3 Decimals: A Peek Beyond the Point

Why This Chapter Matters

Imagine you're building a miniature model of your school. You need to cut a piece of wood that is exactly 12.7 centimeters long. What does that ".7" mean? How is it different from just 12 centimeters? Or, think about your favorite sports car's speed – 250.8 kilometers per hour! Why isn't it just 250? Decimals are everywhere, helping us measure, calculate, and understand the world with incredible precision. Get ready to explore the fascinating world of numbers beyond the whole!

Meet EeeBee.AI

Hello, young mathematicians! I'm EeeBee, your friendly guide through the exciting world of numbers. I love exploring how math helps us understand everything around us, from the tiniest measurements to the vastness of space. In this chapter, we're going on an adventure to discover decimals – those clever numbers that let us be super precise. I'll be here to help you, share cool facts, and make sure you have fun learning. Let's dive in and unlock the secrets of precision together!

Learning Outcomes

By the end of this chapter, students will be able to:

- Define decimals and identify their whole and fractional parts.
- Explain the concept of place value for decimal numbers, extending to thousandths.
- Convert fractions with denominators of 10, 100, and 1000 into decimal form and vice-versa.
- Compare and order decimal numbers using place value and number line representation.
- Perform addition and subtraction operations on decimal numbers accurately.
- Solve real-life problems involving decimal numbers and their applications in measurement and money.

From Last Year's Notebook

- In earlier grades, you learned about **whole numbers** (like 1, 5, 100) which represent complete units.
- You also explored **fractions** (like $\frac{1}{2}, \frac{3}{4}, \frac{7}{10}$) which represent parts of a whole.
- You practiced adding, subtracting, and visually representing simple fractions.
- Decimals are another way to write fractions, especially those with denominators like 10, 100, or 1000 (powers of 10).
- We'll see how place value, which you know for whole numbers, extends to numbers smaller than one.

Real Math, Real Life

- **Everyday Use:** Decimals help us measure things accurately in daily life, like:
  - **Cooking:** Measuring ingredients (e.g., 0.5 kg of flour).
  - **Travel:** Calculating distances on a map (e.g., 15.3 km).
  - **Money:** Handling rupees and paise (e.g., ₹15.75).
- **Professional Fields:** Decimals are crucial in many professions for precision:
  - **Engineering:** Designing accurate structures.
  - **Medicine:** Prescribing exact medication dosages (e.g., 0.25 mg).
  - **Science:** Recording precise experimental data.
  - **Finance:** Managing money and calculations.
  - **Career Opportunities:** Understanding decimals can open doors to exciting careers in fields like Science, Technology, Engineering, Medicine, and Finance.

Quick Prep

1. If you divide a chocolate bar into 10 equal pieces, what fraction of the bar is one piece?
2. A ruler is marked in centimeters. How many small divisions are there between 0 cm and 1 cm?
3. If you have 2 whole pizzas and $\frac{3}{10}$ of another pizza, how many tenths of a pizza do you have in total?
4. Can you think of a situation where measuring something to the nearest whole centimeter isn't accurate enough?
5. If you walk 1 kilometer and then another $\frac{3}{10}$ of a kilometer, what is your total distance walked?

Introduction

Have you ever tried to measure something really small, like the thickness of a coin or the length of a tiny insect? Sometimes, whole numbers just aren't precise enough. This section will introduce you to the idea that we often need to divide our standard units into even smaller parts to get accurate measurements. We'll start by exploring "**tenths**" – the first step into the world of numbers beyond the decimal point. Understanding tenths is crucial because it forms the foundation for understanding all decimal numbers.

Chapter Overview

- **Introduction to Decimals:** Understand the need for precision, exploring tenths, hundredths, and thousandths with real-life examples and basic operations.
- **Place Value System:** Learn decimal notation, reading, and the significance of each place value to the right of the decimal point.
- **Measurement Conversions:** Master converting between various units of length, weight, and currency using decimal representation.
- **Locating & Comparing:** Represent decimals on a number line and develop strategies for effective comparison.
- **Operations:** Perform addition and subtraction of decimals, including estimation for reasonableness.
- **Real-world Applications:** Explore practical implications, common pitfalls, and the historical context of decimals.

From History's Pages

The need to divide units for trade and construction led ancient civilizations, including those in India, Babylon, and Egypt, to develop fractions. Indian mathematicians were pioneers, extending their decimal place value system for whole numbers to include fractions. Figures like **Aryabhata, Śrīdhara**, and **Bhāskara II** significantly contributed to this understanding. The formal decimal point notation, as used today, evolved later in 16th-17th century Europe through mathematicians like **Simon Stevin** and **John Napier**, who recognized the efficiency of this base-10 extension. This chapter explores the practical needs that led to this powerful mathematical invention.

The Need for Smaller Units: A Tenth Part

Sometimes, when we measure things, a whole unit (like 1 centimeter) isn't precise enough. Imagine trying to measure a tiny screw or the exact length of a small insect. If the length falls between two whole numbers, say between 2 cm and 3 cm, how do we describe its exact size? This is where smaller units come in handy! We need a way to talk about parts of a unit. This concept will introduce the idea of dividing a unit into 10 equal parts to achieve greater accuracy, setting the stage for understanding "**tenths**."

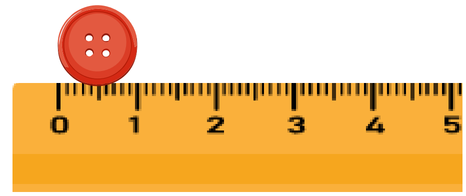
Sub-concepts to be covered

1. Limitations of whole number measurements.
2. Introduction to the idea of fractional parts for precision.
3. Visualizing division of a unit into 10 equal parts.

Limitations of Whole Number Measurements

Whole numbers (0, 1, 2, 3, ...) are great for counting complete items or measuring exact, full units. However, many real-world quantities don't fit perfectly into whole units. For example, a pencil might be longer than 10 cm but shorter than 11 cm. If we only use whole numbers, we have to round, losing precision.

Examples and Illustrations



- **Measuring a small button:** It's clearly less than **1 cm**. How do we describe its length accurately?

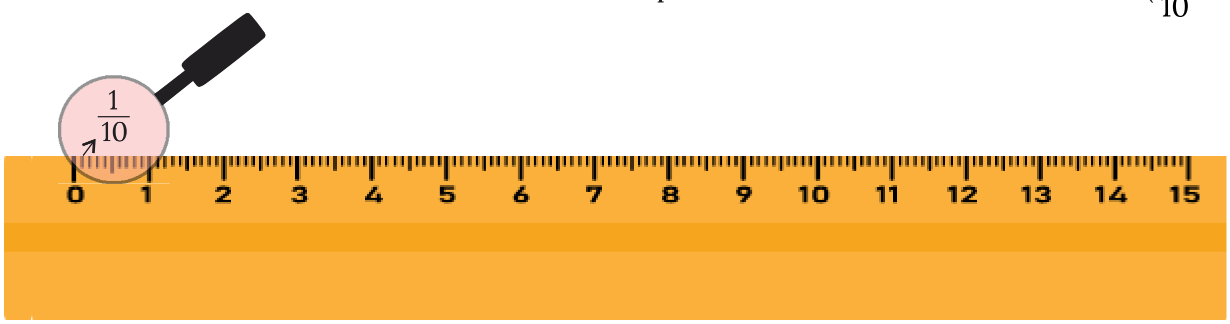
Key points: Whole numbers are insufficient for precise measurements when quantities fall between two consecutive whole units.

Introduction to the Idea of Fractional Parts for Precision

To measure quantities that are "in between" whole numbers, we divide a whole unit into smaller, equal parts. The most common and convenient way to do this in our number system is to divide each unit into 10 equal parts. Each of these parts is called a "tenth."

Examples and Illustrations:

- **A ruler:** Look closely at your ruler. Between any two whole centimeter marks (e.g., 2 cm and 3 cm), there are 10 smaller divisions. Each small division represents one-tenth of a centimeter ($\frac{1}{10}$ cm).



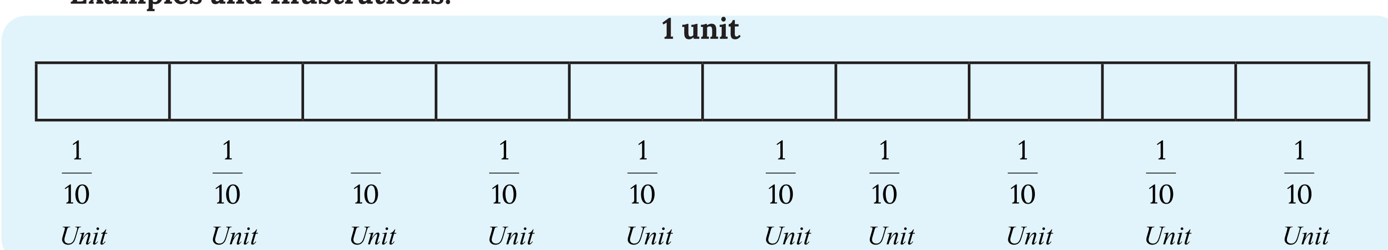
- **Example:** A chocolate bar divided into 10 pieces: Each piece is $\frac{1}{10}$ of the whole bar.

  Key points: Dividing a unit into 10 equal parts allows for more precise measurements. Each part is $\frac{1}{10}$ of the unit.

Visualizing Division of a Unit into 10 Equal Parts

Visualizing these divisions helps solidify the concept of tenths. Imagine a number line. If you mark 0 and 1, then divide the segment between 0 and 1 into 10 equal smaller segments, each mark represents a tenth.

- **Examples and Illustrations:**



  Key points: Visual representations like number lines and divided shapes are powerful tools for understanding fractional parts.

In 2021, the world used about 3,85,000,000 liters of drinking water every hour.
To understand smaller amounts, scientists often break this into tenths of a liter (0.1 L), because people usually drink water in glasses of 0.2 to 0.3 liters.

Mathematical Explanation: Understanding Tenths

When we need to measure something that isn't an exact whole number, we extend our number system to include parts of a whole. The most natural extension, given our base-10 number system, is to divide each whole unit into 10 equal parts. Each of these equal parts is called a tenth.

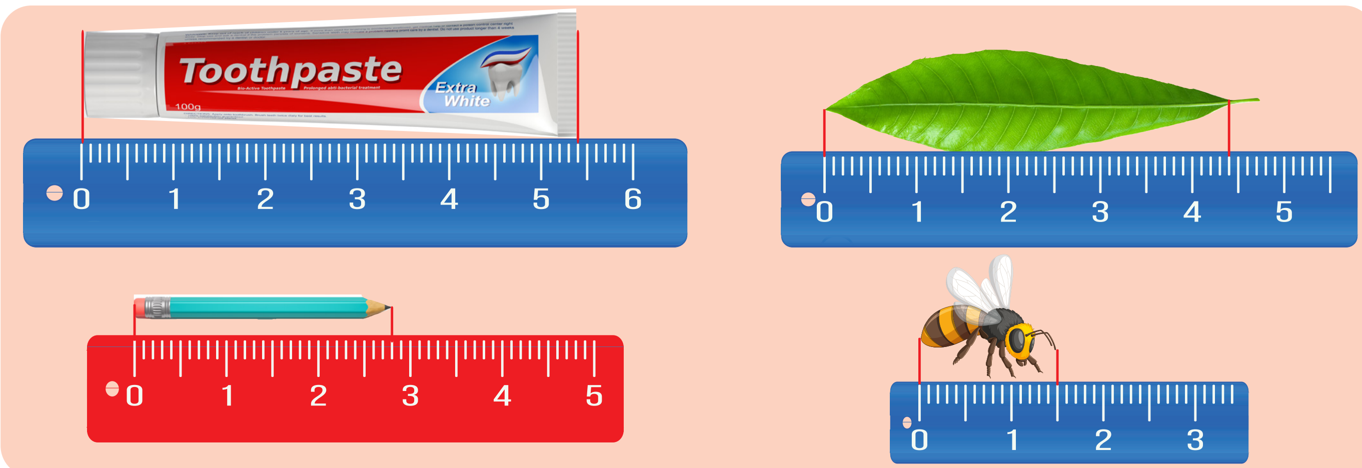
For the objects shown below, write their lengths. (Note that the unit length used in each diagram is not the same).

