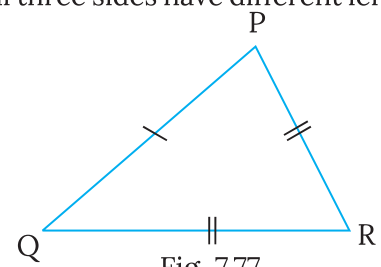


Fig. 7.77

Angles: Because the sides are all different, all three angles inside a scalene triangle will also have different measurements.

Example: Think of a triangle with sides measuring 5 cm, 8 cm, and 10 cm. Since $5 \neq 8 \neq 10$, it's a scalene triangle.

2. **Isosceles Triangle:** A triangle where at least two sides are of equal length. It can have exactly two equal sides, or even all three (which makes it a special type of isosceles).

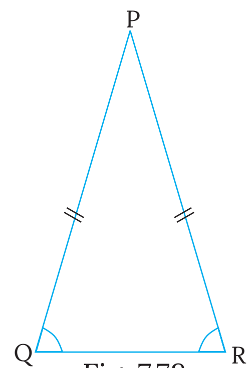


Fig. 7.78

Angles: The two angles that are opposite the equal sides are also equal in measure. These are often called "**base angles**."

Example: Consider a triangle with sides measuring 6 cm, 6 cm, and 9 cm. Since two sides are 6 cm long, it's an isosceles triangle.

3. **Equilateral Triangle:** A triangle where all three sides are of equal length.

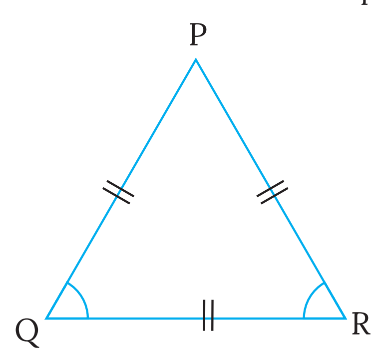


Fig. 7.79

Angles: Since all sides are equal, all three angles are also equal. Because the total sum of angles is 180 degrees, each angle in an equilateral triangle is always 60 degrees ($180° \div 3 = 60°$).

Example: A triangle where every side measures 7 cm. Each angle will automatically be 60°.

Key points: An equilateral triangle is also an isosceles triangle (since it has at least two equal sides).

Classification by Angles

Location of Altitudes

Altitudes are crucial for understanding the "**height**" of a triangle, which is essential for calculating its area (Area = $\frac{1}{2}$ × base × height). The concept of perpendicularity is central to altitudes.

Construction of Altitude from a Vertex A to Side BC:

Step 1. Place the ruler along the side BC (the base).

Step 2. Place the set square such that one of its right-angle edges aligns with the ruler.

Step 3. Slide the set square along the ruler until the other right-angle edge touches vertex A.

Step 4. Draw a line segment from A along the edge of the set square to meet BC (or its extension) at a point D. AD is the altitude.

