5

Intersecting and Parallel Lines

Why This Chapter Matters

Have you ever wondered why railway tracks never meet, or how a carpenter ensures two shelves are perfectly parallel? What about the precise angles a builder needs to create a stable roof? Geometry is all around us, from the intricate patterns in nature to the structures we live and work in. In this chapter, we'll embark on an exciting journey to explore the fundamental building blocks of geometry: lines and angles. We'll discover their hidden relationships and how they shape our world. Get ready to unlock the secrets of shapes and spaces!

Meet EeeBee.AI

Hello, young mathematicians! I'm EeeBee, your friendly guide through the fascinating world of numbers and shapes. I love exploring patterns and solving puzzles, and I'm super excited to embark on this geometric adventure with you. Throughout this chapter, I'll pop up with fun facts, helpful hints, and challenging questions to make learning about lines and angles an absolute blast. Let's discover how geometry shapes everything around us!

Learning Outcomes

By the end of this chapter, you will be able to:

- Define and differentiate between intersecting, perpendicular, and parallel lines.
- Identify and classify various types of angles formed by intersecting lines and transversals (linear pairs, vertically opposite, corresponding, alternate, interior angles).
- Apply the properties of angles formed by parallel lines and a transversal to find unknown angle measures.
- Construct parallel lines using different geometric tools and methods.
- Solve real-life problems involving lines and angles.
- Appreciate the importance of geometric reasoning in everyday life and various professions.

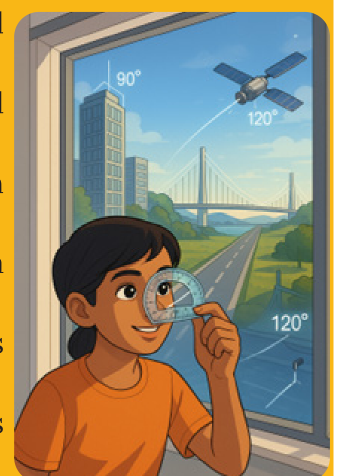
From Last Year's Notebook

- Points, Lines, Line Segments, and Rays
- **Different Types of Angles:** Acute, Obtuse, Right, Straight, and Reflex angles
- Measure angles using a protractor
- Identify adjacent angles
- We also touched upon the idea of Perpendicular Lines.
- This chapter will build upon that foundation, delving deeper into:
- The relationships between lines when they intersect or run parallel
- The special angles formed in these situations
- We will use your understanding of basic angles and their measurements to explore more complex geometric properties.

Real Math, Real Life

Lines and angles are more than just shapes on a page; they are everywhere in our world!

- **Architecture:** Architects use them to design strong and beautiful buildings.
- **Engineering:** Engineers need precise angles to build bridges and machines safely.
- **Art and Design:** Artists use lines to create amazing paintings with depth and perspective.
- **Sports:** Understanding angles can help a footballer score a goal or a billiards player make a perfect shot.
- **Technology:** From your smartphone's design to a satellite's path, lines and angles make our modern technology possible.
- **Foundation of Geometry:** These concepts are the basic building blocks of geometry, helping us understand the physical world around us.



Quick Prep

1. What is the difference between a line, a line segment, and a ray?
2. How many degrees are there in a straight angle?
3. If two angles are adjacent and their sum is 90°, what are they called?
4. Can two straight lines intersect at more than one point? Why or why not?
5. What is the measure of an angle that is complementary to 35°?
6. What is the measure of an angle that is supplementary to 110°?

Introduction

Welcome to the world of lines! Imagine drawing two straight lines on a piece of paper. What are the possibilities? Do they cross each other, or do they run side-by-side forever? This section will introduce you to the fundamental ways lines can interact on a flat surface. Understanding these basic relationships is the first step towards unlocking more complex geometric concepts and seeing the geometry hidden in plain sight all around you.

Chapter Overview

Types of Lines: Intersecting lines (linear pairs, vertically opposite angles, perpendicular) and parallel lines.

Transversals: Angles formed include corresponding, alternate (interior/exterior), and interior angles on the same side, with properties for parallel lines.

Construction: Methods to draw parallel lines using angles, ruler-set square, and paper folding.

Applications: Real-world relevance in architecture, engineering, art, and daily life.

Problem Solving: Techniques for finding unknown angles and proving lines parallel.

From History's Pages

The study of lines and angles began in ancient civilizations such as those of the Egyptians and Babylonians, who used them for surveying and construction. Greeks, especially Euclid in "Elements" (300 BCE), formalized geometry with definitions, postulates, and theorems, including those for parallel lines. His famous "parallel postulate" spurred centuries of inquiry. Indian mathematicians, in "Sulba Sutras," also showed advanced geometric understanding. These fundamental concepts remain crucial in modern mathematics, science, and engineering.

Lines on a Plane Surface

Imagine a perfectly flat surface, like the top of your study table, a blackboard, or a sheet of paper. This flat surface is what mathematicians call a "**plane**." When we draw lines on such a surface, they can interact in different ways. They might cross each other, or they might run alongside each other without ever meeting. Understanding these basic interactions is crucial for exploring the world of geometry. In this section, we will explore how lines behave when drawn on a plane.

Fig. 5.1

Sub-concepts to be Covered

1. **Definition of a Plane:** A flat, two-dimensional surface that extends infinitely in all directions. It has no thickness. Examples include the surface of a wall, a floor, or a calm water body. **(Fig. 5.2)**
2. **Lines on a Plane:** A line is a one-dimensional figure, which has length but no width. It extends infinitely in both directions. When we talk about "lines on a plane," we are considering lines that lie entirely within that flat surface. **(Fig. 5.3)**
3. **Points on a Line:** A point is a location in space, having no size. An infinite number of points lie on a single line. **(Fig. 5.4)**
4. **Collinear Points**: Points that lie on the same straight line are called collinear points. If points do not lie on the same straight line, they are non-collinear. **(Fig. 5.5)**

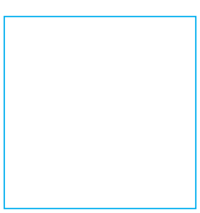


Fig. 5.2

Fig. 5.3

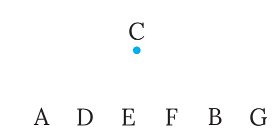


Fig. 5.4

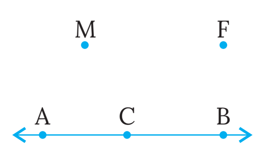


Fig. 5.5

Mathematical Explanation

A **plane** is like an infinitely large, perfectly flat sheet. Any line drawn on it is straight and extends endlessly in both directions, shown with arrows at the ends. For instance, if we draw a line ***l*** on a piece of paper, we are representing a small part of an infinitely long line.

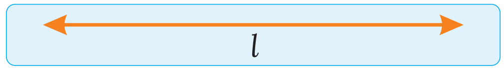


Fig. 5.6

Through any two distinct points on a plane, only one unique straight line can pass. If three or more points lie on the same line, they are **collinear**; otherwise, they are **non-collinear**. Understanding lines on a plane helps us study intersections, parallelism, and angles—the foundation of all geometric shapes and figures.

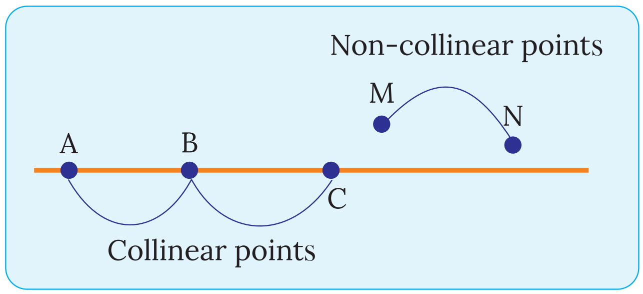


Fig. 5.7

Example 1 : Identify if the following points are collinear: P(1,2), Q(3,4), R(5,6).

Solution: To check for collinearity, we can see if the points lie on the same straight line. Let us plot the points on the graph.

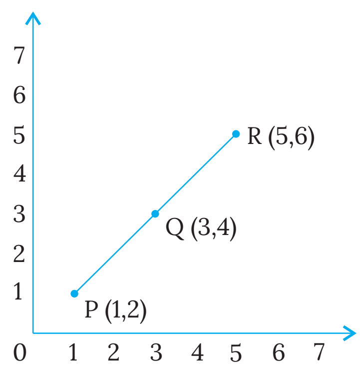
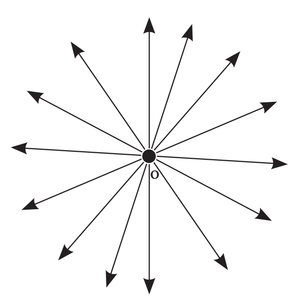


Fig. 5.8

Since they lie on the same straight line, so the points P, Q and R are collinear.

Example 2 : Can you draw more than one straight line passing through a single point?



Solution: Yes, we can draw infinitely many straight lines passing through a single point. From that position, lines can extend in every possible direction.

Therefore, through a single point, countless straight lines can be drawn.

Example 3 : Give two real-life examples of a plane surface.

Solution: 1. The surface of a calm lake.

2. A perfectly smooth, flat wall.

Example 4 : A line segment AB is 8 cm long. If point C lies on the line segment AB such that AC = 3 cm, what is the length of CB?

Solution: Since C lies on the line segment AB, the lengths are additive:

AC + CB = AB

3 cm + CB = 8 cm

CB = 8 cm - 3 cm

CB = 5 cm

Fig. 5.9

Key Terms

- **Plane:** A flat, two-dimensional surface extending infinitely.
- **Line:** A straight, one-dimensional figure extending infinitely in two directions.
- **Point:** A specific location with no dimension.
- **Collinear Points**: Points that lie on the same straight line.
- **Line Segment:** A part of a line with two distinct endpoints.
- **Ray:** A part of a line with one endpoint, extending infinitely in one direction.

Fact Flash

- Did you know that the shortest distance between two points on a plane is always a straight line? This is a fundamental concept in geometry!
- The word "**geometry**" comes from the Greek words "**geo**" (earth) and "**metron**" (measure), literally meaning "**earth measurement**." This shows its ancient roots in surveying land.

Do It Yourself

If you draw a straight line on a spherical surface (like a globe), and then extend it infinitely, would it ever meet itself? How is this different from a line on a flat plane? (Hint: Think about the shortest path between two points on a sphere).

Mental Mathematics

- Do the points (1,2), (3,6) and (6,12) lie on the same line?
- How many lines can pass through one point?
- How many lines can be drawn through two distinct points?
- Do the points (2,3), (4,7), and (7,12) lie on the same line?

Intersecting Lines and Angles

What happens when two straight lines cross each other? They don't just pass through; they create a special point where they meet, and around this point, they form angles. These angles have fascinating relationships that are always true, no matter how the lines intersect. Understanding these relationships is key to solving many geometric puzzles and real-world problems.

Sub-concepts to be Covered

1. **Definition of Intersecting Lines**: Two distinct lines are said to be intersecting if they have exactly one point in common. This common point is called the point of intersection. (**Fig. 5.10**)
2. **Angles Formed by Intersecting Lines:** When two lines intersect, they form four angles around their point of intersection. (**Fig. 5.11**)
3. **Linear Pair of Angles:** A pair of adjacent angles whose non-common sides are opposite rays (form a straight line). The sum of angles in a linear pair is always 180°. They are supplementary. (**Fig. 5.12**)
4. **Vertically Opposite Angles:** When two lines intersect, the angles opposite each other at the point of intersection are called vertically opposite angles. Vertically opposite angles are always equal. (**Fig. 5.13**)

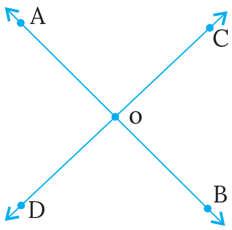


Fig. 5.10

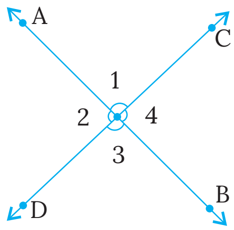


Fig. 5.11

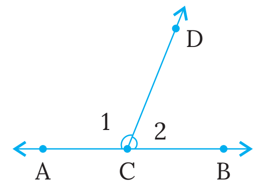


Fig. 5.12

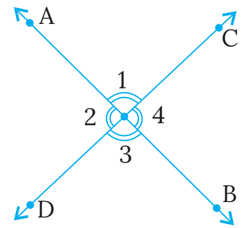


Fig. 5.13

Mathematical Explanation

When two lines, say line l and line **m**, cross each other, they meet at a single point, let's call it **O**. This point **O** is their point of intersection. Around this point **O**, four angles are formed. Let's label them ∠1, ∠2, ∠3, and ∠4 in a clockwise or counter-clockwise manner.

Consider any two adjacent angles, like ∠1 and ∠2. Their non-common sides form a straight line. This pair (∠1, ∠2) is called a **linear pair.** The sum of angles in a linear pair is always 180°. So, ∠1 + ∠2 = 180°. Similarly, (∠2, ∠3), (∠3, ∠4), and (∠4, ∠1) are also linear pairs, and their sums are also 180°.

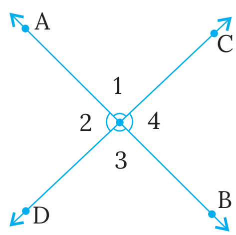


Fig. 5.14

Now, look at the angles that are directly opposite each other, like ∠1 and ∠3, or ∠2 and ∠4. These are called **vertically opposite angles**. A very important property of vertically opposite angles is that they are always equal. So, ∠1 = ∠3 and ∠2 = ∠4.

We can prove this using the linear pair property:

1. ∠1 + ∠2 = 180° (Linear pair)
2. ∠2 + ∠3 = 180° (Linear pair)

   From (1) and (2), we can say:

   ∠1 + ∠2 = ∠2 +∠3

   Subtract ∠2 from both sides:

   ∠1 = ∠3

   Similarly, we can prove∠2 = ∠4.

