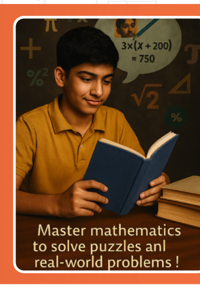
2 Arithmetic Expressions

Why This Chapter Matters

Ever calculated snack costs or playtime? You've used arithmetic expressions! Buy 3 comics (₹50 each) and a ₹20 pen – how to find the total cost in one step? This chapter decodes "arithmetic expressions", our special 'secret codes' for calculations. Master them to solve puzzles and real-world problems like party planning or game scores!



Meet EeeBee.AI

Hello, explorers! I'm EeeBee, your friendly guide on this mathematical adventure! I love numbers and figuring out puzzles. Arithmetic expressions are like the secret language of math, and I'm here to help you decode them. I'll pop in with questions, handy hints, fun facts, and maybe even a challenge or two. Whenever you see me, get ready to think and explore. Let's make learning about arithmetic expressions exciting together!

Learning Outcomes

By the end of this chapter, you will be able to:

- Define arithmetic expressions and identify their components (e.g., terms).
- Evaluate expressions accurately using BODMAS/PEMDAS, including brackets.
- Translate word problems and real-life situations into arithmetic expressions.
- Compare the values of different expressions using <, >, and =.
- Apply properties (commutative, associative, distributive) to simplify and solve.
- Simplify expressions by correctly removing brackets and handling sign changes.
- Create multiple arithmetic expressions that evaluate to the same value.

From Last Year's Notebook

- You mastered operations (+, –, ×, ÷) with whole numbers, fractions, decimals, and met early algebra!
- Now, we'll combine these operations in new, exciting ways.
- Learn math's "grammar" (like BODMAS/PEMDAS) to ensure everyone gets the same answer.
- It's like moving from simple number sentences to powerful mathematical statements!

Real Math, Real Life

Arithmetic expressions are not just numbers and symbols; they are tools we use every single day!

- **Shopping Smarts:** Use them to calculate total costs, especially with discounts (like "50% off, then an extra ₹100 off").
- **Planning Trips:** Figure out trip expenses (fuel, food, tickets) or total travel time.
- **Cooking & Baking:** Easily adjust recipes (like "double the recipe, then add 2 extra eggs").

- **Gaming:** Understand video game scores and complex scoring systems.
- **Instructions:** Follow building instructions for furniture or models.
- **Science & Tech:** Used in scientific formulas and even in computer programming!
- **Making Sense of Numbers:** They help us organize and solve problems involving quantities and operations in a clear way.

Quick Prep

1. If you have 50 rupees and spend 18 rupees, how much is left? (Real-world)
2. If 4 friends share 24 sweets equally, how many does each get?
3. Write a number phrase for "10 more than 23."
4. If a bus travels 40 km every hour, how far does it travel in 3 hours?

Introduction

Welcome to simple expressions – your first step into math phrases! Think of "15 + 7" like a short sentence giving a clear numerical value. In this section, you'll learn to define these expressions, find their value, write them for everyday situations, and compare them using <, >, or =. Mastering these basics is key for tackling complex calculations and understanding how math describes the world. Let's start translating words into numbers and discovering their meaning!

Chapter Overview

Navigating Arithmetic Expressions:

- Start by defining arithmetic expressions, their components (like terms), how to read them, and find their value.
- Learn to compare expressions using <, >, and =, and understand why rules (BODMAS/PEMDAS) are crucial for clarity with multiple operations.
- Explore expression structure, identify terms (including subtraction as adding an inverse), and grasp the order of operations.
- Master using and removing brackets, correctly handling sign changes, and applying properties like commutative, associative, and distributive.
- Ultimately, aim to evaluate and create expressions accurately and efficiently to solve real-world problems.

From History's Pages

Basic arithmetic operations evolved over thousands of years across civilizations like ancient Egyptians and Babylonians. Our modern symbols (+, -, ×, ÷) were standardized much later. The '+' and '-' signs first appeared in Germany in the late 15th century. The '×' symbol was popularized by William Oughtred in the 17th century, around when '÷' also gained traction; the '•' was also used. The concept of an "**expression**" developed alongside these notations, allowing clearer communication of calculations, with early algebra laying systematic groundwork.

What is an Arithmetic Expression?

An arithmetic expression is a mathematical phrase made up of numbers, at least one arithmetic operation (like +, -, ×, ÷), or both. It represents a single value. Even a single number can be considered a very simple expression, but usually, we look at combinations. These expressions are the building blocks for solving mathematical problems. They allow us to write down a calculation in a short, precise way.

- 13 + 7
- 25 – 9
- 6 × 8
- 45 ÷ 5

It's important to understand that an expression is the phrase itself (e.g., 13 + 7), while its value is the result of the calculation (e.g., 20). Different expressions can sometimes have the same value.
For instance, 10 + 2, 15 – 3, 6 × 2, and 24 ÷ 2 all have the value 12. This shows the flexibility of arithmetic expressions in representing numbers.

Writing expressions from word problems is a key skill. You need to identify the numbers involved and the operation(s) connecting them.

For example, If "Ravi has 15 marbles and gets 8 more," the expression for the total number of marbles Ravi has is 15 + 8.
If "Sita had 20 cookies and she gave away 5," the expression for the cookies left is 20 – 5.

Example 1 : Find the value of the expression 34 + 17.

Solution: This expression means 34 plus 17
Adding 34 and 17:

$$\begin{array}{r} 34 \\ +\ 17 \\ \hline 51 \\ \hline \end{array}$$

So, 34 + 17 = 51. The value of the expression is 51.

Example 2 : What is the value of 62 – 28?

Solution: This expression means "62 minus 28".
Subtracting 28 from 62:

$$\begin{array}{r} 62 \\ -\ 28 \\ \hline 34 \\ \hline \end{array}$$

So, 62 – 28 = 34. The value of the expression is 34.

Example 3 : Evaluate 9 × 7.

Solution: This expression means "9 times 7".

Multiplying 9 by 7:

9 × 7 = 63.

The value of the expression is 63.

Example 4 : Find the value of 72 ÷ 8.

Solution: This expression means "72 divided by 8".

Dividing 72 by 8:

72 ÷ 8 = 9.

The value of the expression is 9.

Example 5 : Priya buys 4 notebooks, and each notebook costs ₹30. Write an expression for the total amount she spends and find its value.

Solution: Priya buys 4 notebooks.

Each notebook costs ₹30.

To find the total cost, we need to multiply the number of notebooks by the cost of each notebook.

The expression is 4 × 30.

To find its value: 4 × 30 = 120.

Priya spends ₹120 in total.

Example 6 : Ahmed scored 85 marks in a test. Karan scored 12 marks less than Ahmed. Write an expression for Karan's score and find its value.

Solution: Ahmed's score = 85 marks.

Karan scored 12 marks less than Ahmed.

"**Less than**" indicates subtraction.

The expression for Karan's score is 85 – 12.

To find its value: 85 – 12 = 73.

Karan scored 73 marks.

Knowledge Checkpoint

- What is an arithmetic expression? Give an example.
- Find the value of the expression 14 × 3.
- Write an expression for "20 decreased by 12". What is its value?

Key Terms

- **Arithmetic Expression:** A combination of numbers, at least one arithmetic operator (like +, -, ×, ÷), or both, that represents a numerical value. Examples: 7 + 3, 15 – 6 × 2.
- Value (of an expression): The single number that an arithmetic expression evaluates to. For example, the value of 7 + 3 is 10.
- Operator: A symbol representing a mathematical operation (e.g., +, -, ×, ÷).

Activity

My Favorite Number Expressions!

- Objective: To practice creating different arithmetic expressions that evaluate to the same number.
- Materials: Paper, pen/pencil.
- Steps:

1. Choose your favorite number between 10 and 50. Let's say you choose 20.
2. Now, try to write as many different arithmetic expressions as you can that have this number (20) as their value.
3. Use at least two numbers in each expression and any of the four operations (+, -, ×, ÷).
4. For example, for the number 20:
   10 + 10, 25 − 5, 4 × 5, 40 ÷ 2, 15 + 5, 30 − 10,2 × 10, 60 ÷ 3, 1 + 19, 100 ÷ 5
5. Challenge yourself: Can you use three numbers in an expression?
   (e.g., 2 + 8 + 10 = 20 or 5 × 2 × 2 = 20)

Discussion: Share your expressions with a classmate. Did you find any surprising ones? How many different expressions could you create? This shows that a single value can be represented in many mathematical ways.

Fact Flash

- The word "**arithmetic**" comes from the Greek word "**arithmos**," which means "number."
- The equals sign (=) was invented by Robert Recorde in 1557. He chose two parallel lines because "**noe .2. thynges**, can be moare equalle" (**no two things can be more equal**).

Do It Yourself

Consider the number 18.

- How many different ways can you write an expression using exactly two numbers and one operation (+, -, ×, ÷) that results in 18? (e.g., 9 + 9, 20 − 2, 6 × 3, 36 ÷ 2).
- Can you use fractions or decimals in your expressions to get 18? (e.g., 17.5 + 0.5 or 36 × 0.5).

