

Symmetry

Why This Chapter Matters

Have you ever looked closely at a butterfly's wings and wondered how they are so perfectly matched? Or how an artist creates a beautiful, repeating Rangoli pattern that looks the same from different directions? This perfection is not magic; it's mathematics! Symmetry is the secret ingredient that brings balance and beauty to the world around us. It's in the petals of a flower, the design of a car's logo, and even your own reflection in a mirror.



Meet EeeBee.Al



Hi, I'm EeeBee! I love finding patterns and perfect balance in everything. Symmetry is like a secret code used by nature and people to create amazing designs. I'll be your guide on this exciting journey. I'll pop up with hints, fun facts, and tricky questions to help you become a symmetry expert. Let's explore the world of perfect halves and amazing turns together!



Learning Outcomes

By the end of this chapter, students will be able to:

- Identify and draw lines of reflective symmetry (vertical, horizontal, and diagonal) in various 2D shapes.
- Define rotational symmetry and identify the center, angle, and order of rotation.
- Differentiate between reflective, rotational, and translational symmetry.
- Analyze and compare the symmetry in different geometric figures like triangles, quadrilaterals, and regular polygons.
- Apply the principles of symmetry to create your own artistic patterns, such as mosaics and Rangoli.
- Recognize and explain examples of symmetry in the real world, from nature to technology.

From Last Year's Notebook

Remember Grade 5?

- We explored basic shapes and the idea of "mirror halves" through paper folding.
- Introducing a New Name: In this chapter, we'll formally call these "mirror halves" Symmetry.
- **Going Deeper:** We will move beyond simple folding to understand the mathematical rules that create perfect balance in shapes and patterns.

Real Math, Real Life

Symmetry is a key principle all around us

- **Architecture:** Used to design stable and beautiful buildings like the Taj Mahal.
- **Nature:** Helps biologists understand the growth and function of organisms.
- **Engineering:** Essential for creating efficient car wheels, fan blades, and gears.
- **Branding:** Makes famous logos memorable and visually appealing to customers

Quick Prep

- 1. Look at the letter A. Can you draw a line that splits it into two identical halves?
- 2. If you look at a clock, and the minute hand moves from the 12 to the 3, how many degrees has it turned?
- 3. Imagine a line of ants marching one after another. What do you notice about the spacing between them?
- 4. Which of these objects is most likely to have a "mirror half": a spoon, a teacup, or a pair of sunglasses?
- 5. If you write the number 8 and cut it in half horizontally, are the top and bottom parts identical? What if you cut it vertically?
- 6. Think of a bicycle wheel. As it spins, does it look different at any point? Why or why not?

Introduction

Welcome to the fascinating world of symmetry! In this first section, we will explore the three fundamental types of symmetry that describe how objects can be balanced. Think of it as learning the basic grammar of geometric design. We will learn about mirror images (**reflection**), turning objects (**rotation**), and sliding patterns (**translation**). Understanding these core ideas will give you the tools to see and appreciate the hidden mathematical structure in the world all around you, from a simple leaf to a complex wallpaper design.

Chapter Overview

- **Understanding the Basics:** Explore the main types of symmetry: Reflective (mirror), Rotational (turning), and Translational (sliding), along with their real-world examples like butterflies and fans.
- **Finding Lines of Symmetry:** Investigate symmetry in detail within polygons (squares, triangles) and everyday designs like the English alphabet, numbers, and national flags.
- **Applying Your Skills:** Learn how to combine line and rotational symmetry and use these principles to create beautiful, balanced patterns through paper folding, Rangoli, and mosaics.

From History's Pages

The love for symmetry is as old as humanity. We see it everywhere, from a butterfly's wings to our own reflections. Ancient civilizations, like the **Greeks and Egyptians**, used symmetry to build beautiful and balanced structures, such as the **Parthenon** and the **great Pyramids**. They knew that symmetrical designs were visually pleasing and strong. The line of symmetry that we learn about in mathematics is the key to understanding this perfect balance in the world all around us.

Symmetry

Hello! Let's explore one of the most beautiful and common ideas in both art and mathematics: **Symmetry**.

At its heart, **symmetry means a perfect, balanced arrangement**. It's when one half of an object is the exact mirror image of the other half.

The Mirror Line Test

Imagine you have a picture of a butterfly. Now, picture drawing a straight line right down the middle of its body.

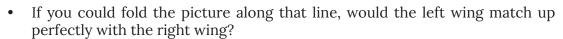




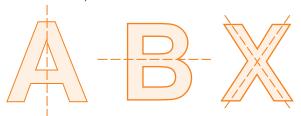
Fig. 9.2

• Yes, it would!

That imaginary folding line is the key to understanding symmetry. We call this the **Line of Symmetry** (or the axis of symmetry).

Key Ideas to Remember

- **Symmetrical Figure:** An object is symmetrical if it has at least one line of symmetry.
- **Line of Symmetry:** This is the imaginary line that divides a figure into two identical, mirror-image halves.
- **Types of Lines:** A line of symmetry can be:
 - **Vertical** (straight up and down, like in the letter **A**)
 - ♦ Horizontal (straight across, like in the letter B)
 - ◆ **Diagonal** (slanted, like in the letter **X**)



Types of Symmetry

Reflective Symmetry (Mirror Magic)

Reflective symmetry is the easiest and most common type of symmetry to spot. It's often called mirror symmetry because one half of an object is the perfect mirror image of the other half. Imagine placing a mirror down the middle of an object. If the reflection in the mirror looks exactly like the other half of the object, then it has reflective symmetry. The imaginary line where you would place the mirror is called the **line of symmetry** or the **axis of symmetry**.

Sub-concepts to be covered

- 1. **Line of Symmetry:** This is the fundamental element of reflective symmetry. It is a line that divides a figure into two congruent (identical in shape and size) parts, where one part is the mirror image of the other.
- 2. **Multiple Lines of Symmetry:** Many shapes have more than one line of symmetry. For example, a square has four, while an equilateral triangle has three. Some shapes, like a scalene triangle, have none.

Mathematical Explanation

Mathematically, a figure has reflective symmetry if there exists a line (the axis of symmetry).

Let's consider the letter **H**. It has a **horizontal line of symmetry**. If you pick any point on the top bar, you can find a corresponding point on the bottom bar that is at the same distance from the central horizontal line. It also has a **vertical line of symmetry** running down the middle.

A circle is a special case. Any line that passes through its center is a line of symmetry. Since you can draw an infinite number of lines through the center, a circle has **infinite lines of symmetry**.

In contrast, a parallelogram (that is not a rectangle or rhombus) has no lines of symmetry. If you try to fold it along a diagonal, the two halves will not match up perfectly. This demonstrates that not all geometric shapes are symmetrical.

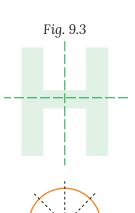


Fig. 9.4

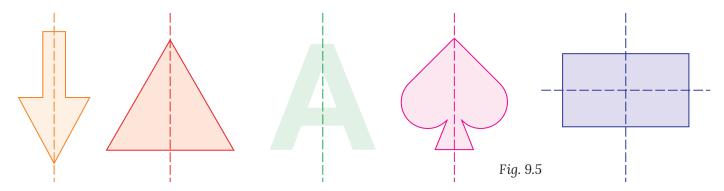
How to Identify Reflective Symmetry:

Step 1: Look for the line that divides the shape or object into two identical halves.