This understanding of linear pairs and vertically opposite angles is fundamental for solving problems involving intersecting lines and forms the basis for more complex angle relationships.

Example 5 : Two lines AB and CD intersect at point O. If ∠AOC = 75°, find the measures of ∠BOD, ∠AOD, and ∠BOC.

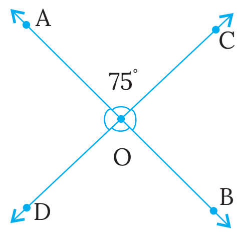


Fig. 5.15

Solution: In the diagram, two intersecting lines AB and CD at point O.

Label angles AOC, BOD, AOD, BOC.

Given: ∠AOC = 75°

1. **Find ∠BOD:**

   ∠AOC and ∠BOD are vertically opposite angles.

   Therefore,∠BOD =∠AOC = 75°.

2. **Find ∠AOD:**

   ∠AOC and ∠AOD form a linear pair on line CD.

   Therefore, ∠AOC + ∠AOD = 180°

   75° + ∠AOD = 180°

   ∠AOD = 180° - 75° = 105°.

3. **Find∠BOC:**

   ∠AOD and ∠BOC are vertically opposite angles.

   Therefore, ∠BOC = ∠AOD = 105°.

(Alternatively, ∠BOD and ∠BOC form a linear pair on line AB: 75° + ∠BOC = 180° => ∠BOC = 105°).

Example 6 : In the given figure, lines PQ and RS intersect at O. If ∠POR = 3x and ∠QOS = 2x + 20°, find the value of x.

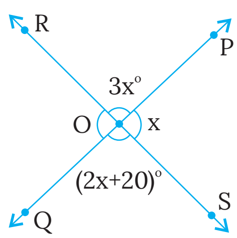


Fig. 5.16

Solution: In the diagram, two intersecting lines PQ and RS intersect at point O.

Label angle POR as 3x and angle QOS as 2x + 20°.

∠POR and ∠QOS are vertically opposite angles.

Therefore, ∠POR = ∠QOS

$3x = 2x + 20°$

$3x - 2x = 20°$

$x = 20°$

Example 7 : Two lines intersect such that one of the angles formed is 90°. What can you say about the other three angles?

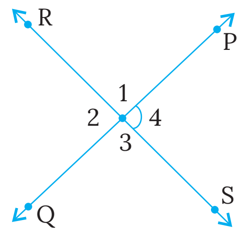


Fig. 5.17

Solution: Let the intersecting lines form angles ∠1, ∠2, ∠3, ∠4.

Assume ∠1 = 90°.

1. ∠1 and ∠2 form a linear pair. So, ∠1 + ∠2 = 180°.
   90° + ∠2 = 180° => ∠2 = 90°.
2. ∠1 and ∠3 are vertically opposite angles. So, ∠3 = ∠1 = 90°.
3. ∠2 and ∠4 are vertically opposite angles. So, ∠4 = ∠2 = 90°.
   Therefore, if one angle is 90°, all four angles formed are 90°. This is the definition of perpendicular lines.

Example 8 : If ∠A and ∠B form a linear pair, and ∠A is twice ∠B, find the measures of ∠A and ∠B.

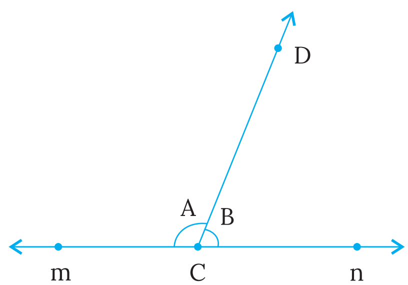


Fig. 5.18

Solution: Let ∠B = x.

Then ∠A = 2x.

Since they form a linear pair, ∠A + ∠B = 180°.

2x + x = 180°

3x = 180°

$$x = \frac{180^\circ}{3}$$

x = 60°

So, ∠B = 60° and ∠A = 2 × 60° = 120°.

Knowledge Checkpoint

- What is the point where two lines cross called?
- If ∠A and ∠B are vertically opposite angles, and ∠A = 65°, what is ∠B?
- Can a linear pair consist of only 2 acute angles? Why or why not?

Activity

Discovering Angle Relationships

Objectives: To help students empirically discover the properties of linear pairs and vertically opposite angles.

Materials: Two transparent plastic strips (or cardboard strips), a thumbtack, a protractor, paper.

Procedure:

1. Place one plastic strip on the paper. Place the second strip on top of the first, crossing it.
2. Use the thumbtack to pivot the top strip through the intersection point.
3. Draw the two intersecting lines on the paper.
4. Using a protractor, measure all four angles formed by the intersection. Record your measurements.
5. Repeat steps 2-4 several times, changing the angle of intersection each time.

Observations:

- What do you notice about the sum of adjacent angles? (They add up to 180°).
- What do you notice about the measures of angles opposite each other? (They are equal).

Conclusion: This hands-on activity allows students to empirically discover the properties of linear pairs and vertically opposite angles.

Key Terms

- **Intersecting Lines:** Lines that cross at one common point.
- **Point of Intersection:** The single point where intersecting lines meet.
- **Linear Pair:** Two adjacent angles that form a straight line (sum 180°).
- **Vertically Opposite Angles:** Non-adjacent angles formed by two intersecting lines that are equal.

Fact Flash

- The concept of vertically opposite angles was known to ancient Greek mathematicians like **Euclid**.
- If you draw a perfect '**X**' shape, the two angles on the top and bottom are equal, and the two angles on the left and right are equal!

Do It Yourself

If three lines intersect at a single point, how many angles are formed around that point? Can you identify any linear pairs or vertically opposite angles in this scenario?

Mental Mathematics

- If one angle in an intersecting pair is 100°, what are the other three angles?
- Two angles form a linear pair. If one is 70°, what is the other?
- If $\angle X$ and $\angle Y$ are vertically opposite, and $\angle X = 40°$, what is $\angle Y$?
- What is the sum of all four angles formed by two intersecting lines?
- If an angle is 120°, what is the measure of its adjacent angle that forms a linear pair?

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Exercise 5.1

1. **Fill in the Blanks**:

   a) A flat, two-dimensional surface that extends infinitely in all directions is called a ______.

   b) Through any two distinct points, exactly ______ straight line(s) can pass.

   c) When two lines intersect, they meet at exactly one ______ of intersection.

   d) Angles that are opposite each other when two lines intersect are called ______ angles.

   e) If two lines intersect and one of the angles formed is 45°, its vertically opposite angle is ______ degrees.

2. **Solve the following questions**:

   a) If a point is located at (2, 3) on a coordinate plane, and another point is at (2, 7), what kind of line connects them (horizontal, vertical, or slanted)?

   b) If an angle is $\frac{1}{3}$ of a straight angle, and it forms a linear pair with another angle, what is the measure of the second angle in degrees?

   c) If two lines intersect and form angles such that one angle is 2 times its adjacent angle, what are the measures of these angles

   d) Two roads cross each other at an intersection. If the angle formed by the roads in the north east direction is 110°, what is the angle formed in the southwest direction?

3. **Bridging the Gap Questions (between theory and practice)**:

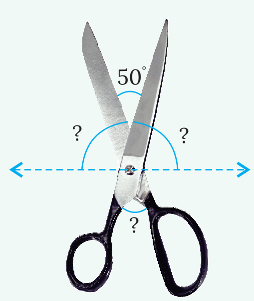
   a) Imagine a perfectly smooth, still swimming pool. If you drop a tiny pebble into it, the surface of the water before the ripples form can be considered what geometric figure?

   b) **A map shows three cities:** City A, City B, and City C. If you can draw a single straight line connecting all three cities on the map, what can you say about the cities' positions relative to each other?

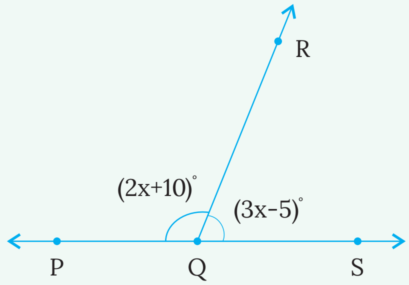
   c) A clock's hands form an angle. At 3:00 PM, what kind of angle do they form? If the minute hand moves slightly past 3, creating a small angle with the hour hand, If these two hands are extended backwards then what kind of angle would be the measure of vertically opposite angle to it?

4. **Concept Carnival: Mixed Question Set:**

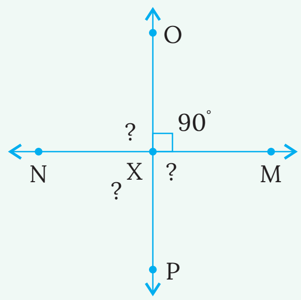
   a) A pair of scissors forms an 'X' shape when open. If the angle between the blades on one side is 50°, what is the angle on the opposite side? What about the sum of the angles adjacent to the 50° angle?



b) If ∠PQR and ∠RQS form a linear pair, and ∠PQR = 2x + 10° and ∠RQS = 3x - 5°, find the value of x and the measure of each angle.



c) Lines MN and OP intersect at point X. If ∠MXO = 90°, what are the measures of ∠NXP, ∠PXM, and ∠NXO? Justify your answers using angle properties.



d) Angles P and Q form a linear pair. If the measure of ∠P is 30° less than twice the measure of ∠Q, find the measures of ∠P and ∠Q.

Perpendicular Lines

We've seen that when lines intersect, they form angles. But what if they intersect in a very specific way, creating perfect "**square**" corners? This special type of intersection leads us to perpendicular lines, which are incredibly important in construction, design, and many other fields. Think about the corners of a room or the cross of a plus sign – these are examples of perpendicularity.

Sub-concepts to be Covered

1. **Definition of Perpendicular Lines:** Two lines are said to be perpendicular if they intersect at a right angle (90°).
2. **Notation:** The symbol '⊥' is used to denote perpendicularity. For example, if line ***l*** is perpendicular to line **m**, we write ***l*** ⊥ **m**.

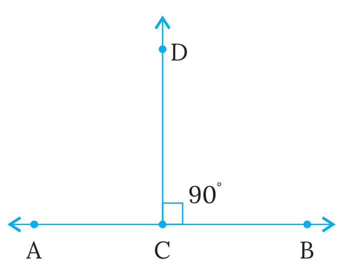


Fig. 5.19

3. **Properties of Perpendicular Lines:** When two lines are perpendicular, all four angles formed at their intersection are right angles (90°). This is a direct consequence of the properties of linear pairs and vertically opposite angles.

Mathematical Explanation

Perpendicular lines are a special case of intersecting lines. While any two lines can intersect, perpendicular lines intersect specifically to form a right angle, which measures exactly 90°.

Let's say line ***l*** and line **m** intersect at point **P**. If one of the angles formed, say ∠1, is 90°, then:

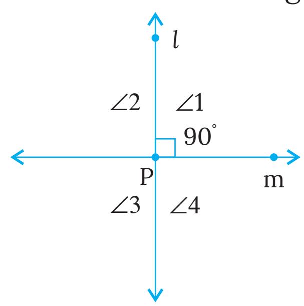


Fig. 5.20

1. ∠1 and ∠2 form a linear pair. So, ∠1 + ∠2 = 180°. Since ∠1 = 90°, then 90° + ∠2 = 180°, which means ∠2 = 90°.
2. ∠1 and ∠3 are vertically opposite angles. Since vertically opposite angles are equal, ∠3 = ∠1 = 90°.
3. ∠2 and ∠4 are vertically opposite angles. Since vertically opposite angles are equal, ∠4 = ∠2 = 90°.

Therefore, if any one angle formed by two intersecting lines is 90°, all four angles are 90°. This is the defining characteristic of perpendicular lines. The symbol '⊥' is a shorthand way to indicate that two lines are perpendicular. For example, if line AB is perpendicular to line CD, we write AB ⊥ CD. Perpendicularity is crucial for creating stable structures and precise measurements.

Example 9 : In the figure, line XY and line MN intersect at O. If ∠XOM = 90°, identify all pairs of perpendicular lines and the measures of all other angles.

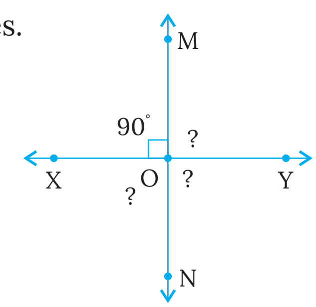


Fig. 5.21

Solution: Given: ∠XOM = 90°.

Since ∠XOM is 90°, the lines XY and MN are perpendicular.

So, XY ⊥ MN.

1. ∠XOM and ∠YON are vertically opposite angles.
   So, ∠YON = ∠XOM = 90°.
2. ∠XOM and ∠XON form a linear pair. So, ∠XOM + ∠XON = 180°.
   90° + ∠XON = 180° => ∠XON = 90°.
3. ∠XON and ∠YOM are vertically opposite angles. So, ∠YOM = ∠XON = 90°.
   All four angles are 90°.

Example 10 : A carpenter is building a square table top. How many pairs of perpendicular lines will be formed by the edges of the table top?

Solution: A square table top has four edges. Each corner of a square forms a right angle.
Consider two adjacent edges. They meet at a corner and are perpendicular.
There are four corners in a square. So, there are 4 pairs of perpendicular lines formed by the adjacent edges.

Example 11 : A diagram showing line AB perpendicular to line CD at P. Draw ray PE such that E is in the angle APD. Mark angle APE as 30°. Find ∠EPC = ?