The length of the toothpaste tube- 5 and $\frac{4}{10}$ units = $5\frac{4}{10}$ units = $\frac{54}{10}$ units

The length of the leaf- 4 and $\frac{4}{10}$ units = $4\frac{4}{10}$ units = $\frac{44}{10}$ units

The length of the pencil-2 and $\frac{8}{10}$ units = $2\frac{8}{10}$ units = $\frac{28}{10}$ units

The length of the honeybee-1 and $\frac{5}{10}$ units = $1\frac{}{10}$ units = $\frac{15}{10}$ units



Mathematically, a tenth can be written as the fraction $\frac{1}{10}$.

Step 1 : If we have, for example, 3 whole units and 4 of these tenths, we can express this as a mixed fraction: $3\frac{4}{10}$.

Step 2 : This mixed fraction can also be written as an improper fraction. Since 1 whole unit is equal to 10 tenths, 3 whole units would be 3 × 10 = 30 tenths. Adding the 4 tenths we already have, the total becomes 30 + 4 = 34 tenths. So, $\frac{}{10}$ is equivalent to $\frac{34}{10}$.

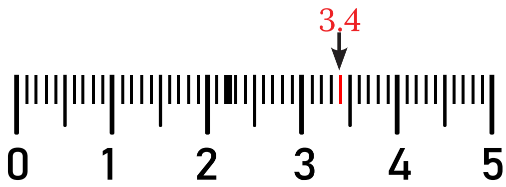
Step 3 : The beauty of the decimal system is that it provides a more compact way to write these numbers. Instead of writing $3\frac{4}{10}$ or $\frac{34}{10}$, we can write it as 3.4. The point (.) is called the **decimal point**, and it separates the whole number part from the fractional part. The digit immediately to the right of the decimal point represents the number of tenths.

So, in 3.4:

- 3 is in the **ones place**, representing 3 **whole units**.
- (.) is the **decimal point**.
- 4 is in the **tenths place**, representing 4 tenths $\frac{4}{10}$.

This means that $3.4 = 3 + \frac{4}{10}$.

We can also think of numbers like 3.4 as a collection of only tenths. Since 1 unit is 10 tenths, 3 units is 30 tenths. Adding the 4 tenths, we get a total of 34 tenths. This is why $3.4 = \frac{34}{10}$.



Addition and Subtraction of Numbers involving tenths

To add numbers with tenths, we can either convert them to mixed fractions, add the whole parts and fractional parts separately, and then combine them, or convert them entirely to improper fractions (tenths) and add.

For example, to add 2.7 + 3.5:

Addition Using Mixed Fractions (Method 1):

$$2.7 = 2\frac{7}{10}, \quad 3.5 = 3\frac{5}{10}$$

$$\text{Sum} = 2\frac{7}{10} + 3\frac{5}{10}$$
$$= (2+3) + \left(\frac{7}{10} + \frac{5}{10}\right)$$
$$= 5 + \frac{12}{10}$$
$$\left(\frac{12}{10} = 1\frac{2}{10}\right)$$

so the total is: $5 + 1\frac{2}{10} = 6\frac{2}{10}$

Addition Using Improper Fractions (Method 2)

$$2.7 = \frac{27}{10}, \quad 3.5 = \frac{35}{10}$$

$$\text{Sum} = \frac{27}{10} + \frac{35}{10}$$
$$= \frac{27+35}{10}$$
$$= \frac{62}{10} = 6\frac{2}{10}$$

Similar to addition, subtraction can be done by converting to mixed fractions or improper fractions.

For example, to subtract 3.5 - 2.7:

Subtraction Using Mixed Fractions (Method 1):

$$3.5 = 3\frac{5}{10}, \; 2.7 = 2\frac{7}{10},$$

We need to subtract $\frac{7}{10}$, from $\frac{5}{10}$, which requires borrowing.

Borrow 1 from 3: $3\frac{5}{10} = 2\frac{15}{10}$

Difference = $2\frac{15}{10} - 2\frac{7}{10} = (2-3) + \left(\frac{15}{10} - \frac{7}{10}\right)$

$$= 0 + \frac{8}{10} - \frac{8}{10}$$

Subtraction Using Improper Fractions (Method 2):

$$3.5 = \frac{35}{10}, \quad 2.7 = \frac{27}{10},$$

Difference $= \frac{35}{10} - \frac{27}{10}$

$$= \frac{35-27}{10} = \frac{8}{10}$$

Example 1 : Measuring and Writing Lengths in Tenths

A small toy car measures 4 units and 6 one-tenths.

a) Write its length as a mixed fraction.

b) Write its length as an improper fraction (in terms of tenths).

c) Write its length using decimal notation.

d) Read the length aloud.

Solution: a) **As a mixed fraction:** 4 units and 6 one-tenths means $4\frac{6}{10}$.

b) **As an improper fraction:** 4 units is 4 times 10 = 40 tenths. Adding 6 tenths, we get 40 + 6 = 46 tenths. So, $\frac{46}{10}$.

c) **Using decimal notation:** The whole part is 4, and the tenths part is 6. So, 4.6.

d) **Reading aloud:** "Four and six-tenths" or "Four point six."

Example 2 : Arranging Lengths in Increasing Order

a) $\frac{7}{10}$ b) $1\frac{3}{10}$ c) $\frac{125}{10}$ d) $12\frac{1}{10}$ e) $9\frac{4}{10}$

Convert all to improper fractions (if needed):

a) Already $\frac{7}{10}$ b) $1\frac{3}{10} = \frac{13}{10}$ c) Already $\frac{125}{10}$ d) $12\frac{1}{10} = \frac{121}{10}$ e) $9\frac{4}{10} = \frac{94}{10}$

Now arrange the fractions in increasing order by numerator (denominators are all 10):

$$\frac{7}{10} < \frac{13}{10} < \frac{94}{10} < \frac{121}{10} < \frac{125}{10}$$

So, the correct order is:

$$\frac{7}{10}, 1\frac{3}{10}, 9\frac{4}{10}, 12\frac{1}{10}, \frac{125}{10}$$

Example 3 : Rohan's height is $1\frac{6}{10}$ meters, and his brother's height is $1\frac{9}{10}$ meters. Find the combined height.

Method 1: Adding Whole and Fractional Parts

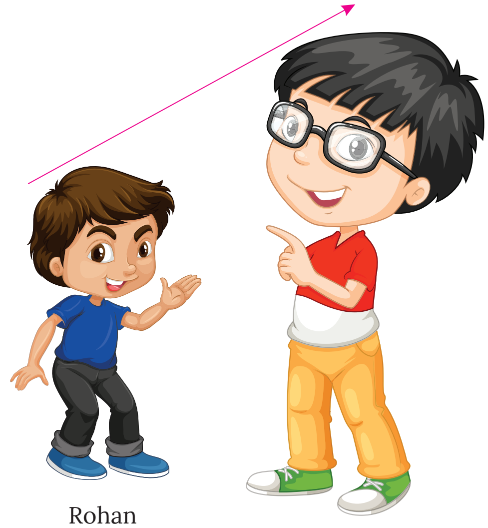
$$1\frac{6}{10} + 1\frac{9}{10} = (1+1) + \left(\frac{6}{10} + \frac{9}{10}\right) = 2 + \frac{15}{10} = 2 + 1\frac{5}{10} = 3\frac{5}{10}$$

Method 2: Converting to Improper Fractions

$$1\frac{6}{10} = \frac{16}{10}, 1\frac{9}{10} = \frac{19}{10}$$

$$\text{Sum} = \frac{16}{10} + \frac{19}{10} = \frac{35}{10} = 3\frac{5}{10}$$

Combined height: $3\frac{5}{10}$



Example 4 : A rope is $8\frac{2}{10}$ meters long. If $3\frac{7}{10}$ meters are cut, what is left?

Method 1: Using Mixed Fractions and Borrowing

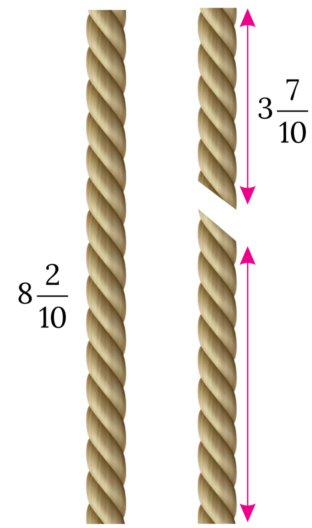
$$8\frac{2}{10}-3\frac{7}{10}$$

Since $\frac{2}{10}-\frac{7}{10}$ is not possible, we borrow 1 unit from 8:

$$8\frac{2}{10}=7\frac{12}{10}$$

Now subtract:

$$7\frac{12}{10}-3\frac{7}{10}=(7-3)+\left(\frac{12}{10}-\frac{7}{10}\right)=4+\frac{5}{10}=4\frac{5}{10}$$



Method 2: Converting to Improper Fractions

$$8\frac{2}{10}=\frac{82}{10},\ 3\frac{7}{10}=\frac{37}{10}$$

Difference $=\frac{82}{10}-\frac{37}{10}=\frac{45}{10}=4\frac{5}{10}$

Remaining rope: $4\frac{5}{10}$ meters

Activity

Decimal Ruler Challenge

Objective: To practice measuring objects accurately using tenths and representing measurements in decimal form.

Materials: Rulers (with millimeter markings), various small objects (e.g., paper clip, eraser, small toy, coin, key), worksheet for recording measurements.

Procedure:

Demonstration: The teacher demonstrates how to measure an object (e.g., a pen) to the nearest tenth of a centimeter using the ruler. Explain how to count the small divisions after the whole centimeter mark.

Measurement Task: Students work individually or in pairs.

- Each student selects 5-7 small objects.
- For each object, they measure its length (or width, if appropriate) to the nearest tenth of a centimeter.
- They record the measurement in a table, first as "___ cm and ___ tenths of a cm," then as a mixed fraction (e.g., $5\frac{3}{10}$ cm), and finally as a decimal (e.g., 5.3 cm).

Discussion:

- Compare measurements for the same object among different students. Discuss why there might be slight variations.
- Ask students to identify the shortest and longest objects they measured.

Extension: Challenge students to find objects that are exactly a certain number of tenths long (e.g., 7.0 cm, 3.5 cm).

Knowledge Checkpoint

- What does the digit '8' represent in the number 5.8?
- Write the decimal form for "nine and four-tenths."
- Is 1.2 greater than or less than 1.5?

Key Terms

- **Tenth:** One of ten equal parts of a whole. Written as $\frac{1}{10}$ or 0.1.
- **Decimal Point:** A dot used to separate the whole number part from the fractional part in a decimal number.
- **Decimal Number:** A number that contains a decimal point, representing a whole number and a fractional part (tenths, hundredths, etc.).

Fact Flash

- Did you know that the word "**decimal**" comes from the Latin word "**decem,**" which means "**ten**"? This is because our decimal system is based on groups of ten!
- The ancient Egyptians used a system of fractions, but they were mostly unit fractions (like $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$). They didn't have a systematic way to write fractions like $\frac{7}{10}$ until much later.
- The idea of using a decimal point to separate whole numbers from fractions was popularized in Europe in the 16th century, making calculations much easier!

