4 Algebraic Expressions: Using Letter-Numbers

Why This Chapter Matters

Imagine you're a detective trying to solve a mystery. You find clues, but they're not always straightforward. Sometimes, you need to use symbols or codes to represent unknown information. Mathematics is similar! What if you could write down a rule for any number, not just a specific one? How would you describe the relationship between the number of sides of a polygon and its perimeter, without picking a specific polygon? This chapter will unlock the secret language of algebra, where letters become powerful tools to describe patterns and solve puzzles!

Meet EeeBee.AI

Hello, young mathematicians! I'm EeeBee, your friendly guide through the exciting world of numbers and patterns. I love exploring how math helps us understand everything around us. In this chapter, I'll be here to ask thought-provoking questions, share fun facts, give helpful hints, and recap important ideas. Let's embark on this algebraic adventure together and discover the power of letter-numbers! Ready to decode the language of algebra? Let's go!

Learning Outcomes

By the end of this chapter, students will be able to:

- Define letter-numbers and algebraic expressions.
- Translate real-life situations into algebraic expressions and vice versa.
- Identify like and unlike terms in algebraic expressions.
- Simplify algebraic expressions by combining like terms.
- Evaluate algebraic expressions for given numerical values of variables.
- Formulate rules for number patterns and geometric patterns using algebraic expressions.
- Solve problems involving arithmetic operations on algebraic expressions.

From Last Year's Notebook

- **Recall various types of numbers:** whole numbers, integers, fractions, and decimals.
- **Remember the four basic arithmetic operations:** addition, subtraction, multiplication, and division.
- Revisit patterns in numbers and shapes (e.g., perimeter of a square = 4 × side).
- Introduce a new method to represent relationships and patterns using letters (variables).
- Extend arithmetic operations to expressions involving these "letter-numbers."
- Develop a more general and versatile mathematical language.

Real Math, Real Life

Algebraic expressions are not just for textbooks; they're powerful tools used everywhere!

- **Real-World Calculations:** They help calculate everyday things, like the total cost of groceries based on how much you buy.
- **Predicting & Designing:** Engineers use them to design bridges, scientists describe natural phenomena, and they even help predict a rocket's path or design video games!
- **Understanding Relationships:** Algebra provides a clear, concise way to show how different things are connected.
- **Future Skills & Careers:** Understanding algebraic expressions is a fundamental skill that opens doors to countless exciting careers (like engineering, economics, science) and helps you truly understand the world around you.

Quick Prep

1. A taxi charges a fixed fare of ₹50 plus ₹10 per kilometer. How much would a 3 km ride cost?
2. Can you find the next two numbers in the sequence: 3, 7, 11, 15, ...? What is the rule for this sequence?
3. What is the perimeter of a rectangle with length 8 cm and width 5 cm?
4. Simplify: 25 + 15 − (7 + 3) × 2.
5. If you have **x** number of apples and your friend gives you 3 more, how many apples do you have now?

Introduction

Welcome to the fascinating world of algebra! Until now, you've primarily worked with specific numbers. But what if you want to describe a general rule or a relationship that applies to any number? This is where algebraic expressions come in. They provide a powerful and concise way to represent unknown quantities and general relationships using letters as placeholders for numbers. This section will introduce you to the fundamental concepts of algebraic expressions, how to form them, simplify them, and use them to describe and solve real-world problems. Get ready to unlock a new level of mathematical thinking!

Chapter Overview

- **Introduction to Variables:** Understand letter-numbers, their purpose (generalization, shorthand), forming simple expressions, and evaluating by substitution.
- **Revisiting Arithmetic:** Reinforce BODMAS and distributive property as foundational for algebraic evaluation.
- **Notation:** Learn standard algebraic notation (e.g., 4n) and avoid common misconceptions.
- **Simplification:** Master like/unlike terms, combining them, and using the distributive property, including real-world applications.
- **Patterns:** Discover how algebraic expressions represent number, geometric, calendar, and matchstick patterns, including "Number Machine" rules.
- **Applications:** Translate word problems, solve real-life scenarios, compare, add, and subtract expressions.
- **Connections:** Bridge arithmetic to algebra, patterns to rules, and simplification concepts, setting groundwork for equations.

From History's Pages

Algebra's roots trace back to ancient Babylonians and Egyptians solving problems with words. The "**father of algebra," al-Khwarizmi** (9th century), gave algebra its name and systematic methods for equations. Later, in the 16^{th}-17^{th} centuries, mathematicians like Viète and Descartes introduced using letters (variables) for unknowns, making algebra symbolic. This revolutionized mathematics, allowing complex problems to be solved and laying the groundwork for modern algebra, where "**letter-numbers**" help describe patterns universally.

The Notion of Letter-Numbers and Algebraic Expressions

Imagine you want to describe a rule that applies to many different situations. For example, how do you find someone's age if they are always 5 years older than their sibling? You could say, "**Sibling's age plus 5**." But what if the sibling's age changes? Instead of writing "**Sibling's age**" every time, we can use a letter, like 's', to represent the sibling's age. Then, the older person's age is simply 's + 5'. These letters that stand for numbers are called letter-numbers or variables, and expressions like 's + 5' are called **algebraic expressions.** They help us write general rules concisely.

Sub-concepts to be covered

1. What are Letter-Numbers (Variables)?
2. Why use Letter-Numbers?
3. Forming Simple Algebraic Expressions (Addition, Subtraction)
4. Evaluating Expressions (Substituting Values)

What are Letter-Numbers (Variables)?

A letter-number, also known as a variable, is a symbol (usually a letter from the alphabet like '**x**', '**y**', '**a**', '**b**', '**n**') that represents an unknown or changing numerical value. Unlike specific numbers (like 5 or 10), the value of a variable can vary.

Explanation: Think of a variable as an empty box where you can put any number. For example, if '**c**' represents the number of chocolates, '**c**' can be 1, 5, 10, or any other whole number.

Example: In "cost = 5 × number of items", '**number of items**' can be represented by '**n**'. So, cost = 5n. Here, '**n**' is a variable.

Key points to remember:

- Variables are usually lowercase letters.
- They represent numbers.
- Their value can change.

Common errors to avoid: Confusing a variable with a unit (e.g., 'm' for meters vs. 'm' for a variable).

Why use Letter-Numbers?

Definition: Letter-numbers are used to express general rules, formulas, and relationships in a concise and universal way, without having to specify particular numerical values.

Explanation: If you want to say "the perimeter of a square is 4 times its side length," you could write it out. But if you use 's' for side length and 'P' for perimeter, you can simply write P = 4s. This is much shorter and applies to any square, regardless of its side length. They allow us to generalize mathematical statements.

Examples:
a) Age relationships: If Rohan is 7 years younger than Priya, and Priya's age is **p**, then Rohan's age is 'p - 7'.
b) Cost calculations: If a book costs ₹150, and you buy 'b' books, the total cost is 150b.

Key points to remember:

- **Conciseness:** Makes mathematical statements shorter.
- **Generalization:** Applies to all possible values.
- **Problem Solving:** Helps in setting up equations for unknown values.

Forming Simple Algebraic Expressions (Addition, Subtraction)

An algebraic expression is a combination of variables, numbers, and arithmetic operations (+, -, ×, ÷). It does not contain an equality sign (=).

Explanation: When we combine variables with numbers using operations, we form expressions.

Examples:

a) "5 more than a number" can be written as 'n + 5'.
b) "3 less than twice a number" can be written as '2x - 3'.
c) "Sum of 'x' and 7": x + 7
d) "Difference of 'y' and 4": y - 4 (if y is larger) or 4 - y (if 4 is larger)
e) "Product of 6 and 'm'": 6m
f) "Quotient of 'p' divided by 2": p/2

Key points to remember:

- Order matters for subtraction and division.
- "**More than**" implies addition, "**less than**" implies subtraction.
- "**Times**" or "**product**" implies multiplication.
- **Common errors to avoid:** Incorrectly translating "**less than**" (e.g., "5 less than x" is x - 5, not 5 - x).

Evaluating Expressions (Substituting Values)

Evaluating an algebraic expression means finding its numerical value by replacing each variable with a given number.

Explanation: Once you have an algebraic expression, you can find out what its value would be for specific numbers. You just "**plug in**" the numbers where the letters are.

Examples:

a) Evaluate '**a + 3**' if a = 23.
b) Evaluate '**2n**' if n = 7.
c) Evaluate '**c × 35 + j × 60**' if c = 7 and j = 4.

Key points to remember:

- Always follow the order of operations (BODMAS/PEMDAS) when evaluating.
- Be careful with negative numbers.
- **Common errors to avoid:** Forgetting to perform multiplication when a number is next to a variable (e.g., 5x when x = 2 is 5 × 2 = 10, not 52).

Mathematical Explanation

The transition from arithmetic to algebra is a significant step in mathematics, allowing for generalization and abstract reasoning. At its core, an algebraic expression is a mathematical phrase that can contain numbers, variables (letter-numbers), and operations. Unlike an equation, it does not have an equals sign and therefore cannot be "solved" in the traditional sense; it can only be "**evaluated**" or "**simplified**."

Variables (Letter-Numbers): A variable is a symbol, typically a letter, that represents a quantity that may change or is unknown. For instance, in the expression x + 5, x is the variable. Its value can be any number. This contrasts with constants, which are fixed numerical values (like the 5 in x + 5). The power of variables lies in their ability to represent a whole range of numbers, making it possible to write general rules or formulas. For example, the formula for the area of a rectangle, Area = length × width, can be concisely written as A = l × w or simply A = lw, where A, l, and w are variables representing area, length, and width, respectively.

A (*fig.* 4.1) diagram showing a balance scale with an unknown weight (labeled '**x**') on one side and known weights (e.g., 5 kg) on the other, illustrating the concept of a variable.

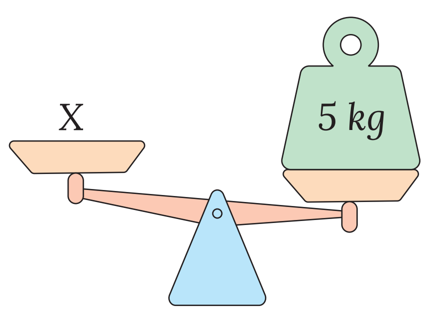

Fig. 4.1

Forming Expressions: Expressions are formed by combining variables and constants using the basic arithmetic operations.

