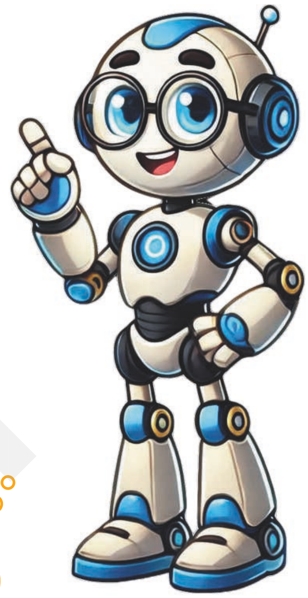
3
Chapter

Rational Numbers

We'll cover the following key points:

- → Rational numbers on a number line
- → Standard (or simplest) form of a rational number
- → Absolute value of a rational number
- → Operation on rational numbers
- → To Find a Relational Number Between Two Given Rational Numbers
- → Multiplication of rational numbers

Still curious? Talk to me by scanning the QR code.



Hi, I'm EeeBee

Learning Outcomes

By the end of this chapter, students will be able to:

- Define and identify rational numbers and distinguish them from other number types, including integers, whole numbers, and irrational numbers.
- Represent rational numbers on a number line accurately, and understand their positions relative to zero and other numbers.
- Express a rational number in its standard form by simplifying fractions and ensuring that the denominator is positive.
- Perform operations (addition, subtraction, multiplication, and division) on rational numbers, applying appropriate methods and rules for each operation.
- Understand the properties of rational numbers, including closure, commutativity, associativity, distributive property, and the identity elements for addition and multiplication.
- Compare and order rational numbers by converting them into a common format (fractions, decimals) and arranging them in ascending or descending order.
- Identify and find the reciprocal of a rational number and use it to perform division.
- Solve real-life problems involving rational numbers, such as calculating distances, temperatures, and financial transactions.
- Apply the concept of rational numbers in solving equations and inequalities, and use rational numbers in algebraic expressions.

Mind Map

RATIONAL NUMBERS

Rational numbers

- A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, q ? 0

✓ p & q are integers

e.g.,

$\frac{3}{5}, \frac{-4}{7}$

p = Numerator

q = Denominator

Positive and Negative Rational Numbers

i. Positive rational number

Both numerator and denominator positive. $\frac{3}{7}, \frac{2}{5}$ etc.

ii. Negative rational numbers

Either numerator or denominator is negative. $\frac{-3}{7}, \frac{2}{-5}$ etc.

Rational numbers in standard form

HCF of Numerator and Denominator is 1.

e.g., $\frac{4}{5}$

HCF of 4 & 5 = 1

Rational number between two rational number

➢ There are unlimited number of rational numbers between any two rational numbers.

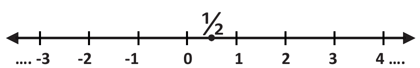
Equivalent rational numbers

e.g.,

$\frac{5}{7} = \frac{5 \times 2}{7 \times 2} = \frac{10}{14}$

$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

Rational number on a number line



e.g.,

$\frac{1}{2}$ is exactly between 0 and 1

Comparison of rational numbers

- Make the denominators equal by taking LCM of denominators then compare numerators.

Right > Left (on number line)

e.g., $\frac{3}{5}, \frac{7}{10}$

LCM of 5 & 10 is 10

$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$

$\frac{7}{10} = \frac{7}{10}$

7 > 6

So, $\frac{7}{10} > \frac{3}{5}$

Operations on rational number

i. Addition

e.g., $\frac{7}{3} + \frac{8}{3} = \frac{7+8}{3} = \frac{15}{3} = \frac{5}{1}$

Additive inverse of $\frac{4}{7}$ is $\frac{-4}{7}$

ii. Subtraction

e.g., $\frac{5}{7} - \frac{3}{4}$

LCM of 7 and 4 is 28

$\frac{5}{7} = \frac{5 \times 4}{7 \times 4} = \frac{20}{28}$

$\frac{3}{4} = \frac{3 \times 7}{4 \times 7} = \frac{21}{28}$

$\frac{5}{7} - \frac{3}{4} = \frac{20}{28} - \frac{21}{28} = \frac{20-21}{28} = \frac{-1}{8}$

iii. Multiplication

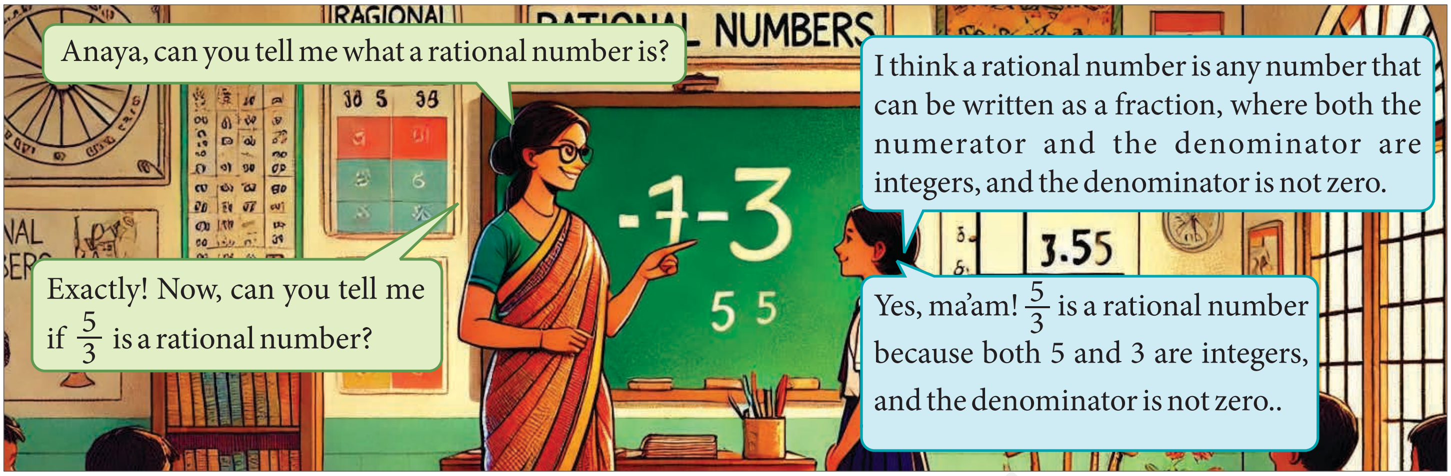
$\frac{\text{Numerator} \times \text{Numerator}}{\text{Denominator} \times \text{Denominator}}$

e.g., $\frac{8}{7} \times \frac{14}{15} = \frac{16}{15}$

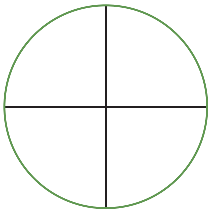
iv. Division

e.g., $\frac{15}{16} \div \frac{3}{4} = \frac{15}{16} \times \frac{4}{3} = \frac{5}{4}$

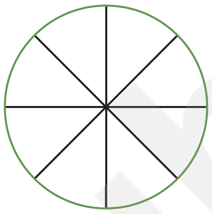
Introduction



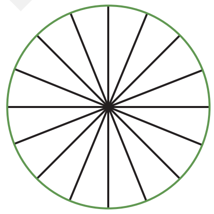
If we divide any thing in two parts then one of the part is called half of one thing and denoted by $\frac{1}{2}$. It means we can divide the same thing into many parts. Like this given below.



