3 Number Play

Why This Chapter Matters

Have you ever thought about the biggest number you can imagine? What about the population of India, the distance to the Moon, or the number of stars in our galaxy? These are all enormous numbers! But how do we read them, compare them, and use them in our daily lives? This chapter is your guide to becoming a master of numbers, big and small. We will explore how numbers are built, uncover their secret patterns, and learn to perform calculations that can solve real-world mysteries.

Meet EeeBee.AI

Hello, explorers! I'm EeeBee, your friendly robot guide. I'm powered by curiosity and numbers! Throughout this chapter, I'll be here to share amazing facts, give you helpful hints, and challenge you with tricky questions. Whenever you see me, get ready for a fun fact or a new way to think about numbers. Let's start our adventure together!

Learning Outcomes

By the end of this chapter, students will be able to:

- Read, write, and compare large numbers using both the Indian and International systems of numeration.
- Apply the rules of divisibility to check if a number can be divided by 2, 3, 5, 9, 10, and 11.
- Estimate sums, differences, and products by rounding numbers to the nearest tens, hundreds, and thousands.
- Analyze number patterns and solve puzzles like the Kaprekar constant and palindromes.

From Last Year's Notebook

- In your previous class, you learned to work with numbers up to one lakh with confidence.
- You revised place value and face value and used them to read and write numbers correctly.
- These skills are the foundation for understanding larger numbers in lakhs, crores, and millions.
- You can confidently use the four basic operations: addition, subtraction, multiplication, and division.
- You have explored the building blocks of numbers: factors and multiples.
- Now, in Grade 6, we will expand these ideas further and discover deeper patterns in numbers.

Real Math, Real Life

- **Shopping & Fun:** Checking the price of a new video game or your favorite snack.
- **In the Kitchen:** Measuring the exact ingredients to bake a perfect cake.
- **Sports & Games:** Calculating the total runs in a cricket match or keeping score.
- **At Home:** Helping to plan your family's monthly budget.
- **Technology:** Writing the code for a new app or website.
- **Understanding Our Country:** Making sense of data about India's population and growth.

Quick Prep

1. What is the largest 5-digit number?
2. Write the number "fifty-four thousand, three hundred twenty-one" in figures.
3. In the number 78,451, what is the place value of the digit 8?
4. What is the expanded form of 9,345? (e.g., 9000 + 300 + 40 + 5)
5. How many hundreds are there in one thousand?

Introduction

Imagine you are an astronomer discovering a new galaxy, or a government official planning the budget for the entire country. The numbers you would work with are huge! To handle these large numbers, we need a system. In this section, we will learn how to read, write, and understand very large numbers using two important systems: the Indian Place Value System and the International Place Value System. Mastering this will allow you to make sense of the big numbers you see in news, science, and finance.

Chapter Overview

This map is our guide for the exciting journey ahead. Here's a sneak peek at what we will explore and master together:

- First, we'll conquer **Large Numbers,** learning to read and write them in both the Indian (Crores) and International (Millions) systems.
- Next, you'll become an expert at **Comparing and Ordering**, arranging any set of numbers and even forming the greatest or smallest numbers from given digits.
- We will then uncover the secrets of divisibility, learn clever **Divisibility Rules**, and explore the building blocks of all numbers: prime factors.
- You'll sharpen your skills by performing all four **Operations on Large Numbers** to solve real-world word problems.
- Finally, we'll learn the practical life skill of **Estimation** and have fun with amazing **Number Patterns and Puzzles**

From History's Pages

Our modern number system is a gift from ancient India. Developed between the 1st and 4th centuries, the **Hindu-Arabic** system introduced the revolutionary concepts of zero and place value, making calculations much easier than with older systems like Roman numerals. Arab mathematicians adopted this efficient method and introduced it to Europe. This system, which uses just ten symbols (**0-9**) to represent any number, is the foundation of all modern mathematics and technology we use today.

Number Play and Place Value Systems

Number Play

Number Play is introduced as a fun and engaging way to build foundational math skills. It focuses on enhancing a student's ability to **work with numbers**, **recognize patterns**, and **develop problem-solving skills**. The activities are designed to help students apply mathematical concepts in real-world scenarios, making learning interactive and enjoyable.

