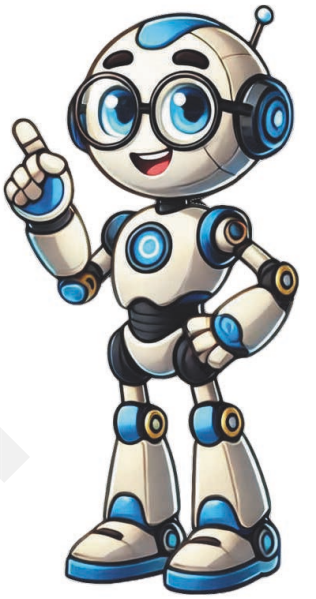
12
Chapter

Data Handling

We'll cover the following key points:

- → Organisation of data
- → Mean of tabulated data
- → Median
- → Mode
- → Bar Graph



Hi, I'm EeeBee

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Do you Remember fundamental concept in previous class.

In class 5th we learnt

→ Bar graphs

In class 4th we learnt

→ Pictorial Representation of Data and Bar Graph

Learning Outcomes

By the end of this chapter, students will be able to:

- Define and identify different types of data, such as qualitative and quantitative data, and distinguish between them.
- Collect, organize, and represent data in various forms, including tables, bar graphs, and pictographs.
- Interpret data presented in bar graphs, histograms, and pie charts, and draw conclusions based on the graphical representation.
- Calculate the mean, median, mode, and range of a data set, and understand their significance in analyzing data.
- Understand the concept of frequency distribution and create frequency tables to organize data.
- Identify and understand the use of scales in graphical representation of data and adjust them appropriately for accurate visualization.
- Analyze and compare different data sets by interpreting the central tendency (mean, median, mode) and the spread (range, variance) of the data.

Mind Map

DATA HANDLING

Arithmetic Mean

Mean =

$$\frac{\text{Sum of all observations}}{\text{Number of observations}}$$

e.g., 30, 20, 40

$$\text{Mean} = \frac{30+20+40}{3} = \frac{90}{3} = 30$$

Mode

- The mode of a set of observations is the observation that occurs most often.

e.g., 1, 1, 1, 2, 2, 3, 4, 4

Mode = 1

Median

- In a given data, arranged in ascending or descending order, the median gives the middle observation.

e.g., 24, 36, 46, 17, 18, 25, 35

Ascending order ? 17, 18, 24, 25, 35, 36, 46

Median = 25

Range

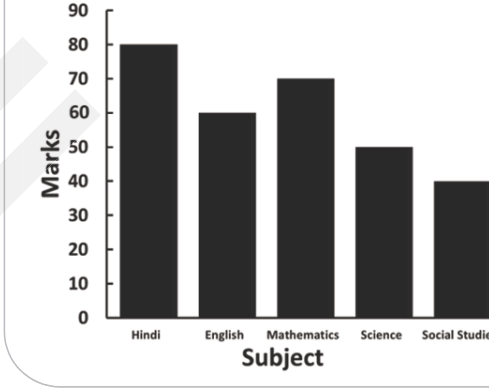
- The difference between the highest and the lowest observation.

e.g., 32, 41, 28, 54, 33, 57

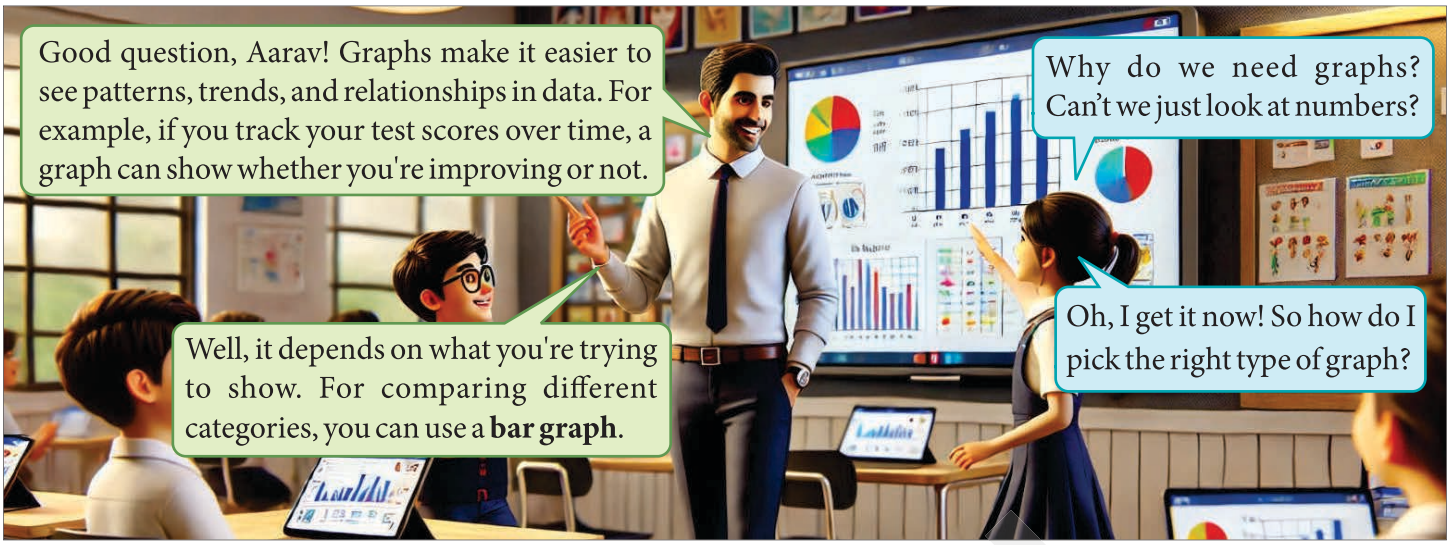
Range = 57 – 28 = 29

Bar graph

Pictorial representation of numerical data in the form of bars created vertically or horizontally with equal spacing between them.



Introduction



Everyday, we come across a wide range of information in the form of numerical, figure, table, graph etc., through the newspaper, television etc. These facts and information collected for the specific purpose are called **data**. We know that there are various types of data. You have learnt to collect data, tabulate and put it in the form of bar graph. When we collect data and present it in tabular form we can draw many inferences in short period on the basis of our experiences. We know that all kind of data gives us some sort of information. The extraction of meaningful information from such data is studied in a branch of mathematics is called **statistics**. For example :

Marks obtain by students (Class VII)

Name of students	Marks (Max -100)
Ojas	55
Taniya	60
Rashmi	35
Sumati	70
Gautam	15
Shreyas	95

Some Important Terms

Data : A collection of numerical figures providing some specific type of information is called data.

For example : The marks secured by 10 students of Class VII in a certain mathematics test are 23, 17, 5, 11, 31, 40, 19, 28, 35 and 40.

These marks constitute the data related to the marks obtained by the students of Class VII in a certain mathematics test.

Observation : Each numerical figure in a data is called an observation (or variable).

Raw data : The numerical data recorded in its original form is called raw data (or ungrouped data). Data are of two types (*i*) primary data and (*ii*) secondary data.

Primary data : The data collected by an investigator or by his/her representative with a definite objective in his/her mind is called primary data.

Secondary data : The data collected by someone other than the investigator is known as secondary data.

Array : The raw data when arranged in an ascending or descending order of magnitude is called array.

Range : The difference between the highest and lowest values in the data is called the range of the data.

Frequency of an observation : The number of times an observation occurs in the given data is called the frequency of the given observation.

Organisation of Data

After collecting the data, it has to be put in a proper table, so that it can be understood and interpreted in an easy way.

Collection of Data

There are two ways of collecting data :

(*I*) When the information is collected by the investigator himself/herself for the specific purpose, the data so obtained are called primary data. These data are quite reliable.

