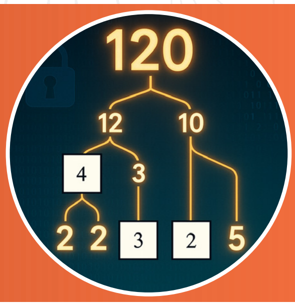
5 Prime Time

Why This Chapter Matters

Have you ever wondered what all the numbers in the world are made of? Just like a house is built from individual bricks, every whole number can be built by multiplying a special set of numbers called prime numbers. These numbers are the secret "atoms" of mathematics. What if you had a secret code that only you and your friend knew? Prime numbers are the key to creating unbreakable codes that protect information online!



Meet EeeBee.AI

I'm a friendly robot who loves finding patterns and solving puzzles. Numbers are like a giant playground to me! Throughout this chapter, I'll be your guide. I'll pop up with helpful hints, ask tricky questions to make you think, and share some amazing facts about the world of prime numbers. Whenever you see me, get ready for a fun challenge or a cool new idea.

Learning Outcomes

By the end of this chapter, you will be able to:

- Define and differentiate between prime and composite numbers.
- Apply divisibility rules to quickly test if a number is a factor of another.
- Break down any composite number into its unique prime factors using prime factorisation.
- Calculate the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of a set of numbers.
- Analyze the relationship between HCF and LCM to solve problems efficiently.
- Solve real-world problems involving factors, multiples, and prime numbers.

From Last Year's Notebook

- Factors divide a number completely. (e.g., The factors of 10 are 1, 2, 5, 10).
- Multiples are the results of multiplying a number. (e.g., Some multiples of 10 are 10, 20, 30).
- We will use this knowledge of factors to explore prime numbers—special numbers that have exactly two factors.
- This will help us understand the basic building blocks of all numbers!

Real Math, Real Life

Prime numbers have amazing jobs outside of your textbook!

- **Keeping Secrets Safe (Cryptography):** Large prime numbers are used to create secret codes that encrypt (lock) your data. This is what keeps your online messages and payments secure.
- **Clever Tricks in Nature (Biology):** Some animals use primes to survive! Cicada insects emerge from the ground every 13 or 17 years—both prime numbers! This clever timing helps them avoid predators.

Quick Prep

1. List all the factors of 24.
2. What are the first five multiples of 7?
3. Which of these numbers can be divided evenly by 2? (15, 28, 30, 41)
4. Which of these numbers can be divided evenly by 5? (52, 75, 83, 90)
5. I am a number between 10 and 15. I am a multiple of 3. What number am I?
6. Find a number that is a factor of 20 and also a factor of 30.
7. What is 6 multiplied by 8?

Introduction

Welcome to the fundamental classification of numbers! In this section, we will divide the world of whole numbers into two main groups: prime and composite. Understanding this difference is the first step to unlocking the structure of any number. We will also revisit factors and multiples, but this time, we'll focus on what's common between different numbers. This will lay the groundwork for solving problems like scheduling two events to happen at the same time or splitting items into identical groups.

Chapter Overview

Get ready to become a number expert! Here is our journey:

Start with the Basics: We'll define Prime and Composite numbers and learn a fun method to find them (the Sieve of Eratosthenes).

Master the Shortcuts: You'll learn the Divisibility Rules to quickly test if a number can be divided by another.

Break Numbers Down: We'll explore Prime Factorisation to find the unique "number DNA" of any number.

Find Common Ground: Using these skills, we will find the Highest Common Factor (HCF) and Lowest Common Multiple (LCM).

See Primes in Action: Finally, we'll discover how these ideas are used in online security and nature!

From History

The concept of division is ancient, used by early societies in **Egypt** and **Mesopotamia** for practical tasks like trade and distributing resources. While their methods were complex, the long division we learn today was standardized over centuries, making the calculation much simpler. The familiar division symbol, '÷' (called an obelus), is relatively modern, first introduced by mathematician Johann Rahn in 1659. This evolution turned a once-difficult task into a fundamental skill accessible to everyone.

Prime Factors

Hello, young mathematicians! Imagine you have a box of LEGO bricks. You can combine small, basic bricks in different ways to build anything you want—a car, a house, or a spaceship! But you can't break the basic bricks themselves into smaller pieces.

In the world of numbers, prime numbers are like those basic LEGO bricks. And prime factors are the specific set of prime number bricks you need to build any other number. Today, we're going to learn how to find the unique set of prime "bricks" that make up every number!

Sub-concepts to be covered

1. Prime Number
2. Common Factors and Common Multiples

Mathematical Explanation

Prime Number

A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself. In other words, prime numbers cannot be divided evenly by any number other than 1 and the number itself.

For example:

- 2 is a prime number because it can only be divided by 1 and 2.
- 3 is also a prime number because it can only be divided by 1 and 3.
- However, 4 is not a prime number because it can be divided by 1, 2, and 4.

Prime Factors

Prime factors are the prime numbers that multiply together to give the original number. To find the prime factors of a number, we break it down into smaller prime numbers.

Steps to Find Prime Factors

Step 1. Start with the number you want to factorize.

Step 2. Divide the number by the smallest prime number (usually 2) and check if the result is a whole number.

Step 3. If it is, continue dividing by the same prime number until it is no longer divisible.

Step 4. Then, move on to the next prime number (3, 5, 7, etc.) and repeat the process.

Step 5. Continue until you can no longer divide by any prime number.

Example: Prime Factors of 18

Step 1. Start with 18.

Step 2. Divide by 2 (the smallest prime number):

$18 \div 2 = 9$

So, $18 = 2 \times 9$.

Step 3. Now divide 9 by 3 (next prime number):

$9 \div 3 = 3$

So, $9 = 3 \times 3$.

Step 4. Now, 3 is a prime number, so the prime factors of 18 are:

$2 \times 3 \times 3$ or 2×3^2.

Example: Prime Factors of 30

Step 1. Start with 30.

Step 2. Divide by 2:

$30 \div 2 = 15$

So, $30 = 2 \times 15$.

Step 3. Now divide 15 by 3 (the next smallest prime):

$15 \div 3 = 5$

So, $15 = 3 \times 5$.

Step 4. 5 is a prime number, so the prime factors of 30 are: $2 \times 3 \times 5$.

Common Factors and Common Multiples

In mathematics, understanding common factors and common multiples is important for solving problems involving numbers. These concepts help us work with numbers that are shared by two or more numbers. Let's take a closer look at what they mean!

Common Factors

Common factors are numbers that are factors of two or more numbers. In other words, common factors are numbers that divide two or more numbers exactly (without leaving a remainder).

Example: Common Factors of 12 and 18

- Factors of 12: 1, 2, 3, 4, 6, 12
- Factors of 18: 1, 2, 3, 6, 9, 18

The common factors of 12 and 18 are: 1, 2, 3, 6.

Example: Common Factors of 15 and 20

- Factors of 15: 1, 3, 5, 15
- Factors of 20: 1, 2, 4, 5, 10, 20

The common factors of 15 and 20 are: 1, 5.

Common Multiples

Common multiples are numbers that are multiples of two or more numbers. In other words, common multiples are numbers that can be divided by both numbers evenly.

Example: Common Multiples of 4 and 6

- Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, ...
- Multiples of 6: 6, 12, 18, 24, 30, ...

The common multiples of 4 and 6 are: 12, 24, ...

Highest Common Factor (HCF)

When we compare two or more numbers, we often look for factors that they share. The Highest Common Factor (HCF) is the largest number that divides each of them exactly. It is also called the Greatest Common Divisor (GCD). **For example**, the HCF of 12 and 18 is 6.

Least Common Multiple (LCM)

The least common multiple is the smallest multiple that both numbers share. **For example**, the LCM of 4 and 6 is 12 (the smallest common multiple).

Jump Treasure

A number line up to 30 was shown to students in a class. They were asked to make jumps of equal size starting from 0 and land exactly on 30. The jump sizes chosen by some students are:

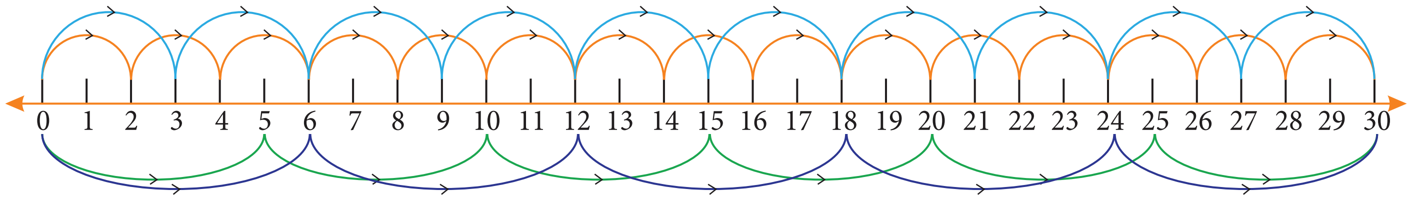


Fig. 5.1

- **Raj:** "I will take jumps of 2."
- **Riya:** "I will take jumps of 3."
- **Aman:** "Let me use jumps of 5."
- **Sonal:** "Oh! I can use jumps of 6."

Solution:

Let's analyze the jumps made by the students:

Raj: Jumps of size 2 (2 → 4 → 6 → 8 → 10 → 12 → 14 → 16 → 18 → 20 → 22 → 24 → 26 → 28 → 30).

Riya: Jumps of size 3 (3 → 6 → 9 → 12 → 15 → 18 → 21 → 24 → 27 → 30).

Aman: Jumps of size 5 (5 → 10 → 15 → 20 → 25 → 30).

Sonal: Jumps of size 6 (6 → 12 → 18 → 24 → 30).

Thus, the factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30.

Example 1 : Determine the common factors of:

(i) 16 and 40 (ii) 24, 36, and 48.