Step 2:Fold the object along that line. If both halves match perfectly, the shape has reflective symmetry.

Step 3:Count how many lines of symmetry are present. For example, a circle has an infinite number of lines of symmetry.

Examples of Reflective Symmetry:



Example 1: Check whether the following have reflective symmetry or not. If yes, then draw the line of symmetry.

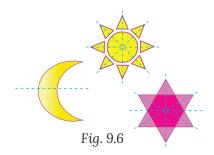






Solution: Draw the lines of symmetry:

- a) Sun has vertical, horizontal and diagonal lines of symmetry.
- b) Moon has one horizontal line of symmetry.
- c) Star has vertical, horizontal and diagonal lines of symmetry.



Example 2: Does the English letter W have any line of symmetry? If yes, draw them.

Solution: Step 1: Visualize the letter W.

Step 2: Try folding it horizontally. The top and bottom parts do not match. So, no horizontal line of symmetry.

Step 3: Try folding it vertically, right down the middle point. The left side is a perfect mirror image of the right side.



Rotational Symmetry (The Turning Trick)

Have you ever spun a pinwheel or watched a fan rotate? You might have noticed that even as they turn, they look the same at certain points. This property is called **rotational symmetry**. An object has rotational symmetry if it looks exactly the same as its original position after being rotated by an angle less than a full 360-degree turn around a central point.

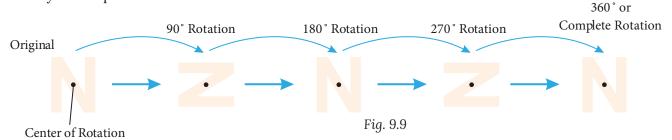


Sub-concepts to be covered

- 1. **Rotation:** The action of turning an object around a fixed point (the center).
- 2. **Center of Rotation:** The pivot point that does not move during the rotation. For a square, it's the point where the diagonals intersect. For a fan, it's the center hub.
- 3. **Order of Rotational Symmetry:** This is a count. If a shape matches its original position 4 times in a full 360° turn (at 90°, 180°, 270°, 360°), its order of rotation is 4. A shape with no rotational symmetry has an order of 1 (as it only matches itself after a full 360° turn).
- 4. **Angle of Rotation:** The measure of the turn. If a square looks the same after a 90° turn, then 90° is an angle of rotation. The smallest such angle is key.

Mathematical Explanation

Center of Rotation: This is the point around which the object rotates. Every object with rotational symmetry has a specific center of rotation.



Order of Rotation (Rotation Symmetry Order): This refers to how many times the object matches its original position.

Here, after 2 rotations letter N looks exactly same as the original shape, so order of rotation is 2.

Angle of Rotation: The angle by which the object must be rotated to look the same again. Here, the angle of rotation for letter N is 180° (since 360° : $2 = 180^{\circ}$).

How to Identify Rotational Symmetry

- **Step 1:** Place the object at its center of rotation and rotate it.
- **Step 2:** Check at different angles (e.g., 90°, 180°, 270°) to see if the object matches its original position.
- **Step 3:** Count how many times the object looks the same during one complete rotation (360°).

Examples of Rotational Symmetry and Lines of Symmetry

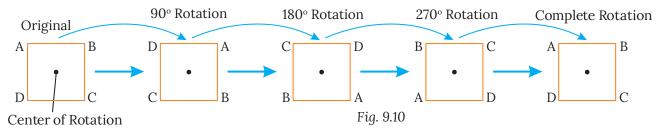
Rotational and Line Symmetry of a Square

Rotational Symmetry of a Square

Consider a square ABCD with a center as indicated in the figure. Upon rotating the square by 90°, 180°, 270°, and 360° about its center, it returns to its original position each time. Therefore, the square exhibits rotational symmetry of order 4, with the angles of symmetry being 90°, 180°, 270°, and 360°.

A square has rotational symmetry of order 4. This means it can be rotated around its center and still look the same at specific angles. These angles are:

- 90° (one-quarter turn)
- 180° (half turn)
- 270° (three-quarters turn)
- 360° (Complete Rotation)

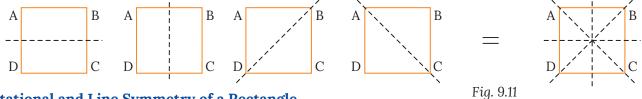


So, when a square is rotated by these angles, it looks identical to its original position.

Line Symmetry of a Square

A **square** has rotational symmetry of **order 4**, which divide it into identical parts. The lines of symmetry in a square are:

- 1. Two diagonal lines (connecting opposite corners).
- 2. Two lines through the midpoints of opposite sides (vertical and horizontal lines).



Rotational and Line Symmetry of a Rectangle

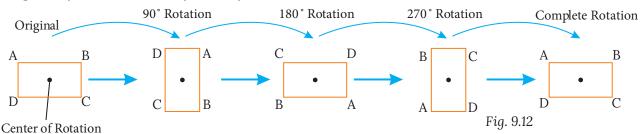
Rotational Symmetry of a Rectangle

Consider a rectangle ABCD with a center as shown in the given figure. When the rectangle is rotated by 180° or 360° about its center, it returns to its original position. Therefore, the rectangle has rotational symmetry of order 2, with symmetry angles of 180° and 360°.

A **rectangle** has rotational symmetry of **order 2**. This means it can be rotated around its center and still look the same at specific angles:

- 0° (no rotation)
- 180° (half turn)
- 360° (Complete Rotation)

After a 90° or 270° rotation, the rectangle will not look the same unless it is a square. Therefore, a rectangle only has rotational symmetry at 0° and 180°.



Line Symmetry of a Rectangle

A rectangle has 2 lines of symmetry, which divide it into identical parts. The lines of symmetry in a rectangle are:

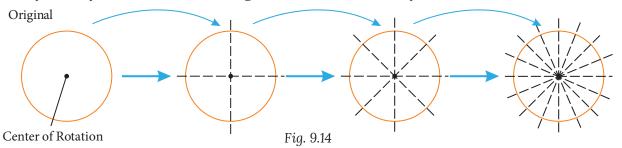
- 1. One vertical line (through the midpoints of opposite sides).
- 2. One horizontal line (through the midpoints of the other two opposite sides).



Rotational and Line Symmetry of a Circle

Rotational Symmetry of a Circle

A circle has infinite rotational symmetry because it looks the same at any angle of rotation. The order of rotational symmetry is infinite, and the angle of rotation can be any value.



Line Symmetry of a Circle

A circle has infinite lines of symmetry. Any straight line that passes through its center divides the circle into two equal halves. These lines can be drawn in any direction, whether vertically, horizontally, diagonally, or at any angle. The number of lines of symmetry is infinite.

