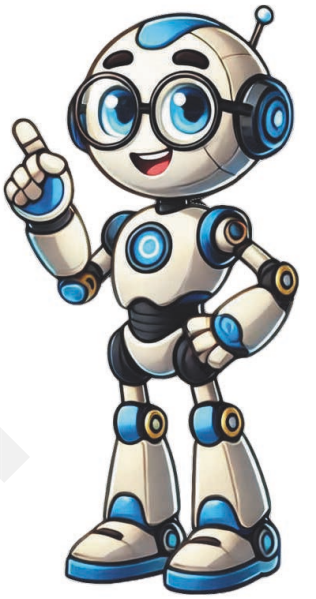
6
Chapter

Comparing Quantities

We'll cover the following key points:

- → Ratio
- → Proportion
- → Unitary method
- → Introduction to percentage
- → Conversion of percentage
- → Profit and loss
- → Simple interest



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Do you Remember fundamental concept in previous class.

In class 5th we learnt

- → Introduction to Profit and Loss
- → Finding Profit and Loss Percent

Learning Outcomes

By the end of this chapter, students will be able to:

- Define and identify ratios and understand their relationship to fractions, expressing one quantity as a ratio to another.
- Simplify ratios by reducing them to their lowest terms, using the greatest common divisor (GCD).
- Represent ratios in different forms, including as fractions, decimals, and percentages, and convert between these forms.
- Understand and apply the concept of proportion, explaining how two ratios are equal and solving problems involving proportions.
- Solve problems using the cross-multiplication method to find unknown values in proportion statements.
- Use ratios and proportions in real-life contexts, such as in recipes, scale models, and financial problems.
- Compare and solve word problems involving direct and inverse proportions, understanding their applications in everyday situations.

Mind Map

COMPARING QUANTITIES

Percentage

- Percent is derived from Latin word 'per – centum' meaning 'per hundred'.

$$1\ \% = \frac{1}{100}$$

- Percentage are numerators of fractions with denominator 100.

$$20\% = \frac{20}{100}$$

Converting decimals to percentage

➢ **By multiplying with 100**

e.g.,

$0.75 \Rightarrow 0.75 \times 100 = 75\ \%$

$0.9 \Rightarrow 0.9 \times 100 = 90\ \%$

Use of percentages

i. Increase or Decrease percent

- % Increase or % Decrease

$$= \frac{\text{amount of change}}{\text{original amount}} \times 100\%$$

General questions

ii. In a city, 30% are females, 40% are males and remaining are children. Then % of children.

$= 100\% - (30\% + 40\%)$

$= 100\% - 70\ \% = 30\ \%$

Profit or Loss

Profit = Selling price – Cost price

Loss = Cost Price – Selling price

$$\text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100\%$$

$$\text{Loss\%} = \frac{\text{Loss}}{\text{CP}} \times 100\%$$

Converting Fractional Numbers to Percentage

By multiplying with 100

e.g.,

$$\frac{1}{4} = \frac{1}{4} \times 100\% = 25\%$$

$$\frac{1}{5} = \frac{1}{5} \times 100\% = 20\%$$

Converting percentage to fractions or decimals

➢ **By dividing from 100**

e.g.,

$$5\ \% = \frac{5}{100} = 0.05$$

$$25\ \% = \frac{25}{100} = 0.25$$

Simple interest

➢ $\text{SI} = \frac{\text{P} \times \text{R} \times \text{T}}{100}$

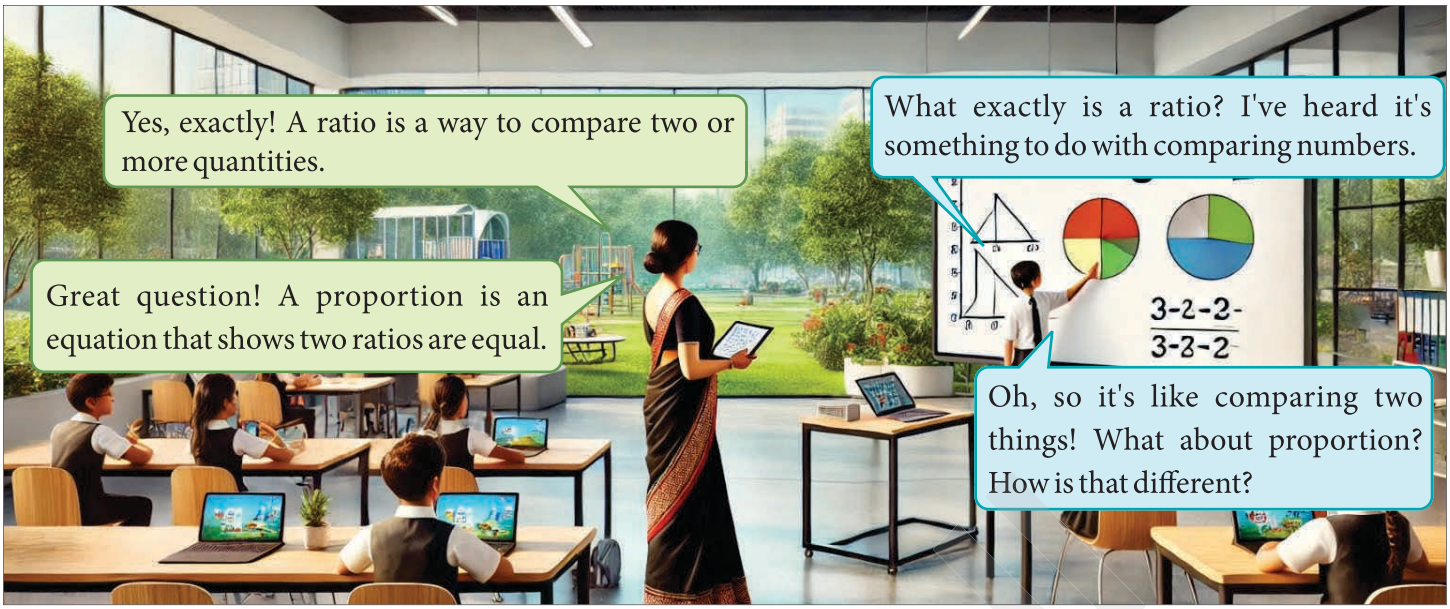
P = Principal

R = Rate

T = Time

Amount = P + SI

Introduction



Ratio

We have learnt how to compare two quantities and we have used the symbols like >, < or = to compare them. e.g., 50 g > 40 g, 5 m < 10 m, 7 sec = 7 sec etc. These comparisons were based on differences. Let us now start comparing quantities by division.