(*ii*) Sometimes it is not possible for an investigator himself/ herself to collect data due to lack of time, money and resources. The data collected by someone else, other than the investigator himself/ herself are known as secondary data. The main sources of secondary data are :

(*i*) Data published by research organisation.

(*ii*) Data published by international organisation.

(*iii*) Data published by central or state governments.

Presentation of Data

After collecting data, the investigator finds out the ways to present them in a form which is meaningful and simple to understand and gives its main features at a glance.

When the data are not arranged in any order, we call it **raw data** or **ungrouped data**. Raw data are not directly fit for interpretation. They fail to give proper information.

For example : The following are the marks obtained by 20 students out of 50 marks: 42, 35, 5, 7, 30, 11, 15, 3, 18, 25, 20, 27, 45, 36, 32, 12, 16, 4, 9, 39.

The above data are not expressed in any order, so they are called **raw data**. Each of the data is called an **observation**.

From the above raw data, can you find the highest and lowest marks easily ?

Of course, it is time consuming to search maximum and minimum marks. Would not it be less time consuming if those marks are arranged in the ascending order or descending order ?

So, let us arrange the marks in ascending order.

3,4, 5, 7, 9, 11, 12, 15, 18, 20, 25, 27, 30, 32, 35, 36, 39, 42 and 45

Now, we can clearly see that the lowest mark is 3 and the highest mark is 45.

The difference between the highest and the lowest values of the observations in a data is called the **range**. In the above data, 45 is the highest observation and 3 is the lowest observation.

$\therefore$ The range = 45 – 3 = 42.

Frequency Distribution

The number of times a particular observation occurs is called its frequency. The number of observations in a particular class (group) is called the frequency of that group.

Ungrouped Frequency Distribution

Presentation of data in ascending or descending order can be quite time consuming, particularly, when the number of observations in an experiment is large. To overcome this shortcoming, we prepare an ungrouped frequency distribution table.

To prepare an, Ungrouped Frequency Distribution Table, follow the steps given below:

Working Rules

Step 1 : *Prepare a table with three columns :* first for variables or items under study such as marks, weights, heights etc., second for Tally marks and third for the Frequency.

Step 2 : Place all the values of the variables in the first column in ascending order.

Step 3 : Take the first observation, match it with the variable in first column and put a bar in the second column. Continue this process till all the observations in the fifth bar as shown in the following |||| or ||||.

Step 4 : Give a suitable title which conveys accurately what the table is about.

Example 1 : The marks obtained by 50 students in a class test of math (out of 15) are given below :

3, 9, 6, 9, 4, 2, 5, 1, 6, 3, 0, 4, 9, 2, 8 1, 3, 3, 0, 5, 2, 7, 3, 9, 9, 2, 8, 2, 5, 4, 5
4, 8, 0, 6, 11, 11, 9, 0, 0, 3, 9, 9, 7, 14, 15, 15, 14, 7, 5

(*i*) Arrange these data and form a frequency table.
(*ii*) What is the heighest mark ?
(*iii*) What is the range of marks ?
(*iv*) Which mark is occurring most frequently ?

Ans (*i*) The data in ascending order is shown below :

0, 0, 0, 0, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, ,4 ,4 ,4, 4, 5, 5, 5, 5, 5,
6, 6, 6, 7, 7, 7, 8, 8, 8, 9, 9, 9, 9, 9, 9, 9, 9, 11, 11, 14, 14, 15, 15

Marks	Tally marks	No. of students
0	𝍸	5
1	||	2
2	𝍸	5
3	𝍸 |	6
4	𝍸	4
5	||||	5
6	|||	3
7	|||	3
8	|||	3
9	𝍸 |||	8
11	||	2
14	||	2
15	||	2

Total - 50

Grouped Frequency Distribution

When the raw data are in large number and the difference between the greatest and the smallest observation is large, we condense the data into classes (groups).

There are two types of Classes :

- **Exclusive Classes :** Class intervals where the upper limits do not belong to the class are called exclusive classes.
- **Inclusive Classes :** Class intervals where the upper limits belong to the class are called inclusive classes.

Exclusive Classes

The classes like 0 – 10, 10 – 20, 20 – 30, 30 – 40, ... etc. are exclusive classes. In the class 0 - 10, 0 is called the lower limit and 10 is the upper limit. Here we observe that the upper limit of one class is the lower limit of the next class. That means if the number 10 belongs to the data, then it is not counted as a member of class 0-10 : but as a member of the class 10 - 20. Therefore, the upper limit is excluded from each class and hence are named as exclusive classes.

Inclusive Classes

The classes like 0 – 9, 10 –19, 20 – 29, 30 – 39, ... etc. are inclussive classes.

Here, the upper limit belong to each class, and hence are known as inclusive classes.

The following inclusive classes are equivalent to the exclusive classes:

0.5 – 9.5, 9.5 – 19.5, 19.5 – 29.5, 29.5 – 39.5....

To prepare a Grouped Frequency Distribution Table, we need follow the steps given below.

Working Rules

Step 1 : Group the data into different classes.

Step 2 : Represent the items in each class by 'tally marks' (*i.e.*, a 'l' for each item).

The four consecutive tally marks are represented by vertical lines. The fifth tally mark is placed diagonally (\) across the four tally marks, so that counting of frequency becomes less labourious.

Step 3 : Count the tally marks in each class.

Step 4 : The number of tally marks in each class is called its frequency.

NOTE : The data can be represented in different ways by choosing different classes.

Example 2 : Marks obtained by 40 students in an examination are given below:

69, 59, 49, 39, 84, 68, 77, 48, 47, 57, 46, 41, 44, 67, 57, 45, 34, 36 87, 89, 65, 41, 84, 78, 52, 49, 75, 37, 38, 42, 73, 31, 34, 37, 56, 59, 64, 85, 81, 62

Prepare a frequency distribution table.

Solution : The lowest mark is 31 and the highest mark is 89.

$\therefore$ Range = 89 – 31 = 58

Let us consider that we wish to distribute the above data in 6 classes. The number greater than and nearest to 58 is 60. Thus the class size is 10 and we can take the classes as :

30 — 40, 40 — 50, 50 — 60, 60, — 70, 70 — 80 and 80 — 90

Now, we count the number of students obtaining marks falling in each class.

This is done by using tally marks.

Now, the required frequency distribution table is prepared as follows :

Class-interval	Tally-Marks	Frequency
30 — 40	𝍸 |||	8
40 — 50	𝍸 𝍸	10
50 — 60	𝍸 |	6
60 — 70	𝍸 |	6
70 — 80	𝍸	5
80 — 90	𝍸	5
		Total = 40

Mean, Median and Mode

We have learnt about collection, organisation and tabulation of the given data. It is not that we have to always study the entire data to '**make sense**' of it. In fact, we can make out some important features of it by considering only certain representatives of the data. This becomes possible by using measures of **central tendency** or **averages**. The commonly used measures of central tendency are

1. Arithmetic mean 2. Median 3. Mode

Arithmetic Mean

The average of a given set of numbers is called the arithmetic mean or simply the mean of the given numbers.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

The following example illustrate how to find out the mean of a given set of data:

Example 3 : Find the mean of each set of data :

(*i*) 38, 42, 25, 38, 77, 47, 76, 59, 42, and 24.

(*ii*) the first eight natural numbers.

(*iii*) the first four prime numbers.

(*iv*) the first five multiples of 4.