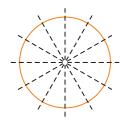


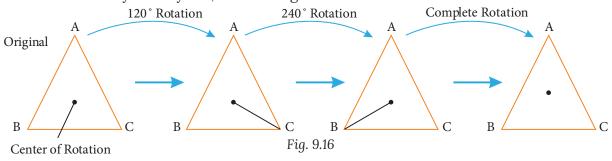
Fig. 9.15

Rotational and Line Symmetry of an Equilateral Triangle

Rotational Symmetry of a Equilateral Triangle

An equilateral triangle has 3 order of rotational symmetry because it looks the same after a rotation of 120°, 240°, and 360°.

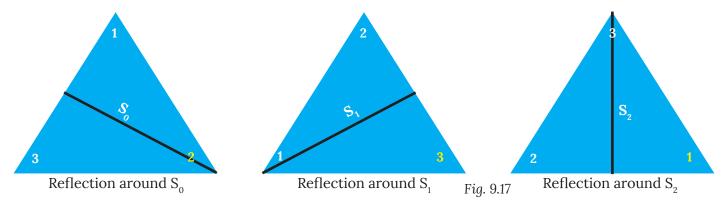
The order of rotational symmetry is 3, and the angle of rotation is 120°.



Line Symmetry (or Reflectional Symmetry) of a Equilateral Triangle

A line of symmetry is a line that divides a shape into two identical halves that are mirror images of each other. Think of it as a "**fold line**."

An equilateral triangle has 3 lines of symmetry.



Example 3: What is the degree and angle of rotation of a five sided star?

Solution: A star with 5 points has 5 orders of rotational symmetry.

The angle of rotation is $360^{\circ}: 5 = 72^{\circ}$

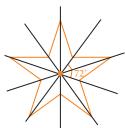
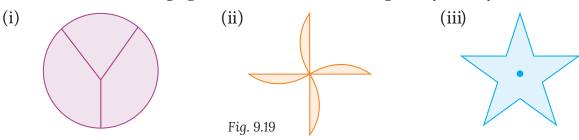


Fig. 9.18

Example 4: Which of the following figures have more than one angle of symmetry?



Solution:

Figure (i) has only one angle of symmetry

Figure (ii) has 4 angles of symmetry (90°, 180°, 270°, 360°)

Figure (iii) has 5 angles of symmetry (72°, 144°, 216°, 288°, 360°)

Example 5: A shape has rotational symmetry of order 2 but no lines of symmetry. Give an example.

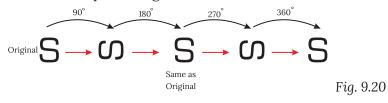
Solution:

Step 1: We need a shape that looks the same after a 180° turn.

Step 2: We also need it to fail the "mirror test".

Step 3: Consider the letter 'S'. If you rotate it 180°, it looks the same. But if you try to draw a line of symmetry (vertical, horizontal, or diagonal), the two halves never match. The letter 'Z' and a standard parallelogram also work.

Answer: The letter 'S' or a parallelogram.



Translational Symmetry (The Sliding Pattern)

Imagine a wallpaper with a repeating flower pattern or a tiled floor. If you slide a section of the pattern in a specific direction and distance, it will land perfectly on top of an identical section. This "**sliding**" property is called **translational symmetry**. It's the symmetry of repetition. Unlike reflection or rotation, translation doesn't happen around a point or a line; it happens along a line.

We will focus on two key aspects:

- **Repeating Unit:** The basic shape or design that is being copied.
- **Translation**: The specific movement (direction and distance) required to get from one unit to the next.

Sub-concepts to be covered

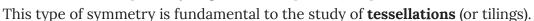
- **1. Translation:** This is the geometric transformation of "**sliding**" every point of a figure by the same distance in the same direction.
- **2. Repeating Patterns:** Translational symmetry is most evident in patterns that cover a plane, like fabric designs, brick walls, and tiled floors. The pattern is created by translating a single base unit (or 'motif') over and over.
- **3. Identifying Translational Symmetry:** This involves finding the smallest repeating unit and the direction/distance it needs to be moved to create the full pattern.

Mathematical Explanation

Translational symmetry occurs when a pattern can be translated (moved) by a certain vector (a specific distance and direction) and remain unchanged.

The key is to identify the **fundamental unit or motif**. This is the smallest part of the pattern which, when translated, can generate the entire pattern.

For example, in a simple checkerboard, the fundamental unit could be a single black square and a single white square next to it. If you translate this two-square unit by the width of two squares, you get the next part of the pattern.



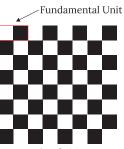


Fig. 9.21

Examples of Translational Symmetry

How to Identify Translational Symmetry:

Step 1: Look at the object or pattern and see if it can be shifted along a straight line (either horizontally, vertically, or diagonally).

Step 2: Translate the object or pattern by a certain distance in a certain direction.

Step 3: Check if the object looks exactly the same after translation. If it does, it has translational symmetry.

Step 4: If the object or pattern can be translated multiple times along the same direction and still looks the same, then the pattern has translational symmetry.

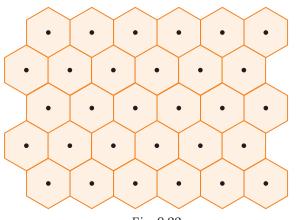


Fig. 9.22



Fig. 9.24

- 1. Wallpaper Patterns: Many wallpaper patterns, such as those with floral designs or geometric shapes, exhibit translational symmetry. If you slide the pattern horizontally or vertically, the design will repeat and look identical.
- **2. Tiling:** A tiled floor made of square tiles demonstrates translational symmetry. You can translate one tile horizontally or vertically, and it will match the next tile perfectly.



Fig. 9.23

- **3. Floor Patterns:** A checkerboard pattern also exhibits translational symmetry. If you slide the pattern horizontally or vertically by one square, it will look the same.
- **4. Geometric Figures:** Some geometric shapes like parallelograms, rectangles, and triangles can exhibit translational symmetry, where you can move them along a straight path, and they will align with their original positions.

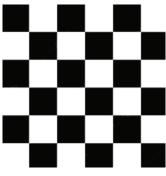


Fig. 9.25

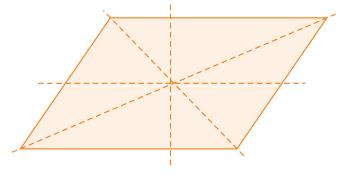


Fig. 9.26

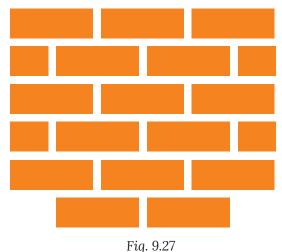
Example 6 : A bricklayer lays bricks in a standard running bond pattern. Describe the translational symmetry.

Solution: Step 1: Identify the repeating unit. A single brick is the basic component.

Step 2: Describe the translations. You can translate a brick horizontally by its full length to get to the brick two places over in the same row.

Step 3: You can also translate a brick diagonally (down one row and over by half a brick's length) to land on the brick directly below it in the pattern.