When we compare two quantities of the same kind (with respect to magnitude) by division, we say that we have formed a ratio of the two quantities.

For example, the ratio of 250 g to 5 kg can be obtained by expressing them in the same unit [i.e., converting 5 kg into 5000 g.]

250 g cake

5 kg cake

(*i*) 1st cake is $\frac{1}{20}$ th part of 2nd cake.

(*ii*) 2nd cake is 20 times of 1st cake.

Thus, the ratio $= \frac{250\text{ g}}{5000\text{ g}} = \frac{1}{20}$.

Usually, the symbol (:) is used to express ratio. Therefore, the above ratio is written as 1 : 20 and read as '1 is to 20'.

Similarly, the ratio between the two numbers, say 50 and 75 is 50 : 75 or 2 : 3.

Usually a ratio is expressed in the simplest form.

The two quantities in a ratio are called the terms of the ratio. The first term is called the **antecedent**, whereas the second term is called the **consequent**.

The order of the terms in a ratio is important. The ratio 2 : 3 is different from the ratio 3 : 2, because the ratio 2 : 3 means $\frac{2}{3}$ and 3:2 means $\frac{3}{2}$. Obviously $\frac{2}{3}$ is not equal to $\frac{3}{2}$.

Example 1 : Find the ratio of the following :

(*i*) 48 to 84 (*ii*) 200 g to 4 kg

Solution : (*i*) The ratio of 48 to 84 = 48 : 84 = 4 : 7

[Dividing the first and second terms by their HCF = 12]

(*ii*) The ratio of 200 g to 4 kg = 200 g : 4 kg

$= \overset{1}{\cancel{200}}\,g : \overset{20}{\cancel{4000}}\,g$ [$\because$ 1 kg = 1000 g $\Rightarrow$ 4 kg = 4000 g]

= 1 : 20

Example 2 : Your father's height is double of your younger sister's height. Find the ratio of your younger sister's height to father's height.

Solution : Let younger sister's height be x units.

Then, father's height = $2x$.

Thus, ratio of younger sister's height to father's height $= \frac{\cancel{x}^{1}}{2\cancel{x}} = \frac{1}{2} = 1:2$

Example 3 : Out of the 720 students in a school, 280 are boys and the remaining are girls. Find the ratio of the number of :

(*i*) boys to that of girls. (*ii*) boys to the total number of students.

(*iii*) total students to that of girls.

Solution : The total number of students = 720

The number of boys = 280

The number of girls = 720 – 280 = 440

(*i*) The ratio of the number of boys to the number of girls = $\frac{280}{440}$

We get, $\frac{280 \div 40}{440 \div 40} = \frac{7}{11} = 7 : 11$

Thus, the ratio of the number of boys to that of the girls is 7 : 11.

(*ii*) Similarly, the ratio of the number of boys to the total number of students

$$= \frac{\cancel{280}^{7}}{\cancel{720}_{18}} = \frac{7}{18} = 7 : 18$$

(*iii*) The ratio of the total number of students to the number of girls.

$$= \frac{\cancel{720}^{18}}{\cancel{440}_{11}} = \frac{18}{11} = 18:11$$

Dividing a Given Number in the Given Ratio

Working Rules

Step 1 : Obtain the given number. Let it be 'a'.
Step 2 : Obtain the ratio in which the number is to be divided. Let the ratio be $a_1 : a_2$.
Step 3 : Find the sum of the terms of the ratio i.e., $a_1 + a_2$.
Step 4 : Use the following formula to obtain the two parts :

$$\textbf{First part} = \frac{a_1}{a_1 + a_2} \times a, \quad \textbf{Second part} = \frac{a_2}{a_1 + a_2} \times a$$

Example 4 : Divide 108 in two parts in the ratio 4 : 5.

Solution : The sum of the terms of the ratio = $4 + 5 = 9$

$$\text{The first part} = \frac{4}{\cancel{9}_{1}} \times \cancel{108}^{12} = 4 \times 12 = 48$$

$$\text{The second part} = \frac{5}{\cancel{9}_{1}} \times \cancel{108}^{12} = 5 \times 12 = 60$$

Hence, the two parts are 48 and 60 respectively.

Note

Similar procedure can be used to divide a number in more than two parts in the given ratio.

Comparison of Ratios

Working Rules

We can compare the given ratios by following the steps given below:
Step 1 : Obtain the given ratios.
Step 2 : Express each ratio in the form of a fraction in the simplest form.
Step 3 : Find the L.C.M of the denominators of the fractions obtained in Step 2.
Step 4 : Obtain the first fraction and its denominator. Divide the L.C.M. obtained in Step 3 by the denominator to get a number x (say).

Now, multiply the numerator and denominator of the fraction by x. Apply the same procedure to the other fractions. Now, the denominators of all the fractions will be the same.

Step 5 : Compare the numerators of the fractions obtained in Step 4. The fraction having the larger numerator will be larger than the other.

Example 5 : Compare the two ratios 5 : 12 and 3 : 8.

Solution : We have, $5 : 12 = \frac{5}{12}$ and $3 : 8 = \frac{3}{8}$.