Each part is $\frac{1}{4}$



Each part is called $\frac{1}{8}$



Each part is called $\frac{1}{16}$

Hence, the number is divided into many parts called Rational Numbers, like :

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9} \ldots.$$

The upper number is called **numerator** and below number is called **denominator**.

The term '**rational**' comes from the word '**ratio**', because rational numbers are the ones that can be written in the ratio form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Before knowing more about rational numbers, let us recall natural and whole numbers.

Natural numbers: Counting numbers 1, 2, 3, 4, 5, ... are called natural numbers.

Whole numbers: Counting numbers together with zero (0), i.e., 0, 1, 2, 3, 4, 5, ... are called whole numbers.

While learning the properties of whole numbers, we observe the following facts.

1. The sum of two whole numbers is a whole number.

   Examples : $6 + 7 = 13, 18 + 19 = 37, 5 + 25 = 30$

2. The product of two whole numbers is a whole number.

Examples: $3 \times 5 = 15, 6 \times 7 = 42, 10 \times 8 = 80$

Can you think of any two whole numbers whose product is 1 ?

When 1 is multiplied by 1, the product is 1. No other whole number has this property.

3. The difference of two whole numbers is not always a whole number.

Examples: $3 - 2 = 1$ [1 is a whole number.]

$7 - 3 = 4$ [4 is a whole number.]

$5 - 7 = (-2)$ [(−2) is not a whole number.]

$6 - 10 = (-4)$ [(−4) is not a whole number.]

Hence, the number system is extended from whole numbers to integers by including negative numbers with the whole numbers.

Integers: Whole numbers together with their negatives are called integers, *i.e.*, $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$ is the set of integers.

Thus, the subtraction of whole numbers led us to integers. Similarly, in the previous chapter, while learning the properties of integers, we observed that the division of one integer by another integer may or may not be an integer.

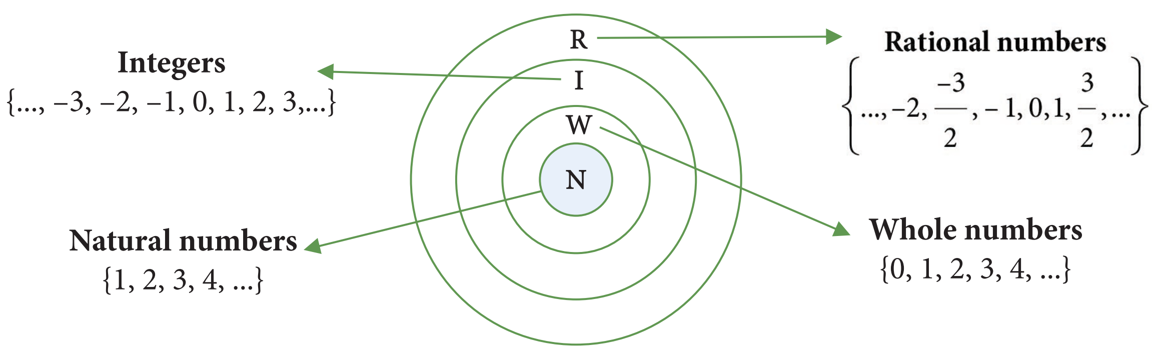
$8 \div 4 = 2$ [2 is an integer.]

$(-12) \div 3 = (-4)$ [(−4) is an integer.]

$3 \div (-4) = \frac{3}{-4}$ [$\frac{3}{-4}$ is not an integer.]

$(-1) \div (-7) = \frac{1}{7}$ [$\frac{1}{7}$ is not an integer.]

Hence, there is again a need to extend our number system in which negative and positive fractions must be included. This new system of numbers is called the **rational number system**, about which you learnt in the beginning of this chapter.



Let us define a rational number as follows:

Rational number: A number which is in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.

In any fraction, the denominator cannot be equal to zero because division by zero is not defined.

Example: (*a*) $\frac{5}{6}$ is a rational number because both 5 and 6 are integers and the denominator 6 is not equal to zero.

(*b*) $\frac{-1}{7}$ is a rational number because both (–1) and 7 are integers and the denominator 7 is not equal to zero.

Remember, all integers are rational numbers since we can write them as rational numbers with the denominator as 1.

Example: $9 = \frac{9}{1}$ Here, $\frac{9}{1}$ is a rational number, where 9 and 1 are integers.

Thus, the system of rational numbers includes all integers and fractions.

There are two types of rational numbers:

(*i*) Positive rational numbers (*ii*) Negative rational numbers

Positive rational numbers: A rational number is said to be positive if its numerator and denominator are both either positive or negative.

Examples: $\frac{2}{7}, \frac{3}{6}, \frac{13}{69}, \frac{0}{9}$ are positive rational numbers.

Also, $\frac{-3}{-5}, \frac{-17}{-21}, \frac{-8}{-9}$ are positive rational numbers. Can you say why?

Because, $\frac{-3}{-5} = \frac{3}{5}, \frac{-17}{-21} = \frac{17}{21}, \frac{-8}{-9} = \frac{8}{9}$

REMEMBER

A rational number $\frac{a}{b}$ is a fraction only when a and b are whole numbers and $b \neq 0$.

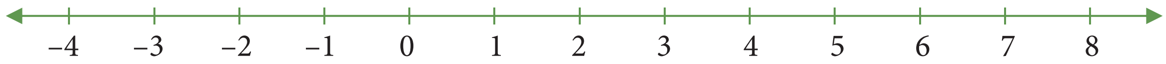
Negative rational numbers: A rational number is said to be negative if either its numerator or denominator is negative.

Examples: $\frac{-8}{17}, \frac{8}{-17}, \frac{-6}{11}, \frac{6}{-11}$ are negative rational numbers because

$$\frac{-8}{17} = \left(-\frac{8}{17}\right), \frac{8}{-17} = \left(-\frac{8}{17}\right), \frac{-6}{11} = \left(-\frac{6}{11}\right), \frac{6}{-11} = -\left(\frac{6}{11}\right).$$

Rational Numbers on a Number Line

We have learnt earlier how to represent fractions, integers, whole numbers, and natural numbers on a number line. Rational numbers can also be represented on a number line. We know that as we move from left to right on the number line, the number increases as shown in the figure.

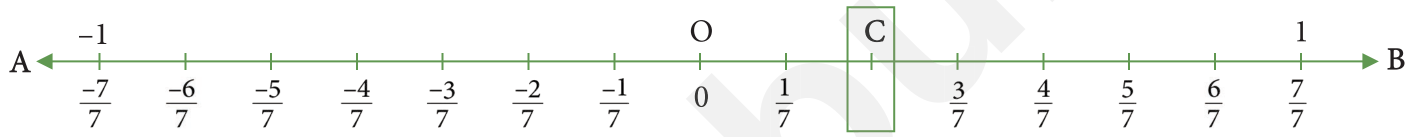


To express a rational number on a number line, we divide each unit length into as many equal parts as the denominator of the rational number. After that we can easily mark the required rational number on the number line.

Let us consider some examples:

Example 1 : Represent $\frac{2}{7}$ on a number line.

Solution : Here, the denominator of the rational number is 7. So, we divide each unit length on the number line AB into 7 equal parts as shown in the figure.



Since the numerator is 2, count 2 parts to the right of zero on the number line and mark it as point C.