Do It Yourself

Imagine you have a very long ribbon, and you want to cut it into pieces that are exactly 0.1 meters long. How many such pieces could you get from a 5-meter ribbon? What if you wanted pieces that were 0.5 meters long? How does thinking about "tenths" help you solve this?

Mental Mathematics

- If you have 1.0 meter of string and cut off 0.4 meters, how much is left?
- Count by tenths starting from 0.1 up to 1.0.
- Which is greater: 0.7 or 0.9?
- If you add 0.2 to 0.8, what whole number do you get?

Gap Analyzer™ Homework

Watch Remedial

Exercise 3.1

1. Convert each of the following mixed numbers to its decimal form.

a) $4\frac{3}{10}$ b) $9\frac{7}{10}$ c) $12\frac{1}{10}$ d) $5\frac{25}{10}$ e) $1\frac{6}{10}$

2. **Convert each of the following decimals to an improper fraction.**

a) 2.3 b) 7.9 c) 10.5 d) 0.6 e) 1.8
f) 5.8 g) 2.7 h) 3.4 i) 1.5 j) 6.4

3. **Solve the following questions:**

a) A tailor used 2.5 meters of fabric for a shirt and 1.8 meters for a pair of shorts. What is the total length of fabric used?

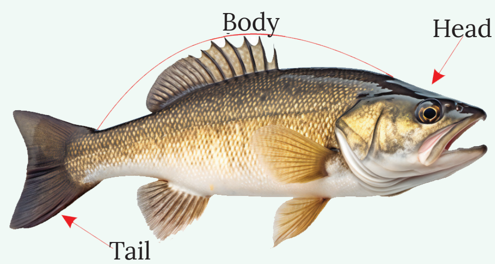
b) A jug contains 3.7 liters of juice. If 1.2 liters are poured into glasses, how much juice is left in the jug?

c) The length of a table is 1.9 meters. If a chair is 0.8 meters long, what is the difference in their lengths?

d) A plant grew 0.3 cm on Monday and 0.6 cm on Tuesday. How much did it grow in total over the two days?

4. **The lengths of the body parts of a Fish are given. Find its total length.**

- Head: $1\frac{7}{10}$
- Body: $5\frac{2}{10}$
- Tail: $2\frac{6}{10}$



5. **Simplify the following expression involving mixed fractions:**

a) $7\frac{4}{5}-3\frac{2}{5}$ b) $1\frac{1}{3}+4\frac{1}{9}$ c) $8\frac{5}{6}-5\frac{1}{3}$ d) $2\frac{1}{2}+3\frac{1}{4}-1$ e) $6\frac{3}{4}-2\frac{1}{2}+1\frac{1}{8}$

6. A water tank can store 100 L. Today, only $\frac{1}{10}$ of it is filled. That means there are just 10 L of water.

If every hour 5 L is added, the tank will slowly fill.

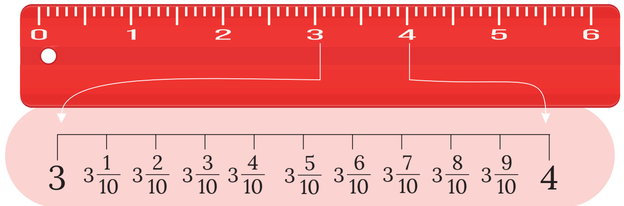
Questions

1. What is $\frac{1}{10}$ of 100 L?
2. How much more water is needed to make the tank half full?
3. If 5 L is added every hour, in how many hours will the tank reach 50 L?
4. Write the total water after 3 hours.

A Hundredth Part

When we divide a unit into 10 parts, we get tenths. If we divide one tenth into 10 smaller parts, we get **hundredths**. A hundredth is 1 out of 100 equal parts of a whole. This helps us extend decimals to **two places** after the point.

We can say that the length lies between $3\frac{4}{10}$ units and $3\frac{5}{10}$ units. But we cannot tell the exact length because there are no smaller markings. Earlier, we divided one unit into 10 equal parts, called tenths, to measure smaller lengths. In the same way, now we can divide each tenth into 10 smaller parts.

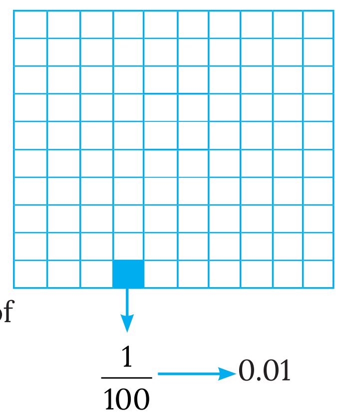


Sub-concepts to be covered

1. Understanding $\frac{1}{100}$ as a unit.
2. Relationship between tenths and hundredths.
3. Representing quantities as "units, tenths, and hundredths."
4. Converting fractions with denominator 100 to decimal form.
5. Addition and subtraction of numbers involving hundredths.

Understanding $\frac{1}{100}$ as a Unit

A hundredth is one part when a whole unit is divided into 100 equal parts. It is written as the fraction $\frac{1}{100}$. In decimal form, it is written as 0.01. The digit '1' is in the second place after the decimal point, which is the hundredths place.



Examples and Illustrations:

- 1 Rupee = 100 Paise. So, 1 Paisa is $\frac{1}{100}$ of a Rupee, or ₹0.01.
- 1 Meter = 100 Centimeters. So, 1 Centimeter is $\frac{1}{100}$ of a Meter, or 0.01 m.

Key points: A hundredth is a very small unit, representing one part out of one hundred.

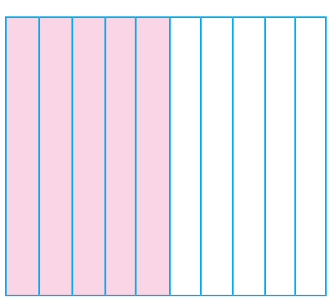
Common errors: Confusing 0.1 (one-tenth) with 0.01 (one-hundredth).

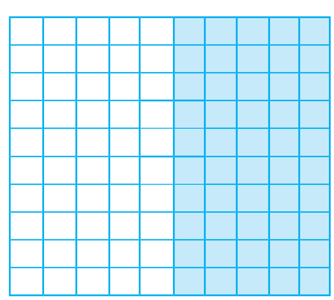
Relationship Between Tenths and Hundredths

Just as 1 whole unit is made of 10 tenths, 1 tenth is made of 10 hundredths. This means that $\frac{1}{10} = \frac{10}{100}$. This relationship is crucial for understanding place value and for performing operations.

Examples and Illustrations:

- If you have a strip representing five-tenth, and you divide it into 10 smaller equal parts, each of those smaller parts is a hundredth.
- 0.5 = 0.50 (one-tenth is equivalent to ten-hundredths). This is why adding a zero at the end of a decimal doesn't change its value.

$\frac{5}{10}$ = 0.5 

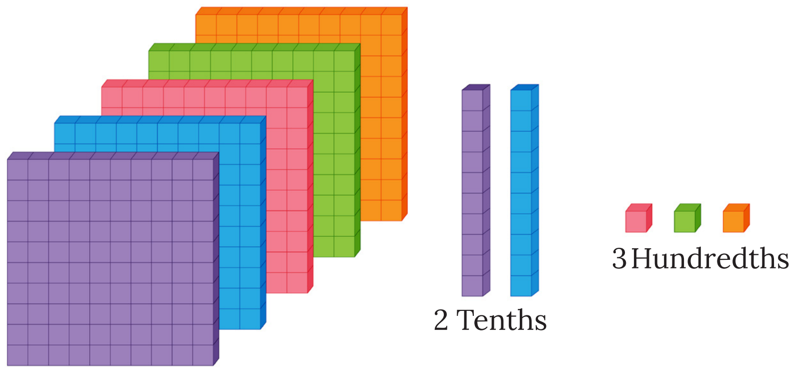
 $\frac{50}{100}$ = 0.50

Key points: The decimal system is based on powers of 10. Each place value is 10 times smaller than the one to its left.

Common errors: Thinking that 0.1 and 0.10 are different values.

Representing Quantities as "Units, Tenths, and Hundredths"

A decimal number with two digits after the decimal point represents a combination of whole units, tenths, and hundredths. For example, 5.23 means 5 whole units, 2 tenths, and 3 hundredths. This can be written as $5+\frac{2}{10}+\frac{3}{100}$.



Key points: The first digit after the decimal point is tenths, the second is hundredths.

Common errors: Misplacing digits in the wrong place value (e.g., writing 5.23 as 5.023).

Converting Fractions with Denominator 100 to Decimal Form

Any fraction with a denominator of 100 can be directly converted to a decimal by placing the numerator after the decimal point, ensuring there are two decimal places. If the numerator is a single digit, a zero is placed before it in the tenths place.

Examples and Illustrations:

- $\frac{7}{100}=0.07$
- $\frac{45}{100}=0.45$
- $\frac{123}{100}=1.23$

Key points: The number of zeros in the denominator (100 has two zeros) indicates the number of decimal places.

Common errors: Writing $\frac{7}{100}$ as 0.7 (confusing hundredths with tenths).

Addition and Subtraction of Numbers Involving Hundredths

To add or subtract decimals involving hundredths, the most important rule is to align the decimal points. This ensures that you are adding or subtracting digits of the same place value (ones with ones, tenths with tenths, hundredths with hundredths). You can add trailing zeros to numbers to make them have the same number of decimal places, which helps in alignment.

Examples and Illustrations:

Example: Add 15.34 + 2.68

15.34 (15 units, 3 tenths, 4 hundredths)
\+ 2.68 (2 units, 6 tenths, 8 hundredths)

Start adding from the rightmost digit (hundredths place):

1. **Hundredths:** 4 + 8 = 12 hundredths. 12 hundredths is 1 tenth and 2 hundredths. Write down '2' in the hundredths place and carry over '1' to the tenths place.
2. **Tenths:** 3 + 6 + 1 (carry-over) = 10 tenths. 10 tenths is 1 unit and 0 tenths. Write down '0' in the tenths place and carry over '1' to the ones place.
3. **Ones:** 5 + 2 + 1 (carry-over) = 8 ones. Write down '8' in the ones place.
4. **Tens:** 1 + 0 = 1 ten. Write down '1' in the tens place.

Result: 18.02

Subtraction of Decimals (with Hundredths):

Similar to addition, align the decimal points vertically. Borrowing works the same way as with whole numbers, but across decimal places.

Example: Subtract 25.90 - 6.47

1. **Hundredths:** We need to subtract 7 from 0. Borrow 1 tenth from the tenths place (9 tenths becomes 8 tenths). The 0 hundredths becomes 10 hundredths. 10 - 7 = 3 hundredths.
2. **Tenths:** Now we have 8 tenths. 8 - 4 = 4 tenths.
3. **Ones:** We need to subtract 6 from 5. Borrow 1 ten from the tens place (2 tens becomes 1 ten). The 5 ones becomes 15 ones. 15 - 6 = 9 ones.
4. **Tens:** Now we have 1 ten. 1 - 0 = 1 ten.