Since L.C.M. of 12 and 8 is 24, we get

$$\frac{5}{12} = \frac{5 \times 2}{12 \times 2} = \frac{10}{24} \text{ and } \frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

Since $10 > 9$, we get $\frac{10}{24} > \frac{9}{24}$ i.e., $\frac{5}{12} > \frac{3}{8}$ i.e., $\mathbf{5 : 12 > 3 : 8}$

Proportion

When two ratios are equal, then their terms are said to be in proportion.

For example, 2 : 3 and 8 : 12 are equal and hence 2, 3, 8 and 12 are in proportion.

In general, the four quantities a, b, c, and d are said to be in proportion, if $a : b = c : d$. The numbers a, b, c, and d are respectively called the first term, second term, third term and fourth term.

Note

$a : b = c : d$ is also written as $a : b :: c : d$ or simply $\frac{a}{b} = \frac{c}{d}$.

Now considering the following picture, we have:

Photograph

$$\frac{\text{Actual height of tree}}{\text{Actual height of house}} = \frac{\text{Height of tree in photograph}}{\text{Height of house in photograph}}$$

In a proportion, the first and fourth terms are called the **extreme** terms or **extremes**, whereas the second the third terms are called the **middle terms** or **means**.

In a proportion, we have the following :

The product of the extremes = The product of the means

For example, if $a : b = c : d$, then $ad = bc$.

Conversely, if $ad = bc$, then a, b, c and d are in proportion.

Example 6 : Are 40, 30, 60, 45 in proportion ?

Solution : We have, $40 : 30 = \frac{40}{30} = \frac{4}{3}$ and $60 : 45 = \frac{60^{4}}{{}_{3}45} = \frac{4}{3}$

Clearly, $40 : 30 = 60 : 45$

Hence 40, 30, 60, 45 are in proportion.

Example 7 : The first, third and fourth terms of a proportion are 12, 8 and 14, respectively. Find the second term.

Solution : Let the second term be x.

Then, $12, x, 8, 14$ are in proportion.

We know that, product of the extreme terms = product of the mean terms

i.e., $12 \times 14 = x \times 8$ or $168 = 8x$

or $8x = 168$ or $\frac{8x}{8} = \frac{168^{21}}{{}_{1}8}$ or $x = 21$

Hence, the second term of the proportion is 21.

Example 8 : If $a : b = 5 : 7$, then find the value of $(3a + 5b) : (5a - 2b)$

(*i*) 45 : 7 (*ii*) 35 : 9 (*iii*) 50 : 11 (*iv*) 32 : 5

Solution : Given, $\frac{a}{b} = \frac{5}{7} = k$ (let)

$\Rightarrow \quad a = 5k$ and $b = 7k$

$\therefore (3a+5b):(5a-2b) = \frac{3a + 5b}{5a - 2b} = \frac{3 \times 5k + 5 \times 7k}{5 \times 5k - 2 \times 7k} = \frac{15k + 35k}{25k - 14k} = \frac{50\,k}{11\,k} = \mathbf{50 : 11}$

Continued Proportional and Mean Proportional

Three numbers a, b, c are said to be in continued proportion, if a, b, b, c are in proportion. Also, if a, b, c are in continued proportion, then a, b, b, c are in proportion i.e., $a : b :: b : c$ or $a : b = b : c$. If a, b, c are in continued proportion, then b is called the mean proportional between a and c. Also, we have $a : b = b : c$.

Now, the product of extreme terms = the product of mean terms.

i.e., $a \times c = b \times b \quad \Rightarrow \quad ac = b^2$ or $b^2 = ac$

Clearly, if b is the mean proportional between a and c, then $\boldsymbol{b^2 = ac}$.

Example 9 : If 3, x, 12 are in continued proportion, then find the value of x.

Solution : Since 3, x, 12 are in continued proportion, 3, x, x, 12 are in proportion.

So, the product of extreme terms = the product of mean terms.

i.e., $3 \times 12 = x \times x$ or $36 = x^2$

or $x^2 = 6^2$ or $x = 6$.

Hence, the value of x is 6.

Example 10 : Find the mean proportional between 9 and 25.

Solution : Let x be the mean proportional between 9 and 25.

Then, $x^2 = 9 \times 25$ or $x^2 = 225$ or $x^2 = (15)^2$

or $x \times x = 15 \times 15$ or $x = 15$

Hence, the mean proportional between 9 and 25 is 15.

Exercise 6.1

1. Which of the following is/are in proportion ?

(*i*) 8, 16, 6, 12 (*ii*) 6, 2, 4, 3 (*iii*) 150, 250, 200, 300

2. Find x in the following proportions :

(*i*) $x : 6 = 55 : 11$ (*ii*) $18 : x = 27 : 3$ (*iii*) $7 : 14 = 15 : x$

3. Compare the ratios, 7 : 6 and 4 : 9.

4. Find the ratio of the following :

(*i*) 3.2 metres to 56 metres (*ii*) 10 metres to 25 cm (*iii*) 25 paise to ₹60

(*iv*) 10 litres to 0.25 litres (*v*) 36 minutes to 2 hours (*vi*) 250 ml to 2 *l*

5. Which ratio is larger in the following pairs ?

(*i*) 3 : 4 or 9 : 16 (*ii*) 15 : 16 or 24 : 25 (*iii*) 4 : 7 or 5 : 8

(*iv*) 9 : 20 or 8 : 13 (*v*) 1 : 2 or 13 : 27

6. Sunita works in a factory and earns ₹955 per month. She saves ₹185 per month from her earnings. Find the ratio of :

(*i*) her savings to her income (*ii*) her income to her expenditure.

(*iii*) her savings to her expenditure.

7. Sumit earned ₹40000 and paid ₹5000 as income tax. Find the ratio of:

   (*i*) income tax to income. (*ii*) income to income tax.

8. Boys and girls in a school are in the ratio 7 : 4. If the total strength of the school is 550, find the number of boys and girls.