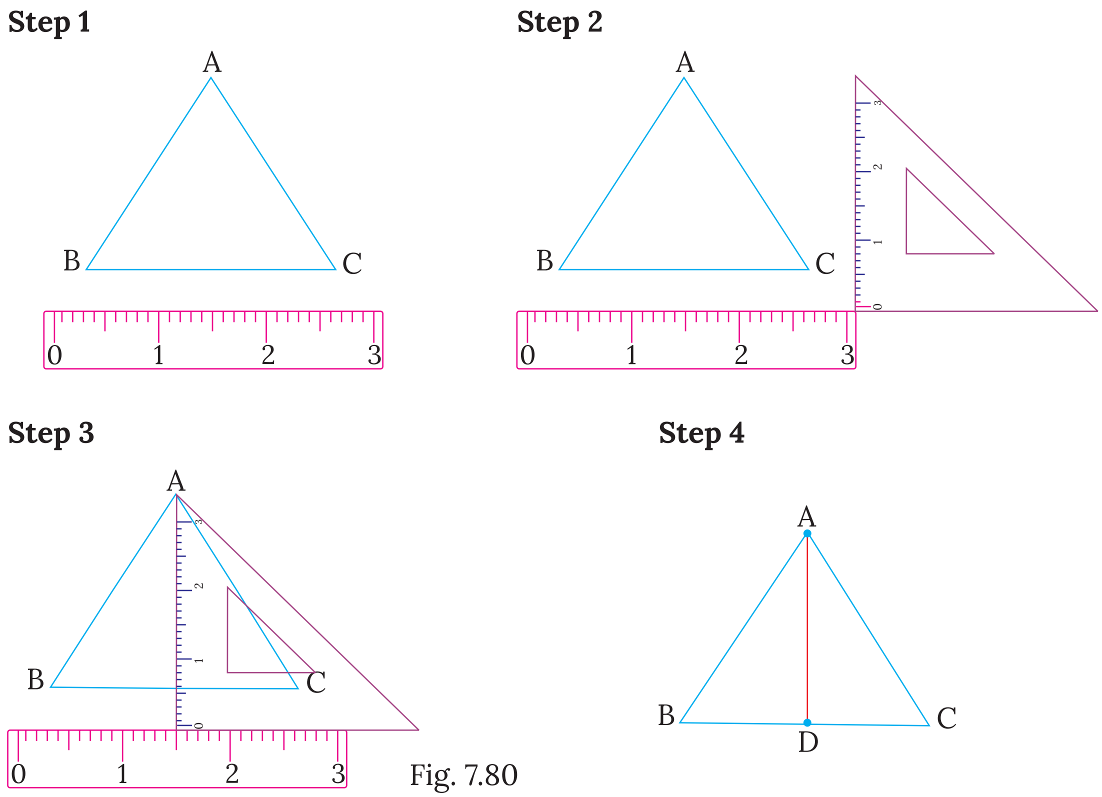


Fig. 7.80

This way of classifying triangles looks at the measurements of their three interior angles.

1. **Acute-angled Triangle:** A triangle where all three interior angles are acute angles. An acute angle is an angle that measures less than 90 degrees (< 90°).

Example: A triangle with angles measuring 50°, 60°, and 70°. All these angles are less than 90°.

A clear diagram showing an acute-angled triangle with all three altitudes drawn, intersecting inside the triangle.

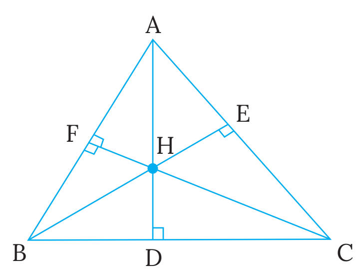


Fig. 7.81

2. **Right-angled Triangle:**

Definition: A triangle where one of its interior angles is exactly a right angle (measures exactly 90 degrees). You'll often see a small square symbol in the corner to show the 90° angle.

Key Feature: The side directly opposite the right angle is called the hypotenuse. It is always the longest side in a right-angled triangle.

Example: A triangle with angles measuring 30°, 60°, and 90°. The 90° angle makes it a right-angled triangle.

A clear diagram showing a right-angled triangle with its three altitudes drawn, highlighting that two sides are altitudes and they intersect at the right-angle vertex.

3. **Obtuse-angled Triangle**

A triangle where one of its interior angles is an obtuse angle. An obtuse angle is an angle that measures more than 90 degrees but less than 180 degrees (> 90° but < 180°).

Note: A triangle can only ever have one obtuse angle because if it had two, the sum of angles would be more than 180 degrees, which is impossible for a triangle.

Example: A triangle with angles measuring 25°, 35°, and 120°. The 120° angle is obtuse, making it an obtuse-angled triangle.

A clear diagram showing an obtuse-angled triangle with its three altitudes drawn, showing two altitudes falling outside the triangle and meeting the extended sides.

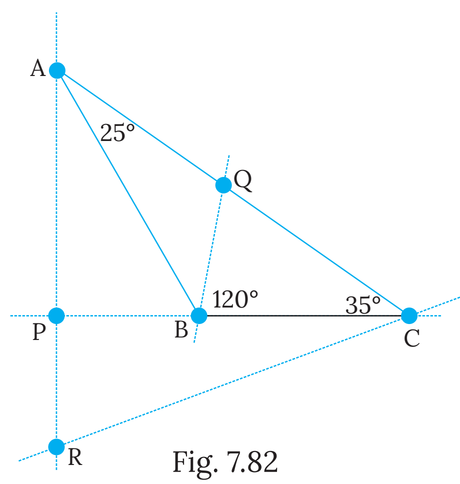


Fig. 7.82

Key points:

- The three altitudes of any triangle always intersect at a single point called the orthocenter.
- A triangle can have at most one right angle or one obtuse angle. If it had two, the sum of angles would exceed 180°.

Relationship between Classifications

- An equilateral triangle is always an acute-angled triangle (all angles are 60°).
- An isosceles triangle can be acute-angled, right-angled, or obtuse-angled.
- Isosceles acute: (e.g., 70°, 70°, 40°)
- Isosceles right: (e.g., 90°, 45°, 45°)
- Isosceles obtuse: (e.g., 100°, 40°, 40°)
- A scalene triangle can also be acute-angled, right-angled, or obtuse-angled.