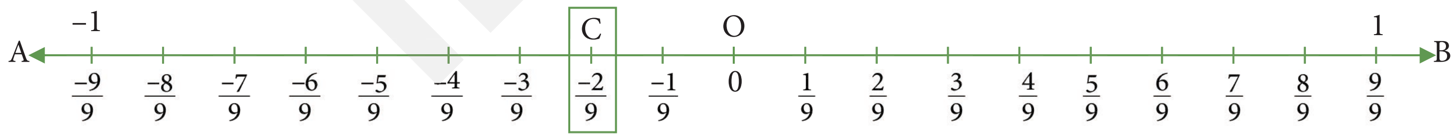
Hence, point C represents $\frac{2}{7}$.

Take a Task

Watch Remedial

Example 2 : Represent $\frac{-2}{9}$ on a number line.

Solution : Here, the denominator is 9. So, we divide each unit length on the number line AB into 9 equal parts as shown in the figure.



Since the numerator is 2, count 2 parts to the left of zero on the number line and mark it as point C. Hence, point C represents $\frac{-2}{9}$.

Standard (or Simplest) form of a Rational Number

A rational number $\frac{p}{q}$ is said to be in standard form if :

(*i*) the denominator (q) is a positive integer.

(*ii*) p and q are co-primes, i.e., have no common factor other then 1.

Example 3 : Express each of the following rational numbers in standard form:

(*i*) $\frac{-15}{36}$ (*ii*) $\frac{45}{-75}$

Solution : (*i*) The denominator of $\left(\frac{-15}{36}\right)$ is positive. For expressing it in standard form, we find the HCF of 15 and 36, which is 3. Now, on dividing both the numerator and denominator by 3, we get

$$\left(\frac{-15}{36}\right) = \frac{(-15)\div 3}{36\div 3} = \frac{-5}{12}.$$

Thus, the standard form of $\frac{-15}{36}$ is $\frac{-5}{12}$.

(*ii*) The denominator of $\left(\frac{45}{-75}\right)$ is negative. To make it positive, we multiply its numerator and denominator by (–1).

$$\frac{45\times(-1)}{(-75)\times(-1)} = \frac{-45}{75}$$

Now, we find the HCF of 45 and 75, which is 15. On dividing the numerator and denominator by 15, we get $\frac{-45}{75} = -\frac{(-45)\div 15}{75\div 15} = \frac{-3}{5}$

Thus, the standard form of $\left(\frac{-45}{75}\right)$ is $\frac{-3}{5}$.

Take a Task

Watch Remedial

Absolute Value of a Rational Number

The absolute value of a rational number is its positive numerical value,

i.e., $\left|\frac{p}{q}\right| = \frac{p}{q}, \left|\frac{-p}{q}\right| = \frac{p}{q}, \left|\frac{p}{-q}\right| = \frac{p}{q}$

Examples : (*i*) $\left|\frac{4}{7}\right| = \frac{4}{7}$

(*ii*) $\left|\frac{-11}{13}\right| = \frac{11}{13}$

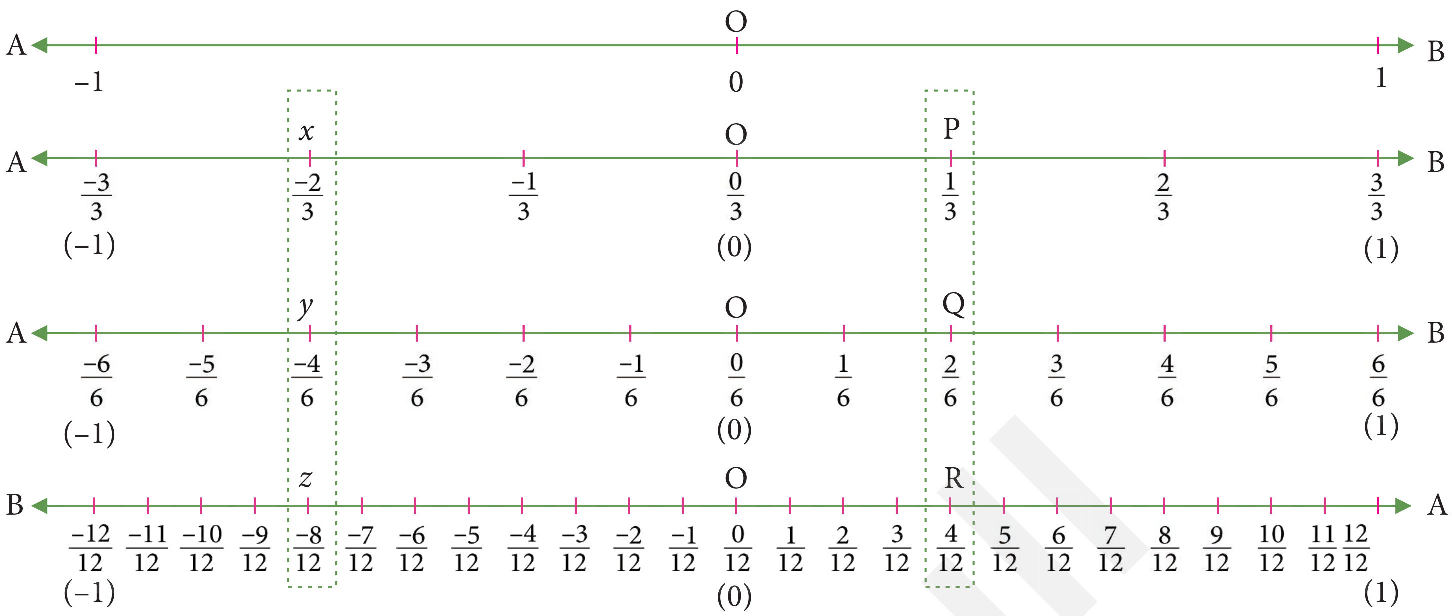
(*iii*) $\left|\frac{12}{-25}\right| = \frac{12}{25}$

Note

The absolute value of a rational number, when represented on a number line, is considered as its distance from zero (irrespective of the direction) and is represented as $\left|\frac{p}{q}\right|$. A positive rational number is always greater than a negative rational number.

Equivalent Rational Numbers

Let us draw number lines as shown in the figure.



We can clearly see that points P, Q, and R, representing the rational numbers $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$, respectively, are equidistant from point O representing zero on the number lines. In other words, the same point corresponds to these three rational numbers. We say rational numbers $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$, are equivalent. Similarly, points x, y, and z representing the rational numbers $\left(\frac{-2}{3}\right)$, $\left(\frac{-4}{6}\right)$, and $\left(\frac{-8}{12}\right)$ are equidistant from point O representing zero on the number lines.

Hence, they are equivalent, i.e., $\frac{-2}{3} = \frac{-4}{6} = \frac{-8}{12}$.

Thus, rational numbers which are represented by the same point on a number line are called **equivalent (or equal) rational numbers**.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers:

(*i*) If $\frac{a}{b} = \frac{c}{d}$ or $ad = bc$, they are equivalent rational numbers.

(*ii*) If $ad > bc$, then $\frac{a}{b} > \frac{c}{d}$.

(*iii*) If $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$.

Equivalent rational numbers are obtained by multiplying (or dividing) the numerator and the denominator of the given rational number by the same non-zero integer.