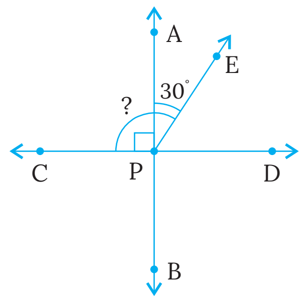


Fig. 5.22

Solution: Given: AB ⊥ CD at P. This means ∠APC = 90°.
We are given ∠APE = 30°.
From the figure, ∠APC is composed of ∠APE and ∠EPC.
So, ∠APE + ∠EPC = ∠APC
30° + ∠EPC = 90°
∠EPC = 90° - 30°
∠EPC = 60°.

Example 12 : Can two lines be perpendicular if they do not intersect?

Solution: No. By definition, perpendicular lines are a type of intersecting lines. They must intersect at a single point to form the 90° angle. If they do not intersect, they are either parallel or skew (in 3D space), but not perpendicular.

Knowledge Checkpoint

- What is the measure of the angle formed by perpendicular lines?
- If two lines are perpendicular, are they also intersecting?
- Draw two perpendicular lines and mark the right angles.

Activity

Creating Perpendicular Lines with Paper Folding

Objectives: To enable students to understand the concept of perpendicular lines and a right angle (90°) through a simple hands-on paper folding activity.

Materials: A rectangular sheet of paper.

Procedure:

1. Take a rectangular sheet of paper.
2. Fold the paper in half lengthwise, making a sharp crease. This crease represents a line.
3. Now, fold the paper again, but this time, make sure the first crease lies exactly on top of itself. This means you are folding perpendicular to the first crease. Make a sharp second crease.
4. Unfold the paper. Observe the two creases.

Observation: The two creases you made are perpendicular to each other. You have created a perfect 90° angle using just paper folding!
Conclusion: This activity demonstrates how perpendicular lines can be formed and reinforces the concept of a right angle.

Key Terms

- **Perpendicular Lines:** Lines that intersect to form a right angle (90°).
- **Right Angle:** An angle measuring exactly 90°.

Do It Yourself

If you have a line segment, how many lines can be drawn perpendicular to it at its midpoint? What if the line is infinitely long?

Fact Flash

- **The Great Pyramid of Giza** is an ancient marvel of engineering, and its base is almost perfectly square, with its sides forming nearly perfect right angles.
- Many ancient civilizations used simple tools like **ropes and pegs** to construct right angles for their buildings and fields, long before protractors existed!

Mental Mathematics

- If two lines are perpendicular, and one angle is 90°, what is the sum of the other three angles?
- What is the complement of a right angle?
- If you draw a square, how many right angles does it have?
- Can an acute angle be formed by perpendicular lines?
- If a line is perpendicular to another, and that second line is horizontal, what is the orientation of the first line?

Parallel Lines

Imagine two straight roads running side-by-side, never getting closer or farther apart. Or think about the opposite edges of a ruler. These are examples of parallel lines. Unlike intersecting lines that meet at a point, parallel lines maintain a constant distance from each other and never cross, no matter how far they extend. This concept is fundamental in geometry and has countless applications in the real world.

Fig. 5.23

Sub-concepts to be Covered

1. **Definition of Parallel Lines:** Two lines in a plane are said to be parallel if they do not intersect, no matter how far they are extended in either direction.
2. **Notation:** The symbol '||' is used to denote parallel lines. For example, if line l is parallel to line **m**, we write l || **m**.

3. **Distance Between Parallel Lines:** The perpendicular distance between any two points on parallel lines is always constant. This means they are always the same distance apart.

4. **Lines on the Same Plane:** It's crucial to remember that parallel lines must lie on the same plane. Lines that do not intersect and are not in the same plane are called **skew lines** (a concept for higher grades).

Mathematical Explanation

Parallel lines are a distinct type of line relationship. The key characteristic is that they never intersect. If you extend two parallel lines infinitely in both directions, they will always remain equidistant from each other. This constant distance is a defining feature.

Think of the opposite edges of a rectangular blackboard. If you were to draw lines along these edges and extend them, they would never meet. This is because they are parallel.

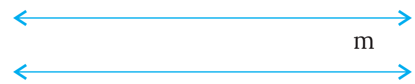


Fig. 5.24

The notation ***l* || m** is a concise way to state that line ***l*** is parallel to line **m**. In diagrams, parallel lines are often indicated by small arrows placed on the lines themselves, pointing in the same direction. If there are multiple sets of parallel lines in a diagram, different numbers of arrows (e.g., single arrow, double arrow) can be used to distinguish between the sets.

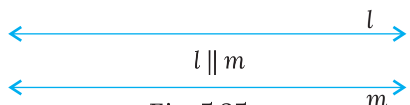


Fig. 5.25

It's important to differentiate parallel lines from lines that simply don't intersect in a diagram. For lines to be truly parallel, they must lie on the same flat surface (plane) and never meet. This concept is vital for understanding shapes like rectangles, squares, and parallelograms, which are built upon parallel sides.

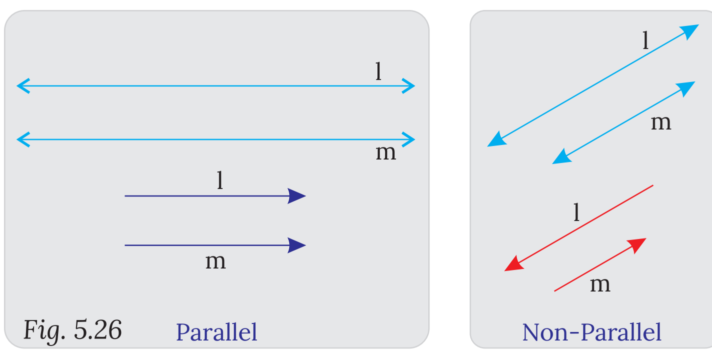


Fig. 5.26

Example 13 : Identify pairs of parallel lines in the following real-life objects:

a) Railway tracks
b) Opposite edges of a ruler
c) The rungs of a ladder

Solution: a) Railway tracks: The two rails of a railway track are designed to be parallel so that the train wheels can run smoothly on them without falling off.

b) Opposite edges of a ruler: The top and bottom edges of a standard ruler are parallel, al lowing you to draw straight, parallel lines.

c) The rungs of a ladder: The horizontal steps (rungs) of a ladder are parallel to each other, ensuring a stable climb.

Example 14 : Can two lines that are perpendicular to the same line be parallel to each other? Explain with a diagram.

Solution: A diagram showing a vertical line **t**. Draw two horizontal lines ***l*** and **m** that are both perpendicular to **t**.

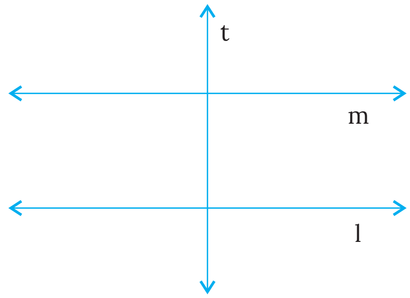


Fig. 5.27

Yes, they can.

Let line ***t*** be a transversal. If line ***l*** is perpendicular to **t**, and line **m** is also perpendicular to **t**, then both ***l*** and **m** form 90° angles with **t**.

These 90° angles are corresponding angles (or alternate interior angles, or interior angles on the same side). Since corresponding angles are equal (both 90°), lines ***l*** and **m** must be parallel. This is a key property often used in constructions.

Example 15 : A student draws two lines on a graph paper. Line A passes through (1,2) and (3,2). Line B passes through (1,5) and (3,5). Are these lines parallel?

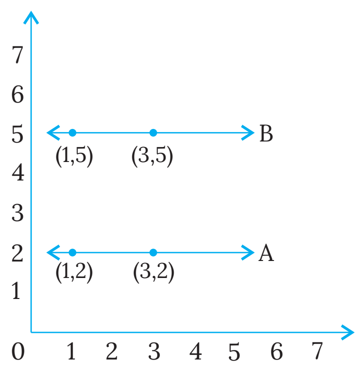


Fig. 5.28

Solution: Line A passes through (1,2) and (3,2). Both points have the same y-coordinate (2), so Line A is a horizontal line.

Line B passes through (1,5) and (3,5). Both points have the same y-coordinate (5), so Line B is also a horizontal line.

Horizontal lines on a graph paper are always parallel to each other. They will never intersect. Therefore, Line A and Line B are parallel.

Example 16 : A rectangular field has sides of length 50 meters and 30 meters. How many pairs of parallel sides does it have?

Solution: A rectangle has four sides. Opposite sides of a rectangle are always parallel and equal in length.

So, the two sides of length 50 meters are parallel to each other.

And the two sides of length 30 meters are parallel to each other.

Therefore, the rectangular field has 2 pairs of parallel sides.

Knowledge Checkpoint

- What is the main characteristic of parallel lines?
- What symbol is used to denote parallel lines?
- Give one real-life example of parallel lines.

Do It Yourself

If two lines are parallel, and a third line is perpendicular to one of them, what can you say about the relationship between the third line and the other parallel line?

Key Terms

- **Parallel Lines:** Lines in a plane that never intersect.
- **Equidistant:** Maintaining a constant distance apart.

Fact Flash

- The concept of parallel lines is so fundamental that **Euclid's fifth postulate** (the parallel postulate) was one of the most debated and influential ideas in the history of mathematics, leading to the development of non-Euclidean geometries!
- If you look at the lines of latitude on a globe, they are all parallel to the equator and to each other!

Mental Mathematics

- Are the opposite sides of a parallelogram parallel?
- If two lines are not parallel, what must they eventually do?
- Can a horizontal line and a vertical line ever be parallel?
- If you have three lines, and the first is parallel to the second, and the second is parallel to the third, what is the relationship between the first and third lines?
- Imagine a set of stairs. Are the steps parallel to each other?

Gap Analyzer™ Homework

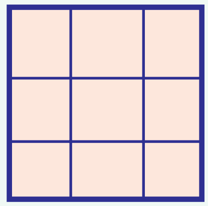
Watch Remedial

Exercise 5.2

1. Fill in the Blanks:

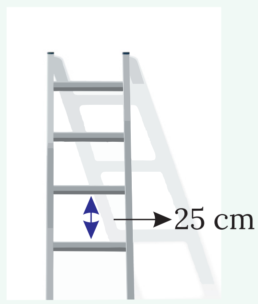
a) Two lines are perpendicular if they intersect at a ______ angle.
b) If line P is perpendicular to line Q, then the angle between them is ______ degrees.
c) When two lines are perpendicular, all ______ angles formed are 90°.
d) The perpendicular distance between parallel lines is always ______.
e) Opposite sides of a rectangle are always ______ to each other.

2. A wall meets the floor in your room. What kind of lines do the edge of the wall and the edge of the floor form?
3. A crosswalk is painted on a road. The lines forming the crosswalk are parallel, but the lines that mark the beginning and end of the crosswalk are perpendicular to the road lines. Draw a simple diagram to illustrate this.
4. In a game of tic-tac-toe, the grid lines form what kind of angles at their intersections?

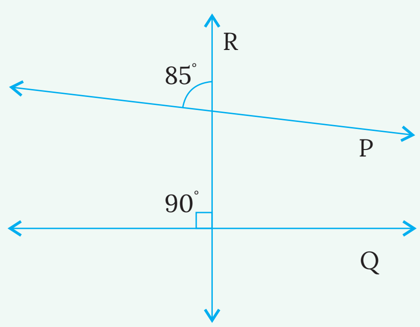


Tic-Tac-Toe

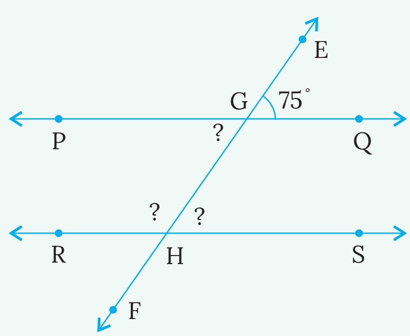
5. A ladder is leaning against a wall. The rungs of the ladder are parallel to each other. If the distance between the first and second rung is 25 cm, what is the distance between the third and fourth rung?



6. A student drew two lines, P and Q, and a transversal R. They measured a pair of corresponding angles and found them to be 85° and 90°. What can you say about lines P and Q based on this measurement? Why?



7. Lines PQ and RS are parallel. The transversal line EF intersects them at G and H respectively. If ∠EGQ = 75°, find the measures of:
   a) ∠GHS
   b) ∠PGH
   c) ∠RHG



Transversal and Angles Formed

At each crossing point, angles are formed. A transversal is simply a line that intersects two or more other lines at distinct points. When a transversal cuts across lines, it creates a total of eight angles. These angles have specific names and relationships, which are especially important when the lines being intersected are parallel.

Sub-concepts to be covered

1. **Definition of a Transversal**: A line that intersects two or more lines at different points. The lines being intersected do not necessarily have to be parallel.
2. **Angles Formed:** When a transversal intersects two lines, eight angles are formed. These angles can be categorized based on their position relative to the transversal and the two lines.
3. **Interior Angles:** The angles that lie between the two lines intersected by the transversal. (e.g., $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$ in a standard diagram).
4. **Exterior Angles:** The angles that lie outside the two lines intersected by the transversal. (e.g., $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$ in a standard diagram).

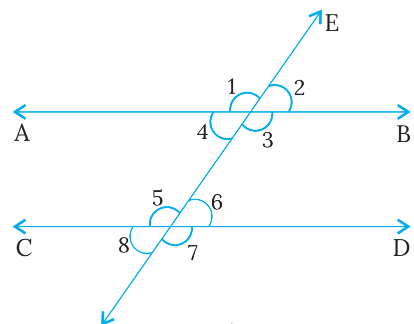


Fig. 5.29

Pairs of Angles:

Corresponding Angles: Angles that are in the same relative position at each intersection.
Alternate Interior Angles: Interior angles on opposite sides of the transversal.
Alternate Exterior Angles: Exterior angles on opposite sides of the transversal.
Interior Angles on the Same Side of the Transversal (Consecutive Interior Angles): Interior angles on the same side of the transversal.