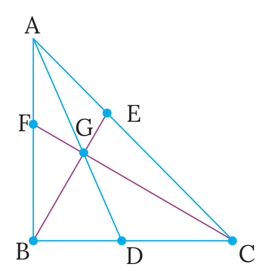


Fig. 7.83

Classification Type	Sub-type	Description (Sides)	Description (Angles)
By Sides	Scalene	All 3 sides are different lengths	All 3 angles are different measures
	Isosceles	At least 2 sides are equal lengths	Angles opposite equal sides are equal
	Equilateral	All 3 sides are equal lengths	All 3 angles are equal (each 60°)
By Angles	Acute-angled	(Sides can vary)	All 3 angles are acute (<90°)
	Right-angled	(Sides can vary, hypotenuse is longest	Exactly 1 angle is a right angle (= 90°)
	Obtuse-angled	(Sides can vary)	Exactly 1 angle is an obtuse angle (> 90° but < 180°)

Example : Draw an acute-angled triangle and construct one of its altitudes.

Step 1: Draw a triangle ABC with angles, say, 70°, 60°, and 50°. (All angles < 90°).

Step 2: Place the ruler along BC.

Step 3: Place the set square along the ruler and slide it until its perpendicular edge aligns with vertex A.

Step 4: Draw a line segment from A to BC, meeting at D. AD is the altitude. (Observe AD is inside the triangle).

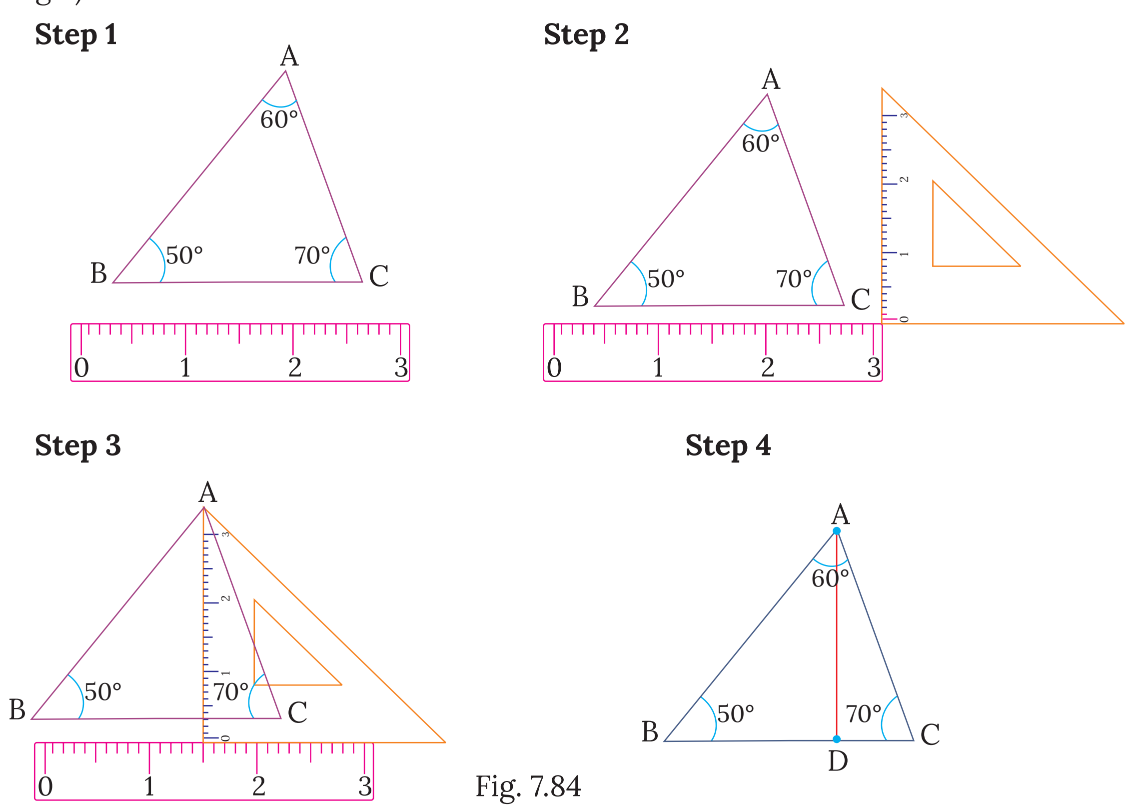


Fig. 7.84

Example: Draw a right-angled triangle and identify its altitudes.

Step 1: Draw a right-angled triangle PQR with ∠Q = 90°.

Step 2: Identify the sides forming the right angle: PQ and QR.

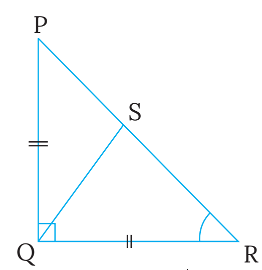


Fig. 7.85

Observation: PQ is perpendicular to QR. So, PQ is the altitude from P to QR, and QR is the altitude from R to PQ. The third altitude from Q to PR would be inside.

Conclusion: In a right-angled triangle, the two legs are also altitudes.