Examples: (*a*) $\dfrac{2}{9} = \dfrac{2 \times 4}{9 \times 4} = \dfrac{8}{36}$ So, $\dfrac{2}{9} = \dfrac{8}{36}$ (*b*) $\dfrac{-5}{11} = \dfrac{(-5 \times 3)}{11 \times 3} = \dfrac{-15}{33}$ So, $\dfrac{-5}{11} = \dfrac{-15}{33}$

Example 4: Are rational numbers $\dfrac{8}{12}$ and $\dfrac{10}{15}$ equivalent?

Solution: We know that $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are equivalent if $ad = bc$.

Let $\dfrac{a}{b} = \dfrac{8}{12}$ and $\dfrac{c}{d} = \dfrac{10}{15}$

$ad = 8 \times 15 = 120$ and $bc = 12 \times 10 = 120$

Since $ad = bc = 120$, Thus, $\dfrac{8}{12}$ and $\dfrac{10}{15}$ are equivalent.

Example 5: Write three equivalent rational numbers of $\dfrac{2}{7}$.

Solution: $\dfrac{2}{7} = \dfrac{2 \times 2}{7 \times 2} = \dfrac{4}{14}$, $\dfrac{2}{7} = \dfrac{2 \times 3}{7 \times 3} = \dfrac{6}{21}$, $\dfrac{2}{7} = \dfrac{2 \times 4}{7 \times 4} = \dfrac{8}{28}$

So, the three equivalent rational numbers of $\dfrac{2}{7}$ are $\dfrac{4}{14}$, $\dfrac{6}{21}$, and $\dfrac{8}{28}$.

Example 6: Express $\dfrac{-48}{80}$ as a rational number with numerator 12.

Solution: First, find a number which when divided by (−48) gives 12.

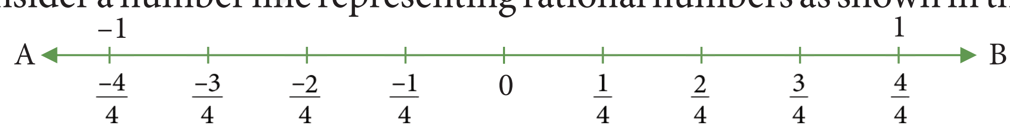
Since $(-48) \div 12 = (-4)$, we divide the numerator and denominator of $\dfrac{-48}{80}$ by (−4).

$$\frac{-48}{80} = \frac{(-48) \div (-4)}{80 \div (-4)} = \frac{12}{-20}$$

So, $\dfrac{-48}{80} = \dfrac{12}{-20}$

Comparison of Rational Numbers

Let us consider a number line representing rational numbers as shown in the figure.



From the figure,

$$\frac{-4}{4} < \frac{-3}{4} < \frac{-2}{4} < \frac{-1}{4} < 0 < \frac{1}{4} < \frac{2}{4} < \frac{3}{4} < \frac{4}{4} \quad \text{(Ascending order)}$$

$$\frac{4}{4} > \frac{3}{4} > \frac{2}{4} > \frac{1}{4} > 0 > \frac{-1}{4} > \frac{-2}{4} > \frac{-3}{4} > \frac{-4}{4} \quad \text{(Descending order)}$$

The above rational numbers have the same denominator. So, by comparing the numerators we can find out which is greater or which is smaller.

Example: $\frac{3}{4} > \frac{-1}{4}$ (Since $3 > -1$)

or, $\frac{-1}{4} < \frac{3}{4}$ (Since $-1 < 3$)

If rational numbers have different denominators, we change them as rational numbers having the same denominator and then compare.

Example 7: Which is smaller, $\frac{13}{23}$ or $\frac{8}{23}$?

Solution: Since both $\frac{13}{23}$ and $\frac{8}{23}$ have the same denominator, we compare their numerators.

Here, $8 < 13$

So, $\frac{8}{23} < \frac{13}{23}$.

Example 8: Which of the two rational numbers $\frac{5}{-6}$ and $\frac{-3}{4}$ is greater ?'

Solution: First, we rewrite the rational number $\frac{5}{-6}$ with its denominator as positive.

So, $\frac{5}{-6} = \frac{5}{-6} \times \frac{(-1)}{(-1)} = \frac{-5}{6}$

We now express $\frac{-5}{6}$ and $\frac{-3}{4}$ as rational numbers with common denominator.

So, $\frac{-5}{6} = \frac{-5 \times 2}{6 \times 2} = \frac{-10}{12}, \frac{-3}{4} \times \frac{3}{3} = \frac{-9}{12}$

Since, $-9 > -10$

$\therefore \frac{-9}{12} > \frac{-10}{12}$ i.e., $\frac{-3}{4} > \frac{-5}{6}$

Thus, $\frac{-3}{4} > \frac{5}{-6}$, i.e. $\frac{-3}{4}$ is the greater one.

Exercise 3.1

1. (*a*) **Which of the following are rational numbers?**

$\frac{3}{1}, \frac{-7}{5}, \frac{0}{3}, \frac{-8}{-1}, \frac{0}{0}, \frac{0}{8}, \frac{9}{0}, \frac{4}{-1}$

(*b*) **Write the following rational numbers as integers:**

$\frac{-8}{-1}, \frac{0}{-2}, \frac{-11}{1}, \frac{4}{-2}, \frac{-5}{-1}$

(*c*) **Do both $\frac{-5}{7}$ and $\frac{5}{-7}$ represent the same negative rational number?**

2. **Identify the following as negative or positive rational numbers:**

   (*i*) $\frac{-6}{-15}$ (*ii*) $\frac{8}{-13}$ (*iii*) $\frac{2}{3}$ (*iv*) $\frac{3}{-19}$

3. **Represent the following on a number line:**

   (*i*) $\frac{3}{2}$ (*ii*) $\frac{-4}{3}$ (*iii*) $\frac{-2}{5}$ (*iv*) $\frac{2}{7}$

4. **Express the following in standard form:**

   (*i*) $\frac{-24}{36}$ (*ii*) $\frac{-21}{-49}$ (*iii*) $\frac{-16}{80}$ (*iv*) $\frac{96}{-120}$

5. **Find the absolute value of the following:**

   (*i*) $\frac{-7}{10}$ (*ii*) $\frac{-3}{8}$ (*iii*) $\frac{-3}{-5}$ (*iv*) $\frac{-6}{13}$

6. **Write the first four equivalent rational numbers to the following:**

   (*i*) $\frac{1}{7}$ (*ii*) $\frac{-5}{8}$ (*iii*) $\frac{4}{9}$ (*iv*) $\frac{6}{13}$

7. **Find the value of p in the following:**

   (*i*) $\frac{-2}{3} = \frac{6}{p}$ (*ii*) $\frac{6}{-7} = \frac{42}{p}$ (*iii*) $\frac{9}{-5} = \frac{p}{25}$ (*iv*) $\frac{12}{48} = \frac{p}{-12}$

8. **Fill in the blanks with correct symbol using >, < or = :**

   (*i*) $\frac{5}{-7}$ ☐ $\frac{-5}{8}$ (*ii*) $\frac{-6}{11}$ ☐ $\frac{5}{8}$ (*iii*) $\frac{3}{5}$ ☐ $\frac{7}{12}$ (*iv*) $\frac{5}{-8}$ ☐ $\frac{-6}{13}$

To Find a Relational Number Between Two Given Rational Numbers

Let us find some rational numbers between two rational numbers, say, $\frac{-5}{9}$ and $\frac{5}{9}$.

We know that there are nine integers between –5 and 5. They are –4, –3, –2, –1, 0, 1, 2, 3, 4.