Mental Mathematics

- What is 12 + 8 ÷ 2 − 3?
- Calculate: (−5) × (−3) + 10
- If the temperature is −10° and it rises by 12°C, what is the new temperature?
- What is 40 ÷ (2 × 4)?
- Find the value of 7 × 6 − 20 + 5.

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Exercise 2.1

1. **Find the value of the expression**

a) 45 + 23 b) 81 − 37 c) 12 × 6 d) 96 ÷ 8 e) 58 + 35

f) 49 − 75 g) 15 × 5 h) 108 ÷ 9 i) 33 + 47 j) 84 ÷ 7

2. **Calculated values for Simple Expressions in Everyday Scenarios**

a) A baker bakes 6 trays of cookies, with 9 cookies on each tray. Write an expression for the total number of cookies baked and find its value.

b) Sunil had 150 stamps. He gave 35 stamps to his sister. Write an expression for the number of stamps Sunil has left and find its value.

c) There are 28 students in a class. If they are divided into 7 equal groups for a project, write an expression for the number of students in each group and find its value.

d) Maria bought a book for ₹120 and a pen for ₹25. Write an expression for the total amount she spent and find its value.

e) If a movie ticket costs about ₹180 and you buy 3 tickets, write an expression for the approximate total cost.

3. You have about ₹500 and want to buy a game that costs around ₹380. Write an expression for the approximate money left.

4. If a large pizza has 8 slices and you estimate you need 20 slices for a party, write an expression for approximately how many pizzas you need (think division).

5. **Exploring Simple Miscellaneous Expressions:**

a) Write three different arithmetic expressions that have a value of 24.

b) Can an arithmetic expression involve more than two numbers? Give an example.

c) If $x + 5 = 12$, what is the value of x? (Simple equation as expression context)

6. A fruit seller sold baskets of fruits. Each basket had 12 fruits. He sold 8 baskets in the morning. In the afternoon, he sold 5 baskets. Each fruit costs ₹15.

Write the expression and find the value of the following:

i. The total number of fruits sold in the morning

ii. The total number of fruits sold in the afternoon

iii. The total fruits sold in the whole day

iv. The money earned in the morning

v. The money earned in the whole day

Comparing Expressions

Just as we compare numbers to see which is larger or smaller, we can also compare arithmetic expressions. To do this, we first find the value of each expression and then compare these values. We use the comparison symbols:

- > (greater than)
- < (less than)
- = (equal to)

For example, To compare 7 + 5 and 10 + 1, we find their values: 7 + 5 = 12 and 10 + 1 = 11. Since 12 is greater than 11, we write 7 + 5 > 10 + 1.

Mathematical Explanation

Comparing expressions involves a two-step process:

1. **Evaluate each expression:** Calculate the numerical value of the expression on the left side of the comparison and the expression on the right side.
2. **Compare the values:** Use the appropriate symbol (<, >, or =) to show the relationship between the two values.

Let's look at some examples:

Comparing 15 − 6 and 4 + 3:

- Value of 15 − 6 = 9.
- Value of 4 + 3 = 7.
- Since 9 > 7, we write 15 − 6 > 4 + 3.

Comparing 3 × 8 and 30 − 6:

- Value of 3 × 8 = 24.
- Value of 30 − 6 = 24.
- Since 24 = 24, we write 3 × 8 = 30 − 6.

Comparing 20 ÷ 4 and 2 + 5:

- Value of 20 ÷ 4 = 5.
- Value of 2 + 5 = 7.
- Since 5 < 7, we write 20 ÷ 4 < 2 + 5.

Sometimes, we can compare expressions without fully calculating them, especially if they share common parts or if the changes are obvious. For example, consider comparing 100 + 50 and 100 + 60. Since both expressions start with 100 and 60 is greater than 50, we know that 100 + 50 < 100 + 60 without needing to find the exact sums.

This skill is useful in many situations, like determining which deal is better when shopping, or who scored more in a game.

Example 7 : Compare the expressions 18 + 9 and 30 − 5.

Solution: First, evaluate 18 + 9: 18 + 9 = 27.

Next, evaluate 30 − 5: 30 − 5 = 25.

Now compare the values 27 and 25.

Since 27 > 25, we can write 18 + 9 > 30 − 5.

Example 8 : Is 6 × 7 greater than, less than, or equal to 50 − 8?

Solution: Evaluate 6 × 7: 6 × 7 = 42.
Evaluate 50 − 8: 50 − 8 = 42.
Now compare the values 42 and 42.
Since 42 = 42, we can write 6 × 7 = 50 − 8.

Example 9 : Place the correct symbol (<, >, or =) in the blank: 45 ÷ 5 _____ 3 × 4.

Solution: Evaluate 45 ÷ 5: 45 ÷ 5 = 9.
Evaluate 3 × 4: 3 × 4 = 12.
Now compare the values 9 and 12.
Since 9 < 12, we fill the blank with <.
So, 45 ÷ 5 < 3 × 4.

Example 10 : Ria has 500 + 75 marbles. Sam has 500 + 60 marbles. Who has more marbles?

Solution: Ria's marbles: 500 + 75.
Sam's marbles: 500 + 60.
Both start with 500 marbles. Ria gets an additional 75, while Sam gets an additional 60.
Since 75 > 60, Ria gets more additional marbles.
Therefore, 500 + 75 > 500 + 60.
Ria has more marbles. We didn't need to calculate the exact totals (575 and 560) to know this.

Example 11 : Store A sells a toy for ₹300 − ₹20 (discount). Store B sells the same toy for ₹300 − ₹30. Which store sells it cheaper?

Solution: Store A's price: 300 − 20.
Store B's price: 300 − 30.
Both stores start with the same original price of ₹300.
Store A gives a discount of ₹20. Store B gives a discount of ₹30.
A larger discount means a lower final price.
Since Store B offers a larger discount (30 > 20), its final price will be lower.
So, 300 − 20 > 300 − 30.
Store B sells it cheaper.

Example 12 : Fill in the blank to make the expressions equal: 13 + 8 = _______ + 6.

Solution: First, evaluate the left side: 13 + 8 = 21.
So, we need the right side to also equal 21.
Let the blank be x. So, x + 6 = 21.
To find x, we can think: what number added to 6 gives 21?
x = 21 − 6
x = 15.
So, the blank should be 15.
Check: 15 + 6 = 21.
Thus, 13 + 8 = 15 + 6.

Knowledge Checkpoint

- Compare 22 − 7 and 5 × 3. Use <, >, or =.
- Is 40 + 15 greater than 60 − 10?
- Arrange these expressions from smallest to largest value: 2 + 3, 10 − 4, 2 × 2.

Activity

The Expression Race

- **Materials:** A six-sided die, paper and pen for each group, blackboard/whiteboard for scoring.
- **Setup (5 mins):**

Divide the class into four teams (A–D). Assign each team an expression:

Team A: x + 6 (steady growth)

Team B: 3x (multiplicative growth)

Team C: 12 − x (decreasing value)

Team D: 2x + 1 (mix of multiplication and addition)

The Race (15–20 mins):

A student rolls the die to give a value of x. Teams substitute x into their expression. The team with the highest result wins the round and scores 1 point.

Example (x = 4): A = 10, B = 12, C = 8, D = 9 → Team B wins.

Play 4–5 rounds, each time rolling for a new x.

- **Discussion (10 mins):**

  After the game, ask:
  - Did the same team always win? Why not?
  - How did Team C's score change with high vs. low x?
  - Which grows faster: x + 6 or 3x?
  - If x = 10, who would win? Why?

Key Terms

- **Greater Than (>):** A symbol used to indicate that the value on its left is larger than the value on its right.
- **Less Than (<):** A symbol used to indicate that the value on its left is smaller than the value on its right.
- **Equal To (=):** A symbol used to indicate that the values on both its sides are the same.
- **Comparison (of expressions):** The process of determining the relationship (greater than, less than, or equal to) between the values of two expressions.

Do It Yourself

Two ticket plans:

- **Plan A:** 50 + 12x (₹), **Plan B:** 30 + 15x (₹).
  Which is cheaper for small x? For large x? Find the break-even x.

Fact Flash

- The symbols < and > for "**less than**" and "**greater than**" were introduced by English mathematician Thomas Harriot in his book published posthumously in 1631.
- Before these symbols, people wrote out "**is greater than**" or used other notations!
- An alligator's mouth is sometimes used to remember the < and > signs – it always wants to eat the bigger number!

Mental Mathematics

Instructions: Solve these quickly in your head and compare the two expressions using <, >, or =.