9. Divide ₹1250 between Aman and Amit in the ratio 2 : 3.

10. The sides of a triangle are in the ratio 1 : 2 : 3. If the perimeter is 36 cm, find its sides.

HOTS (Higher Order Thinking Skills)

Critical Thinking

1. 72 persons are working in an office. Out of them, if 28 are men and the remaining are women, find the ratio of the number of:

   (*i*) men to that of women. (*ii*) men to the total number of persons.

   (*iii*) total number of persons to that of women.

2. The ratio of the length of a school ground to its width is 5 : 2. Find its length, if the width is 40 metres.

Unitary Method

The unitary method is one of the most important principles in arithmetic which allows us to find the value associated to individual object and then to determine the value of a group of objects.

For example : If the cost of 1 dozen books is ₹720, then determine the cost of 10 books.

This problem is solved in two parts. First, find out the cost of one book from the given information by division.

i.e., the cost of 12 books = ₹720

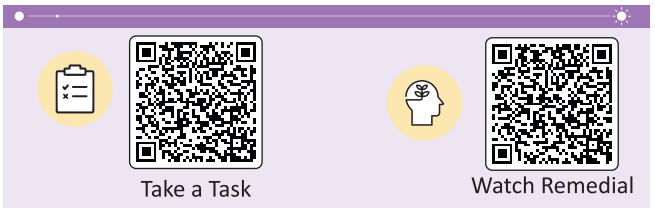
or the cost of 1 book = ₹720 ÷ 12 = ₹60

In the second part of the problem, we have to find the cost of required number of books (i.e. 10 books) by multiplication.

i.e., the cost of 1 book = ₹60

∴ the cost of 10 books = 10 × ₹60 = ₹600

Thus, the cost of 10 books is ₹600.



For solving a problem using unitary method :

Working Rules

Step 1 : Express the given data in a mathematical sentence such that the quantity which is to be found comes at the end of the sentence.

NOTE : Cost of 1 article =

Step 2 : Determine the quantity corresponding to one unit.

Step 3: Lastly, calculate the quantity corresponding to the required number of units.

Example 11 : A worker earns ₹1625 in 10 days. What is his income is in 18 days?

Solution : Earning in 10 days = ₹1625

Earning in 1 day = ₹1625 ÷ 10 = ₹162.50

The earning in 18 days = 18 × ₹162.50 = ₹2925

Thus, the income of the worker in 18 days is ₹2925.

Note

Less number of days less earning.
More number of days more earning.

Example 12 : An aeroplane flies 4000 km in 5 hours. How far does it travel in 3 hours?

Solution : Distance travelled in 5 hours = 4000 km

Distance travelled in 1 hour = $\frac{4000}{5}$ km = 800 km [∵ Less the time less the distance covered]

The distance travelled in 3 hours = (800 × 3) km = 2400 km

Thus, the aeroplane travels 2400 km in 3 hours.

Exercise 6.2

1. A train journey of 90 km costs ₹315. How much will be the cost for the journey of 120 km?
2. If the sales tax on a purchase worth ₹80 is ₹5.60, what will be the sales tax on a purchase worth ₹150 ?
3. The weight of 72 books is 9 kg.

   (*i*) What is the weight of 80 such books ? (*ii*) How many such books weigh 6 kg ?

4. A car travels 165 km in 3 hours.
   (*i*) How much time will it take to travel 440 km ?
   (*ii*) How far will it travel in $6\frac{1}{2}$ hours ?
5. The price of 3 metres of cloth is ₹79.50. Find the price of 15 metres of such cloth.
6. A train runs at a speed of 40 km/h. How many kilometres does it run in 7 hours ?
7. The yield of wheat from 6 hectares land is 280 quintals. Find the number of hectares required for a yield of 225 quintals.
8. Fifteen cards cost ₹2.25. What will be the cost of 36 such cards? How many such cards can we buy in ₹45 ?

Introduction to Percentage

A fraction with its denominator as 100 is called a **percent**.

Percent is an abbreviation of the **Latin** phrase **per centum**, meaning per hundred or hundredths.

For convenience, the symbol '%' is used for percent and it indicates multiplication with $\frac{1}{100}$.

For example, $\frac{18}{100} = 18 \text{ hundredths} = 18 \text{ per hundred}$

$= 18 \times \frac{1}{100} = 18 \text{ percentage} = 18\%$

$\therefore \quad \frac{18}{100}$ can also be expressed as 18 : 100.

Now, we can say that a percentage is simply a ratio of two terms in which the second term is arranged to be 100.

30% is equivalent to the ratio 30 : 100.

Also, 30% is equivalent to the fraction $\frac{30}{100}$ or $\frac{3}{10}$.

Conversion of Percentage

There are various conversions of percentage. Let us consider the following cases to know how a fraction is converted into a percentage.

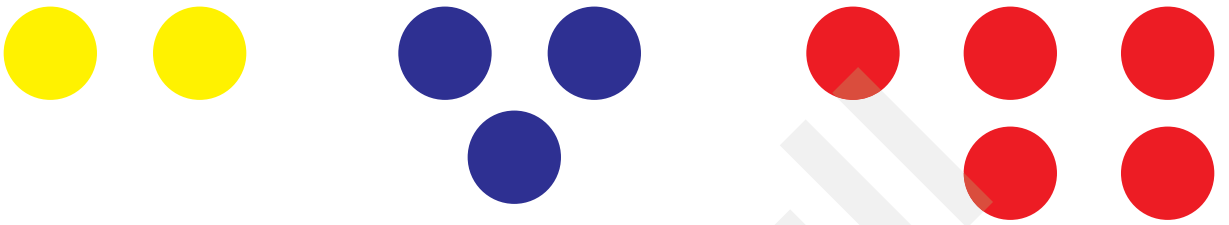
Case I : In the following figure,

$\frac{2}{5}$th part is shaded *i.e.*, $\frac{2}{\cancel{5}_{1}} \times \cancel{100}^{20}\% = 40\%$ part is shaded.