Solution : (*i*) $\text{Mean} = \dfrac{\text{Sum of all observations}}{\text{Number of observations}}$

$$\text{Mean} = \frac{38+42+25+38+77+47+76+59+42+24}{10} = \frac{\cancel{468}}{\cancel{10}} = 46.8$$

(*ii*) The first eight natural numbers are 1, 2, 3, 4, 5, 6, 7 and 8.

$$\therefore \text{ Mean} = \frac{1+2+3+4+5+6+7+8}{8} = \frac{36}{8} = 4.5$$

(*iii*) The first four prime numbers are 2, 3, 5 and 7.

$$\therefore \text{Their mean} = \frac{2+3+5+7}{4} = \frac{17}{4} = 4.25$$

(*iv*) The first five multiples of 4 are 4, 8, 12, 16 and 20.

$$\therefore \text{Their mean} = \frac{4+8+12+16+20}{5} = \frac{60}{5} = 12$$

Example 4 : The mean of six numbers 6, 8, 5, 7, x and 4 is 7. Find the value of x.

Solution:

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

$$\Rightarrow \quad 7 = \frac{6+8+5+7+x+4}{6} \qquad [\because \text{ Mean} = 7 \text{ (given)}]$$

$$\Rightarrow \quad 7 = \frac{30+x}{6} \quad \Rightarrow \quad 42 = 30 + x \quad \Rightarrow \quad x = 12$$

Example 5 : The rainfall (in mm) in a city on 7 days of a week was recorded as follows :

Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Rainfall (in *mm*)	2.2	10.3	8.5	0.1	9.5	7.0	3.0

(*i*) Find the range of the rainfall in the given data.

(*ii*) Find the mean rainfall for the week.

(*iii*) On how many days was the rainfall less than mean rainfall ?

Solution: (*i*) Range of the rainfall = Highest rainfall – lowest rainfall

$$= 10.3\ mm - 0.1\ mm = 10.2\ mm$$

(*ii*) $\text{Mean rainfall for the week} = \dfrac{\text{Sum of all observations}}{\text{Number of observations}}$

$$= \frac{(2.2+10.3+8.5+0.1+9.5+7.0+3.0)mm}{7}$$

$$= \frac{\cancel{38.6}^{40.6}\ mm}{\cancel{7}} = 5.8\ mm$$

(*iii*) The rainfall on Monday. Thursday and Sunday was less than 5.5 *mm*.

Example 6 : The mean of 10 numbers is 25. If 4 is subtracted from each number, then what will be the new mean ?

Solution: Let $x_1, x_2, x_3, x_4, ...$, be ten numbers whose mean is 25.

Then, $25 = \frac{x_1 + x_2 + x_3 + ... + x_{10}}{10}$

$\Rightarrow 25 \times 10 = x_1 + x_2 + x_3 + x_4 + ... + x_{10}$

$\Rightarrow 250 = x_1 + x_2 + x_3 + x_4 + ... + x_{10}$

New ten numbers are $x_1 - 4, x_2, -4, x_3 - 4, ..., x_{10} - 4$

$\therefore$ New mean $= \frac{x_1 - 4 + x_2 - 4 + x_3 - 4 + ... + x_{10} - 4}{10}$

$= \frac{(x_1 + x_2 + x_3 + ... + x_{10}) - 4 \times 10}{10}$

$= \frac{250 - 40}{10} = \frac{210}{10} = 21$

Hence, the new mean = 21 $\quad$ (25 – 4 = original mean –4)

Example 7 : The mean weight of 6 girls in a group is 48 kg wt. The individual weight of five of them is 44 kg wt, 51 kg wt, 45 kg wt, 46 kg wt and 49 kg wt respectively. Find the weight of the sixth girl.

Solution: Let the weight of the girl be x kg wt .

$$\text{Mean weight} = \frac{\text{Sum of weight of all the girls}}{\text{Number of girls}}$$

$\Rightarrow 48 \text{kg wt} = \frac{(44 + 51 + 45 + 46 + 49 + x) \text{kg wt}}{6}$ $\quad [\because \text{Mean} = 48, \text{given}]$

$\Rightarrow 48 \text{ kg wt} \times 6 = (235 + x) \text{ kg wt}$

$\Rightarrow 288 = 235 + x$

$\Rightarrow x = 288 - 235 = 53$

Hence, the weight of the sixth girl is 53 kg wt.

Example 8 : The mean of 5 numbers is 25.
If one of the numbers is excluded, the mean gets reduced by 2.
Determine the excluded number.

Solution: Mean of 5 numbers = 25

$\therefore$ Sum of 5 numbers = $25 \times 5 = 125$

Mean of 4 numbers = $25 - 2 = 23$

$\therefore$ Sum of 4 numbers = $23 \times 4 = 92$

$\therefore$ Excluded (5th) number = $125 - 92 = 33$

Mean of Tabulated Data

If $x_1, x_2, x_3, \ldots, x_n$ are n observations with frequencies $f_1, f_2, f_3, \ldots f_n$ respectively, then the mean $\bar{x}$ of these observations is given by:

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \ldots + f_nx_n}{f_1 + f_2 + f_3 + \ldots + f_n} = \frac{\Sigma f_1x_1}{\Sigma f_1}$$

where Σ (called sigma) is the Greek letter representing summation.

Example 9 : Find the mean of the following distribution:

x_1	5	10	15	20	25	30	35
f_1	2	4	6	9	11	3	5

Solution :

x_1	f_1	f_1x_1
5	2	10
10	4	40
15	6	90
20	9	180
25	11	275
30	3	90
35	5	175
Total	$\Sigma f_1=40$	$\Sigma f_1x_1=860$

$$\text{Mean} = \frac{\Sigma f_1x_1}{\Sigma f_1} = \frac{860}{40} = 21.5.$$

Exercise 12.1

1. Organise the following marks in a class assessment in a tabular form:

   7, 6, 5, 8, 7, 3, 1, 1, 2, 3, 4, 5, 7, 7, 8, 3, 1, 7, 8, 7, 3, 9, 5, 4, 4, 3, 7, 2, 1, 9

   (*i*) Arrange the data to form a frequency table.

   (*ii*) Which number is the highest ?

   (*iii*) What is the range of the data ?

2. The rainfall (in *mm*) recorded for a city on 15 days is as follows:

   3.4, 5.0, 3.8, 8.1, 1.8, 0.5, 2.7, 4.7, 5.1, 2.3, 2.9, 2.6, 2.5, 2.2, 3.9

   (*i*) Rearrange the data in descending order.

   (*ii*) Determine the highest rainfall.

   (*iii*) Determine the lowest rainfall.

   (*iv*) Determine the range of the rainfall.

   (*v*) On how many days was the rainfall less than 2.5 mm ?

3. Find the mean of

   (*i*) the first six whole numbers.

   (*ii*) the prime numbers between 5 and 25.

   (*iii*) the first five multiples of 9 ?

   (*iv*) all the factors of 24

4. The quantity of oil consumed in five households in a week is 22L, 24 L, 25 L, 21 L and 34 L. Find the mean weekly consumption of oil by these households. Also find the number of households whose consumption is more than the mean weekly consumption.

5. The mean of 4, 8, x, 7, 10 and 12 is 8. Find the value of x.

6. The mean of $x + 8, x + 6, x + 4, x + 2$ and x is 24. Find the value of x.

7. The mean of 15 numbers is 5. If 2 is added to each number, then what will be the new mean ?

8. The mean of a hundred observations was found to be 40. Later on, it was discovered that a score of 27 was misread as 57. Find the correct mean.

9. Find the mean of the following distribution :

x_1	5	10	15	20	25
f_1	2	8	15	10	5

10. The mean of 5 numbers is 22. If one more number is included, then the mean of the new set of numbers becomes 24. Find the included number.

Median

Take a Task

Watch Remedial

The median is the middle value of set of observations arranged in either ascending or descending order.

