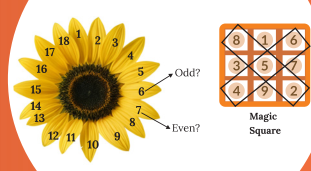
# 6 Number Play

## Why This Chapter Matters

Have you ever noticed how numbers seem to have hidden rules? What if I told you that numbers can tell us secrets about the world around us, from the petals on a flower to the rhythm of a song? We often use numbers without thinking about their deeper properties. But understanding these properties, like whether a number is 'even' or 'odd', can unlock a whole new way of looking at mathematics and solving puzzles! Get ready to explore the surprising patterns and playful secrets that numbers hold.




## Meet EeeBee.AI

Hello, young mathematicians! I'm EeeBee, your curious and clever guide through this exciting journey into the world of numbers. I love discovering patterns and solving puzzles, just like you will in this chapter. I'll be popping up with interesting questions, helpful hints, and fun facts to make your learning adventure even more enjoyable. Let's explore the magic of numbers together!

## Learning Outcomes

**By the end of this chapter, you will be able to:**

- Define and identify even and odd numbers.
- Explain the concept of parity and its significance.
- Apply rules of parity to predict the outcome of addition and subtraction operations.
- Formulate algebraic expressions to represent even and odd numbers.
- Analyze and solve number puzzles, including cryptarithms and grid-based problems, using logical reasoning.
- Discover and appreciate the historical and natural occurrences of number sequences like the Virahānka–Fibonacci numbers.

## From Last Year's Notebook

**From Previous Grades, You Remember:**

- **Number Types:** Natural numbers (1, 2, 3...), whole numbers (0, 1, 2, 3...), and integers (...-1, 0, 1...).
- **Basic Operations:** How to add, subtract, multiply, and divide numbers.
- **Counting & Grouping:** How to organize and count objects.
- **Even Numbers:** You've already learned to identify numbers that can be divided exactly by 2! This was your first step into understanding "even."

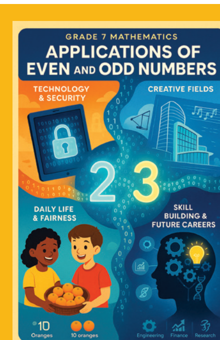
**In This Chapter, We Will:**

- **Deepen Understanding:** Dive much deeper into the fascinating properties of numbers, especially even and odd numbers.
- **Solve Problems:** Discover how these properties help us solve more complex mathematical challenges.
- **Uncover Patterns:** Explore and find amazing patterns hidden within numbers.
- **Introduction to Algebra:** Briefly touch upon simple algebraic expressions, building on any past encounters.

## Real Math, Real Life

Mathematics is more than just calculations; it helps us understand the world! Even and odd numbers (parity) are fundamental concepts with wide-ranging uses:

- **Technology:** Essential in computer science (binary code) and data security.
- **Daily Life:** Used for sharing fairly and planning.
- **Creative Fields:** Architects use number patterns, musicians use sequences for rhythm.
- **Skill Building:** Understanding them boosts critical thinking and problem-solving.
- **Future Careers:** These skills are vital for technology, engineering, finance, and research.




## Quick Prep

1. Can you divide 15 chocolates equally among two friends? What about 12 chocolates?
2. Look at the numbers: 7, 10, 13, 16, 19. What is the pattern? What would be the next two numbers?
3. If you add two numbers, say 4 and 6, what kind of number do you get (even or odd)? What about 3 and 5?
4. Imagine you have a stack of 20 books. Can you arrange them in pairs without any book left over?
5. Think of any two numbers. If you add them, will the sum always be even? Or always odd? Or can it be either?

## Introduction

Numbers are everywhere, from counting your friends to measuring ingredients for a recipe. But have you ever thought about the basic nature of numbers themselves? Some numbers can be perfectly divided into two equal groups, while others always leave one leftover. This fundamental property, known as parity, is what defines numbers as either even or odd.

*Fig. 6.1*

In this section, we will delve into the world of even and odd numbers, explore their unique characteristics, and understand how they behave when we perform basic operations like addition and subtraction. This understanding forms the bedrock for solving many mathematical puzzles and appreciating deeper number patterns.

## Chapter Overview

This chapter explores number properties, starting with observing patterns and defining even and odd numbers.

- **Understanding Parity:** Learn definitions (2n, 2n-1), visual representations, and properties of even/odd numbers through addition and subtraction. Explore consecutive numbers and parity.
- **Number Puzzles & Logic:** Dive into grid puzzles, Magic Squares (history, properties, algebraic generalization), and Cryptarithms, using logical reasoning.
- **Special Number Sequences:** Discover Virahānka–Fibonacci numbers – their origin (Indian contributions), occurrence in nature, and problem-solving applications.
- **Chapter Wrap-Up:** Summarize key concepts, answer review questions, and apply your new knowledge.

## From History's Pages

The ideas of even and odd numbers are ancient, observed by early humans when sharing. The ancient Greeks (Pythagoreans), around 500 BCE, deeply studied and classified them, even giving them philosophical meanings. Indian mathematicians in texts like the Sulba Sutras (800-500 BCE) and later scholars like Aryabhata and Brahmagupta ($5^{th}$-$7^{th}$ CE) also explored these concepts within number theory. The algebraic definitions (2n, 2n-1) evolved over centuries with global contributions, leading to our modern understanding. This history shows how mathematical ideas develop over time and across cultures!

## Even and Odd Numbers

Numbers are fundamental to mathematics, and one of their most basic classifications is whether they are even or odd. This property, called parity, helps us understand how numbers behave in operations and forms the basis for many mathematical puzzles and patterns.

An even number is any integer that can be divided by 2 without leaving a remainder. Think of it as a number of items that can be perfectly grouped into pairs.

An odd number, on the other hand, is any integer that leaves a remainder of 1 when divided by 2. These are numbers of items where, if you try to group them into pairs, there will always be one item left over.

We will explore the definitions, visual representations, algebraic forms, and the fascinating properties of these numbers under addition and subtraction.

**Sub-concepts to be covered**

1. Even Numbers
2. Odd Numbers
3. Visual Representation of Even and Odd Numbers
4. Algebraic Representation of Even and Odd Numbers
5. Properties of Even and Odd Numbers under Addition
6. Properties of Even and Odd Numbers under Subtraction
7. Parity of Consecutive Numbers

## Mathematical Explanation

### Even Numbers

An even number is an integer that is exactly divisible by 2. This means that when you divide an even number by 2, the remainder is 0.

**Examples:** 0, 2, 4, 6, 8, 10, 12, ... Any number ending with 0, 2, 4, 6, or 8 is an even number.

**Key point to remember:** Zero is considered an even number because it fits the definition (0 ÷ 2 = 0, with no remainder).

### Odd Numbers

An odd number is an integer that is not exactly divisible by 2. This means that when you divide an odd number by 2, the remainder is 1.

**Examples:** 1, 3, 5, 7, 9, 11, 13, ... Any number ending with 1, 3, 5, 7, or 9 is an odd number.

**Key point to remember:** Odd numbers are always one more or one less than an even number.

### Visual Representation of Even and Odd Numbers

Visualizing numbers helps in understanding their properties.

**Even Numbers:** Imagine you have a collection of objects. If you can arrange all the objects into perfect pairs, with no object left alone, then the total number of objects is even.

A diagram showing 6 circles arranged in 3 pairs.

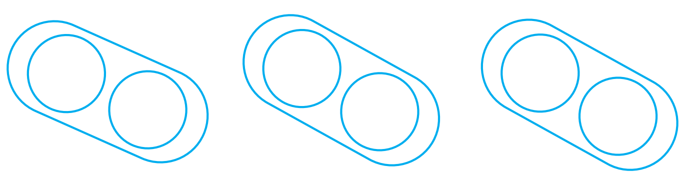


*Fig. 6.2* Even Number (6) - Can be paired

A diagram showing 10 squares arranged in 5 pairs.

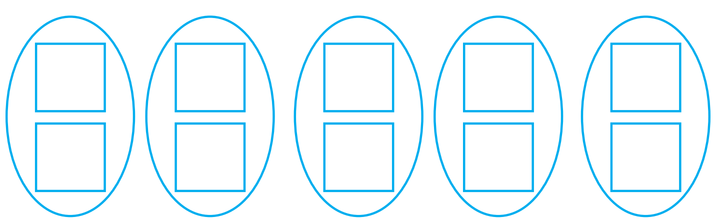


*Fig. 6.3* Even Number (10) - All paired up

**Odd Numbers:** If you try to arrange a collection of objects into pairs, and there's always one object left over, then the total number of objects is odd.