Solution: (i) 16 and 40

Factors: Factors of 16: 1, 2, 4, 8, 16

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

Common factors: 1, 2, 4, 8

(ii) 24, 36, and 48

Factors: Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Common factors: 1, 2, 3, 4, 6, 12

Example 2 : Write the first 5 multiples of: 12 and 14

Solution: Multiples of 12: 12 × 1 = 12 12 × 2 = 24 12 × 3 = 36 12 × 4 = 48 12 × 5 = 60

Multiples of 12: 12, 24, 36, 48, 60

Multiples of 14: 14 × 1 = 14 14 × 2 = 28 14 × 3 = 42 14 × 4 = 56 14 × 5 = 70

Multiples of 14: 14, 28, 42, 56, 70

Example 3 : Find the Common Factors of 24 and 36 and Determine the Greatest Common Factor (GCF).

Solution: Step 1: List the factors of 24 and 36.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Step 2: Identify the common factors.

The common factors of 24 and 36 are the numbers that appear in both lists: 1, 2, 3, 4, 6, 12

Step 3: Find the Greatest Common Factor (GCF).

The GCF is the largest number that is a common factor.

In this case, the largest common factor is 12.

Therefore, the GCF of 24 and 36 is 12.

Example 4 : A traffic light at one crossing changes every 20 seconds, while at another crossing it changes every 30 seconds.

After how many seconds will both lights change together again?

Solution: Multiples of 20 = 20, 40, 60, 80, 100, 120 ...

Multiples of 30 = 30, 60, 90, 120 ...

Common multiples = 60, 120 ...

LCM = 60

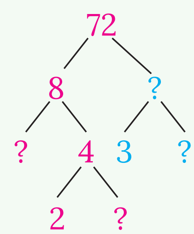
Both lights will change together after 60 seconds.

Knowledge Checkpoint

- Find the prime factorization of the following numbers using the Division Method:

  a) 48 b) 81 c) 100
- Create a Factor Tree for the following numbers:

  a) 60 b) 98 c) 54
- Fill in the missing numbers in this factor tree for 72:



- **A number's secret code is its prime factorization:** 2 × 3 × 3 × 5. What is the number?

Activity

The Prime Brick Challenge

- **Materials:** Different colored blocks (like LEGOs or wooden cubes) and sticky notes.
- **Task:**

  Assign a prime number to each color (e.g., Red = 2, Blue = 3, Yellow = 5, Green = 7). Give students a composite number (e.g., 30). Their challenge is to build the number using the correct "prime bricks."

Key Terms

- **Prime Factor:** A factor that is also a prime number.
- **Prime Factorization:** Expressing a composite number as the product of its prime factors.
- **Factor Tree:** A diagram used to break down a number into its prime factors.
- **Composite Number:** A number with more than two factors.

Do It Yourself

- Can a prime factor ever be a composite number? Why or why not?
- Your friend says the prime factors of 24 are 4 and 6. Are they correct? Explain the mistake.
- How can you use the prime factors of a number to quickly tell if it is a perfect square (like 36 or 49)? (**Hint:** Look at the prime factors of 36 (2,2,3,3) and 49 (7,7). Do you see a pattern?)

Facts Flash

- **A Unique Fingerprint:** Every composite number has exactly one set of prime factors! This is called the **Fundamental Theorem of Arithmetic**. It means the prime factorization of 12 ($2 \times 2 \times 3$) is as unique to it as a fingerprint is to a person.
- **Secret Codes:** Prime factorization is a super important concept in **computer science** and **cryptography**. The security of online banking and shopping depends on the fact that it is very, very difficult to find the prime factors of extremely large numbers!
- **The Smallest Brick:** The number 2 is the only even prime number, which makes it a prime factor for all even numbers.

Mental Mathematics

Quickly answer these in your head!

- What are the prime factors of 10?
- Is 3 a prime factor of 20?
- A number is made of the prime factors 2, 2, and 5. What is the number?
- What is the smallest prime factor of 45?
- What is the largest prime factor of 21?

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Exercise 5.1

1. Find the common factors of:

(a) 12 and 20 (b) 16 and 24 (c) 9 and 27 (d) 6 and 15
(e) 45, 60, and 90 (f) 16, 20, and 24 (g) 40, 60, and 80 (h) 18, 27, and 36

2. Write the first five multiples of:

(a) 9 (b) 6 (c) 13 (d) 8
(e) 7 (f) 12 (g) 16 (h) 20

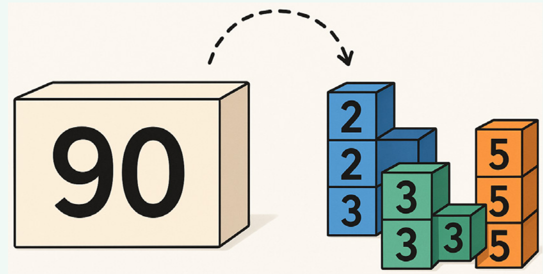
3. Write all the factors of the following numbers:

(a) 36 (b) 18 (c) 50 (d) 48
(e) 28 (f) 13 (g) 100 (h) 7

4. Find the first three common multiples of:

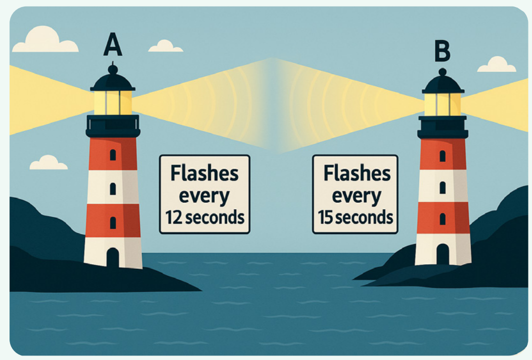
(a) 3 and 5 (b) 4 and 10 (c) 6 and 9 (d) 12 and 15
(e) 4, 5, and 8 (f) 2, 3, and 7 (g) 6, 9, and 12 (h) 5, 7, and 10

5. You need to build a tower that represents the number 90 using only the prime bricks. The final tower will be a product of its prime factors.



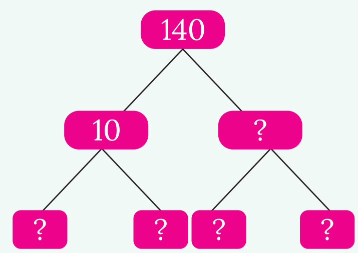
   a) Using the division method or a factor tree, find the prime factorization of 90.
   b) How many "2" bricks, "3" bricks, and "5" bricks will you need to build the tower for 90?
   c) If you were given a composite "brick" labeled "6", could you use it in a prime factorization? Why or why not?
   d) What is the smallest number you can build that uses at least one of each of the available prime bricks (2, 3, and 5)?

6. A coastline with two lighthouses. Lighthouse A: "Flashes every 12 seconds". Lighthouse B: "Flashes every 15 seconds". At exactly 9:00:00 PM, both lighthouses flash at the same time.



   a) Write the prime factors for the time interval of Lighthouse A (12) and Lighthouse B (15).
   b) To find out when they will next flash together, what mathematical concept must you use?
   c) After how many seconds will both lighthouses flash together again?
   d) How many times will Lighthouse A have flashed by the time they both flash together again for the first time?

7. Complete the factor tree by filling in the missing numbers



   a) Look at the first split from 140 into 10 and another number. What is the missing number in the box?
   b) Complete the entire factor tree by filling in all the missing numbers in the boxes and circles.
   c) Write the final prime factorization of 140 using the prime numbers at the bottom of your completed tree.

   (d) Your friend says the prime factors of 140 are 2, 7, and 10. Explain why your friend is incorrect.

Prime and Composite Numbers

Every whole number greater than 1 falls into one of two categories. If a number has exactly two factors (1 and itself), it's a special number called a **prime number**. Think of it as a basic element that can't be broken down further. If a number has more than two factors, it is called a **composite number**. It's "**composed**" of other factors. In this part, we will explore these two types and learn about their unique relatives: Twin Primes and Co-primes.

Sub-concepts to be covered

1. Prime Numbers
2. Composite Numbers
3. Twin Primes
4. Prime Triplets
5. Co-primes (or Relatively Prime Numbers)
6. Sieve of Eratosthenes

Mathematical Explanation

Prime Numbers

- A prime number is a number that has exactly two distinct positive divisors: 1 and itself.
- In other words, a prime number can only be divided by 1 and the number itself without leaving a remainder.

Example: 2 is prime because it can only be divided by 1 and 2.

3 is prime because it can only be divided by 1 and 3.

5 is prime because it can only be divided by 1 and 5.

The smallest prime number is 2, and it is the only even prime number. All other prime numbers are odd.

Composite Numbers

- A composite number is a number that has more than two distinct divisors.
- In other words, a composite number can be divided by 1, itself, and at least one other number.

Example: 4 is composite because it can be divided by 1, 2, and 4.

6 is composite because it can be divided by 1, 2, 3, and 6.

9 is composite because it can be divided by 1, 3, and 9.

Key Differences

- **Prime Numbers have only 2 divisors:** 1 and the number itself.
- Composite Numbers have more than 2 divisors.

For example

2, 3, 5, 7, 11, 13, etc., are prime.

4, 6, 8, 9, 10, 12, etc., are composite.

Twin Primes

- Twin primes are pairs of prime numbers that have a difference of 2.
- In other words, if you subtract the smaller prime number from the larger one in the pair, the result is 2.

Example: (3, 5): 5 - 3 = 2

(5, 7): 7 - 5 = 2

(11, 13): 13 - 11 = 2

(17, 19): 19 - 17 = 2

Twin primes are rare, and as numbers get larger, they become less frequent.

Prime Triplets

A prime triplet consists of three prime numbers that are close together, usually differing by just 2 or 4 between them. **Like this:** (P, P + 2, P + 4) where all three numbers are prime.

Example: (3, 5, 7): These are three consecutive prime numbers.

(5, 7, 11): These are three prime numbers, with the difference between 5 and 7 being 2, and between 7 and 11 being 4.