Median of an ungrouped data

Working Rules

1. Arrange the given data in ascending or descending order of magnitude.
2. Determine the total number of observations, say, n.
3. If n is odd, then median $= \left(\frac{n+1}{2}\right)$th observation.
4. If n is even, then $\frac{\left(\frac{n}{2}\right)^{th} \text{ observation} + \left(\frac{n}{2}+1\right)^{th} \text{ observation}}{2}$.

Example 10 : A student obtained the following marks in fifteen class tests:

25, 22, 29, 24, 21, 25, 22, 24, 27, 20, 28, 27, 29, 27

Find the median marks.

Solution: Arranging the marks in ascending order, we get

20, 21, 22, 22, 24, 24, 25, 25, 27, 27, 27, 28, 29, 29, 29

Here, the number of observations is $n = 15$, which is odd.

$$\therefore \quad \text{Median} = \left(\frac{n+1}{2}\right)\text{th term}$$

$$\Rightarrow \quad \text{Median (marks)} = \left(\frac{15+1}{2}\right)\text{th term} = 8\text{th term} = 25.$$

Example 11 : The following are the ages (in years) of 12 persons:

35, 36, 27, 32, 23, 39, 40, 55, 28, 32, 46, 45

Find the median age of these persons.

Solution : Arranging the ages (in years) in ascending order, we get

23, 27, 28, 32, 32, 35, 36, 39, 40, 45, 46, 55

Here, the number of observations is $n = 12$, which is even.

$$\therefore \quad \text{Median age} = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$\Rightarrow \quad \text{Median age} = \frac{6\text{th observation} + 7\text{th observation}}{2}$$

$$= \left(\frac{35+36}{2}\right)\text{years} = \frac{71}{2}\text{years}$$

$$= 35.5 \text{ years.}$$

Check Your Progress

Experiential Learning

1. Find the median for the digits 3, 1, 5, 4, 2 _____ .
2. Find the median of 75, 90, 70, 50, 70, 50, 75, 90, and 75.
3. The following observations have been arranged in an ascending order : 18, 20, 25, 26, 30, x, 57, 38, 39, 48. If the median of the data is 35, then find the value of x.

Example 12 : The following data have been arranged in ascending order : 24, 27, 28, 31, x, 34 40, 42, 45, If the median of the data is 35, then find x.

If in the given data. 45 is replaced by 33, then find the new median.

Solution: Here, the number of observations is $n = 10$, which is even.

$$\therefore \quad \text{Median} = \frac{\left(\frac{n}{2}\right)\text{th observation} + \left(\frac{n}{2}+1\right)\text{th observation}}{2}$$

$$\Rightarrow \quad \text{Median} = \frac{5\text{th observation} + 6\text{th observation}}{2}$$

$$\Rightarrow \quad 35 = \frac{34+x}{2} \qquad [\text{Median} = 35, \text{given}]$$

$$\Rightarrow \quad 70 = 34 + x$$

$$\Rightarrow \quad x = 36$$

If 45 is replaced by 33, then the observations arranged in ascending order are 24, 27, 28, 31, 33, 34, 36, 37, 40, 42.

New median = Average of 5th observation and 6th observation

$$= \frac{33+34}{2} = 33.5$$

Hence, $x = 36$ and the new median is 33.5.

Exercise 12.2

1. Find the median of the following:

(*i*) 64, 41, 87, 77, 58, 55, 90, 46, 35, 33, 92.

(*ii*) 42, 40, 39, 50 , 52, 46, 38, 47, 49, 44, 41, 42, 43.

(*iii*) 83, 36, 40, 248, 0, 69, 73, 105.

(*iv*) 28, 35, 32, 26, 27, 35, 36, 28 , 29, 32, 37, 24, 33, 26, 25, 31, 22, 36, 27.

2. The ages (in years) of 10 students participating in a English drama are 10, 6, 8, 19, 12, 8, 12, 15, 10, 18, 19, 15. Find the median age of the participants.

3. The weight (in kg wt) 10 students are as follows: 45, 32, 36, 30, 43, 27, 29, 31, 37, 42,. Find the median weight.

4. The median of the following observations, arranged in ascending order, is 22. Find x. 8, 11, 13, 15, $x+1$, $x+3$, 30, 35, 40, 43

5. The weight (in kg wt) of 15 students are as follows :

42, 27, 43, 30, 45, 32, 34, 36, 35, 44, 37, 31, 29, 41, 28

Find the median. If 27 kg wt is replaced by 25 kg wt and 44 kg wt by 46 kg wt, then find the new median.

6. The following data have been arranged in ascending order :

12, 14, 17, 20, 22, x, 26, 28, 32, 36

If the median of the data is 23, then find x. In the given data if 32 is replaced by 23, then find the new median.

Mode

The mode is the score (observation) that appears most often in a given set of observations. Hence, it is that value of the observation which occurs most frequently.

Example 13 : Find the mode for each of the following distributions :

26, 41, 8, 30, 26, 20, 26, 24, 13, 17, 24

Solution: Arranging the observations in ascending order, we have

8, 13, 17, 20, 24, 26, 26, 26, 30, 41

Here 26 occurs most frequently, i.e., three times.

Hence, the mode is 26.

Example 14 : Find the mode for the following data:

x	8	12	13	16	17	20
f	5	10	23	17	12	19

Solution: Score 13 has the maximum frequency (i.e., = 23).

13 occurs the maximum number of times.

Hence, the mode is 13.

Example 15 : Khushi scored the following marks in various class tests during a term, each test being of 20 marks.

15, 17, 16, 7, 10, 12, 14, 16, 19, 12, 16

What are her model marks ?

Solution: Arranging the marks in ascending order, we have

7, 10, 12, 12, 14, 15, 16, 16, 16, 17, 19

Here, 16 occurs most frequently, i.e., three times.

Hence, Khushi model marks = 16.

Check Your Progress

Experiential Learning

1. Find mode of 4, 6, 7, 8, 12, 11, 13, 9, 13, 9, 7, 8, 9 .
2. The numbers 2, 3, 4, 4, $2x + 1$, 7, 7, 8 and 9 are written in ascending order. If the median is 7, then find mode of this data.
3. Median of the following observations arranged in ascending order is 22, 8, 11, 13, 15, $x + 1$, $x + 3$, 30, 35, 40, 43. Then find the value of x.

Example 16 : Find the mean, median and mode of the following marks (in ascending order) secured by 16 students in a class test of 10 marks:

0, 0, 2, 2, 3, 3, 3, 4, 5, 5, 5,5, 6, 6, 7, 8

Solution: $\text{mean} = \dfrac{\text{Sum of observations}}{\text{Numberofobservations}}$

$$= \frac{0+0+2+2+3+3+3+4+5+5+5+5+6+6+7+8}{16} = \frac{64}{16} = 4$$

Here, the number of observations is $n = 16$, which is even.

$\therefore$ Median = Average value of $\left(\frac{n}{2}\right)$th termand and $\left(\frac{n}{2}+1\right)$th term

$\Rightarrow$ Median = Average value of 8th term and 9th term $= \frac{4+5}{2} = 4.5$

In the given data, 5 occurs most frequently, i.e., four times. So, the mode is 5.

Hence, mean = 4, median = 4.5 and mode = 5.

Example 17 : The number 2, 3, 3, 4, $2x + 1$, 5, 5, 6 and 8 are written in ascending order. If the median is 5, then find x. Hence, find the mode.

Solution: Here, the number of observations is $n = 9$, which is odd.