Answer: The pattern has translational symmetry both horizontally and diagonally. The repeating unit is a single brick.



Example 7: Which of these shows translational symmetry?

- a) A single circle.
- b) A chain of paper clips linked together.
- c) A five-pointed star.
- **Solution:** a) A single circle has rotational and reflective symmetry, but you cannot slide it to a new position where it overlaps with another identical circle (as there isn't one). No translational symmetry.
 - b) A chain of paper clips is a repeating pattern. You can slide one paper clip along the chain to the position of the next one. Yes, this has translational symmetry.
 - c) A single star has rotational and reflective symmetry, but not translational symmetry.

Answer: b) A chain of paper clips.



Knowledge Checkpoint

- 1. What is the order of rotational symmetry for the letter 'Z'?
- 2. A regular octagon has 8 sides. What is its angle of rotation?
- 3. What is the order of rotation for a plus sign (+)?
- 4. Give an example of an object in your school that has translational symmetry.
- 5. What is the difference between rotational and translational symmetry?



The Symmetry Spinner

Objective: To physically test for rotational symmetry.

Materials: Cardboard, a pin, scissors, a protractor, a marker.

STEPS:

- **Step 1:** Cut out a shape from the cardboard, for example, a square or an equilateral triangle.
- **Step 2**: Mark the top corner of the shape with a small dot.
- **Step 3**: Find the center of the shape and push a pin through it into a piece of scrap cardboard below, so the shape can spin.
- **Step 4**: Trace the shape's outline on the scrap cardboard.
- **Step 5:** Slowly rotate the shape. Count how many times it fits perfectly into its outline before the dot returns to the top. This count is the order of rotation.

Inquiry Question: Use a protractor to measure the angle of the first turn. Does it match the formula 360°/order? Try this with different shapes like a rectangle and a regular pentagon.



Do It Yourself

Why are so many man-made patterns (like tiles, bricks, and fabrics) based on translational symmetry? What are the practical advantages of creating things with repeating units? Think about cost, efficiency, and ease of construction.

Key Terms

- **Center of Rotation:** The fixed point around which the rotation occurs.
- **Angle of Rotation:** The smallest angle of turn for which the shape looks the same.
- **Translation:** The geometric transformation of sliding a figure.
- **Repeating Unit (Motif):** The smallest part of a pattern that can be translated to create the entire pattern.

Facts Flash

- A shape with point symmetry is another name for a shape with rotational symmetry of order 2. The letters S, N, and Z have point symmetry.
- Some words, when written in capital letters, have a horizontal line of symmetry, like "CHOICE-BOX" or "DECIDE". These are called ambigrams.
- The DNA molecule has a helical structure, which involves both rotation and translation along an axis, creating a type of 3D symmetry called screw symmetry.

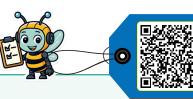


Mental Mathematics

- A pattern is ABCABCABC.... What is the repeating unit? (ABC)
- Is a flight of stairs an example of translational symmetry?
- Quickly say "yes" or "no" if the letter has a line of symmetry: T, S, O, L, M, P, H.



Exercise 9.1





(Watch Remedia



- 1. Provide the missing information in the blanks:
 - a) A square has ___ lines of symmetry.
 - b) A rectangle has rotational symmetry of ___ degrees.
 - c) A circle has ___ lines of symmetry.
 - d) The number of orders of rotational symmetry of a square is ___.
 - e) An equilateral triangle has rotational symmetry at $___$ degrees.
 - f) A figure with radial arms has rotational symmetry at ___ degrees for each arm.
- 2. List all the letters of the English alphabet which have rotational symmetry.
- 3. State whether the figure shows rotational symmetry. If yes, then what is the order of rotational symmetry?

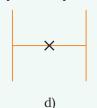


4. Which of the following figures have rotational symmetry of order more than 1:













5. Draw the other half of the given images.







6. Match the following figures with their number of rotational symmetries.

Figures	Number of Rotational Symmetries
	4
	2
<u> </u>	5

7. Repeat the pattern 4 times right side and then 4 times downwards to form the tessellation.



8. Deeksha is trying to read the number of a car whose reflection is given below. She placed a mirror on the left side of the number. Can you guess the number? Which letters and numbers are looking exactly same in the reflection also?

8808 WA C8 LD

Lines of Symmetry

In our first section, we met the line of symmetry as the star of reflective symmetry. Now, we're going to put it under a microscope! This section is a deep dive into the fascinating properties of lines of symmetry.

We will explore the different types of lines of symmetry:

- Vertical Line of Symmetry: A line that runs up and down.
- Horizontal Line of Symmetry: A line that runs from left to right.
- Diagonal Line of Symmetry: A slanted line.

Types of Lines of Symmetry

- 1. Vertical Line of Symmetry
- A line that divides the shape into two equal parts from **top to bottom**.

Example: Letter A has one vertical line of symmetry.



Fig. 9.28

2. Horizontal Line of Symmetry

• A line that divides the shape into two equal parts from **left to right**.

Example: A square has both a horizontal and a vertical line of symmetry.

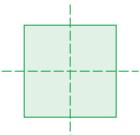


Fig. 9.29

3. Diagonal Line of Symmetry:

• A line that divides the shape into two equal parts at an angle. These lines are not necessarily **vertical or horizontal**.

Example: An equilateral triangle has 3 lines of symmetry, all diagonal.

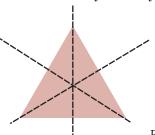


Fig. 9.30

Symmetry in Polygons

Polygons are closed 2D shapes with straight sides. Their symmetry is directly related to their properties, like the length of their sides and the size of their angles.

Sub-concepts to be covered with properties

1. Symmetry in Triangles

- Scalene Triangle: No equal sides, no equal angles. It has 0 lines of symmetry.
- ◆ **Isosceles Triangle:** Two equal sides, two equal angles. It has **1** line of symmetry, which runs from the vertex between the equal sides to the midpoint of the base.
- Equilateral Triangle: All three sides and angles are equal. It has 3 lines of symmetry, each running from a vertex to the midpoint of the opposite side.

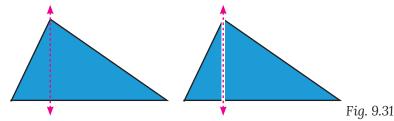
2. Symmetry in Quadrilaterals

- **Square:** 4 equal sides, 4 right angles. It has **4** lines of symmetry (2 through opposite corners, 2 through midpoints of opposite sides).
- **Rectangle:** Opposite sides equal, 4 right angles. It has **2** lines of symmetry (through midpoints of opposite sides only).
- Rhombus: 4 equal sides, opposite angles equal. It has 2 lines of symmetry (its two diagonals).
- **Parallelogram:** Opposite sides parallel. It has **0** lines of symmetry.
- **3. Symmetry in Regular Polygons:** A regular polygon is a polygon where all sides are equal and all angles are equal. There is a beautiful rule for them:
 - A regular polygon with 'n' sides has 'n' lines of symmetry.

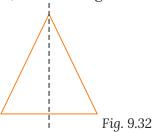
- Regular Pentagon (5 sides): 5 lines of symmetry.
- **Regular Hexagon (6 sides):** 6 lines of symmetry.