Note that prime triplets are quite rare compared to twin primes.

Co-primes (or Relatively Prime Numbers)

- Co-primes (or relatively prime numbers) are two numbers that do not have any common divisor other than 1.
- In other words, the greatest common divisor (GCD) of two co-prime numbers is 1.
- Co-primes may or may not be prime numbers themselves, but they are always numbers whose only common factor is 1.

Example: **(8, 15):** GCD of 8 and 15 is 1, so they are co-primes.

(9, 28): GCD of 9 and 28 is 1, so they are co-primes.

(14, 25): GCD of 14 and 25 is 1, so they are co-primes.

Sieve of Eratosthenes

The Sieve of **Eratosthenes** is a simple and efficient way to find **all prime numbers** up to a given number **N**. It was invented by the ancient Greek mathematician **Eratosthenes**.

Steps to Find Prime Numbers Using the Sieve of Eratosthenes

1. Write down all numbers from 2 to N.
2. Start with the first number (2) and mark all its multiples (except itself).
3. Move to the next unmarked number (3) and mark all its multiples.
4. Repeat this process for the next number 5 and mark all its multiples..
5. Continue this process for the next unmarked number 7 and mark all its multiples
6. The unmarked numbers left are prime numbers.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	~~17~~	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Fig. 5.2

Example: Find Prime Numbers Up to 30

1. Write numbers from 2 to 30.
2. Start with 2 (smallest prime) and mark 4, 6, 8, 10, 12, 14, ..., 30.
3. Move to the next unmarked number 3 and mark 6, 9, 12, 15, 18, ..., 30.
4. Next, 5 and mark 10, 15, 20, 25, 30.
5. Next, 7 and mark 14, 21, 28.
6. The remaining unmarked numbers are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29 → These are prime numbers.

Rules to Check Whether a Number is Prime

Case 1: If a number lies "between" 1 to 100

- **Use the definition of a prime number:** A number is prime if it has exactly two factors: 1 and itself.
- Alternatively, use the Sieve of Eratosthenes to check if the number is encircled (Prime) or crossed out (Composite).

Case 2: If a number lies "between" 100 to 200

- Check if it is divisible by any prime number less than 15 (i.e., 2, 3, 5, 7, 11, 13).
- If it is divisible by any of these, it is not prime; otherwise, it may be prime.

Case 3: If a number lies "between" 200 to 400

- Check if it is divisible by any prime number less than 20 (i.e., 2, 3, 5, 7, 11, 13, 17, 19).
- If it is not divisible by any of these, then it is prime.

Example 5 : Which of the following numbers are prime?

(a) 29 (b) 45 (c) 37 (d) 91

Solution: (a) 29: Factors of 29 are only 1 and 29. Hence, 29 is a prime number.

(b) 45: 45 is divisible by 1, 3, 5, 9, 15, and 45. Hence, 45 is a composite number.

(c) 37: Factors of 37 are only 1 and 37. Hence, 37 is a prime number.

(d) 91: 91 is divisible by 7 (91 ÷ 7 = 13). Hence, 91 is a composite number.

Example 6 : Write three pairs of prime numbers less than 30 whose sum is divisible by 10.

Solution: The required pairs are:

1. 3, 7 : 3 + 7 = 10 (divisible by 10)
2. 11, 19 : 11 + 19 = 30 (divisible by 10)
3. 13, 17 : 13 + 17 = 30 (divisible by 10)

Example 7 : Find all prime numbers between 40 and 60.

Solution: The prime numbers between 40 and 60 are: 41, 43, 47, 53, 59

These numbers have no divisors other than 1 and themselves.

Example 8 : Which of the following numbers are co-prime pairs?

(a) 8,15 (b) 16,28 (c) 9,28 (d) 14,35

Solution: (a) 8 and 15: The GCD of 8 and 15 is 1. Hence, they are co-prime.

(b) 16 and 28: The GCD of 16 and 28 is 4. Hence, they are not co-prime.

(c) 9 and 28: The GCD of 9 and 28 is 1. Hence, they are co-prime.

(d) 14 and 35: The GCD of 14 and 35 is 7. Hence, they are not co-prime.

Knowledge Checkpoint

- Is 57 a prime or composite number? Explain your reasoning.
- What is the difference between twin primes and co-primes?
- Why is 2 the only even prime number?

Activity

The Sieve Challenge

- **Objective:** To find all prime numbers up to 100 using the Sieve of Eratosthenes.
- **Materials:** A 10 × 10 grid with numbers 1-100, different colored pencils.
- **Steps:**
  1. Cross out 1, as it is not prime.
  2. Circle 2 with a red pencil. Now, cross out every multiple of 2 (4, 6, 8...) in the grid.
  3. Take the next available number, 3. Circle it with a blue pencil. Cross out every multiple of 3 (6, 9, 12...). Some will already be crossed out!
  4. Take the next available number, 5. Circle it with a green pencil. Cross out all its multiples.
  5. Do the same for 7 with a yellow pencil.
  6. Circle all the remaining numbers in the grid.
- **Inquiry Question:** What do you notice about the numbers you had to check (2, 3, 5, 7)? Why didn't you need to check for multiples of 4, 6, 8, or 9?

Key Terms

- **Prime Number:** A natural number greater than 1 with exactly two distinct factors: 1 and itself.
- **Composite Number:** A natural number greater than 1 that has more than two factors.
- **Factors:** Numbers that divide another number without a remainder.
- **Twin Primes:** A pair of prime numbers that have a difference of 2.
- **Co-primes:** Two numbers whose highest common factor is 1.

Do It Yourself

- Why do you think prime numbers seem to get farther apart as you look at larger and larger numbers? Is there an end to them? (**Hint:** Remember Euclid's proof!)
- If you pick any prime number greater than 3, square it, and then subtract 1, the result is always divisible by 24. Try it with 5 or 7. Why do you think this happens?

Facts Flash

- The Goldbach Conjecture is an unsolved problem in mathematics that states every even integer greater than 2 is the sum of two primes. For example, 10 = 3 + 7 and 20 = 7 + 13. It has been tested for numbers up to 4 quintillion (a 4 with 18 zeros!) but has never been proven for all numbers.
- The largest known prime number (as of late 2023) has over 24 million digits! It would take a whole shelf of books just to write it down.

Mental Mathematics

- Which is the smallest prime number?
- Is 91 a prime or composite number?
- What is the next prime number after 29?
- Write the first three composite numbers.
- **Every even number greater than 2 is composite. True or False**

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Exercise 5.2

1. **Which of the following numbers are prime numbers?**

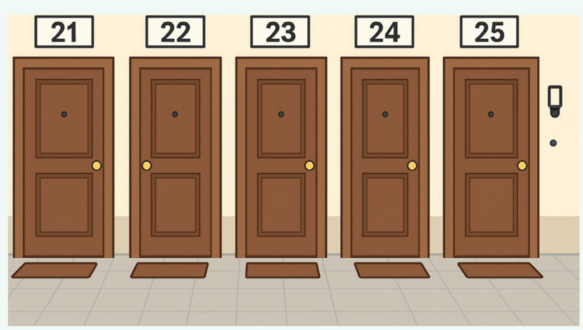
   (a) 17 (b) 21 (c) 31 (d) 29

   (e) 44 (f) 49 (g) 101 (h) 143

   (i) 113 (j) 67 (k) 121 (l) 83

2. Use the image to answer the following question

   a) A special prize is given to the resident of the apartment whose number is prime. Which apartment number shown in the image wins the prize?



   b) The building manager needs to do a maintenance check on all apartments with composite numbers. Which apartment numbers will be checked?

   c) The residents of apartments #22 and #25 are friends. Are their apartment numbers co-prime? Show your work.

   d) The pair of apartments (23, 25) are next to each other. Is this a twin prime pair? Explain why or why not.

3. **A local library has a "Reader's Spotlight" event on all prime-numbered dates of the month. Let's consider the month of April, which has 30 days.**
   a) List all the dates in April on which the "Reader's Spotlight" event will be held.
   b) The library wants to hold a special two-day author festival on a pair of "twin prime" dates. What are the possible pairs of dates in April they could choose?
   c) A visitor says they will come on the 27th of April. Is the number 27 prime or composite? Justify your answer by listing its factors.
   d) Is the total number of days in April (30) a prime or composite number? Explain why.
4. **The school's marching band is practicing for the Republic Day parade.**
   a) On Monday, there are 23 students present for practice. Can the band director arrange them in a rectangular formation that has more than one row and more than one student in each row? Explain your answer using the concept of factors.
   b) On Tuesday, one more student joins, making it 24 students. List all the possible rectangular formations the director can now create. Is 24 a prime or a composite number?
5. Rohan and his friends have started a "**Nature Explorers**" club. To enter their clubhouse (a tree-house!), members must give a password. The rule for the password is: "You must say a two-digit composite number where both digits are the same." What are the possible numbers that could be a valid password?
6. **A 10 × 5 grid showing numbers from 1 to 50. The number 1 is crossed out. The number 2 is circled, and all its multiples are crossed out in red. The number 3 is circled, and all its multiples are crossed out in blue.)**
   a) Looking at the grid, what is the very next number after 3 that is not yet crossed out and should be circled?
   b) After circling the number from part (a), list the first three numbers you would cross out with a new color (that haven't already been crossed out).
   c) Find the number 31 in the grid. Following the Sieve method, will it end up being circled or crossed out? What does this tell you about the number 31?
   d) Both 9 and 15 are composite numbers. Explain why 9 was crossed out when processing multiples of 3, but 15 was also crossed out.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Test for Divisibility of Numbers

Imagine you have a huge number like 7,452 and you need to know if it can be divided evenly by 3 or 4 without actually doing the long division. Is there a shortcut? Yes! In this section, we will learn a set of powerful "divisibility rules." These are simple tricks that let you test for divisibility by common numbers (like 2, 3, 4, 5, 6, 9, and 10) just by looking at the digits. Mastering these rules will make you faster at calculations and better at spotting factors.