A diagram showing 7 triangles arranged in 3 pairs with 1 triangle left alone.

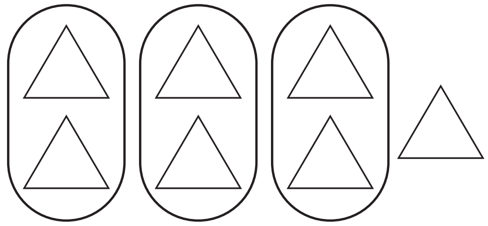


*Fig. 6.4* Odd Number (7) - One left over.

A diagram showing 9 stars arranged in 4 pairs with 1 star left alone.

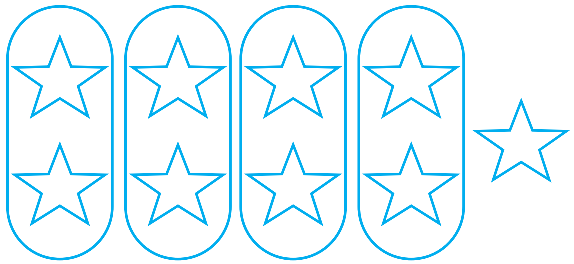


*Fig. 6.5* Odd Number (9) - Always a leftover!

### Algebraic Representation of Even and Odd Numbers

We can represent even and odd numbers using algebraic expressions, which helps us generalize their properties.

**Even Numbers:** Any even number can be expressed in the form 2n, where 'n' is any integer (0, 1, 2, 3, ...).

- If n = 0, 2n = 0 (Even)
- If n = 1, 2n = 2 (Even)
- If n = 5, 2n = 10 (Even)

**Odd Numbers:** Any odd number can be expressed in the form 2n - 1 or 2n + 1, where 'n' is any integer (0, 1, 2, 3, ...).

**Using 2n - 1:**

If n = 1, 2n - 1 = 2(1) - 1 = 1 (Odd)

If n = 3, 2n - 1 = 2(3) - 1 = 5 (Odd)

**Using 2n + 1:**

If n = 0, 2n + 1 = 2(0) + 1 = 1 (Odd)

If n = 2, 2n + 1 = 2(2) + 1 = 5 (Odd)

**Common errors to avoid:** Students sometimes confuse 'n' with the number itself. 'n' is a placeholder for any integer that generates the even or odd number.

### Solved Example:

**Example 1 :** Classify the following numbers as even or odd: 28, 45, 100, 71, 0, 329.

**Solution:** **28:** Ends with 8, which is an even digit. 28 ÷ 2 = 14 (remainder 0). So, 28 is **Even**.

**45:** Ends with 5, which is an odd digit. 45 ÷ 2 = 22 (remainder 1). So, 45 is **Odd**.

**100:** Ends with 0, which is an even digit. 100 ÷ 2 = 50 (remainder 0). So, 100 is **Even**.

**71:** Ends with 1, which is an odd digit. 71 ÷ 2 = 35 (remainder 1). So, 71 is **Odd**.

**0:** 0 ÷ 2 = 0 (remainder 0). So, 0 is **Even**.

**329:** Ends with 9, which is an odd digit. 329 ÷ 2 = 164 (remainder 1). So, 329 is **Odd**.

### Properties of Even and Odd Numbers under Addition

Understanding how even and odd numbers behave when added together is crucial for many mathematical problems.

| Operation | Result |
|---|---|
| Even + Even | Even |
| Odd + Odd | Even |
| Even + Odd | Odd |
| Odd + Even | Odd |

*Fig. 6.6*

**Even + Even = Even:**

- **The Rule:** When you add two even numbers, the sum is always an even number.

**Example:** 4 + 6 = 10 (Even)

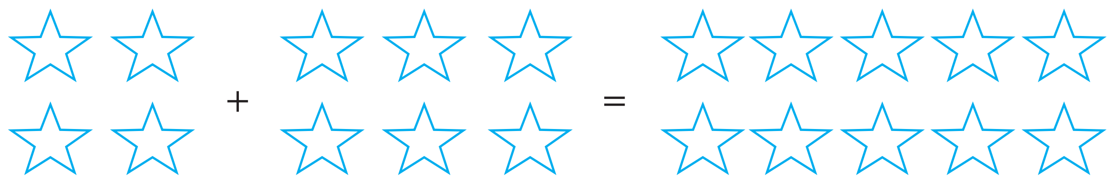


*Fig. 6.7*

**Algebraic Proof:** Let the two even numbers be 2a and 2b. Their sum is 2a + 2b = 2(a + b). Since (a + b) is an integer, 2(a + b) is of the form 2n, which is an even number.

**Odd + Odd = Even:**

- **The Rule:** When you add two odd numbers, the sum is always an even number.

**Example:** 3 + 5 = 8 (Even)

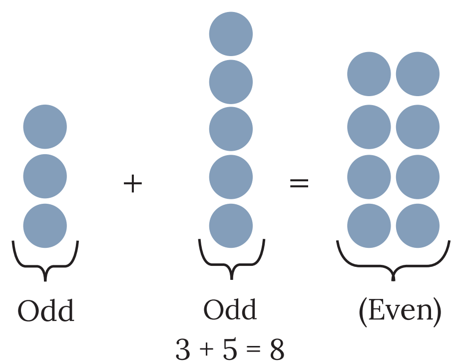



*Fig. 6.8*

**Algebraic Proof:** Let the two odd numbers be (2a + 1) and (2b + 1). Their sum is (2a + 1) + (2b + 1) = 2a + 2b + 2 = 2(a + b + 1). Since (a + b + 1) is an integer, 2(a + b + 1) is of the form 2n, which is an even number.

**Even + Odd = Odd:**

- **The Rule:** When you add an even number and an odd number, the sum is always an odd number.

**Example:** 4 + 3 = 7 (Odd)

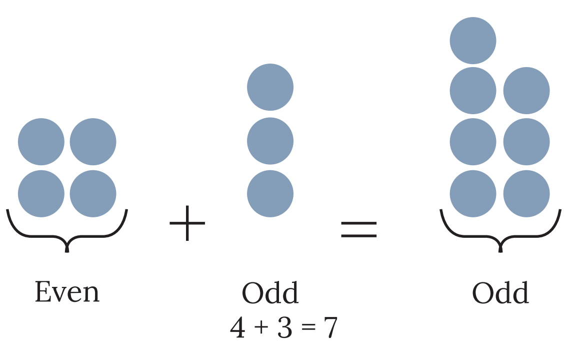



*Fig. 6.9*

**Algebraic Proof:** Let the even number be 2a and the odd number be (2b + 1). Their sum is 2a + (2b + 1) = 2(a + b) + 1. Since (a + b) is an integer, 2(a + b) + 1 is of the form 2n + 1, which is an odd number.

**Odd + Even = Odd:**

- **The Rule:** This is the same as Even + Odd due to the commutative property of addition.

## Properties of Even and Odd Numbers under Subtraction

The rules for subtraction are similar to addition in terms of parity.

| Operation | Result |
|---|---|
| Even - Even | Even |
| Odd - Odd | Even |
| Even - Odd | Odd |
| Odd - Even | Odd |

*Fig. 6.10*

**Even - Even = Even**

- **The Rule:** When you subtract an even number from another even number, the answer is always Even.

  **Example:** 12 - 6 = 6

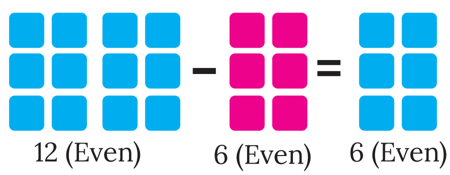



*Fig. 6.11*

**Algebraic Proof:**

- Any even number can be written in the form 2a and 2b, where a and b are any integers.

Let the two even numbers be 2a and 2b. Their difference is 2a - 2b. Factoring out the common 2, we get 2(a - b). Since a and b are integers, (a - b) is also an integer. Therefore, 2(a - b) is of the form 2n, which is an even number.