- Compare: 7 + 2 × 3 and 20 – 5
- Compare: 12 – (–3) and (–2) × (–5)
- Compare: 25 ÷ 5 – 1 and 3 × 2
- Compare: 3 × (10 – 2) and 50 ÷ 2

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Exercise 2.2

1. **Calculated values for Simple Expressions in Everyday Scenarios:**
   a) Team A scored 25 + 10 points in a game. Team B scored 40 – 8 points. Which team scored more?
   b) Anil ran 3 × 2 kilometers. Sunita ran 10 – 3 kilometers. Who ran a longer distance?
   c) A red ribbon is 50 – 12 cm long. A blue ribbon is 15 + 22 cm long. Which ribbon is shorter?
   d) Shop X offers a discount of ₹100 – ₹15. Shop Y offers a discount of ₹50 + ₹30. Which shop offers a better discount (i.e., a larger amount reduced)?
   e) To bake a cake, Recipe 1 needs 2 + 3 cups of flour. Recipe 2 needs 6 – 2 cups of flour. Which recipe needs more flour?

2. Company A charges a fixed fee of ₹50 plus ₹8 per kilometer for a taxi ride. Company B charges a fixed fee of ₹30 plus ₹10 per kilometer.
   a) Write an expression for the total cost of a ride of k kilometers for Company A.
   b) Write an expression for the total cost of a ride of k kilometers for Company B.
   c) If you need to travel 10 kilometers, which company would be cheaper? Show your calculations.

3. **Solve the clue craft puzzle style:**
   i. If ☀️ = (15 + 25), and ☯️ = (60 ÷ 3), find ☀️ + ☯️.
   ii. If ☸️ = (40 × 2), and 🏺 = (90 – 30), compare ☸️ and 🏺.
   iii. If ⚽ = (50 ÷ 5), and ♦️ = (20 × 3), what is ⚽ + ♦️?

4. **Find the following problem puzzle questions**:
   a) Is 200 + 90 greater or less than 200 + 85?
   b) A flight ticket costs ₹4500 + ₹300 (taxes). Another flight costs ₹4500 + ₹250. Which is cheaper?
   c) You have ₹1000 – ₹150 after one purchase. Your friend has ₹1000 – ₹200 after their purchase. Who has more money left, assuming you both started with ₹1000?
   d) Compare 10 × 19 and 10 × 21. Which is larger?

5. Fill in the blank with a number to make the statement true: 15 + 7 > 20 + __. (Any number greater than or equal to 2 will make it false or equal, so any number 0 or 1).
6. A cinema hall sells balcony tickets at ₹120 each and normal tickets at ₹80 each. One family bought 3 balcony and 4 normal tickets. Another family bought 5 balcony and 7 normal tickets.

   Questions:

   1. Write the two expressions for the families' costs.
   2. Simplify each expression.
   3. Compare the values of the two expressions.
   4. Who spent more money?

Reading and Evaluating Complex Expressions

"Why can 10 + 5 × 2 give different answers like 30 or 20? This won't do in mathematics, where we need one correct value! This section tackles complex expressions, introducing the universal BODMAS/PEMDAS rules and the power of brackets. These tools ensure everyone calculates consistently. We'll also explore 'terms' and learn how properties like commutativity, associativity, and distributivity help us skillfully manipulate and simplify expressions."

Sub-concepts to be covered

1. Need for Rules and Brackets
2. Terms in Expressions
3. Order of Operations (DMAS/BODMAS/PEMDAS)
4. Evaluating Expressions with Multiple Operations

Mathematical Explanation

Need for Rules and Brackets

Why can an expression like 7 + 3 × 4 give different answers—40 (if adding first) or 19 (if multiplying first)? This confusion is why mathematicians follow a standard order of operations. **Brackets**—like parentheses **( )**, square brackets **[ ]**, or curly braces **{ }** —are vital tools to clarify this order, dictating exactly what to calculate first.

Consider the expression 30 + 5 × 4.

- If Rahul adds first, the expression must be written with brackets: (30 + 5) × 4 = 35 × 4 = 140.
- If Meena multiplies first (which aligns with the standard order), we can write 30 + (5 × 4) for extra clarity, resulting in 30 + 20 = 50. Without established rules and the use of brackets, ambiguity arises.

This is similar to language: "Rohan saw a man on a hill with a telescope." Who has the telescope? Commas can clarify. In mathematics, brackets serve this clarifying purpose, ensuring any operation inside them is performed first.

Always simplify the part inside brackets down to a single value before using that value in the rest of the expression. If you encounter nested brackets (brackets inside other brackets), like in 100 − (50 − (10 + 5)), solve the innermost part (10 + 5) first, then work your way outwards. Brackets are essential for removing ambiguity and ensuring one clear, correct interpretation of any expression.

Example 13 : Evaluate 15 + (6 × 3).

Solution: The expression has brackets. We must evaluate the part inside the brackets first.

Inside the brackets: 6 × 3 = 18.

Now substitute this value back into the expression: 15 + 18.

Perform the addition: 15 + 18 = 33.

So, 15 + (6 × 3) = 33.

Example 14 : Evaluate (25 – 10) × 2.

Solution: Evaluate the part inside the brackets first: 25 – 10 = 15.

Now substitute this value back: 15 × 2.

Perform the multiplication: 15 × 2 = 30.

So, (25 – 10) × 2 = 30.

Example 15 : Irfan wants to buy a comic book for ₹75 and a pack of crayons for ₹40. He has a ₹200 note. Write an expression using brackets for the change he will receive and find its value.

Solution: First, find the total cost of the items. This needs to be calculated before subtracting from ₹200. So, we put the sum in brackets.

Total cost = (75 + 40) = 75 + 40 = 115.

The expression for the change is 200 – (total cost), which is 200 – (75 + 40).

Substitute the value of the bracket: 200 – 115.

Perform the subtraction: 200 – 115 = 85.

Irfan will receive ₹85 in change.

So, 200 – (75 + 40) = 85.

Example 16 : Evaluate 50 – (30 ÷ (2 + 4)).

Solution: This expression has nested brackets. We start with the innermost bracket.

Innermost bracket: (2 + 4) = 6.

Substitute this back into the expression: 50 – (30 ÷ 6).

Now evaluate the remaining bracket: (30 ÷ 6) = 5.

Substitute this back: 50 – 5.

Perform the final subtraction: 50 – 5 = 45.

So, 50 – (30 ÷ (2 + 4)) = 45.

Example 17 : Place brackets in the expression 4 × 5 + 3 to make its value 32.

Solution: The expression is 4 × 5 + 3.

If we multiply first: 4 × 5 = 20, then 20 + 3 = 23. This is not 32.

Let's try putting brackets around 5 + 3: 4 × (5 + 3).

Evaluate inside the bracket: 5 + 3 = 8.

Now the expression is 4 × 8.

4 × 8 = 32. This is the desired value.

So, the expression with brackets is 4 × (5 + 3).

Terms in Expressions

When we look at an arithmetic expression, we can break it down into smaller parts called terms. Essentially, terms are the parts of an expression separated by addition (+) signs. Understanding terms is vital: it helps us see the expression's structure, apply properties like commutativity (rearranging terms) to simplify calculations, and correctly follow the order of operations.

Formally, terms are the addends—the quantities being added.

- In 12 + 7 + 5, the terms are simply 12, 7, and 5.
- For expressions with subtraction, like 83 − 14, we can rewrite it as 83 + (−14). Now it's clear: the terms are 83 and −14.
- Similarly, -18 − 3 becomes -18 + (−3), making the terms −18 and −3.

What if an expression involves multiplication or division, like 6 × 5 + 3? Here, the product 6 × 5 is treated as a single block that must be evaluated before the addition. Thus, 6 × 5 forms one term, and 3 is the other. Key idea: Parts of an expression connected by × or ÷ are generally considered part of the same term if they are not separated by a + or − sign that is outside of any brackets.

For example, in 2 − 10 + 4 × 6, by rewriting subtractions, we get 2 + (−10) + (4 × 6). The terms are 2, −10, and (4 × 6). In 10 + 7 × 8 ÷ 2 − 3, the terms are 10, (7 × 8 ÷ 2), and −3 (from + (−3)).

Identifying terms correctly is a crucial first step. We generally evaluate each term to a single numerical value first, and then perform the additions (and subtractions as additions of negatives) of these resulting term values.

Example 18 : Identify the terms in the expression 25 + 13 − 7.

Solution: Rewrite the expression showing all additions: 25 + 13 + (−7).
The parts separated by + signs are the terms.
The terms are 25, 13, and −7.

Example 19 : What are the terms in 5 × 9 + 12 ÷ 2?

Solution: The expression has an addition sign separating 5 × 9 and 12 ÷ 2.

The part 5 × 9 is one term.

The part 12 ÷ 2 is another term.

The terms are 5 × 9 and 12 ÷ 2.

Example 20 : List the terms in the expression (10 + 5) − 7 × 2 + 24 ÷ (6 − 3).