Divisibility Rules from 2 to 11

Divisibility rules are efficient shortcuts to determine if a number can be divided by another without a remainder. Instead of performing long division, we can apply a simple test based on the digits of the number. This is like having a set of secret codes to quickly understand a number's properties. We will learn the rules for the numbers 2, 3, 4, 5, 6, 8, 9, 10, and 11, which are the most commonly used in calculations.

Sub-concepts to be covered

1. Divisibility by 2
2. Divisibility by 3
3. Divisibility by 4
4. Divisibility by 5
5. Divisibility by 6
6. Divisibility by 7
7. Divisibility by 8
8. Divisibility by 9
9. Divisibility by 10
10. Divisibility by 11

Mathematical Explanation

Table For The Tests of Divisibility of Numbers

Number	Divisibility Test	Example
2	A number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8.	348 → last digit 8 → divisible by 2
3	A number is divisible by 3 if the sum of its digits is divisible by 3.	258 → 2 + 5 + 8 = 15 → divisible by 3
4	A number is divisible by 4 if its last two digits form a number divisible by 4.	716 → last two digits 16 → divisible by 4
5	A number is divisible by 5 if its last digit is 0 or 5.	325 → last digit 5 → divisible by 5
6	A number is divisible by 6 if it is divisible by both 2 and 3.	312 → divisible by 2 (last digit 2) and 3 (sum 6) → divisible by 6
7	A quick way is to double the last digit, subtract it from the rest of the number, and see if the result is divisible by 7.	203 → Double the last digit (3 × 2 = 6). Subtract 6 from 20 → 20 - 6 = 14. Since 14 is divisible by 7, 203 is divisible by 7.
8	A number is divisible by 8 if its last three digits form a number divisible by 8.	5,416 → last three digits 416 → divisible by 8
9	A number is divisible by 9 if the sum of its digits is divisible by 9.	7,182 → 7 + 1 + 8 + 2 = 18 → divisible by 9
10	A number is divisible by 10 if its last digit is 0.	430 → last digit 0 → divisible by 10
11	A number is divisible by 11 if the difference between the sum of digits at odd and even places is divisible by 11 (or equal to 0).	2728 → Sum of digits at odd places (1^{st}, 3^{rd}) = 2 + 2 = 4. Sum of digits at even places (2nd, 4th) = 7 + 8 = 15 Difference = 15 - 4 = 11, which is divisible by 11 So, 2728 is divisible by 11.

Let's test the number 4,386 with our rules.

- **Rule for 2:** The last digit is 6, which is even. So, 4,386 is divisible by 2.
- **Rule for 3:** Sum of digits = 4 + 3 + 8 + 6 = 21. Since 21 is divisible by 3 (21 ÷ 3 = 7), 4,386 is divisible by 3.
- **Rule for 4:** The last two digits form the number 86. Is 86 divisible by 4? 86 ÷ 4 = 21 with a remainder of 2. So, 4,386 is NOT divisible by 4.
- **Rule for 5:** The last digit is 6, not 0 or 5. So, 4,386 is NOT divisible by 5.
- **Rule for 6:** The number is divisible by 2 (yes) AND by 3 (yes). Since it passes both tests, 4,386 is divisible by 6.
- **Rule for 9:** The sum of digits is 21. Is 21 divisible by 9? No. So, 4,386 is NOT divisible by 9.

Let's test 51,898 with the rule for 11.

- **Rule for 11:**
- **Digits at odd places (1st, 3rd, 5th):** 5, 8, 8. Sum = 5 + 8 + 8 = 21.
- **Digits at even places (2nd, 4th):** 1, 9. Sum = 1 + 9 = 10.
- Difference = 21 - 10 = 11.
- Since the difference is 11, 51,898 is divisible by 11.

Example 9 : Check the divisibility of 5,624 by 2, 3 and 5.

Solution:

Divisibility by 2:

- A number is divisible by 2 if its units digit is even.
- The units digit of 5,624 is 4, which is even.
- So, 5,624 is divisible by 2.

Divisibility by 3:

- A number is divisible by 3 if the sum of its digits is divisible by 3.
- Sum of the digits of 5,624 is 5 + 6 + 2 + 4 = 17
- Since 17 is not divisible by 3, 5,624 is not divisible by 3.

Divisibility by 5:

- A number is divisible by 5 if its units digit is either 0 or 5.
- The units digit of 5,624 is 4, which is neither 0 nor 5.
- So, 5,624 is not divisible by 5.

Example 10 : What is the smallest number that must replace * to make the number 5 * 2 divisible by 2?

Solution:

- We know that a number is divisible by 2 if its units digit is even.
- In the number 5 * 2, the units digit is 2, which is even.
- Therefore, any number placed in * will make the number divisible by 2.
- The smallest number to replace * is 0.
- So, the smallest number that must replace * is 0, and the number becomes 502.

Example 11 : Test the divisibility of: (i) 124 by 2 (ii) 543 by 3 (iii) 1250 by 5

Solution: (i) We know that a number is divisible by 2 if its units digit is divisible by 2.
In the given number 124, its units digit is 4.
Since 4 is divisible by 2, 124 is divisible by 2.

(ii) We know that a number is divisible by 3 if the sum of its digits is divisible by 3.
In the given number 543, the sum of its digits is 5 + 4 + 3 = 12, which is divisible by 3.
So, 543 is divisible by 3.

(iii) We know that a number is divisible by 5 if its units digit is either 0 or 5.
In the given number 1250, the units digit is 0.
So, 1250 is divisible by 5.

Example 12 : What is the smallest digit that must replace in 53_42 to make it divisible by 9?

Solution:

For the number to be divisible by 9, the sum of its digits must be a multiple of 9.

Sum of known digits = 5 + 3 + 4 + 2 = 14.

So, the total sum is 14 + ...

The next multiple of 9 after 14 is 18.

To make the sum 18, we need 14 + = 18.

Therefore, = 18 - 14 = 4.

The smallest digit is 4. The number is 53,442.

Example 13 : A number is divisible by both 5 and 3. What is the smallest two-digit number that satisfies this condition?

Solution: If a number is divisible by 5, it must end in 0 or 5.

If it's divisible by 3, the sum of its digits must be a multiple of 3.

Let's check two-digit numbers ending in 0 or 5:

10: Sum is 1 (not div by 3).

15: Sum is 1 + 5 = 6 (div by 3). This is a candidate.

20: Sum is 2 (not div by 3).

25: Sum is 7 (not div by 3).

The smallest two-digit number is 15.

Knowledge Checkpoint

- Without dividing, check if 2,985 is divisible by 3 and 5.
- What is the difference between the rule for 3 and the rule for 9?
- A number is divisible by 4. Is it guaranteed to be divisible by 2? Why?

Activity

Divisibility Bingo

- **Objective:** To practice identifying numbers divisible by 2, 3, 5, and 10 quickly.
- **Materials:** Bingo cards with a 4 × 4 grid filled with random 2 and 3-digit numbers. A set of caller cards ("**Divisible by 2**," "**Divisible by 3**," "**Divisible by 5,**" "**Divisible by 10**").
- **Steps:**
  1. Each student gets a bingo card.
  2. The teacher draws a caller card, for example, "Divisible by 3."
  3. Students check their bingo cards for any number that is divisible by 3 (**e.g.,** 123, 75, 312). They can mark one such number per turn.
  4. The first student to get four in a row (horizontally, vertically, or diagonally) shouts "**Bingo!**" and wins.
- **Inquiry Question:** Which rule was the fastest to check? Which was the slowest? Why is a number divisible by 10 also always divisible by 2 and 5?

Key Terms

- **Divisible:** A number is divisible by another if the division results in a whole number with no remainder.
- **Divisibility Rule:** A shortcut method to determine if a number is a factor of another number.
- **Sum of Digits:** The result of adding up all the digits in a number.

Facts Flash

- There's a divisibility rule for 7, but it's a bit tricky! Take the last digit, double it, and subtract it from the rest of the number. If the result is 0 or divisible by 7, the original number is too! **Example:** 343 -> 34 - (3 × 2) = 34 - 6 = 28. Since 28 is divisible by 7, so is 343.
- Any 3-digit number repeated to make a 6-digit number (like 257,257) is always divisible by 7, 11, and 13!

Do It Yourself

- The rule for 6 works because 6 = 2 × 3, and 2 and 3 are co-prime. Does a similar rule work for 8? Can you say a number divisible by 2 and 4 is always divisible by 8? Why or why not? (**Hint:** Try it with 12).
- Why does the "sum of digits" rule work for 3 and 9? It has to do with our base-10 system. Think about what a number like 531 really means: 5(100) + 3 (10) + 1(1). Can you connect this to 5(99 + 1) + 3(9 + 1) + 1?

Mental Mathematics

- Without dividing, check if 246 is divisible by 2.
- The number 205 ends in 5. By which number is it divisible?
- Without dividing, check if 312 is divisible by 4.
- **The sum of the digits of 543 is 12. Is it divisible by 3?**

Gap Analyzer™ Homework

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Exercise 5.3

1. Provide the missing information in the blanks.

a) A number is divisible by 2 if its last digit is ___________, by 3 if the sum of its digits is divisible by ___________, and by 5 if its last digit is ___________ or ___________.

b) A number is divisible by 4 if the number formed by its last ___________ digits is divisible by ___________, by 6 if it is divisible by both ___________ and ___________.

c) A number is divisible by 9 if the sum of its digits is divisible by ___________, and divisible by 10 if its last digit is ___________.

d) A number is divisible by 7 if the result of dividing it by 7 gives a remainder of ___________, and divisible by 11 if the alternating sum of its digits is divisible by ___________.

e) A number is divisible by 12 if it is divisible by both ___________ and ___________, and divisible by 15 if it is divisible by both ___________ and ___________.