Mathematical Explanation

Let's consider two lines, ***l*** and **m**, and a third line, **t**, which intersects both ***l*** and **m** at distinct points. Line **t** is called the transversal.

When the transversal **t** intersects line ***l***, it forms four angles. Let's call them $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$. Similarly, when **t** intersects line **m**, it forms another four angles, let's call them $\angle 5$, $\angle 6$, $\angle 7$, $\angle 8$. In total, eight angles are formed.

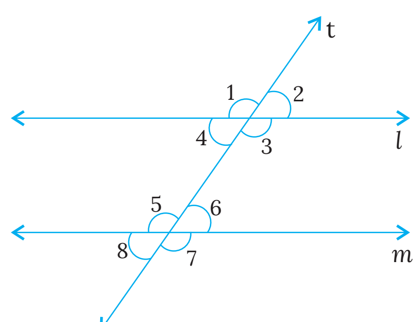


Fig. 5.30

These eight angles can be classified:

Interior Angles: These are the angles that lie between lines ***l*** and **m**. In our example, these are ∠3, ∠4, ∠5, and ∠6.

Exterior Angles: These are the angles that lie outside lines ***l*** and **m**. In our example, these are ∠1, ∠2, ∠7, and ∠8.

Now, let's look at the special pairs of angles:

Corresponding Angles: These angles are in the "**same spot**" at each intersection. They are on the same side of the transversal and either both above or both below the lines.

Pairs: (∠1, ∠5), (∠2, ∠6), (∠3, ∠7), (∠4, ∠8).

Alternate Interior Angles: These are interior angles on opposite sides of the transversal.

Pairs: (∠3, ∠6), (∠4, ∠5).

Alternate Exterior Angles: These are exterior angles on opposite sides of the transversal.

Pairs: (∠1, ∠8), (∠2, ∠7).

Interior Angles on the Same Side of the Transversal (Consecutive Interior Angles): These are interior angles that are on the same side of the transversal.

Pairs: (∠3, ∠5), (∠4, ∠6).

Understanding how to identify these pairs is the first step. Their properties become particularly significant when the lines \`l\` and \`m\` are parallel, which we will explore next.

Example 17 : In the given figure, identify the transversal and list all the interior and exterior angles.

A diagram showing two non-parallel lines 'a' and 'b' intersected by a line 'c'. Label theangles 1 through 8.

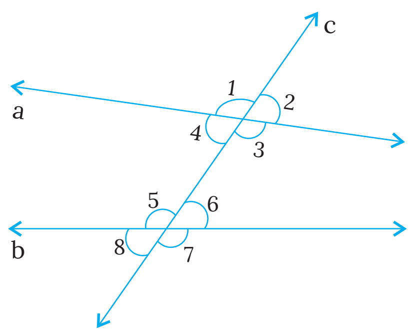


Fig. 5.31

Solution: The transversal is line c, as it intersects lines 'a' and 'b' at distinct points.

Interior Angles: ∠3, ∠4, ∠5, ∠6 (These are between lines 'a' and 'b').

Exterior Angles: ∠1, ∠2, ∠7, ∠8 (These are outside lines 'a' and 'b').

Example 18 : In the figure from Example 1, identify all pairs of corresponding angles.

Solution: (∠1, ∠5), (∠2, ∠6), (∠3, ∠7), (∠4, ∠8)

Example 19 : In the figure from Example 1, identify all pairs of alternate interior angles and alternate exterior angles.

Solution: **Alternate Interior Angles:**

(∠3, ∠6)

(∠4, ∠5)

Alternate Exterior Angles:

(∠1, ∠8)

(∠2, ∠7)

Example 20 : In the figure from Example 1, identify all pairs of interior angles on the same side of the transversal.

Solution: $(\angle 3, \angle 5)$, $(\angle 4, \angle 6)$

Knowledge Checkpoint

- What is a transversal?
- How many angles are formed when a transversal intersects two lines?
- Name one pair of interior angles.

Activity

Exploring Angles with a Transversal

Objectives: To enable students to investigate and understand the angle relationships formed when a transversal intersects two lines.

Materials: Two strips of paper (representing lines), one long strip of paper (representing a transversal), glue or tape, protractor.

Procedure:

1. Place the two shorter strips of paper on a larger sheet, making sure they are not parallel.
2. Place the long strip (transversal) across the two shorter strips, intersecting both. Glue/tape it down.
3. Carefully label all 8 angles formed (1 through 8).
4. Using a protractor, measure all 8 angles and record their values.
5. Now, identify and compare the measures of:
   - Corresponding angles (e.g., $\angle 1$ and $\angle 5$). Are they equal?
   - Alternate interior angles (e.g., $\angle 3$ and $\angle 6$). Are they equal?
   - Interior angles on the same side (e.g., $\angle 4$ and $\angle 6$). What is their sum?

Observation: You will notice that these angle pairs are generally not equal or supplementary when the lines are not parallel. This sets the stage for understanding the special properties when lines are parallel.

Key Terms

- **Transversal:** A line that intersects two or more other lines at distinct points.
- **Interior Angles:** Angles between the two lines.
- **Exterior Angles:** Angles outside the two lines.
- **Corresponding Angles:** Angles in the same relative position.
- **Alternate Interior Angles:** Interior angles on opposite sides.
- **Alternate Exterior Angles:** Exterior angles on opposite sides.
- **Consecutive Interior Angles (Interior Angles on Same Side):** Interior angles on the same side.

Fact Flash

- The word "**transversal**" comes from the Latin word "**transversus**," meaning "**lying or being across**."
- The concept of a transversal is not limited to two lines; it can intersect any number of lines!

Do It Yourself

If a transversal intersects three parallel lines, how many angles are formed in total? How many pairs of corresponding angles would there be?

Mental Mathematics

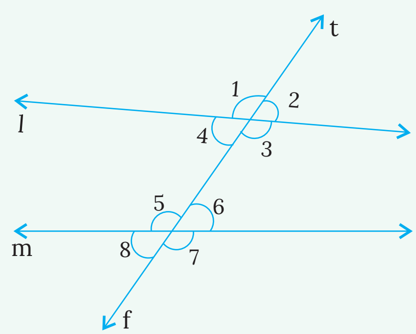
- If a transversal cuts two lines, and one exterior angle is 60°, what is its vertically opposite angle?
- If an interior angle is 110°, and its adjacent angle forms a linear pair, what is the measure of the adjacent angle?
- How many pairs of corresponding angles are formed by a transversal intersecting two lines?
- How many pairs of alternate interior angles are formed?
- If a transversal cuts two lines, and one angle is 90°, can you conclude anything about the other angles without knowing if the lines are parallel?

Gap Analyzer™ Homework

Watch Remedial

Exercise 5.3

1. **Fill in the Blanks:**
   a) A line that intersects two or more lines at distinct points is called a ______.
   b) When a transversal intersects two lines, a total of ______ angles are formed.
   c) Angles that lie between the two lines intersected by a transversal are called ______ angles.
   d) Angles that are in the same relative position at each intersection are called ______ angles.
2. **Conversion-Based Questions:**
   a) If a transversal forms 8 angles, and 4 of them are interior, how many are exterior?
   b) If a transversal intersects three lines, how many intersection points are there, and how many total angles are formed?
3. Two non-parallel lines (*l* and m) intersected by a transversal (t). All 8 angles formed are clearly numbered (1 through 8).
   Observe the figure below. Identify and name the following pairs of angles:

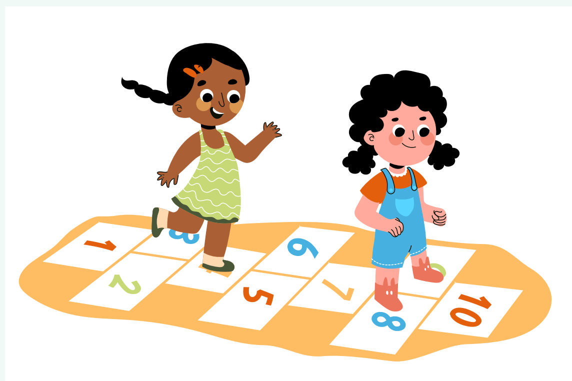


   a) A pair of corresponding angles.
   b) A pair of alternate interior angles.
   c) A pair of consecutive interior angles (or same-side interior angles).
   d) A pair of vertically opposite angles.
   e) A pair of angles that forms a linear pair.

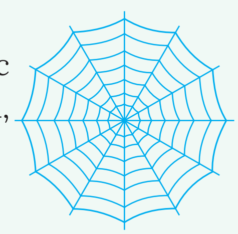
4 **Bridging the Gap Questions (between theory and practice):**
   a) A carpenter is building a wooden frame. They have two vertical beams and want to connect them with a diagonal brace. The diagonal brace acts as a transversal. What kind of angles are formed between the brace and the vertical beams?

b) A street map shows two parallel streets intersected by a cross street. The cross street acts as a transversal. If you are standing at one corner, what will be the measure of angle formed by the cross street and the parallel street you are on? What will be the measure of angle formed by the cross street and the other parallel street at the corresponding corner?

c) In a game of hopscotch, the lines on the ground form a grid. If you consider two horizontal lines and a vertical line crossing them, what geometric concept does the vertical line represent?

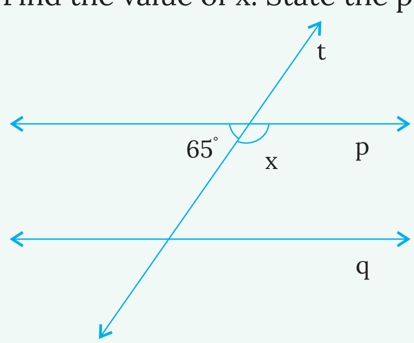


d) A spider web has radial lines extending from the center and concentric circles. If you consider two radial lines and a concentric circle crossing them, can the concentric circle be considered a transversal? Why or why not?



5. Concept Carnival: Mixed Question Set:

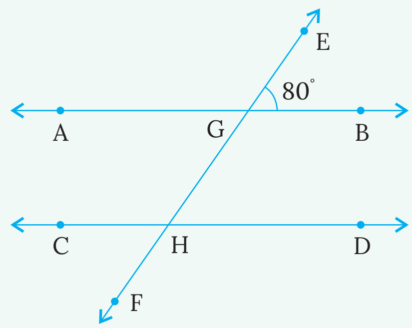
a) In the figure, line p || line q. Find the value of x. State the property you used.



b) Draw a diagram showing two lines and a transversal, and label one pair of corresponding angles and one pair of alternate interior angles.

c) Lines AB and CD are parallel lines cut by a transversal EF. If ∠EGB = 80°, find the measures of the following angles, giving reasons for each:

i) ∠AGH ii) ∠GHD iii) ∠FHC



Properties of Angles with Parallel Lines

The real magic happens when a transversal intersects parallel lines. In this special case, the eight angles formed develop very specific and predictable relationships. These properties are fundamental to geometry and are used extensively in various fields, from architecture to computer programming. Understanding these relationships allows us to find unknown angles and even prove if lines are parallel.

Sub-concepts to be covered

1. **Corresponding Angles Property:** If two parallel lines are intersected by a transversal, then each pair of corresponding angles is equal.
2. **Converse:** If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel.
3. **Alternate Interior Angles Property:** If two parallel lines are intersected by a transversal, then each pair of alternate interior angles is equal.
4. **Converse:** If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.
5. **Interior Angles on the Same Side of the Transversal Property (Consecutive Interior Angles):** If two parallel lines are intersected by a transversal, then each pair of interior angles on the same side of the transversal is supplementary (their sum is 180°).
6. **Converse:** If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.
7. **Relationship between Exterior Angles:**
   - Alternate Exterior Angles are equal when lines are parallel.
   - Exterior angles on the same side of the transversal are supplementary when lines are parallel.

Mathematical Explanation

Let lines ***l*** and **m** be parallel lines (***l*** **||** **m**), and let **t** be a transversal intersecting them. The eight angles formed (∠1 to ∠8) now have specific relationships:

Corresponding Angles are Equal

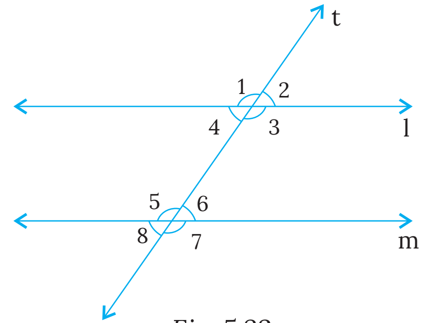


Fig. 5.32

($\angle 1 = \angle 5$), ($\angle 2 = \angle 6$), ($\angle 3 = \angle 7$), ($\angle 4 = \angle 8$)

Why? Imagine sliding line ***l*** along the transversal until it perfectly overlaps line **m**. The angles would perfectly match. This property is also used to prove lines are parallel. If you find even one pair of corresponding angles to be equal, then the lines must be parallel.

Alternate Interior Angles are Equal

($\angle 3 = \angle 5$), ($\angle 4 = \angle 6$)

Why? Let's prove $\angle 3 = \angle 5$. We know $\angle 3 = \angle 1$ (vertically opposite angles).

We also know $\angle 1 = \angle 5$ (corresponding angles).

And $\angle 5 = \angle 7$ (vertically opposite angles).

So, $\angle 3 = \angle 1 = \angle 5 = \angle 7$. Therefore, $\angle 3 = \angle 5$. This property is also used to prove lines are parallel.

Interior Angles on the Same Side of the Transversal are Supplementary

($\angle 3 + \angle 5 = 180°$), ($\angle 4 + \angle 6 = 180°$)

Why? We know $\angle 3 + \angle 4 = 180°$ (linear pair).

We also know $\angle 4 = \angle 5$ (alternate interior angles).