**Odd - Odd = Even**

- **The Rule:** When you subtract an odd number from another odd number, the answer is always Even.

  **Example:** 15 - 7 = 8

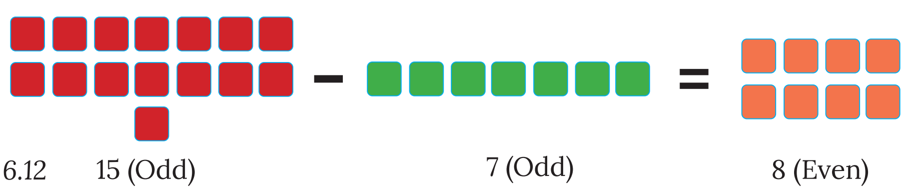


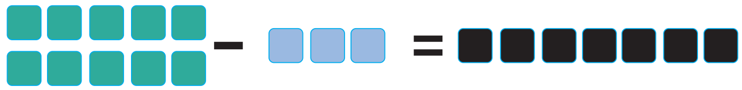

*Fig. 6.12*

**Algebraic Proof:** Let the two odd numbers be (2a + 1) and (2b + 1). Their difference is (2a + 1) - (2b + 1). Distributing the negative sign, we get 2a + 1 - 2b - 1. The +1 and -1 cancel out, leaving 2a - 2b. Factoring out the common 2, we get 2(a - b). Since (a - b) is an integer, 2(a - b) is of the form 2n, which is an even number.

**Even - Odd = Odd**

- **The Rule:** When you subtract an odd number from an even number, the answer is always Odd.

  **Example:** 10 - 3 = 7



10 (Even) 3 (Odd) 7 (Odd)

*Fig. 6.13*

**Algebraic Proof:** Let the even number be 2a and the odd number be (2b + 1). Their difference is 2a - (2b + 1). Distributing the negative sign, we get 2a - 2b - 1. We can rewrite this as (2a - 2b - 2) + 1. Factoring out 2 from the terms in the bracket, we get 2(a - b - 1) + 1. Since (a - b - 1) is an integer, 2(a - b - 1) + 1 is of the form 2n + 1, which is an odd number.

**Odd - Even = Odd**

- **The Rule:** When you subtract an even number from an odd number, the answer is always Odd.

  **Example:** 9 - 2 = 7

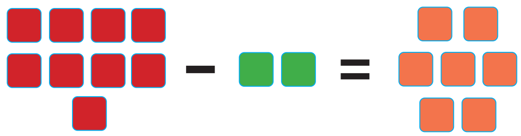


*Fig. 6.14* 9 (Odd) 2 (Even) 7 (Odd)

**Algebraic Proof:** Let the odd number be (2a + 1) and the even number be 2b. Their difference is (2a + 1) - 2b. Rearranging the terms, we get 2a - 2b + 1. Factoring out 2 from the first two terms, we get 2(a - b) + 1. Since (a - b) is an integer, 2(a - b) + 1 is of the form 2n + 1, which is an odd number.

### Parity of Consecutive Numbers

Consecutive numbers are numbers that follow each other in order, like 7 and 8, or 14 and 15.

**Key Property:** In any pair of consecutive integers, one number will always be even, and the other will always be odd.

**Example:** 11 (Odd), 12 (Even); 20 (Even), 21 (Odd).

**Sum of Consecutive Numbers:** Because one is even and one is odd, their sum will always be an odd number (Even + Odd = Odd).

**Example:** 11 + 12 = 23 (Odd); 20 + 21 = 41 (Odd).

**Example 2 :** Without actually calculating the sum, determine whether the result of each operation will be even or odd:

a) 124 + 38 b) 57 + 91 c) 205 + 68 d) 49 − 21 e) 76 - 35

**Solution:** a) 124 (Even) + 38 (Even): Even + Even = Even.
b) 57 (Odd) + 91 (Odd): Odd + Odd = Even.
c) 205 (Odd) + 68 (Even): Odd + Even = Odd.
d) 49 (Odd) - 21 (Odd): Odd - Odd = Even.
e) 76 (Even) - 35 (Odd): Even - Odd = Odd.

**Example 3 :** a) Write an expression for the 15th even number.
b) Write an expression for the 20th odd number.
c) If 'n' is an integer, what is the parity of the expression 4n + 3?

**Solution:** a) The nth even number is 2n. So, the 15th even number is 2 × 15 = 30.
b) The nth odd number is 2n - 1. So, the 20th odd number is 2 × 20 - 1 = 40 - 1 = 39.
c) The expression is 4n + 3.
4n is always an even number (since it's a multiple of 2).
3 is an odd number.
Even + Odd = Odd.

Therefore, the expression 4n + 3 will always result in an odd number, regardless of the integer value of 'n'.

**Example 4 :** A group of students is playing a game where they form pairs. If there are 37 students, how many pairs can be formed, and how many students will be left out? What does this tell you about the number 37?

*Fig. 6.15*

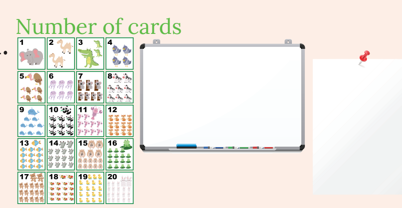
**Solution:** To form pairs, we need to divide the total number of students by 2.
37 ÷ 2 = 18 with a remainder of 1.
This means 18 pairs can be formed, and 1 student will be left out.
Since there is a remainder of 1 when 37 is divided by 2, this tells us that 37 is an odd number.

**Example 5 :** The sum of two consecutive numbers is 85. What are the numbers? What is the parity of their sum?

**Solution:** Let the first number be x.
The next consecutive number will be x + 1.
Their sum is $x + (x + 1) = 85$
$2x + 1 = 85$
$2x = 85 - 1$
$2x = 84$
$x = 84 \div 2$
$x = 42$
So, the first number is 42.
The second number is x + 1 = 42 + 1 = 43.
The two consecutive numbers are 42 and 43.
Now, let's check the parity of their sum:
42 is an Even number.
43 is an Odd number.
Even + Odd = Odd.
The sum, 85, is indeed an Odd number. This confirms the property that the sum of two consecutive numbers is always odd.

## Activity

**Parity Prediction Game**




**Objective:** To reinforce the rules of parity for addition and subtraction.
**Materials:** Number cards (1-20, two sets), a whiteboard or large paper.
**Procedure:**
1. Divide the class into small groups.
2. Each group draws two number cards.
3. Before calculating, they predict whether the sum of the two numbers will be even or odd, and whether the difference will be even or odd. They must state the parity rule they used (e.g., "Odd + Even = Odd").
4. They then calculate the actual sum and difference to verify their prediction.
5. Repeat several rounds. The group with the most correct predictions wins.

**Inquiry-based:** Encourage students to discuss why the rules work, perhaps by drawing small diagrams or thinking about pairing.

## Knowledge Checkpoint

- Is 146 an even or odd number? How do you know?
- If you add an odd number to an even number, what kind of number will the sum be?
- Give an example of two odd numbers whose sum is an even number.

## Key Terms

- **Even Number:** An integer that is exactly divisible by 2.
- **Odd Number:** An integer that is not exactly divisible by 2; it leaves a remainder of 1 when divided by 2.
- **Parity:** The property of an integer being either even or odd.
- **Consecutive Numbers:** Numbers that follow each other in order, with a difference of 1.
- **Remainder:** The amount left over after a division.

## Do It Yourself

- If you multiply two odd numbers, what is the parity of the product? What about two even num bers? Or an even and an odd number? (**Hint:** Think about the algebraic forms 2n and 2n + 1).
- Consider a very large number, like 5,789,342. How can you quickly tell if it's even or odd without dividing it by 2?

## Fact Flash

- The only prime number that is even is 2. All other prime numbers are odd.
- The sum of any number of even numbers is always even.
- The sum of an even number of odd numbers is always even. (e.g., 3 + 5 + 7 + 9 = 24)
- The sum of an odd number of odd numbers is always odd. (e.g., 3 + 5 + 7 = 15)
- The word "**odd**" comes from an Old Norse word meaning "**point**" or "**triangle**," possibly referring to the single point left over when numbers are arranged in pairs.