$\therefore$ Median $= \left(\frac{n+1}{2}\right)$th term

$\Rightarrow$ Median $= \left(\frac{9+1}{2}\right)$th term

Median = 5th term

$\Rightarrow$ $5 = 2x + 1$ [$\therefore$ Median = 5, given]

$\Rightarrow$ $4 = 2x$

$\Rightarrow$ $x = 2$

Now, the numbers written in ascending order are

2, 3, 3, 4, $(2 \times 2 + 1)$, 5, 5, 6 and 8 i.e., 2, 3, 4, 4, 5, 5, 5, 6 and 8

Here, 5 occurs most frequently, i.e., three times.

$\therefore$ Mode = 5

Hence, $x = 2$ and mode = 5.

Exercise 12.3

1. A batsman scored the following runs in 10 one-day matches :

   35, 0, 28, 70, 72, 26, 85, 50, 82, 39. Find his modal score.

2. Find the value of x if the mode of the following data is 7 :

   3, 5, 7, 4, 7, 8, 3, 6, 7, 4, x and 3

3. The ages (in years) of employees in a car wash are 17, 18, 17, 17, 46, 20, 19. Find the modal age.

4. For the data 1, 5, 7, $x + 1$, 9, $x - 2$, 3, if the mean is 4, then find the value of x. Hence, find the mode.

5. For what value of p the mode of the following data be 9 ?

   3, 5, 6, 3, 6, 9, 7, 8, 5, 9, p and 4

   If 3 is added to each element of the above data, find the new mode.

6. The size of 10 shirts (in *cm*) are as follows :

   90, 95, 100, 90, 90, 95, 95, 100, 90, 105

   Find the modal size. If the size of one shirt is misread as 90, instead of 95, find the correct modal size.

7. For each set of data find mean, median and mode :

   (*a*) 9, 7, 11, 3, 5, 8, 9, 7, 6 (*b*) 30, 22, 14, 18, 38, 22, 23, 25, 29, 30

Bar Graph

A Bar Graph is a pictorial representation of numerical data using bars of different heights. Its features include:

1. Rectangular bars are drawn on one axis depicting the variables.
2. The value of those variables is shown on the other axis.
3. The bars are equally distanced from each other.
4. Height of the bar is dependent on the value of the variable.
5. The width of the bar should be uniform.

Examples 18 : The following table gives the data about the number of hours spent on various activities by Surabhi in a day:

Activities	Number of hours
College	5
TV	2
Gadgets and websites	3
Sleeping	8
Others	6

Solution : Draw a bar graph to represent the data.

Working Rules

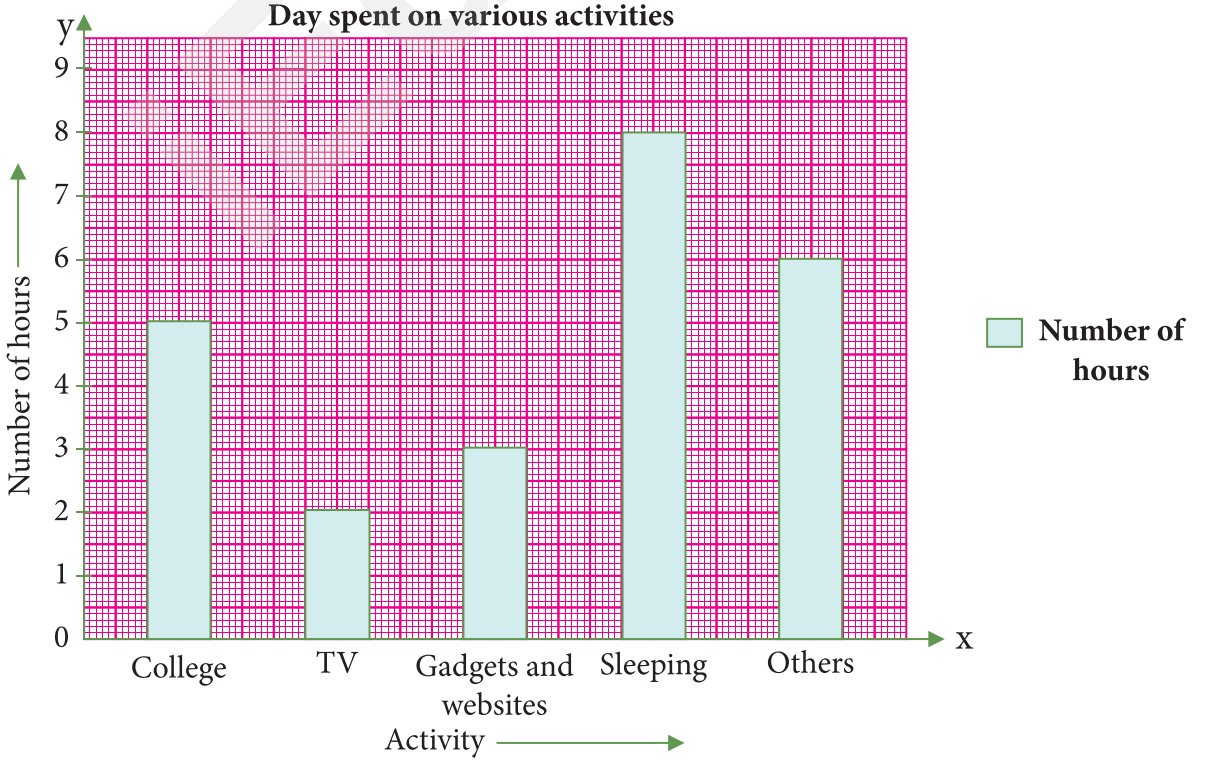
Step 1 : Construct two perpendicular lines–horizontal and vertical on a graph paper. Name the horizontal axis as X–axis and vertical as Y–axis.

Step 2 : Take 'Activities' along X-axis and its is number of hour frequency along Y–axis.

Step 3 : Choose a uniform width and gap of the bars along X-axis.

Step 4 : Choose a suitable scale to determine the height of the bar. Here, take 1 cm as 1 hour,

Step 5 : The bar graph showing the number hours spent on an activity is as follows:



Double Bar Graph

As we recalled the construction of bar graph before, we will now learn about composite or double bar graph wherein two different values of the same variable are considered.

Example 19 : The performance of two students Rahul and Prachi is given below. Draw a double bar graph for the data.

Subject	English	Science	Maths	S.St	Hindi
Rahul	80	75	93	77	67
Prachi	84	82	95	86	74

Solution:

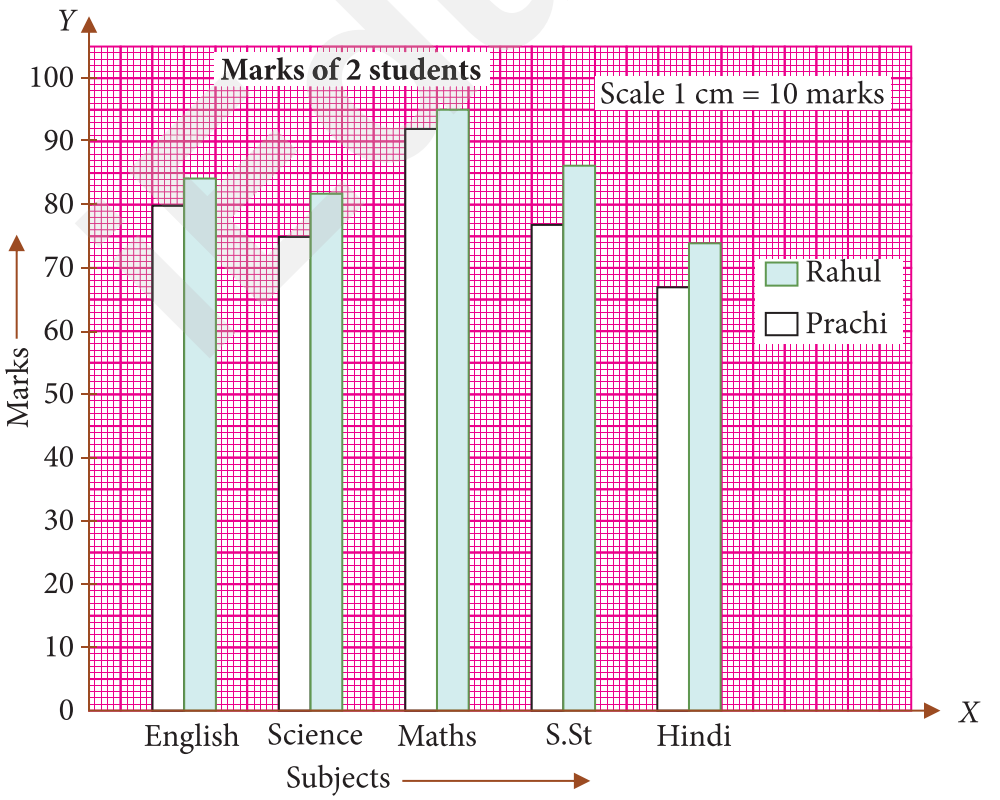
Working Rules

Step 1 : Draw two perpendicular lines representing X and Y axis as OX and OY respectively.

Step 2 : Along OX, mark the subjects at equal widths and gaps; along OY mark the 'marks obtained', choosing a suitable scale.

Step 3 : Along OY, choose the scale: 1 *cm* = 10 marks.

Step 4 : Draw bars of equal widths corresponding to the marks obtained by both the students as shown in the figure.



Histogram

A histogram represents a grouped frequency distribution graphically. It is a bar graph in which the bars are placed next to each other *i.e.*, there is no gap between any two successive rectangles.

Example 20 : Create a histogram to display the data on the frequency table below.

Number of Minutes on the Computer	Tally marks	Frequency
0–5	|||	3
6–10	||	2
11–15	|||	3
16–20	||	2
21–25	|	1
26–30	|	1
31–35	|	1
36–40	|	1
41–45	||	2
46–50	|	1
51–55	|	1
56–60	||	2

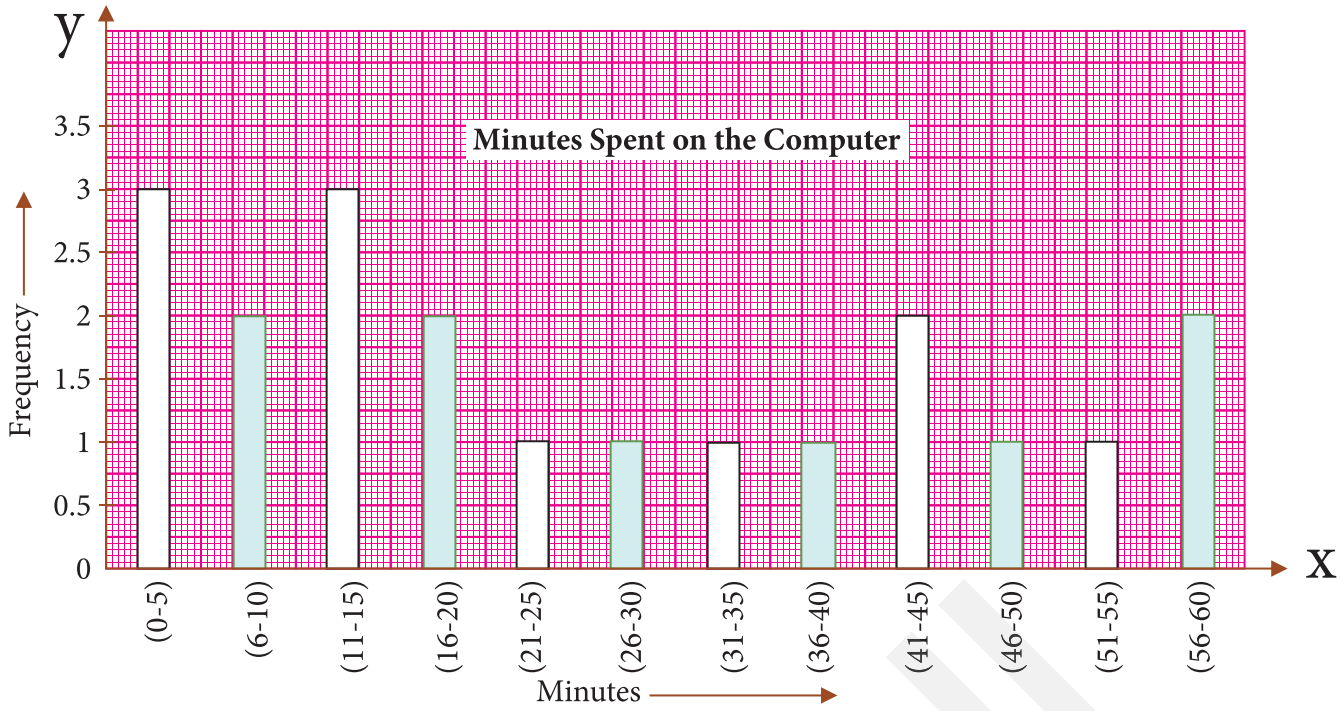
Solution : To create a histogram:

Working Rules

Step 1 : Draw the horizontal (x) and vertical (y) axis.

Step 2 : Label the horizontal axis "Minutes." List the intervals across the horizontal axis. Title the vertical axis "Frequency." Label the axis by halves (0.5).

Step 3 : For each interval on the horizontal access, draw a vertical column to the appropriate frequency value. Recall that on a histogram, there are no spaces in between vertical columns.



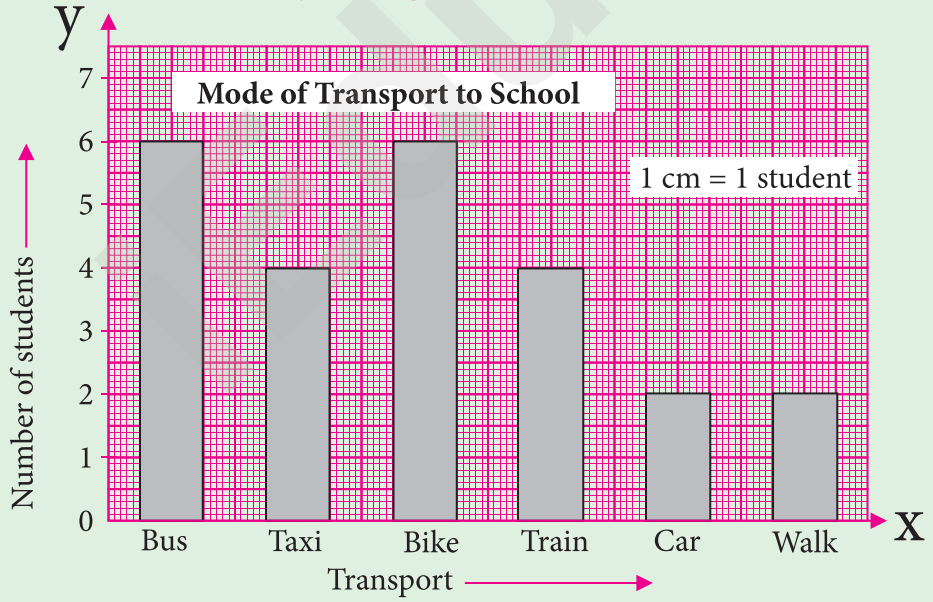
Exercise 12.4

1. A survey of 150 boys revealed their favourite game:

Game	Cricket	Football	Badminton	Volleyball	Kabaddi	Hockey
Boys	35	30	12	25	40	8

Represent the above data by a bar graph.