Substituting $\angle 5$ for $\angle 4$ in the linear pair equation, we get $\angle 3 + \angle 5 = 180°$.

This property is also used to prove lines are parallel.

These three properties are the most commonly used. Additionally:

- **Alternate Exterior Angles are Equal:**
  ($\angle 1 = \angle 8$), ($\angle 2 = \angle 7$)
- **Exterior Angles on the Same Side of the Transversal are Supplementary:**
  ($\angle 1 + \angle 7 = 180°$), ($\angle 2 + \angle 8 = 180°$)

These properties are incredibly powerful because they allow us to deduce the measures of all eight angles if we know just one angle and that the lines are parallel. They also provide criteria to determine if two lines are indeed parallel.

Example 21 : In the given figure, line **p** is parallel to line **q**, and **t** is a transversal. If $\angle 1 = 115°$, find the measures of all other angles.

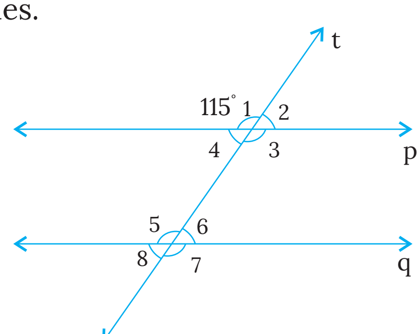


Fig. 5.33

Solution: Given: **p || q** and $\angle 1 = 115°$.

1. $\angle 5$: $\angle 1$ and $\angle 5$ are corresponding angles. Since p || q, $\angle 5 = \angle 1 = 115°$.
2. $\angle 3$: $\angle 1$ and $\angle 3$ are vertically opposite angles. So, $\angle 3 = \angle 1 = 115°$.
3. $\angle 7$: $\angle 3$ and $\angle 7$ are corresponding angles. Since p || q, $\angle 7 = \angle 3 = 115°$. (Alternatively, $\angle 7$ and $\angle 5$ form a linear pair, or $\angle 7$ and $\angle 1$ are alternate exterior angles).

4. $\angle 2$: $\angle 1$ and $\angle 2$ form a linear pair. So, $\angle 1 + \angle 2 = 180°$.
   $115° + \angle 2 = 180° \Rightarrow \angle 2 = 180° - 115° = 65°$.
5. $\angle 4$: $\angle 2$ and $\angle 4$ are vertically opposite angles. So, $\angle 4 = \angle 2 = 65°$.
6. $\angle 6$: $\angle 2$ and $\angle 6$ are corresponding angles. Since p || q, $\angle 6 = \angle 2 = 65°$.
7. $\angle 8$: $\angle 4$ and $\angle 8$ are corresponding angles. Since p || q, $\angle 8 = \angle 4 = 65°$. (Alternatively, $\angle 8$ and $\angle 6$ are vertically opposite angles).

Summary of angles: $\angle 1=115°$, $\angle 2=65°$, $\angle 3=115°$, $\angle 4=65°$, $\angle 5=115°$, $\angle 6=65°$, $\angle 7=115°$, $\angle 8=65°$.

Example 22 : In the figure, are lines **a** and **b** parallel? Justify your answer.

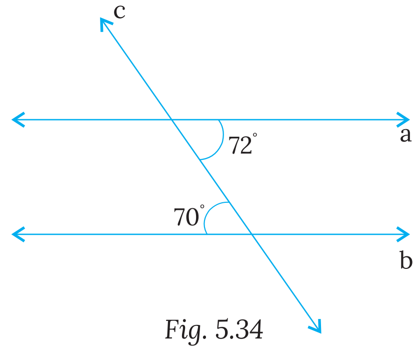


Fig. 5.34

Solution: The angles given are alternate interior angles (70° and 72°).
For lines **a** and **b** to be parallel, their alternate interior angles must be equal.
Here, $70° \neq 72°$.
Therefore, lines **a** and **b** are not parallel.

Example 23 : Find the value of **x** in the given figure, where ***l* || m**.

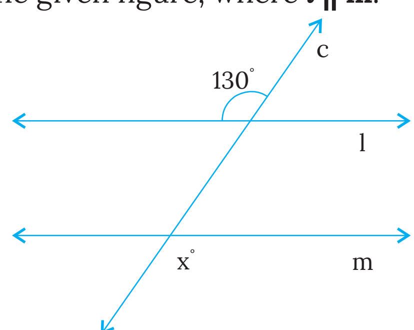


Fig. 5.35

Solution: The angle marked 130° and angle **x** are alternate exterior angles. Since lines ***l*** and **m** are parallel, alternate exterior angles are equal.
Therefore, x = 130°.

Example 24 : In the figure, AB || CD. Find the value of **y**.

Solution: The angles 4y and 100° are interior angles on the same side of the transversal. Since AB || CD, the sum of interior angles on the same side is 180°.

$4y + 100° = 180°$
$4y = 180° - 100°$
$4y = 80°$
$y = 80°/4$
$y = 20$.

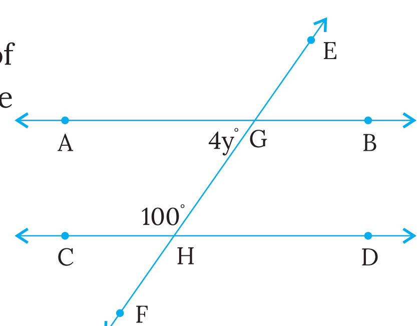


Fig. 5.36

Knowledge Checkpoint

- If two parallel lines are cut by a transversal, and one corresponding angle is 80°, what is the measure of the other corresponding angle?
- If alternate interior angles are 100° and 95°, are the lines parallel?
- What is the sum of interior angles on the same side of a transversal when it cuts parallel lines?

Activity

Verifying Parallel Line Properties

Objectives: To verify the properties of angles formed when a transversal intersects two parallel lines.

Materials: Two parallel rulers or two strips of cardboard taped parallel, a third strip of cardboard (transversal), protractor, paper.

Procedure:

1. Place the two parallel rulers/cardboard strips on a sheet of paper.
2. Place the third strip (transversal) across them, intersecting both.
3. Trace the lines onto the paper.
4. Carefully label all 8 angles formed (1 through 8).
5. Using a protractor, measure all 8 angles and record their values.
6. Now, verify the properties:
    - Are corresponding angles equal? (e.g., $\angle 1 = \angle 5$?)
    - Are alternate interior angles equal? (e.g., $\angle 3 = \angle 6$?)
    - Is the sum of interior angles on the same side 180°? (e.g., $\angle 4 + \angle 6 = 180°$?)

Observation: You should find that these properties hold true, confirming the theoretical rules.

Conclusion: This activity provides empirical evidence for the properties of angles formed by a transversal intersecting parallel lines.

Key Terms

- **Corresponding Angles Property:** Equal when lines are parallel.
- **Alternate Interior Angles Property:** Equal when lines are parallel.
- **Interior Angles on Same Side Property:** Supplementary (sum 180°) when lines are parallel.
- **Converse:** The reverse statement of a theorem.

Fact Flash

- The properties of parallel lines and transversals are so fundamental that they are often called "**Euclid's Postulates**" or "**Euclidean Geometry**."
- If you draw a perfect rectangle, all its opposite sides are parallel, and its adjacent sides are perpendicular, forming 90° angles!

Do It Yourself

Can you think of a real-life situation where you might need to check if two lines are parallel by measuring angles? (Hint: Think about setting up a fence or a bookshelf).

Mental Mathematics

- If $l \parallel m$, and a corresponding angle is 70°, what is the alternate interior angle?
- If **p || q**, and an interior angle on the same side is 110°, what is the other interior angle on that side?
- If two lines are cut by a transversal, and alternate interior angles are 45° and 45°, are the lines parallel?
- If an exterior angle is 130°, and the lines are parallel, what is the corresponding interior angle?
- If a transversal forms a right angle with one of two parallel lines, what kind of angle does it form with the other parallel line?

Gap Analyzer™ Homework

Watch Remedial

Exercise 5.4

1. Fill in the Blanks:

a) If two parallel lines are cut by a transversal, corresponding angles are _______.

b) If alternate interior angles are equal, then the lines are _______.

c) The sum of interior angles on the same side of a transversal intersecting parallel lines is _______ degrees.

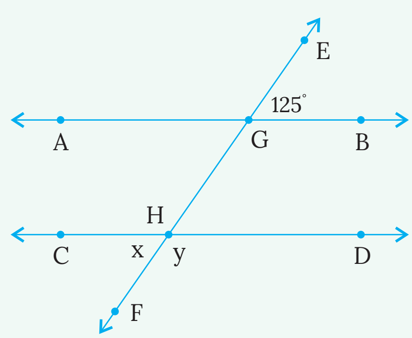
d) If line A || line B, and a transversal forms an angle of 60° with line A, its alternate interior angle with line B will be _______ degrees.

2. Bridging the Gap Questions (between theory and practice):

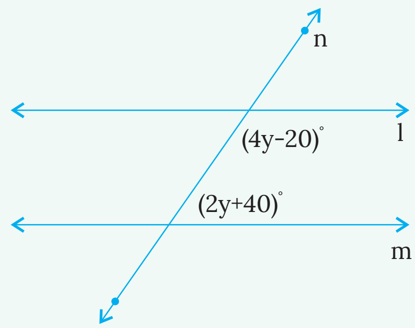
a) Two parallel roads are connected by a diagonal road. If the angle between one parallel road and the diagonal road is 120°, what is the angle between the other parallel road and the diagonal road on the same side?

b) A railway crossing has two parallel tracks. A barrier arm comes down, acting as a transversal. If the angle between the barrier and the top track is 95°, what is the angle between the barrier and the bottom track on the same side?

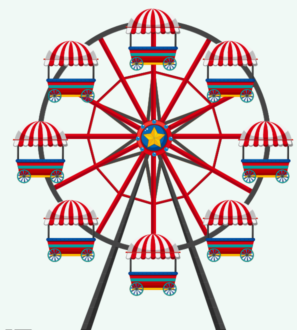
3. In the figure, line AB || line CD. The transversal EF intersects them at G and H respectively. If ∠EGB = 125°, find the measures of: a) ∠AGH b) ∠GHD c) ∠CHF (marked as 'x') d) ∠DHF (marked as 'y')



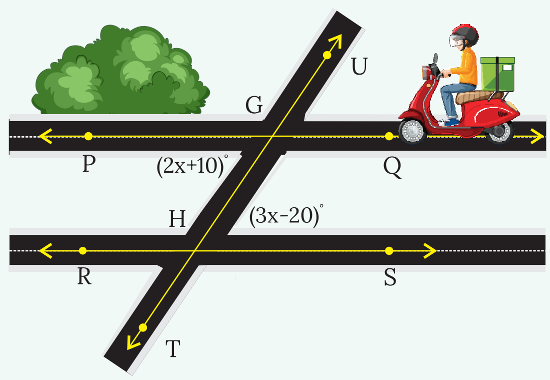
4. In the given figure, line l || line m. Determine the value of 'y'. Also, find the measure of the angle (4y - 20)°



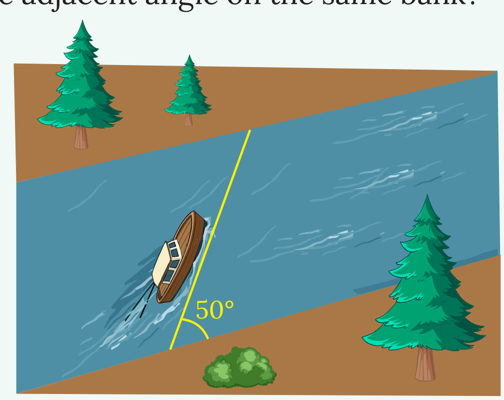
5. **We have a Ferris Wheel with cabins placed equally spaced around the center. This means all the central angles at the hub (center of the wheel) are equal.**
   i. What is the angle between two adjacent cabins?
   ii. Find the angle between two adjacent cabins. (8 cabins)
   iii. How many such angles are there around the center? (12 cabins)
   iv. In a Ferris wheel, the central angle between two adjacent cabins is 45°. How many cabins are there on the wheel?



6. **Concept Carnival: Mixed Question Set:**
   a) Lines PQ and RS are parallel. Find the value of x and the measure of ∠RHT



   b) **Two parallel riverbanks are cut by a slanting boat's path across the river.**
      i. If the boat's path makes an angle of 50° with one bank, what is the measure of the corresponding angle it makes with the other bank?
      ii. In the same situation, if the boat's path makes an angle of 50° with one bank on one side, what is the measure of the adjacent angle on the same bank?



Construction of Parallel Lines

Now that we understand the properties of parallel lines and the angles formed by a transversal, let's put that knowledge into practice! How can we actually draw a line that is perfectly parallel to another given line? This section will teach you practical methods using common geometric tools and even simple paper folding, demonstrating how the theoretical properties translate into real-world constructions.

Drawing Parallel Lines using Angle Properties

One of the most elegant ways to draw parallel lines is by using the angle properties we just learned Specifically, if we can create a pair of equal corresponding angles (or equal alternate interior angles), we can guarantee that the lines we draw will be parallel. This method relies on precise measurement and construction.

Sub-concepts to be covered

1. **Method using Corresponding Angles:** This method involves drawing a transversal through a given point, then copying an angle to create an equal corresponding angle at another point, thus ensuring parallelism.
2. **Method using Alternate Interior Angles:** Similar to the corresponding angles method, but involves copying an angle to create an equal alternate interior angle.

   Tools Required: Ruler (straightedge), protractor (or compass for more advanced constructions), pencil.

Mathematical Explanation

Let's say you have a line ***l*** and a point **P** not on line ***l***. Your goal is to draw a line **m** through **P** such that **m || *l***.