Example 9 : Classify the following triangles based on their sides and angles:

a) Sides: 7 cm, 7 cm, 7 cm

b) Sides: 3 cm, 4 cm, 5 cm

c) Angles: 100°, 40°, 40°

Solution: **a) Sides: 7 cm, 7 cm, 7 cm:**

Side classification: Equilateral (all sides equal).

Angle classification: Acute-angled (all angles are 60°, which is acute).

b) Sides: 3 cm, 4 cm, 5 cm:

Side classification: Scalene (all sides different).

Angle classification: Right-angled (since $3^2 + 4^2 = 9 + 16 = 25 = 5^2$, it's a right-angled triangle by Pythagorean theorem, which you'll learn later, but for now, it's a common example).

c) Angles: 100°, 40°, 40°:

Angle classification: Obtuse-angled (one angle, 100°, is obtuse).

Side classification: Isosceles (since two angles are equal, the sides opposite to them must also be equal).

Example: Draw an obtuse-angled triangle and construct an altitude that lies outside the triangle.

Step 1: Draw a triangle ABC with an obtuse angle, say ∠B = 110°. (e.g., A = 40°, B = 110°, C = 30°).

Step 2: Extend the side BC to a point D. Place the ruler along the extended line BD.

Step 3: Place the set square along the ruler and slide it until its perpendicular edge aligns with vertex A.

Step 4: Draw a line segment from A perpendicular to the extended line BD, meeting at E. AE is the altitude from A to BC, and it lies outside the triangle.

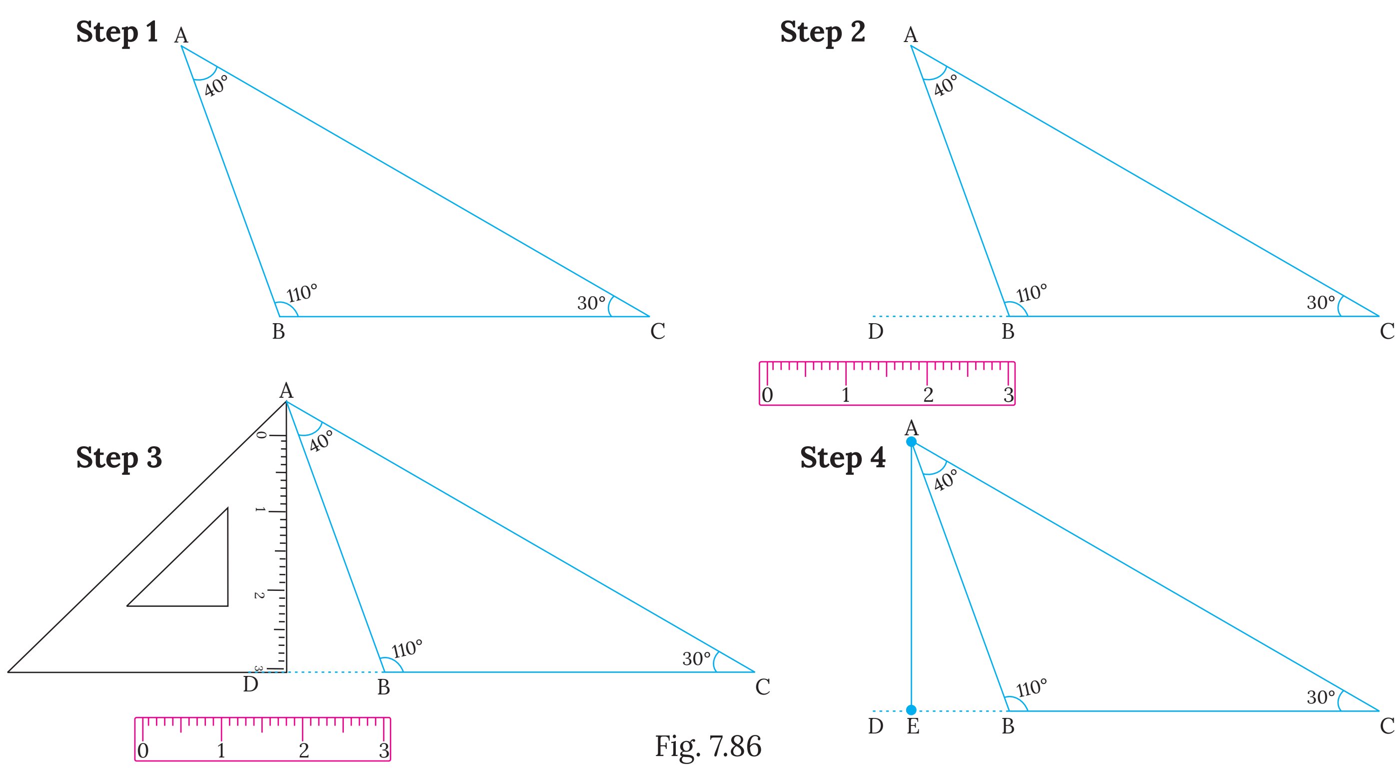


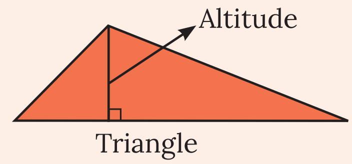
Fig. 7.86

Activity

Altitude Exploration

Objectives: To help students visually understand the position of altitudes (inside, outside, or on the triangle) depending on the triangle type.