Result: 19.43

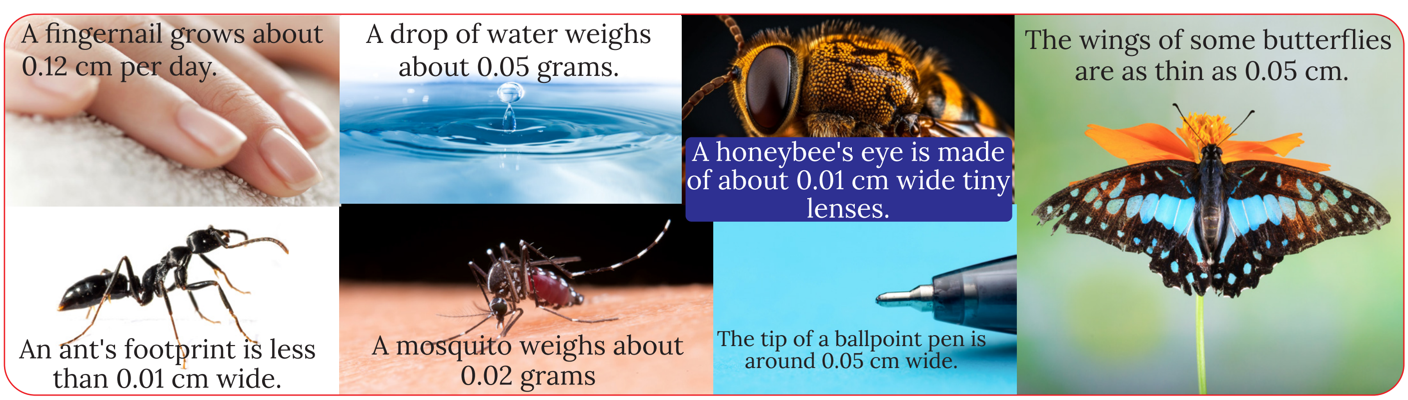
Key points: Always align decimal points before adding or subtracting. Add trailing zeros if necessary to ensure equal decimal places.

Common errors: Not aligning decimal points, leading to incorrect place value addition / subtraction.

Mathematical Explanation

As we move to more precise measurements, we introduce the concept of a hundredth. A hundredth is derived by dividing a whole unit into 100 equal parts. This can also be thought of as dividing each tenth into 10 smaller equal parts.

Mathematically, a hundredth is written as the fraction $\frac{1}{100}$. In decimal notation, this is represented as 0.01. The digit '1' is placed in the second position to the right of the decimal point, which is the hundredths place.



Place Value Extension

Our decimal system is built on powers of 10.

- 10 ones make 1 ten.
- 10 tenths make 1 one.
- 10 hundredths make 1 tenth.

This means:

Step-by-step in fraction form:

1. **Start with:**

$$1 = 10 \times \frac{1}{10}$$

→Because ten tenths make one whole.

2. **Also:**

$$\frac{1}{10} = 10 \times \frac{1}{100}$$

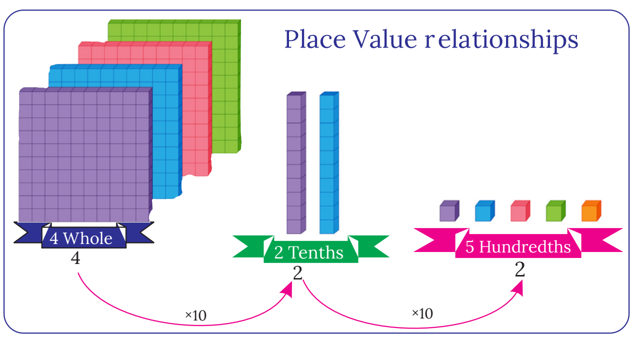
→Because ten hundredths make one tenth.

3. **Therefore:**

$$1 = 10 \times \left(10 \times \frac{1}{100}\right) = 100 \times \frac{1}{100}$$

This confirms that 100 hundredths make one whole unit.

A number like 4.25 is read as "four and twenty-five hundredths" or "four point two five".



Example 5 : Find the sum of 12.45 and 3.78.

Solution: Align the decimal points and add:

```
  12.45
+  3.78
-------
  16.23
-------
```

- **Hundredths:** 5 + 8 = 13. Write 3, carry over 1.
- **Tenths:** 4 + 7 + 1 (carry-over) = 12. Write 2, carry over 1.
- **Ones:** 2 + 3 + 1 (carry-over) = 6. Write 6.
- **Tens:** 1 + 0 = 1. Write 1.

The sum is 16.23.

Breaking it down by place value:

- 4 is in the ones place (4 units).
- 2 is in the tenths place (2 tenths or $\frac{2}{10}$).
- 5 is in the hundredths place (5 hundredths or $\frac{5}{100}$).

Convert 4.25 to fraction:

We start with:

$$4.25 = 4 + \frac{2}{10} + \frac{5}{100}$$

Now, to combine the fractional parts, we find a common denominator

$$\frac{2}{10} = \frac{20}{100}$$

So

$$\frac{20}{100} + \frac{5}{100} = \frac{25}{100}$$

Now add to the whole number:

$$4.25 = 4 + \frac{25}{100} = 4\frac{25}{100}$$

Final Answer: $4.25 = 4\frac{25}{100}$

This also means that 4.25 can be thought of as 425 hundredths, or $\frac{425}{100}$.

Example 6 : What is the difference: 18.50 - 9.23?

Solution: Align the decimal points and subtract:

```
 18.50
 -9.23
-------
  9.27
-------
```

- **Hundredths:** We need to subtract 3 from 0. Borrow 1 tenth from 5 (5 becomes 4). 0 hundredths becomes 10 hundredths. 10 - 3 = 7.
- **Tenths:** Now we have 4 tenths. 4 - 2 = 2.
- **Ones:** We need to subtract 9 from 8. Borrow 1 ten from 1 (1 becomes 0). 8 ones becomes 18 ones. 18 - 9 = 9.
- **Tens:** Now we have 0 tens. 0 - 0 = 0.

The difference is 9.27.

Example 7 : Writing Lengths with Hundredths

A very thin wire has a length of 1 unit, 1 tenth, and 4 hundredths.

a) Write its length as a mixed fraction.

b) Write its length as a fraction with denominator 100.

c) Write its length using decimal notation.

d) Read the length aloud.

Solution:

a) As a Mixed Fraction:

You are given $1\frac{1}{10}\ \frac{4}{100}$

Now convert $\frac{1}{10}$ to hundredths

$$\frac{1}{10} = \frac{10}{100}$$

Now add the two fractional parts:

$$\frac{1}{100} + \frac{4}{100} = \frac{14}{100}$$

So the full number becomes

$$1\frac{14}{100}$$

b) As a fraction with Denominator 100:

- $1 = \frac{100}{100}$
- Add the fractional part: $\frac{100}{100} + \frac{14}{100} = \frac{114}{100}$

c) Using decimal notation: The whole part is 1. The tenths digit is 1, and the hundredths digit is 4. So, 1.14.

d) Reading aloud: "One and fourteen-hundredths" or "One point one four."

Knowledge Checkpoint

- How many hundredths are there in one tenth?
- Write the decimal form for $\frac{18}{100}$.
- Which is greater: 0.4 or 0.39?

Key Terms

- **Hundredth:** One of one hundred equal parts of a whole. Written as $\frac{1}{100}$ or 0.01.
- **Place Value (Hundredths):** The second digit to the right of the decimal point, representing parts out of one hundred.

Activity

Hundredths Grid Challenge

Objective: To visually understand hundredths and practice representing them as decimals and fractions.

Materials: Printed 10 × 10 grids (hundreds charts), colored pencils or markers, worksheet with decimal numbers and fractions.

Shading Task: Students receive several blank 10 × 10 grids.

- For each grid, they are given a decimal number (e.g., 0.07, 0.30, 0.68) or a fraction (e.g., $\frac{1}{100}$, $\frac{3}{10}$, $\frac{42}{100}$).
- They must shade the correct number of squares to represent the given value.
- They then write the value in both decimal and fractional form (if not already given).

Comparison and Discussion:

- Students compare their shaded grids with classmates.
- Discuss questions like: "Which is larger, 0.20 or 0.02? How can you tell from your grids?"
- EeeBee asks: "How many small squares do you need to shade to make one full column? What does that tell you about tenths and hundredths?"

Extension: Challenge students to represent sums or differences (e.g., shade 0.15, then shade another 0.20 on the same grid, and find the total).

Do It Yourself

If you have a digital weighing scale that shows weight to two decimal places (e.g., 1.23 kg), what is the smallest change in weight it can detect? How is this related to hundredths? Why is this precision important for things like weighing medicines or precious metals?

Fact Flash

- The symbol for cent (¢), used in many currencies, comes from the Latin word "centum," meaning one hundred. This is directly related to hundredths!
- Before decimals became common, people used complex fractions to represent parts of a whole. Decimals made calculations much simpler and faster!
- The first known use of a decimal point in a mathematical text was by the Italian mathematician Giovanni Bianchini in the 15th century.

Mental Mathematics

- If you have ₹1.00 and spend ₹0.30, how much is left?
- Count by hundredths starting from 0.01 up to 0.10.
- Which is smaller: 0.08 or 0.18?
- If you add 0.05 to 0.95, what whole number do you get?

Gap Analyzer™ Homework

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Exercise 3.2

1. **Identify the digit in the hundredths place for each of the following numbers and state its value:**
   a) 45.672 b) 0.091 c) 123.405 d) 78451.854 e) 254.510
2. **Convert the following fractions to decimal form:**
   a) $\frac{8}{10}$ b) $\frac{45}{100}$ c) $\frac{9}{100}$ d) $\frac{25}{100}$ e) $\frac{3}{10}$
   f) $\frac{70}{100}$ g) $\frac{6}{100}$ h) $\frac{21}{10}$ i) $\frac{50}{100}$ j) $\frac{1}{100}$
3. **Convert the following decimals to fractions with the specified denominator (or the smallest power of 10):**
   a) Convert 0.15 to a fraction with denominator 100.
   b) Convert 0.003 to a fraction with denominator 1000.
   c) Convert 0.025 to a fraction with denominator 1000.
   d) Convert 2.1 to a fraction with denominator 10.
   e) Convert 0.750 to a fraction with denominator 1000.
4. A bottle holds 1 litre of water. If we divide it into 10 equal parts, each part is 0.1 litre. If we divide it into 100 equal parts, each part is 0.01 litre. So, 0.25 litre means one-fourth of the bottle, and 0.75 litre means three-fourths of the bottle.

   Questions:
   i. How many tenths make 0.50 litre?
   ii. Write 0.90 litre as hundredths.
   iii. What fraction of the bottle is 0.40 litre?
   iv. How many hundredths make 1 litre?

5. **Simplifying the following questions:**
   a) A piece of cloth is 5.65 meters long. Another piece is 3.28 meters long. What is the total length of the cloth?
   b) A packet of biscuits weighs 0.25 kg. If 0.12 kg of biscuits are eaten, how much is left?
   c) A swimmer completed a race in 58.75 seconds. Another swimmer completed it in 59.05 seconds. What is the difference in their timings?
   d) A shopkeeper had 15.50 kg of rice. He sold 8.75 kg. How much rice is left?
6. A thermometer shows 1 whole degree as 1.00. If it is divided into 10 parts, each part is 0.1 degree. If one part is further divided into 10 smaller parts, each is 0.01 degree. So, 0.25 degree means 25 hundredths of a degree, and 0.90 degree means 90 hundredths of a degree.

   Questions:
   i. How many tenths are there in 1 degree?
   ii. How many hundredths are equal to 0.30 degree?
   iii. Write 0.60 degree in tenths and hundredths.
   iv. What fraction of a degree is 0.25?

7. **Add the following decimal numbers:**

a) 3.45 + 2.12
b) 15.8 + 6.3
c) 0.75 + 0.23
d) 8.9 + 5.14
e) 12.05 + 3.7 + 0.82
f) 4.678 + 1.2
g) 23.01 + 9.99
h) 0.005 + 0.12 + 1.5
i) 7.00 + 3.45
j) 10.10 + 0.99 + 5.01

8. **A digital scale shows a weight of 3.491 kg.**
   a) What is the value of the digit in the hundredths place?
   b) If you add 0.01 kg to this weight, what will the new reading be?
   c) If you remove 0.001 kg, how many hundredths will be in the new reading?
9. A petrol pump sells fuel. One full unit of fuel is written as 1.00 litre. If the quantity is 0.1 litre, it means one tenth of a litre. If the quantity is 0.01 litre, it means one hundredth of a litre. For example, 0.25 litre is one-fourth of a litre, and 0.75 litre is three-fourths of a litre.