- **Addition:** "**A number increased by 8**" translates to n + 8.
- **Subtraction:** "**12 less than a number**" translates to x - 12. Note the order: the number from which 12 is subtracted comes first.

- **Multiplication:** "**5 times a number**" translates to 5 × y or 5y. Conventionally, the number (coefficient) is written before the variable, and the multiplication symbol is often omitted.
- **Division:** "**A number divided by 3**" translates to z ÷ 3 or z/3. The fraction notation is more common in algebra.

A flowchart showing "Input (n)" -> "Rule (+5)" -> "Output (n+5)", demonstrating how expressions are formed.

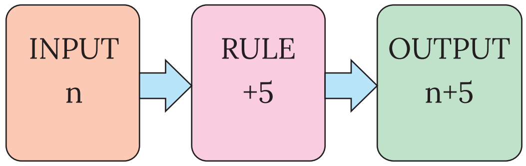

Fig. 4.2

Evaluating Expressions: To evaluate an algebraic expression, we substitute the given numerical value for each variable and then perform the operations according to the order of operations (BODMAS/PEMDAS).

Example: To evaluate 3p - 7 when p = 5:

Solution: Substitute p with 5: 3 × 5 - 7
Perform multiplication: 15 - 7
Perform subtraction: 8

The ability to evaluate expressions is crucial for checking solutions, testing hypotheses, and applying formulas to specific scenarios. The process reinforces the understanding that variables are indeed placeholders for numbers.

Example 1 : Shyam is 3 years older than Rohit.

a) Write an algebraic expression for Shyam's age if Rohit's age is 'a' years.
b) If Rohit is 18 years old, what is Shyam's age?
c) Write an algebraic expression for Rohit's age if Shyam's age is 's' years.
d) If Shyam is 20 years old, what is Rohit's age?

Solution: a) Let Rohit's age be 'a' years.
Since Shyam is 3 years older than Rohit, Shyam's age = Rohit's age + 3.
So, the expression for Shyam's age is a + 3.

b) Given Rohit's age (a) = 18 years.
Substitute a = 18 into the expression: Shyam's age = 18 + 3 = 21 years.

c) Let Shyam's age be 's' years.
Since Rohit is 3 years younger than Shyam, Rohit's age = Shyam's age - 3.
So, the expression for Rohit's age is s - 3.

d) Given Shyam's age (s) = 20 years.
Substitute s = 20 into the expression: Rohit's age = 20 - 3 = 17 years.

Example 2 : Parnav makes patterns with matchsticks. Each 'L' shape uses 2 matchsticks.

a) Write an algebraic expression for the number of matchsticks needed to make 'n' L-shapes.
b) How many matchsticks are needed to make 15 L-shapes?
c) If Parnav used 48 matchsticks, how many L-shapes did he make?

Solution: a) Number of matchsticks per L-shape = 2.

If 'n' is the number of L-shapes, then total matchsticks = 2 × n.

The expression is 2n.

b) To make 15 L-shapes, substitute n = 15 into the expression:

Total matchsticks = 2 × 15 = 30 matchsticks.

c) If Parnav used 48 matchsticks, we have the equation 2n = 48.

To find 'n', divide both sides by 2: n = 48 / 2 = 24 L-shapes.

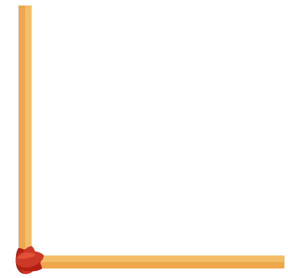

Fig. 4.3

Example 3 : Palvi prepares coconut-jaggery laddus. A coconut costs ₹40 and 1 kg jaggery costs ₹75.

a) Write an algebraic expression for the total cost if she buys 'c' coconuts and 'j' kg of jaggery.

b) Calculate the total cost if she buys 6 coconuts and 3 kg of jaggery.

Solution: a) Cost of 1 coconut = ₹40. Cost of 'c' coconuts = 40 × c = 40c.

Cost of 1 kg jaggery = ₹75. Cost of 'j' kg jaggery = 75 × j = 75j.

Total cost = Cost of coconuts + Cost of jaggery.

The expression for total cost is 40c + 75j.

b) To calculate the total cost for 6 coconuts and 3 kg jaggery, substitute c = 6 and j = 3 into the expression:

Total cost = (40 × 6) + (75 × 3)

Total cost = 240 + 225

Total cost = ₹465.

Example 4 : The perimeter of a square is 4 times the length of its side.

a) Write an algebraic expression for the perimeter of a square with side length 'q'.

b) Find the perimeter of a square with side length 9 cm.

Solution: a) Let the side length of the square be 'q'.

Perimeter = 4 × side length.

The expression for the perimeter is 4q.

b) Given side length (q) = 9 cm.

Substitute q = 9 into the expression: Perimeter = 4 × 9 = 36 cm.

Knowledge Checkpoint

- What is a variable? Give an example.
- Translate "The product of 8 and a number 'z', decreased by 5" into an algebraic expression.
- Evaluate the expression 4p + 10 when p = 3.

Activity

Algebraic Expression Builder

Objective: To understand how to form algebraic expressions from verbal statements and evaluate them.

Materials: Index cards or small slips of paper, markers.

Procedure:

1. Divide the class into small groups (3-4 students).

2. Each group prepares two sets of cards:
    - **Set A (Verbal Statements):** Write down 5-7 different verbal statements that can be translated into algebraic expressions (e.g., "A number increased by 10," "Twice a number decreased by 3," "The sum of two different numbers," "Half of a number").
    - **Set B (Numerical Values):** Write down 5-7 different numerical values for variables (e.g., "x = 5," "y = 12," "a = -3," "b = 0.5").
3. **Round 1:** Translation: Groups exchange Set A cards. Each group picks a card and writes the corresponding algebraic expression. Discuss and verify answers as a class.
4. **Round 2:** Evaluation: Groups exchange Set B cards. Each group picks an algebraic expression (from Round 1 or provided by the teacher, e.g., 2m + 7) and a numerical value card. They substitute the value into the expression and calculate the result. Discuss and verify answers.
    - Inquiry-based questions:
    - Did everyone get the same expression for a given statement? Why or why not?
    - What happens if the order of words changes in a statement (e.g., "5 less than x" vs. "x less than 5")?
    - Why is it important to follow the order of operations when evaluating expressions?

Key Terms

- **Letter-number (Variable):** A symbol, usually a letter, representing an unknown or changing numerical value.
- **Algebraic Expression:** A combination of variables, numbers, and arithmetic operations.
- **Constant:** A fixed numerical value in an expression.
- **Coefficient:** The numerical factor multiplying a variable (e.g., in 5x, 5 is the coefficient).
- **Evaluate:** To find the numerical value of an expression by substituting given numbers for variables.

Fact Flash

- The word "**algebra**" comes from the Arabic word "**al-jabr**," meaning "**reunion of broken parts**" or "**bone-setting**."
- Did you know that the equals sign (=) was invented by Robert Recorde in 1557? He said, "**no two things can be more equal**" than two parallel lines of the same length. Before that, mathematicians wrote out "**is equal to**" in words!
- Variables aren't always '**x**' or '**y**'. Sometimes mathematicians use Greek letters like alpha (α) or Beta (β) as variables!

Do It Yourself

- Can an algebraic expression have no variables? If yes, what would it look like?
- Is 2x + 3 the same as 3 + 2x? Why or why not? What about 2x - 3 and 3 - 2x?
- If you have 'n' friends and each friend gives you 'k' candies, how would you write an expression for the total candies you receive? What if you then eat 5 candies?

Mental Mathematics

- What is 5 more than 'x' if x = 10?
- If a pen costs ₹8, how much do 'p' pens cost if p = 4?
- Evaluate 2y + 1 when y = 6.
- If you have 'a' apples and give away 2, how many are left if a = 7?

Gap Analyzer™ Homework

Watch Remedial

Exercise 4.1

1. **Write an algebraic expression for each of the following statements:**

   a) A number 'n' increased by 10.
   b) The product of 'x' and 'y'.
   c) 7 subtracted from the number 'p'.
   d) The number 'z' multiplied by itself.
   e) One-fourth of the sum of numbers 'a' and 'b'.
   f) 9 taken away from twice the number 'q'.

2. **Word Problem questions:**

   a) A bus travels at a speed of 'v' km/h. How far will it travel in 5 hours?
   b) A fruit seller sells apples for ₹25 each and oranges for ₹18 each. If a customer buys 'a' apples and 'o' oranges, write an expression for the total cost.
   c) You have 'm' marbles. Your friend gives you twice the number of marbles you already have. Write an expression for the total number of marbles you now have.
   d) The temperature in a city was 'T' degrees Celsius. It dropped by 4 degrees in the evening. What is the new temperature?

3. **Estimation Questions (Real-life Estimation)**

   a) Estimate the value of 3x + 10 if x is approximately 7.
   b) If a movie ticket costs '**t**' rupees, and popcorn costs '**p**' rupees, estimate the total cost for 4 tickets and 2 popcorns if t is around 200 and p is around 150.
   c) A car travels '**d**' km in '**h**' hours. Estimate its average speed (d/h) if d is about 300 km and h is about 5 hours.
   d) If a person earns 'E' rupees per day and spends 'S' rupees per day, estimate their savings in 30 days (30 × (E - S)) if E is around 800 and S is around 500.

4. **What is the total amount Meena has, if she has the following numbers of coins of ₹10, ₹5 and ₹2? Complete the table:**

No. of ₹10 coins	No. of ₹5 coins	No. of ₹2 coins	Expression and total amount
4	3	6	4 × 10 + 3 × 5 + 6 × 2 = 68
7	y	5	
p	q	8	

5. **A magician says: "Think of a number, multiply it by 6, add 5, then double the result."**

   (i) Write the expression step by step.
   (ii) Write the final simplified expression without the × sign.

Revisiting Arithmetic Expressions and Omission of Multiplication Symbol

Before we dive deeper into algebraic expressions, let's quickly refresh our memory on how we handle arithmetic expressions, especially with brackets and the order of operations. This foundation is crucial because the same rules apply when variables are involved. We'll also learn a common shorthand in algebra: how the multiplication symbol often disappears, making expressions even more compact. This makes reading and writing algebraic expressions much faster!