2. **Can you give an example of a number that is divisible by...**
   a) Divisible by 2 but not by 5 ______________________
   b) Divisible by 3 but not by 9 ______________________
   c) Divisible by 6 but not by 9 ______________________
   d) Divisible by 3 but not by 6 ______________________
   e) Divisible by 5 but not by 10 ______________________
   f) Divisible by 4 but not by 8 ______________________

3. A librarian has received a large donation of 783 new books. She wants to arrange them on shelves.
   a) She first considers shelves that hold 3 books each. Will she be able to shelve all 783 books in groups of 3 with none left over? Use the divisibility rule for 3 to find out.
   b) Next, she thinks about using larger shelves that hold 4 books each. Is 783 divisible by 4? Explain how you know without dividing.
   c) Based on your answers for parts (a) and (b), can the librarian arrange the books in equal stacks of 6? Why or why not?
   d) If the donation was actually 780 books, by which of these numbers (2, 3, 4, 5, 6, 9, 10) would the new total be divisible?

4. A mail carrier has special packages for apartments whose numbers follow certain rules.
   a) A package is for an apartment number divisible by 9. Which mailbox gets this package?
   b) Another package is for a number divisible by 5. Which apartment gets it?

432	555	678	901
Name	Name	Name	Name
Apartment	Apartment	Apartment	Apartment
Notes	Notes	Notes	Notes

   c) A third package goes to an address that is divisible by 6. Which apartment number is this? Show the two tests you performed.
   d) The mail carrier has a notice for a number that is divisible by 4. Which apartment gets the notice?

5. A restaurant menu with four items and their prices. Use divisibility tests. Tick all that apply and show your working.
   a) The restaurant is running a "Lucky 3" offer. Any bill amount divisible by 3 gets a discount. Which item's price is divisible by 3?
   b) A "Fabulous 5" discount applies to any bill divisible by 5. Which items qualify for this discount?
   c) The "Super 6" combo discount is for bills divisible by 6. Which item's price is divisible by 6? Explain how you checked.
   d) To win a grand prize, the bill amount must be divisible by 4. Which item's price would win the grand prize?

MENU

Burger .... ₹252
Pizza .... ₹455
Pasta .... ₹330
Sandwich ₹148

Prime Factorisation

Every composite number has a secret identity—a unique product of prime numbers. Prime factorisation is the process of revealing this identity. No matter how you start breaking down a number, you will always end up with the same set of prime factors. We will explore two popular methods to find this prime "**DNA**": the Division Method, which is a step-by-step division process, and the Factor Tree Method, which is a more visual way of branching out factors until only primes remain.

Sub-concepts to be covered

1. **Prime Factorisation:** The process of expressing a given number as a product of its prime factors.
2. **Division Method:** A method where we repeatedly divide a number by the smallest possible prime number until the quotient becomes 1.
3. **Factor Tree Method:** A method where a number is split into two factors, and each factor is further split until all branches of the tree end in a prime number.

Mathematical Explanation

Let's find the prime factorisation of the number 84.

Division Method

We start by dividing 84 by the smallest prime number, which is 2.

We stop when we reach 1. The prime factors are the numbers on the left: 2, 2, 3, and 7.

So, the prime factorisation of 84 is $2 \times 2 \times 3 \times 7$ or $2^2 \times 3 \times 7$.

2	84
2	42 (84 ÷ 2 = 42)
3	21 (42 ÷ 2 = 21)
7	7 (21 ÷ 3 = 7, 2 is not a factor)
	1 (7 ÷ 7 = 1)

Fig. 5.3

Factor Tree Method

We can start by splitting 84 into any two factors, for example, 4 and 21.

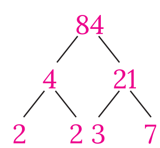

Fig. 5.4

Now we look at the ends of the branches. The numbers are 2, 2, 3, and 7. All of them are prime, so we stop.

The prime factorisation is $2 \times 2 \times 3 \times 7$. Notice we get the same result, proving the uniqueness of the factorisation. You could have started with 2 and 42, or 6 and 14, and you would still end up with the same set of prime factors.

Prime Factorisation

Prime factorisation is the process of breaking down a number into its smallest prime numbers that, when multiplied together, give the original number. A prime number is a number that is greater than 1 and has only two factors: 1 and itself. For example, 2, 3, 5, 7, and 11 are prime numbers.

Steps for Prime Factorization

1. Start with the smallest prime number (2) and divide the given number.
2. Keep dividing the number by 2 until it is no longer divisible by 2.
3. Move to the next prime number (3, 5, 7, etc.), and repeat the process.
4. When you can no longer divide, the remaining numbers are prime factors.

Example: Prime factorisation of 36.

1. Start with 36.
2. Divide by 2 (the smallest prime number):
   • 36 ÷ 2 = 18
3. Divide 18 by 2 again:
   • 18 ÷ 2 = 9
4. Now divide 9 by the next smallest prime number, 3:
   • 9 ÷ 3 = 3
5. Finally, divide 3 by 3:
   • 3 ÷ 3 = 1

2	36
2	18
3	9
3	3
	1

So, the prime factorisation of 36 is: $36 = 2 \times 2 \times 3 \times 3$

Division Method

This is the most common method, where we divide the number by prime numbers until we reach 1.

Steps:

1. Start with the given number.
2. Divide it by the smallest prime number (2).
3. Keep dividing the quotient by the smallest prime until it is no longer divisible.
4. Move to the next prime number (3, 5, 7, etc.) and continue dividing.
5. When the quotient is 1, the prime factorization is complete.

Example: Prime factorisation of 48 using the division method:

2	48
2	24
2	12
2	6
3	3
	1

So, the prime factorisation of 48 is: $48 = 2 \times 2 \times 2 \times 2 \times 3$

Factor Tree Method

In this method, you create a factor tree, where you break the number down into factors step by step until you reach prime numbers.

Steps:

1. Write the number at the top.
2. Split it into any two factors (preferably prime factors).
3. Continue breaking down the factors until all of them are prime numbers.
4. The prime factors, when multiplied together, will give the original number.

Example: Prime factorisation of 48 using the factor tree method:

Prime factorisation of 48 in different ways as:

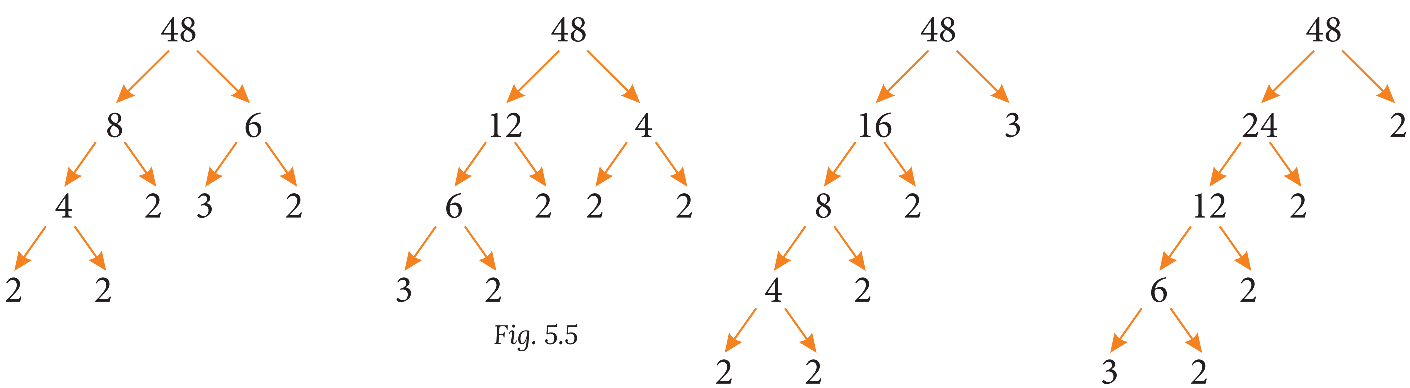

Fig. 5.5

Example: Find the prime factorisation of the following numbers:

(i) 60 (ii) 144

(i)

2	60
2	30
3	15
5	5
	1

Prime Factorization: $60 = 2 \times 2 \times 3 \times 5$

(ii)

2	144
2	72
2	36
2	18
3	9
3	3
	1

Prime Factorization: $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Example 14 : Find the smallest number having four different prime factors.

Solution: The smallest four distinct prime numbers are 2, 3, 5 and 7.

The required number is: $2 \times 3 \times 5 \times 7 = 210$

Example 15 : Which of the following numbers are the product of exactly four distinct prime numbers: 210, 420, 231, 1155, 462?

Solution:
1. $210 = 2 \times 3 \times 5 \times 7$ (Four distinct primes: 2, 3, 5, 7)
2. $420 = 2 \times 2 \times 3 \times 5 \times 7$ (Four primes, but 2 is repeated)
3. $231 = 3 \times 7 \times 11$ (Three distinct primes: 3, 7, 11)
4. $1155 = 3 \times 5 \times 7 \times 11$ (Four distinct primes: 3, 5, 7, 11)
5. $462 = 2 \times 3 \times 7 \times 11$ (Four distinct primes: 2, 3, 7, 11)

Thus, 210, 1155, and 462 are products of exactly four distinct prime numbers.

Knowledge Checkpoint

- Find the prime factors of 40.
- What is the main difference between the division method and the factor tree method?
- Why can't a prime factorisation include the number 9?

Activity

Factorisation Race

- **Objective:** To practice finding prime factors quickly and accurately.
- **Materials:** A set of cards with 2-digit and 3-digit composite numbers (e.g., 56, 81, 110, 144). Whiteboard or paper for each team.
- **Steps:**
  1. Divide the class into two teams.
  2. A student from each team comes to the front.
  3. The teacher shows a number card (e.g., 56).
  4. The two students race to find the complete prime factorisation of the number. The first one to write it correctly (e.g., $2 \times 2 \times 2 \times 7$) wins a point for their team.
  5. Students can choose whichever method they prefer (division or factor tree).
- **Inquiry Question:** For which types of numbers was the factor tree easier? For which was the division method easier? Is there a "best" method?