## Mental Mathematics

- Quickly state the parity of the sum: 12 + 18, 25 + 31, 40 + 17.
- Is 200 an even or odd number?
- What is the parity of the number of days in February (non-leap year)?
- If you start at 1 and count up by 3s (1, 4, 7, 10, ...), what is the parity pattern?
- Without calculating, what is the last digit of the sum of 13 and 27? What is its parity?

Gap Analyzer™ Homework

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## Exercise 6.1

**1. Fill in the Blanks**

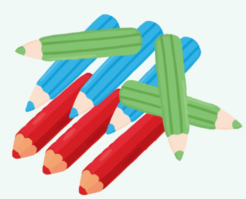
a) An odd number leaves a remainder of _______ when divided by 2.
b) The sum of two odd numbers is always an _______ number.
c) The product of an even number and any other integer is always an _______ number.
d) The algebraic expression for any even number is _______, where 'n' is an integer.

2. **Determine whether each of the following numbers is even or odd:**

| Number | Odd/Even |
|---|---|
| 99 | |
| 124 | |
| 108 | |

| Number | Odd/Even |
|---|---|
| 111 | |
| 17 | |
| 305 | |

3. **Solve the following questions:**
   a) Write a general algebraic expression for the nth odd number, where 'n' is a positive integer. Using this expression, what is the 12th odd number?
   b) Write a general algebraic expression for the nth even number, where 'n' is a positive integer. Using this expression, what is the 25th even number?
   c) If 'k' is an integer, determine the parity (even or odd) of the expression 2k + 7. Justify your answer.
   d) Consider the expression 6m - 1, where 'm' is an integer.
      i) Is 6m an even or odd number?
      ii) What is the parity of the entire expression 6m - 1? Explain your reasoning.
4. The sum of three integers is 50. Two of the integers are odd. What can you say about the third integer? Is it even or odd? Explain your reasoning.
5. **Let A be an even number represented as 2k, and B be an odd number represented as 2m + 1.**
   a) Write an algebraic expression for the sum of A and B.
   b) Using your expression from part (a), explain why the sum of an even and an odd number is always odd.
6. Two friends, Rohan and Meera, are comparing their ages. Rohan says, "If you add both our ages, it comes to 85." But Meera reminds him that she is exactly 5 years younger than Rohan. Is this possible? Why or why not?
7. **Rita has three types of pencils in her box.**
   - She has an odd number of red pencils,
   - an odd number of blue pencils, and
   - an even number of green pencils.

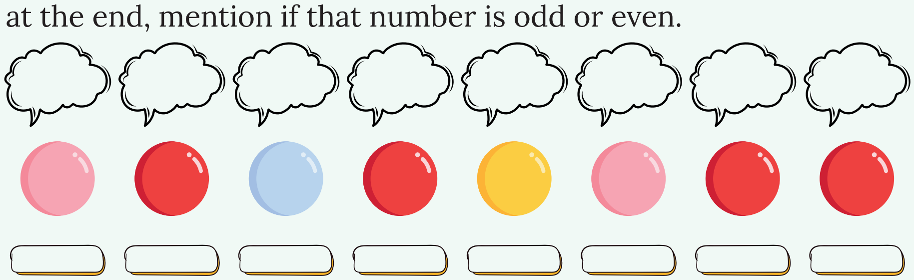


When she counted them all together, she got 28 pencils in total. Is this possible? Why or why not?

If possible, give one combination of red, blue, and green pencils.

☐ + ☐ + ☐ = ☐

8. The sum of three consecutive numbers is 63. What are the numbers? What is the parity of each number?
9. Write the number for each ball such that the number represents the number of red balls in front of them. Add at the end, mention if that number is odd or even.



## Number Puzzles and Sequences

Numbers are not just tools for calculation; they are also the stars of many intriguing puzzles and the building blocks of beautiful patterns called sequences.

In this section, we will explore how logical thinking, combined with our understanding of number properties like parity, can help us solve these challenges.

### Sub-concepts to be covered

1. Grid Puzzles
2. Magic Squares (Definition, Properties, Historical Context, Algebraic Generalization)
3. Cryptarithms (Alphametics)
4. Virahānka–Fibonacci Numbers (Definition, Origin, Natural and Artistic Occurrences)

### Grid Puzzles

Grid puzzles are a type of puzzle where numbers, letters, or symbols are placed in a grid (rows and columns) by following certain rules. They can be a square or rectangle. These puzzles help develop logical reasoning and systematic problem-solving skills.

Rules are different for each kind of grid puzzle.

**Example:** Sudoku (fill numbers 1 – 9), Crossword (fill words), KenKen or Kakuro (number sums).

**Magic Square:** No repetition of numbers 1 to 9 in each row, column and diagonally. And sum of each row, column and diagonal must be the same. *Fig. 6.16*

**Sudoku:** No repetition of numbers 1 to 9 in each row, column and in each bold 9 × 9 box. *Fig. 6.17*

**Kenken:** A square grid puzzle in which number have to be filled according to the given operation and number. *Fig. 6.18*

**3 × 3 Magic Grid puzzle**

| | | | |
|---|---|---|---|
| **4** | **9** | **2** | **15** |
| **3** | **5** | **7** | **15** |
| **8** | **1** | **6** | **15** |
| **15** | **15** | **15** | |

*Fig. 6.16 Magic Square*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 9 | 4 | 1 | 2 | 6 | 3 | 5 |
| 4 | 1 | 5 | 6 | 9 | 3 | 7 | 2 | 8 |
| 2 | 3 | 6 | 5 | 7 | 8 | 9 | 1 | 4 |
| 5 | 2 | 1 | 7 | 3 | 4 | 8 | 9 | 6 |
| 6 | 4 | 3 | 8 | 2 | 9 | 1 | 5 | 7 |
| 8 | 9 | 7 | 1 | 5 | 6 | 2 | 4 | 3 |
| 9 | 5 | 8 | 3 | 6 | 1 | 4 | 7 | 2 |
| 1 | 7 | 8 | 2 | 8 | 5 | 3 | 6 | 9 |
| 3 | 6 | 2 | 9 | 4 | 7 | 5 | 8 | 1 |

*Fig. 6.17 Sudoku*

| | | | |
|---|---|---|---|
| 2÷ **2** | 3 **3** | 4× **1** | **4** |
| **3** | 2÷ **2** | 7+ **3** | **1** |
| 3 **3** | **1** | **4** | 7+ **2** |
| 3− **1** | **4** | **2** | **3** |

*Fig. 6.18 Kenken*

### Magic Squares

A magic square is a square grid where each cell contains a distinct number, and the sum of the numbers in each row, each column, and both main diagonals is the same. This constant sum is called the **magic sum**.

A square array of numbers where the sum of numbers in every row, every column, and both main diagonals is identical.

**Properties of a 3 × 3 Magic Square (using numbers 1-9):**

**Magic Sum:** For a 3 × 3 magic square using numbers 1 to 9, the sum of all numbers is 1 + 2 + ... + 9 = 45. Since there are 3 rows (or columns), and each must sum to the same value, the magic sum must be 45 ÷ 3 = 15.

- **Center Number:** The number in the center cell of a 3 × 3 magic square (using numbers 1 - 9) is always 5. This is because the center number is part of four sums (middle row, middle column, and both diagonals).
- **Corner Numbers:** The numbers 2, 4, 6, 8 (even numbers) are typically in the corner positions.
- **Side Numbers:** The numbers 1, 3, 7, 9 (odd numbers excluding 5) are typically in the middle of the sides.

**Magic Sum**

| | | | |
|---|---|---|---|
| **4** | **9** | **2** | **15** |
| **3** | **5** | **7** | **15** |
| **8** | **1** | **6** | **15** |
| **15** | **15** | **15** | |

*Fig. 6.19*

**Historical Context:**

- **Lo Shu Square (Ancient China):** The oldest known magic square, a 3 × 3 grid with numbers 1 - 9, having a magic sum of 15. Legend says it appeared on the back of a turtle.
- **Algebraic Generalization of Magic Squares:**We can use variables to represent the numbers in a magic square, especially if the numbers are consecutive or follow a pattern. For a 3 × 3 magic square where the numbers are consecutive and the center number is **m**, the magic sum is 3m. The other numbers can be expressed in terms of **m** and constants.