Sub-concepts to be covered

1. Number Patterns
2. Basic Operations
3. Problem Solving and Logical Reasoning

Number Patterns

Recognizing and creating sequences of numbers like even and odd numbers, multiples, factors, and prime numbers.

- **Patterns in Multiplication and Division:** Exploring multiplication tables and understanding division patterns.
- **Series and Sequences:** Identifying arithmetic and geometric progressions, number series, and finding the missing terms in a sequence.

Basic Operations

Performing operations (addition, subtraction, multiplication, division) with multi-digit numbers.

- **Order of Operations:** Learning the correct sequence of operations (PEMDAS/ BODMAS) for solving expressions.

Problem Solving and Logical Reasoning

- Applying mathematical concepts to solve word problems and puzzles
- Using number patterns and operations to reason through challenges and derive solutions.
- Understanding Numbers (Place Value for Large Numbers)
- Place value is a fundamental concept in mathematics that helps us understand the value of each digit in a number based on its position. For large numbers, place value becomes even more critical, as it enables us to read, write, and compare numbers efficiently.

Place Value Systems

Place value is the idea that a digit's value depends on its position in a number. The '5' in 50 is different from the '5' in 500. As numbers get bigger, we need a structured way to name and read them. We will explore two such structures.

Sub-concepts to be covered

1. Understanding Numbers (Place Value for Large Numbers)
2. Method of Writing/Reading Large Numbers in Words
3. Reading and Writing Large Numbers.

Mathematical Explanation

Understanding Numbers (Place Value for Large Numbers)

Place value is a fundamental concept in mathematics that helps us understand the value of each digit in a number based on its position. For large numbers, place value becomes even more critical, as it enables us to read, write, and compare numbers efficiently.

What is Place Value?

- Place value refers to the value of a digit depending on its position in a number.
- For example, in the number 4,672, the digit 4 is in the thousands place, so its value is 4,000.

Place Value Chart for Large Numbers

To understand large numbers, we use a place value chart that organizes digits into groups or periods. In the Indian and International systems of numeration, the grouping differs slightly.

Method of Writing/Reading Large Numbers in Words

1. Indian Place Value System

The Indian system groups digits into ones, thousands, lakhs, and crores:

- **Ones Period:** Ones, Tens, Hundreds
- **Thousands Period:** Thousands, Ten Thousands
- **Lakhs Period:** Lakhs, Ten Lakhs
- **Crores Period:** Crores, Ten Crores

Table of the Indian Place Value System

Name of Place	Value of the Place
TKh (Ten kharabs)	10,00,00,00,00,000
Kh (Kharabs)	1,00,00,00,00,000
TA (Ten arabs)	10,00,00,00,000
A (Arabs)	1,00,00,00,000
TC (Ten crores)	10,00,00,000
C (Crores)	1,00,00,000
TL (Ten lakhs)	10,00,000
L (Lakhs)	1,00,000
TTh (Ten thousands)	10,000
Th (Thousands)	1,000
H (Hundreds)	100
T (Tens)	10
O (Ones)	1

Table no. 3.1

Place	Ten Crores	Crores	Ten Lakhs	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Ones
Example (3,47,85,291)		3	4	7	8	5	2	9	1

Table no. 3.2

Three crore forty-seven lakh eighty-five thousand two hundred ninety-one.

2. International Place Value System

The International system groups digits into ones, thousands, and millions:

- **Ones Period**: Ones, Tens, Hundreds

- **Thousands Period:** Thousands, Ten Thousands, Hundred Thousands
- **Millions Period**: Millions, Ten Millions, Hundred Millions

Table of the International Place Value System

Name of Place	Value of the Place
Ten billions (Ten Arabs)	10,000,000,000
Billions (One Arab)	1,000,000,000
Hundred millions (Ten Crores)	100,000,000
Ten millions (One Crore)	10,000,000
Millions (Ten Lakhs)	1,000,000
Hundred thousands (One Lakh)	100,000
Ten Thousands	10,000
Thousands	1,000
Hundreds	100
Tens	10
Ones	1

Table no. 3.3

Place	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
Example (347,852,291)	3	4	7	8	5	2	2	9	1

Table no. 3.4

Three hundred forty-seven million eight hundred fifty-two thousand two hundred ninety-one.