   Questions:
   1. How many hundredths are there in 0.30 litre?
   2. How many tenths make 1 litre?
   3. Write 0.40 litre as tenths and hundredths.
   4. How many hundredths are equal to $\frac{1}{2}$ litre?

Decimal Place Value & Units of Measurement

We know that in whole numbers, each place value is 10 times greater than the place value to its right (e.g., tens are 10 times ones). The decimal system beautifully extends this pattern to numbers smaller than one. The decimal point acts as a separator, and to its right, we have place values that are successively divided by 10: **tenths, hundredths, thousandths**, and so on. Understanding this structure is key to correctly writing, reading, comparing, and performing operations with decimals. It also directly links to how we convert between different units of measurement.

Hundreds	Tens	Ones	Decimal	Tenths	Hundredths	Thousandths
H	T	O	.	t	h	th
Whole Part				Decimal Part		

Sub-concepts to be covered

1. Extension of Indian Place Value System to decimals.
2. Understanding tenths, hundredths, and thousandths as place values.
3. Notation, writing, and reading of decimal numbers.
4. Significance of trailing zeros.
5. Conversion between units of measurement (Length, Weight, Money).

Extension of Indian Place Value System to Decimals

The Indian Place Value System is based on powers of 10. For whole numbers, we have Units (1), Tens (10), Hundreds (100), Thousands (1000), etc. Each place is 10 times the one to its right. This pattern continues to the right of the decimal point.

- The first place to the right of the decimal point is the Tenths place $\left(\frac{1}{10}\right)$.
- The second place is the Hundredths place $\left(\frac{1}{100}\right)$.
  - The third place is the Thousandths place $\left(\frac{1}{1000}\right)$.

  Examples and Illustrations:
  - A place value chart showing: Lakhs, Ten Thousands, Thousands, Hundreds, Tens, Ones . Tenths, Hundredths, Thousandths. Populate with an example like 122345.67

Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Ones	Decimals	Tenth	Hundredths
1	2	2	3	4	5	.	6	7

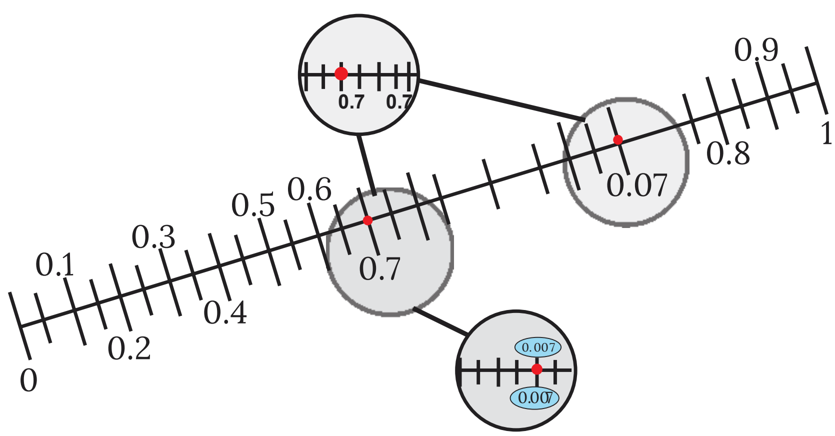
Key points: The decimal point separates the whole number part from the fractional part. Each place value to the right of the decimal point is $\frac{1}{10}$ of the place value to its left.

Common errors: Confusing the order of decimal places (e.g., thinking hundredths comes before tenths).

Understanding Tenths, Hundredths, and Thousandths as Place Values

- **Tenths (0.1):** The first digit after the decimal point. Represents how many groups of $\frac{1}{10}$ are present.
- **Hundredths (0.01):** The second digit after the decimal point. Represents how many groups of $\frac{1}{100}$ are present.
- **Thousandths (0.001):** The third digit after the decimal point. Represents how many groups of $\frac{1}{1000}$ are present.

Examples and Illustrations:
- **0.7:** 7 tenths
- **0.07:** 7 hundredths
- **0.007:** 7 thousandths



Key points: The position of a digit after the decimal point determines its value.

Common errors: Reading 0.23 as "**two point three**" instead of "**zero point two three**."
  - Here is an image of a number line with magnified sections to show the difference between 0.7, 0.07, and 0.007

Notation, Writing, and Reading of Decimal Numbers

- **Notation:** We use a decimal point ('.') to separate the whole number part from the fractional part.
- **Writing:** To write a decimal, place the digits according to their place value. If a place value is empty, use a zero as a placeholder (e.g., 0.05 for five hundredths).
- **Reading:** Read the whole number part first. Then say "point" and read the digits after the decimal point individually. For example, 12.345 is read as "twelve point three four five," NOT "twelve point three hundred forty-five." Alternatively, it can be read as "twelve and three hundred forty-five thousandths."

Examples and Illustrations:

- "Four and six tenths" is written as 4.6.
- "Zero and twenty-five hundredths" is written as 0.25.
- "Seventy-eight thousandths" is written as 0.078.

Key points: The decimal point is crucial for distinguishing whole from fractional parts. Read digits individually after the decimal point.

Common errors: Incorrectly reading decimal numbers, especially those with zeros (e.g., 0.05 as 0.5).

Significance of Trailing Zeros

Adding zeros to the right end of a decimal number (after the last non-zero digit) does not change its value. This is because 0.5 is 5 tenths, which is equivalent to 50 hundredths (0.50) or 500 thousandths (0.500). These are all the same quantity.

Examples and Illustrations:

- 0.7 = 0.70 = 0.700
- 12.3 = 12.30 = 12.300
- This is useful when comparing decimals or performing addition/subtraction, as it helps align place values.

Key points: Trailing zeros after the decimal point do not change the value of the number.

Common errors: Thinking that 0.5 and 0.50 are different values.

Conversion Between Units of Measurement (Length, Weight, Money)

The decimal system makes unit conversions straightforward because most standard units (metric system) are also based on powers of 10.

Length:

- 1 cm = 10 mm implies 1 mm = $\frac{1}{10}$ cm = 0.1 cm
- 1 m = 100 cm implies 1 cm = $\frac{1}{100}$ m = 0.01 m
- 1 m = 1000 mm implies 1 mm = $\frac{1}{1000}$ m = 0.001 m

Weight:

- 1 kg = 1000 g implies 1 g = $\frac{1}{1000}$ kg = 0.001 kg
- 1 g = 1000 mg implies 1 mg = $\frac{1}{1000}$ g = 0.001 g

Money:

- 1 Rupee = 100 Paise implies 1 Paisa = $\frac{1}{100}$ Rupee = ₹0.01

  Examples and Illustrations:

- Convert 25 mm to cm: 25 times 0.1 cm = 2.5 cm.
- Convert 150 cm to m: 150 times 0.01 m = 1.50 m.
- Convert 500 g to kg: 500 times 0.001 kg = 0.500 kg.
- Convert 75 paise to rupees: 75 times 0.01 = ₹0.75.

  Key points: To convert a smaller unit to a larger unit, divide by the conversion factor (or multiply by its decimal equivalent). To convert a larger unit to a smaller unit, multiply by the conversion factor.

  Common errors: Multiplying instead of dividing (or vice-versa) during conversion. Forgetting the correct conversion factor.

Mathematical Explanation

Decimal Place Value and Conversions

The decimal system is a powerful extension of our familiar place value system for whole numbers. Just as each place to the left of the ones place represents a power of 10 (tens, hundreds, thousands), each place to the right of the ones place represents a fractional power of 10.

Decimal Place Value Chart: Let's use the number 5,234.789 to illustrate

Thousands	Hundreds	Tens	Ones	Decimals	Tenths	Hundredths	Thousandths
5	2	3	4	.	7	8	9

Understanding the Relationship:

- Moving one place to the left means multiplying by 10.
- Moving one place to the right means dividing by 10 (or multiplying by $\frac{1}{10}$).

For example, in the number 54.321:

- 5 is in the tens place (5 times 10 = 50)
- 4 is in the ones place (4 times 1 = 4)
- 3 is in the tenths place (3 times $\frac{1}{10}$ = 0.3)
- 2 is in the hundredths place (2 times $\frac{1}{100}$ = 0.02)
- 1 is in the thousandths place (1 times $\frac{1}{1000}$ = 0.001)

So, 54.321 = 50 + 4 + 0.3 + 0.02 + 0.001.

Example: A large, clear place value chart extending from Thousands to Thousandths, with an example number (e.g., 4567.891) filled in, and arrows showing "×**10**" for leftward movement and "**/10**" for rightward movement.

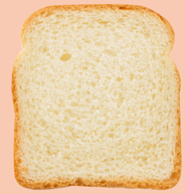
← ×10 ———— ÷10

Thousands	Hundreds	Tens	Ones		Tenth	Hundredths	Thousandths
4	5	6	7	.	8	9	1

A pencil is about 14.75 cm long

The distance covered in one step is about 0.65 m

A coin is about 2.25 cm wide



A slice of bread weighs about 32.50 grams

A cup of milk may hold 125.75 ml

Example 8 : Writing Decimal Numbers from Place Value

a) 3 ones, 7 tenths, 2 hundredths

b) 1 ten, 5 ones, 8 hundredths

c) 0 ones, 4 tenths, 9 thousandths

Solution:

a) 3 ones, 7 tenths, 2 hundredths = 3.72

b) 1 ten, 5 ones, 8 hundredths = 15.08 (Note the 0 in the tenths place as there are no tenths)

c) 0 ones, 4 tenths, 9 thousandths = 0.409 (Note the 0 in the hundredths place)

Example 9 : Converting Units of Length

a) Convert 45 mm to cm.

b) Convert 3.2 meters to cm.

c) Convert 75 cm to meters.

Solution:

a) We know 1 cm = 10 mm. So, to convert mm to cm, divide by 10.

$45 \text{ mm} = \frac{45}{10} \text{ cm} = 4.5 \text{ cm}.$

b) We know 1 m = 100 cm. So, to convert meters to cm, multiply by 100.

$3.2 \text{ m} = 3.2 \times 100 \text{ cm} = 320 \text{ cm}.$

c) We know 1 m = 100 cm. So, to convert cm to meters, divide by 100.

$75 \text{ cm} = \frac{75}{100} \text{ m} = 0.75 \text{ m}.$

Example 10 : Converting Units of Weight

a) Convert 1250 grams to kilograms.

b) Convert 0.8 kilograms to grams.

Solution: a) We know 1 kg = 1000 g. So, to convert grams to kilograms, divide by 1000.

$1250 \text{ g} = \frac{1250}{1000} \text{ kg} = 1.250 \text{ kg}.$

b) We know 1 kg = 1000 g. So, to convert kilograms to grams, multiply by 1000.

$0.8 \text{ kg} = 0.8 \times 1000 \text{ g} = 800 \text{ g}.$

Example 11 : Converting Money

a) Convert 65 paise to rupees.

b) Convert ₹15.20 to paise.

Solution:

a) We know ₹1 = 100 paise. So, to convert paise to rupees, divide by 100.

65 paise = $\frac{65}{100}$ rupees = (₹)0.65.

b) We know ₹1 = 100 paise. So, to convert rupees to paise, multiply by 100.

₹15.20 = $\frac{15.20}{100}$ × 100 paise = 1520 paise.

Example 12: A scooter uses 2.35 litres of petrol for 1 day. In 1 week, it uses 16.45 litres of petrol. A car uses 35.8 litres in the same time.

Questions:

1. How much petrol does the scooter use in 3 days?
2. How much more petrol does the car use than the scooter in 1 week?
3. Write 2.35 litres in millilitres.
4. Round 16.45 litres to the nearest litre.