2.



The bar graph shows the usual mode of transport to school for the students in a class. Answer following questions by studying the bar graph:

(*i*) How many students are there altogether in the class?

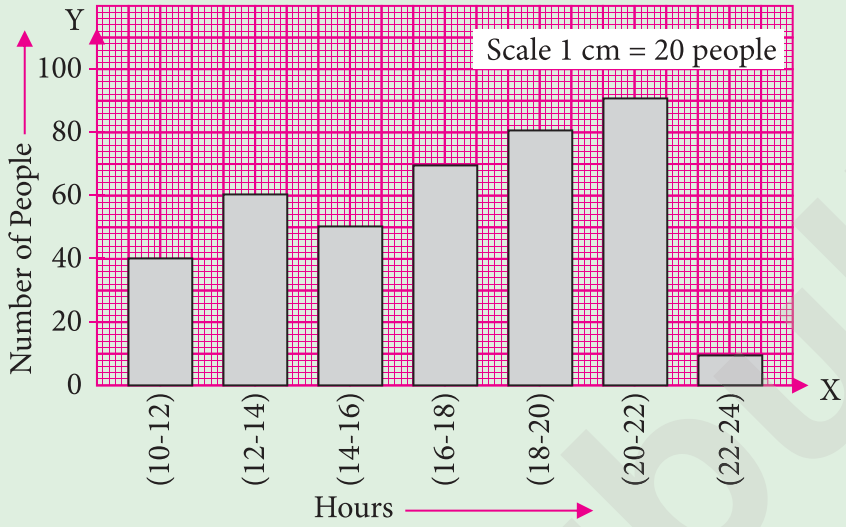
(*ii*) How many students preferred traveling by train?

(*iii*) Which is the most and the least transport system used by the students?

(*iv*) What percent of students prefer to walk over go by the bus?

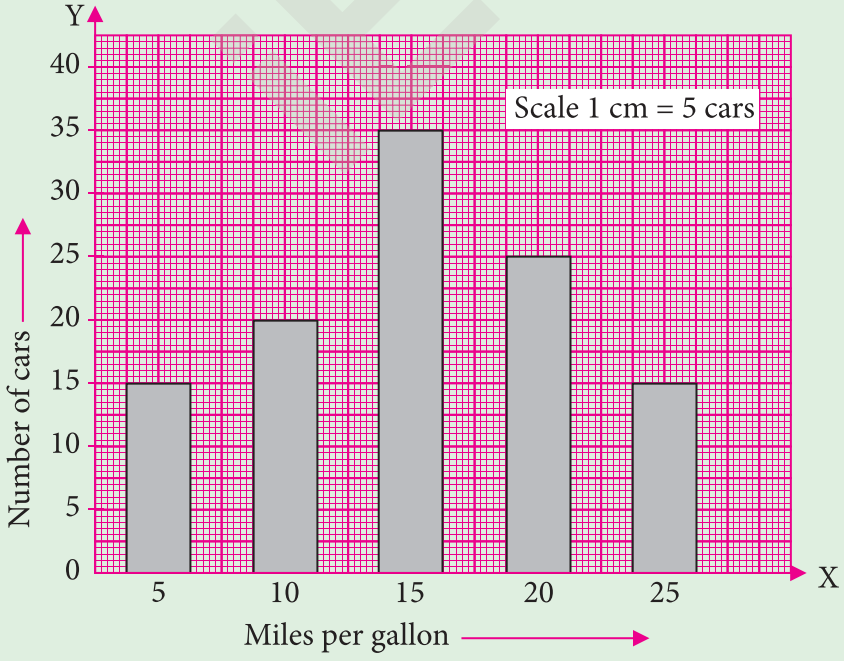
3. **A survey showed the average daily expenditures of 25 households in a city:**
211, 222, 227, 238, 230, 221, 245, 216, 222, 219, 229, 244, 236, 250, 214, 215, 237, 240, 227, 233, 210, 225, 248, 215, 242.
Form a frequency distribution table using equal class size of ₹5 and draw a histogram for the data.

4. **The graph represents the number of people entering into a mall between different times in a day. Read the histogram and answer the questions.**



(*i*) Use the graph to construct a frequency distribution table.
(*ii*) How many people entered the mall between 12 to 6 pm?
(*iii*) When did the maximum number of people enter?
(*iv*) During what time did around 80 people enter the mall?

5. **The histogram below shows the efficiency level (in miles per gallons) of 110 cars.**



(*i*) How many cars have an efficiency between 15 and 25 miles per gallon?

(*ii*) How many cars have an efficiency more than 10 miles per gallon?

(*iii*) What percentage of cars have an efficiency less than 20 miles per gallon?

Pie Chart

A Pie Chart is a circular diagram in which the observations are represented by non-intersection sections of a circle. The size of each sector corresponds to the magnitude or the frequency of each observation and the whole circle represents the total sum of the observations. It is also known as a **circular graph**.

Central angle for a component

The central angle for a component is given by:

$$\text{Central angle for a component} = \left(\frac{\text{Value of the component}}{\text{Sum of the values of all components}} \times 360^\circ\right)$$

How to make a pie chart or graph?

1. Calculate the central angle for each component, given by:

   $$\text{Central angle of component} = \left(\frac{\text{Value of the component}}{\text{Total value}} \times 360^\circ\right)$$

2. Draw a circle of convenient radius.
3. Within this circle, draw a horizontal radius.
4. Starting with the horizontal radius, draw radii making central angles corresponding to the values of the respective components, till all the components are exhausted. These radii divide the whole circle into various sectors.
5. Shade each sector with different colors or designs.

Example 21 : The following table shows the expenditure in percentage incurred on the construction of a house in a city:

Item	Brick	Cement	Steel	Labour	Miscellaneous
Expenditure (in percentage)	15%	20%	10%	25%	30%

Represent the above data by a pie chart.

Solution : Total percentage = 100.

$$\text{Central angle for a component} = \left(\frac{\text{Value of the component}}{100} \times 360^\circ\right)$$

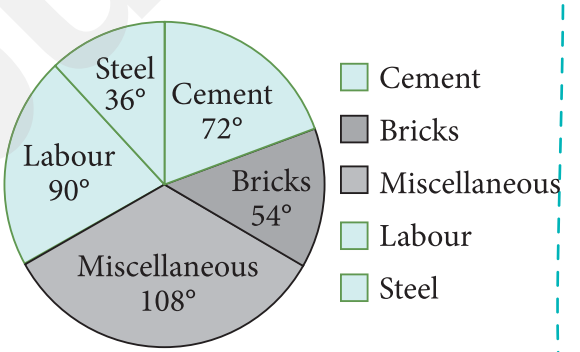
Calculation of central angles

Item	Expenditure (in percentage)	Central Angle
Brick	15%	$(15/100 \times 360)° = 54°$
Cement	20%	$(20/100 \times 360)° = 72°$
Steel	10%	$(10/100 \times 360)° = 36°$
Labour	25%	$(25/100 \times 360)° = 90°$
Miscellaneous	30%	$(30/100 \times 360)° = 108°$

Working Rules

Steps of construction:

1. Draw a circle of any convenient radius.
2. Draw a horizontal radius of the circle.
3. Draw sectors starting from the horizontal radius with central angles of 54°, 72°, 36°, 90° and 108° respectively.
4. Shade the sectors differently using different colors and label them.