Sub-concepts to be covered

1. Order of Operations (BODMAS/PEMDAS)
2. Use of Brackets
3. Distributive Property (revisited)
4. Standard Notation (omitting multiplication symbol)
5. Evaluating Expressions with Omitted Symbols

Order of Operations (BODMAS/PEMDAS)

BODMAS (Brackets, Orders/Of, Division, Multiplication, Addition, Subtraction) or PEMDAS (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) is the rule that dictates the sequence in which mathematical operations should be performed in an expression to ensure a unique result.

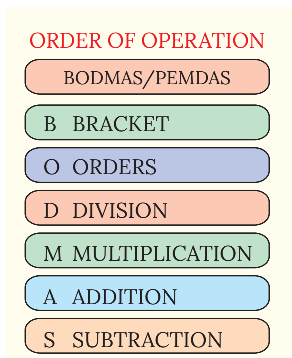

Fig. 4.4

Explanation: When an expression has multiple operations, we need a standard order to solve it. For example, in $5 + 2 \times 3$, if you add first, you get $7 \times 3 = 21$. If you multiply first, you get $5 + 6 = 11$. BODMAS tells us to multiply first.

Examples:

a) $23 - 10 \times 2 = 23 - 20 = 3$ (Multiplication before Subtraction)

b) $15 + 4 \times (8 - 3) = 15 + 4 \times 5 = 15 + 20 = 35$ (Brackets first, then Multiplication, then Addition)

Key points to remember

- Operations within brackets are always performed first.
- Multiplication and Division have equal priority (from left to right).
- Addition and Subtraction have equal priority (from left to right).

Common errors to avoid: Not following the order, especially with multiplication/division and addition/subtraction.

Use of Brackets

Brackets (parentheses, square brackets, curly braces) are used to group parts of an expression, indicating that the operations within them should be performed first.

Explanation: Brackets change the order of operations. For example, $5 \times (2 + 3)$ means 5 times the sum of 2 and 3, which is $5 \times 5 = 25$. Without brackets, $5 \times 2 + 3$ would be $10 + 3 = 13$.

Examples:

a) $68 - (18 + 13) = 68 - 31 = 37$

b) When a negative sign is outside a bracket: $68 - (18 + 13) = 68 - 18 - 13 = 50 - 13 = 37$.

c) (The negative sign applies to all terms inside the bracket).

Key points to remember

- Operations inside brackets are prioritized.
- A negative sign outside a bracket changes the sign of every term inside when the bracket is removed.

Common errors to avoid: Not distributing the negative sign correctly when removing brackets.

Distributive Property (revisited)

The distributive property states that multiplying a sum (or difference) by a number is the same as multiplying each addend (or subtrahend) by the number and then adding (or subtracting) the products.

For numbers: $a \times (b + c) = a \times b + a \times c$.

Explanation: This property is fundamental for simplifying expressions. It allows us to "**distribute**" a multiplication over addition or subtraction.

Examples:

a) $7 \times (4 + 6) = 7 \times 4 + 7 \times 6 = 28 + 42 = 70$. (Also, $7 \times 10 = 70$)

b) In algebra: $5(x + 2) = 5x + 5 \times 2 = 5x + 10$.

Key points to remember:

- Applies to both addition and subtraction inside the bracket.
- Crucial for expanding and simplifying algebraic expressions.

Common errors to avoid: Only multiplying the first term inside the bracket (e.g., $5(x + 2) = 5x + 2$, which is incorrect).

Standard Notation (Omission of Multiplication Symbol)

In algebraic expressions, the multiplication symbol (×) is often omitted between a number and a variable, or between two variables.

Explanation: This is a convention to make expressions more compact and easier to read.

For examples: a) $4 \times n$ is written as $4n$. $a \times b$ is written as ab. The number written before the variable is called the coefficient.

b) $7 \times k$ is written as $7k$.

c) $p \times q$ is written as pq.

d) $3 \times x \times y$ is written as $3xy$.

Key points to remember:

- The number (coefficient) always comes before the variable(s).
- The multiplication symbol is not omitted between two numbers (e.g., 2×3 is not written as 23).

Common errors to avoid: Misinterpreting 23 as 2×3 (it's twenty-three).

Evaluating Expressions with Omitted Symbols

When evaluating expressions where the multiplication symbol is omitted, remember that it implies multiplication.

Explanation: If you have 7k and k = 4, you must remember it means 7 multiplied by 4.

Example: Evaluate 7k when k = 4.
Solution: 7 × 4 = 28.
Evaluate 5m + 3 when m = 2. Solution: 5 × 2 + 3 = 10 + 3 = 13.

Key points to remember:

- Always re-insert the multiplication symbol mentally or explicitly before performing the calculation.
- Follow BODMAS/PEMDAS.

Common errors to avoid: Treating 5m as 5 + m or 5 - m.

Mathematical Explanation

The foundation of algebra rests on a solid understanding of arithmetic operations and their properties. When we move from numerical expressions to algebraic expressions, the rules for evaluating and simplifying remain consistent.

Order of Operations: The universally accepted order of operations (BODMAS/PEMDAS) ensures that any mathematical expression yields a unique result. This hierarchy is crucial:

1. **Brackets (or Parentheses):** Operations enclosed within any type of bracket are performed first.
2. **Orders (or Exponents):** Powers and roots are evaluated next. (Less common in Grade 7, but good to know).
3. **Division and Multiplication:** These are performed from left to right. They have equal priority.
4. **Addition and Subtraction:** These are performed from left to right. They also have equal priority.

Applying this order consistently is vital, whether the expression contains only numbers or a mix of numbers and variables. For instance, 2x + 3y means (2x) + (3y), not 2 (x + 3) y.

Brackets and Negative Signs: Brackets serve as grouping symbols. When an expression contains brackets, the operations inside them are prioritized. A common point of error arises when a negative sign precedes a bracket. The negative sign must be distributed to every term inside the bracket. For example, A - (B + C) becomes A - B - C, not A - B + C. This is equivalent to multiplying the entire bracket by -1: -1 (B + C) = -1 B + -1 C = -B - C. This property is an extension of the distributive property.

Omission of Multiplication Symbol: To simplify notation, algebra adopts conventions. One significant convention is the omission of the multiplication symbol (×) between a number and a variable, or between two variables.

- 5 × x is written as 5x. Here, 5 is the coefficient of x.
- a × b is written as ab.
- 2 × p × q is written as 2pq.

This shorthand is universally understood to imply multiplication. However, it's crucial to remember that this omission does not apply between two numbers (e.g., 2 × 3 cannot be written as 23, which represents the number twenty-three). When evaluating such expressions, one must mentally re-insert the multiplication sign. For example, if 5x and x = 4, then 5x = 5 × 4 = 20. This convention makes algebraic expressions more compact and efficient for writing and reading complex mathematical relationships.

Examples 5 : Evaluate the following arithmetic expressions:

a) $35 - 12 \times 2$ b) $90 + 30 - 15 + 35$ c) $50 - (20 + 10)$ d) $8 \times 5 + 10 \times 7$

Solution: **a) $35 - 12 \times 2$**

$= 35 - 24$ (Multiplication first)

$= 11$ (Subtraction)

b) $90 + 30 - 15 + 35$

$= 120 - 15 + 35$ (Addition/Subtraction from left to right)

$= 105 + 35$

$= 140$

c) $50 - (20 + 10)$

$= 50 - 30$ (Brackets first)

$= 20$ (Subtraction)

Alternatively, distributing the negative sign:

$50 - (20 + 10) = 50 - 20 - 10 = 30 - 10 = 20$

d) $8 \times 5 + 10 \times 7$

$= 40 + 70$ (Multiplication first)

$= 110$ (Addition)

Examples 6 :

a) Find the value of the expression 9k when k = 5.

b) Find the value that the expression 6m + 4 takes when m = 3.

c) Evaluate 2xy when x = 5 and y = 2.

Solution: **a) Expression: 9k**

Substitute k = 5:

$9 \times 5 = 45$

b) Expression: 6m + 4

Substitute m = 3:

$= 6 \times 3 + 4$

$= 18 + 4$ (Multiplication first)

$= 22$ (Addition)

c) Expression: 2xy

Substitute x = 5 and y = 2:

$= 2 \times 5 \times 2$

$= 10 \times 2 = 20$

Examples 7 : Observe each of the following and identify if there is a mistake. If so, explain and correct it.

1. If a = -5, then 10 - a = 5.

Mistake: When substituting a negative number, the two negative signs become positive.

Correction: $10 - (-5) = 10 + 5 = 15$.

2. If d = 7, then 3d = 37.

Mistake: 3d means 3 multiplied by d, not 3 and 7 together.

Correction: $3d = 3 \times 7 = 21$.

3. If s = 8, then 3s - 2 = 22.

Mistake: Calculation error.

Correction: $3s - 2 = 3 \times 8 - 2 = 24 - 2 = 22$. (This one was correct, good for practice)

4. If r = 9, then 2r + 1 = 19.

Mistake: Calculation error.

Correction: $2r + 1 = 2 \times 9 + 1 = 18 + 1 = 19$. (This one was correct, good for practice)

5. If m = -7, then 3(m + 1) = 18.

Mistake: Calculation error, specifically with negative numbers inside the bracket.

Correction: $3(m + 1) = 3(-7 + 1) = 3(-6) = -18$.

6. If t = 5, b = 4, then 2t + b = 14.

Mistake: Calculation error.

Correction: $2t + b = (2 \times 5) + 4 = 10 + 4 = 14$. (This one was correct, good for practice)

7. If h = 6, n = 7, then h - (3 - n) = 4.

Mistake: Calculation error, specifically with negative numbers and distributing the negative sign.

Correction: $h - (3 - n) = 6 - (3 - 7) = 6 - (-4) = 6 + 4 = 10$.

Knowledge Checkpoint

- Evaluate $40 - 5 \times 6$.
- What is the value of $3(x - 2)$ when $x = 7$?
- Explain why $2a$ is not the same as $2 + a$.

Activity

Distributive Property Discovery

Objective: To visually understand and apply the distributive property.

Materials: Graph paper, colored pencils.

Procedure:

1. Draw a rectangle with length 5 units and width (3 + 2) units on graph paper.
2. Calculate its area: $5 \times (3 + 2)$.
3. Now, divide the rectangle into two smaller rectangles: one with dimensions 5×3 and another with 5×2.
4. Calculate the area of each smaller rectangle and add them: $(5 \times 3) + (5 \times 2)$.
5. Compare the total areas.
6. Repeat with an algebraic example: Draw a rectangle with length 'x' and width (4 + 3). Show how its area can be $x(4 + 3)$ or $4x + 3x$.