Solution:

1. Petrol used by the scooter in 3 days
   Per day = 2.35 L
   For 3 days: 2.35 × 3 = 7.05 L
2. How much more the car uses than the scooter in 1 week
   Car in a week = 35.8 L
   Scooter in a week = 16.45 L
   Difference: 35.8 − 16.45 = 19.35 L
3. Convert 2.35 litres to millilitres
   1 L = 1000 mL
   2.352.35 L = 2.35 × 1000 = 2350 mL

Activity

Decimal Place Value Card Sort

Objective: To reinforce understanding of decimal place values and their corresponding fractional values.

Materials: Index cards or small slips of paper. Each set of cards should include:

- Decimal numbers (e.g., 0.3, 0.07, 0.45, 1.2, 2.08, 0.006, 3.125)
- Corresponding fractional forms (e.g., $\frac{3}{10}, \frac{7}{100}, \frac{45}{100}, \frac{12}{10}, \frac{208}{100}, \frac{6}{1000}, \frac{3125}{1000}$)
- Corresponding word forms (e.g., "three tenths", "seven hundredths", "forty-five hundredths", "one and two tenths", "two and eight hundredths", "six thousandths", "three and one hundred twenty-five thousandths")

Preparation: Prepare sets of these cards. Each set should have 5-7 different decimal numbers with their corresponding fraction and word cards.

Card Sort:

- Students work in small groups.
- Each group receives a shuffled set of cards.
- Their task is to sort the cards into sets of three (decimal, fraction, word form) that represent the same value.
- Encourage discussion within groups about why certain cards match.

Verification and Discussion:

- Once groups have sorted their cards, they can check their answers (e.g., using a provided answer key or by having groups swap and check each other's work).
- Discuss common challenges: "Why is 0.06 different from 0.6?" "How do you know if it's hundredths or thousandths from the decimal?"

Extension: Challenge students to create their own sets of decimal, fraction, and word cards for classmates to sort.

Knowledge Checkpoint

- In the number 12.345, what is the place value of the digit '5'?
- Write "fifty-two hundredths" as a decimal.
- Convert 250 grams to kilograms.

Key Terms

- **Place Value:** The value of a digit based on its position in a number.
- **Thousandth:** One of one thousand equal parts of a whole. Written as $\frac{1}{1000}$ or 0.001.
- **Metric System:** A system of measurement based on units of 10, 100, 1000, etc., used for length (meter), mass (gram), and capacity (liter).
- **Conversion Factor:** The number used to multiply or divide when changing from one unit of measurement to another.

Do It Yourself

If you have a digital thermometer that reads temperature to one decimal place (e.g., 37.5°C), what is the smallest temperature change it can show? What if it read to two decimal places (e.g., 37.52°C)? Why might a doctor need a thermometer that is more precise (shows more decimal places) than one used for cooking?

Fact Flash

- The metric system, which uses decimals for conversions, was first proposed in France in the late 18th century to standardize measurements and simplify trade.
- The word "**millimeter**" literally means "**thousandth of a meter**" (milli- means thousandth). Similarly, "**centi**-" means hundredth, and "**deci**-" means tenth.
- Some ancient civilizations, like the Babylonians, used a base-60 number system, which also had ways to represent fractions, but it was much more complex than our base-10 decimal system!

Mental Mathematics

- How many millimeters are in 0.5 cm?
- Convert 1.5 meters to centimeters.
- How many paise are in ₹0.75?
- What is 0.01 times 10?
- If you have 250 grams, how many kilograms is that (as a decimal)?

Gap Analyzer™ Homework

Watch Remedial

Exercise 3.3

1. **Practice Questions: Unit Conversions**

   a) Convert 560 cm to meters.
   b) Convert 4.75 liters to milliliters.
   c) Convert 90 minutes to hours.
   d) Convert 0.25 km to meters.
   e) Convert 750 grams to kilograms.
   f) Convert ₹12.50 to paise.
   g) Convert 180 seconds to minutes.
   h) Convert 3.2 meters to centimeters.
   i) Convert 4500 ml to liters.
   j) Convert 65 paise to rupees.

2. **A shopkeeper weighs sugar as 2.5 kg. He adds another packet of 1.75 kg. Later, a customer buys 0.8 kg from it.**

   Questions

   i. What is the total sugar in both packets?
   ii. How much sugar is left after selling 0.8 kg?
   iii. Express 1.75 kg in grams.
   iv. Round 2.5 kg to the nearest whole number.

3. **Estimation Questions (Real-life Estimation)**

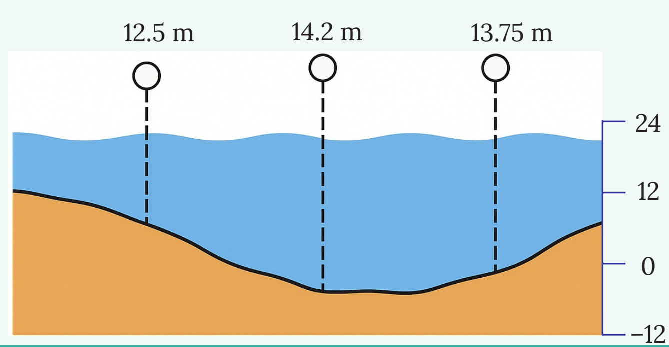
   a) A road sign says the next town is 15.9 km away. Approximately how far is it in whole kilometers?
   b) A bottle of juice contains 0.98 liters. Is this closer to 1 liter or 0.5 liters?
   c) If a person's height is 1.72 meters, approximately what is their height in whole meters?
   d) A small packet of spices weighs 0.045 kg. Is this closer to 0 kg or 0.1 kg?
   e) A movie ticket costs ₹185.50. Approximately how much does it cost in whole rupees?

4. A water tank has a capacity of 1200 liters. If it is 0.75 full, how many liters of water are currently in the tank?

5. The depth of a river at three points is 12.5 m, 14.2 m, and 13.75 m.

   Questions

   i. Arrange the depths in descending order.
   ii. Write 14.2 as a mixed fraction (tenths).
   iii. Round 13.75 to 1 decimal place.
   iv. Find the range of depths.



6. Convert 750 paise to rupees. If you have ₹8.25, how much more money do you need to have exactly ₹10? Express your answer in rupees.

7. **Miscellaneous Questions**
   a) Write the decimal number for "seven tens, three ones, zero tenths, five hundredths, and one thousandth."
   b) What is the place value of the digit '6' in the number 23.469?
   c) Arrange the following in descending order: 0.123, 0.132, 0.012, 0.213.
   d) A piece of wood is 2.5 meters long. How many pieces of 0.5 meters can be cut from it?
   e) If a bag of sugar weighs 5.005 kg, what is its weight in grams?

Locating and Comparing Decimals

Just like whole numbers, decimal numbers can be placed on a number line. This visual representation helps us understand their order and relative size. When comparing two decimals, we need a systematic approach to determine which one is larger or smaller. This involves looking at their place values, starting from the largest. Once we can confidently locate and compare decimals, we can move on to performing basic arithmetic operations: addition and subtraction. These operations follow rules similar to whole numbers, with the crucial addition of aligning the decimal points to ensure correct place value addition or subtraction.

Sub-concepts to be covered

- Locating Decimals on a Number Line.
- Comparing Decimals (using place value and trailing zeros).
- Addition of Decimals.
- Subtraction of Decimals.
- Estimating Sums and Differences.

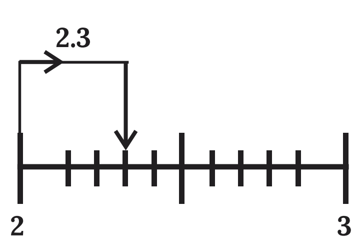
Mathematical Explanation

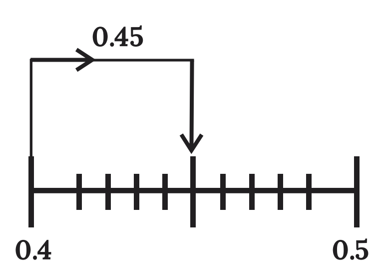
Locating Decimals on a Number Line

A number line is a straight line on which numbers are placed at equal intervals. Decimals can be located on a number line by dividing the segments between whole numbers into tenths, then hundredths, and so on. This helps visualize their position and relative value.

Examples and Illustrations:

- **To locate 2.3:** Find 2 on the number line. Divide the segment between 2 and 3 into 10 equal parts. Count 3 parts from 2.
- **To locate 0.45:** Find 0.4 on the number line. Divide the segment between 0.4 and 0.5 into 10 equal parts (representing hundredths). Count 5 parts from 0.4.





Key points: The number line provides a visual representation of decimal values and their order. Magnification helps locate more precise decimal values.

Comparing Decimals (using place value and trailing zeros)

To compare two decimal numbers, follow these steps:

1. Compare the whole number parts: The number with the larger whole number part is greater.
2. If whole parts are equal, compare the tenths digits: The number with the larger tenths digit is greater.
3. If tenths are equal, compare the hundredths digits: The number with the larger hundredths digit is greater.
4. Continue this process for subsequent decimal places (thousandths, etc.) until a difference is found.

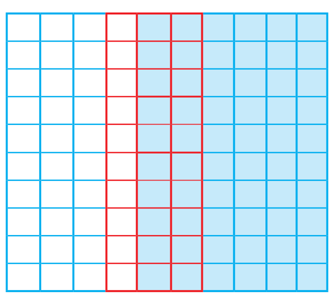
- **Using Trailing Zeros:** It's often helpful to add trailing zeros to the decimal with fewer decimal places so that both numbers have the same number of decimal places. This makes comparison easier by aligning the place values. For example, to compare 3.4 and 3.38, rewrite 3.4 as 3.40. Now compare 3.40 and 3.38.

  Key points: Compare digits from left to right, starting with the largest place value. Trailing zeros can simplify comparison.

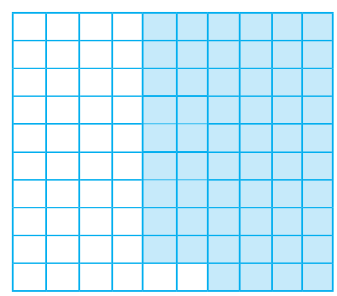
  Examples and Illustrations:

- **Compare**: A visual comparison of two decimals (e.g., 0.60 and 0.58) using two 10 × 10 grids. Shade 60 squares for 0.60 and 58 squares for 0.58 to clearly show which is larger.

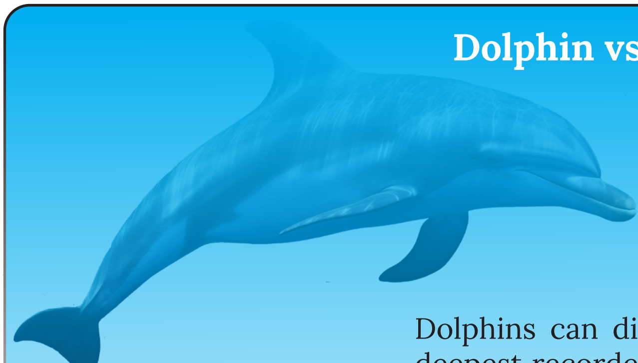
0.60

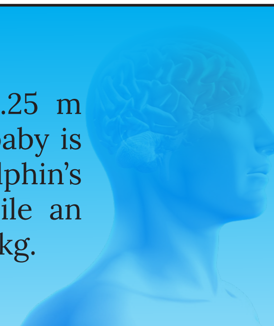


0.58

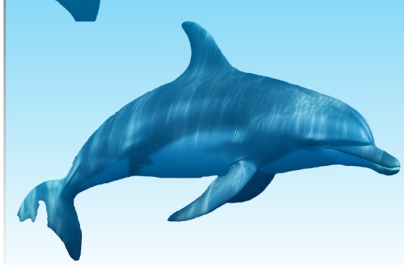


Dolphin vs Human





A newborn dolphin is about 1.25 m long, while a newborn human baby is about 0.50 m long. An adult dolphin's brain weighs about 1.6 kg, while an average human brain weighs 1.4 kg.