Method 1: Using Corresponding Angles

1. **Draw a transversal:** Draw any line **t** passing through point **P** and intersecting line ***l*** at a point, say **Q**.
2. **Identify an angle:** Measure the angle formed by line ***l*** and transversal **t** (e.g., ∠PQX, where X is a point on line ***l***). This is your reference angle.
3. **Copy the angle:** At point **P**, on the same side of the transversal **t** as your reference angle, construct an angle equal to ∠PQX. Let the new ray formed be **PY**.

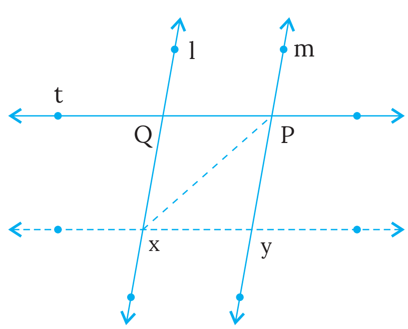


Fig. 5.37

4. **Draw the parallel line:** The line **PY** is your line **m**. Since ∠PQX and ∠YPX are corresponding angles and you've made them equal, line **m** will be parallel to line ***l***.

Method 2: Using Alternate Interior Angles

1. **Draw a transversal:** Draw any line **t** passing through point **P** and intersecting line ***l*** at a point, say `**Q**`.
2. **Identify an angle:** Measure an interior angle formed by line ***l*** and transversal **t** (e.g., ∠PQX, where X is a point on line ***l***).

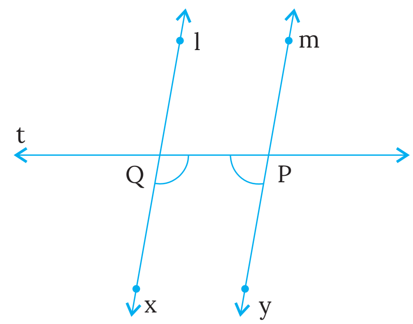


Fig. 5.38

3. **Copy the angle:** At point **P**, on the opposite side of the transversal **t** from your reference angle, construct an angle equal to ∠PQX. Let the new ray formed be **PY**.
4. **Draw the parallel line:** The line **PY** is your line **m**. Since ∠PQX and ∠YPQ are alternate interior angles and you've made them equal, line **m** will be parallel to line ***l***.

Both methods rely on the converse of the angle properties: if the specific angle pairs are equal, then the lines are parallel.

Example 25 : Draw a line parallel to a given line segment AB, passing through a point C not on AB, using the corresponding angles method.

Solution: A line segment AB. A point C above AB.

Steps: 1. Draw a line segment AB.

2. Mark a point C not on AB.
3. Draw a line (transversal) from C intersecting AB at D.
4. Measure ∠CDB (the angle formed by the transversal and AB). Let's say it's 60°.
5. At point C, using a protractor, draw an angle of 60° on the same side of the transversal CD as ∠CDB. Let this new ray be CE.
6. Extend CE to form a line. This line CE is parallel to AB.

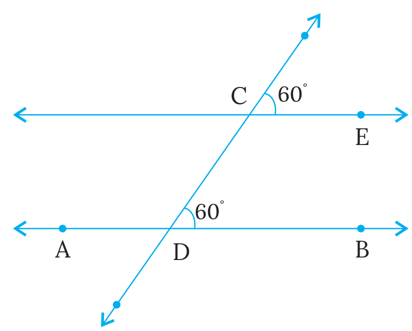


Fig. 5.39

Step-by-step diagrams showing the construction:

1. Line AB, point C.
2. Transversal CD drawn.
3. Angle CDB measured.
4. Angle DCE constructed equal to CDB.
5. Line CE drawn and labeled parallel to AB.

Example 26 : Draw a line *l*. Mark a point P not on the line. Using the property of corresponding angles, draw a line through point P parallel to *l*.

Solution:

Steps: 1. Draw a line l and choose a point P above it.

2. From point P, draw a transversal line intersecting l at any angle (say 60°).

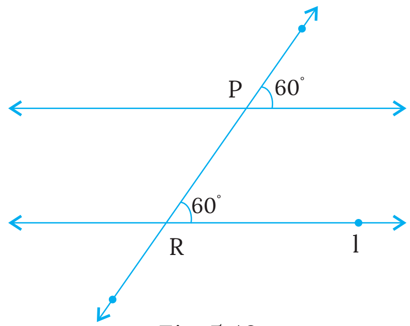


Fig. 5.40

3. At point P, using a protractor, construct a corresponding angle of 60° on the same side of the transversal.
4. Draw the new line through P using this angle.

This line is parallel to line *l* because corresponding angles are equal.

Example 27 : Draw a line m and a transversal. Using alternate interior angles, construct another line n parallel to m.

Solution:

Steps: 1. Draw line m and a transversal intersecting it at point A.

2. At point A, draw an angle (e.g., 70°) on the interior side.
3. Move along the transversal and mark point B.

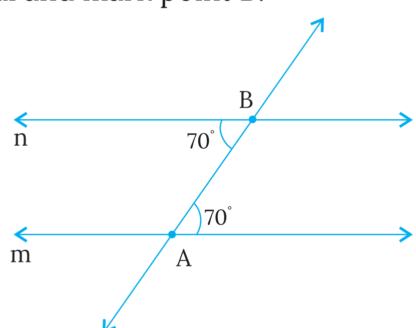


Fig. 5.41

4. At B, on the opposite side of the transversal, construct an angle of 70° (alternate interior).
5. Draw line n through point B using the constructed angle.

Line n is parallel to line m because alternate interior angles are equal.

Knowledge Checkpoint

- What angle property is used when you copy an angle to draw a parallel line on the same side of the transversal?
- If you use the alternate interior angles method, where do you construct the new angle relative to the transversal?
- What tools are typically used for these constructions?

Key Terms

- **Construction:** The process of drawing geometric figures accurately using tools.
- **Converse:** The reverse statement of a theorem (e.g., if corresponding angles are equal, then lines are parallel).

Fact Flash

- The ancient Egyptians used a tool called a "**gnomon**" to create right angles and parallel lines for their massive construction projects.
- The ability to draw parallel lines accurately was a significant challenge for early

Do It Yourself

If you were given a line and a point, and you could only use a ruler (straightedge) and a compass, how would you construct a line parallel to the given line through the point? (This is a classic Euclidean construction problem!).

Mental Mathematics

- If you are constructing a parallel line using corresponding angles, and the original angle is 80°, what angle do you need to draw?
- If you are using alternate interior angles, and the original angle is 120°, what angle do you need to draw?
- What is the primary reason these angle-based constructions work?
- Can you construct a parallel line if you don't have a protractor?
- If you draw a line and then draw two lines perpendicular to it, what can you say about those two perpendicular lines?

Drawing Parallel Lines using Ruler and Set Square

While angle properties provide the theoretical basis, practical tools like a ruler and a set square offer a quick and efficient way to draw parallel lines. This method is widely used in technical drawing and everyday situations where precision is needed. It cleverly uses the concept of perpendicularity and sliding motion to achieve parallelism.

Sub-concepts to be covered

1. **Method 1: Using Perpendiculars;** Draw a line, then draw two lines perpendicular to it at different points. These two perpendicular lines will be parallel to each other.
2. **Method 2: Sliding Set Square;** Place a set square against a ruler. Draw a line along one edge of the set square. Then, slide the set square along the ruler and draw another line along the same edge. These two lines will be parallel.

Tools Required: Ruler (straightedge), set square, pencil.

Mathematical Explanation

Method 1: Drawing two perpendiculars to a common line

1. Draw a line **t**.
2. At two different points on line **t**, say **A** and **B**, draw lines ***l*** and **m** respectively, such that $l \perp t$ and $m \perp t$.

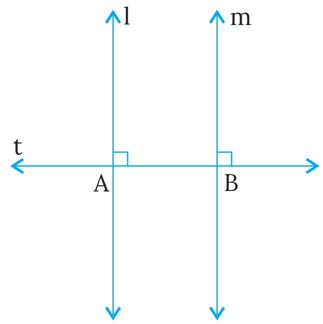


Fig. 5.42

3. Since both *l* and **m** form a 90° angle with the transversal **t**, their corresponding angles (or alternate interior angles, or interior angles on the same side) are equal (all 90°).
4. Therefore, line ***l*** will be parallel to line **m** ($l \parallel m$). This method is very accurate if your perpendiculars are precise.

Method 2: Sliding a Set Square along a Ruler

1. Place a ruler firmly on your paper. This ruler acts as a guide.
2. Place a set square against the edge of the ruler.
3. Draw a line along one of the edges of the set square (e.g., the longer edge or one of the shorter edges). Let's call this line ***l***.

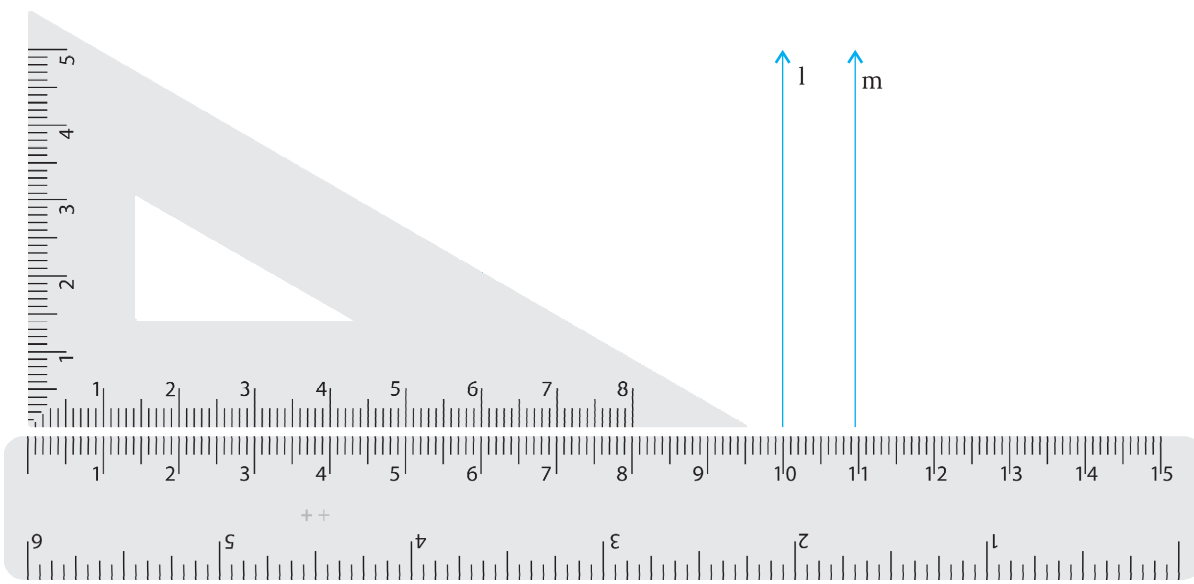


Fig. 5.43

4. Hold the ruler firmly and slide the set square along the ruler's edge, keeping the same edge of the set square against the ruler.
5. Draw another line along the same edge of the set square in its new position. Let's call this line **m**.

6. Since the angle between the set square's edge and the ruler's edge remains constant as you slide it, the lines ***l*** and **m** will be parallel. The ruler acts as a transversal, and the angle formed by the set square's edge and the ruler's edge is a constant corresponding angle.

   This sliding set square method is particularly efficient for drawing multiple parallel lines quickly.

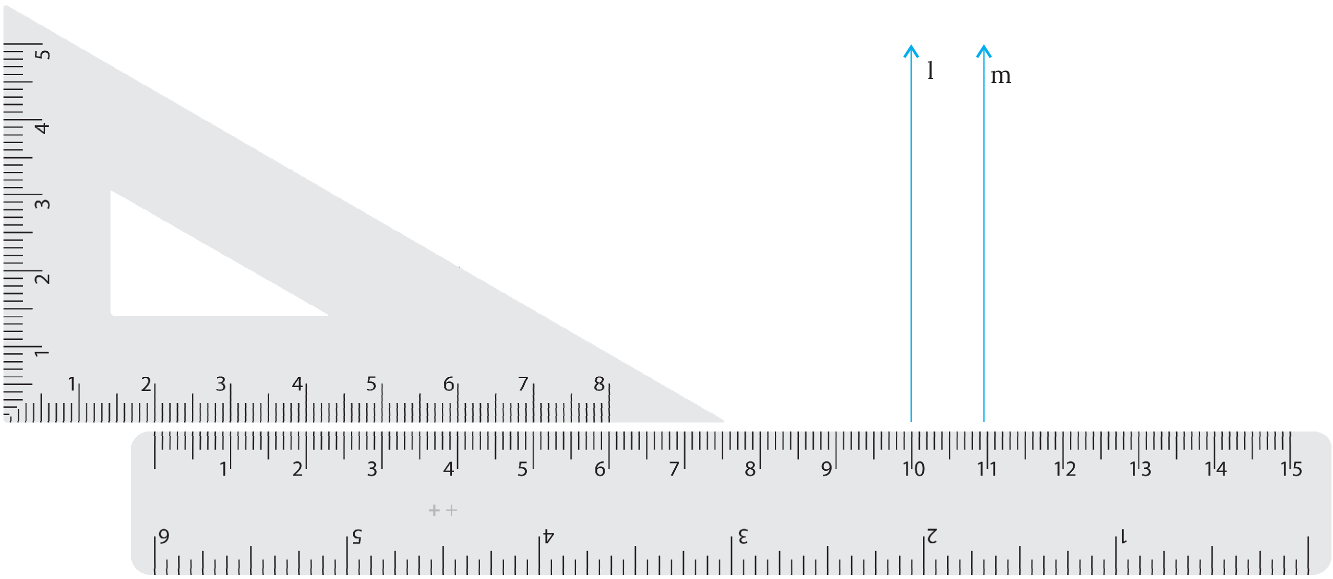


Fig. 5.44

Example 28 : Draw a line parallel to a given line `XY` through a point `Z` not on `XY`, using the ruler and set square method (sliding method).