Also, $\frac{3}{5}$th part is unshaded *i.e.*, $\frac{3}{\cancel{5}_{1}} \times \cancel{100}^{20}\% = 60\%$ part is unshaded.

We can also say that out of 100 % region, if 40 % region is shaded, then 100 % – 40 % = 60 % is unshaded.

Case II : Count the number of balls in each colour category as shown in the following.



Yellow balls Blue balls Red balls

Counting the no. of balls, we can prepare the following table.

Colour	No. of Balls	Rate	Fraction	Percentage
1. Yellow	2	2 per 10	$\frac{2}{10} = \frac{1}{5}$	$\frac{1}{5} = \frac{1}{5} \times \frac{20}{20} = \frac{20}{100} = 20\%$
2. Blue	3	3 per 10	$\frac{3}{10}$	$\frac{3}{10} = \frac{3}{10} \times \frac{10}{10} = \frac{30}{100} = 30\%$
3. Red	5	5 per 10	$\frac{5}{10} = \frac{1}{2}$	$\frac{1}{2} = \frac{1}{2} \times \frac{50}{50} = \frac{50}{100} = 50\%$

In the two cases, we have seen that how numbers in the fractional form are converted into the numbers in the percentage form.

Now, let us consider a comparative study using the conversion of percentage. Mohan has got 66 marks out of 75 in Maths. Gita has got 45 out of 50 marks in Maths. Whose result is better ?

Let us find out the result by calculating the percentage of each of them.

$$\% \text{ marks of Mohan} = \frac{\cancel{66}^{22}}{\cancel{75}_{\cancel{3}_{1}}} \times \cancel{100}^{4}\% = 88\%$$

$$\% \text{ marks of Gita} = \frac{45}{\cancel{50}_{1}} \times \cancel{100}^{2}\% = 90\%$$

Clearly, Gita has got more percentage of marks than Mohan, although her total marks obtained (45) is less than Mohan's marks (66) in Maths. Hence, the result of Gita is better than that of Mohan.

Therefore, it is easier to compare the two quantities using the concept of percentage.

Now, let us discuss some types of conversions of percentage as follows :

(*i*) Converting a Fraction into a Percentage.

Working Rules

Step 1 : Obtain the fraction. Let it be

Step 2: Multiply the fraction by 100 and put the percent sign '%' to obtain the required percentage.

For example, the percentage of $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$.

Example 13 : Express each of the following fractions as percent :

(*i*) $\frac{4}{5}$ (*ii*) $5\frac{1}{4}$

Solution : (*i*) $\frac{4}{5} = \left(\frac{4}{\cancel{5}} \times \cancel{100}^{20}\right)\% = 80\%$ (*ii*) $5\frac{1}{4} = \frac{21}{4} = \left(\frac{21}{\cancel{4}} \times \cancel{100}^{25}\right)\% = 525\%$

(*ii*) Converting a Ratio into a Percentage.

Working Rules

Step 1 : Obtain the ratio, say, $a : b$.

Step 2 : Convert the given ratio into the fraction, say $a : b = \frac{a}{b}$.

Step 3 : Multiply the fraction obtained in Step 2 by 100 and put the percent, sign '%'.

Example 14 : Express the following as percent :

(*i*) 6 : 5 (*ii*) 30 : 80

Solution : (*i*) $6 : 5 = \frac{6}{5} = \left(\frac{6}{\cancel{5}} \times \cancel{100}^{20}\right)\% = 120\%$

(*ii*) $30 : 80 = \frac{30}{80} = \left(\frac{30}{80} \times 100\right)\% = 37.5\%$

(*iii*) Converting a Decimal into a Percentage.

Working Rules

Step 1 : Obtain the number in decimal form.

Step 2: Convert it into a fraction by removing the decimal point. In order to remove the decimal, divide by 10 or 100 or 1000 according to the number of digits on the right side of the decimal point.

Step3 : Multiply the converted fraction by 100 and put '%' sign to obtain the required percentage.

Example 15 : Express each of the following as percent :

(*i*) 0.037 (*ii*) 0.002

Solution : (*i*) $0.037 = \frac{37}{1000} = \left(\frac{37}{10\cancel{00}} \times 1\cancel{00}\right)\% = \frac{37}{10}\% = 3.7\%$

(*ii*) $0.002 = \frac{2}{1000} = \left(\frac{2}{10\cancel{00}} \times 1\cancel{00}\right)\% = \frac{2}{10}\% = 0.2\%$

(*iv*) Converting a Percentage into a Fraction or a Decimal.

Working Rules

Step 1 : Obtain the number in percentage form.

Step 2 : Change the percent into a fraction or a decimal just by writing $\frac{1}{100}$ in place of %.

Example 16 : Express each of the following percents as fractions in the simplest form:

(*i*) 115% (*ii*) $12\frac{1}{2}\%$

Solution : (*i*) $115\% = 115 \times \frac{1}{100} = \frac{\cancel{115}^{23}}{\cancel{100}_{20}} = \frac{23}{20} = 1\frac{3}{20}$

(*ii*) $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{\cancel{25}^{1}}{2 \times \cancel{100}_{4}} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Example 17 : Express the following percents as the decimal fractions :

(*i*) 25.5% (*ii*) 15%

Solution : (*i*) $25.5\% = 25.5 \times \frac{1}{100} = \frac{25.5}{100} = 0.255$ (*ii*) $15\% = 15 \times \frac{1}{100} = \frac{15}{100} = 0.15$

(*v*) Finding the Percentage of a Number.

In day-to-day life, we come across various situations where finding the percentage of numbers is needed.

Let us follow the Working Rules given below to find the percentage of a number.

Working Rules

Step 1 : Obtain the given number, say x.

Step 2 : Obtain the required percent, say P %.

Step 3 : Multiply x by P and divide the product $P \times x$ by 100 to obtain the required

$P\%$ of x *i.e.*, $P\%$ of $x = \frac{P}{100} \times x$.