Dolphins can dive as deep as 299.5 m, but the deepest recorded human free dive is about 214.0 m. A dolphin can stay underwater for 9.8 minutes, while an average human can hold breath for 2.5 minutes.

Dolphins swim at speeds of up to 36.8 km/h, while the fastest human swimmer reaches about 9.7 km/h.

Addition of Decimals

To add decimal numbers:

1. **Align the decimal points:** This is the most crucial step. Write the numbers one below the other so that their decimal points are in a vertical line. This ensures that digits of the same place value are aligned.
2. **Add trailing zeros (optional but recommended):** If the numbers have different numbers of decimal places, add zeros to the end of the shorter decimal(s) so they all have the same number of decimal places. This helps prevent errors.
3. **Add as you would with whole numbers:** Start adding from the rightmost column (smallest place value).
4. **Place the decimal point:** Bring down the decimal point directly into the sum, aligning it with the decimal points above.

Examples and Illustrations:

- 12.34 + 5.6

```
   12 . 34
 +  5 . 60  (added trailing zero)
 ---------
   17 . 94
 ---------
```

- 0.75 + 1.25

```
    0 . 75
 +  1 . 25
 ---------
    2 . 00
 ---------
```

Key points: Align decimal points. Add from right to left. Place decimal point in the sum.

Common errors: Not aligning decimal points, leading to incorrect sums.

Subtraction of Decimals

To subtract decimal numbers:

1. **Align the decimal points:** Similar to addition, align the decimal points vertically.
2. **Add trailing zeros (optional but recommended):** If the numbers have different numbers of decimal places, add zeros to the end of the shorter decimal(s) so they all have the same number of decimal places. This is especially important in subtraction to avoid errors when borrowing.
3. **Subtract as you would with whole numbers:** Start subtracting from the rightmost column. Borrow from the left if necessary, just like with whole numbers.
4. **Place the decimal point:** Bring down the decimal point directly into the difference, aligning it with the decimal points above.

Examples and Illustrations:

(i) 25.8 - 12.35

```
   25 . 80  (added trailing zero)
 - 12 . 35
 ---------
   13 . 45
 ---------
```

(ii) 10.00 - 0.75

```
   10 . 00
 -  0 . 75
 ---------
    9 . 25
 ---------
```

Key points: Align decimal points. Add trailing zeros if needed. Subtract from right to left. Place decimal point in the difference.

Common errors: Not aligning decimal points. Incorrect borrowing across the decimal point.

Estimating Sums and Differences

Estimating sums and differences of decimals involves rounding the decimal numbers to the nearest whole number or to a specific decimal place before performing the operation. This helps in quickly checking if the calculated answer is reasonable.

Examples and Illustrations:

- Estimate 12.7 + 5.2: Round 12.7 to 13, and 5.2 to 5. Estimated sum: 13 + 5 = 18. (Actual sum: 17.9)
- Estimate 20.15 - 8.9: Round 20.15 to 20, and 8.9 to 9. Estimated difference: 20 - 9 = 11. (Actual difference: 11.25)

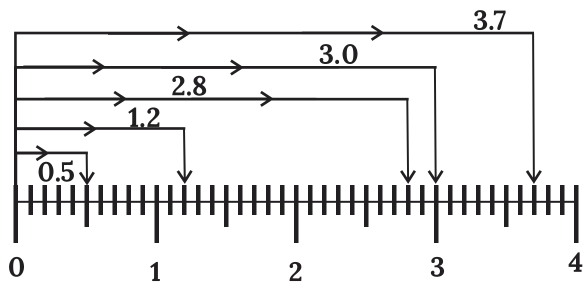
Key points: Rounding simplifies calculations and provides a quick check for accuracy.

Common errors: Rounding incorrectly or not understanding when to round up or down.

Example 13 : Mark the following decimal numbers on a number line: 0.5, 1.2, 2.8, 3.0, 3.7.

Solution: A number line from 0 to 4.

- Mark 0, 1, 2, 3, 4 with larger ticks.
- Between each whole number, draw 9 smaller ticks to represent tenths.
- Place a dot and label for:
- 0.5 (halfway between 0 and 1)
- 1.2 (2 ticks after 1)
- 2.8 (2 ticks before 3)
- 3.0 (at the 3 mark)
- 3.7 (7 ticks after 3)



Example 14 : Comparing Decimals

Which is greater:

a) 4.15 or 4.51? b) 0.9 or 0.89? c) 12.001 or 12.010?

Solution: **a) Compare 4.15 and 4.51:**

- Whole number parts are equal (4).
- Compare tenths digits: 1 (in 4.15) vs 5 (in 4.51). Since 5 > 1, 4.51 is greater.
- So, 4.51 > 4.15.

b) Compare 0.9 and 0.89:

- Add a trailing zero to 0.9 to make it 0.90. Now compare 0.90 and 0.89.
- Whole number parts are equal (0).
- Compare tenths digits: 9 (in 0.90) vs 8 (in 0.89). Since 9 > 8, 0.90 is greater.
- So, 0.9 > 0.89.

c) Compare 12.001 and 12.010:

- Whole number parts are equal (12).
- Tenths digits are equal (0).
- Compare hundredths digits: 0 (in 12.001) vs 1 (in 12.010). Since 1 > 0, 12.010 is greater.
- So, 12.010 > 12.001.

Example 15 : Find the sum of 7.85, 12.3, and 0.09.

Solution: Align the decimal points and add trailing zeros to ensure all numbers have the same number of decimal places (two in this case):

$$\begin{array}{r} 7.85 \\ 12.30 \\ +\ 0.09 \\ \hline \mathbf{20.24} \\ \hline \end{array}$$

- **Hundredths:** 5 + 0 + 9 = 14. Write 4, carry over 1.
- **Tenths:** 8 + 3 + 0 + 1 (carry-over) = 12. Write 2, carry over 1.
- **Ones:** 7 + 2 + 0 + 1 (carry-over) = 10. Write 0, carry over 1.
- **Tens:** 1 + 1 (carry-over) = 2. Write 2.

The sum is 20.24.

Example 16 : Ribbons of length 3.45 m, 2.75 m, and 4.25 m are cut for decoration.

i. Arrange 3.45 m, 2.75 m, 4.25 m in ascending order.

ii. What is the total length of all ribbons?

iii. If one piece of 2.15 m is used, how much length is left from total?

Solution:

1. Compare decimal numbers:
   - 2.75 has the smallest whole part (2) → smallest.
   - Between 3.45 and 4.25, clearly 3.45 < 4.25.

   Ascending order = 2.75 m, 3.45 m, 4.25 m
2. What is the total length of all ribbons?

   3.45 + 2.75 + 4.25

   Step 1: 3.45 + 2.75=6.20

   Step 2: 6.20 + 4.25 = 10.45

   Total length = 10.45 m
3. If one piece of 2.15 m is used, how much length is left from total?

   Total length = 10.45 m

   Used = 2.15 m

   10.45 − 2.15 = 8.30

   Remaining length = 8.30 m (or 8.3 m)

Knowledge Checkpoint

- Place 1.7 on a number line.
- Which is smaller: 0.25 or 0.3?
- Calculate: 4.5 + 2.35.
- Calculate: 10.0 - 3.7.

Think About It!

You are planning a road trip. Your car's fuel tank holds 40.0 liters. If you start with 15.5 liters and then add 20.75 liters, will your tank be full? How much more fuel do you need, or how much extra do you have? How would you estimate this quickly before doing the exact calculation?

Activity

Decimal Race on the Number Line

Objective: To practice locating and comparing decimals on a number line and reinforce addition/subtraction.

Materials: Large number line drawn on the floor or a long strip of paper (0 to 10, marked with tenths), dice (one standard 6-sided, one custom with +0.1, +0.2, -0.1, -0.2, +0.5, -0.5), small markers for each player.

Setup:

- Players place their markers at 0 on the number line.
- Decide on a target number (e.g., 5.0 or 7.0).

Gameplay:

- Players take turns rolling both dice.
- The standard die determines the number of moves (e.g., 1 to 6).
- The custom die determines the decimal increment/decrement (e.g., +0.1, -0.2).
- **Players calculate their new position:** Current Position + (Standard Die Roll × Custom Die Value). For example, if at 1.5, roll 3 and +0.2, new position is 1.5 + (3 × 0.2) = 1.5 + 0.6 = 2.1
- If a move takes them exactly to the target number, they win. If they overshoot, they must wait for a turn where they can land exactly on the target.

Discussion:

- Discuss strategies for landing exactly on the target.
- Ask players to compare their current position with another player's position.

Extension: Introduce a third die for hundredths (+0.01, -0.01, etc.) for more advanced players.

Key Terms

- **Number Line:** A line on which numbers are represented as points, ordered by their value.
- **Comparing Decimals:** Determining which of two decimal numbers is greater or smaller.
- **Aligning Decimal Points:** The crucial step in adding and subtracting decimals, where the decimal points are placed directly below each other.
- **Trailing Zeros:** Zeros added to the end of a decimal number (after the last non-zero digit) without changing its value, often used for alignment.

Fact Flash

- The first known use of the decimal point in print was in 1593 by **Christopher Clavius**, a German Jesuit mathematician.
- In some countries, a comma is used instead of a decimal point (e.g., 3,14 for pi). This is called a **decimal comma**.
- The concept of "**significant figures**" in science and engineering is closely related to decimals, indicating the precision of a measurement.

Mental Mathematics

- 0.6 + 0.3 =
- Which is greater: 0.4 or 0.40?
- If you have ₹5.50 and spend ₹2.00, how much is left?
- 1.0 - 0.2 =
- Estimate 5.9 + 2.1.

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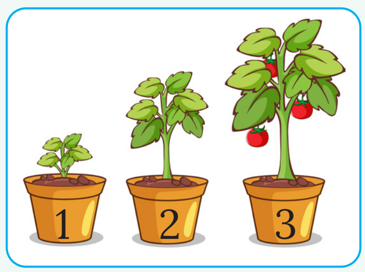
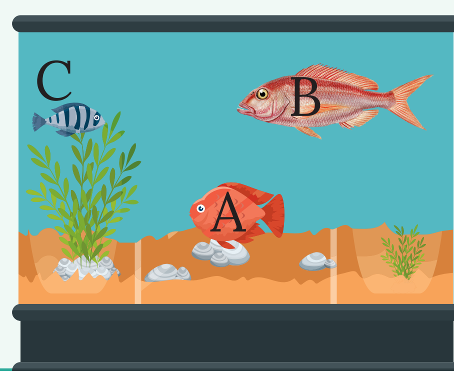
Exercise 3.4

1. **Simplifying the following questions:**
   a) Calculate the sum of 12.345 and 6.78.
   b) Add 0.057, 1.2, and 15.34.
   c) Find the sum: 20.009 + 5.6 + 0.12.
   d) Subtract 8.95 from 23.4.
   e) Calculate: 100.0 - 75.63.
   f) Find the difference between 9.001 and 3.56
2. **Word Problems (Real-life Context)**
   a) A cyclist covered 15.7 km on Monday and 18.9 km on Tuesday. What is the total distance covered in two days?
   b) A cylindrical container has a capacity of 2.5 liters. It currently contains 1.875 liters of water. How much more water is needed to fill the container completely? Express your answer in milliliters as well.
   c) A bag of apples weighs 2.35 kg, and a bag of oranges weighs 1.9 kg. Which bag is heavier, and by how much?
   d) A runner completed a race in 45.28 seconds. Another runner completed the same race in 44.9 seconds. What is the difference in their finishing times?
3. Point A is at 3.6 on a number line. Point B is 1.25 units to the right of Point A. Point C is 0.8 units to the left of Point B. What decimal number does Point C represent?
4. **Three friends recorded their long jump distances: Rahul jumped 4.15 meters, Priya jumped 3.9 meters, and Amit jumped 4.08 meters.**
   a) What is the total distance jumped by Rahul and Amit?
   b) Is this total distance more or less than twice Priya's jump distance?
5. The growth of a plant in a week is recorded as follows:
   Week 1: 3.25 m **Week 2:** 4.15 m **Week 3:** 2.75 m



   Questions:
   a) Arrange the growth amounts in ascending order.
   b) Find the total growth of the plant in three weeks.
   c) Round each week's growth to the nearest whole meter.
   d) If the plant grows another 1.35 m next week, what will be its total height?
6. Lengths of three fish in a Aquarium Fish tank (in cm):
   Fish A: 3.45 cm
   Fish B: 4.15 cm
   Fish C: 2.75 cm



   Questions:
   a) Arrange the lengths in ascending order.
   b) Find the total length of all fish.
   c) If Fish B grows 0.25 cm more, what is its new length?
   d) Round each length to the nearest centimeter.