Solution: First, simplify within brackets if possible, but for identifying terms, we look at the main structure. The expression can be seen as: (10 + 5) is one block, − 7 × 2 is another, and + 24 ÷ (6 − 3) is the third.
Rewriting as sum of terms:
(10 + 5) + (−7 × 2) + (24 ÷ (6 − 3))

The terms are:

1. (10 + 5)
2. (−7 × 2) (or simply -7 × 2 if we understand it's being added as a negative)
3. (24 ÷ (6 − 3)

Order of Operations (DMAS/BODMAS/PEMDAS)

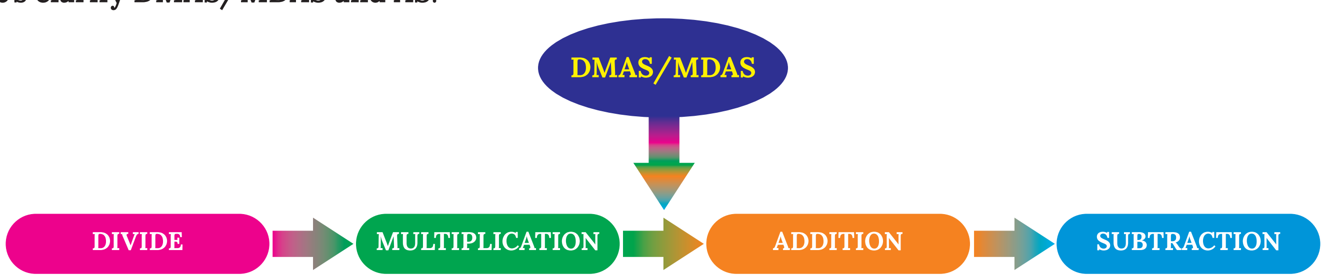
Now that we understand brackets and terms, we're ready for the full Order of Operations. This is a set of rules that ensures everyone evaluates a complex arithmetic expression in the same way, leading to a single, correct answer. Common mnemonics to remember this order are **BODMAS** (Brackets, Orders, Division, Multiplication, Addition, Subtraction) or **PEMDAS** (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction— **Common in the US**). The core idea is to handle brackets first, then any exponents (orders), then multiplication and division (from left to right), and finally addition and subtraction (from left to right).

BODMAS		PEMDAS
Brackets →	(or)	← Parentheses
Order →	√ (or) X^2	← Exponents
Division →	÷ (or) ×	← Multiplication
Multiplication →	× (or) ÷	← Division
Addition →	+ (or) +	← Addition
Subtraction →	− (or) −	← Subtraction

The universally accepted order of operations can be summarized as follows:

1. **Brackets (or Parentheses):** Evaluate expressions inside any grouping symbols like ( ), [ ], { } first. If there are nested brackets, work from the innermost outwards.
2. **Orders (or Exponents):** Evaluate any powers or square roots (e.g., $3^2 = 9$). (Note: Grade 7 might focus less on exponents in this chapter, but it's part of the full rule).
3. **Division and Multiplication:** Perform all divisions and multiplications as they appear, working from left to right. These two operations have equal priority.
4. **Addition and Subtraction:** Perform all additions and subtractions as they appear, working from left to right. These two operations also have equal priority.

Let's clarify DMAS/MDAS and AS:



- **Multiplication and Division (M/D)**: These are at the same level of priority. When you have a string of multiplications and divisions, you perform them from left to right. For example, in 20 ÷ 2 × 5, you do 20 ÷ 2 = 10 first, then 10 × 5 = 50. You don't do multiplication before division universally; you do them in order of appearance from left to right.
- **Addition and Subtraction (A/S):** Similarly, these are at the same level. Perform them from left to right. In 10 − 4 + 3, you do 10 − 4 = 6 first, then 6 + 3 = 9.

Connection to Terms:

Identifying terms helps with the DMSA/AS part. Once brackets and orders are handled, each term (which might involve multiplications or divisions) is evaluated to a single number. Then, these resulting term values are added or subtracted from left to right.

For example, in 30 + 5 × 4 − 12 ÷ 2:

- No brackets, no orders.
- Terms are 30, 5 × 4, and −12 ÷ 2.
- Evaluate terms involving × or ÷:
- 5 × 4 = 20
- 12 ÷ 2 = 6
- Expression becomes: 30 + 20 − 6.
- Now perform addition/subtraction from left to right:
- 30 + 20 = 50
- 50 − 6 = 44.

Commutative and Associative Properties of Addition: (Swapping and Grouping Terms)

Once an expression is broken down into its terms, we can often rearrange or group these terms without changing the overall value of the expression. This is due to two important properties of addition: the commutative property and the associative property. These properties make calculations more flexible and sometimes simpler.

Commutative property of addition	Associative property of addition
a + b = b + a	**a + (b + c) = (a + b) + c**

Commutative Property of Addition: This property states that the order in which you add numbers does not change their sum.

- For any numbers a and b, a + b = b + a.

Example: 5 + 8 = 13 and 8 + 5 = 13.

This property extends to terms in an expression. If you have an expression like Term1 + Term2 + Term3, you can swap the order of any two terms, and the sum will remain the same.

Example: 10 + (-3) = 7 and (-3) + 10 = 7.

Associative Property of Addition: This property states that when adding three or more numbers, the way you group them (which pair you add first) does not change their sum.

- For any numbers a, b, and c, (a + b) + c = a + (b + c).

Example: (2 + 3) + 4 = 5 + 4 = 9 and 2 + (3 + 4) = 2 + 7 = 9.

This property also applies to terms in an expression. You can group terms differently using brackets, and the sum will be the same.

Example: (5 + (-2)) + 7 = 3 + 7 = 10 and 5 + ((-2) + 7) = 5 + 5 = 10.

These properties are incredibly useful because they allow us to reorder and regroup terms in an expression to make calculations easier, especially when dealing with positive and negative numbers.

Example 21 : Expression: 15 + (-7) (Commutative Property with Terms)

Solution: 15 + (-7) = 8

Swapped Order: (-7) + 15 = 8

Conclusion: The value remains the same.

Example 22 : Expression: (10 + 5) + (-3) (Associative Property with Terms)

Solution: (10 + 5) + (-3) = 15 + (-3) = 12

Regrouped: 10 + (5 + (-3)) = 10 + 2 = 12

Conclusion: The value remains the same.

Example 23 : Expression: 20 + 5 × 2 – 8 (Combining Properties)

Solution: Terms: 20, 5 × 2 (which is 10), -8. So, 20 + 10 + (-8).

Original Order: (20 + 10) + (-8) = 30 + (-8) = 22

Swapped Terms: 10 + 20 + (-8) = (10 + 20) + (-8) = 30 + (-8) = 22

Regrouped: 20 + (10 + (-8)) = 20 + 2 = 22

Conclusion: The value remains the same regardless of swapping or grouping.

Example 24 : Evaluate 30 ÷ 5 + 4 × (7 – 3).

Solution: 1. Brackets: (7 – 3) = 4.

Expression becomes: 30 ÷ 5 + 4 × 4.

2. No Orders.

3. Division (from left to right): 30 ÷ 5 = 6.

Expression becomes: 6 + 4 × 4.

4. Multiplication: 4 × 4 = 16.

Expression becomes: 6 + 16.

5. Addition: 6 + 16 = 22.

So, 30 ÷ 5 + 4 × (7 – 3) = 22.

Example 25 : A shopkeeper calculates daily sales. On Monday, he sold 1500, on Tuesday ₹2000, and on Wednesday ₹1800. Does the order he adds these amounts change the total weekly sales?

Expression: 1500 + 2000 + 1800

Solution: (1500 + 2000) + 1800 = 3500 + 1800 = 5300

1500 + (2000 + 1800) = 1500 + 3800 = 5300

Conclusion: The total sales remain ₹5300, demonstrating the associative property. The order of adding daily sales doesn't matter for the total.

Example 26 : Evaluate 100 – [20 + (5 × 3 – 5)].

Solution: **1. Innermost Brackets (5 × 3 – 5):**

Multiplication inside: 5 × 3 = 15.

Becomes (15 – 5).

Subtraction inside: 15 – 5 = 10.

Expression becomes: 100 – [20 + 10].

2. Next Brackets [20 + 10]:

Addition inside: 20 + 10 = 30.

Expression becomes: 100 – 30.

3. Subtraction: 100 – 30 = 70.

So, 100 – [20 + (5 × 3 – 5)] = 70.

Example 27 : Evaluate -7 + 15 + (-10) + 5. (Commutativity/Associativity)

Solution: We can reorder and group terms for easier calculation.

Group positive numbers and negative numbers:

(15 + 5) + (−7 + (−10))

20 + (−17)

20 − 17 = 3.

Alternatively, left to right:

-7 + 15 = 8

8 + (-10) = -2

-2 + 5 = 3.

The result is the same.

Evaluating Expressions with Multiple Operations

Using **terms, properties of addition**, and **BODMAS/PEMDAS**, we evaluate complex expressions by first simplifying each term (multiplication/division) and then combining them with **addition** and **subtraction**.

When an expression contains multiple operations and no brackets explicitly dictate the order, we follow these steps:

1. **Identify the Terms:** Break the expression down into its individual terms. Remember, terms are separated by '+' or '-' signs. Any multiplication or division within a term stays together.
2. **Evaluate Each Term:** For each term, perform any multiplication or division operations from left to right. This will reduce each term to a single numerical value.
3. **Combine the Terms:** Once all terms have been evaluated to single numbers, perform the addition and subtraction operations from left to right.

   This systematic approach ensures that the expression is evaluated correctly and consistently, leading to the unique correct value.