Example 18 : Find each of the following :

(*i*) 30% of ₹180 (*ii*) 75% of 400

Solution : (*i*) 30% of ₹180 $= ₹\left(\frac{30}{100} \times 180\right) = ₹54$ (*ii*) 75% of 400 $= \left(\frac{75}{100} \times 400\right) = 300$

Exercise 6.3

1. Express each of the following fractions as the percent :

(*i*) $\frac{1}{5}$ (*ii*) $\frac{2}{3}$ (*iii*) $\frac{1}{8}$ (*iv*) $\frac{3}{4}$ (*v*) $\frac{3}{8}$ (*vi*) $\frac{53}{100}$ (*vii*) $\frac{5}{7}$

2. Express each of the following ratios as the percent :

(*i*) 60 : 80 (*ii*) 2 : 25 (*iii*) 4 : 5 (*iv*) 2 : 3 (*v*) 1 : 5 (*vi*) 11 : 125

3. Express each of the following decimals as the percent :

(*i*) 0.54 (*ii*) 1.545 (*iii*) 9.18 (*iv*) 0.004 (*v*) 0.24 (*vi*) 0.02 (*vii*) 0.275

4. Express each of the following as whole numbers or mixed numbers :

(*i*) 136% (*ii*) 250% (*iii*) 300%

5. Express each of the following percents as fractions in the simplest form :

(*i*) 28% (*ii*) 52% (*iii*) 0.4% (*iv*) $6\frac{2}{3}\%$

6. Express each of the following as a decimal :

(*i*) 65% (*ii*) 12% (*iii*) 7.4% (*iv*) 0.1%

7. Find each of the following :

(*i*) 5% of ₹1400 (*ii*) 7% of ₹7150 (*iii*) 40% of 400 kg

8. Find the quantities:

(*i*) 20% of 15.125 litres (*ii*) 16% of 25 litres (*iii*) $4\frac{1}{2}\%$ of ₹1800

Profit and Loss

In our day-to-day routine, we buy various articles from shops. The shopkeepers purchase these articles either from the wholesalers or directly from the manufacturers by paying a certain price. Generally, the shopkeepers sell these articles at different prices. Various terms related to these prices are involved such as cost price, selling price, profit, loss, etc.

Cost Price : The price at which an article is purchased is called the cost price and is abbreviated as CP.

Selling Price : The price at which an article is sold is called the selling price and is abbreviated as S P.

Profit : If the difference between the selling price and the cost price is positive, then the difference is called the profit.

Profit = Selling Price – Cost Price

Loss : If the difference between the selling price and cost price is negative, then the difference is called loss.

Loss = Cost Price – Selling Price

Usually a merchant has to spend some money on freight, labour or maintenance of the purchased articles. These extra expenditures are called overheads. Since overheads are an essential part of cost price.

Cost price = (payment made while purchasing the articles) + overhead charges.

Generally, the profit or loss is expressed as profit % or loss %. It is to be kept in mind that the profit % or loss % is calculated on the cost price.

$$i.e., \text{Profit } \% = \frac{\text{Profit}}{\text{CP}} \times 100 \text{ and Loss } \% = \frac{\text{Loss}}{\text{CP}} \times 100$$

Example 19 : If the profit made on a packet of tea is ₹4 and the cost price of the packet is ₹20, then how much is the profit percentage ?

Problem Solving

Solution : It is given that, CP = ₹20 and profit = ₹4.

So, $\text{Profit\%} = \frac{\text{Profit}}{\text{CP}} \times 100$ CP

$$= \left(\frac{4}{\cancel{20}} \times \cancel{100}^{5} \right)\% = 20\%$$

Hence, the required profit percentage is 20%.

Example 20 : A man sells his scooter for ₹18000 making a profit of 20%.

How much did the scooter cost him?

Problem Solving

Solution : Let the cost price of the scooter be ₹100.

Then, profit = ₹20 [Since profit = 20% given]

Now, SP = CP + Profit = ₹100 + ₹20 = ₹120

Now, if the SP is ₹120, then CP = ₹100.

If the SP is ₹18000, then CP = ₹ $\left(\frac{100}{\cancel{120}} \times \cancel{18000}^{150} \right)$ = ₹15,000

Hence, the cost of the scooter is ₹15,000.

Example 21 : Find the unknown values from the following:

(*i*) CP = ₹500, SP = ₹600, Profit/Loss = ?

(*ii*) CP = ?, SP = ₹2390, Profit = ₹120.50

Solution : (*i*) We have, CP = ₹500, SP = ₹600

Clearly, SP > CP.

So, there will be profit given by

Profit = SP – CP = ₹(600 – 500) = ₹100

(*ii*) It is given that SP = ₹2390 and Profit = ₹120.50.

We know that Profit = SP – CP

i.e., CP = SP – Profit = ₹2390 – ₹120.50

= ₹(2390 – 120.50) = ₹2269.50

Hence, CP = ₹2269.50

Example 22 : Neha bought 1600 bananas at ₹3.75 a dozen. She sold 900 of them at 2 for ₹1 and the remaining at 5 for ₹2. Find her gain or loss percent.

Solution: Cost of 1 dozen (*i.e.* 12) bananas = ₹3.75

$$\text{Cost of 1600 bananas} = ₹\left(\frac{3.75}{12}\times 1600\right) = ₹\frac{\cancel{375}^{125}\times\cancel{16}^{4}}{\cancel{12}_{\cancel{3}_1}} = ₹\,500$$

Thus, CP of 1600 bananas = ₹500.

Now, the selling price (SP) of 900 bananas at the rate of 2 for ₹1

$$= ₹\frac{900}{2} = ₹450$$

The selling price of the remaining *i.e.* 1600 – 900 = 700 bananas at the rate of 5 for ₹2 $= ₹\left(\frac{2}{\cancel{5}}\times\cancel{700}^{140}\right) = ₹280$

$\therefore$ SP of 1600 bananas = ₹(450 + 280) = ₹730.