Inquiry-based questions:

a) What did you observe about the total area in both methods?
b) How does this activity help you understand why $a(b + c) = ab + ac$?
c) Can you think of a real-life scenario where you might use the distributive property?

Key Terms

- **Distributive Property:** A property linking multiplication and addition/subtraction, allowing a factor to be distributed over terms inside a bracket.
- **Coefficient:** The numerical factor that multiplies a variable (e.g., in 7k, 7 is the coefficient).

Fact Flash

- The symbol for division (÷) is called an **obelus**.
- The word "**minus**" comes from the Latin word "**minor**," meaning "**less.**"
- Some mathematicians argue that the order of operations is a convention, not a fundamental law, but it's a very useful convention for clear communication!

Do It Yourself

- If you see x/y + z, does it mean (x/y) + z or x/(y + z)? How can you make it clear?
- Why do we write 2x instead of x2? Is there a mathematical reason or just a convention?

Mental Mathematics

- Evaluate 10 + 3 × 2.
- What is 4(5 - 2)?
- If a = 5, what is 6a?
- Calculate 20 - (8 + 2).
- What is 3x + 5 if x = 1?

Gap Analyzer™ Homework

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Exercise 4.2

1. **Simplify the following expressions:**

   a) $18 \div 6 + 4 \times (10 - 7)$

   b) $5^2 - 3 \times (12 + (-4))$

   c) $\left(\frac{2}{3}+\frac{1}{6}\right) \div \left(\frac{5}{12}-\frac{1}{12}\right)$

   d) $25 - [15 + \{8 \times (6 - 4) \div 2\}]$

   e) $1.5 \times 4 + (7.2 \div 0.8) - 3.5$

   f) $(-10) + 15 \div (-3) \times 2 - (-5)$

   g) $\left(\frac{3}{4}-\frac{1}{2}\right) \times \left(\frac{8}{3}+\frac{5}{6}\right)$

   h) $7 \times 3^2 - (40 \div 8 + 12)$

   i) $100 - [25 - \{10 - (5 - 2.5)\} + 15]$

   j) $(6.25 + 3.75) \div 2.5 \times 1.2$

2. You are decorating for a birthday party. You need 5 balloons for each guest who attends. You also have a packet of 20 spare balloons.

   a) If g guests attend the party, write an expression for the total number of balloons you will need.

   b) If the cost of one balloon is b rupees, write an expression for the total cost of all the balloons needed for g guests (not including the spare ones).

3. Complete the table by writing the arithmetic expressions (with multiplication sign omitted) and their simplified values.

Statement	Expression (with ×)	Expression (without ×)	Simplified Value
5 multiplied by a number x	5 × x	5x	___________
Twice the sum of a and b	2 × (a + b)	2(a + b)	___________
Product of 7 and the square of m	7 × m × m	$7m^2$	___________
Half of the product of p and q	½ × (p × q)	(pq)/2	___________
3 multiplied by (x + y + z)	3 × (x + y + z)	3(x + y + z)	___________

4. **Word Problems (Real-life Context)**
   a) A student buys 'p' pencils at ₹7 each and 'e' erasers at ₹3 each. Write an expression for the total cost. Then, calculate the total cost if p = 10 and e = 5.
   b) A car travels for 't' hours at a speed of 's' km/h. It then stops for 1 hour. Write an expression for the total time elapsed. If s = 60 and t = 4, what is the total time?
   c) A rectangular garden has length 'l' meters and width 'w' meters. Write an expression for its perimeter. If l = 15 and w = 8, find the perimeter.
   d) A shop offers a discount of ₹10 on every item. If an item originally costs 'c' rupees, write an expression for its discounted price. If c = 150, what is the discounted price?
   e) A factory produces 'x' toys per hour. It works for 8 hours a day. Write an expression for the total toys produced in 'd' days. If x = 50 and d = 5, how many toys are produced?
5. **Estimation Questions (Real-life Estimation)**
   a) Estimate the value of 2(a + b) if a is about 10 and b is about 15.
   b) A recipe calls for 'c' cups of sugar. If you want to make half the recipe, estimate the sugar needed (c/2) if c is around 3.5 cups.
   c) Estimate the total score (S + 2B) if 'S' is your score in a test (around 70) and 'B' is bonus points (around 15).
   d) Estimate the cost of 'n' items at ₹49 each (50n) if n is around 7.
6. **Miscellaneous Questions**
   a) Evaluate the following expressions:
      i) 45 − 15 ÷ 3 ii) 18 + 7 × (12 − 5) iii) 75 − (25 + 15)
   b) Find the value of the expression 8p - 5 when p = -2.
   c) Are the expressions 6u and 6 + u equal to each other? Justify your answer by evaluating them for u = 1 and u = 5.

Simplification of Algebraic Expressions

Just like you can simplify arithmetic expressions (e.g., 5 + 3 + 7 = 15), you can also simplify algebraic expressions. This involves combining terms that are "**alike**." Imagine you have 3 apples and 2 bananas, and then you get 4 more apples and 1 more banana. You wouldn't say you have "**3 apples + 2 bananas + 4 apples + 1 banana**." Instead, you'd combine them to say "**7 apples + 3 bananas**." In algebra, we learn to do this with variables, identifying like terms and combining them to make expressions shorter and easier to work with.

Sub-concepts to be covered

1. Understanding Like and Unlike Terms
2. Combining Like Terms (Addition, Subtraction)
3. Using Distributive Property for Simplification
4. Simplifying Expressions with Brackets (Positive and Negative Signs)

Understanding Like and Unlike Terms

Like terms: Terms that have the same variables raised to the same powers. Only their numerical coefficients can be different.

Unlike terms: Terms that have different variables or the same variables with different powers.

Explanation: Think of terms as categories. '**Apples**' and '**bananas**' are different categories. Similarly, '**x**' and '**y**' are different categories. '**x**' and 'x^2' are also different categories. You can only combine items within the same category.

Examples:

a) **Like terms:** 5x, 3x, -10x (all have '**x**' to the power of 1)
b) **Like terms:** 2ab, -7ab, ab (all have '**ab**')
c) **Unlike terms:** 5x, 3y (different variables)
Unlike terms: 5x, $5x^2$ (same variable, different powers)
Unlike terms: 28c, 21d

Key points to remember

- The variable part (and its power) must be identical for terms to be like terms.
- Constants (e.g., 5, -12) are considered like terms with each other.

Common errors to avoid: Mistaking x and x^2 as like terms.

Combining Like Terms (Addition, Subtraction)

Combining like terms means adding or subtracting their numerical coefficients while keeping the variable part the same.

Explanation: This is based on the distributive property. For example, 5x + 3x can be thought of as (5 + 3) x = 8x. You're essentially counting how many '**x**'s you have in total.

Examples:

a) 5c + 3c + 10c = (5 + 3 + 10)c = 18c
b) 12n - 4n = (12 - 4)n = 8n
c) 7p + 8p + 6p = (7 + 8 + 6)p = 21p
d) -3q - 4q - 2q = (-3 - 4 - 2)q = -9q

Key points to remember

- Only like terms can be combined.
- The variable part does not change during addition/subtraction.
- Pay attention to the signs of the coefficients.

Common errors to avoid: Changing the variable part (e.g., $x + x = x^2$, which is incorrect; x + x = 2x).

Using Distributive Property for Simplification

The distributive property a(b + c) = ab + ac is used to remove brackets in algebraic expressions, which often creates like terms that can then be combined.

Explanation: When a number or variable is multiplied by an expression in a bracket, it must be multiplied by each term inside the bracket. This step is often necessary before combining like terms.

Examples:

a) $4(x + y) = 4x + 4y$

b) $3(2a - 5b) = 3 \times 2a - 3 \times 5b = 6a - 15b$

c) $p(q + r) = pq + pr$

Key points to remember

- Distribute the term outside the bracket to all terms inside.
- Be careful with signs when multiplying.

Common errors to avoid: Forgetting to multiply the second (or third) term inside the bracket.

Simplifying Expressions with Brackets (Positive and Negative Signs)

When simplifying expressions involving multiple brackets, especially with positive or negative signs outside them, we first remove the brackets by applying the distributive property (if a multiplier is present) and then combine like terms.

Explanation: If a + sign is outside a bracket, the terms inside retain their original signs when the bracket is removed. $A + (B + C) = A + B + C$.

- If a - sign is outside a bracket, the sign of each term inside the bracket changes when the bracket is removed. $A - (B + C) = A - B - C$.

Examples:

a) $(40x + 75y) - (6x + 10y)$

$= 40x + 75y - 6x - 10y$ (distribute negative sign)

$= (40x - 6x) + (75y - 10y)$ (group like terms)

$= 34x + 65y$

b) $7p - 3q + 8p - 4q + 6p - 2q$ (after removing brackets)

$= (7p + 8p + 6p) + (-3q - 4q - 2q)$ (group like terms)

$= 21p - 9q$

Key points to remember:

- Always handle brackets first.
- Carefully apply the sign rules when removing brackets.
- Then, identify and combine like terms.

Common errors to avoid: Sign errors when distributing a negative sign.

Mathematical Explanation

Simplification of algebraic expressions is a core skill in algebra, analogous to reducing a fraction to its lowest terms. The goal is to rewrite an expression in its most compact and readable form without changing its value. This process primarily involves identifying and combining "like terms."

Like and Unlike Terms: The concept of like terms is fundamental. Terms are "like" if they have the exact same variable part, including the same exponents for each variable.

For example, $5x$, $-2x$, and $(1/2)x$ are like terms because they all contain x raised to the power of 1.

However, $5x$ and $5x^2$ are unlike terms because the power of x is different. Similarly, 3ab and 7ba are like terms (due to commutativity of multiplication, ab is the same as ba), but 3ab and 3ac are unlike terms because their variable parts (ab vs. ac) are different. Constants (numbers without variables) are also considered like terms among themselves.