Thus, we can say that the rational numbers $\frac{-4}{9}, \frac{-3}{9}, \frac{-2}{9}, \frac{-1}{9}, \frac{0}{9}, \frac{1}{9}, \frac{2}{9}, \frac{3}{9}$ and $\frac{4}{9}$ lie between $\frac{-5}{9}$ and $\frac{5}{9}$. This is not the limit, we can write many more rational numbers between $\frac{-5}{9}$ and $\frac{5}{9}$.

Since $\frac{-5}{9} = \frac{-50}{90}$ and $\frac{5}{9} = \frac{50}{90}$, therefore $\frac{-49}{90}, \frac{-48}{90}, \frac{-47}{90}, \frac{-46}{90}, \frac{-45}{90}, \frac{-44}{90}$,....., $\frac{-2}{90}, \frac{-1}{90}, \frac{-0}{90}, \frac{1}{90}, \frac{2}{90}, \frac{3}{90}$,....., upto $\frac{49}{90}$ all lie between $\frac{-50}{90}$ and $\frac{50}{90}$. That is, they lie between $\frac{-5}{90}$ and $\frac{5}{90}$. Similarly by writing $\frac{-5}{9} = \frac{-500}{900}$ and $\frac{5}{9} = \frac{500}{900}$, we can insert other such rational numbers.

Hence, we can say that there are infinite rational numbers between any two rational numbers.

General Method of Comparing Two Rational Numbers

Working Rules

Step 1 : Rewrite all the given rational numbers such that their denominators are positive.

Step 2 : Express the given rational numbers with a common denominator.

Step 3 : Next, compare the numerators such that the greater number has the greater numerator.

Example 9 : Arrange the following in ascending order:

$$\frac{-2}{3}, \frac{5}{-6}, \frac{-8}{9}, \frac{3}{-4}$$

Solution : There are four rational numbers, such as $\frac{-2}{3}, \frac{3}{-4}, \frac{5}{-6}$ and $\frac{-8}{9}$.

Now, $\frac{-2}{3} = \frac{-2 \times 12}{3 \times 12} = \frac{-24}{36}, \frac{3}{-4} = \frac{-3 \times 9}{4 \times 9} = \frac{-27}{36}$,

$\frac{5}{-6} = \frac{-5 \times 6}{6 \times 6} = \frac{-30}{36}, \frac{-8}{9} = \frac{-8 \times 4}{9 \times 4} = \frac{-32}{36}$

$\because \quad -24 > -27 > -30 > -32$

$\therefore \quad \frac{-24}{36} > \frac{-27}{36} > \frac{-30}{36} > \frac{-32}{36} \Rightarrow \frac{-2}{3} > \frac{3}{-4} > \frac{5}{-6} > \frac{-8}{9}$

Take a Task

Watch Remedial

Example 10 : Find any ten rational numbers between $\frac{2}{3}$ and $\frac{-7}{9}$.

Solution : $\frac{2}{3}$ can be written as $\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$.

Since $\frac{6}{9}$ and $\frac{-7}{9}$ have the same denominator, all the numbers

$\frac{-6}{9}, \frac{-5}{9}, \frac{-4}{9}, \frac{-3}{9}, \frac{-2}{9}, \frac{-1}{9}, \frac{0}{9}, \frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \frac{4}{9}, \frac{5}{9}$ lie between $\frac{-7}{9}$ and $\frac{6}{9}$.

Check Your Progress

Experiential Learning

1. Arrange the following rational numbers in ascending order:

   (*i*) $\frac{3}{10}, \frac{6}{15}, \frac{-11}{5}, \frac{1}{6}$ (*ii*) $\frac{-9}{10}, \frac{-5}{20}, \frac{7}{12}, \frac{2}{5}$

2. Arrange the following rational numbers in descending order:

   (*i*) $\frac{5}{5}, \frac{7}{6}, \frac{-3}{8}, \frac{11}{12}$ (*ii*) $\frac{7}{9}, \frac{5}{6}, \frac{11}{12}, \frac{-1}{3}, \frac{-5}{6}$

Operations on Rational Numbers

Addition of Rational Numbers

(*a*) When Denominators are Same.

To add rational numbers with the same denominator, simply add their numerators keeping the common denominator as such.

NOTE: In case of rational numbers with negative denominator, we first convert each of them into a rational number with a positive denominator.

For example, $\frac{3}{-5} + \frac{1}{5}$ is the same as $\frac{-3}{5} + \frac{1}{5}$.

Example 11: Add: (*i*) $\frac{7}{5}$ and $\frac{-3}{5}$ (*ii*) $\frac{7}{-5}$ and $\frac{3}{5}$

Solution: (*i*) $\frac{7}{5} + \frac{(-3)}{5} = \frac{7+(-3)}{5} = \frac{7-3}{5} = \frac{4}{5}$

Hence, the sum of $\frac{7}{5}$ and $\frac{-3}{5}$ is $\frac{4}{5}$.

(*ii*) Here, $\frac{7}{-5}$ is with a negative denominator. Therefore, first we convert it with a positive denominator.

$\therefore \quad \frac{7}{-5} = \frac{7}{-5} \times \frac{(-1)}{(-1)} = \frac{-7}{5}$

Now, $\frac{7}{-5} + \frac{3}{5} = \frac{-7}{5} + \frac{3}{5} = \frac{(-7)+3}{5} = \frac{-4}{5}$

Hence, $\frac{7}{-5} + \frac{3}{5} = \frac{-4}{5}$.

Take a Task

Watch Remedial

(*b*) When Denominators are Different.

Working Rules

Step 1: Find the LCM of the denominators.

Step 2: Write the equivalent rational numbers with the LCM as their common denominator.

Step 3: Add them as rational numbers with same denominator.

Example 12: Add the rational numbers $\frac{-4}{9}$, $\frac{-5}{12}$ and $\frac{11}{18}$.

Solution : The denominators of the given rational numbers are 9, 12 and 18 respectively.

Therefore, LCM of 9, 12 and 18 is 36.

Now, $\frac{-4}{9} = \frac{-4 \times 4}{9 \times 4} = \frac{-16}{36}$

$\frac{-5}{12} = \frac{-5 \times 3}{12 \times 3} = \frac{-15}{36}$

$\frac{11}{18} = \frac{11 \times 2}{18 \times 2} = \frac{22}{36}$

2	9	12	18
3	9	6	9
3	3	2	3
	1	2	1

$\therefore$ L.C.M. $= 2 \times 2 \times 3 \times 3$

$= 36$

Now, $\frac{-4}{9} + \left(\frac{-5}{12}\right) + \frac{11}{18} = \frac{-16}{36} + \left(\frac{-15}{36}\right) + \frac{22}{36}$

$= \frac{(-16) + (-15) + 22}{36}$

$= \frac{-31 + 22}{36} = \frac{-\cancel{9}^{1}}{\cancel{36}_{4}} = \frac{-1}{4}$

Properties of Addition of Rational Numbers

1. The Additive Inverse of a Rational Number.

   If $\frac{a}{b}$ is a rational number, then its negative $\frac{-a}{b}$ is called the additive inverse of $\frac{a}{b}$.

2. Additive Identity or the Identity Element for the Addition of Rational Numbers.

   The sum of any rational number and 0 is the rational number itself. For example, if $\frac{a}{b}$ is any rational number, then $\frac{a}{b} + 0 = 0 + \frac{a}{b} = \frac{a}{b}$.