Now, CP of 1600 bananas = ₹500 and SP of 1600 bananas = ₹730.

Clearly, SP > CP. So, there is a gain given by

Gain = SP – CP = ₹(730 – 500) = ₹230

Hence, gain percent $= \left(\frac{\text{Gain}}{\text{CP}}\times 100\right)\% = \left(\frac{\cancel{230}^{46}}{\cancel{500}}\times\cancel{100}\right) = 46\%$

Hence, the required gain percent is 46%.

Exercise 6.4

Problem Solving

1. A vendor bought oranges at ₹26 per dozen and sold them at 5 for ₹13. Find his gain percent.

2. **Naresh bought 4 dozen pencils at ₹10.80 a dozen and sold them for 80 paise each. His loss percent is**

   (*i*) $\frac{10}{3}\%$ (*ii*) $\frac{9}{100}\%$ (*iii*) $\frac{100}{9}\%$ (*iv*) 10%

3. **Some values are given in the following. Find the unknown values :**

   (*i*) CP = ₹1200, SP = ₹1350, Profit/Loss = ?

   (*ii*) CP = ?, SP = ₹1254, Loss = ₹32

   (*iii*) CP = ₹72, SP = ? Loss = ₹15.60

4. Ravinder purchased a house for ₹45200 and spent ₹2800 on its repairs. He had to sell it for ₹46800. Find his loss and loss percent.

5. Riya bought 100 eggs for ₹50. Out of these, 4 eggs were found to be broken and he sold the remaining eggs at the rate of ₹8.50 per dozen. Find his gain or loss percent.

6. Shivam bought an article for ₹1215 and spent ₹35 on its transportation. At what price should he sell the article to have a gain of 16% ?

7. A farmer sells his product at a loss of 8%. If his SP is ₹27600, what is his actual loss? What is his cost price ?

8. Ishita sold a book worth ₹200 at a profit of 20% to Rohan. Rohan sells it at a loss of 10% to Rishi. What did Rishi pay for his book ?

HOTS (Higher Order Thinking Skills)

Critical Thinking

1. The cost price of 8 books is equal to the selling price of 6 books. Find the gain percent.
2. A grocer buys eggs at 10 for ₹8 and sells at 8 for ₹10. Find his gain or loss percent.

Simple Interest

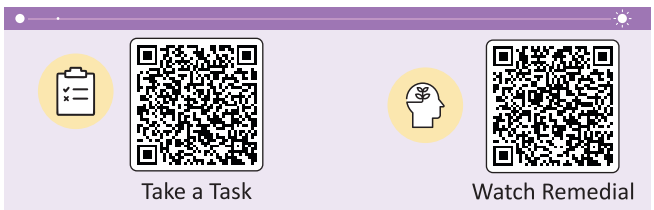
Whenever we borrow money from some lending sources such as banks or financial institutions etc., we have to pay some extra money for the service of lending. This extra money depends on the sum we want to borrow and the period of time for which we wish to borrow it. This extra money is called the interest. It is usually a percentage of the borrowed money, which is called principal.

On the other hand, when we deposit money in a bank for safe keeping, we earn interest. Interest is calculated according to an agreement which specifies the rate of interest. Generally the rate of interest is taken as "percent per annum" which means "per ₹100 per year."

For example, a rate of '10% per annum' means ₹10 on ₹100 for 1 year.

When interest is calculated simply on the original principal, it is known as **simple interest**. When the interest for a specific period is added to the principal, then the sum is called the **amount**.

Now, we have the following :



(*i*) $\text{Amount} = \text{Principal} + \text{Interest}$

(*ii*) $\text{Simple Interest (S.I.)} = \dfrac{P \times R \times T}{100}$

where P = Principal, R = Rate percent per annum, T = Time

Example 23 : A sum of ₹800 is lent for one year at the rate of 18% per annum. Find the interest.

Solution : Here, we have : principal (P) = ₹800, rate (R) = 18% per annum, time (T) = 1 year

Now, interest on ₹100 for 1 year = ₹18

Interest on ₹1 for 1 year = $₹\frac{18}{100}$

Interest on ₹800 for 1 year = $₹\left(\frac{18}{\cancel{100}} \times 8\cancel{00}\right) = ₹144.$

Hence, the required interest is ₹144.

Example 24 : Anita borrowed ₹400 from her friend at the rate of 12% per annum for $2\frac{1}{2}$ years. Find the interest and amount paid by her.

Solution : It is given that principal (P) = ₹400, rate of interest (R) = 12% and time (T) = $2\frac{1}{2}$ years = $\frac{5}{2}$ years.

Now, interest rate = 12% per annum

i.e., interest on ₹100 for 1 year = ₹12

interest on ₹100 for $\frac{5}{2}$ years = $₹\left(\frac{\cancel{12}^{6} \times 5}{\cancel{2}}\right) = ₹30$

Then, interest on ₹1 for $\frac{5}{2}$ years = $₹\frac{30}{100}$

Hence, interest on ₹400 for $\frac{5}{2}$ years = $₹\left(\frac{30}{\cancel{100}} \times 4\cancel{00}\right) = ₹120$

Thus, interest paid = ₹120.

Now, amount to be paid = principal + interest

= ₹400 + ₹120 = ₹520

Thus, interest and amount paid by Anita were ₹120 and ₹520 respectively.

Example 25 : A moneylender wants $\frac{1}{5}$th of the amount loaned every year as interest. What will be the rate of interest, if a farmer borrows ₹5000 for 1 year from the moneylender? What is the amount that he has to pay back altogether?

Solution : The moneylender wants $\frac{1}{5}$th of the amount loaned every year as interest.

So, if the money lender gives a loan of ₹100,

then interest after one year $= ₹\left(\frac{1}{\cancel{5}} \times \cancel{100}^{20}\right) = ₹20$

Thus, the rate of interest = 20% per annum.

Now, interest on ₹100 for one year = ₹20.