Solution: A line XY. A point Z above XY.

Steps: 1. Place the ruler firmly along the line XY.

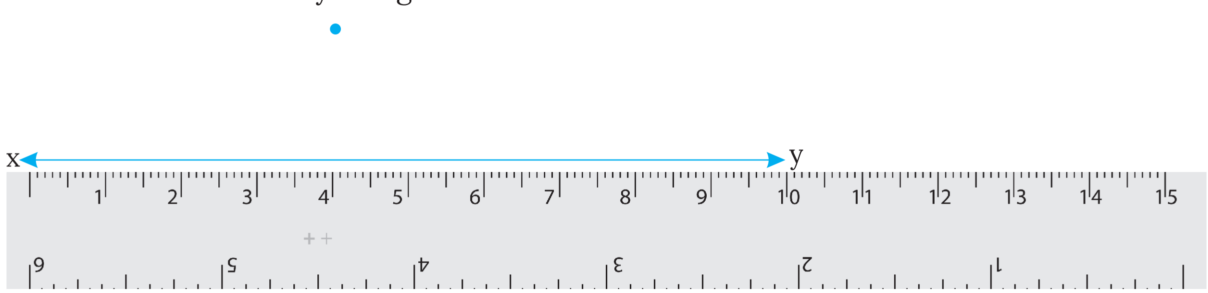


Fig. 5.45

2. Place the set square with one of its edges (the one you'll draw along) against the ruler, and the other edge touching point Z.

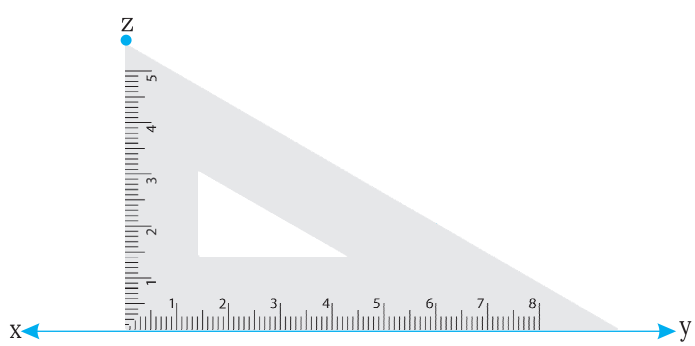


Fig. 5.46

3. Slide the set square along the ruler until the drawing edge passes through point Z.
4. Draw a line along the drawing edge of the set square, passing through Z. This line will be parallel to XY.

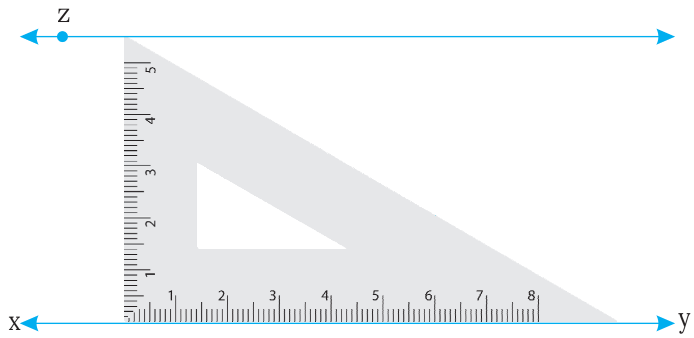


Fig. 5.47

Example 29 : Explain why the method of drawing two lines perpendicular to a common line results in parallel lines.

Solution: Let the common line be **t**. Let line *l* be perpendicular to **t** at point P, and line **m** be perpendicular to **t** at point Q.
This means that the angle formed by *l* and **t** at P is 90°, and the angle formed by **m** and **t** at Q is 90°.
These two 90° angles are corresponding angles (or alternate interior angles, or interior angles on the same side, depending on how you view them).
Since the corresponding angles are equal (both 90°), by the converse of the corresponding angles property, lines ***l*** and **m** must be parallel.

Knowledge Checkpoint

- What are the two main tools used in the "ruler and set square" method for drawing parallel lines?
- How does the "two perpendiculars" method ensure parallelism?
- Why is it important to hold the ruler firmly when using the sliding set square method?

Activity

Parallel Lines Challenge
Objectives: To construct parallel lines using multiple geometric methods and compare their accuracy.
Materials: Paper, ruler, set square, protractor, pencil.
Procedure:

1. Draw a straight line **L** on your paper.
2. Mark a point **P** about 5 cm away from line **L**.
3. **Challenge 1:** Draw a line parallel to **L** passing through **P** using only your ruler and set square (sliding method).
4. **Challenge 2:** Draw another line parallel to **L** passing through **P** using the corresponding angles method (using protractor).
5. **Challenge 3:** Draw a third line parallel to **L** passing through **P** by drawing two lines perpendicular to a common transversal.
6. Compare the three "parallel" lines you drew. Are they perfectly aligned? Discuss any discrepancies.

Observation: This activity allows students to practice different construction methods and compare their accuracy. They will likely find slight variations, highlighting the importance of precision.
Conclusion: Reinforces the practical application of geometric tools and the underlying principles of parallel lines.

Key Terms

- **Set Square:** A triangular drawing instrument with a right angle.
- **Ruler:** A straightedge for drawing lines and measuring.

Fact Flash

- The set square, as we know it, evolved from ancient measuring tools used by builders and stonemasons.
- **Leonardo da Vinci**, the famous artist and inventor, used geometric principles extensively in his designs and drawings, often relying on precise parallel and perpendicular lines.

Do It Yourself

If you only had a compass and a straightedge, how would you construct a line parallel to a given line through a given point? (This is a more advanced construction, but it's good to ponder!)

Drawing Parallel Lines through Paper Folding

Paper folding is not just a fun activity; it's a powerful way to explore geometric concepts in a hands-on, intuitive manner. You can create perfectly straight lines, perpendicular lines, and even parallel lines just by folding a piece of paper. This method helps visualize the underlying geometric principles without needing complex tools.

Sub-concepts to be Covered

Method 1:

Folding Perpendiculars: Fold a line, then fold a perpendicular to it. Then fold another perpendicular to the first perpendicular. The last two folds will be parallel.

Method 2:

Repeated Parallel Folds: Fold a line. Then, without unfolding, make another fold parallel to the first. This creates two parallel lines.

Paper image

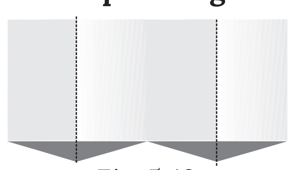


Fig. 5.48

Underlying Principle: Paper folding relies on the fact that a crease forms a straight line, and folding a line onto itself creates a perpendicular. Repeated perpendiculars lead to parallel lines.

Mathematical Explanation

1. Take a rectangular sheet of paper.
2. Fold the paper anywhere to create a straight crease. Let's call this crease $\boldsymbol{l}$. This is your first line.
3. Now, fold the paper again such that line $\boldsymbol{l}$ folds exactly onto itself. This means the new crease will be perpendicular to $\boldsymbol{l}$. Make a sharp crease. Let's call this crease **t**. So, $\mathbf{t} \perp \boldsymbol{l}$.
4. Now, without unfolding, fold the paper a third time such that crease **t** folds exactly onto itself, but at a different point than the first fold. Make a sharp crease. Let's call this crease **m**. So, $\mathbf{m} \perp \mathbf{t}$.

5. Unfold the paper completely. You will see three creases: ***l***, **t**, and **m**.

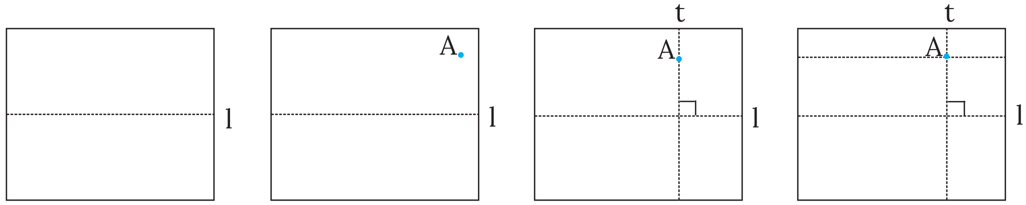


Fig. 5.49

Observation: Since both ***l*** and m are perpendicular to the same line **t**, they must be parallel to each other (***l*** **|| m**). This demonstrates the property that two lines perpendicular to the same line are parallel.

Example 30 : Describe how to create two parallel lines using paper folding, based on the principle that two lines perpendicular to the same line are parallel.

Solution:

Step 1: Fold a Center Line (Reference Line)

- Fold the paper horizontally or vertically in half.
- Crease the fold well and then unfold it.
- This fold is your reference line (Line L).

Step 2: Fold the First Perpendicular Line

- Take one side of the paper.
- Fold the paper so that an edge of the paper touches the reference line at a right angle (90°).
- Crease the fold and then unfold it.
- This is your first perpendicular line (Line A).

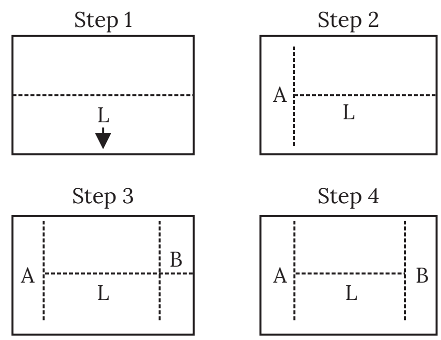


Fig. 5.50

Step 3: Fold the Second Perpendicular Line on the Opposite Side

- Now take the opposite side of the paper.
- Again, fold the paper so that this edge touches the same reference line at a right angle (90°).
- Crease the fold and then unfold it.
- This is your second perpendicular line (Line B).

Step 4: Observe the Parallel Lines

- The two fold lines (Line A and Line B) are each perpendicular to the same reference line (Line L).
- By the geometric principle, two lines perpendicular to the same line are parallel.
- Therefore, Line A and Line B are parallel lines.

Example 31 : Can you create a rectangle by making only two folds on a piece of paper? Explain.

Solution: Yes, you can create a rectangle using only two folds on a piece of paper. Here's how:

Step 1: Fold the Paper in Half Horizontally (or Vertically)

- Take a rectangular paper and fold it in half from top to bottom (horizontal fold) or side to side (vertical fold).
- Crease the fold well and unfold.
- This forms one straight fold line.

Step 2: Fold the Paper in Half the Other Way

- Now fold the paper in half in the perpendicular direction to the first fold.
- Crease it well and unfold.
- This creates a second straight fold line, perpendicular to the first one.

Final Result:

- The two fold lines intersect at right angles, dividing the paper into four rectangles.
- The area where the fold lines form a corner is itself a rectangle.
- The entire paper also remains rectangular in shape.

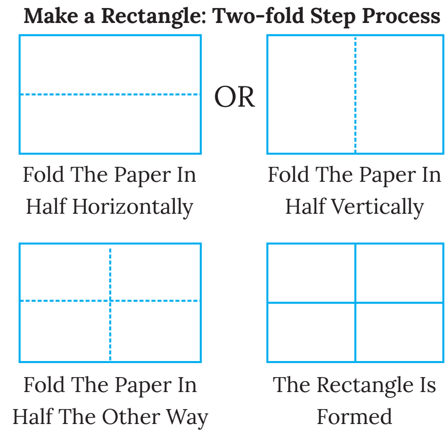


Fig. 5.51

Knowledge Checkpoint

- When you fold a paper, and then fold it again so the first crease lies on itself, what kind of angle is formed between the two creases?
- If you want to create two parallel lines using paper folding, what is one simple method?
- Why are the creases formed by paper folding considered "lines" in geometry?

Key Terms

- **Crease:** The line formed by folding paper.
- **Origami:** The art of paper folding, often demonstrating geometric principles

Fact Flash

- The **art of origami**, which originated in Japan, is deeply rooted in geometric principles, including the creation of parallel and perpendicular lines.
- Mathematicians have proven that any geometric construction that can be done with a compass and straightedge can also be done with paper folding (though some are very complex!).

Do It Yourself

If you fold a paper to create a line, and then fold it again to create a line perpendicular to the first, and then fold it a third time to create a line parallel to the first, how many distinct creases will you have?

Mental Mathematics

- If you fold a paper in half, how many creases do you get?
- If you fold a paper in half, and then in half again (perpendicular to the first fold), how many right angles are formed at the center?
- Can you make a curved line by folding paper?
- What is the relationship between the top and bottom edges of a folded piece of paper?
- If you fold a paper to create a line, and then fold it again to create a line 3 cm away and parallel to the first, what is the distance between the two creases?

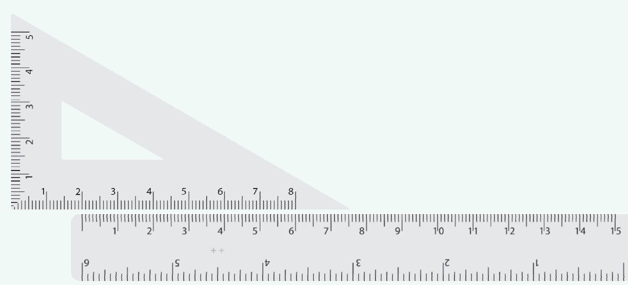
Gap Analyzer™ Homework

Watch Remedial

Exercise 5.5

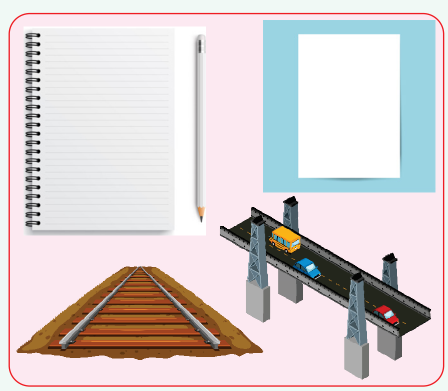
1. **Fill in the Blanks:**
   a) When drawing parallel lines using the alternate interior angles method, we ensure that the angles formed by the transversal are ___________, which confirms the lines are parallel.
   b) If two lines are perpendicular to the same third line, then they are ______ to each other.
   c) The sliding set square method works because it maintains a constant ______ angle with the guide.
   d) When you fold a paper and then fold it again so the first crease lies on itself, the second crease is ______ to the first.
   e) If two creases are both perpendicular to a third crease, then the first two creases are ______ to each other.
2. **Bridging the Gap Questions (between theory and practice):**
   a) Two parallel zebra crossings are painted on a straight road. A traffic light pole stands beside the road, and its shadow falls across both zebra crossings. If the angle the shadow makes with the first zebra crossing is 110°, what is the angle the shadow makes with the second zebra crossing on the opposite side?
   b) Parallel lines never meet, no matter how far they are extended. Using a ruler and a set square, we can easily draw parallel lines. In nature, we often see examples of parallel lines—like railway tracks, the edges of a cricket pitch, or the lanes of a running track. Recognising and drawing parallel lines helps us understand geometry in real life.