   Hence, **0 is called the additive identity** or the identity element for the addition of rational numbers.

   For example : (*i*) $\frac{7}{2} + 0 = \frac{7}{2}$ (*ii*) $\frac{-3}{4} + 0 = \frac{-3}{4}$

Example 13 : What should be added to $\frac{3}{4}$ so that the sum may be zero?

Solution : We know that the sum of a rational number and its negative is zero.

Since the given rational number is $\frac{3}{4}$, its negative is $\frac{-3}{4}$.

Now, $\frac{3}{4} + \frac{(-3)}{4} = 0$.

Hence, the required number $= \frac{-3}{4}$.

Subtraction of Rational Numbers

We know that subtraction is the inverse of addition.

For example: To subtract $\frac{-5}{7}$ from $\frac{3}{4}$ $\left[\text{i.e., } \frac{3}{4} - \left(\frac{-5}{7}\right)\right]$ means to add $\frac{3}{4}$ to $\frac{5}{7}$, which is also the same as to add $\frac{5}{7}$ to $\frac{3}{4}$ i.e., $\frac{3}{4} + \frac{5}{7} = \frac{21+20}{28} = \frac{41}{28}$.

Note

Subtraction is inverse of addition *i.e.*, we add the negative of number.

To Subtract a Fraction $\boldsymbol{\frac{c}{d}}$ from another Fraction $\boldsymbol{\frac{a}{b}}$.

Working Rules

First Method

Step 1: Express the unlike fractions $\frac{a}{b}$ and $\frac{c}{d}$ as the like fractions with the same denominator using LCM of b and d.

Step 2: Change the sign of $\frac{c}{d}$ and add it to

Example 14: Subtract the sum of $\frac{-2}{3}$ and $\frac{1}{3}$ from the sum of $\frac{5}{3}$ and $\frac{-4}{3}$.

Solution: The sum of $\frac{-2}{3}$ and $\frac{1}{3} = \frac{-2}{3} + \frac{1}{3} = \frac{-2+1}{3} = \frac{-1}{3}$

The sum of $\frac{5}{3}$ and $\frac{-4}{3} = \frac{5}{3} + \frac{(-4)}{3} = \frac{5-4}{3} = \frac{1}{3}$

Now, $\left(\frac{5}{3} + \frac{-4}{3}\right) - \left(\frac{-2}{3} + \frac{1}{3}\right) = \frac{1}{3} - \left(\frac{-1}{3}\right) = \frac{1}{3} + \frac{1}{3} = \frac{1+1}{3} = \frac{2}{3}$.

Example 15: The sum of two rational numbers is $\frac{-17}{27}$. If one of them is $\frac{-11}{27}$, find the other.

Solution: The sum of the two rational numbers $= \frac{-17}{27}$.

One of the numbers $= \frac{-11}{27}$

The other number = Sum of the numbers – the given number

$$= \frac{-17}{27} - \left(\frac{-11}{27}\right) = \frac{-17}{27} + \frac{11}{27} = \frac{-17}{27} + \frac{11}{27}$$

$$= \frac{-17+11}{27} = \frac{-6}{27} = -\frac{2}{9}$$

Exercise 3.2

1. **Represent the following rational numbers on the number line :**

(i) $-\frac{1}{2}$ (ii) $-\frac{1}{3}$ (iii) $\frac{2}{3}$ (iv) $\frac{-3}{5}$ (v) $\frac{-3}{4}$

2. **Which of the following represent the same number on the number line ?**

(i) $\frac{4}{7}$ and $\frac{-4}{7}$ (ii) $\frac{0}{4}$ and $\frac{0}{3}$ (iii) $\frac{18}{-3}$ and $\frac{-3}{18}$

(iv) $\frac{-1}{2}$ and $\frac{5}{-10}$ (v) $\frac{-4}{5}$ and $\frac{-5}{20}$ (vi) $\frac{3}{-5}$ and $\frac{9}{-15}$

3. **Arrange the following rational numbers in ascending order :**

(i) $\frac{-5}{4}, \frac{6}{14}, \frac{3}{-5}$ (ii) $\frac{2}{3}, \frac{-3}{2}, \frac{4}{-7}, \frac{7}{4}$ (iii) $\frac{-10}{15}, \frac{1}{2}, \frac{-12}{10}$ (iv) $0, -2, \frac{-1}{2}$

4. **Arrange the following rational numbers in descending order :**

(i) $\frac{-2}{3}, \frac{5}{8}, \frac{2}{-5}$ (ii) $\frac{-5}{3}, \frac{-13}{15}, \frac{2}{-5}$ (iii) $\frac{7}{11}, \frac{-3}{5}, \frac{-2}{3}$

5. **Find ten rational numbers between $\frac{1}{7}$ and $\frac{-5}{7}$.**

6. **Find the sum:**

(i) $\frac{-3}{4} + \frac{4}{5}$ (ii) $\frac{-2}{15} + \frac{3}{4}$ (iii) $\frac{-5}{6} + \frac{7}{15}$

(iv) $\frac{1}{3} + \frac{-5}{6} + \frac{1}{18}$ (v) $\frac{-9}{21} + \frac{5}{-7} + \frac{2}{3}$ (vi) $3\frac{1}{7} + \frac{-5}{14} + \frac{-5}{21}$

7. **Subtract the following:**

(i) 2 from $\frac{-19}{9}$ (ii) $\frac{-18}{11}$ from 2 (iii) $\frac{3}{4}$ from $\frac{2}{3}$

(iv) $\frac{7}{5}$ from $\frac{11}{5}$ (v) $\frac{-5}{8}$ from $\frac{9}{10}$ (vi) $\frac{-6}{21}$ from $\frac{5}{63}$

8. **Find the sum of the following rational numbers :**

(i) $\frac{4}{9} + \frac{9}{4}$ (ii) $\frac{-4}{5} + \frac{5}{4}$ (iii) $\frac{1}{9} + \frac{-5}{6} + \frac{-2}{3}$

(iv) $\frac{-4}{5} + \frac{13}{10} + \frac{-7}{15}$ (v) $\frac{-2}{3} + \frac{4}{6} + \frac{3}{4}$ (vi) $\frac{-5}{7} + \frac{3}{-4} + \frac{2}{3}$

9. **Verify the following :**

(i) $\frac{5}{7}+\frac{-12}{5}=\frac{-12}{5}+\frac{5}{7}$ (ii) $\frac{-3}{4}+\frac{17}{8}+\frac{-1}{2}=\frac{-1}{2}+\frac{-3}{4}+\frac{17}{8}$

(iii) $\frac{2}{-9}+\frac{-3}{5}+\frac{1}{3}=\frac{-3}{5}+\frac{-2}{9}+\frac{1}{3}$

10. What should be added to $\frac{-11}{17}$ to get 0 ?

HOTS (Higher Order Thinking Skills)

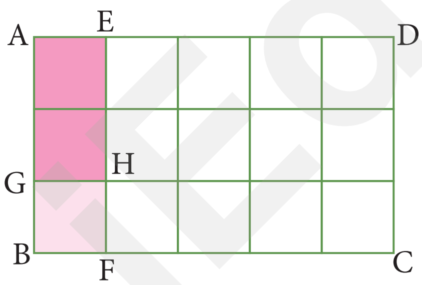
Experiential Learning

1. What number should be added to $\frac{8}{14}$ to get $\frac{-2}{7}$?
2. The sum of two rational numbers is $\frac{7}{5}$. If one of them is $\frac{-2}{15}$, find the other one.
3. The sum of two rational numbers is $\frac{3}{2}$. If one of the numbers is $\frac{-3}{10}$, find the other.