Example 28 : 20 + 4 × 5 - 10 ÷ 2

1. **Identify Terms:**
   - Term 1: 20
   - Term 2: 4 × 5
   - Term 3: -10 ÷ 2 (or + (-10 ÷ 2))

2. **Evaluate Each Term:**
   - Term 1: 20
   - Term 2: 4 × 5 = 20
   - Term 3: -10 ÷ 2 = -5

3. **Combine Terms:**
   - 2**0 +** 20 + (-5)
   - 40 + (-5)
   - 35

   So, the value of the expression 20 + 4 × 5 - 10 ÷ 2 is 35.

Example 29 : Expression: 15 - 3 × 2 + 8 ÷ 4 (Evaluating with Terms)

Solution: 1. **Identify Terms:** 15, -3 × 2, 8 ÷ 4

2. **Evaluate Terms:**
   - 15
   - -3 × 2 = -6
   - 8 ÷ 4 = 2
3. **Combine Terms:** 15 + (-6) + 2 = 9 + 2 = 11
   - Value: 11.

Example 30 : Expression: 5 × 6 ÷ 3 + 10 - 2 × 4 (Expression with Multiple Multiplications/Divisions)

Solution: 1. **Identify Terms:** 5 × 6 ÷ 3, 10, -2 × 4

2. **Evaluate Terms (left to right within terms):**

- 5 × 6 ÷ 3 = 30 ÷ 3 = 10
- 10
- -2 × 4 = -8

3. **Combine Terms:** 10 + 10 + (-8) = 20 + (-8) = 12
   - Value: 12.

Example 31 : A school ordered 10 boxes of chalk, with 50 pieces per box. They also ordered 2 packs of markers, with 12 markers per pack. If they used 150 pieces of chalk and 5 markers, how many items are left?

Expression: (10 × 50) + (2 × 12) - 150 - 5

Solution: 1. **Identify Terms:** 10 × 50, 2 × 12, -150, -5

2. **Evaluate Terms:**
   - 10 × 50 = 500 (chalk pieces)
   - 2 × 12 = 24 (markers)
   - -150
   - -5
3. **Combine Terms:** 500 + 24 + (-150) + (-5) = 524 - 150 - 5 = 374 - 5 = 369
   - Value: 369 items are left.

Knowledge Checkpoint

- Solve the expression 15 + 5 × 2 in two different ways. Which way is correct according to the rules of mathematics, and why?
- How many terms are there in the expression 12xy - 7xy + xy - 9? List them
- Identify the numerical coefficient for each term in the expression: x - 6xy + 8xy - 11.
- Evaluate the expression 3a + (b ÷ 2) if a = 5 and b = -14
- Solve the expressions 100 – (4 × 15) + 10

Activity

BODMAS Challenge Relay

- **Objective:** To practice evaluating expressions using the correct order of operations in a fun, competitive way.
- **Materials:** Whiteboard/chart paper, markers, list of 5-6 complex expressions, stopwatch.
- **Preparation:** Divide the class into teams (e.g., 4-5 teams). Write one complex expression on the board, large enough for everyone to see.
- **Steps:**

1. The first player from each team comes to the board.
2. On "**Go!**", they must perform ONLY the very first step of solving the expression according to BODMAS and write down the new, simplified expression. (e.g., if expression is 10 + (6-2) × 3, first player writes 10 + 4 × 3).
3. They run back and hand the marker to the next teammate.
4. The second player performs the NEXT single step in the calculation and writes the new expression. (e.g., from 10 + 4 × 3, player 2 writes 10 + 12).

5. This continues until a team arrives at the final single-number answer.
6. The first team to correctly evaluate the expression wins the round.
7. Teacher checks the steps for correctness. If a team makes a mistake, they can be paused or a small time penalty added, giving other teams a chance.
8. Repeat with new expressions for several rounds.

- **Discussion:**
  - Which steps were most commonly mistaken?
  - Did seeing only one step at a time help focus on the correct order?
  - How crucial is each step for getting the final answer right?

Key Terms

- **Parentheses:** ( ) - most commonly used brackets.
- **Square Brackets**: [ ] - often used for an outer layer if parentheses are already inside.
- **Curly Braces:** { } - often used for a further outer layer.
- **Term**: A single number, a variable (in algebra), or numbers and variables multiplied together. In an arithmetic expression, terms are separated by + or − signs (where subtraction is treated as adding the inverse). Operations like × and ÷ happen within terms before the terms are added or subtracted.
- **Additive Inverse:** The opposite of a number. When a number and its additive inverse are added, the sum is 0 (e.g., the additive inverse of 14 is -14, because 14 + (-14) = 0).

Do It Yourself

- When you see an expression like
  6 + (12 ÷ 3 × 2) − (4 + 5)
  what should you do first? How does BODMAS/PEMDAS guide you?
- Can you think of a real-life example (like calculating bills, discounts, or distances) where evaluating a complex expression in the wrong order would give a completely different result?

Fact Flash

- The word "**term**" comes from the Latin word "**terminus**," meaning an end, boundary, or limit. In an expression terms are like distinct sections.
- The word "**parenthesis**" comes from Greek words meaning "**to place beside**."
- Different types of brackets ( ), [ ], { } are often used to make expressions with many **nested** parts easier to read. You solve the innermost ones first!
- **Nested Brackets:** Brackets placed inside other brackets.
- While **BODMAS/PEMDAS** are common in English-speaking countries, other countries might use slightly different mnemonics (like **BIDMAS** where 'I' is Indices for Orders/Exponents), but the underlying mathematical rules are the same worldwide!

Mental Mathematics

- (12 + 6) × (18 ÷ 6)
- Calculate: 15 ÷ 3 − 2 × 1
- Evaluate: 15 − (3 × 4) + 2
- Evaluate: 45 ÷ (9 − 4) + 7
- 4 × 2 + 5 ÷ 20 − 7
- What is the value of: 20 − (5 + 3)
- Evaluate: 10 × [(−2) + 7] − 5
- Evaluate: 3 + 8 × 2 − (12 ÷ 4)

Gap Analyzer™ Homework

Watch Remedial

Exercise 2.3

1. **Evaluate the following expressions.**

   a) 80– [30 – (2 × 7 + 1)]   b) 7 × (12 – 5) + 40 ÷ 8 – 1   c) 100 ÷ 10 + (15 – 7) × 3

   d) 50 – [10 + (25 ÷ 5 × 2)]   e) 60 - (5 × 7) + (24 ÷ 8)   f) 75 – [15 + (4 × 6 – 9)]

2. **Identify the terms in each expression and complete the table.**

Expression	Expression as the sum of its terms	Terms
12 – 4 + 7	(12) + (-4) + (7)	12,-4,7
8 + 5 × 2	(_) + (_)	________
20 – 3 + 11	(_) + (_) + (_)	________
15 + 9 – 6	(_) + (_) + (_)	________
30 – 5 × 3 + 2	(_) + (_) + (_)	________

3. **Place brackets (parentheses) in each expression to make the equation true.**

   a) 10 + 2 × 3 to make its value 36.   b) 15 - 6 ÷ 3 to make its value 3.

   c) 20 ÷ 5 + 5 to make its value 2.   d) 7 × 8 - 2 to make its value 42.

4. **Insert the correct mathematical operations (+, -, ×, ÷) in the blanks to make the equation true:**

   a) 36_4_5 = 14   b) 10_2_3 = 16   c) 48_6_2 = 16   d) 15_3_7 = 12

5. A theme park charges ₹200 per entry ticket, ₹50 per ride, and ₹100 for food.

   - For one visitor, the total cost is written as (1 × 200) + (3 × 50) + (1 × 100).
   - Another visitor spends (1 × 200) + (5 × 50) + (2 × 100).

   These expressions represent the money spent by the visitors.

   Questions

   i. Evaluate both expressions and find the two totals.

   ii. Who spent more and by how much?

   iii. Which part (ticket, rides, or food) contributes the most in the second visitor's bill?

   iv. If the first visitor takes 2 more rides, what will be the new total?

6. Imagine you have 20 candies. You eat 5 candies, then share the remaining equally among 3 friends.

   a. Write an expression (using numbers and operations) that represents this situation without using brackets, if possible. Explain why it might be difficult or lead to the wrong answer.

   b. Write an expression that correctly represents the situation using brackets.

7. **Expression: (4 × 120) + (3 × 80) + (2 × 50) – 100**

   Represents 4 shirts, 3 trousers, 2 caps, and a discount of ₹100.

   Questions:

   i. Write all the terms in the expression.

   ii. Which term is the discount?

   iii. Evaluate the expression.

   iv. If the discount was only ₹50, what would the new total be?

Removing Brackets – I (Simplifying Expressions)

We've seen that brackets () tell us to calculate what's inside them first. Sometimes, it's helpful to rewrite an expression without brackets, especially when it simplifies the calculation or when we move into algebra. There are specific rules for removing brackets, and these rules depend on what's directly in front of the bracket – particularly if it's a plus sign +, a minus sign −, or a number/variable implying multiplication. This concept focuses on removing brackets preceded by + or −.