  A 3 × 3 *grid with algebraic expressions:*

| | | |
|---|---|---|
| (m + 3) | (m − 4) | (m + 1) |
| (m − 2) | (m) | (m + 2) |
| (m − 1) | (m + 4) | (m − 3) |

*Fig. 6.20*

(**Note:** This is a common algebraic representation, but the specific constants might vary based on the chosen starting point or pattern. The key is to show 'm' at the center and other cells as **m +/- x**.)

A clear, labeled diagram of a 3 × 3 magic square (e.g., the standard 8,1,6 / 3,5,7 / 4,9,2) with arrows indicating rows, columns, and diagonals, and the magic sum clearly stated

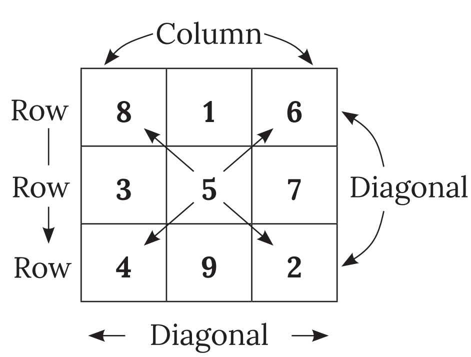



*Fig. 6.21*

Magic Sum = 15

### Cryptarithms (Alphametics)

Cryptarithms are mathematical puzzles where digits (0 - 9) are replaced by letters. Each letter represents a unique digit, and the goal is to find which digit each letter stands for to make the mathematical equation true.

**Rules**

- Each letter represents a unique digit from 0 to 9.
- No two letters can represent the same digit.
- The first letter of a number cannot be zero (e.g., in "HOPE + HOPE = SMILE", H and S cannot be 0).
- The puzzle is usually presented as an arithmetic problem (addition, subtraction, multiplication, or division).

**Solving Strategies**

- **Look for obvious clues:** For example, in addition, if two identical letters add up to a two-digit number, the carry-over must be 1.
- **Consider parity:** If A + A = B, then B must be an even number.
- **Analyze column by column:** Start from the rightmost column (units place) and work your way left, considering carry-overs.
- **Trial and error:** Sometimes, you might need to try a few possibilities and eliminate those that don't fit the rules.

## Virahānka–Fibonacci Numbers

The Virahānka–Fibonacci sequence is a special sequence of numbers where each number is the sum of the two preceding ones. The sequence typically starts with 1, 1, or 1, 2.

**Sequence:** 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

$F_n = F_{n-1} + F_{n-2}$, where $F_n$ is the nth term in the sequence.

- **Origin and Indian Contributions:** This sequence was first described in detail by Indian mathematicians and scholars, particularly in the context of Sanskrit prosody (the study of poetic meters).
- **Pingala (c. 300 BCE):** An ancient Indian prosodist whose work **Chhandah-shastra** implicitly contains the sequence in his method for counting combinations of long and short syllables.
- **Virahānka (c. 700 CE):** Explicitly described the sequence in his work on prosody. He was interested in finding the number of ways to form a meter of a given length using combinations of long (2 beats) and short (1 beat) syllables.
- **Gopala (c. 1135 CE) and Hemachandra (c. 1150 CE):** Other Indian mathematicians who studied this sequence in similar contexts.
- **Leonardo Fibonacci (1202 CE):** An Italian mathematician who introduced the sequence to the Western world in his book **Liber Abaci**, using a problem about rabbit population growth. Due to his popularization, it became widely known as the "**Fibonacci sequence**" in the West. It is more accurate to call them "**Virahānka–Fibonacci numbers**" to acknowledge their true origin.

**Occurrence in Nature and Art**

**Botany:** The numbers appear in the arrangement of leaves on a stem, the branching of trees, the spirals of seeds on a sunflower head, the scales of a pinecone, and the number of petals on many flowers (e.g., daisies often have 13, 21, or 34 petals).

*Fig. 6.22*

***A photograph of a sunflower head showing clear spirals.***

- **Art and Architecture:** The Golden Ratio (approximately 1.618), which is closely related to the Fibonacci sequence (the ratio of consecutive terms approaches the Golden Ratio), has been used in art and architecture for centuries to create aesthetically pleasing proportions.

- **Music:** The sequence can be found in musical compositions, particularly in the structure of melodies and rhythms.
- **Problem-Solving with Fibonacci Sequence:** The sequence is useful for solving problems that involve counting combinations where each step depends on the previous two steps, such as climbing stairs where you can take 1 or 2 steps at a time.

## Mathematical Explanation

**Grid Puzzles**

Imagine a grid is a perfectly organized shelf. The rules tell you how you can place your items (numbers) so that everything is in order and nothing clashes. Your job is to use logic to figure out the one and only correct arrangement.

**Example:** A Simple 4 × 4 Sudoku Let's look at a 4 × 4 Sudoku. The rules are:

1. Each row must contain the numbers 1, 2, 3, and 4 exactly once.
2. Each column must contain the numbers 1, 2, 3, and 4 exactly once.
3. Each 2 × 2 box (the four smaller squares) must contain the numbers 1, 2, 3, and 4 exactly once.

### Initial Puzzle

**Solving Process (A thought process to model for students)**

| | | | |
|---|---|---|---|
| | | 3 | 2 |
| 3 | | | 1 |
| | 4 | | 3 |
| 1 | | 2 | 4 |

*Fig. 6.23*

- "Look at the top-right 2 × 2 box. It has a 3 and a 2. It needs a 1 and a 4. The first row already has a 2, so the empty cell in that row must be a 1. That means the cell below it must be a 4."
- This step-by-step logical deduction, with no guessing, is the key skill being developed.

### Solved Puzzle

| | | | |
|---|---|---|---|
| 4 | 1 | 3 | 2 |
| 3 | 2 | 4 | 1 |
| 2 | 4 | 1 | 3 |
| 1 | 3 | 2 | 4 |

*Fig. 6.24*

### Magic Squares

**The Magic Constant:** For a magic square of order 'n' (meaning n rows and n columns) that uses numbers from 1 to $n^2$, the magic constant (M) can be calculated with the formula: $M = \frac{n(n^2+1)}{2}$

1. **Example for a 3 × 3 square:** Here, n = 3. $M = \frac{3(3^2+1)}{2} = \frac{3(9+1)}{2} = \frac{3 \times 10}{2} = \frac{30}{2} = 15$.

   So, any standard 3 × 3 magic square will have a magic constant of 15.

### Algebraic Generalization

We can use algebra to understand why a magic square works. Let's represent a 3 × 3 square with variables:

**The rules mean:**

- a + b + c = M
- d + e + f = M
- a + d + g = M
- a + e + i = M (and so on...)

| a | b | c |
|---|---|---|
| d | e | f |
| g | h | i |

*Fig. 6.25*

This shows that algebra is a powerful tool for describing rules and patterns. For Grade 7, you don't need to solve this generally. The goal is to show the connection between the puzzle and the concepts in their 'Simple Equations' chapter.

## Cryptarithms (Alphametics)

**Example:** A Classic Alphametic

```
  H O P E
+ H O P E
---------
 S M I L E
```

**Logical Deduction Process:**

1. **Focus on right-most column:** E + E = E. There can be two possibilities, E = 0 or E = 5, let us consider E=0 first for simplicity.
2. **Now, move to P + P = L:** Sum of two same numbers is some different number. Taking any random number, other than 0 (already used) to avoid the carry over. 2 + 2 = 4

   So, P = 2 and L = 4
3. **Move on the O + O = I:** Sum of two same numbers is some different number. Taking any random number, other than 0, 2, 4 (already used) to avoid the carry over. 3 + 3 = 6

   So, O = 3 and I = 6
4. **Move on the H + H = SM:** Sum of two same numbers is some different number, with carry over. Here, S can only be 1.

   Taking any random number, other than 0, 2, 3, 4, 6 (already used) with 1 as carry over.

   7 + 7 = 14 (But, 4 is already used)

   8 + 8 = 16 (But, 6 is already used)

   9 + 9 = 18, So, H = 9, M = 8 and S = 1

   **Solution:** E = 0, P = 2, L = 4, O = 3, I = 6, H = 9, M = 8 and S = 1

## Virahānka–Fibonacci Numbers

The sequence starts with 0 and 1. The next number is found by adding the two before it.