Reading and Writing Large Numbers

1. Use Commas for Clarity

- **Indian System:** Write commas (,) after every three digits starting from the right (after hundreds) and then after every two digits.
  - **Example:** 3,47,85,291
- **International System:** Write commas (,) after every three digits starting from the right.
  - **Example: 347,852,291**

2. Read Each Period Separately

- **Indian System:** Read periods as lakhs, crores, etc.
- **International System:** Read periods as thousands, millions, etc.

Group the Digits According to Place Value

In the Indian System, group digits as follows:

- **Example:** 56,78,910 → 56 Lakh 78 Thousand 910.

In the International System, group digits into periods of three (from right to left):

- **Example:** 56,789,010 → 56 Million 789 Thousand 10.

Read the Number from Left to Right

Read each group of numbers (period) along with its place value.

- **Example (Indian System):** 4,32,560 → Four Lakh Thirty-Two Thousand Five Hundred Sixty.
- **Example (International System):** 432,560 → Four Hundred Thirty-Two Thousand Five Hundred Sixty.

This step-by-step method helps students build confidence in reading and writing large numbers accurately.

Example 1 : Write the corresponding numerals in figures and in words for each of the following.

4,00,00,000 + 30,00,000 + 6,00,000 + 40,000 + 5,000 + 100 + 8

Solution: In Figures: 4,36,45,108

In Words: Four crore thirty-six lakh forty-five thousand one hundred eight.

Example 2 : Write the corresponding numerals in figures and in words for each of the following.

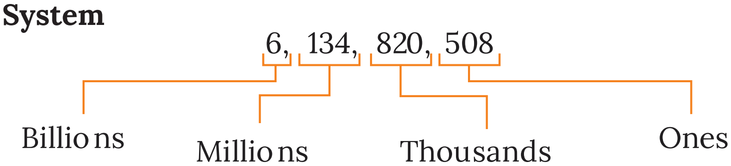
1,00,00,000 + 20,00,000 + 9,00,000 + 30,000 + 7,000 + 500 + 4

Solution: In Figures: 1,29,37,504

In Words: One crore twenty-nine lakh thirty-seven thousand five hundred four.

Example 3 : Write the number name for the numeral 6134820508 in International and Indian systems of numeration.

Solution: **International System**



Indian System

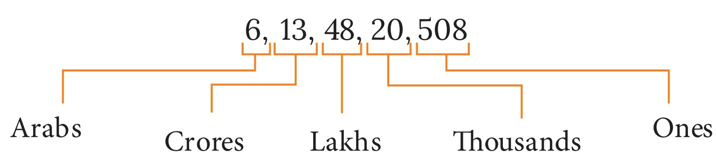


Fig. 3.1

Example 4 : In the number 54,32,109 calculate the difference between the place value and face value of 3.

Solution: Face value of 3 in 54,32,109 = 3.

Place value of 3 in 54,32,109 =3 × 10000 =30000.

Difference = 30000 – 3 = 29997.

Example 5 : In the number 9,42,315 how many times is the place value of 9 greater than the place value of 4?

Solution: In the number 9,42,315 we need to compare the place values of the digits 9 and 4

Place Value of 9:

Place value of 9 = 9 × 100,000 = 900,000.

Place Value of 4:

Place value of 4 = 4 × 10,000 = 40,000.

Comparison: To find how many times the place value of 9 is greater than the place value of 4, divide the place value of 9 by the place value of 4:

$$\frac{900,000}{40,000} = 22.5$$

Final Answer: The place value of 9 is 22.5 times greater than the place value of 4.

Knowledge Checkpoint

- Write 45,678,901 in the Indian system with commas.
- What is the period of the digit '8' in the number 987,654,321 in the International system?
- Is 1 Lakh greater than or less than 1 Million?