Multiplication of Rational Numbers

Consider an example, say, $\frac{2}{3}\times\frac{1}{5}$.

Let us represent this multiplication with the help of a diagram.



Take a Task

Watch Remedial

The shaded part ABFE in figure shows $\frac{1}{5}$.

The double-shaded part AGHE in the figure shows $\frac{2}{3}$ out of $\frac{1}{5}$.

From the figure, it is clear that the double-shaded part shows $\frac{2}{15}$.

So, $\frac{2}{3}$ of $\frac{1}{5}=\frac{2}{15}$ Or, $\frac{2}{3}\times\frac{1}{5}=\frac{2}{15}$

Also, we observe that $\frac{2}{3}\times\frac{1}{5}=\frac{2\times 1}{3\times 5}$

Hence, we conclude that the product of two rational numbers is a rational number whose numerator is the product of the two numerators and whose denominator is the product of the two denominators. If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers, then

$$\frac{p}{q} \times \frac{r}{s} = \frac{p \times r}{q \times s} = \frac{\text{Product of numerators}}{\text{Product of denominators}}$$

Example 16 : Find the product of $\frac{-8}{9}$ and $\frac{5}{7}$.

Solution : $\frac{-8}{9} \times \frac{5}{7} = \frac{(-8) \times 5}{9 \times 7} = \frac{-40}{63}$

Example 17 : Find the product of $\frac{3}{4}$, $\frac{-2}{7}$, and $\frac{3}{5}$.

Solution : $\frac{3}{4} \times \frac{-2}{7} \times \frac{3}{5} = \frac{3 \times (-2) \times 3}{4 \times 7 \times 5} = \frac{-18}{140} = \frac{-9}{70}$

Division of Rational Numbers

We know that division is the inverse of multiplication, i.e., if a and b are two integers, then $a \div b = a \times \frac{1}{b}$. It means we multiply the dividend by the multiplicative inverse of the divisor. We apply the same rule for division of rational numbers. If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers, then

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} \quad \text{or, } \frac{p}{q} \div \frac{r}{s} = \frac{p \times s}{q \times r} \qquad \left(\frac{r}{s} \neq 0\right)$$

Or,

Example 18 : Divide $\frac{3}{5}$ by $\frac{4}{9}$.

Solution : $\frac{3}{5} \div \frac{4}{9} = \frac{3}{5} \times \frac{9}{4} = \frac{3 \times 9}{5 \times 4} = \frac{27}{20}$ $\left(\text{Multiplicative inverse of } \frac{4}{9} \text{ is } \frac{9}{4}.\right)$

REMEMBER

If $\frac{p}{q} + \frac{r}{s} = 0$, then $\frac{p}{q}$ is the additive inverse of $\frac{r}{s}$ and vice versa.

The product of any rational number and zero gives zero, i.e., $\frac{p}{q} \times 0 = 0$.

$\frac{p}{q} \times \frac{r}{s} = \frac{p \times r}{q \times s}$; $\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{p \times s}{q \times r}$, where $\left(\frac{r}{s} \neq 0.\right)$

Exercise 3.3

1. Find the product of the following:

(i) $\frac{8}{10} \times \frac{-19}{6} \times \frac{12}{19}$ (ii) $\frac{8}{2} \times \frac{-14}{15}$ (iii) $\left(\frac{6}{-5}\right) \times \frac{9}{11}$

(iv) $\frac{3}{-5} \times \frac{-5}{3} \times \frac{1}{7}$ (v) $\frac{3}{7} \times \frac{-2}{5} \times \frac{5}{11}$ (vi) $\frac{16}{-5} \times \frac{3}{8} \times \frac{10}{3}$

2. Find the quotient of the following:

(i) $(-4) \div \frac{12}{8}$ (ii) $\frac{2}{8} \div \frac{5}{12}$ (iii) $\frac{-4}{5} \div 12$

(iv) $\frac{12}{-13} \div \frac{1}{26}$ (v) $\frac{3}{13} \div \frac{-4}{65}$ (vi) $\frac{21}{-36} \div \frac{30}{44}$

3. Simplify the following:

(i) $\frac{12}{32} \times \frac{15}{75} \div \frac{4}{9}$ (ii) $\frac{4}{5} - \frac{7}{6} + \frac{-2}{3}$ (iii) $\frac{2}{3}\left[\frac{-1}{3} + \frac{5}{6}\right]$

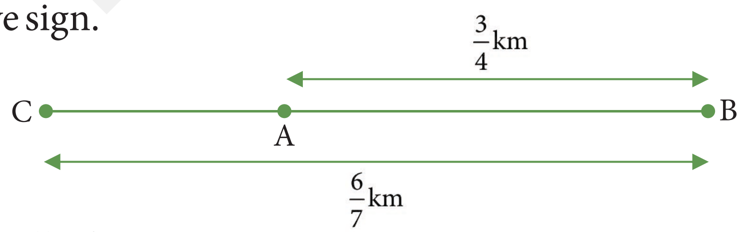
(iv) $2\frac{1}{5}\left(\frac{6}{7} + 1\frac{1}{7}\right)$ (v) $\left(\frac{2}{13} \div \frac{1}{7}\right) \times \frac{26}{14}$ (vi) $\frac{8}{15} + \frac{-13}{25} + \frac{-11}{45}$

(vii) $\left(\frac{3}{5} \times \frac{-3}{2}\right) + \left(\frac{+3}{10} \times \frac{11}{8}\right)$ (viii) $2\frac{3}{4} - \frac{5}{8} + \frac{-5}{12} + 1\frac{1}{6}$

Word Problems

Example 19: From a starting point A, Rahul walks $\frac{3}{4}$ km towards east and then $\frac{6}{7}$ km towards west to reach point C. Where will he be now from the starting point A ?

Solution : Distance towards east is denoted by the positive sign and towards west by the negative sign.



Rahul walks from A to B.

So, $\overline{AB} = \frac{3}{4}$ km

Rahul walks from B to C in the opposite direction.

So, $\overline{BC} = \frac{6}{7}$ km

Now, Rahul is at C. So, the distance from A to C is $\overline{AC}$.

$$AC = AB - BC = \frac{3}{4} - \frac{6}{7}$$

$$\frac{3}{4} + \left(\frac{-6}{7}\right) = \frac{3 \times 7}{4 \times 7} + \frac{-6 \times 4}{7 \times 4} = \frac{21}{28} + \frac{-24}{28} = \frac{21-24}{28} = \frac{-3}{28}$$

It means that Rahul is at a distance of $\frac{-3}{28}$ km towards east from A, i.e., he is at $\frac{3}{28}$ km towards west from A.

Example 20: Suman reads $\frac{1}{3}$ of a storybook on the first day and $\frac{1}{4}$ of the book on the second day. What part of the book is yet to be read by Suman?

Solution: Suman reads on the first day $\frac{1}{3}$ of the storybook and on the second day $\frac{1}{4}$ of the storybook.