The Recursive Formula If $F_n$ is the term we want to find, then:

- **The 'previous number'** is $F_{n-1}$ (the term at position n-1)
- **The 'number before the previous'** is $F_{n-2}$ (the term at position n-2)

So, our rule **Next Number = Previous Number + Number Before Previous becomes:**

**Definition:** $F(n) = F(n-1) + F(n-2)$

**With starting terms:**

- F (0) = 0, F (1) = 1
- F (2) = F (1) + F (0) = 1 + 0 = 1

- F (3) = F (2) + F (1) = 1 + 1 = 2
- F (4) = F (3) + F (2) = 2 + 1 = 3
- F (5) = F (4) + F (3) = 3 + 2 = 5
- F (6) = F (5) + F (4) = 5 + 3 = 8

...and so on. The sequence is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

## Natural and Artistic Occurrence

### Seed Heads

The seeds in a sunflower head are arranged in two sets of spirals curving in opposite directions. The number of spirals in each direction is almost always a pair of consecutive Fibonacci numbers, like 34 and 55. This is the most efficient way to pack seeds.

### In Art and Architecture

The ratio of consecutive Fibonacci numbers (e.g., $\frac{8}{5} = 1.6$, $\frac{13}{8} = 1.625$) gets closer and closer to the **Golden Ratio** (approx. 1.618). This ratio is considered to be aesthetically pleasing and has been used by artists and architects for centuries, from the Parthenon in ancient Greece to Leonardo da Vinci's Mona Lisa.

**Example 6 :** Fill the 3 × 3 grid using numbers from 1 to 9, each used once, such that the sum of each row and each column is 15.

**Solution:** This is a standard 3 × 3 magic square. We know the center must be 5.
Let's try to place numbers systematically.
The sum of 1 to 9 is 45. 45 ÷ 3 = 15. So, the magic sum is 15.
Center number is 5.

Pairs that sum to 15 (excluding 5): (1, 14 - not possible), (2, 13 - not possible), (3, 12 - not possible), (4,11 - not possible), (6, 9), (7, 8).

Pairs that sum to 10 (with 5): (1, 9), (2, 8), (3, 7), (4, 6). These pairs must be opposite to each other through the center 5.
One possible solution (the standard magic square):

| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

*Fig. 6.26*

**Verification:**
**Rows:** 8 + 1 + 6 = 15, 3 + 5 + 7 = 15, 4 + 9 + 2 = 15
**Columns:** 8 + 3 + 4 = 15, 1 + 5 + 9 = 15, 6 + 7 + 2 = 15
**Diagonals:** 8 + 5 + 2 = 15, 6 + 5 + 4 = 15

**Example 7 :** A certain type of flower always has a number of petals that is part of the Virahānka–Fibonacci sequence (1, 2, 3, 5, 8, 13, 21, 34, ...). If you find a flower with 55 petals, what would be the next possible number of petals for a flower of this type?

**Solution:** The Virahānka–Fibonacci sequence is generated by adding the two previous numbers.

The sequence given is: 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
To find the next number after 55, we need to add the two numbers preceding it: 34 and 55.
Next number = 34 + 55 = 89.
So, the next possible number of petals for a flower of this type would be 89.

**Example 8 :** Using the algebraic form for a 3 × 3 magic square with center 'm':

| | | |
|---|---|---|
| m+3 | m-4 | m+1 |
| m-2 | m | m+2 |
| m-1 | m+4 | m-3 |

*Fig. 6.27*

If the magic sum is 27, find the value of 'm' and fill the magic square.

**Solution:** We know that for this algebraic form, the magic sum is 3m.
Given magic sum = 27.
So, 3m = 27
m = 27 ÷ 3
m = 9.
Now substitute m = 9 into the algebraic expressions:

**Row 1:** m + 3 = 9 + 3 = 12
m - 4 = 9 - 4 = 5
m + 1 = 9 + 1 = 10

**Row 2:** m - 2 = 9 - 2 = 7
m = 9
m + 2 = 9 + 2 = 11

**Row 3:** m - 1 = 9 - 1 = 8
m + 4 = 9 + 4 = 13
m - 3 = 9 - 3 = 6

The filled magic square is:

| | | |
|---|---|---|
| 12 | 5 | 10 |
| 7 | 9 | 11 |
| 8 | 13 | 6 |

*Fig. 6.28*

**Verification:**
**Rows:** 12 + 5 + 10 = 27, 7 + 9 + 11 = 27, 8 + 13 + 6 = 27
**Columns:** 12 + 7 + 8 = 27, 5 + 9 + 13 = 27, 10 + 11 + 6 = 27
**Diagonals:** 12 + 9 + 6 = 27, 10 + 9 + 8 = 27
All sums are 27.

## Knowledge Checkpoint

- What is the magic sum of a 3 × 3 magic square that uses the numbers 1 to 9?
- In the cryptarithm "A + A = B", if A and B are single digits, what can you say about the parity of B?
- List the first 7 terms of the Virahānka–Fibonacci sequence starting with 1, 2.

## Activity

**Constructing Your Own Magic Square**

**Objective:** To understand the properties and construction of magic squares.

**Materials:** Grid paper, pencil, number cards (1-9).

**Procedure:**

1. Provide students with a blank 3 × 3 grid.
2. Challenge them to create a magic square using numbers 1-9.
3. Guide them by reminding them of the magic sum (15) and the center number (5).
4. Encourage them to try different placements for the remaining numbers, using trial and error and logical deduction.
5. Once they complete one, challenge them to find another unique 3 × 3 magic square (there are 8 unique solutions, ignoring rotations and reflections).

**Inquiry-based:** Ask students to explain their strategy. "What was the hardest part? What did you learn about number placement?"

## Key Terms

- **Grid Puzzle:** A problem involving arranging numbers in a grid according to specific rules.
- **Magic Square:** A square grid where the sum of numbers in each row, column, and main diagonal is the same.
- **Magic Sum:** The constant sum in a magic square.
- **Cryptarithm (Alphametic):** A mathematical puzzle where letters represent unique digits.
- **Virahānka–Fibonacci Numbers:** A sequence where each number is the sum of the two preceding ones (e.g., 1, 2, 3, 5, 8...).
- **Recursive Sequence:** A sequence where each term is defined by one or more preceding terms.

## Do It Yourself

- Can you create a 3 × 3 grid that is not a magic square, but where all rows sum to the same number? What about all columns?
- If you have a cryptarithm like "ABC + DEF = GHI", how many possible digits are there for each letter? What makes solving these puzzles challenging?
- How would the Virahānka–Fibonacci sequence change if it started with 0, 1 instead of 1, 2? Would it still have the same properties?

## Fact Flash

- There are 8 unique 3 × 3 magic squares using numbers 1-9 (if rotations and reflections are considered the same).
- The sum of the numbers in any magic square is always equal to the magic sum multiplied by the number of rows (or columns).
- The ratio of consecutive Virahānka–Fibonacci numbers (e.g., $\frac{89}{55}$, $\frac{144}{89}$) gets closer and closer to the Golden Ratio (approximately 1.618). This ratio is often represented by the Greek letter phi (ϕ).

## Mental Mathematics

- What is the 8th term in the Virahānka–Fibonacci sequence (starting 1, 2, 3...)?
- If a 3x3 magic square has a magic sum of 30, what is the center number?
- In the cryptarithm "AA + BB = CC", what is the smallest possible value for C?
- If you have a 5-step staircase and can take 1 or 2 steps at a time, how many ways can you climb it?
- Quickly identify the next two numbers in the sequence: 2, 4, 6, 10, 16, ... (This is a variation of Fibonacci where you add the previous two, but starting with evens).

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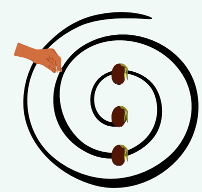
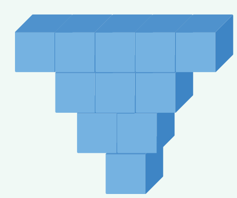
## Exercise 6.2

1. **Fill in the Blanks**
   a) In a magic square, the sum of numbers in each row, column, and main diagonal is called the ___________.
   b) The earliest known magic square is the ___________ Square from ancient China.
   c) In a cryptarithm, each letter represents a ___________ digit.
   d) The Virahānka–Fibonacci sequence is generated by adding the _______ preceding numbers.
   e) The 10th-century 4 × 4 magic square found in Khajuraho is known as the ___________.
2. If the magic sum of a 3 × 3 magic square is 21, what is the number at the center of the square?
3. A gardener is planting seeds in a spiral pattern. If the first two layers have 1 and 2 seeds respectively, and each subsequent layer has a number of seeds equal to the sum of the previous two layers, how many seeds will be in the 7th layer?