Activity

Number Card Shuffle

Materials: A set of 10 cards, each with a single digit (0-9).

Procedure:

Divide the class into small groups.

Ask each group to draw 7 cards randomly.

- ♦ **Task 1:** Arrange the cards to form the largest possible 7-digit number. Write its name in both Indian and International systems.
- ♦ **Task 2:** Arrange the cards to form the smallest possible 7-digit number (remember, 0 cannot be the first digit!). Write its name.
- ♦ **Task 3:** Swap one card with another group and see how it changes their largest and smallest numbers.

- **Inquiry Question:** How does the position of the digit '0' affect the value of the number you form?

Do It Yourself

If you were to invent a new place value system for an alien race that has 12 fingers instead of 10, how would it work? What would the place values be called? How would you represent the number we call '15'?

Key Terms

- **Place Value:** The value of a digit based on its position in a number.
- **Face Value:** The intrinsic value of a digit (e.g., the face value of 5 in 543 is 5).
- **Indian System of Numeration:** A place value system using periods of Ones, Thousands, Lakhs, Crores.
- **International System of Numeration:** A place value system using periods of Ones, Thousands, Millions, Billions.
- **Period:** A group of places in a number (e.g., the thousands period).

Facts Flash

- A '**Googol**' is the number 1 followed by 100 zeros! It's a number so large that it's bigger than the number of atoms in the known universe.
- The term '**Lakh**' used in the Indian system comes from the Sanskrit word "**laksh**a," which means "**a mark**" or "**a target,**" and also "**100,000**."

Mental Mathematics

- Write the place value of 7 in the number 5,74,236 (Indian system).
- Write 4,58,209 in words in the International System.
- What is the difference between the place value and face value of 3 in 54,32,109?
- A popular YouTuber's video just crossed 8 million views. How many lakhs is that?

Gap Analyzer™ Homework

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Exercise 3.1

1. Determine Place Value of bold Digit in Indian and International Systems:

a) 1234**5**6789 (b) 2035062**7**4 (c) 7**6**482059 (d) 12345**8**96

e) 2**9**5678401 (f) 87**6**543210 (g) 12345**6**789 (h) 43827649**1**

2. Write Numbers in Words in the given instructions:

a) Write 76,89,540 in words using the Indian system of numeration.

b) Express 1,23,45,678 in words using the International system of numeration.

c) Write 5,61,29,834 in words and identify the place value of 6.

d) Convert 2,48,76,509 into words and identify the place value of 8.

3. The final match of a major cricket tournament has just concluded. You are a sports journalist reporting on the event.

Data Provided:

- Winning Team's Prize Money: ₹20,00,00,000
- Runner-up's Prize Money: ₹12,50,00,000
- Total Viewership on TV and Digital: 5,59,08,210 people
- Highest Individual Score by a Batsman: 129 runs

Questions:

i. Write the winning team's prize money in words according to the Indian System of Numeration.

ii. By how much money did the winning team's prize exceed the runner-up's prize? Write your final answer in words.

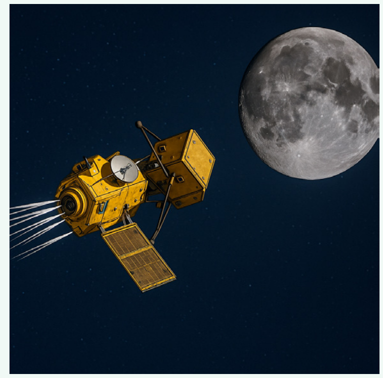
iii. Look at the total viewership number (5,59,08,210). What is the place value of the digit '9' in this number? What is its face value?

iv. For a newspaper headline, you need to round the viewership number to the nearest lakh. What number would you report?

4. The Chandrayaan-3 spacecraft traveled nearly 3,80,000 km from Earth to the Moon.
   - The launch vehicle carried about 7,542 kg of weight.
   - India spent nearly ₹6,15,00,000 on the mission.