Materials: Cardboard triangles (pre-cut, one acute, one right, one obtuse), rulers, set squares, pencils.



Procedure:

1. Students receive one of each type of triangle.
2. For each triangle, they identify the vertices and sides.
3. They then attempt to construct all three altitudes for each triangle using the set square and ruler.
4. Students observe and record where the altitudes fall (inside, outside, or on the triangle) and where they intersect for each type of triangle.

Inquiry Focus: Hands-on experience with altitude construction and a visual understanding of their location in different triangle types.

Knowledge Checkpoint

- What is the main characteristic of an altitude?
- Where do the altitudes of an acute-angled triangle intersect?
- Can a triangle be both equilateral and right-angled? Why or why not?

Key Terms

- **Altitude:** A perpendicular line segment from a vertex to the opposite side (or its extension). Represents the height.
- **Orthocenter:** The point where the three altitudes of a triangle intersect.
- **Acute-angled Triangle:** A triangle where all three angles are acute (less than 90°).
- **Right-angled Triangle:** A triangle with one angle exactly 90°.
- **Obtuse-angled Triangle:** A triangle with one angle greater than 90°.
- **Hypotenuse:** The side opposite the right angle in a right-angled triangle (always the longest side).
- **Set Square:** A triangular ruler with a right angle, used for drawing perpendicular and parallel lines.

Do It Yourself

- If you know the base and the area of a triangle, how can you find its altitude?
- Can a triangle have two obtuse angles? Why or why not, based on the angle sum property?

Fact Flash

- The orthocenter (where altitudes meet), centroid (where medians meet), and circumcenter (where perpendicular bisectors meet) are all collinear (lie on the same straight line) in any triangle. This line is called the **Euler line!**
- The word "**altitude**" comes from the Latin word "**altitudo**," meaning "**height**."

Mental Mathematics

- A triangle has angles 60°, 60°, 60°. What type of triangle is it by sides?
- A triangle has angles 90°, 45°, 45°. What type of triangle is it by sides?
- If a triangle has one angle of 120°, can it be a right-angled triangle?

Gap Analyzer™ Homework

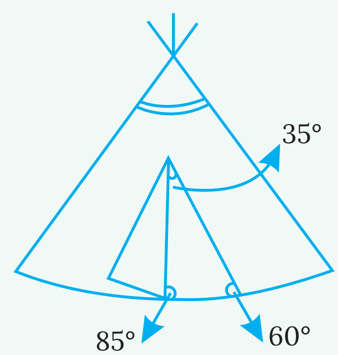
Watch Remedial

Exercise 7.5

1. **Fill in the Blanks:**
   a) An altitude is a line segment from a vertex _______ to its opposite side.
   b) In a right-angled triangle, the two _______ are also altitudes.
   c) A triangle with all angles less than 90° is called an _______ triangle.
   d) A triangle with two equal sides is called an _______ triangle.

2. **Classify the following triangles based on their sides and angles:**
   a) Sides: 6 cm, 8 cm, 6 cm
   b) Angles: 30°, 60°, 90°
   c) Sides: 10 cm, 12 cm, 15 cm
   d) Angles: 75°, 75°, 30°
   e) Sides: 5.5 cm, 5.5 cm, 5.5 cm
   f) Angles: 110°, 35°, 35°

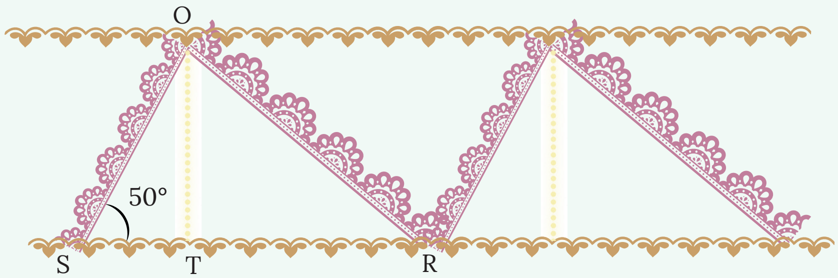
3. **"Bridging the Gap Questions (between theory and practice)":**
   a) A triangular sail needs a reinforcement strip from the top corner straight down to the base, making a 90° angle with the base. What geometric term describes this strip? How would you mark its position?
   b) A carpenter is cutting a triangular piece of wood for a roof support. If the angles are 45°, 45°, and 90°, what kind of triangle is it? How would its altitudes behave?
   c) A tent has a triangular front. If the angles of this triangle are 85°, 60°, and 35°, what kind of triangle is it?