Interest on ₹1 for one year $= ₹\frac{20}{100}$

Interest on ₹5000 for one year $= ₹\left(\frac{20}{\cancel{100}} \times 50\cancel{00}\right) = ₹1000$

Amount to be paid = principal + interest

= ₹5000 + ₹1000 = ₹6000

Hence, the farmer has to pay back ₹6000 altogether to the moneylender.

Exercise 6.5

Problem Solving

1. Ramesh borrowed ₹7,000 from a bank for 1 year at the rate of 10% per annum. Find the interest to be paid after one year.
2. Find the interest on ₹500 for a period of 4 years at the rate of 8% per annum. Also, find the amount to be paid at the end of the period.
3. Find the interest on ₹25,000 for a period of 4 years at the rate of 9% per annum. Also, find the amount to be paid at the end of the period.
4. Ravi borrowed ₹2000 at 2% per annum and ₹1000 at 5% per annum. He cleared his debt after 2 year's by giving ₹2800 and a watch. What is the cost of the watch ?
5. A farmer borrowed ₹2400 at an interest of 12% per annum. At the end of $2\frac{1}{2}$ years, he cleared his account by paying ₹1200 and a cow. Find the cost of the cow.
6. Shikha deposited ₹2000 in a bank which pays 6% simple interest. She withdrew ₹700 at the end of the first year. What will be her balance after 3 years ?
7. What principal will amount to ₹20800 in 2 years at the simple interest of 2% per annum ?

Chapter-end Exercise

Gap Analyzer™

A. Tick (✓) the correct option

1. If $x : 5 = 24 : 40$, then the value of x is

   (a) 3 ☐ (b) 12 ☐ (c) 5 ☐ (d) 2 ☐

2. 16 men can reap a field in 30 days. In how many days will 20 men reap the same field ?

   (a) 18 days ☐ (b) 20 days ☐ (c) $15\frac{1}{3}$ days ☐ (d) 24 days ☐

3. On selling a book for ₹48, a shopkeeper loses 20%. In order to gain 20%, what would be the selling price?

(*a*) ₹50 ☐ (*b*) ₹70 ☐ (*c*) ₹68 ☐ (*d*) ₹72 ☐

4. The simple interest on ₹5000 for 6 months at 4% per annum is

(*a*) ₹110 ☐ (*b*) ₹120 ☐ (*c*) ₹100 ☐ (*d*) ₹105.80 ☐

B. True / False:

Knowledge Application

1. The ratio of 0.15 kg to 180 g is 5 : 6. ☐
2. If the weight of 10 mathematics books is 5 kg, then the weight of such 14 books will be 7 kg. ☐
3. The percent of 36 minutes in 1 day is 2.5%. ☐
4. Ratio can be expressed as a fraction or as a percentage. ☐
5. The length and breadth of a strip are 2 m and 25 cm respectively. The ratio of breadth to length is 11 : 8. ☐

C. Answer the following questions:

Problem Solving

1. Three ferry loads are needed to carry 150 people across a river. How many people will be carried by 4 ferry loads?
2. Express each of the following ratios as the percents:

   (*i*) 36 : 75 (*ii*) 7 : 5 (*iii*) 2 : 5 (*iv*) 12 : 25
3. Express each of the following decimals as the percent:

   (*i*) 5.575 (*ii*) 2.05 (*iii*) 1.2 (*iv*) 0.063
4. Hari bought a second-hand typewriter for ₹1200 and spent ₹200 on its repairs. He sold it for ₹1680. Find his profit or loss. What was his profit or loss percentage?
5. Krish bought oranges at ₹5 a dozen. He had to sell them at a loss of 4%. Find the selling price of one orange.
6. Find the interest on ₹1200 at 6% per annum for 146 days.
7. Amit and Sumit share their chocolates in the ratio of 5 : 8. If Sumit has 56 chocolates, how many does Amit have?

Custom Learning Path

Scan to Create Your Own Learning Path

HOTS (Higher Order Thinking Skills)

Critical Thinking

1. If in a proportion, the first, second and fourth terms are 32, 112 and 217 respectively, find the third term.
2. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each, the ratio becomes 4 : 5. Find the numbers.
3. If $x : y = 3 : 2$, find $(2x + 3y) : (3x + 5y)$.

Assertion and Reason

Critical Thinking

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.
Study both the statements and state which of the following is correct:
(*a*) Both A and R are true and R is the correct explanation of A.
(*b*) Both A and R are true but R is not the correct explanation of A.
(*c*) A is true but R is false.
(*d*) A is false but R is true.

1. **Assertion (A) :** A ratio is a fraction and has no unit.
   Reason (R) : A ratio is expressed in the simplest from.
2. **Assertion (A) :** A fraction with its denominator as 100 is called a percent.
   Reason (R) : 6 : 5 is equal to 128%.
3. **Assertion (A) :** 30% of ₹180 is equal to ₹55.
   Reason (R) : 75% of ₹400 is equal to ₹300.
4. **Assertion (A) :** Profit = Selling Price – Cost Price.
   Reason (R) : The price at which an article is purchased is called the cost price.
5. **Assertion (A) :** The ratio of ₹8 and 80 paise is 10 : 1.
   Reason (R) : If an item is sold for ₹990 at a profit of 10%, then its cost price is ₹950.

Activity

Collaboration

Unitary Method :

- Ask 1 student to come forward and give a packet of balls to him.
- Ask him to find the value of 1 ball if whole packet of ₹60. Help the students to recognise that to find the price of 1 ball, we divide the cost by number of balls.
- When teacher gets the answer of 10, ask another students to come. Now ask the other student to tell the cost of 15 such balls.
- Help the students to recognise that they already have the cost of 1 ball and now have to find the cost of 15 such balls, so they will need to multiply the cost of 1 ball by number of balls.
- Now explain to the students that such method of calculating the price of various units is known as Unitary Method. Students will enjoy performing the activity and the rest of the students will enjoy observe it.