   Questions

   i. Write 3,80,000 in words in both Indian and International systems.
   ii. What is the place value of 7 in 7,542?
   iii. Expand 6,15,00,000 in the Indian system.
   iv. Write the successor of 3,80,000.



5. **Application Corner: The Odometer Challenge**

   The odometer displays the number [0 9 8 5 4 6] km.

   Questions:

   i. Aman's father has just returned from a long trip. How many kilometers has the car run in total? Write this number in words.
   ii. If Aman's father drives 4 more kilometers to the market, what new number will the odometer show?
   iii. The car needs a major service at 1,00,000 km. Approximately how many more kilometers can the car run before this service? To find this, first round the current reading to the nearest thousand.

6. **During an international cricket match in Kolkata, the stadium had 66,732 spectators.**
   - On TV, nearly 12,54,000 viewers watched live.
   - The total money collected from ticket sales was ₹3,42,65,000.
   - These numbers show how place value helps us understand very large data.

   Questions

   i. Write 66,732 in the International number system.
   ii. What is the place value of 2 in 12,54,000?
   iii. Expand 3,42,65,000 in the Indian system.
   iv. Arrange the three numbers in ascending order.

Comparing and Ordering Numbers

Now that we can read and write large numbers, how do we know which one is bigger? Imagine choosing between two mobile plans: one offers 28,999 MB of data, and the other offers 31,500 MB. To make the right choice, you need to compare them. In this section, we will learn simple but powerful rules for comparing and arranging numbers in order, from smallest to largest (ascending) and largest to smallest (descending). We will also learn how to use given digits to create the largest or smallest possible numbers.

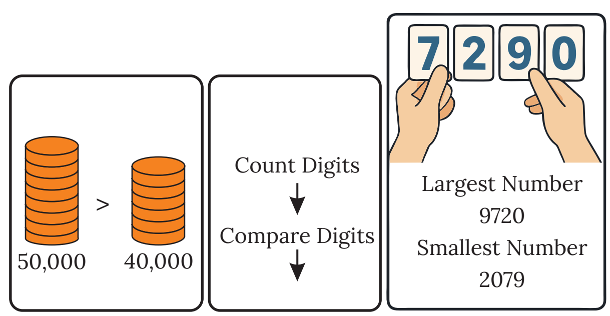


Fig. 3.2

Ordering and Forming Number is a fundamental skill that helps us make decisions. We will learn the definitive rules to determine if a number is greater than, less than, or equal to another. Once we can compare, we can arrange any set of numbers in a specific order. We will also explore the fun challenge of using a given set of digits to construct the largest or smallest possible numbers.

Sub-concepts to be covered

1. Patterns of Numbers on the Number Line
2. Comparison of Numbers
3. Ascending and Descending Order

Mathematical Explanation

The logic behind comparing numbers is tied directly to the place value system. A digit in a higher place value position contributes more to the total value of the number. That's why the number of digits is the first and most important check. A 5-digit number, even the smallest one (10,000), will always be greater than the largest 4-digit number (9,999).

When the digit count is the same, we start our comparison from the left because that's where the most "valuable" digits are. In the number 78,500, the '7' represents 70,000, while the '8' represents only 8,000. So, when comparing 78,500 and 69,999, we only need to look at the ten thousands place. Since 7 > 6, we immediately know that 78,500 is greater, regardless of all the other digits.

This same logic applies to forming numbers. To make the largest number, we want our most valuable digits (the largest ones) in the most valuable positions (the leftmost places). To make the smallest number, we do the opposite, placing the smallest digits in the leftmost places, with the special rule for zero to ensure the number has the correct number of digits.

Patterns of Numbers on the Number Line

Understanding number patterns on a number line helps students visualize how numbers increase or decrease. Here's a simplified explanation, focusing on common patterns found on a number line:

Pattern: Rule: add 1200 then add 1000 repeat

Example: 1050, 2250, 3250, 4450, 5450, 6650, 7650 ...