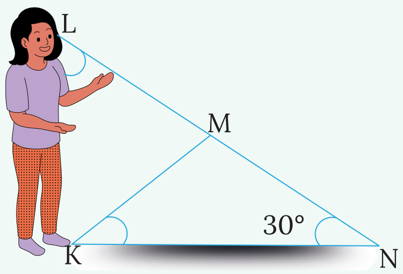
   d) An architect is designing a building with a triangular facade. If one angle of the triangle is 110°, what type of triangle is it? Where would its altitudes fall?

4. **Read the following Assertion and Reason and choose the correct option:**
   Assertion: AD is an altitude of ΔABC. It divides the side BC and ΔBAC into two equal parts.
   Reason: An altitude of a triangle is a line drawn from a vertex perpendicular to the opposite side (or the line containing the opposite side).
   A. Both Assertion (A) and Reason (R) are true, and R is the correct explanation of A.
   B. Both Assertion (A) and Reason (R) are true, but R is not the correct explanation of A.
   C. Assertion (A) is true, but Reason (R) is false.
   D. Assertion (A) is false, but Reason (R) is true.

5. **Estimation Questions**
   a) If a triangle has angles 60°, 60°, 60°, what can you estimate about its sides?
   b) If a triangle has one angle of 90°, and two sides are equal, estimate the other two angles.
   c) If a triangle has sides 3, 4, 5, what type of angle would you expect the largest angle to be?
   d) If a triangle has one angle of 130°, will any of its altitudes fall outside the triangle?
6. **Seema has made the given pattern from a lace, she wants to continue with the same pattern repeating it 10 more times. To make the exactly same triangle she decides to take the measurement of angle of first triangle she made. She observed that the triangle formed is a right-angled triangle with ∠SOR = 90°. Also, ∠OSR = 50°.**



   a) Determine the value of ∠ORS.
   b) What will be the value of ∠TOR?
7. **The shadow formed makes an angle of 30° with the line of sight of the girl.**
   a) Determine the value of the ∠NLK.
   b) If KM forms an altitude, then is the value of ∠MKN equal to the value of angle ∠NLK?



Common Misconceptions

Misconception: The sum of angles in a triangle is always 360 degrees.

Correction: This is a common confusion with quadrilaterals. The Angle Sum Property states that the sum of the three interior angles of any triangle is always 180 degrees. You can verify this by tearing off the corners of a paper triangle and placing them together on a straight line.

Misconception: Any three side lengths can form a triangle.

Correction: Not true! The Triangle Inequality Property must be satisfied: the sum of the lengths of any two sides must be greater than the length of the third side. If you try to construct a triangle with sides 3 cm, 4 cm, and 8 cm, you'll find the arcs don't meet because 3 + 4 is not greater than 8.

Misconception: An altitude always falls inside the triangle.

Correction: While this is true for acute-angled triangles, it's not always the case. In a right-angled triangle, two of the altitudes are the sides forming the right angle. In an obtuse-angled triangle, two of the altitudes fall outside the triangle, meeting the extension of the opposite side.

Real-life Triangle: Mathematical Applications

1. **Architecture & Bridge Design:** Triangles are used in designing trusses and bridges because they distribute weight evenly and provide structural strength. In roof designs, triangular shapes help stabilize structures.
2. **Surveying & Land Mapping:** Surveyors divide plots of land into triangles to measure distances and areas accurately using triangulation. In map making, locations are found by intersecting three distance lines (triangulation).
3. **Navigation & GPS Systems:** GPS systems use triangulation, where three satellites' signals intersect to locate a position on Earth. Ships and aircraft navigate using angles between intersecting directional lines, forming triangles.
4. **Mechanical & Structural Engineering:** Triangular components are used in machine parts and cranes to handle force efficiently without bending. Engineers use the properties of triangles to create stable and rigid frameworks.

Gap Analyzer™
Complete Chapter Test

EXERCISE

A. Choose the correct answer.

1. Which of the following sets of lengths can form a triangle?
   a) 2 cm, 3 cm, 6 cm ☐ b) 5 cm, 7 cm, 12 cm ☐
   c) 4 cm, 6 cm, 9 cm ☐ d) 1 cm, 2 cm, 3 cm ☐
2. If two angles of a triangle are 45° and 75°, what is the measure of the third angle?
   a) 60° ☐ b) 70° ☐
   c) 80° ☐ d) 90° ☐
3. In a triangle, if one angle is 110°, the triangle is classified as:
   a) Acute-angled. ☐ b) Right-angled ☐
   c) Obtuse-angled ☐ d) Equilateral. ☐
4. An altitude of a triangle is a line segment from a vertex to the opposite side that is:
   a) Equal to the median ☐ b) Perpendicular to the side ☐
   c) Bisects the angle ☐ d) Connects midpoints ☐
5. Which criterion is used to construct a triangle when two angles and the included side are given?
   a) SSS ☐ b) SAS ☐
   c) ASA ☐ d) RHS ☐

Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct:

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

1. **Assertion (A):** A triangle with sides 7 cm, 8 cm, and 16 cm cannot be constructed.