Symmetry in Triangles

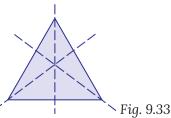
• Scalene Triangle: A scalene triangle has **no lines of symmetry**. This is because all three sides of a scalene triangle have different lengths, and all three angles have different measures. Therefore, it's impossible to draw a line that divides the triangle into two mirror-image halves.



• Isosceles Triangle: An isosceles triangle has **one line of symmetry**. This line runs from the vertex where the two equal sides meet (the apex) down to the midpoint of the opposite side (the base), dividing the triangle into two identical, mirror-image halves.



• Equilateral Triangle: An equilateral triangle is a triangle in which all three sides are equal in length and all angles are equal (60° each). Because of its perfect balance in size and shape, the equilateral triangle has 3 lines of symmetries.



Symmetry in Quadrilaterals

• **Square:** A square is a special quadrilateral with four equal sides and four right angles (90° each). Its perfect shape makes it one of the most symmetrical figures in geometry.

Line Symmetry: A square has 4 lines of symmetry

- 2 lines through the midpoints of opposite sides (horizontal & vertical)
- 2 lines through opposite corners (diagonals)











Fig. 9.34

• Rectangle: A rectangle is a quadrilateral with opposite sides equal and four right angles (90° each). Although it shares some features with a square, it has slightly less symmetry.

Line Symmetry: A rectangle has 2 lines of symmetry

- One horizontal line through the middle
- One vertical line through the middle

(The diagonals are not lines of symmetry unless all sides are equal, as in a square.)







Fig. 9.35

• Rhombus: A rhombus is a quadrilateral with all sides equal, but the angles are not necessarily 90°.

Line Symmetry: 2 lines of symmetry

Along its diagonals, which bisect each other at right angles.

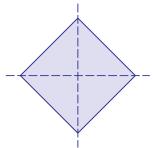


Fig. 9.36

• Parallelogram: A parallelogram is a quadrilateral where opposite sides are equal and parallel, but angles and sides may vary.

Line Symmetry: 0 lines of symmetry

No line divides it into two mirror-image halves.



Fig. 9.37

Symmetry in Regular Polygons

• Regular Pentagon: A regular pentagon has five equal sides and five equal angles.

Line Symmetry: 5 lines of symmetry

Each passes through a vertex and the midpoint of the opposite side.

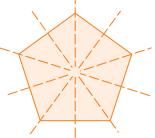


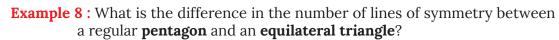
Fig. 9.38

• Regular Hexagon: A regular hexagon has six equal sides and six equal angles.

Line Symmetry: 6 lines of symmetry

- ♦ 3 lines connecting opposite vertices
- ♦ 3 lines connecting midpoints of opposite sides

When a shape has two or more lines of symmetry, it means the shape can be divided into two or more identical parts by multiple lines. Each of these lines reflects the shape's symmetry, making it look identical on both sides of the line(s).



Solution: Step 1: A regular pentagon has 5 sides. The number of lines of symmetry is 5.

Step 2: An equilateral triangle is a regular polygon with 3 sides. The number of lines of symmetry is 3.

Step 3: The difference is 5 - 3 = 2.

Answer: The difference is 2.

Example 9 : A quadrilateral has 4 equal sides but its angles are not 90°. What is this shape and how many lines of symmetry does it have?

Solution: Step 1: A quadrilateral with 4 equal sides and non-right angles is a rhombus.

Step 2: The lines of symmetry for a rhombus are its diagonals.

Step 3: A rhombus has two diagonals.

Answer: The shape is a rhombus, and it has 2 lines of symmetry.

Example 10 : Draw a hexagon with exactly two lines of symmetry.

Solution: Step 1: A regular hexagon has 6 lines of symmetry. We need an irregular one.

Step 2: Think of a shape like a rectangle but with its corners clipped in a symmetrical way. Or, draw a long rectangle. Then draw a smaller rectangle centered on top of it. This shape has a vertical and a horizontal line of symmetry.

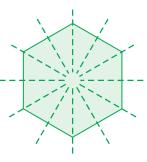


Fig. 9.39

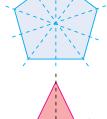


Fig. 9.40



Fig. 9.41

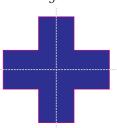


Fig. 9.42



Knowledge Checkpoint

- 1. The center of rotation is the point around which a shape is turned. (True or False)
- 2. A regular octagon (8 sides) has an order of rotational symmetry of 8. (True or False)
- 3. Draw a regular pentagon and show all its lines of symmetry using dotted lines.
- 4. Draw a shape that has rotational symmetry but no lines of symmetry. (**Hint:** Think about one of the quadrilaterals from our activity!)



Do It Yourself

Real-World Design: Why are most manhole covers on the street circular? (**Hint:** It has to do with symmetry! A circular lid can't fall into the hole, no matter how you rotate it. A square lid, if turned diagonally, could.)



The Real-Life Symmetry Hunt

Objective: To identify real-life objects that have 1, 2, 3, or 4 lines of symmetry.

Materials:

- A notebook or worksheet
- A pencil
- (Optional) A camera/tablet to take pictures of objects found

Procedure:

Part A: Finding Lines of Symmetry

- 1. Look around your classroom, school, or home.
- 2. Try to spot objects that have 1, 2, 3, or 4 lines of symmetry.

Example:

- a. $1 \text{ line} \rightarrow \text{a leaf}$, a heart shape
- b. $2 \text{ lines} \rightarrow \text{a rectangle (non-square book cover)}$
- c. $3 \text{ lines} \rightarrow \text{an equilateral traffic sign (triangle)}$
- d. 4 lines \rightarrow a square tile, window pane
- 3. Use the **"folding in your mind" test** imagine folding the object along a line. If both sides match, that's a line of symmetry.
- 4. Record your findings in a table like this:

Number of Lines of Symmetry	Real-Life Object	Sketch / Picture
1	Leaf	
2	Notebook cover	
3	Traffic triangle	
4	Square tile	

Part B: Reflection

- 1. Which type of symmetry (1, 2, 3, or 4 lines) was easiest to find?
- 2. Did you notice that **regular shapes** (like square, equilateral triangle, circle) often appear in real-life objects with multiple lines of symmetry?

Key Terms

- **Polygon:** A closed 2D shape made of three or more straight line segments.
- **Regular Polygon:** A polygon where all sides are of equal length and all interior angles are of equal measure. (e.g., Square, Equilateral Triangle).
- Irregular Polygon: A polygon that does not have all sides and all angles equal. (e.g., Rectangle, Scalene Triangle).
- **Line of Symmetry:** An imaginary line that divides a figure into two identical, mirror-image halves.
- Order of Rotational Symmetry: The number of times a shape fits into its own outline during a complete 360° rotation.