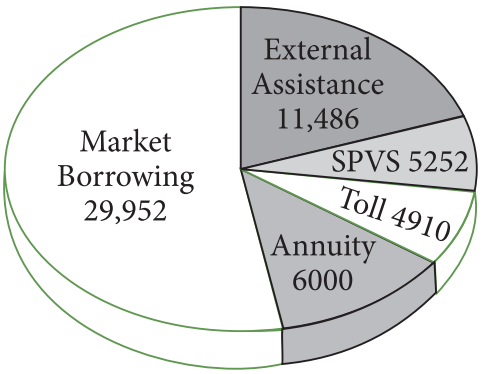


This we obtain the required pie chart, shown in the adjoining figure.

Example 22 : The following pie chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects.

Study the pie-chart and answers the question that follow.

Sources of funds to be arranged by NHAI for Phase II projects (in crores ₹)



(*i*) Where is the maximum number of funds required?

(*ii*) What percentage of funds is required for Toll?

(*iii*) What is the central angle corresponding to Annuity?

(*iv*) What is the approximate ratio of funds to be arranged through SPVS and Market Borrowing?

Solution : (*i*) Looking at the pie chart, we can conclude that market borrowing requires that maximum funds.

(*ii*) Funds for Toll : 4910

Total funds : 57,600

Percentage of funds for Toll $= \left(\frac{4910}{57600}\right) \times 100 = 8.5\%$

(*iii*) Central angle corresponding to Annuity $= \left(\frac{6000}{57600}\right) \times 360° = 37.5°$

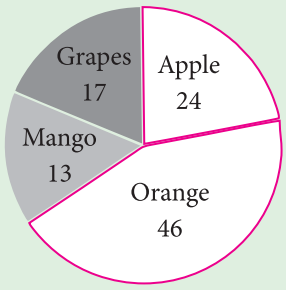
(*iv*) Required ratio = Funds for SPVS/funds for Market Borrowing

$= \frac{5252}{29952} = 101 : 576$

Exercise 12.5

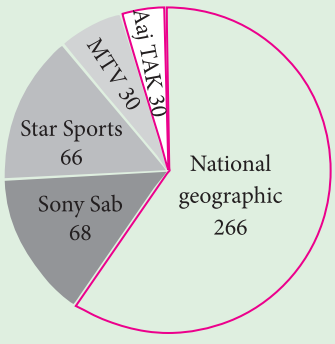
1. A group of 100 children were asked to name their favourite fruit.

Fruit	No. of pupils
Apple	24
Orange	46
Grapes	17
Mango	13



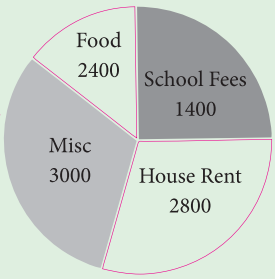
2. The favourite TV channels of 550 children are given in the table.

TV Channel	No. of children
G.TV	266
Star Sports	66
Sony	68
MTV	30
Aaj Tak	20



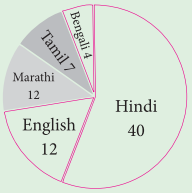
3. The monthly expenditure on various items of a family is given below.

Item	Food	House Rent	Misc	School Fees
Amount Spent	₹2400	₹2800	₹3000	₹1400

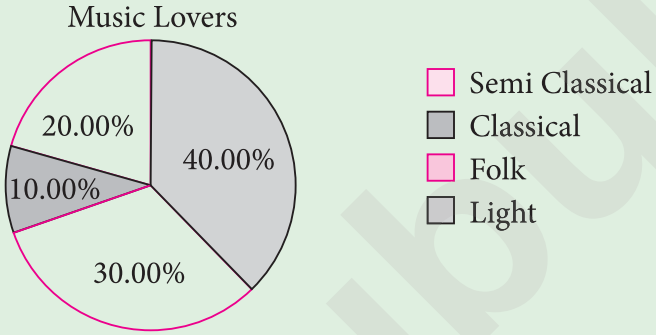


4. The number of students in a hostel, speaking different languages is given below.

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of students	40	12	9	7	4	72



5. A survey was made to find the type of music that a certain group of young people liked in a city. Following pie chart shows the findings of this survey.



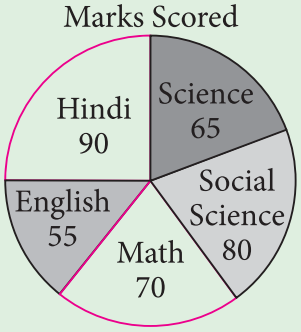
From this pie chart answer the following:

(*i*) If 45 people liked classical music, how many young people were surveyed?

(*ii*) Which type of music is liked by the minimum number of people?

(*iii*) If a cassette company were to make 2000 CD's, how many of each type would they make?

6. The following pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the student was 540, answer the following questions. All values are shown in degrees.



(*i*) In which subject did the student score 65 marks?

(*ii*) How many more marks were obtained by the student in Hindi than in English?

(*iii*) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.

Project

Collaboration

Observe the daily schedule of you and your best friend.

Activity	You	Your Best Friend
School		
Sleeping		
Playing		
Watching TV		
Studying		

Draw a bar graph for the above data and find out the following:

1. Who sleeps more and by how much?
2. Who spends more time playing?
3. Who is studying less?
4. According to you, who has a better schedule?

Chapter-end Exercise

Gap Analyzer™

A. Multiple Choice Questions (MCQs)

Tick (✓) the correct option.

1. A group of observations which is classified specifically in a frequency distribution is known as:

   (*a*) range ☐ (*b*) class mark ☐ (*c*) class limit ☐ (*d*) class ☐

2. A data is such that its maximum value is 222 and the range is 52, then the minimum value is:

   (*a*) 170 ☐ (*b*) 70 ☐ (*c*) 172 ☐ (*d*) 52 ☐

3. The upper limit of the class 52.4 – 62.4 is:

   (*a*) 26.4 ☐ (*b*) 62.4 ☐ (*c*) 52.4 ☐ (*d*) 10 ☐

4. The class mark of the class 2.2 – 7.6 is:

   (*a*) 4.5 ☐ (*b*) 2.4 ☐ (*c*) 4.9 ☐ (*d*) 4.2 ☐

5. A pie chart represents a total of 180 students opting for their favourite clubs in school. If the central angles for Music and Dance are 150° and 100° respectively, then how many more students opted for Music?

(*a*) 30 ☐ (*b*) 60 ☐ (*c*) 25 ☐ (*d*) 15 ☐

Custom Learning Path

Scan to Create Your Own Learning Path

Mental Maths

Critical Thinking

1. The mean of six observations is 17.5. If five of them are 25, 23, 10, 9 and 14, then find the sixth observation.
2. The mean of five observations is 15. If the mean of the first three observations is 14 and the mean of the last three observations is 17, then find the third observation.
3. If the mean of the following distribution is 6, then find the value of P.

x_1	2	4	6	10	P +5
f_1	3	2	3	1	2

Activity

Collaboration

Constructing Bar Graph

Teacher will distribute one graph paper and five different coloured strips to each student. Tell the students that they are going to represent the marks obtained by them in the last test on a Bar graph. Now, give the following instructions to the students to make a bar graph.

Step 1 : Draw two lines perpendicular to each other on the given graph paper.

Step 2 : Mark the horizontal line as X-axis and vertical line as Y-axis.

Step 3 : Now write 'Subjects' horizontally under the X-axis and then write 'Marks' obtained' vertically along the Y-axis.

Step 4 : Take on appropriate scale, like if 1 square shows 10 marks, then write "1 unit = 10 marks".

Step 5 : Now start cutting ad pasting the strips to represent the marks obtained in each subject in a sequence and make sure that all bars are equidistant.

To make teaching-learning process a success, teacher will make sure that almost all the students in the class will participate in this activity.