Like Terms	Unlike Terms
$2x + 19x$	$2x + 19a$
$4w - 10w$	$4w - 10w^2$
$14.2r - 12r$	$12r - 12s$
$32a^2 + 9a^2$	$32a^2 + 9a^3$
$8y + 5y$	$8y + 5$

Combining Like Terms: The process of combining like terms is an application of the distributive property. When we have $5x + 3x$, we can factor out the common variable x to get $(5 + 3)x$, which simplifies to $8x$. This means we simply add or subtract the numerical coefficients of the like terms, keeping the variable part unchanged.

- $7y - 4y = (7 - 4)y = 3y$
- $-2p + 9p = (-2 + 9)p = 7p$
- $6a + 2b - 3a = (6a - 3a) + 2b = 3a + 2b$
  (Here, 2b is an unlike term and cannot be combined further).

A collection of different colored blocks (fig. 4.5). Some blocks are labeled '**x**', others '**y**', and some '**x²**'. This visually demonstrates like and unlike terms (e.g., you can combine all '**x**' blocks, but not '**x**' and '**y**' blocks).

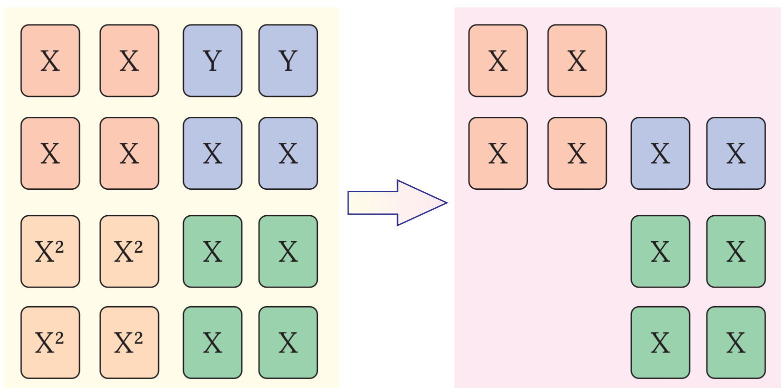

Fig. 4.5

Combining Like Terms

Simplifying Expressions with Brackets: Brackets indicate that the enclosed operations should be treated as a single entity. When simplifying expressions that contain brackets, the first step is usually to remove the brackets.

Distributive Property: If a number or variable is multiplying a bracket, the distributive property is applied: $k(a + b) = ka + kb$. For example, $4(x + y) = 4x + 4y$.

Signs Preceding Brackets:

a) If a + sign precedes a bracket, the terms inside retain their original signs when the bracket is removed: $A + (B - C) = A + B - C$.

b) If a - sign precedes a bracket, the sign of every term inside the bracket changes when the bracket is removed: $A - (B - C) = A - B + C$. This is equivalent to multiplying each term inside the bracket by -1.

After removing all brackets, the final step in simplification is to identify and combine any remaininglike terms. This systematic approach ensures that expressions are reduced to their simplest, most manageable form.

Example 8 : The perimeter of a rectangle is l + b + l + b. Simplify this expression.

Solution: Perimeter = l + b + l + b

Group like terms: = (l + l) + (b + b)

Combine like terms: = 2l + 2b

The simplified expression for the perimeter is 2l + 2b.

Example 9 : A bookshop sells pencils (price 'c') and erasers (price 'd').

Day 1: 5 pencils, 4 erasers

Day 2: 3 pencils, 6 erasers

Day 3: 10 pencils, 1 eraser

Write and simplify the expression for the total money earned.

Solution: Money from pencils:

Day 1: 5c, **Day 2:** 3c, **Day 3:** 10c

Total money from pencils = 5c + 3c + 10c = (5 + 3 + 10)c = 18c

Money from erasers:

Day 1: 4d, **Day 2:** 6d, **Day 3:** 1d (or just d)

Total money from erasers = 4d + 6d + 1d = (4 + 6 + 1)d = 11d

Total money earned = Total money from pencils + Total money from erasers

Total money earned = 18c + 11d.

This expression cannot be simplified further as 18c and 11d are unlike terms.

Example 10 : A large rectangle is split into two smaller rectangles. Their areas are 5v sq. units and 3v sq. units. Write an expression for the area of the bigger rectangle.

Solution: Area of bigger rectangle = Area of first smaller rectangle + Area of second smaller rectangle

Area = 5v + 3v

Combine like terms: Area = (5 + 3)v = 8v sq. units.

Example 11 : Simplify the expression: (50x + 80y) − (8x + 15y)

Solution: (50x + 80y) − (8x + 15y)

Remove brackets, distributing the negative sign:

= 50x + 80y − 8x − 15y

Group like terms: (50x − 8x) + (80y − 15y)

Combine like terms: (50 − 8)x + (80 − 15)y

= 42x + 65y

Example 12 : Simplify the expression: 5(a + b) − 2b

Solution: 5(a + b) − 2b

Apply distributive property to remove the bracket:

= 5a + 5b − 2b

Group like terms: 5a + (5b − 2b)

Combine like terms: 5a + (5 − 2)b = 5a + 3b

Knowledge Checkpoint

- Are 6p and $6p^2$ like terms? Explain.
- Simplify: 10a + 4b - 3a + b.
- Remove the brackets and simplify: 5(x - y) + 2x.

Activity

Expression Simplification Race

Objective: To practice simplifying algebraic expressions efficiently.

Materials: Whiteboard or large paper, markers, pre-written expressions of varying difficulty.

Procedure:

1. Divide the class into two teams.
2. Write an expression on the board.
3. The first student from each team rushes to the board to simplify it. The first one to correctly simplify gets a point.
4. Include expressions with brackets, negative signs, and multiple variables.

Inquiry-based questions:

- What was the first step you took to simplify this expression?
- Did you encounter any sign errors? How did you fix them?
- What strategies did you use to quickly identify like terms?

Key Terms

- **Like Terms:** Terms with the same variables raised to the same powers.
- **Unlike Terms:** Terms with different variables or variables with different powers.
- **Simplification**: The process of rewriting an expression in a more compact form by combining like terms and removing unnecessary brackets.

Fact Flash

- The idea of combining like terms is very intuitive and was used even before formal algebra existed, just not with letters!
- Some mathematicians call variables "**indeterminates**" because their value is not determined.
- The concept of "**terms**" in algebra is similar to "**word**s" in a sentence – they are the building blocks of expressions.

Do It Yourself

- Can you always simplify an algebraic expression? When is it not possible?
- If you have x + y + z, can you combine any of these terms? Why or why not?
- How is simplifying an algebraic expression similar to simplifying a fraction? How is it different?

Mental Mathematics

- Simplify: 5x + 3x.
- Remove brackets: 2(a + 4).
- Simplify: 10 - (x + 3).
- Simplify: 7y - 2y.
- Simplify: 4p + q - p.

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Exercise 4.3

1. **For each of the following expressions, identify and list the Like Terms and Unlike Terms separately:**

   a) 7x + 5y - 3x + 2
   b) -4a + 9b - a - 6b
   c) 15m - 8n + 2m + 10n - 5
   d) p + 3q - 7p + q + 11
   e) 6xy + 4x - 2y + xy - 7
   f) -10z + 12 - 3z - 9 + 5z
   g) 9x - 7y + 4z - x + 3y - 2z
   h) c - 8d + 14 - 3c + 5d + 2
   i) 2ab + 5a - 3b - ab + 8a
   j) $\frac{1}{2}h + \frac{3}{4}k - \frac{1}{4}h + \frac{1}{2}k$

2. **Word Problems (Real-life Context):**

   a) A shopkeeper sold 'x' red pens and 'y' blue pens on Monday. On Tuesday, he sold '3x' red pens and '2y' blue pens. Write an expression for the total number of red pens and blue pens sold over two days. Simplify it.

   b) A farmer has a rectangular field with length 'L' meters and width 'W' meters. He decides to extend the length by 5 meters and the width by 2 meters. Write an expression for the new perimeter and simplify it.

   c) Your current savings are 'S' rupees. You earn 'E' rupees and spend 'P' rupees. Write an expression for your new savings. If you repeat this for 3 days, write and simplify an expression for your total savings.

   d) A student scores 'a' marks in English, 'b' marks in Hindi, and 'c' marks in Math in the first term. In the second term, their marks are 'a+5' in English, 'b-2' in Hindi, and 'c +10' in Math. Write and simplify an expression for their total marks in both terms combined.

   e) A box contains 'r' red balls and 'g' green balls. You add 2 more boxes of the same type. Then you remove 5 red balls and 3 green balls from the total. Write and simplify an expression for the number of red and green balls remaining.

3. **Simplify each of the following expressions:**

   i) p + p + p + q   ii) p + q + p - q   iii) 2d - d - d - c   iv) 2d - d - (d - c)

4. **Forming and Simplify Algebraic Expressions:**
   - A rectangle has length (2x + 3) cm and breadth (x + 5) cm.
   - The cost of 1 notebook is (5x + 2) rupees.
   - The perimeter of a triangle with sides (x + 4) cm, (2x + 1) cm and (3x − 2) cm is obtained by adding the sides.
   - The total marks of a student in two tests are (45 + x) and (50 + 2x).

   Questions

   i. Simplify the perimeter of the triangle with sides (x + 4), (2x + 1), (3x − 2).

   ii. A rectangle has length (2x + 3) and breadth (x + 5).

(a) Write the expression for its perimeter. (b) Simplify it.

iii. The cost of 1 notebook is (5x + 2). Find the cost of 3 notebooks. Simplify your answer.

iv. A student scores (45 + x) marks in the first test and (50 + 2x) in the second.

(a) Write the expression for the total marks. (b) Simplify it.

5. Let's say your current age is y years.

a) Write an expression for your age after 5 years.

b) Your father is three times your current age. Write an expression for your father's age.

c) Your younger sister is 4 years younger than you. Write an expression for her age.

d) Write an expression for the sum of your age, your father's age, and your sister's age.

Pick Patterns and Reveal Relationships

Mathematics is often called the science of patterns. From the way numbers grow in a sequence to the arrangement of shapes, patterns are everywhere. Algebraic expressions are incredibly powerful tools for describing these patterns in a general way. Instead of just saying "**the next number is 4 more than the previous one**," we can write a formula that tells us any number in the sequence! This section will teach you how to observe patterns, express them using variables, and even use algebra to prove why certain patterns always hold true.