Facts Flash

- **Symmetry in Nature:** Nature loves symmetry! The beautiful six-pointed shape of a snowflake has hexagonal symmetry. A starfish has 5-fold rotational symmetry, just like a regular pentagon!
- **Symmetry in Alphabets:** Look at the capital letters of the English alphabet. Which ones have a horizontal line of symmetry (like **B**, **C**, **D**, **E**)? Which have a vertical line of symmetry (like **A**, **M**, **T**, **U**)? Which have both (like **H**, **I**, **O**, **X**)?

× ×

Mental Mathematics

- 1. Give 2 real life examples with only 1 line of symmetry?
- 2. I am a triangle with no equal sides. How many lines of symmetry do I have?
- 3. I am a regular polygon with 10 sides. What is my order of rotational symmetry?
- 4. A rectangle has two lines of symmetry. What is its order of rotational symmetry?
- 5. How many lines of symmetry does a parallelogram have?



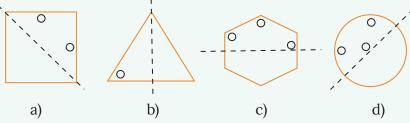


Gap Analyzer™ Homework

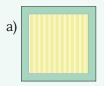
Watch Remedia



1. Given the line (s) of symmetry, find the other hole (s):



- 2. Answer the following questions:
 - a) Does a parallelogram have a line of symmetry?
 - b) How many lines of symmetry does a circle have?
 - c) Which shape has two lines of symmetry: a rectangle or a rhombus?
 - d) Does the flag of Switzerland have rotational symmetry? If so, what is the order?
 - e) Why does the flag of the United States have no lines of symmetry?
- 3. Find the number of line(s) of symmetry for each of the following shapes:

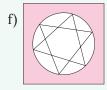


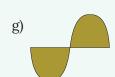




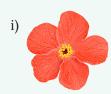




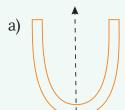


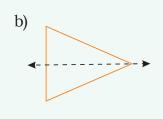


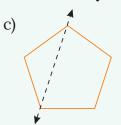


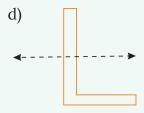


4. Find in which of the following, the dotted line is a line of symmetry.



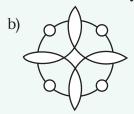




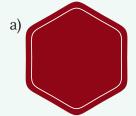


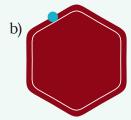
5. Find the difference between the lines of symmetries of the given images.





6. Find the difference between the lines of symmetries of the given images.





Symmetry in Real-World Designs

Symmetry isn't just for math class polygons; it's a design principle used everywhere! In this concept, we'll become symmetry detectives, uncovering the hidden lines of symmetry in the English alphabet, numbers, and even the flags of different countries.

Sub-concepts to be covered

1. Symmetry in the English Alphabet

- ♦ Vertical line
- ♦ Horizontal line
- Both: (Vertical and Horizontal line)

2. Symmetry in Numbers

- ♦ Vertical line
- ♦ Horizontal line
- ♦ Both: (Vertical and Horizontal line)

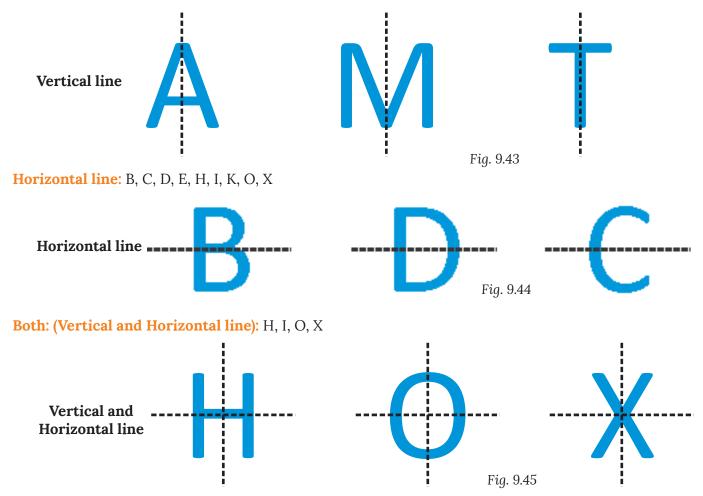
3. Symmetry in National Flags

Mathematical Explanation

Symmetry in the English Alphabet

Analyzing which capital letters have vertical, horizontal, both, or no lines of symmetry.

Vertical line: A, H, I, M, O, T, U, V, W, X, Y



None Symmetry in the English Alphabet: F, G, J, L, N, P, Q, R, S, Z

Symmetry in Numbers

Investigating the digits 0-9 for lines of symmetry.

• Vertical line: 0, 8

• Horizontal line: 0, 3, 8

• Both: (Vertical and Horizontal line): 0, 8



Symmetry in National Flags

Observing how countries use symmetry in their flags. This includes vertical symmetry (e.g., Canada, France), horizontal symmetry (e.g., Germany, India), and multiple lines of symmetry (e.g., Switzerland, UK's Union Jack).

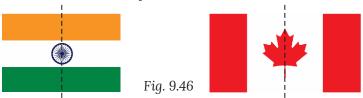
Lines of Symmetry in National Flags

National flags often feature symmetry in their designs, reflecting balance, unity, and aesthetic appeal. The number and type of lines of symmetry depend on the layout of the flag.

Examples of National Flags with Lines of Symmetry

1. Flags with Vertical Line of Symmetry

- **India:** The flag of India has a vertical line of symmetry along its center.
- **Canada:** The flag of Canada, with its maple leaf and red bars, has one vertical line of symmetry.



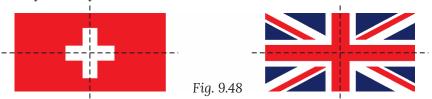
2. Flags with Horizontal Line of Symmetry

- **Ukraine:** The blue and yellow horizontal stripes in Ukraine's flag create a horizontal line of symmetry.
- **Argentina:** The flag of Argentina has a horizontal line of symmetry, dividing the white central stripe and sun from the blue stripes above and below.



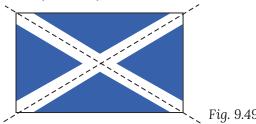
3. Flags with Both Vertical and Horizontal Lines of Symmetry

- **Switzerland:** The square flag of Switzerland, featuring a white cross on a red background, has both vertical and horizontal lines of symmetry.
- **England:** The flag of England (red cross on a white background) also has both vertical and horizontal lines of symmetry.



4. Flags with Diagonal Lines of Symmetry

• **Scotland:** The flag of Scotland, with a white diagonal cross (X-shaped saltire) on a blue background, has two diagonal lines of symmetry.



5. Flags with No Lines of Symmetry

- Some flags do not exhibit any symmetry due to their complex or asymmetrical designs.
 - **Nepal:** The flag of Nepal, with its unique two-triangle design, has no lines of symmetry.
 - **United States:** The flag of the United States, with 50 stars and 13 stripes, does not have a line of symmetry due to the arrangement of the stars and stripes.





Fig. 9.50

Example 11: Write down three capital letters that have exactly one horizontal line of symmetry.

Solution:

Step 1: Go through the alphabet mentally or by writing it down.