So, the total part read so far $= \frac{1}{3} + \frac{1}{4}$

The part yet to be read $= 1 - \left(\frac{1}{3} + \frac{1}{4}\right)$

$$= \frac{1}{1} - \left(\frac{1 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3}\right) = \frac{1}{1} - \left(\frac{4+3}{12}\right)$$

$$= \frac{1}{1} - \frac{7}{12} = \frac{1 \times 12}{1 \times 12} - \frac{7 \times 1}{12 \times 1} = \frac{12-7}{12} = \frac{5}{12}$$

Exercise 3.4

Problem Solving

1. What should be added to $\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)$ to get 4?
2. If 35 shirts of equal size can be stitched from $\frac{49}{2}$ metres of cloth, calculate the length of cloth required for each shirt.
3. Divide the sum of $\frac{12}{5}$ and $\frac{13}{7}$ by the product of $\frac{-4}{7}$ and $\frac{-1}{2}$.
4. Cheena spent $\frac{3}{4}$ of her pocket money. She spends $\frac{1}{2}$ of it on a book, $\frac{1}{6}$ on a movie, and the rest on a dress. What part of her pocket money did she spend on the dress?
5. Anil, Amit and Sumit went out for dinner in a restaurant. Anil paid $\frac{1}{3}$ of the amount of the bill, and Sumit had enough money to pay $\frac{1}{4}$ of the bill. Amit paid the remaining part. What part of the bill was paid by Amit?

Chapter-end Exercise

Gap Analyzer™

A. Tick (✓) the correct option

1. The multiplicative inverse of $\frac{-3}{7}$ is ____________ .

(a) $\frac{3}{7}$ ☐ (b) $\frac{-3}{7}$ ☐ (c) $\frac{7}{3}$ ☐ (d) $\frac{7}{-3}$ ☐

2. $\frac{-102}{119}$ in a standard form is ___________ .

(a) $\frac{-4}{7}$ ☐ (b) $\frac{-6}{7}$ ☐ (c) $\frac{-6}{17}$ ☐ (d) none ☐

3. The sum of two rational numbers is 5. If one of the numbers is $\left(\frac{13}{6}\right)$, then the other number is ______________ .

(a) $\frac{-17}{6}$ ☐ (b) $\frac{17}{6}$ ☐ (c) $\frac{43}{6}$ ☐ (d) $\frac{-43}{6}$ ☐

B.

1. Write down three rational numbers equivalent to each of the following:

(i) $\frac{2}{-5}$ (ii) $\frac{-9}{13}$ (iii) $\frac{6}{7}$ (iv) $\frac{5}{11}$

2. (a) Express $\left(-\frac{3}{7}\right)$ as a rational number with:

(i) denominator (–35). (ii) numerator 63.

(b) Express $\frac{-81}{135}$ as a rational number with:

(i) denominator 15. (ii) numerator 27.

3. Write the following rational numbers in standard form:

(i) $\frac{27}{81}$ (ii) $-\frac{9}{108}$ (iii) $\frac{-42}{343}$ (iv) $\frac{-75}{225}$

4. Identify which of the following pairs of rational numbers are equivalent:

(i) $\frac{12}{-18}$ and $\frac{-4}{6}$ (ii) $\frac{-9}{27}$ and $\frac{12}{25}$ (iii) $\frac{-65}{-30}$ and $\frac{13}{6}$ (iv) $\frac{17}{35}$ and $\frac{34}{60}$

5. Add the following:

(i) $\left(\frac{-3}{7}\right)$ and $\left(+\frac{4}{5}\right)$ (ii) 3 and $\frac{-5}{9}$ (iii) $\frac{3}{5}$ and $\frac{2}{7}$ (iv) $\frac{15}{-6}$ and $\frac{7}{6}$

6. Subtract the following:

(i) $\frac{-7}{24}-\frac{-8}{18}$ (ii) $\frac{5}{63}-\frac{-8}{21}$ (iii) $\frac{9}{14}-\frac{4}{21}$

7. Find the product of the following:

(*i*) $\frac{-3}{4} \times \frac{-8}{15}$ (*ii*) $\frac{-9}{2} \times \frac{-2}{9}$ (*iii*) $\frac{-6}{11} \times \frac{44}{12}$

Custom Learning Path

Scan to Create Your Own Learning Path

8. Find the quotient of the following:

(*i*) $\frac{15}{7} \div \frac{-5}{7}$ (*ii*) $\left(\frac{-9}{11}\right) \div \left(-\frac{2}{3}\right)$ (*iii*) $\frac{-9}{16} \div \frac{1}{4}$ (*iv*) $\frac{-3}{13} \div \frac{-4}{65}$

C. Fill in the blanks:

1. A rational number which is neither positive nor negative is ________________ .
2. The product of a rational number and its reciprocal is equal to ________________ .
3. If any rational number is multiplied by 1, then the product is ________________ .
4. The reciprocal of a positive rational number is ________________ .
5. ________________ is a rational number whose additive inverse is the number itself.

Project

Purpose : To check if rational numbers follow various properties like Closure property, Commutative Property, Associative property of Addition, Subtraction, Multiplication and Division.

Procedure :

- Work with your Partner.
- Recall the Closure property, Commutative property, Associative property of Addition, Subtraction Multiplication and Division, you have learnt for integers.
- Now, check if these properties apply for rational numbers.
- Prepare a chart on the results obtained.

Assertion and Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.
Study both the statements and state which of the following is correct:

(*a*) Both A and R are true and R is the correct explanation of A.
(*b*) Both A and R are true but R is not the correct explanation of A.
(*c*) A is true but R is false.
(*d*) A is false but R is true.

1. **Assertion (A) :** The additive inverse of $\frac{a}{b}$ is $\frac{-a}{b}$, where $b \neq 0$.

   Reason (R) : $\frac{1}{2} - \frac{1}{7}$ is equal to $\frac{-3}{8}$.

2. **Assertion (A) :** The product of $\frac{-7}{8}$ and $\frac{5}{6}$ is $\frac{-25}{48}$.

   Reason (R) : $\left(\frac{-8}{15}\right) \div \left(\frac{-4}{3}\right)$ is equal to $\frac{2}{5}$.

3. **Assertion (A) :** The value of $\frac{-7}{9} - \left(\frac{-2}{5}\right)$ is equal to $\frac{-17}{45}$.

   Reason (R) : The multiplicative inverse is also called the reciprocal.

4. **Assertion (A) :** There are infinite number of rational numbers between any two rational numbers.

   Reason (R) : All integers and fractions belong to the group of rational numbers.

5. **Assertion (A) :** The product of a rational number and its reciprocal is equal to one.

   Reason (R) : The sum of a rational number and its additive inverse is 1.

Activity

Presentation of Numbers on Number Line and Comparison of Rational Numbers

- Divide the class into groups of 5. Distribute one A4 sheet to each group. Now give a set of rational numbers to each group. Like.
- Group 1 : $\frac{3}{7}, \frac{-2}{7}, \frac{5}{7}, \frac{-4}{7}, 0$.
- Group 2 : $\frac{4}{11}, \frac{-2}{11}, \frac{-6}{11}, \frac{-10}{11}, \frac{3}{11}$ and so on.
- Now ask each group to work in team and represent these rational numbers on the number line.
- When all the groups have done their activity, ask them to exchange their sheets and check whether the work of other team is correct or not.
- Now explain the students that to compare the rational numbers, make the denominators of the given two rational numbers same (if not) using L.C.M. of both denominators and then compare the numerators.
- After this again give any 5 rational numbers to each group and ask them to compare and arrange in ascending order. The group which take least time to arrange the rational numbers will be the winner.
- To make teaching-learning process a success, teacher will make sure that almost all the students in the class will participate in this activity.