   Reason (R): The sum of the two shorter sides (7+8=15) is less than the longest side (16).

2. **Assertion (A):** If two angles of a triangle are 60° and 60°, then the third angle is also 60°.

   Reason (R): The sum of angles in a triangle is 180°.

3. **Assertion (A):** In a right-angled triangle, two of its altitudes are the sides forming the right angle.

   Reason (R): An altitude is a line segment from a vertex perpendicular to the opposite side.

Case Study

The Great Pyramid of Giza

The Great Pyramid of Giza is one of the most iconic structures in the world, known for its massive and stable triangular faces. Each face is an isosceles triangle. Imagine one such triangular face has a base of approximately 230 meters and two equal sides of approximately 186 meters.

a) Verify if these dimensions satisfy the Triangle Inequality Property.
b) What type of triangle (by sides and angles) is each face of the pyramid? (You may need to infer angle type based on the side lengths, or state if it's not possible to determine without more info).
c) If you were to draw a scale model of one face, what scale factor would you use if your paper allows a base of 23 cm?
d) Explain why a triangular shape was chosen for the pyramid's faces, relating it to the properties of triangles you have learned.

Skill: Apply all learned concepts to a real-world scenario (Analyze, Evaluate)

Project

"Triangle in My World" Portfolio

Objective: To identify, analyze, and represent various types of triangles found in the real world, applying the concepts of construction, properties, and classification learned in this chapter.

Task: Create a digital or physical portfolio showcasing at least five different real-world examples of triangles. For each example, you must:

1. **Photograph/Sketch:** Capture a clear photograph or draw a detailed sketch of the object/structure containing the triangle.
2. **Identify:** Clearly point out the triangle(s) in your image/sketch.

3. **Classify:** Classify each identified triangle based on:
   Sides: Equilateral, Isosceles, or Scalene.
   Angles: Acute-angled, Right-angled, or Obtuse-angled.

Justify your classification based on visual estimation or assumed properties (e.g., "This looks like an isosceles triangle because two sides appear equal," or "This is a right-angled triangle because it forms a perfect corner").

4. **Analyze Properties:**

- For at least two examples, estimate the approximate side lengths and angle measures.
- For one example, demonstrate how the Angle Sum Property applies (e.g., "If two angles are X and Y, the third must be 180 - X - Y").
- For another example, explain how the Triangle Inequality Property would apply to its side lengths.

5. **Construction (Scale Model):** Choose one of your real-world examples and construct a scale model of its triangular component on paper using a ruler, compass, and protractor. Clearly state the scale you used (e.g., 1 cm = 1 meter).
6. **Reflection:** Write a short paragraph explaining why triangles are so prevalent in the real world, especially in structures and designs, based on their unique properties.

Possible Examples: Bridge trusses, roof frames, road signs, pizza slices, musical instruments (e.g., guitar pick), patterns in architecture, sports fields, hangers, etc.

Submission: A portfolio (digital presentation, poster board, or bound report) with clear labels, images, constructions, and explanations.

Source-Based Question

The pyramids of Egypt fascinated travellers and conquerors in ancient times and continue to inspire wonder in the tourists, mathematicians, and archeologists who visit, explore, measure, and describe them.

Tombs of early Egyptian kings were bench-shaped mounds called mastabas. Around 2780 BCE, King Djoser's architect, Imhotep, built the first pyramid by placing six mastabas, each smaller than the one beneath, in a stack to form a pyramid rising in steps. This Step Pyramid stands on the west bank of the Nile River at Sakkara near Memphis. Like later pyramids, it contains various rooms and passages, including the burial chamber of the king.

Adapted from reports published in Trends in International Mathematics and Science Study

The transition from the Step Pyramid to a true, smooth-sided pyramid took place during the reign of King Snefru, founder of the Fourth Dynasty (2680–2560 BCE). At Medum, a step pyramid was built, then filled in with stone, and covered with a limestone casing. Nearby at Bahshur, construction was begun on a pyramid apparently planned to have smooth sides. About halfway up, however, the angle of incline decreases from over 51 degrees to about 43 degrees, and the sides rise less steeply, causing it to be known as the Bent Pyramid. The change in angle was probably made during construction to give the building more stability. Another great pyramid was built at Dahshur with its sides rising at an angle of somewhat over 43 degrees, resulting in a true, but squat looking pyramid.