Bracket preceded by +	Bracket preceded by -
a + (b - c)	a - (b - c)
↓	↓
a + b - c	a − b + c
	Signs Inside Change

Sub-concepts to be covered

1. Rule 1: Brackets preceded by a Plus Sign (+)
2. Rule 2: Brackets preceded by a Minus Sign (−)

Mathematical Explanation

Rule 1: Brackets preceded by a Plus Sign (+)

If a bracket is preceded by a + sign, or if there's no sign (implying + at the start of an expression), the brackets can be removed without changing the signs of the terms inside the bracket.

Example: 28 + (35 − 10)

This is 28 + (35 + (−10)).

Removing the bracket: 28 + 35 − 10.

The terms 35 and −10 inside the bracket keep their signs.

Calculation: 28 + 35 = 63, then 63 − 10 = 53.

Original with bracket: 28 + (25) = 53. The value is the same.

Example: (15 + 7) + 12

Removing bracket: 15 + 7 + 12.

Rule 2: Brackets preceded by a Minus Sign (−)

If a bracket is preceded by a − sign, when the brackets are removed, the sign of each term inside the bracket changes: + becomes −, and − becomes +. This is because subtracting a quantity is like adding its negative, and the negative sign distributes over all terms inside. Think of it as multiplying each term inside the bracket by −1.

Example: 200 – (40 + 3)

The terms inside the bracket are +40 and +3.

When removing the bracket, +40 becomes −40 and +3 becomes −3.

So, 200 – (40 + 3) = 200 – 40 – 3.

Calculation: 200 – 40 = 160, then 160 – 3 = 157.

Original with bracket: 200 – (43) = 157. The value is the same.

Example: 500 – (250 – 100)

The terms inside the bracket are +250 and −100.

When removing the bracket:

+250 becomes −250.

−100 becomes +100.

So, 500 – (250 – 100) = 500 – 250 + 100.

Calculation: 500 – 250 = 250, then 250 + 100 = 350.

Original with bracket: 500 – (150) = 350. The value is the same.

Understanding these rules is crucial for simplifying expressions correctly, especially in algebra.

Example 32 : Simplify 70 + (15 – 5 + 3) by removing brackets.

Solution: The bracket (15 – 5 + 3) is preceded by a + sign.

So, we can remove the brackets without changing the signs of the terms inside.

70 + (15 – 5 + 3) = 70 + 15 – 5 + 3.

Now, evaluate from left to right:

70 + 15 = 85

85 – 5 = 80

80 + 3 = 83.

Value of original expression: 70 + (10 + 3) = 70 + 13 = 83. Matches.

Example 33 : Simplify 120 – (50 + 20 – 10) by removing brackets.

Solution: The bracket (50 + 20 – 10) is preceded by a – sign.

When removing the brackets, change the sign of each term inside:

+50 becomes −50.

+20 becomes −20.

−10 becomes +10.

So, 120 – (50 + 20 – 10) = 120 – 50 – 20 + 10.

Now, evaluate from left to right:

120 – 50 = 70

70 – 20 = 50

50 + 10 = 60.

Value of original expression: 120 – (70 – 10) = 120 – 60 = 60. Matches.

Example 34 : Remove brackets and evaluate: 300 – (150 – 70).

Solution: The bracket (150 – 70) is preceded by a – sign.

Terms inside are +150 and −70.

Change signs: +150 becomes −150, −70 becomes +70.

300 – (150 – 70) = 300 – 150 + 70.

Evaluate: 300 – 150 = 150

150 + 70 = 220.

Value of original expression: 300 – (80) = 220. Matches.

Example 35 : Remove brackets: x + (y – z). (Algebraic context)

Solution: Bracket is preceded by +. Signs inside do not change.

x + (y – z) = x + y – z.

Example 36 : Remove brackets: p – (q + r – s). (Algebraic context)

Solution: Bracket is preceded by –. Signs of all terms inside change.

+q becomes –q.

+r becomes –r.

–s becomes +s.

p – (q + r – s) = p – q – r + s.

Knowledge Checkpoint

- What happens to the signs of terms inside a bracket if the bracket is preceded by a + sign and then removed?
- Simplify 60 – (20 + 10) by removing brackets. What is its value?
- Rewrite a – (–b + c) without brackets.

Activity

Bracket Busters!

- **Objective:** To practice removing brackets correctly and see how signs (+, –) affect the terms inside.
- **Materials:** Blackboard/whiteboard, chalk/markers, student notebooks.
- **Steps:**
  1. Write these expressions on the board:
     - 45 + (25 – 10)
     - 200 – (60 + 15)
     - 500 – (250 – 100)
  2. Ask each group to remove the brackets and simplify.

  Remind them:
     - If the bracket has a + sign → signs inside stay the same.
     - If the bracket has a – sign → change every sign inside.
  3. Groups solve and compare answers on the board.
  4. Discuss: Why did signs change (or not change) when brackets were removed?

Key Terms

- **Removing Brackets:** The process of rewriting an expression without brackets, following specific rules based on the sign or term preceding the bracket.
- **Sign Change:** When a bracket preceded by a minus sign is removed, the positive terms inside be come negative, and negative terms inside become positive.

Do It Yourself

- Try this: 100 – (40 + 20 – 10). What happens if you don't change the signs correctly?
- Can you think of a real-life situation where removing brackets (or grouping carefully) matters, like calculating discounts or expenses?

Fact Flash

- The rule for changing signs when a bracket is preceded by a minus sign is an application of the distributive property: a – (b + c) is like a + (–1) × (b + c) = a + (–1) b + (–1) c = a – b – c.
- Mastering bracket removal makes complex-looking expressions much simpler to handle!
- It's like unpacking a box: if the label says "FRAGILE - HANDLE WITH CARE" (like a minus sign), you treat everything inside differently!

Mental Mathematics

- Simplify: 6(12 – 3)
- Simplify: –4(16 × 5 – 65)
- Simplify: 3(400 – 300 + 150)
- Simplify: –2(28 – 12 ÷4)
- Simplify: 10 – (12 + 30 ÷ 5 × 2)
- Simplify: 120 + 600 – 465

Gap Analyzer™ Homework

Watch Remedial

Exercise 2.4

1. Simplify the following:

a) 6 × (10 – 7) b) 4 + (8 + 9) c) 5 × (7 + 4) – 6 d) 3 × (2 + 5) + 4

e) 2 × (3 + 4 + 5) f) 95 + (40 – 15) g) 210 – (100 + 50) h) 500 – (300 – 100 + 50)

2. Ravi had ₹2000. He went shopping and bought a shirt for ₹(500 + 50) and a book for ₹(200 - 25). Write a single mathematical expression to represent the money Ravi has left. Simplify the expression to find the final amount.

3. A fruit seller had 100 apples. He sold (40 + 10) apples in the morning and (20 – 5) in the afternoon. Simplify the expression to find the number of apples left.

4. A car travels (120 – 30) km in the morning and (80 + 10) km in the evening. Simplify the expression to find the total distance travelled.

5. A shopkeeper sells a packet of pens for ₹50 and a packet of pencils for ₹30. Write and simplify the expression for the total cost if you buy one packet of pens and (two packets of pencils – one packet of pens).

6. A student spends (3 × 2) hours studying Maths and (4 + 2) hours studying Science. Simplify to find the total study time.

7. **A basket contains 3 identical groups of (5 − 2) chocolates.**

   Another basket contains 2 identical groups of (4 + 3) chocolates.

   Both baskets are combined together.

   Now we need to know the total chocolates after removing brackets.

   Question

   Simplify: 3 × (5 − 2) + 2 × (4 + 3)

Removing Brackets – II (Distributive Property)

We've seen how to remove brackets preceded by + or −. What if a bracket is preceded by a number or a variable that's multiplying it, like 5 × (10 + 3) or a × (b − c)? This is where the Distributive Property comes in. It tells us how to "distribute" the multiplication over the terms inside the bracket. This property is one of the most fundamental and useful rules in arithmetic and algebra for expanding and simplifying expressions.

The **Distributive Property** states that multiplying a sum (or difference) by a number is the same as multiplying each term of the sum (or difference) by that number and then adding (or subtracting) the results.

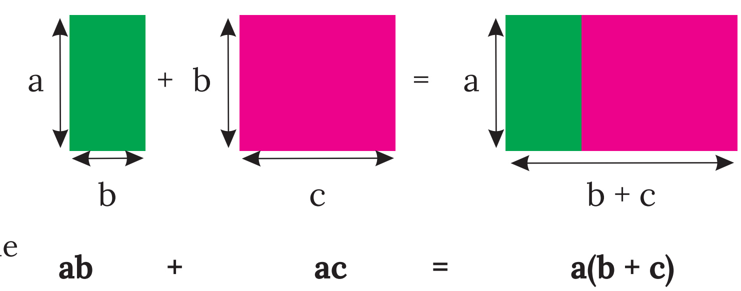
Sub-concepts to be covered

1. Distribution of Multiplication over Addition
2. Distribution of Multiplication over Subtraction

Mathematical Explanation

Distribution of Multiplication over Addition

a × (b + c) = (a × b) + (a × c)



To evaluate a × (b + c):

- You can first add b and c, and then multiply the sum by a.
- OR, you can multiply a by b, multiply a by c, and then add these two products. The result will be the same.