Sub-concepts to be covered

1. Identifying Patterns in Sequences (Number patterns, Geometric patterns)
2. Formulating Rules for Patterns using Algebraic Expressions
3. "Number Machine" Problems (Input-Output Rules)
4. Calendar Patterns (2 × 2 squares, other shapes)
5. Matchstick Patterns
6. Verifying Patterns using Algebra

Identifying Patterns in Sequences (Number patterns, Geometric patterns)

A sequence is an ordered list of numbers or objects that follow a specific rule or pattern.

Explanation: We look for how each term relates to the previous one, or how each term relates to its position in the sequence. This could be adding a constant number, multiplying by a constant number, or a more complex relationship.

Examples: **a) Number sequence:** 4, 8, 12, 16, ... (adding 4 each time, or multiples of 4)

b) Geometric pattern: Matchstick figures where the number of sticks increases by a fixed amount for each step.

Key points to remember

- Look for the difference between consecutive terms.
- Look for the ratio between consecutive terms.
- Relate the term to its position (1st, 2nd, 3rd, ...).

Common errors to avoid: Jumping to conclusions about a pattern based on only a few terms; not checking the rule for all given terms

Formulating Rules for Patterns using Algebraic Expressions

Once a pattern is identified, an algebraic expression (often involving 'n' for the nth term or 'step number') is created to describe the rule for any term in the sequence.

Explanation: If a sequence is 4, 8, 12, 16, ... and we see it's multiples of 4, the nth term is $4 \times n$ or $4n$. If a sequence is 3, 5, 7, 9, ... (adding 2 each time, starting from 3), the nth term can be $2n + 1$ (since $2 \times 1 + 1 = 3$, $2 \times 2 + 1 = 5$, etc.).

Example: Matchstick pattern: 3, 5, 7, 9, 11, 13... (Step 1 = 3, Step 2 = 5). Rule: $2n + 1$.

Key points to remember:

- Use a variable (like 'n') to represent the term number or step number.
- Test your formulated rule with several terms from the sequence.

Common errors to avoid: Incorrectly identifying the starting point or the common difference/multiplier.

"Number Machine" Problems (Input-Output Rules)

These problems present a "machine" that takes an input number(s), performs a fixed operation(s), and produces an output number. The task is to find the algebraic expression that describes the machine's rule.

Explanation: You are given several input-output pairs. You need to deduce the mathematical operation(s) that transform the input(s) into the output.

Examples:

a) Input 'a', Output 'a + 5'. Rule: Add 5.
b) Input 'a', 'b'; Output '2a - b'. Rule: Two times the first number minus the second number.

Key points to remember:

- Look for consistent operations across all input-output pairs.
- Test your proposed rule with all given examples.

Common errors to avoid: Finding a rule that works for one pair but not others.

Calendar Patterns (2 × 2 squares, other shapes)

Exploring numerical relationships within a calendar grid, often using algebraic expressions to prove general properties.

Explanation: The calendar has a fixed structure: numbers increase by 1 horizontally and by 7 vertically. This allows us to represent any date in relation to another using variables. For example, if the top-left date in a 2×2 square is '**a**', the dates are:

a	a+1
a+7	a+8

Examples:

a) Diagonal sums in a 2 × 2 square: $(a + (a + 8))$ vs. $((a + 1) + (a + 7))$. Both simplify to $2a + 8$, proving they are always equal.

b) Sum of numbers in a cross shape.

Key points to remember:

- **Horizontal movement:** +1 (right), -1 (left).
- **Vertical movement:** +7 (down), -7 (up).
- **Diagonal movement:** +8 (down-right), +6 (down-left), etc.

Common errors to avoid: Incorrectly adding/subtracting for horizontal/vertical movements.

Matchstick Patterns

Analyzing patterns formed by matchsticks (or other objects) to find a general rule for the number of sticks needed for any given step or figure.

Explanation: These patterns often involve a constant increase in the number of sticks per step, leading to a linear algebraic expression.

Examples:

a) **L-shapes:** 2n sticks for 'n' L-shapes.

b) **Triangles:** 3, 5, 7, 9, ... sticks. Rule: 2n + 1.

Key points to remember

- Count sticks carefully for the first few steps.
- Identify the constant difference between consecutive steps.
- Relate the number of sticks to the step number.

Common errors to avoid:

- Miscounting sticks, or not accounting for overlapping sticks if applicable.
- A sequence of matchstick figures (*fig.* 4.6) (e.g., squares or triangles) with the number of sticks clearly counted for each step.

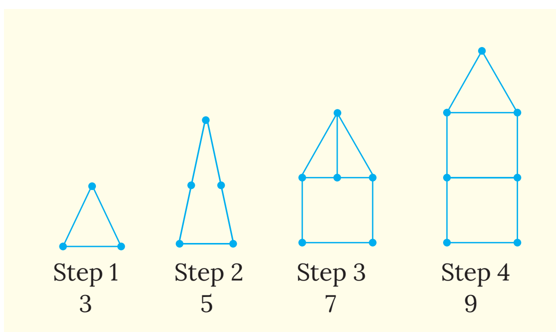


Fig. 4.6

Verifying Patterns using Algebra

Using algebraic expressions and properties to mathematically prove that an observed pattern will hold true for all cases, not just the examples.

Explanation: Instead of just observing that diagonal sums in a 2 × 2 calendar square are equal for a few examples, we use variables to represent the numbers. When the algebraic expressions for the sums simplify to the same form, it's a mathematical proof that the pattern is universal.

Examples:

a) Proving calendar diagonal sums are equal (as above).

b) Proving that the sum of three consecutive integers is always a multiple of 3.

(Let integers be n, n + 1, n + 2. Sum = 3n + 3 = 3 (n + 1), which is a multiple of 3).

Key points to remember:

- Represent the general case using variables.
- Apply algebraic operations (simplification, distributive property).
- Show that the expressions lead to the same result or desired property.

Common errors to avoid: Only showing examples, not providing a general algebraic proof.

Mathematical Explanation

The ability to identify, describe, and generalize patterns is a hallmark of mathematical thinking. Algebraic expressions provide the perfect language for this generalization.

Sequences and Rules: Many patterns appear as sequences of numbers or geometric figures. To describe these, we look for a rule that relates each term to its position in the sequence (often denoted by n, where n = 1 for the first term, n = 2 for the second, and so on).

Arithmetic Sequences: An arithmetic sequence (also called an arithmetic progression) is a list of numbers where the difference between any two consecutive terms is always the same. The general form of the nth term is $a + (n - 1)d$, where a is the first term and d is the common difference.

For example, In the sequence 4, 8, 12, 16, ..., a = 4 and d = 4.

So, the nth term is $4 + (n - 1)4$

$= 4 + 4n - 4 = 4n$.

Geometric Patterns: Geometric patterns are designs or arrangements made by repeating shapes, numbers, or objects in a specific order.

They can be based on:

Shape Patterns

Created by repeating or changing shapes in size, color, or orientation.

Example: ▲▲▲ → ▼▼▼ → ▲▲▲ (triangles flipping).

Number Patterns (Geometric Sequences)

Each term is obtained by multiplying the previous term by the same number (called common ratio r).

Example: 2, 4, 8, 16, 32, ... (here r = 2).

Number Machines and Input-Output Rules: These problems reinforce the concept of a function, where an input is transformed into an output by a specific rule. By analyzing several input-output pairs, we deduce the algebraic expression that represents the machine's operation. This involves reverse-engineering the arithmetic operations.

For example, if input a gives output $2a - b$ when input b is also given, it means the machine multiplies the first input by 2 and then subtracts the second input.

Calendar Patterns: Calendars offer a rich source of numerical patterns due to their fixed grid structure. The key insight is that moving one day to the right adds 1 to the date, and moving one week down adds 7 to the date. This allows us to represent any date in a square or other shape in terms of a single variable (e.g., the top-left date 'a').

By applying algebraic simplification, we can prove properties like the equality of diagonal sums in a 2 × 2 square. If the top-left date is a, the square is:

a	a + 1
a + 7	a + 8

The sum of one diagonal is a + (a + 8) = 2a + 8. The sum of the other diagonal is (a + 1) + (a + 7) = 2a + 8. Since both expressions simplify to the same form, the property holds for any 2×2 square in the calendar. This demonstrates the power of algebra in providing universal proofs, rather than just empirical observations.

A calendar page with a 2 × 2 square highlighted, showing (Fig. 4.7) the dates '**a**', '**a + 1**', '**a + 7**', '**a + 8**'.

6	7	8	9	10
13	14	a	a + 1	17
20	21	a + 7	a + 8	24
27	28	29	30	31
3	4	5	6	7

Fig. 4.7

Note
The correct rule for a 2 × 2 calendar square is 2a + 8, where '**a**' is the smallest number.

Example 13 : Find the formula for the number machine where:

Input (x, y) → Output (6, 2) → 16, (10, 3) → 27, (4, 1) → 11, (12, 5) → 31

Solution:

Let's analyze the relationship between inputs (x, y) and the output.

For (6, 2) → 16: We need to find a combination of operations on 6 and 2 that results in 16.

Try simple operations:

- x + y = 6 + 2 = 8 (No)
- x - y = 6 - 2 = 4 (No)
- x × y = 6 × 2 = 12 (Close, but not 16)

Let's consider multiplying one of the inputs by a constant. What if we multiply '**x**' by something?

- **If we try 2x:** 2 × 6 = 12. To get 16, we need to add 4. Can we get 4 from '**y**'? Yes, 2 × y = 2 × 2 = 4.
  So, 2x + 2y = 12 + 4 = 16. This looks promising!

Let's test this rule (2x + 2y) for other pairs:

For (10, 3): 2 × 10 + 2 × 3 = 20 + 6 = 26. Hold on, the expected output is 27. So, the rule 2x + 2y doesn't work for this pair. This means we need to re-evaluate our hypothesis.

Let's go back to (6, 2) → 16. We tried 2x = 12. To get 16, we need to add 4. What if the operation involves a constant or a different multiple of '**y**'? Consider 3x: 3 × 6 = 18. To get 16, we need to subtract 2. And '**y**' is 2. So, 3x - y = 18 - 2 = 16. This works for the first pair!

Let's test this new rule (3x - y) for the other pairs:

For (10, 3): 3 × 10 − 3 = 30 − 3 = 27. (Works!)

For (4, 1): 3 × 4 − 1 = 12 − 1 = 11. (Works!)

For (12, 5): 3 × 12 − 5 = 36 − 5 = 31. (Works!)

The formula for the number machine is 3x - y.