4. A puzzle requires you to arrange numbers in a 4x4 grid such that each row and column sums to 34. If you have the numbers 1 to 16, can you create such a grid? (**Hint:** This is a famous magic square).
5. A secret code uses a cryptarithm where each letter in "MATH + FUN = PLAY" represents a unique a (leading letters are non-zero). If M = 3 and L = 0, what could be the value of U?
6. **Use the Fibonacci rule to find the missing terms in the given sequences**:
   a) 21, 24, 45,_____, 114, _____
   b) 52, 63, ______, 178, ______, 471
   c) 12, 16, 28, ______, _______, _______
7. If you are building a tower with blocks, and each layer has a number of blocks from the Virahānka–Fibonacci sequence, approximately how many layers would you need to use over 100 blocks in total?



8. Solve the cryptarithm: `IT + IS = FUN`. (Each letter represents a unique digit. I and F cannot be 0).
9. Fill in the numbers 10, 11, 12, 13, 14, 15, 16, 17, 18 in the given magic square. What is its magic sum?

| | | |
|---|---|---|
| | | |
| | | |
| | | |

10. Find out which number is wrong in the given magic grid. Cross that out and write the correct number.

| | | |
|---|---|---|
| 16 | 21 | 14 |
| 15 | 17 | 19 |
| 20 | 12 | 18 |

## Common Misconceptions

**Misconception:** Zero is neither even nor odd.

**Correction:** Zero is an even number. According to the definition, an even number is any integer that is divisible by 2 without a remainder. 0 ÷ 2 = 0, with a remainder of 0. Therefore, 0 fits the definition of an even number.

**Misconception:** All prime numbers are odd.

**Correction:** While most prime numbers are odd, the number 2 is a prime number and it is even. It is the only even prime number.

**Misconception:** The sum of any three numbers is always odd or always even.

**Correction:** The parity of the sum of three numbers depends on the parity of each individual number. For example, Odd + Odd + Odd = Odd, but Even + Even + Odd = Odd, and Even + Odd + Odd = Even. Students need to apply the rules step-by-step.

## Real-life Number Play: Mathematical Applications

Understanding number parity (even/odd) and recognizing patterns are fundamental skills that develop logical reasoning and prediction. These applications make number theory relevant, fostering critical thinking and problem-solving, aligning with NEP 2020's focus on experiential learning:

1. **Scheduling & Cycles:** Identifying alternating events or recurring sequences, like odd/even days for traffic restrictions or duty rosters. (Helps organize and predict events).
2. **Resource Distribution:** Ensuring fair division or identifying remainders, such as distributing items equally or determining if something can be split into pairs. (Fundamental for sharing and grouping).
3. **Game Strategies:** Using parity to predict outcomes or develop winning moves in simple games, like separating objects into even/odd piles. (Develops strategic thinking and foresight).
4. **Digital Checksums & Error Detection:** Understanding the concept behind how computers ensure data integrity (though complex, the underlying idea of parity checks is relevant). (Introduces foundational computational logic).
5. **Predicting Sequences:** Extending number patterns in various contexts, from growing plant arrays to predicting the next number in a series. (Enhances observational skills and logical inference).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# EXERCISE

**A. Choose the correct answer.**

1. Which of the following numbers is an odd number?
   a) 246 ☐ b) 380 ☐ c) 517 ☐ d) 992 ☐
2. The sum of an even number and an odd number is always:
   a) Even ☐ b) Odd ☐
   c) Can be either even or odd ☐ d) Zero ☐
3. If 'k' is an integer, which expression always represents an odd number?
   a) 2k ☐ b) 2k + 2 ☐ c) 3k ☐ d) 2k - 1 ☐
4. What is the magic sum of a 3x3 magic square using numbers 1 to 9?
   a) 9 ☐ b) 15 ☐ c) 27 ☐ d) 45 ☐
5. The next term in the Virahānka–Fibonacci sequence 1, 2, 3, 5, 8, 13, ... is:
   a) 18 ☐ b) 20 ☐ c) 21 ☐ d) 26 ☐

## Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.
Study both the statements and state which of the following is correct:

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

**1.** **Assertion (A):** The sum of 10 odd numbers is always an even number.
**Reason (R):** The sum of an even number of odd numbers is always even.

**2.** **Assertion (A):** The number 144 is an even number.
**Reason (R):** Any number ending with 4 is an even number.

**3.** **Assertion (A):** The center number of any 3x3 magic square using numbers 1-9 is always 5.
**Reason (R):** The magic sum for such a square is 15, and 5 is the middle number of the sequence 1-9.

## Case Study

Rohan has a laundry basket full of socks. He knows he put in 15 pairs of socks (30 socks total) and then added 7 single socks that didn't have a match. After washing, he pulls out 28 socks. He wants to know if he has an even or odd number of socks left in the basket without counting them all. He also wonders if it's possible that all the socks he pulled out were perfectly paired.

1. What is the total number of socks Rohan initially had in the basket? Is this number even or odd?
2. If Rohan pulled out 28 socks, is this an even or odd number?
3. Without counting, determine the parity of the socks remaining in the basket. Explain your reasoning using parity rules.
4. Is it possible that all 28 socks Rohan pulled out were perfectly paired (meaning they formed 14 pairs)? Why or why not?

## Project

**The Great Number Pattern Exhibition**

**Objective:** To synthesize understanding of parity, number puzzles, and sequences, and present them creatively.

**Task:** Work individually or in pairs to create an exhibit for a "Great Number Pattern Exhibition." Your exhibit should showcase at least three different concepts from this chapter (e.g., parity, magic squares, Virahānka–Fibonacci numbers, cryptarithms).

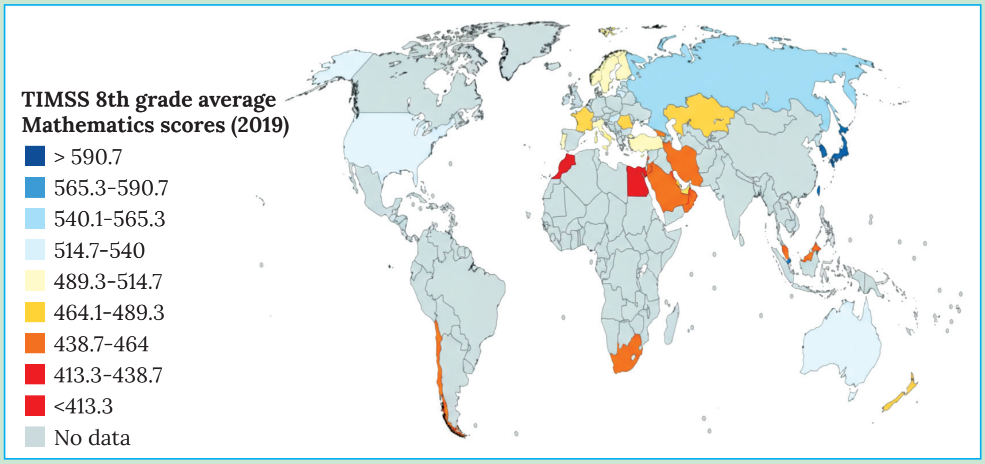
**Your exhibit must include:**

1. **A Title:** Catchy and relevant to your chosen concepts.
2. **Introduction:** A brief overview of the mathematical concepts you are presenting.
3. **Interactive Element:**
   - **For Parity:** A "Parity Challenge" board where visitors guess the parity of sums/differences.
   - **For Magic Squares:** A blank 3 × 3 magic square for visitors to try filling, with a hidden solution.
   - **For Cryptarithms:** A simple cryptarithm for visitors to solve, with hints available.
   - **For Virahānka–Fibonacci:** A display of natural objects (photos or drawings) showing the sequence, or a "staircase climbing" puzzle.
4. **Visuals:** Clear diagrams, illustrations, or photographs relevant to your concepts.
5. **Explanation:** Concise and clear explanations of the mathematical principles behind your chosen concepts.
6. **Real-World Connection:** A short paragraph explaining how these concepts are used in daily life, science, art, or technology.
7. **"Did You Know?" Fact:** An interesting historical or fun fact related to your exhibit.