Example: 5 × (10 + 3)

- **Method 1 (Brackets first):** 5 × (13) = 65.
- **Method 2 (Distributive Property):** (5 × 10) + (5 × 3) = 50 + 15 = 65.

Distribution of Multiplication over Subtraction

a × (b − c) = (a × b) − (a × c)

a(b − c) = ab − ac

OR

a × (b − c) = (a × b) − (a × c)

Example: 7 × (8 − 2)

- **Method 1 (Brackets first):** 7 × (6) = 42.
- **Method 2 (Distributive Property):** (7 × 8) − (7 × 2) = 56 − 14 = 42.

  This property also works if the multiplier is on the right:

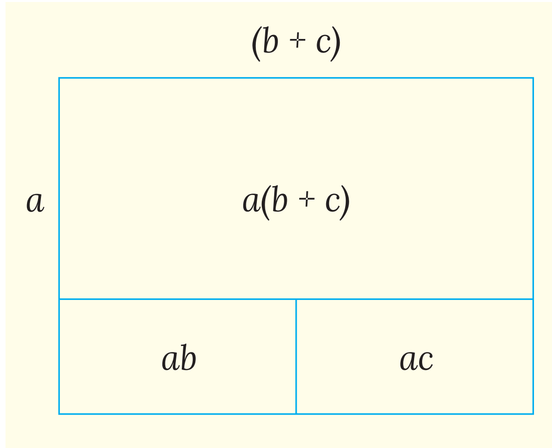
  (b + c) × a = (b × a) + (c × a)

Visualizing with Area:

Imagine a rectangle with width a and length (b + c).

Its area is a × (b + c).

You can also see this rectangle as two smaller rectangles side-by-side:



- One with width a and length b (Area = a × b).
- Another with width a and length c (Area = a × c).

The total area is the sum of these two areas: (a × b) + (a × c).

Thus, a × (b + c) = (a × b) + (a × c).

The distributive property is key for:

- **Mental Math:** e.g., 7 × 102 = 7 × (100 + 2) = 700 + 14 = 714.
- **Expanding Algebraic Expressions:** e.g., 3(x + 2y) = 3x + 6y.
- **Factoring Expressions:** The reverse of distributing (e.g., 4x + 4y = 4(x+y)).

Example: Evaluate 6 × (10 + 4) using the distributive property and also by calculating brackets first.

Solution: **Method 1 (Brackets first):**

6 × (10 + 4) = 6 × 14.

6 × 14 = 84.

Method 2 (Distributive Property):

6 × (10 + 4) = (6 × 10) + (6 × 4)

= 60 + 24

= 84.

Both methods give the same result.

Example 37 : Expand 9 × (7 − 3) using the distributive property and verify.

Solution: 9 × (7 − 3) = (9 × 7) − (9 × 3)

= 63 − 27

= 36.

Verification (Brackets first):

9 × (7 − 3) = 9 × 4 = 36.

The results match.

Example 38 : Use the distributive property to calculate 15 × 98 mentally.

Solution: We can write 98 as (100 − 2).

So, 15 × 98 = 15 × (100 − 2).

Using distributive property:

15 × (100 − 2) = (15 × 100) − (15 × 2)

= 1500 − 30

= 1470.

Example 39 : Simplify 3 × (a + 5) + 2 × (a − 1). (Algebraic)

Solution: Apply distributive property to both parts:

3 × (a + 5) = (3 × a) + (3 × 5) = 3a + 15.

2 × (a − 1) = (2 × a) − (2 × 1) = 2a − 2.

Now substitute these back into the expression:

(3a + 15) + (2a − 2).

Remove remaining brackets (preceded by +):

3a + 15 + 2a − 2.

Combine like terms (terms with 'a', and constant terms):

(3a + 2a) + (15 − 2)

5a + 13.

Knowledge Checkpoint

- State the distributive property of multiplication over addition.
- Use the distributive property to expand 4 × (y − 6).
- Calculate 25 × 102 using the distributive property.

Activity

Distributive Detective with Dot Paper

- **Objective:** To visualize and verify the distributive property using dot paper or grid paper.
- **Materials:** Dot paper or grid paper, colored pencils.
- **Steps:**

1. Let's verify 3 × (4 + 2) = (3 × 4) + (3 × 2).
2. **Represent 3 × (4 + 2):**
   - On the dot paper, (4 + 2) represents a length of 6 units.
   - 3 × (4 + 2) means a rectangle with width 3 units and length 6 units.
   - Draw this rectangle (3 rows of 6 dots, or a 3x6 grid). Count the total dots/squares. It should be 18.
3. **Represent (3 × 4) + (3 × 2):**
   - On the same or a new paper, draw a rectangle for 3 × 4 (3 rows of 4 dots). Color it, say, red. Count dots (12).
   - Adjacent to it, or separately, draw a rectangle for 3 × 2 (3 rows of 2 dots). Color it, say, blue. Count dots (6).
   - The total number of dots is the sum of dots in the red and blue rectangles: 12 + 6 = 18.
4. Compare the total dots from step 2 and step 3. They should be equal, visually demonstrating the property.
   - Discussion: How does the visual representation help understand why the property works? Can you explain it in terms of combining or splitting areas?

Key Terms

- **Distributive Property:** A property that relates multiplication and addition (or subtraction). It states that a × (b + c) = ab + ac and a × (b − c) = ab − ac.
- **Expanding (an expression):** Using the distributive property to remove brackets by multiplying the term outside the bracket with each term inside.
- **Factoring (an expression):** The reverse of distributing; finding common factors and rewriting an expression as a product (e.g., ab + ac = a(b+c)).

Do It Yourself

- Which is easier: calculating 25 × 99 directly, or rewriting it as 25 × (100 − 1)?
- Can you think of daily life situations—like shopping in bulk or calculating area—where using the distributive property makes calculations simpler?

Fact Flash

- The distributive property is what connects the two main arithmetic operations of addition and multiplication in a fundamental way.

Mental Mathematics

Use the distributive property for quick mental calculations:

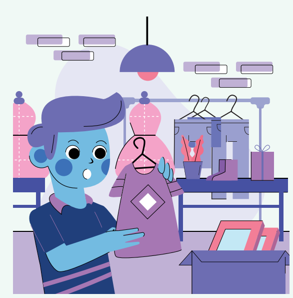
- 7 × 103 = 7 × (100 + 3)
- 15 × 19 = 15 × (20 − 1)
- 9 × 58 = 9 × (60 − 2)
- 6 × (100 + 50 + 2)
- 12 × 25 = 12 × (20 + 5)
- 14 × (140 + 100 + 20)

Gap Analyzer™ Homework

Watch Remedial

Exercise 2.5

1. **Estimation Questions (Mental application of distributivity):**
   a) 9 × 102 is approximately 9 × 100 plus a little more. What is that "little more"?
   b) 11 × 49 is approximately 11 × 50 minus a little. What is that "little"?
   c) Is 5 × (20 + 8) equal to 5 × 20 + 5 × 8?
   d) To calculate 19 × 7, you can think (20 − 1) × 7. This is (20 × 7) − (1 × 7). True or False?
2. **Each student in a class of 25 needs 3 notebooks and 2 pens.**
   a) Calculate the total notebooks: 25 × 3. Calculate total pens: 25 × 2. Total items: (25 × 3) + (25 × 2).
   b) Alternatively, each student needs (3 + 2) items. Total items: 25 × (3 + 2).
   Show that both expressions give the same total number of items.
3. A shopkeeper sells T-shirts for ₹200. He offers a discount of ₹15 per T-shirt if you buy 5 or more. If you buy 6 T-shirts, the cost per T-shirt is (200 − 15). Write an expression for the total cost of 6 T-shirts using the distributive property. Calculate the total cost.



4. Find out the sum of the numbers given in the picture below in at least two different ways. Describe how you solved it through expressions.

3	7	3	7	3	7
3	7	3	4	6	4
6	4	6	4	4	6
6	4	6	4	6	4

5. A rectangular garden is 7 meters wide. Its length is (10 + 3) meters (10m main part, 3m extension). Use the distributive property to write an expression for the area in two ways and find the area.
6. A shop is offering a discount of ₹50 on every item. Sunita buys 3 T-shirts that were originally priced at ₹450 each and 2 pairs of jeans that were also originally priced at ₹450 each. What is the total amount she paid after the discount?

7. Two friends, Priya and Simran, are making **decorative kits** to sell at a craft fair. Each kit requires 9 beads and 4 ribbons. Priya makes 12 kits and Simran makes 8 kits. What is the total number of items (beads and ribbons) they used altogether?
8. **A school orders 14 boxes, and each box has (100 + 50) pencils. Find the total pencils using distributive property.**

9. **Solve the following questions:**
   i. 4 × (100 + 50) =
   ii. 12 × (80 + 40) =
   iii. 20 × (90 + 30) =
   iv. 15 × (60 + 20) =

Common Misconceptions

Misconception: Addition is always done before multiplication
Correction: Follow the order of operations (BODMAS/PEMDAS). Multiplication and division are performed before addition and subtraction unless brackets indicate otherwise.
Example: 2 + 3 × 4 is 14, not 20.