Common Misconceptions

Misconception: 0.5 is smaller than 0.25 because 5 is less than 25
Correction: In decimals, the place value after the decimal point matters.
✓ 0.5 is greater than 0.25 because 0.5 = 50/100 and 0.25 = 25/100.

Misconception: Adding decimals is just like whole numbers (line up digits instead of decimal points)
Correction: When adding or subtracting decimals, always line up the decimal points, not just the digits.
✓ Write numbers in place value columns (tenths, hundredths, etc.) for accuracy.

Misconception: More digits after the decimal point means a larger number
Correction: The value of a decimal depends on place value, not the number of digits.
✓ 0.9 is greater than 0.89, even though 89 has more digits

Real-Life Decimals: Mathematical Application

Decimals are essential for precise measurements and calculations in everyday life, extending beyond whole numbers. These applications make decimals relevant, fostering accuracy and practical problem-solving, aligning with NEP 2020's focus on experiential learning:

1. **Money & Transactions:** Calculating costs, change, and currency exchange rates, e.g., ₹50.50 + ₹25.75. (Fundamental for financial literacy).
2. **Measurements (Length, Weight, Capacity):** Recording and comparing precise values like heights, weights, or liquid volumes, e.g., 1.75 meters or 2.3 kg. (Crucial for science, engineering, and daily tasks).
3. **Sports Statistics:** Analyzing performance metrics such as batting averages, race times, or scores, e.g., 9.81 seconds for a sprint. (Quantifies and compares achievements).
4. **Weather & Temperature:** Expressing precise temperatures, rainfall, or wind speeds, e.g., 32.5°C or 15.2 cm of rain. (Helps understand environmental data).
5. **Discounts & Percentages:** Calculating exact discount amounts or tax, as percentages often result in decimal values, e.g., 15% of ₹250.00 is 0.15 × 250. (Enables smart purchasing decisions).

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EXERCISE

A. Multiple Choice Questions (MCQs)

1. What is the decimal form of $\frac{45}{100}$?
   a) 4.5 ☐ b) 0.45 ☐ c) 45.0 ☐ d) 0.045 ☐
2. Which of the following is the smallest decimal number?
   a) 0.8 ☐ b) 0.08 ☐ c) 0.88 ☐ d) 0.808 ☐
3. The sum of 12.3 + 5.75 is:
   a) 18.05 ☐ b) 18.5 ☐ c) 18.08 ☐ d) 18.55 ☐
4. How many centimeters are there in 2.5 meters?
   a) 25 cm ☐ b) 250 cm ☐ c) 0.025 cm ☐ d) 2500 cm ☐
5. If you subtract 3.4 from 7.0, the result is:
   a) 3.6 ☐ b) 4.6 ☐ c) 3.4 ☐ d) 10.4 ☐

Assertion & Reason

Direction: In the following questions, a statement of Assertion (A) is given, followed by a corresponding statement of Reason (R). Choose the correct option.

a) Both A and R are true, and R is the correct explanation of A.
b) Both A and R are true, but R is not the correct explanation of A.
c) A is true, but R is false.
d) A is false, but R is true.

1. **Assertion (A):** 0.7 is greater than 0.65.
Reason (R): When comparing decimals, we compare the digits from left to right, and 7 in the tenths place is greater than 6 in the tenths place.

2. **Assertion (A):** The sum of 2.5 and 3.5 is 6.0.
Reason (R): When adding decimals, we add the whole number parts and the fractional parts separately.

3. **Assertion (A):** 1 meter is equal to 1000 millimeters.
Reason (R): The metric system is based on powers of 10, and 'milli' means one thousandth.

Case Study

The annual school sports day was held, and the results for the 100-meter sprint were very close.

- Runner A finished in 13.25 seconds.
- Runner B finished in 13.09 seconds.
- Runner C finished in 13.20 seconds.
- Runner D finished in 13.19 seconds.

a) Who won the 100-meter sprint? Justify your answer.
b) What is the difference in time between the first and second-place runners?
c) If Runner E finished 0.05 seconds faster than Runner A, what was Runner E's time?
d) Arrange all five runners' times (A, B, C, D, and E) in ascending order.

Project

The Ultimate Sports Analyst

Objective: This project taps into students' interest in sports to analyze and compare athletic performance using decimals. It reinforces the idea that decimals are crucial for measuring performance with high precision.

Materials Needed:

- Access to the internet (for sports statistics websites like official Olympics, ESPN, etc.).
- Poster board or digital tool to create an infographic (like Canva).
- Pen, paper, and markers.

Step-by-Step Procedure:

1. Choose Your Arena: Select a sport where results are measured in decimals. Good examples include:
   - Athletics: 100m sprint (e.g., 9.58 s), Long Jump (e.g., 8.95 m).
   - Swimming: 100m Freestyle (e.g., 46.91 s).
   - Formula 1 Racing: Lap times (e.g., 1:21.046 minutes, which can be converted to seconds).
2. Research the Champions: Find the results for the top 5 athletes in a specific event (e.g., the 100m final at the last Olympics). Record their names, countries, and times/distances.

3. Rank the Athletes: Create a results table and list the athletes in order from 1st to 5th place. This requires careful comparison of decimals.
4. Analyze the Difference: Calculate the difference in time/distance between:
   - The 1st place winner and the 2nd place winner.
   - The 1st place winner and the 5th place finisher.
   - This shows how small decimal differences can separate a champion from the rest!
5. Create an Infographic: Design a visually appealing poster or digital infographic titled "A Race to the Hundredths!" or similar. It should include:
   - The name of the event.
   - Your ranked table of the top 5 athletes.
   - Highlight the calculated differences with interesting call-outs (e.g., "Only 0.12 seconds between Gold and Silver!").
   - Pictures of the athletes or sport.

Source-Based Question

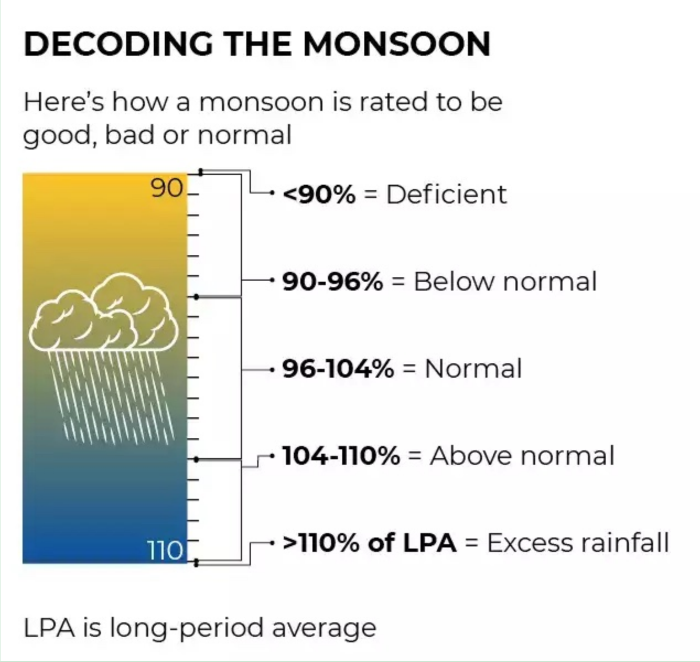
Analyzing India's Monsoon Rainfall

Directions: The monsoon is crucial for India's agriculture and water supply. The India Meteorological Department (IMD), a Government of India agency, tracks rainfall across the country. The table below shows a sample of the actual rainfall received in four major cities during a specific monsoon period, compared to what is considered the normal rainfall for that period. All measurements are in millimeters (mm).

Monsoon Rainfall Data for Major Cities (in mm)

City	Actual Rainfall (mm)	Normal Rainfall (mm)
Mumbai	985.4	840.7
Delhi	632.8	653.6
Chennai	480.5	447.1
Kolkata	1150.2	1020.9

Source Text: Adapted from data provided by the India Meteorological Department (IMD), Government of India.



Questions on the Data

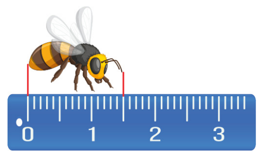
1. Which of the four cities received the highest actual rainfall during this period? Which city received the lowest?
2. Rainfall is considered 'surplus' if the actual rainfall is more than the normal rainfall, and 'deficient' if it is less. How much surplus rainfall (in mm) did Kolkata receive compared to its normal rainfall?
3. What was the total actual rainfall received by Delhi and Chennai combined? Was this combined amount greater or less than the actual rainfall received by Mumbai alone?
4. We know that 1 centimeter (cm) = 10 millimeters (mm). Convert Mumbai's actual rainfall from millimeters (mm) to centimeters (cm).
5. Based on the data, identify which of the four cities experienced 'deficient' rainfall (where actual rainfall was less than normal rainfall).

Mind Map

Decimals: A Peek Beyond the Point

A Tenth Part

- **Forms:**
- Fraction: $\frac{1}{10}$
- Mixed: $3\frac{4}{10}$
  - ✓ Improper: $\frac{34}{10}$
  - ✓ Decimal: 3.4 (3 = ones, . = point, 4 = tenths)
- **Operations:** Addition, Subtraction



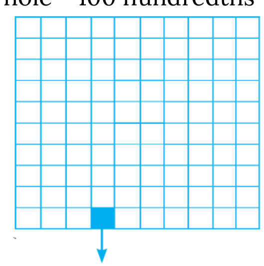
Decimal Place Value & Units of Measurement (System & Application)

- **System:** Ones, Tenths, Hundredths ...
- **Zeros:** Trailing zeros don't change value (e.g., 2.5 = 2.50)
- **Applications:** Metric conversions in length, mass, money

Hundreds	Tens	Ones	Decimal	Tenths	Hundredths	Thousandths
H	T	O	.	t	h	th
Whole Part				Decimal Part		

A Hundredth Part (Increasing Precision)

- **Relation:** 1 tenth = 10 hundredths; 1 whole = 100 hundredths
- **Forms:**
  - ✓ **Fraction:** $\frac{1}{100}$
  - ✓ **Decimal:** 0.01
  - ✓ **Example:** $5.23 = 5 + \frac{2}{10} + \frac{3}{100}$
- **Operations:** Addition, Subtraction



Locating and Comparing Decimals (Ordering and Operations)

- **On Number Line:**
  - ✓ Between whole numbers → divide into tenths → divide further into hundredths
- **Comparing:**
  - ✓ Whole parts
  - ✓ Tenths
  - ✓ Hundredths ...
- **Operations:** Addition, Subtraction, Estimation