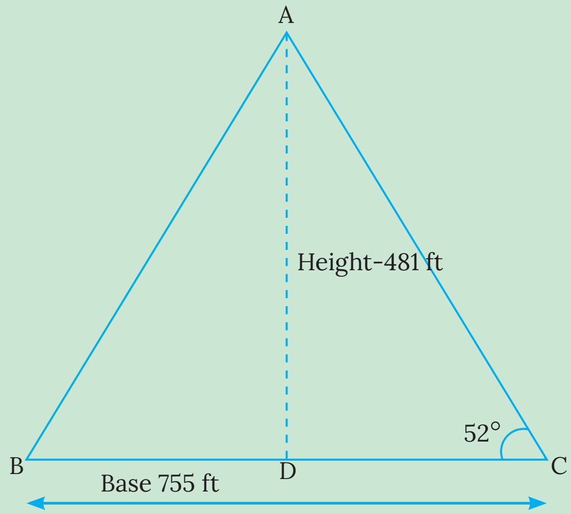
The largest and most famous of all the pyramids, the Great Pyramid at Giza, was commissioned by Snefru's son, Khufu, known also as Cheops, the later Greek form of his name. The pyramid's base covered over 13 acres and its sides rose at an angle of 51 degrees 52 minutes and were over 755 feet long. It originally stood over 481 feet high; today it is 450 feet high. Scientists estimate that its stone blocks average over two tons apiece, with the largest weighing as much as fifteen tons each. Two other major pyramids were built at Giza, for Khufu's son, King Khafre (Chephren), and a successor of Khafre, Menkaure (Mycerinus). Also located at Giza is the famous Sphinx, a massive statue of a lion with a human head, carved during the time of Khafre.

Directions:

The **Great Pyramid of Giza** (Khufu/Cheops) provides classic geometric data useful for triangle problems. Measurements below are drawn from the Smithsonian "Ancient Egypt" spotlight page.

Pyramid	Height	Base Length	Angle of incline
Great Pyramid, Giza (Khufu/Cheops)	481 ft	755 ft (Approx.)	51° 52′

Modeling note (for problems): In a vertical cross-section through the center, the altitude bisects the base, forming an isosceles triangle. Use Half-base = 377.5 ft (for calculations).



Questions on the Data

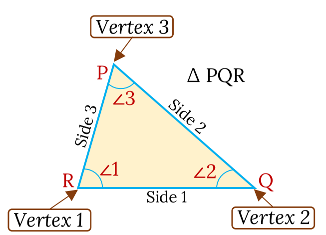
1. Draw a triangle representing a vertical cross-section of the Great Pyramid. Mark its base = 755 ft, height = 481 ft, and the slant edge making an angle of 52° with the base.
2. Identify which kind of triangle is formed (isosceles, scalene, or equilateral) in this central cross-section.
3. Find the slant height of the pyramid's face using the vertical height and half of the base (assume the altitude bisects the base). (Hint: right triangle with legs 481 ft and 377.5 ft.)

Mind Map

Triangle: Three Intersecting Lines

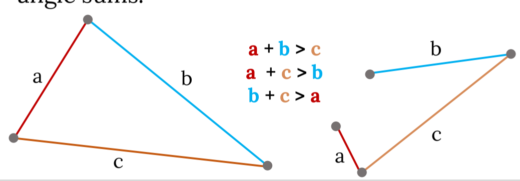
Introduction to Triangles and Construction with Given Side Lengths

- ❖ Definition and components of a triangle (vertices, sides, angles).
- ❖ Naming conventions for triangles.
- ❖ Construction of equilateral triangles (all sides equal).
- ❖ Construction of general triangles (all sides different).



The Triangle Inequality Property

- ❖ Construction using SAS (Side-Angle-Side) criterion.
- ❖ Construction using ASA (Angle-Side-Angle) criterion.
- ❖ Conditions for existence of triangles based on angle sums.

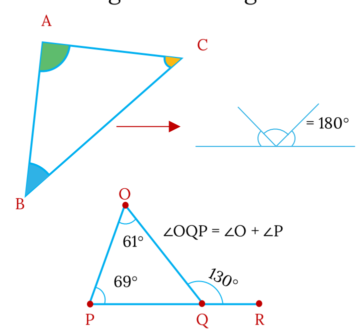


Construction with Given Sides and Angles

- ❖ Construction using SAS (Side-Angle-Side) criterion.
- ❖ Construction using ASA (Angle-Side-Angle) criterion.
- ❖ Conditions for existence of triangles based on angle sums.

Angle Sum Property and Exterior Angle Property

- ❖ Angle Sum Property of a triangle: Proof and Verification.
- ❖ Exterior Angle of a triangle.



Altitudes of Triangles and Types of Triangles

- ❖ Altitude of a triangle
- ❖ Construction of altitudes using a set square and ruler.