Key Terms

- **Prime Factorisation:** The process of writing a number as a product of its prime factors.
- **Factor Tree:** A diagram used to break down a number into its prime factors.
- **Fundamental Theorem of Arithmetic:** The principle that every integer greater than 1 is either a prime number itself or can be represented as a unique product of prime numbers.

Facts Flash

- Perfect numbers are numbers that are equal to the sum of their proper divisors (all divisors except the number itself). The first perfect number is 6 (1+2+3=6). The second is 28 (1 + 2 + 4 + 7 + 14 = 28). All known perfect numbers are related to a special type of prime called Mersenne primes.
- The number 1 has no prime factors, which is another reason it is not considered a prime number.

Do It Yourself

- If you know the prime factorisation of a number, can you quickly find out how many factors it has in total (both prime and composite)? Hint: Look at the exponents in the prime factorisation of 12 ($2^2 \times 3^1$). The exponents are 2 and 1. Add 1 to each (3 and 2) and multiply them ($3 \times 2 = 6$). Does 12 have 6 factors? Try it for 30 ($2^1 \times 3^1 \times 5^1$).
- What kind of numbers have an odd number of total factors? (**Hint:** Think about 36 and 49).

Mental Mathematics

- Write the prime factors of 36.
- What is the prime factorisation of 48?
- A number is made of the prime factors 2, 3, and 5. What is the number?
- Which prime factors make up 81?
- What are the prime factors of 210?

Gap Analyzer™ Homework

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Exercise 5.4

1. Which of the following expressions has the correct prime factorization?

   (a) $48 = 2 \times 2 \times 2 \times 2 \times 3$ (b) $120 = 2 \times 2 \times 3 \times 5 \times 5$ (c) $81 = 3 \times 3 \times 3 \times 3$

   (d) $56 = 2 \times 2 \times 2 \times 3 \times 7$ (e) $45 = 3 \times 3 \times 5$ (f) $72 = 2 \times 2 \times 2 \times 3 \times 3$

2. Which of the following numbers is the product of exactly three distinct prime numbers?

   30, 60, 105, 210, 660

3. A factory warehouse has received a large, non-standard order for 360 identical machine parts. The logistics manager needs to figure out how to pack them. Instead of guessing, she first finds the prime factorisation of 360 to understand all its building blocks. Find the prime factorisation of 360 using a factor tree. How could this help the manager decide on the size of smaller boxes (e.g., could she pack them in boxes of 7? How about boxes of 12?)?

4. Imagine numbers have families. A number's "parents" are any two numbers that multiply to make it, its "grandparents" multiply to make the parents, and so on. The oldest "**ancestors**" of any number are always prime numbers. Your task is to find the prime ancestors of the number 98. Draw a family tree (a factor tree) to trace its lineage all the way back to its prime number ancestors.

5. In a video game, you complete a level and earn a final score of 420 points. The game shows you a "Score DNA" which is the prime factorisation of your score.

   a) What is the prime factorisation (the "Score DNA") of 420?

   b) The game awards bonus achievements for each distinct prime factor in your score. How many achievements did you unlock for this score?

   c) Your friend scored 512 points. Find the prime factorisation of 512.

   d) Compare the "Score DNA" of 420 and 512. What is a key difference between the types of prime factors that make up these two numbers?

6. A partially completed division method ladder for the number 126. The next divisor and result are missing. Complete the ladder.

2	126
?	63
?	?
7	7
	1

   a) The number 63 is not divisible by 2. What is the next smallest prime number that can divide 63?
   b) Fill in all the missing numbers in the division ladder.
   c) Using the completed ladder, write the prime factorisation of 126.
   d) Could you have used the divisibility rule for 9 at the second step (for 63)? Would that be a valid step in finding prime factors? Explain.

Finding Connections: HCF and LCM

Now that we can break numbers down into their prime factors, we can explore the relationships between them. In this final section, we will learn about two very important concepts: the Highest Common Factor (HCF) and the Lowest Common Multiple (LCM). The HCF helps us find the largest number that can divide a set of numbers, which is useful for splitting things into equal groups. The LCM helps us find the smallest number that is a multiple of a set of numbers, perfect for figuring out when events will happen at the same time.

Highest Common Factor (HCF) and Lowest Common Multiple (LCM)

The Highest Common Factor (HCF) is the greatest number that is a factor of two or more numbers. It's the biggest "**piece**" you can use to measure out other numbers perfectly.

The Lowest Common Multiple (LCM) is the smallest number that is a multiple of two or more numbers. It's the first time their "**cycles**" will match up.

We will learn how to calculate both HCF and LCM using prime factorisation and discover the elegant relationship that connects them.

Sub-concepts to be covered

1. Highest Common Factor (HCF)
2. Methods for finding HCF
   - Listing all common factors.
   - Using prime factorisation.
3. Lowest Common Multiple (LCM)
4. Methods for finding LCM
   - Listing common multiples.
   - Using prime factorisation.
   - Common Division Method.
5. **Relationship between HCF and LCM:** For any two positive integers '**a**' and '**b**', the product of their HCF and LCM is equal to the product of the numbers themselves. HCF (a, b) × LCM (a, b) = a × b.

Mathematical Explanation

Let's find the HCF and LCM of two numbers: 36 and 48.

Step 1: Prime Factorisation

- $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
- $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3^1$

Step 2: Finding the HCF

To find the HCF, we take the common prime factors with the lowest power.

- The common prime factors are 2 and 3.
- The lowest power of 2 is 2^2 (from 36).
- The lowest power of 3 is 3^1 (from 48).
- HCF = $2^2 \times 3^1 = 4 \times 3 = 12$.

Step 3: Finding the LCM

To find the LCM, we take all prime factors with the highest power.

- The prime factors involved are 2 and 3.
- The highest power of 2 is 2^4 (from 48).
- The highest power of 3 is 3^2 (from 36).
- LCM = $2^4 \times 3^2 = 16 \times 9 = 144$.

Step 4: Verifying the Relationship

- HCF × LCM = 12 × 144 = 1728
- Product of numbers = 36 × 48 = 1728
- The relationship HCF × LCM = Product of Numbers holds true!

Common Division Method for LCM (e.g., for 15, 20, 30):

2	15, 20, 30
2	15, 10, 15
3	15, 5, 15
5	5, 5, 5
	1, 1, 1

LCM = 2 × 2 × 3 × 5 = 60.

Fig. 5.6

Highest Common Factor (HCF)

The Highest Common Factor (HCF) is the largest number that divides two or more numbers exactly (without leaving any remainder). In simple words, it is the biggest number that is a factor of all the given numbers.

Key Terms:

- **Factor:** A factor is a number that divides another number exactly, leaving no remainder.
- **Common Factor:** A common factor is a number that divides two or more numbers exactly.

Example: Let's say you have two numbers: 12 and 18.

- The factors of 12 are: 1, 2, 3, 4, 6, 12
- The factors of 18 are: 1, 2, 3, 6, 9, 18

The common factors of 12 and 18 are: 1, 2, 3, 6

The highest common factor (HCF) is the largest of these common factors, which is 6.

So, the HCF of 12 and 18 is 6.

The Highest Common Factor (HCF) is the largest number that divides two or more numbers without leaving a remainder. To find the HCF of two or more numbers, follow these steps:

Method 1: Listing the Factors

1. List the factors of each number: Factors of a number are the numbers that divide it exactly (without leaving a remainder).
2. Find the common factors: Look for the numbers that appear in both lists.
3. Choose the highest factor: The largest of the common factors is the HCF.

Example: Find the HCF of 18 and 24 using the Listing Factors Method

Solution:

Step 1: List the factors

Factors of 18 = 1, 2, 3, 6, 9, 18

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Step 2: Find the common factors

Common factors = 1, 2, 3, 6

Step 3: Choose the highest factor

The largest common factor = 6

Therefore, HCF (18, 24) = 6

Method 2: Prime Factorization

1. Write the prime factorization of each number.
2. Identify the common prime factors with the smallest power.
3. Multiply these common factors to get the HCF.

Example: Find the HCF of 36 and 48 using Prime Factorization

Solution:

Step 1: Write the prime factorization

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3^1$

Step 2: Identify the common prime factors with the smallest power

For 2 → powers are 2^2 and 2^4 → take 2^2

For 3 → powers are 3^2 and 3^1 → take 3^1

Step 3: Multiply these common factors

$HCF = 2^2 \times 3^1 = 4 \times 3 = 12$

Therefore, HCF (36, 48) = 12

Properties of HCF (Highest Common Factor)

1. HCF of 1 and any number is 1.
2. HCF of two numbers is always less than or equal to the smaller number.
3. HCF of a number and itself is the number.
4. HCF divides both the numbers exactly.
5. HCF of two co-prime numbers is 1.

Example: Find the HCF of 18 and 42.

Step 1: List the factors of each number.

- Factors of 18: 1, 2, 3, 6, 9, 18
- Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

Step 2: Find the common factors. The common factors of 18 and 42 are: 1, 2, 3, 6

Step 3: Choose the highest common factor. The highest common factor (HCF) is 6.

Final Answer: HCF = 6

Example: A baker has two trays, one weighing 56 kg and the other 98 kg. What is the maximum weight of dough that can be measured on both trays exactly?

To find the HCF (Highest Common Factor) of 56 and 98, we can use the factorization method or listing the factors. Let's solve this step-by-step.

Step 1: Find the factors of 56 and 98.

- Factors of 56: 56 = 1, 2, 4, 7, 8, 14, 28, 56
- Factors of 98: 98 = 1, 2, 7, 14, 49, 98

Step 2: Identify the common factors.

The common factors of 56 and 98 are: 1, 2, 7, 14

Step 3: Choose the highest common factor.

The highest common factor (HCF) is 14.

The maximum weight of dough that can be measured on both trays exactly is 14 kg.