Step 2: Test each letter for a horizontal line of symmetry.

'B': Yes. 'C': Yes. 'D': Yes. 'E': Yes. 'K': Yes.

Step 3: Check if they have any other lines of symmetry. 'B', 'C', 'D', 'E', 'K' do not have vertical symmetry.

Answer: B, C, D (other answers are possible).

Example 12: The flag of Jamaica has a diagonal cross. How many lines of symmetry does it have?

Solution:

Step 1: The flag has a saltire (a diagonal cross) dividing it into four triangles.

Step 2: The two diagonals of the flag are lines of symmetry. If you fold along either diagonal, the colours and shapes match up.

Step 3: A horizontal or vertical fold would not work.

Answer: The flag of Jamaica has 2 diagonal lines of symmetry.

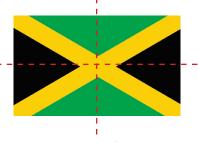


Fig. 9.51

V

Knowledge Checkpoint

- 1. Write the word **"BOX"**. Which letters have vertical symmetry? Which have horizontal symmetry? Which have both?
- 2. Look at the number 8. Describe its symmetry (line and rotational).
- 3. List three capital letters that have **no symmetry** at all.
- 4. List two numbers that have **rotational symmetry**.
- 5. Your friend wants to create a simple, symmetrical logo for their club, "**WOW**". Is this a good name for a symmetrical logo? Why or why not? Draw the logo to show its symmetry.

Key Terms

- Vertical Symmetry: The property of an object being symmetrical across a vertical line.
- Horizontal Symmetry: The property of an object being symmetrical across a horizontal line.
- **Line of Symmetry:** The imaginary "**mirror line**" that divides a shape into two identical halves.
- **Rotational Symmetry:** The property where a shape looks the same after being rotated by less than 360°.
- **Asymmetry:** The absence of symmetry. Many objects in the real world are asymmetrical!



The Real-World Symmetry Hunt

Objective: To identify and categorize line symmetry (vertical, horizontal) and rotational symmetry in letters, numbers, and national flags.

Materials:

- A list of English capital letters (A-Z)
- A list of numbers (0-9)
- Images of a few national flags (e.g., Canada, Switzerland, United Kingdom, Japan)
- A small, rectangular mirror (optional, but very helpful for checking line symmetry)
- Pencil and notebook/worksheet

Procedure:

Part A: The Alphabet Symmetry Challenge

- 1. Write down the capital letters of the English alphabet.
- 2. For each letter, determine the type of line symmetry it has.
- ◆ Vertical Line of Symmetry: Can you draw a vertical line down the middle and have the two halves be mirror images? (e.g., A, M, T)
- ♦ Horizontal Line of Symmetry: Can you draw a horizontal line across the middle? (e.g., B, C, E)
- ◆ **Both**: Some letters have both! (e.g., **H**, **I**, **X**)
- 3. Record your findings in a table like the one below.
- 4. Now, check for Rotational Symmetry. Which letters look the same if you rotate them 180° (half a turn)? (e.g., **H**, **I**, **N**, **S**, **X**, **Z**)

Part B: The Flag Explorer

- 1. Look at the image of the flag of **Canada CA**. It has a maple leaf in the center. Does it have a line of symmetry? If so, is it vertical or horizontal?
- 2. Now look at the flag of **Switzerland CH**. It's a white cross on a red square. How many lines of symmetry can you find? What is its order of rotational symmetry?
- 3. Examine the flag of the **United Kingdom GB**. This one is tricky! It has rotational symmetry (order 2), but surprisingly, it does not have any lines of symmetry. Can you see why? (The red diagonal lines are not perfectly mirrored).

Do It Yourself

Asymmetry in Logos: Some of the most famous logos in the world, like the Nike "swoosh" or the Apple logo, are asymmetrical. Why might a designer choose an asymmetrical design instead of a symmetrical one? (Hint: To create a sense of movement, energy, or to be more unique).

Facts Flash

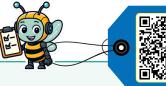
- **Symmetry in Architecture:** The world-famous **Taj Mahal** in Agra is a stunning example of bilateral (vertical) symmetry in architecture. Its perfect balance is a key reason for its beauty.
- **Symmetry in Culture:** Traditional Indian art forms like **Rangoli** and **Kolam** are beautiful examples of geometric and rotational symmetry. They are created to be visually appealing and harmonious.



Mental Mathematics

- What is the order of rotational symmetry for the letter 'S'?
- True or False: The number '7' has a line of symmetry.
- Name a letter that has both horizontal and vertical lines of symmetry.
- What type of symmetry does the letter 'D' have?
- The word "TOOT" is written on a piece of glass. If you look at it from the other side, will it still read "TOOT"? Why?

Exercise 9.3





Gap Analyzer™ Homework

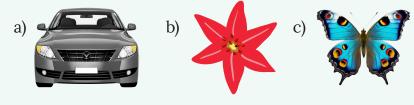
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Switzerland

- 1. Fill in the Blanks (Recall & Understanding) Complete the following sentences:
 - a) The digits ___ and ___ are the only digits (from 0-9) that have both horizontal and vertical lines of symmetry.
 - b) The digit 3 has a _____ one.
 - c) The capital letters H, I, and X are unique because they have both a _____ and a _____ line of symmetry.
 - d) If you want to create a symmetrical password using only numbers, the digit 8 is a good choice because it has _____ lines of symmetry, while the digit 3 has only one _____ line of symmetry.
 - e) A line of symmetry divides a figure into two identical parts that are perfect ______ of each other.
- 2. Using only capital letters that have a vertical line of symmetry, create a 3-letter, 4-letter, or 5-letter meaningful English word. The word itself should look symmetrical. Example: MAT. Can you find another one? Canada
- 3. Look at the national flags of Canada and Switzerland. Which flag has more lines of symmetry?
- 4. You are designing a security keypad for a high-security vault. Only the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 can be chosen. For extra safety, the keypad only uses digits that follow
 - the below mentioned rules:
 - a) A number with 2 lines of symmetry, but not 0.
 - b) Greatest multiple of 2 with no line of symmetry.
 - c) A number which is neither prime nor composite.
 - d) A number with exactly 1 line of symmetry.
- Design a simple, new flag for your school's Sports Day. The flag must be rectangular and must have exactly two lines of symmetry. Draw the flag, its central logo or pattern, and use dotted lines to show the two lines of symmetry.
- 6. Write 5 such English letters which have horizontal line of symmetry.
- 7. Write 4 English letters which have vertical lines of symmetry

8. Draw all possible lines of symmetry in the figures given below:









Common Misconceptions

Misconception: A rectangle has 4 lines of symmetry, just like a square.

Correction: A rectangle only has 2 lines of symmetry (one horizontal, one vertical). Its diagonals are NOT lines of symmetry. You can test this by folding a rectangular paper along its diagonal – the corners won't match up! A square has 4 because all its sides are equal.

Misconception: If a shape has rotational symmetry, it must also have a line of symmetry.