**Presentation:** Prepare to present your exhibit to the class, explaining your choices and the mathematical ideas behind them. This project encourages creativity, research, critical thinking, and communication skills.

## Source-Based Question

The International Association for the Evaluation of Educational Achievement (IEA)'s **Trends in International Mathematics and Science Study (TIMSS)** is a series of international assessments of the mathematics and science knowledge of students around the world. The participating students come from a diverse set of educational systems (countries or regional jurisdictions of countries) in terms of economic development, geographical location, and population size. In each of the participating educational systems, a minimum of 4,000 to 5,000 students is evaluated. Contextual data about the conditions in which participating students learn mathematics and science are collected from the students and their teachers, their principals, and their parents via questionnaires




TIMSS is one of the studies established by IEA aimed at allowing educational systems worldwide to compare students' educational achievement and learn from the experiences of others in designing effective education policy. This assessment was first conducted in 1995, and has been administered every four years thereafter. Therefore, some of the participating educational systems have trend data across assessments from 1995 to 2023. TIMSS assesses 4th and 8th grade students, while TIMSS Advanced assesses students in the final year of secondary school in advanced mathematics and physics.

### Definition of terms

"Eighth grade" in the United States is approximately 13–14 years of age and equivalent to:

- Year 9 (Y9) in England and Wales
- 2nd Year in the Republic of Ireland
- Form 2 in Hong Kong
- Year 9 in New Zealand
- 2nd Year (S2) in Scotland
- 1st Year in South Africa
- 4ème in France
- Form 2 in Malaysia

### TIMSS 2023

TIMSS 2023 was the eighth cycle of TIMSS and reported overall achievement as well as results according to international benchmarks, by major content domains (number, algebra, and geometry in mathematics, and earth science, biology, and chemistry in science) and by cognitive domains (knowing, applying, reasoning). TIMSS 2023 collected detailed information about curriculum and curriculum implementation of participating countries and published this information the TIMSS 2023 Encyclopedia: Education Policy and Curriculum in Mathematics and Science.

TIMSS 2023 results are summarized in TIMSS 2023 International Results in Mathematics and Science. This detailed report presents achievement and contextual data from participating countries and benchmarking entities. **The TIMSS 2023 Encyclopedia:** Education Policy and Curriculum in Mathematics and Science describes various features of the education systems in each participating country, including the mathematics and science curriculum, professional development requirements for teachers, and methods of monitoring student progress in mathematics and science. Each country's "chapter" in the encyclopedia was authored by that country's TIMSS representative.

### Eighth grade – Mathematics

| Rank | Country | Average scale score | Change over 4 years | Rank | Country | Average scale score | Change over 4 years |
|---|---|---|---|---|---|---|---|
| 1 | Chinese Taipei | 602 | ▼ 10 points | 25 | Azerbaijan | 479 | N/A |
| 2 | South Korea | 596 | ▼ 11 points | 26 | France | 479 | ▼ 4 points |
| 3 | Japan | 595 | ▲ 1 point | 27 | Portugal | 475 | ▼ 25 points |
| 4 | Hong Kong | 575 | ▼ 3 points | 28 | Georgia | 467 | ▲ 6 points |
| 5 | England | 525 | ▲ 10 points | 29 | Kazakhstan | 454 | ▼ 34 points |
| 6 | Ireland | 522 | ▼ 2 points | 30 | Qatar | 451 | ▲ 8 points |
| 7 | Czech Republic | 518 | N/A | 31 | Bahrain | 426 | ▼ 55 points |
| 8 | Sweden | 517 | ▲ 14 points | 32 | Iran | 423 | ▼ 20 points |
| 9 | Lithuania | 514 | ▼ 6 points | 33 | Uzbekistan | 421 | N/A |
| 10 | Austria | 512 | N/A | 34 | Chile | 416 | ▼ 25 points |
| 11 | Australia | 509 | ▼ 8 points | 35 | Oman | 411 | 0 |
| 12 | Turkey | 509 | ▲ 13 points | 36 | Malaysia | 411 | ▼ 50 points |
| 13 | Hungary | 506 | ▼ 11 points | 37 | Kuwait | 399 | ▼ 4 points |
| 14 | Finland | 504 | ▼ 5 points | 38 | Saudi Arabia | 397 | ▲ 3 points |
| 15 | Norway | 501 | ▼ 2 points | 39 | South Africa | 397 | ▲ 8 points |
| 16 | Italy | 501 | ▲ 4 points | 40 | Jordan | 388 | ▼ 32 points |
| **International average** | | **500** | - | 41 | Palestine | 382 | N/A |
| 18 | Malta | 499 | N/A | 42 | Brazil | 378 | N/A |
| 19 | Romania | 496 | ▲ 17 points | 43 | Morocco | 388 | ▼ 10 points |
| 20 | Cyprus | 494 | ▼ 7 points | 44 | Ivory Coast | 263 | N/A |
| 21 | United Arab Emirates | 489 | ▲ 16 points | **Benchmarking participants** | | | |
| 22 | United States | 488 | ▼ 27 points | - | Dubai (United Arab Emirates) | **546** | **▲ 9 points** |
| 23 | New Zealand | 485 | ▼ 14 points | - | Sharjah (United Arab Emirates) | **499** | **N/A** |
| 24 | Israel | 487 | ▼ 32 points | - | Abu Dhabi (United Arab Emirates) | **454** | **▲ 18 points** |

Source Text: **Adapted from reports published in Trends in International Mathematics and Science Study**

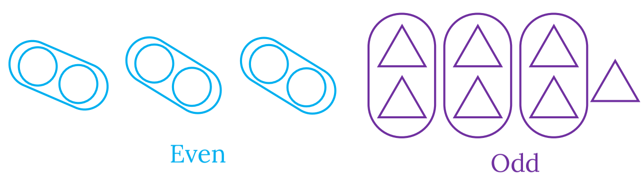
### Questions on the Data

1. Using the above data, write how many “Average Scale Score” are even and how many are odd?
2. Using the above data, write how many numbers given in “Change over 4 years” are even and how many are odd?
3. From the section ”Benchmarking participants” take the greatest and the smallest number and make a Fibonacci sequence upto 6th place.
4. From the column “Change over 4 years” take smallest 9 numbers and made a 3×3 magic square.

# Number Play

## Even and Odd Numbers

- **Even Numbers:** Visual and Algebraic Representation
- **Odd Numbers:** Visual and Algebraic Representation
- **Properties of Even and Odd Numbers under Addition and Subtraction**
- **Parity of Consecutive Numbers**




| Operation | Parity |
|---|---|
| Even + Even | Even |
| Odd + Odd | Even |
| Even + Odd | Odd |
| Odd + Even | Odd |

| Operation | Parity |
|---|---|
| Even – Even | Even |
| Odd – Odd | Even |
| Even – Odd | Odd |
| Odd – Even | Odd |

## Number Puzzles and Sequences

- **Grid Puzzles**
- **Magic Squares**
- **Cryptorithms (Alphametics)**
- **Virahānka–Fibonacci Numbers**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 8 | 9 | 4 | 1 | 2 | 6 | 3 | 5 |
| 4 | 1 | 5 | 6 | 9 | 3 | 7 | 2 | 8 |
| 2 | 3 | 6 | 5 | 7 | 8 | 9 | 1 | 4 |
| 5 | 2 | 1 | 7 | 3 | 4 | 8 | 9 | 6 |
| 6 | 4 | 3 | 8 | 2 | 9 | 1 | 5 | 7 |
| 8 | 9 | 7 | 1 | 5 | 6 | 2 | 4 | 3 |
| 9 | 5 | 8 | 3 | 6 | 1 | 4 | 7 | 2 |
| 1 | 7 | 8 | 2 | 8 | 5 | 3 | 6 | 9 |
| 3 | 6 | 2 | 9 | 4 | 7 | 5 | 8 | 7 |

Sudoku

| | | | |
|---|---|---|---|
| 2+ 2 | 3 3 | 4× 1 | 4 |
| 3 | 2+ 2 | 7+ 3 | 1 |
| 3 3 | 1 | 4 | 7+ 2 |
| 3– 1 | 4 | 2 | 3 |

Kenken

| | | |
|---|---|---|
| 17 | 12 | 15 |
| 10 | 14 | 16 |
| 15 | 18 | 11 |

Magic Square

Mind Map