   Questions



   i. Using a ruler and set square, draw two parallel lines on your notebook. Mark them as AB and CD.
   ii. The railway tracks are 1.435 m apart (standard gauge). Draw two parallel lines 5 cm apart to represent this distance on paper.
   iii. The edges of a football ground are parallel. If its length is 105 m, and width is 68 m, draw a rectangle to represent it and mark the parallel sides.
   iv. Why do you think engineers always keep railway tracks and bridge beams parallel? Write one reason.
3. By folding a sheet such that one crease is made and then folding again so the new crease is at an equal distance, we get two parallel lines. This method helps students see the idea of parallelism in a hands-on way.

   Questions



   i. Take a rectangular sheet of paper. Fold it once to make a crease (line AB). Now fold it again so the new crease (line CD) is at an equal distance from AB.
      - Are AB and CD parallel? Explain.
   ii. A notebook page has 20 horizontal lines, each equally spaced 0.8 cm apart.
      - Show how these lines are parallel by folding the page.

- What is the total distance covered from the top line to the last line?

iii. Why is it important that railway tracks, bridge beams, and walls of buildings are kept parallel? Write one reason.

4. Concept Carnival: Mixed Question Set:

a) Explain in your own words what makes two lines parallel. How can you be sure, mathematically, that they will never intersect?

b) Besides railway tracks and zebra crossings, name two other real-world examples where you can observe parallel lines. For one of your examples, briefly explain how a transversal might interact with them.

Common Misconceptions

Misconception: Vertically opposite angles are always supplementary.

Correction: Vertically opposite angles are always equal, not supplementary (unless they are both 90°). It's linear pairs that are supplementary (sum to 180°). Students often confuse these two properties.

Misconception: If a transversal cuts two lines, then corresponding angles are always equal.

Correction: Corresponding angles are only equal if the two lines intersected by the transversal are parallel. If the lines are not parallel, corresponding angles will generally not be equal. This is a key condition for proving lines parallel.

Misconception: Interior angles on the same side of the transversal are always equal.

Correction: Interior angles on the same side of the transversal are supplementary (sum to 180°) only if the lines are parallel. If the lines are not parallel, their sum will not be 180°.

Real-Life (Lines): Mathematical Applications

Encouraging 21st-Century Skills: Promoting observation, critical thinking, and problem solving. For instance, we could explore applications related to:

- **Architecture and Design:** Identifying different types of angles in buildings, bridges, or furniture.
- **Art and Geometry:** Analyzing angles in patterns, mandalas, or famous artworks.
- **Sports:** How angles are crucial in games like billiards, football, or basketball.
- **Navigation and Maps:** Understanding angles in directions and routes.
- **Everyday Objects:** Finding angles in scissors, clocks, letterforms, or even a simple door.

ON AIR

HINDI

ENGLISH

Gap Analyzer™
Complete Chapter Test

EXERCISE

A. Choose the correct answer.

1. If two lines intersect and one angle formed is 55°, what is the measure of its vertically opposite angle?

   a) 35° b) 125° c) 55° d) 180°

2. Which of the following statements is true about parallel lines?

   a) They intersect at one point.
   b) They are always perpendicular.
   c) They never intersect and are on the same plane.
   d) They form right angles with a transversal.

3. When a transversal intersects two parallel lines, what is the relationship between alternate interior angles?

   a) They are supplementary. b) They are complementary.
   c) They are equal d) Their sum is 90°.

4. If two lines are perpendicular, the angle between them is:

   a) Acute b) Obtuse
   c) Straight d) Right

Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.
Study both the statements and state which of the following is correct:

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

1. **Assertion (A):** If two lines intersect, then the sum of any two adjacent angles is 180°.
Reason (R): Adjacent angles always form a linear pair.

2. **Assertion (A):** If a transversal intersects two lines such that alternate interior angles are equal, then the lines are parallel.
Reason (R): This is the converse of the alternate interior angles theorem.

3. **Assertion (A):** Railway tracks are an example of parallel lines.
Reason (R): Parallel lines never intersect and maintain a constant distance between them.

Case Study

The Leaning Tower of Pisa

The Leaning Tower of Pisa is famous for its unintended tilt. Originally designed to be perfectly vertical (perpendicular to the ground), it began to lean due to unstable foundations.

1. If the tower were perfectly vertical, what angle would it make with the flat ground?
2. As it started to lean, did the angle it made with the ground become acute or obtuse on the side it was leaning towards? (**Use: Interior Angle**)
3. Imagine a perfectly straight, vertical flagpole next to the Leaning Tower. Are the flagpole and the tower parallel to each other? Why or why not?
4. If a horizontal rope is tied from the top of the flagpole to the top of the tower, what kind of angle would the rope make with the flagpole?

Project

Designing a City Block

Objective: To apply concepts of parallel, perpendicular, and intersecting lines, along with angle properties, to design a functional city block.

Task: Imagine you are an urban planner. Design a small city block on a large sheet of chart paper. Your design must include:

1. Main Roads: At least two main roads that are perfectly parallel to each other.
2. Cross Streets: At least two cross streets that are perpendicular to the main roads.
3. Diagonal Avenue: One diagonal avenue that acts as a transversal, intersecting both main roads and at least one cross street.
4. Buildings/Parks: Mark areas for at least 3 buildings or parks within your block.
5. Angle Analysis:
   - Choose one intersection where the diagonal avenue crosses a main road. Label all 8 angles formed.
   - Assume one angle measure (e.g., 60°). Calculate the measures of all other 7 angles at that intersection, justifying each step using the properties of parallel lines and transversals (corresponding, alternate interior, interior on same side, linear pair, vertically opposite).
   - Identify at least one pair of perpendicular lines and one pair of parallel lines in your design.

Materials: Chart paper, ruler, protractor, pencil, colored pencils/markers.

Presentation:

- Present your city block design to the class.
- Explain how you used parallel and perpendicular lines to create an organized layout.
- Clearly show your angle calculations and justifications for the chosen intersection.
- Discuss any challenges you faced and how you overcame them.

This project encourages synthesis of knowledge, creativity, and real-world application of geometric concepts.

Source-Based Question

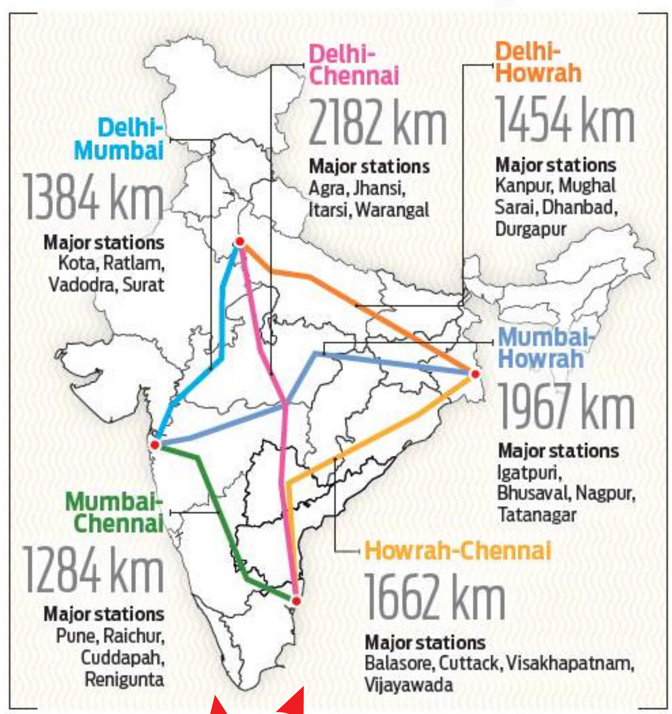
India on the Fast Track: The Diamond Quadrilateral

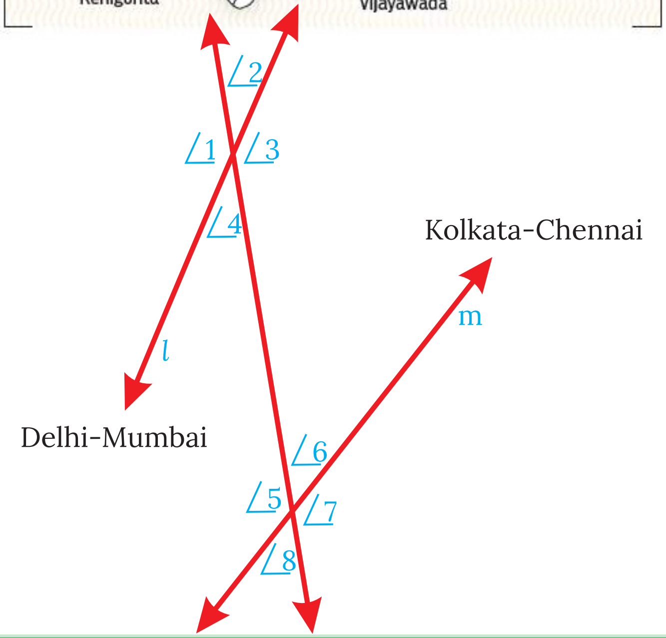
The Government of India has envisioned a high-speed rail network called the Diamond Quadrilateral. This ambitious project aims to connect the four mega-cities of India: **Delhi, Mumbai, Chennai, and Kolkata**, much like the Golden Quadrilateral highway network.

For our geometric study, let's look at a simplified model of this network. Imagine the proposed railway track connecting Delhi to Mumbai and the track connecting Kolkata to Chennai. For efficient planning and signaling over long distances, these two tracks are designed to be parallel to each other.

A new "feeder line" is also planned to connect cities in central India to this high-speed network. Let's consider one such feeder line running from Nagpur to Secunderabad, which crosses both the main high-speed tracks. This feeder line acts as a transversal.

Source Text: Adapted from data and project outlines related to the Diamond Quadrilateral project, Ministry of Railways, Government of India.





Questions Based on the Source

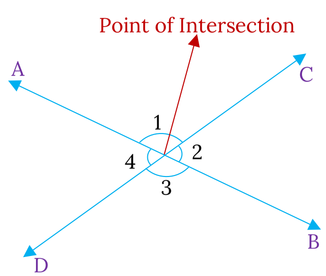
1. Based on the description, the Nagpur-Secunderabad feeder line is represented by line t. What is the geometric term for a line that intersects two or more parallel lines at distinct points?
2. During a survey, it was found that the angle the feeder line makes with the Delhi-Mumbai track (∠1 in the diagram) is 110°. Since lines l and m are parallel, what would be the measure of the corresponding angle at intersection Q (the upper-right angle at Q)? State the property you used.
3. Using the information that ∠1 = 110°, find the measures of: (a) ∠2 (the alternate interior angle to the one vertically opposite ∠1) (b) ∠3 (the angle that forms a linear pair with ∠2) Justify each answer with the correct geometric property.
4. For a different section of the track, engineers are using a model where the two interior angles on the same side of the transversal are represented by (2x + 15)° and (x + 30)°. For the main lines to be perfectly parallel, what must be the value of 'x'?
5. Another proposed cross-country line intersects the Delhi-Mumbai line (l) and the Kolkata-Chennai line (m). A surveyor reports that a pair of alternate interior angles formed by this new line are 89° and 91°. Based on this data, can the surveyor conclude that lines l and m are parallel in that section? Explain your reasoning.

Mind Map

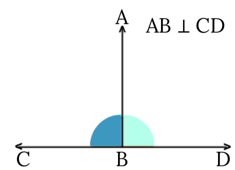
Intersecting and Parallel Lines

Lines on a Plane Surface
- Definition of a Plane
- Lines on a Plane
- Points on a Line
- Collinear Points

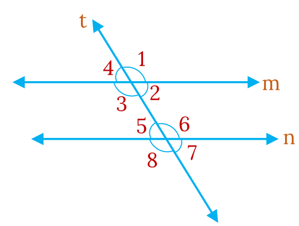
Intersecting Lines and Angles
- Definition of Intersecting Lines
- Angles Formed by Intersecting Lines
- Linear Pair of Angles
- Vertically Opposite Angles



Perpendicular Lines
- Definition of Perpendicular Lines
- Properties of Perpendicular Lines



Parallel Lines
- Definition of Parallel Lines
- Distance Between Parallel Lines
- Lines on the Same Plane



Transversal and Angles Formed
- Definition of a Transversal
- Interior Angles
- Exterior Angles
- Pairs of Angles (Corresponding Angles, Alternate Interior and Exterior Angles, Interior Angles on the Same Side of the Transversal)
- Properties of Angles with Parallel Lines

Construction of Parallel Lines
- Drawing Parallel Lines using Angle Properties
- Drawing Parallel Lines using Ruler and Set Square
- Drawing Parallel Lines through Paper Folding