Misconception: Parentheses don't affect the answer
Correction: Parentheses always take priority and must be solved first in any arithmetic expression.
Example: 2 + 3 × 4 is 14, not 20.
Example: (2 + 3) × 4 is 20, while 2 + 3 × 4 is 14.

Misconception: 2x means 2 + x
Correction: In mathematics, 2x means 2 multiplied by x, not addition.
Tip: Always treat a number next to a variable as multiplication unless a sign is shown.

Real-Life Arithmetic Expressions: Mathematical Application

Arithmetic expressions, using numbers and operations, are fundamental for modeling real-world situations. These applications make arithmetic relevant, fostering computational fluency and practical problem-solving, aligning with NEP 2020's focus on experiential learning:

1. **Shopping & Budgeting:** Calculating total costs or change, e.g., 500 - (2 × 120 + 3 × 35). (Helps manage personal finances).
2. **Recipe Scaling:** Adjusting ingredient quantities for different servings, like (2 cups / 4 servings) × 10 servings. (Ensures correct proportions in cooking).
3. **Sports Scoring:** Determining total points in games, such as (5 baskets × 3 points) + (7 free throws × 1 point). (Quantifies performance).
4. **Travel & Distance:** Calculating total distance or fuel cost for journeys, e.g., (Distance A to B) + (Distance B to C). (Aids in travel planning).
5. **Smart Shopping & Discounts:** Finding final prices after discounts, like 200 - (0.20 × 200). (Enables wise consumer choices).

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Complete Chapter Test

EXERCISE

A. Choose the correct answer.

1. The value of the expression 20 − 3 × (4 + 1) is:

   a) 85 ☐ b) 11 ☐ c) 5 ☐ d) 3 ☐

2. Which of the following expressions is equal to 15 + 5 × 2?

   a) (15 + 5) × 2 ☐ b) 20 × 2 ☐ c) 15 + 10 ☐ d) 5 × (15 + 2) ☐

3. When simplifying 100 − (x − y), it becomes:

   a) 100 − x − y ☐ b) 100 − x + y ☐ c) 100 + x − y ☐ d) 100 + x + y ☐

4. The distributive property states that a × (b − c) equals:

   a) ab − c ☐ b) a − bc ☐ c) ab − ac ☐ d) ac − ab ☐

5. Which operation should be performed first in 50 ÷ [10 + 3 × (2 + 1)]?

   a) 50 ÷ 10 ☐ b) 10 + 3 ☐ c) 3 × 2 ☐ d) 2 + 1 ☐

Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct:

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

1. **Assertion (A):** The value of $10 + 20 \div 5 \times 2$ is 18.

   Reason (R): In order of operations, multiplication is performed before division, and addition is performed before multiplication.

2. **Assertion (A):** $5 \times (12 - 2)$ is equal to $5 \times 12 - 5 \times 2$.

   Reason (R): This is an example of the distributive property of multiplication over subtraction.

3. **Assertion (A):** When simplifying $50 - (10 + 5)$, the expression becomes $50 - 10 + 5$.

   Reason (R): If a bracket is preceded by a minus sign, the signs of all terms inside the bracket are changed when the bracket is removed.

Case Study

The School Fair Budget

The Grade 7 class is organizing a stall at the school fair.

- They plan to sell 100 cups of lemonade. The cost to make each cup is ₹3. They will sell each cup for ₹10.
- They also plan to sell 50 sandwiches. The ingredients for each sandwich cost ₹8. They will sell each sandwich for ₹20.
- They need to pay ₹200 as a fixed stall rental fee to the school.

a) Write an expression for the total cost of making all the lemonade.

b) Write an expression for the total money earned from selling all the lemonade.

c) Write an expression for the profit from lemonade (money earned - cost).

d) Repeat (a), (b), (c) for the sandwiches.

Project

Task:

- You and three friends (total 4 people) are planning a day out. You need to create a budget and calculate the total cost and the cost per person.
- Activities to budget for (choose at least 3):
- Travel (e.g., bus tickets, fuel for a car)
- Entry tickets to a place (e.g., museum, amusement park, zoo)
- Food (e.g., lunch, snacks)
- Souvenirs or other small purchases

Steps:

1. Research approximate costs for each chosen activity for 4 people.
2. For each activity, write an arithmetic expression to calculate its total cost for the group.
   For example, If bus tickets are ₹30 per person one way, the travel cost might be (30 × 4) × 2 for a round trip.
3. Write a single, complex arithmetic expression that represents the total combined cost for all chosen activities for the group. Use brackets where necessary to ensure correct calculation.
4. Evaluate this total cost expression step-by-step, showing your work.
5. Write an expression to calculate the cost per person. Evaluate it.

Source-Based Question

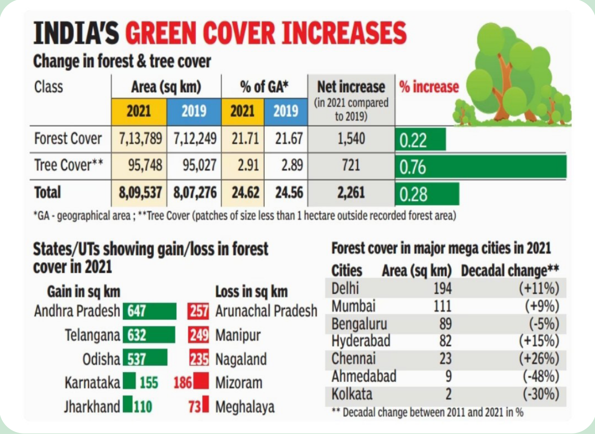
India's Green Cover

Directions: Read the following data carefully and answer the questions that follow.

The India State of Forest Report 2021 provides an assessment of India's forest and tree resources.

- Total Forest and Tree Cover (2021): The total forest and tree cover in the country is 8,09,537 square kilometers (sq km).
- Increase since 2019: There has been an increase of 2,261 sq km in the total forest and tree cover of the country compared to the assessment of 2019.
- Breakdown of Increase: The increase in the forest cover is 1,540 sq km and the increase in tree cover is 721 sq km.
- State-wise Data:
  - o The state with the largest forest cover in the country is Madhya Pradesh, with 77,493 sq km.
  - o Maharashtra has a forest cover of 50,798 sq km.

Source Text: Data adapted from the India State of Forest Report (ISFR) 2021, published by the Press Information Bureau, Government of India.



Questions

1. The total forest and tree cover in 2021 was 8,09,537 sq km, which was an increase of 2,261 sq km from 2019. Write a single arithmetic expression to represent the total forest and tree cover in 2019 and find its value.
2. The report states that the total increase in cover (2,261 sq km) is composed of an increase in 'forest cover' (1,540 sq km) and 'tree cover' (721 sq km). Write an arithmetic expression to check if the sum of the individual increases matches the total increase. Is the statement correct?
3. Write an arithmetic expression to find the difference between the forest cover of Madhya Pradesh and Maharashtra. Which state has more forest cover, and by how much?
4. To further increase green cover, imagine the government starts a special plantation drive.
   - In Madhya Pradesh, they plant trees in 200 new plots, with 150 trees in each plot.
   - In Maharashtra, they plant trees in 300 new plots, with 120 trees in each plot. Write a single arithmetic expression using brackets to find the total number of new trees planted in both states combined. Then, evaluate the expression.
5. A state decides to spend money on afforestation in 150 different locations. The cost for each location is ₹25,000 for saplings and ₹10,000 for labour and maintenance. Write an expression for the total cost using the distributive property a × (b + c). Calculate the total cost of the project.

Mind Map

Arithmetic Expressions

What is an Arithmetic Expression?

❖ **Numbers**: Values

❖ **Operators:** +, −, ×, ÷

Comparing Expressions

❖ **Symbols:** > , < , =

✓ **Example**: 18 + 9 = 27, 30 − 5 = 25 → 27 > 25

Reading & Evaluating Complex Expressions

❖ **Order of Operations (BODMAS/ PEMDAS):**

✓ Brackets ( ), [ ], { }

✓ Orders/Exponents

✓ Division & Multiplication (L→R)

✓ Addition & Subtraction (L→R)

❖ **Properties:**

✓ Commutative: a + b = b + a

✓ Associative: (a + b) + c = a + (b + c)

BODMAS		PEMDAS
Brackets →	(or)	← Parentheses
Order →	$\sqrt{}$ (or) X^2	← Exponents
Division →	÷ (or) ×	← Multiplication
Multiplication →	× (or) ÷	← Division
Addition →	+ (or) +	← Addition
Subtraction →	− (or) −	← Subtraction

Removing Brackets – I (Simplifying with + and –)

❖ **+ before bracket:** Keep signs same

❖ **– before bracket:** Change signs

Bracket preceded by +	Bracket preceded by –
a + (b - c) ↓ a + b - c	a - (b - c) ↓ a - b + c Signs Inside Change

Removing Brackets – II (Distributive Property)

❖ **Over Addition:** a × (b + c) = ab + ac

❖ **Over Subtraction:** a × (b − c) = ab − ac