Example 14 : A traffic light at a busy intersection cycles through colours in a fixed sequence: Red, Yellow, Green, Orange, Red, Yellow, Green, Orange...

a) Write an expression for the position in the sequence where the 'Red' light occurs for the n^{th} time.

b) Which colour light will be displayed at Position 75 in the sequence?

Solution: The colours repeat every 4 lights. Let's list the positions for each colour:

- **Red (R):** Positions 1, 5, 9, ...
- **Yellow (Y):** Positions 2, 6, 10, ...
- **Green (G):** Positions 3, 7, 11, ...
- **Orange (O):** Positions 4, 8, 12, ...

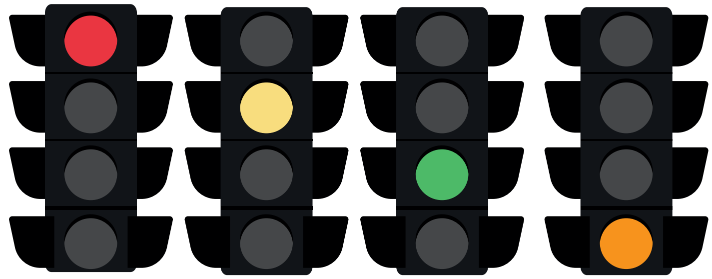


Fig. 4.8

a) For the 'Red' light:

- The positions where 'Red' occurs are 1, 5, 9, ...
- This forms an arithmetic progression.
  The first term (a) = 1.
  The common difference (d) = 4 (because the pattern repeats every 4 colours).
- Using the formula for the nth term of an arithmetic progression: $a_n = a + (n - 1)d$
  $a_n = 1 + (n - 1)4$
  $a_n = 1 + 4n - 4$
  $a_n = 4n - 3$
- So, the 'Red' light occurs at position 4n - 3 for the nth time.

b) To find which colour light appears at Position 75:

- We need to determine where Position 75 falls within the repeating cycle of 4 colours. We can do this by dividing 75 by 4.
- $75 \div 4$
- $75 = 4 \times 18 + 3$
  This means there are 18 full cycles of the 4 colours ($4 \times 18 = 72$ positions).
  After 72 positions, we are at the beginning of the next cycle. The remainder of 3 tells us it's the 3rd position within that new cycle.
- Let's check the colours based on their position in a cycle:
  Position 1 (Remainder 1 when divided by 4): Red
  Position 2 (Remainder 2 when divided by 4): Yellow
  Position 3 (Remainder 3 when divided by 4): Green
  Position 4 (Remainder 0 when divided by 4): Orange
- Since the remainder is 3, the colour at Position 75 is Green.

Example 15 : You are building structures using identical square blocks. Observe the pattern of blocks used in each step:

Step 1: You build a structure using 1 block.
Step 2: You build a structure using 3 blocks.
Step 3: You build a structure using 5 blocks.
How many blocks will be needed for Step 'n'? (b) How many blocks will be needed for Step 40?

Solution:

For Step 'n':

- Observe the pattern of blocks: 1, 3, 5, ...
- The number of blocks increases by 2 for each step.
- This is an arithmetic sequence:
  The first term (a) = 1.

The common difference (d) = 2 (because 3 - 1 = 2, and 5 - 3 = 2).

- Using the formula for the n^{th} term of an arithmetic sequence (which students can derive or be guided to understand as a rule): $a_n = a + (n-1)d$

  $a_n = 1 + (n - 1)2$

  $a_n = 1 + 2n - 2$

  $a_n = 2n - 1$
- So, for Step 'n', the number of blocks needed is 2n - 1.

To find the number of blocks for Step 40:

- Substitute n = 40 into the expression 2n - 1:
- Number of blocks = 2(40) - 1
- Number of blocks = 80 - 1
- Number of blocks = 79
- So, 79 blocks will be needed for Step 40.

Example 16 : Consider a 3 × 3 square in a calendar. If the top-left number in this square is '**n**', prove that: "The sum of the numbers in the middle row is equal to three times the number in the center of the square."

n	n + 1	n + 2
n + 7	n + 8	n + 9
n + 14	n + 15	n + 16

Solution:

1. **Representing the Numbers in the 3×3 Square:**

   Let the top-left number in the 3 × 3 square be '**n**'. Using the calendar properties (horizontal +1, vertical +7), we can represent all the numbers in the square:
2. **Identify the Middle Row and its Numbers:**

   The numbers in the middle row are: (n + 7), (n + 8), and (n + 9)
3. **Calculate the Sum of the Numbers in the Middle Row:**

   Sum of Middle Row = (n + 7) + (n + 8) + (n + 9) = n + 7 + n + 8 + n + 9 = (n + n + n) + (7 + 8 + 9) = 3n + 24
4. **Identify the Number in the Center of the Square:**

   The number in the center of the square is: (n + 8)
5. **Calculate Three Times the Number in the Center:**

Three times the number in the center = 3 × (n + 8) = 3n + 24

Since the "**Sum of the Numbers in the Middle Row**" (3n + 24) is equal to "Three Times the Number in the Center" (3n + 24), the statement is proven. This shows that for any 3x3 square in a calendar, this relationship always holds true.

Knowledge Checkpoint

- What is the 5th term of the sequence 10, 13, 16, 19, ...?
- Write an algebraic expression for the nth term of the sequence 6, 11, 16, 21, ...
- If a number machine takes input 'x' and gives output '3x + 2', what is the output for input 7?

Do It Yourself

Stair Numbers Look at the sequence: 3, 6, 9, 12, ...

- Write the next 5 numbers.
- Find the rule used in this sequence.
- If the 50th number is written, what will it be?

Activity

Calendar Magic

Objective: To explore and prove calendar patterns using algebra.

Materials: Printouts of blank calendar grids (or actual calendars), markers.

Procedure:

1. Students choose any 3 × 3 square on the calendar.
2. Let the center number be 'c'. Write down all 9 numbers in terms of 'c'.
3. Calculate the sum of all 9 numbers.
4. Observe the relationship between the sum and 'c'.
5. Now, use the algebraic expressions for the 9 numbers (in terms of 'c') to prove that this relation ship always holds true.

Inquiry-based questions:

- What is the relationship between the center number and the numbers directly above/below it? To its left/right?
- How did simplifying the algebraic sum help you prove the pattern?
- Can you find other patterns in the calendar (e.g., sum of numbers in a row, sum of numbers in a column)?

Key Terms

- **Sequence:** An ordered list of numbers or objects following a rule.
- **Pattern:** A regular, repeating, or predictable arrangement or sequence.
- **nth term:** The general term in a sequence, expressed using 'n' (the term number).
- **Generalization:** Expressing a specific observation or rule in a broader, universal form using variables.

Fact Flash

- The Fibonacci sequence (1, 1, 2, 3, 5, 8, ...) is a famous pattern found throughout nature, from pinecones to sunflowers!
- Some patterns are so complex that mathematicians are still trying to find rules for them, like the distribution of prime numbers.
- The study of patterns is a huge part of advanced mathematics, leading to fields like **chaos theory** and **fractals**.

Mental Mathematics

- What is the next number in the sequence: 4, 9, 14, 19, ___?
- If the rule is "add 5 and then multiply by 2", what is the output for an input of 7?
- Observe the pairs: (3, 9), (5, 15), (7, 21). If the input is 10, what is the output?
- Fill in the blank: 1, 4, 9, ___, 25 (Hint: Think about types of numbers)
- The sum of two numbers is always 20. If one number is 'x', what is the other number in terms of 'x'?

Gap Analyzer™ Homework

Watch Remedial

Exercise 4.4

1. **Solve the following calendar pattern question:**

   2 × 2 Square Sum:

   - Choose any 2 × 2 square of numbers from the calendar.
   - Add all four numbers in the square.
   - Now, multiply the smallest number in your square by 4, and then add 16 to the result.
   - What do you observe about the two results? Try this with another 2 × 2 square.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

2. You are a secret agent and have found a two-step "Number Machine" used by spies to encode numbers. You must figure out its secret rules.

   Machine A:

   Input (x): 3 → Output (y): 10

   Input (x): 5 → Output (y): 16

   Input (x): 10 → Output (y): 31

   a) The machine performs two operations: first, it multiplies the input x by a secret number, and then it adds another secret number. Can you find the two-step rule? Write it as an algebraic expression for y in terms of x.

   b) What will be the output if the input is x = 7?

   c) What was the input x if the output was y = 25?

3. A snail climbs 'u' cm during the day and slips 'd' cm during the night. Write an expression for its net climb in one day. If this happens for 10 days, write an expression for its total climb.
4. Radha cycles 5 km daily in the first week. Every week she increases the daily distance by 'z' km. How many kilometers would Radha have cycled daily in the 3rd week? Write an expression for the daily distance in the 'w'th week.
5. A baker makes '**c**' cookies on the first day. Each day after that, he makes 5 more cookies than the previous day. Write an expression for the number of cookies he makes on the 'd'th day.
6. A school auditorium has '**r**' rows. The first row has 10 seats. Each subsequent row has 2 more seats than the row before it. Write an expression for the number of seats in the 'n'th row.
7. **Matchstick Patterns**

   A row of houses is built using matchsticks:

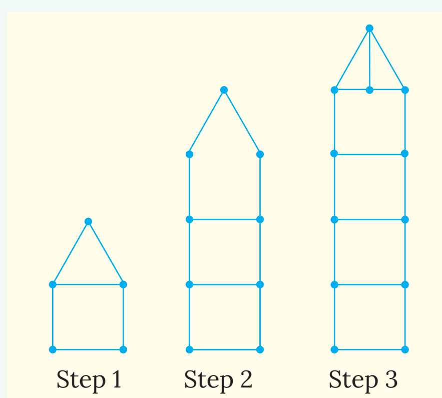
   1^{st} house → 6 matchsticks

   2^{nd} house → 11 matchsticks

   3^{rd} house → 16 matchsticks

   (a) How many matchsticks for the 10th house?

   (b) Write a formula for nth house



Common Misconceptions

Misconception: **5x means 5 + x.**

Correction: In algebra, when a number is written next to a variable without an operation symbol, it implies multiplication. So, 5x means 5 multiplied by x. If $x = 2$, then $5x = 5 \times 2 = 10$, not $5 + 2 = 7$.

Misconception: **$x + x = x^2$.**

Correction: When adding like terms, we add their coefficients and keep the variable part the same. $x + x$ means $1x + 1x$, which is $(1+1)x = 2x$. x^2 means x multiplied by x.