Lowest Common Multiple (LCM)

The Lowest Common Multiple (LCM) of two or more numbers is the smallest multiple that is divisible by all of them. It is a key concept in number theory and is widely used in solving problems that involve fractions, ratios, and divisibility.

- For example, the multiples of 4 are: 4, 8, 12, 16, 20, 24 ...
- The multiples of 6 are: 6, 12, 18, 24, 30 ...

The LCM of 4 and 6 is the smallest number that appears in both lists, which is 12.

Methods for finding LCM

Method 1: Listing Common Multiples

Example: Find the LCM of 4 and 6

Step 1: Write the multiples

- Multiples of 4: 4, 8, 12, 16, 20, 24, ...
- Multiples of 6: 6, 12, 18, 24, 30, ...

Step 2: Identify common multiples

- Common multiples = 12, 24, 36, ...

Step 3: Choose the smallest one

- Smallest common multiple = 12

Therefore, LCM (4, 6) = 12

Method 1: Prime Factorisation Method

This method involves breaking down each number into its prime factors (the smallest numbers that can multiply together to give the original number). After finding the prime factors, we multiply them back together, taking the highest power of each factor that appears.

To calculate the LCM of two or more numbers, follow these steps:

1. **Obtain the Numbers:** Identify the numbers for which the LCM is required.
2. **Prime Factorization:** Perform the prime factorization of each number.

For example:

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$60 = 2 \times 2 \times 3 \times 5$

3. **List All Prime Factors:** Write down all the prime factors of the numbers. For the LCM, include each prime factor the maximum number of times it occurs in any single number's factorization.

In the example:

- 2 occurs a maximum of three times (from 72),
- 3 occurs a maximum of two times (from 72),
- 5 occurs a maximum of one time (from 60).

4. Multiply the Factors

Multiply all the factors together to find the LCM : $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

So, the LCM of 72 and 60 is 360.

Method 2: Common Division Method (or Division Ladder Method)

This method involves repeatedly dividing the numbers by common divisors, and recording the quotients. You keep dividing until no more common divisors can be found. Follow these steps:

1. Arrange the given numbers in a row, separated by commas.

   Example: 72, 60.

2. Choose a number that divides at least two of the numbers exactly. Write the quotients of the division below each number. If a number is not divisible, carry it down unchanged.

Example:

- Divide by 2: 72, 60 → 36, 30
- Divide by 2: 36, 30 → 18, 15
- Divide by 3: 18, 15 → 6, 5
- Divide by 2: 6, 5 → 3, 5

2	72, 60
2	36, 30
3	18, 15
2	6, 5
	3, 5

3. Continue dividing until no two numbers have a common factor.
4. Multiply all the divisors and any numbers left in the final row to find the LCM.

Example: Divisors: 2, 2, 3, 2, 3, 5.

$LCM = 2 \times 2 \times 3 \times 2 \times 3 \times 5 = 360.$

So, the LCM of 72 and 60 is 360.

This approach provides two systematic methods to calculate the LCM effectively.

Properties of LCM

1. LCM of Two Numbers is Always Greater Than or Equal to the Larger Number
2. LCM of Two Numbers is Divisible by each of the Numbers
3. LCM of a Number and Itself is the Number
4. LCM of Two Prime Numbers is Their Product
5. LCM of Two Numbers May Not Always Be Their Product
6. Relationship between LCM and HCF

   For any two numbers, the product of their LCM and HCF is equal to the product of the two numbers.

Formula: HCF × LCM = Product of the Two Numbers

Example: Numbers : 4 and 6.

HCF = 2, LCM = 12.

2 × 12 = 4 × 6 = 24

7. The product of the HCF and the LCM of two numbers is always equal to their product. For example, if 5 and 20 are two numbers then, HCF × LCM = 5 × 20 or, HCF = $\frac{5 \times 20}{LCM}$ or, LCM = $\frac{5 \times 20}{HCF}$

Example 16 : Determine the Least Common Multiple (LCM) of the given numbers using the Prime Factorization Method.

1. 24 and 56. 2. 50, 75 and 100.

Solution: Finding the LCM Using the Prime Factorization Method

1. LCM of 24 and 56

Prime Factorization of

- 24 = 2 × 2 × 2 × 3
- 56 = 2 × 2 × 2 × 7

We find the maximum number of times the factor 2 occurs in the prime factorisation of 24 and in the prime factorisation of 56.

LCM = 2 × 2 × 2 × 3 × 7 = 8 × 3 × 7 = 168

LCM of 24 and 56 is 168.

2. LCM of 50, 75, and 100

Prime Factorization of

- 50 = 2 × 5 × 5
- 75 = 3 × 5 × 5
- 100 = 2 × 2 × 5 × 5

We see that 5 comes 2 times, 2 comes 2 times, and 3 one time.

LCM = 2 × 2 × 3 × 5 × 5 = 4 × 3 × 25 = 300

LCM of 50, 75, and 100 is 300.

Example 17 : Calculate the Least Common Multiple (LCM) of using the Common Division Method.

1. 72 and 108 2. 18, 24, and 30

Solution:

1. **LCM of 72 and 108 using the Common Division Method:**

2	72, 108
2	36, 54
3	18, 27
3	6, 9
	2, 3

Hence. the LCM of 72 and 108

2 × 2 × 2 × 3 × 3 × 3

Now, multiply all the prime divisors together to get the LCM:

LCM = 2 × 2 × 2 × 3 × 3 × 3 = 216

Thus, the LCM of 72 and 108 is 216.

LCM of 18, 24, and 30 using the Common Division Method:

2	18, 24, 30
3	9, 12, 15
2	3, 4, 5
	3, 2, 5

Hence, the LCM of 18, 24, and 30

$2 \times 2 \times 2 \times 3 \times 3 \times 5$

Now, multiply all the prime divisors together to get the LCM:

LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Thus, the LCM of 18, 24, and 30 is 360.

Example 18 : Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times.

Solution: We need to find the HCF of 850 and 680.

$850 = 85 \times 10 = 5 \times 17 \times 2 \times 5 = 2 \times 5^2 \times 17$

$680 = 68 \times 10 = 2 \times 34 \times 2 \times 5 = 2 \times 2 \times 17 \times 2 \times 5 = 2^3 \times 5 \times 17$

Common factors: 2, 5, 17. Lowest powers: 2^1, 5^1, 17^1.

HCF = $2 \times 5 \times 17 = 10 \times 17 = 170$.

The maximum capacity of the container is 170 litres.

Knowledge Checkpoint

- Find the HCF and LCM of 12 and 20.
- If the HCF of two numbers is 1, what is their LCM?
- Why is the HCF of two numbers never greater than the smaller of the two numbers?

Activity

The Tiling Puzzle

- **Objective:** To understand HCF in a real-world context.
- **Materials:** Grid paper, scissors, rulers.
- **Steps:**
  1. Draw a rectangle with dimensions 18 cm by 24 cm on the grid paper.
  2. The challenge is to find the largest possible square tile that can cover this entire rectangle perfectly without any cutting or overlap.
  3. Students can try cutting out squares of different sizes (2x2, 3x3, 4x4, etc.) and see which ones fit perfectly along both the 18 cm side and the 24 cm side.
  4. They will discover that the side length of the largest possible square is the HCF of 18 and 24.
- **Inquiry Question:** What do you notice about the side lengths of all the squares that work (not just the largest)? How are they related to the HCF?

Key Terms

- **Highest Common Factor (HCF):** The largest number that divides two or more numbers without a remainder.
- **Lowest Common Multiple (LCM):** The smallest number that is a multiple of two or more numbers.
- **Common Division Method:** A method to find the LCM of a set of numbers by dividing them simultaneously by their common prime factors.

Do It Yourself

- The formula HCF × LCM = Product of Numbers works for two numbers. Does it work for three numbers? Let's test it. Find the HCF and LCM of 4, 6, and 8. Is HCF(4,6,8) × LCM(4,6,8) equal to 4 × 6 × 8? What do you conclude?
- If you know the HCF of two numbers is 10, what can you say about the last digit of both numbers?

Facts Flash

- The HCF of any two consecutive numbers (like 15 and 16) is always 1. They are always co-prime.
- The LCM of any two prime numbers is simply their product. For example, the LCM of 7 and 13 is 7 × 13 = 91.

Mental Mathematics

- **HCF by Observation:** For numbers like (15, 30), the HCF is the smaller number, 15, because 15 is a factor of 30.
- **LCM by Observation:** For numbers like (5, 20), the LCM is the larger number, 20, because 20 is a multiple of 5.
- **Quick LCM for Co-primes:** The LCM of 8 and 9 (which are co-prime) is simply 8 × 9 = 72.

Gap Analyzer™ Homework

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Exercise 5.5

1. Find the LCM by Prime Factorisation Method:

   (a) 30 and 45 (b) 48 and 72 (c) 24 and 32

   (d) 80 and 100 (e) 14, 35, and 49 (f) 56, 84, and 126
2. Solve for HCF and LCM using the relationship:

   If the product of two numbers is 360 and their HCF is 12, find their LCM.

   (a) Product of two numbers = 144, HCF = 24 (b) Product of two numbers = 240, HCF = 8
3. A factory warehouse has received a large, non-standard order for 360 identical machine parts. The logistics manager needs to figure out how to pack them. Instead of guessing, she first finds the prime factorisation of 360 to understand all its building blocks. Find the prime factorisation of 360 using a factor tree. How could this help the manager decide on the size of smaller boxes (e.g., could she pack them in boxes of 7? How about boxes of 12?)?
4. The school library has two reading clubs. The "Junior Readers" club meets every 4th day, and the "Senior Scholars" club meets every 6th day. Both clubs had a meeting on March 1st.

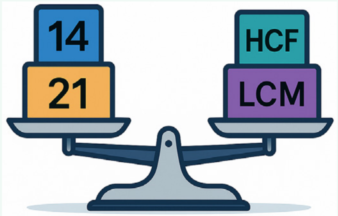
   a) On which date in March will they next have a meeting on the same day?

   b) What mathematical concept (HCF or LCM) did you use to solve this? Why?

   c) The librarian wants to give a welcome gift to both clubs on their next joint meeting day. She has 60 bookmarks and 90 pencils. What is the maximum number of identical gift bags she can make?

   d) How many bookmarks and pencils will be in each gift bag?