Correction: This is not true. Think of the letter 'Z' or 'S'. You can rotate them 180° (a half-turn) and they look the same, so they have rotational symmetry of order 2. However, they have no lines of symmetry at all.

Misconception: All triangles have at least one line of symmetry.

Correction: Only some triangles do. An equilateral triangle has 3 lines of symmetry and an isosceles triangle has 1. A scalene triangle, where all three sides have different lengths, has no lines of symmetry.

Rea

Real-Life Symmetry: Mathematical Applications

Symmetry is nature's and humanity's favorite design tool. It creates balance, beauty, and stability. Here's where we see it:

- Nature's Design: From the perfect mirror-image wings of a butterfly to the petals of a flower and
 the design of a snowflake, nature uses symmetry everywhere. Even our own bodies have a line of
 symmetry right down the middle!
- **Architecture and Art:** Famous buildings like the Taj Mahal are celebrated for their perfect symmetry. Artists use it to create pleasing compositions, and traditional patterns like Rangoli are beautiful examples of intricate symmetrical designs.
- **Engineering for Stability:** Engineers design cars and airplanes to be symmetrical. This isn't just for looks; it provides balance, ensuring a car drives straight and an airplane flies steadily.
- **Everyday Objects:** Look around! A pair of glasses, a dining plate, a T-shirt, and even letters of the alphabet like A, H, M, and O are all examples of symmetry we use daily.











Ga	p Analyzer™
Tal	ke an Exam

1.	Which of these letters has both horizontal and vertical lines of symmetry?	
	a) A	
2.	What is the order of rotational symmetry for a standard parallelogram?	
	a) 0	
3.	A regular nonagon (9 sides) has:	
	a) 9 lines of symmetry and order of rotation 9.	
	b) 2 lines of symmetry and order of rotation 2.	
	c) 0 lines of symmetry and order of rotation 9.	
	d) 9 lines of symmetry and order of rotation 1.	
4 .	Which of the following has translational symmetry?	
	a) A human face	
	c) A brick wall d) A single triangle	
5 .	The angle of rotation for a shape with rotational symmetry of order 5 is:	
	a) 90°	

Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.

Study both the statements and state which of the following is correct:

- a) Both A and R are true and R is the correct explanation of A.
- b) Both A and R are true but R is not the correct explanation of A.
- c) A is true but R is false.
- d) A is false but R is true.
- **1. Assertion** (A): A rectangle has two lines of symmetry.

Reason (R): The diagonals of a rectangle are its lines of symmetry.

2. Assertion (A): An equilateral triangle has rotational symmetry of order 3.

Reason (R): A regular polygon with 'n' sides has rotational symmetry of order 'n'.

3. Assertion (A): The letter 'S' has reflective symmetry.

Reason (R): The letter 'S' has rotational symmetry of order 2.

Case Study

The Symmetrical School Garden:

Mr. Sharma, the school principal, is planning to redesign the school garden. He wants it to be beautiful and symmetrical to make it more appealing. He has asked the Class 6 students to help him by identifying symmetrical patterns. Mr. Sharma also wants a path in the garden that is symmetrical, and he plans to plant flowers in circular flower beds at the center of the garden. He has drawn two possible layouts for the path using square tiles.



- **Layout A:** A path that forms a perfect rectangle in the center of the garden.
- **Layout B:** A path shaped like a star with five equal arms.
- 1. Which layout (A or B) will Mr. Sharma need to choose to ensure the path has both horizontal and vertical lines of symmetry?
- 2. If Mr. Sharma chooses Layout A, how many lines of symmetry will the path have? Draw the lines of symmetry on the garden layout.
- 3. How many lines of symmetry will this circular flower bed have?

Project

The Symmetry Scrapbook

Task: Create a physical or digital scrapbook titled "Symmetry Around Us." Your mission is to find and document at least 10 different examples of symmetry from your home, school, and community.

For each example, you must:

- 1. Include a picture (a photo you take, a drawing, or an image from the internet).
- 2. Identify the type of symmetry present (Reflective, Rotational, and Translational).
- 3. If it has reflective symmetry, draw the line(s) of symmetry on the picture.
- 4. If it has rotational symmetry, state its order of rotation.
- 5. Write one sentence explaining where you found it and why you think it was designed to be symmetrical.

Examples could include:

A company logo, a flower, a floor tile pattern, a building facade, a plate, a piece of fruit, etc. This project encourages you to apply your classroom knowledge to observe and analyze the mathematical principles in your everyday environment.

Source-Based Question

The architecture of a modern city is very diverse, has multifaceted and unique forms that are characteristic of this particular urban environment [1-4]. It should be noted that the architecture of many cities in the world is subject to the laws of harmony and disharmony. The architecture of the city, like music, is a combination of harmony and elements of disharmony. But over time, these elements can already be considered as the development of harmony, but already on a new environment perception. So, the contemporaries of Beethoven did not always understand his music, they believed that it was devoid of harmony. But time has shown exactly how genius his music is and completely obeys the laws of harmony. It is necessary to understand these patterns of our world development, including architecture development (Fig.1, 2).



Fig. 1 Beauty and harmony of the lotus flower.



Fig. 2 Beauty and harmony of the Lotus Temple .

Harmony in architecture is not only the well-established traditions of building structures, but also a look into the future with a change in the rules in accordance with the inner state of a person and his environment [5-7]. Digital technologies and innovations help to create the buildings and complexes of structures based on an understanding of the harmony and disharmony of our world. It is the three-dimensional modeling methods that make it possible to create digital 3D models for the designed object analysis.

Harmony in architecture - symmetry and asymmetry

Symmetry is proportionality and balance in the arrangement of parts of the whole in space, while asymmetry is its opposite. In architecture, symmetry creates completeness and rhythm, making structures appear harmonious and stable.



Fig. 3 Starfish with 5th order ratational symmetry

A **starfish** is an example of a living organism with 5th-order rotational symmetry (Fig. 3). Symmetry is perceived by humans as a manifestation of natural laws, and this perception is reflected in architectural forms.

Objects with a symmetrical arrangement are generally more stable, but the **unity of symmetrical and asymmetrical elements** in buildings can create truly remarkable designs (Fig. 4).



Fig. 4 Residential complex "Via 57 West" by architect B. Ingels in New York.

The architecture of the **Taj Mahal** is based on absolute symmetry, where each element harmoniously fits into the structure of the temple complex (Fig. 5). At the same time, the combination of symmetry and asymmetry gives uniqueness to certain monuments, such as **St. Basil's Cathedral in Moscow** (Fig. 6).



Fig. 5 Temple complex Taj Mahal, India



Fig. 6 Cathedral of St. Basil the B lessed in Moscow

Thus, both symmetry and asymmetry contribute to architectural harmony, producing buildings that are beautiful, balanced, and unique.

Source Text: Adapted from research on architectural harmony and symmetry (International Journal of Architecture and Urban Studies).

Questions on the data:

- 1. What type of symmetry does a starfish have?
- 2. Which famous monument is an example of absolute symmetry?
- 3. Name a monument that shows both symmetry and asymmetry in its design.
- 4. How does symmetry make a building appear?