Misconception: **$2(x + 3) = 2x + 3$.**

Correction: This is a common error in applying the distributive property. The number outside thebracket must be multiplied by every term inside the bracket. So, $2(x + 3) = 2 \times x + 2 \times 3 = 2x + 6$.

Misconception: **$10 - (x + 5) = 10 - x + 5$.**

Correction: When a negative sign precedes a bracket, the sign of every term inside the bracket changes when the bracket is removed. So, $10 - (x + 5) = 10 - x - 5$.

Real-Life Algebra: Mathematical Applications

Algebraic expressions, using letter-numbers, are vital for representing real-world relationships. For Grade 7, applications include:

1. **Cost & Budgeting:** Calculating total costs for items (e.g., 30p + 150 for p packets and a fixed cost).
2. **Distance-Speed-Time:** Expressing distance covered (e.g., 10t for 10 km/hr over t hours).
3. **Patterns:** Generalizing numerical/visual sequences (e.g., 3n + 1 for matchstick patterns).
4. **Age Problems:** Representing current/future ages (e.g., y + 5 for age after 5 years).
5. **Geometry (Perimeter):** Describing perimeters with unknown side lengths (e.g., 2l + 2w for a rectangle).

These applications make algebra relevant, fostering critical thinking and problem-solving, aligning with NEP 2020's focus on experiential learning.

Gap Analyzer™
Complete Chapter Test

EXERCISE

A. Choose the correct answer.

1. If $x = 5$ and $y = -2$, what is the value of the expression $3x - 4y$?

   a) 7 ☐ b) 23 ☐ c) 15 ☐ d) 13 ☐

2. Simplify: $8p + 3q - 5p + q$.

   a) $3p + 2q$ ☐ b) $3p + 4q$ ☐ c) $13p + 4q$ ☐ d) $13p + 2q$ ☐

3. The nth term of a sequence is given by $4n + 1$. What is the 7th term?

   a) 28 ☐ b) 29 ☐ c) 30 ☐ d) 31 ☐

4. A shopkeeper sells each apple for ₹x and each mango for ₹y. If a customer buys 4 apples and 3 mangoes, and pays ₹85, which equation represents this situation?

   a) $4x + 3y = 85$ ☐ b) $4x - 3y = 85$ ☐ c) $4y + 3x = 85$ ☐ d) $7(x + y) = 85$ ☐

5. If $p = 4$ and $q = 2$, evaluate $3p^2 - 2q$.

   a) 44 ☐ b) 46 ☐ c) 48 ☐ d) 52 ☐

6. The product of 7 and the square of y can be expressed as:

   a) $y^2 + 7$ ☐ b) $7y^2$ ☐ c) $(7y)^2$ ☐ d) $y^2 - 7$ ☐

Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct:

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

1. **Assertion (A):** The expression $7x + 2y - 3x$ can be simplified to $4x + 2y$.
Reason (R): Only like terms can be combined in an algebraic expression.

2. **Assertion (A):** If $p = -3$, then $2p + 5 = -1$.
Reason (R): When evaluating expressions, multiplication is performed before addition.

3. **Assertion (A):** The expression $5(a - b)$ is equal to $5a - 5b$.
Reason (R): The distributive property states that $a(b - c) = ab - ac$.

Case Study

The School Canteen's Daily Earnings

The school canteen sells different items. On a particular day, they sold:

- 'p' plates of Pulao at ₹35 each
- 'd' plates of Dosa at ₹25 each
- 's' plates of Samosa at ₹10 each

In the evening, they had to give a refund for 2 plates of Pulao and 1 plate of Dosa due to a quality issue.

a) Write an algebraic expression for the total earnings from Pulao, Dosa, and Samosa before refunds.

b) Write an algebraic expression for the total refund amount.

c) Write and simplify an algebraic expression for the canteen's net earnings for the day.

d) If p = 50, d = 70, and s = 100, calculate the net earnings for the day.

e) The next day, the canteen decided to increase the price of Pulao by ₹5 and Dosa by ₹3. Write a new expression for the net earnings for the next day, assuming the same quantities (p, d, s) are sold and the same refund policy applies.

Project

Algebra in My World

Objective: To identify and apply algebraic expressions in real-world contexts, fostering creativity and problem-solving skills.

Task: Choose one of the following project options:

My Daily Routine in Algebra

- Identify at least three different activities in your daily routine (e.g., time spent on homework, playing, eating, traveling).
- Assign variables to unknown or changing quantities in these activities (e.g., 'h' for hours of homework, 'g' for games played, 't' for travel time).
- Create algebraic expressions to represent different aspects of your routine (e.g., "Total time spent on homework for 'd' days," "Cost of snacks for 's' days," "Total distance traveled in 'k' trips").
- Write a short paragraph explaining each expression and what its variables represent.
- Choose specific values for your variables and evaluate one of your expressions.

Source-Based Question

The Khelo India Medal Tally Challenge

Introduction for Students: The Khelo India Youth Games are a national-level multi-sport event held in India for young athletes. The goal is to encourage a sporting culture and identify talent at the grassroots level. States compete for medals, and their final ranking is often determined by a points system based on the number of Gold, Silver, and Bronze medals they win.

Let's look at the final medal tally for the top three states from the 2023 games held in Madhya Pradesh.

State	Gold Medals	Silver Medals	Bronze Medals
Maharashtra	57	48	53
Haryana	35	22	46
Uttar Pradesh	11	13	18

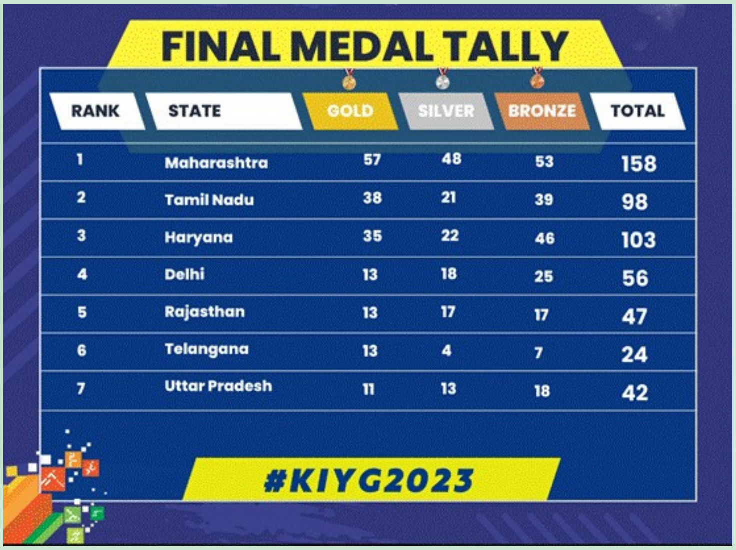
For this challenge, we will use a common points system:

- Gold Medal = 5 points
- Silver Medal = 3 points
- Bronze Medal = 1 point

Now, let's use our algebra skills to analyze the results!

Source Text: **Press Information Bureau (PIB), Government of India. Data from the Khelo India Youth Games 2023.**

Questions Based on the Source



FINAL MEDAL TALLY

RANK	STATE	GOLD	SILVER	BRONZE	TOTAL
1	Maharashtra	57	48	53	158
2	Tamil Nadu	38	21	39	98
3	Haryana	35	22	46	103
4	Delhi	13	18	25	56
5	Rajasthan	13	17	17	47
6	Telangana	13	4	7	24
7	Uttar Pradesh	11	13	18	42

#KIYG2023

1. Using the variables g for the number of Gold medals, s for Silver, and b for Bronze, write a single algebraic expression to calculate the total points for any state based on the given points system.

2. Use the expression you created in Question 1 to calculate the total points scored by the top-ranked state, Maharashtra. Show the values you substitute for g, s, and b.

3. Which state scored more points: Haryana or Uttar Pradesh? Use your algebraic expression to calculate the points for both states and find the difference in their scores.

4. Imagine for the next Khelo India Games, the organizing committee proposes a new rule to encourage winning Gold: Gold medals will be worth 7 points, while Silver and Bronze points remain the same.

   a) Write a new algebraic expression for the total points under this rule.

   b) How many points would Maharashtra have scored with this new rule?

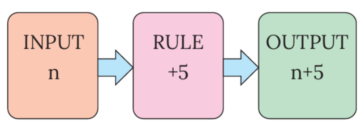
5. The state of Tamil Nadu finished in 4th place. They won 38 Gold medals and 21 Silver medals. If their final total point score was 98, how many Bronze medals did they win? Use your original algebraic expression from Question 1 to find the answer. Show your steps clearly.

Mind Map

Algebraic Expressions: Using Letter-Numbers

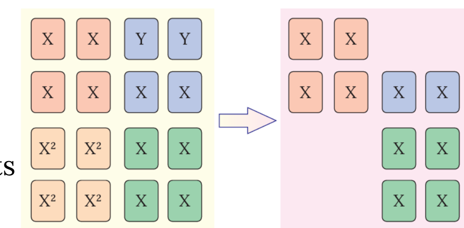
Algebraic Expressions

- **Variables (Letter-Numbers):** Represent numbers
- **Purpose:** Express rules/relationships
- **Form:** Variables + numbers + operations (no "=")
- **Evaluate:** Substitute values (e.g., 2n, n = 5 → 10)



Understanding Like and Unlike Terms

- **Like Terms:** Same variables/powers
- **Unlike Terms:** Different variables/powers
- **Steps:**
  - ✓ Remove brackets
  - ✓ Group like terms
  - ✓ Combine coefficients



Order of Operations (BODMAS/PEMDAS)

- **Order (BODMAS):** Brackets → Orders → ÷/× → +/−
- **Brackets:** Group terms, manage negatives
- **Distributive Law:** $a(b + c) = ab + ac$
- **Notation:** Omit "×" in 2x, xy

Identifying Patterns in Sequences

- **Number sequence:** 4, 8, 12, 16, ... (adding 4 each time, or multiples of 4)
- **Number Machine: Deduce rule (e.g., x → x + 5)**
- **Applications:**
  - ✓ Calendar 2 × 2 block → diagonal sums equal
  - ✓ Matchsticks for houses → 2n + 1
- **Algebraic Proofs:** General validity (**e.g.,** sum of 3 consecutive integers = multiple of 3)