5. Organizers of a health camp need to prepare identical first-aid kits. They have 108 bandages and 72 antiseptic wipes. They want to make the maximum number of kits possible with no items left over.
   a) To find the maximum number of identical kits they can make, should they calculate the HCF or the LCM of 108 and 72? Explain your choice.
   b) Using prime factorisation, find the HCF of 108 and 72.
   c) How many bandages and how many antiseptic wipes will be in each first-aid kit?
   d) The product of the two numbers is 108 × 72 = 7776. Use the relationship HCF × LCM = Product of Numbers to find the LCM of 108 and 72
6. The scale is perfectly balanced, representing the formula: Number 1 × Number 2 = HCF × LCM.
   a) Calculate the value on the left pan of the scale (14 × 21).
   b) Find the HCF of 14 and 21. This is the value of the "HCF" block.
   c) Using the balance formula and your answers from (a) and (b), what must be the value of the "LCM" block?
   d) Calculate the LCM of 14 and 21 using the prime factorisation method to verify your answer for part (c).



Common Misconceptions

Misconception: The number 1 is a prime number.
Correction: A prime number must have exactly two distinct factors. The number 1 has only one factor (itself), so it does not fit the definition. It is a unique number that is neither prime nor composite.

Misconception: All odd numbers are prime numbers.
Correction: While all prime numbers except 2 are odd, not all odd numbers are prime. For example, 9, 15, 21, and 25 are all odd, but they are composite because they have factors other than 1 and themselves (e.g., 9 is divisible by 3).

Misconception: To find the HCF, you multiply all the prime factors. To find the LCM, you multiply only the common ones.
Correction: This is the reverse of the correct method. For HCF, we multiply the common prime factors with the lowest powers. For LCM, we multiply all prime factors (both common and uncommon) using their highest powers.

Real-Life Prime Time: Mathematical Application

Here's how prime numbers play a crucial, often hidden, role in our world:

- **Internet Security:** The safety of your online passwords and messages relies on cryptography, which uses very large prime numbers. Multiplying two huge primes creates a nearly unbreakable code, protecting our digital lives.
- **Nature's Survival Trick:** Some species of cicada insects emerge from the ground only every 13 or 17 years. Because these are prime numbers, it's difficult for predators with shorter life cycles to consistently appear at the same time.
- **Engineering and Machinery:** In complex machines, engineers often design gears with a prime number of teeth. This ensures the gears wear down evenly over time because the same teeth don't line up too frequently.
- **Creating Unique Patterns:** Primes are used in computer graphics and music to generate sequences that seem random and don't repeat too quickly, creating more natural-looking textures and sounds.

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EXERCISE

A. Multiple Choice Questions:

1. Which of the following is a pair of co-prime numbers?
   a) (12, 15) ☐ b) (14, 21) ☐ c) (10, 21) ☐ d) (6, 9) ☐
2. The prime factorisation of 132 is:
   a) 2×66 ☐ b) $2^2 \times 3 \times 11$ ☐ c) $2 \times 3 \times 22$ ☐ d) $4 \times 3 \times 11$ ☐
3. What is the HCF of 48 and 64?
   a) 8 ☐ b) 12 ☐ c) 16 ☐ d) 192 ☐
4. A number is divisible by 11 if:
   a) Its last digit is 1. ☐ b) The sum of its digits is divisible by 11. ☐
   c) It is a prime number. ☐
   d) The difference of the sums of alternate digits is 0 or a multiple of 11. ☐
5. The LCM of 15 and 25 is:
   a) 5 ☐ b) 75 ☐ c) 125 ☐ d) 375 ☐

Assertion & Reason

Direction: In the following questions, a statement of Assertion (A) is given, followed by a corresponding statement of Reason (R). Choose the correct option.

a) Both A and R are true, and R is the correct explanation of A.
b) Both A and R are true, but R is not the correct explanation of A.
c) A is true, but R is false. d) A is false, but R is true.

1. **Assertion (A):** The HCF of 17 and 23 is 1.
Reason (R): 17 and 23 are prime numbers.

2. **Assertion (A):** The number 528 is divisible by 6.
Reason (R): A number divisible by 3 is always divisible by 6.

3. **Assertion (A):** The LCM of 9 and 12 is 36.
Reason (R): The LCM is the product of all prime factors (Common and Uncommon) raised to their highest powers.

Case Study

A bakery is preparing gift boxes. They have 140 cookies and 168 brownies. They want to create identical gift boxes, with each box having the same number of cookies and the same number of brownies. To be profitable, they want to make the maximum number of gift boxes possible.

1. What mathematical concept should they use to solve this problem? (HCF)
2. How many identical gift boxes can they make?
3. How many cookies and how many brownies will be in each box?

Project

Plan a Community Event

- **Scenario:** You are helping to organize a community health camp. You have three activities:
- A dental check-up station that can see a new person every 6 minutes.
- A blood pressure station that can see a new person every 8 minutes.
- A nutrition advice station that can see a new person every 10 minutes.
- All three stations start at 9:00 AM. A special announcement is made every time all three stations start with a new person at the exact same moment.
- **Your Task:**
  1. Calculate when the first special announcement will be made after 9:00 AM. (This requires finding the LCM).
  2. You have 240 registration forms and 360 information pamphlets to distribute. You want to create identical welcome packets with the same number of forms and pamphlets in each, with none left over. What is the maximum number of welcome packets you can create? How many forms and pamphlets will be in each? (This requires finding the HCF).
- **Deliverable:** A one-page report showing your calculations and answering the questions clearly.

Source-Based Question

The 'Har Ghar Jal' Mission

Directions: Read the following text and analyze the data table about the Government of India's Jal Jeevan Mission. Then, answer the questions that follow.

Source Text

The Jal Jeevan Mission (JJM) is a major initiative launched by the Government of India to ensure that every rural household has access to safe and adequate drinking water through individual household tap connections by 2024. This mission, also known as 'Har Ghar Jal' (Water in Every Home), aims to improve the quality of life in rural areas.

Logistics teams work hard to manage supplies for the mission. They have to plan the distribution of items like pipes, water tanks, and testing kits efficiently across thousands of villages. Mathematics, especially concepts like factors and multiples, is crucial for this large-scale planning.

Data Table: Mission Progress in Three States (as of a specific date)

State	Total Rural Households (Approx.)	Households with Tap Connections
Uttar Pradesh	2,62,00,000	1,80,36,000
Bihar	1,65,00,000	1,59,08,175
Rajasthan	1,05,00,000	45,67,500

(Source: Adapted from official data published by the Jal Jeevan Mission, Ministry of Jal Shakti, Government of India)

Questions

1. In Rajasthan, how many rural households were still waiting to get a tap water connection according to the data table?
2. Look at the number of households with tap connections in Uttar Pradesh (1,80,36,000). Without performing actual division, use the divisibility rules to check if this number is divisible by:

   a) 3 b) 4 c) 9

   (Show your work for each rule.)
3. A logistics team has two bundles of pipes. One bundle contains pipes that are 36 metres long, and the other contains pipes that are 48 metres long. They need to cut the pipes into pieces of equal length with no wastage. What is the greatest possible length of each piece they can cut?
4. To ensure water quality, a mobile testing van visits a group of villages every 10 days. A maintenance team for the water pumps visits the same villages every 12 days. If both teams visited today, after how many days will they be in the same group of villages on the same day again?
5. A project manager in Bihar has a team of 105 engineers. For a special survey project, she wants to divide them into smaller groups of equal size. The mission guidelines state that for this special project, the number of engineers in each group must be a prime number.

   a) Find the prime factorisation of 105.

   b) Based on the prime factors, what are the possible sizes for each group?

Mind Map

Prime Time

Prime & Composite Numbers

- **Prime:** >1, only two factors (1 & itself).
  - ✓ **Examples:** 2, 3, 5, 7... (Note: 2 is the only even prime).
- **Composite:** >1, more than two factors.
  - ✓ **Examples:** 4, 6, 9, 12...
- **Number 1:** Neither prime nor composite.
- **Finding Primes:** Sieve of Eratosthenes.
- **Special Groups:**
  - ✓ Twin Primes: (5, 7), (11, 13).
  - ✓ Co-primes: Only common factor = 1 (e.g., 8 & 15).
  - ✓ Prime Triplets: (3, 5, 7).

Tests for Divisibility

- **Quick Rules:**
  - ✓ ÷2 → Last digit even
  - ✓ ÷3 → Sum of digits ÷ 3
  - ✓ ÷4 → Last 2 digits ÷ 4
  - ✓ ÷5 → Last digit 0 or 5
  - ✓ ÷6 → Divisible by 2 & 3
  - ✓ ÷8 → Last 3 digits ÷ 8
  - ✓ ÷9 → Sum of digits ÷ 9
  - ✓ ÷10 → Last digit 0
  - ✓ ÷11 → (Sum of alternate digits) difference ÷ 11

Prime Factorisation

- **Definition:** Breaking a composite number into primes.
- **Key Idea:** Every number has a unique set of prime factors.
- **Methods:**
  - ✓ **Factor Tree** → branch until all primes.
  - ✓ **Division (Ladder)** → divide step by step by smallest primes.

HCF & LCM

- **HCF (Greatest Common Factor):** Largest common divisor.
  - ✓ **Methods:** List factors / Prime factorisation (lowest powers).
  - ✓ **Use:** Splitting items equally.
- **LCM (Lowest Common Multiple):** Smallest common multiple.
  - ✓ **Methods:** List multiples / Prime factorisation (highest powers) / Common division.
  - ✓ **Use:** Finding events that repeat together (e.g., schedules).
- **Golden Rule:** HCF × LCM = Product of the two numbers.