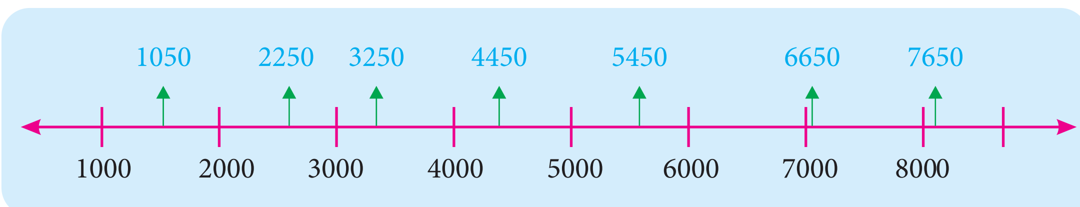


Fig. 3.3

Visualizing the Patterns on a Number Line

- **Zero (0):** The starting point of the number line.
- **Positive Numbers:** Move to the right of 0 (increasing numbers).
- **Negative Numbers:** Move to the left of 0 (decreasing numbers).
- **Intervals:** The space between numbers indicates the order and distance between them.

Example: Plot the following on a number line and find the number that is greater than 54 but less than 58: 52, 55, 57, 60.

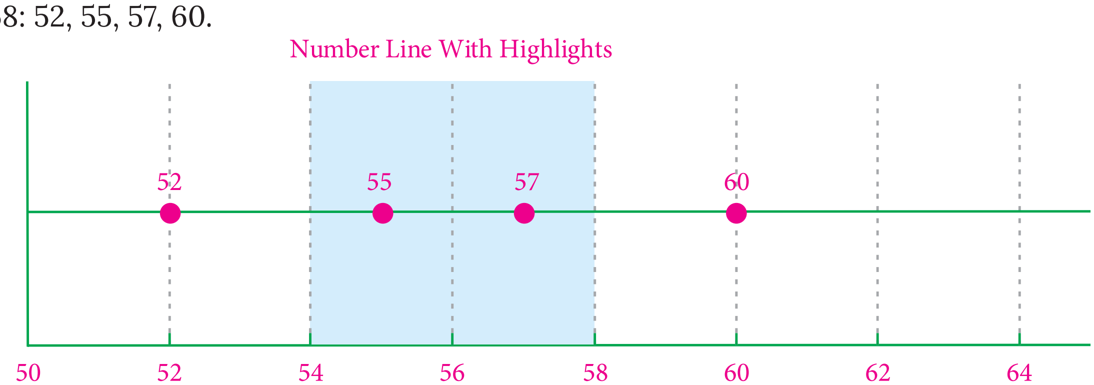


Fig. 3.4

On the number line, the numbers that are greater than 54 but less than 58 are **55** and **57**.

Comparison of Numbers

Comparing numbers helps in determining which number is greater, smaller, or equal. Here's a detailed explanation on how to compare numbers:

Steps to Compare Numbers

1. **Compare the Number of Digits:** The number with more digits is always greater.
   - **Example:** 345 (3 digits) is less than 12345 (5 digits).
2. **Compare from Left to Right**
   - Start by comparing the digits at the leftmost place (highest place value).
   - If the leftmost digits are the same, move to the next digit, and so on.
3. **Use of Symbols:**

   Greater than: >

   Example: 8 > 5 (8 is greater than 5)

   Less than: <

   Example: 4 < 9 (4 is less than 9)

   Equal to : =

   Example: 7 = 7 (both numbers are equal)

Methods for Comparing Different Types of Numbers:

1. **Comparing Whole Numbers**

Example: Compare 54894 and 45956.

Solution: Both numbers have 5 digits.

Start from the left: 5 (ten thousands) vs 4 (ten thousands).

Since 5 is greater than 4, 54894 > 45956

Example: Compare 23456 and 2345.

Solution: 23456 has 5 digits, and 2345 has 4 digits.

Since 23456 has more digits, it is greater.

23456 > 2345.

2. **Comparing Decimals Numbers:** Compare digits from left to right, starting with the whole part, then the decimal part.

Example: Compare 3.56 and 3.65.

Solution: Both numbers have the same whole part (3).

Compare the tenths place: 5 (in 3.56) vs 6 (in 3.65).

Since 6 > 5, 3.65 > 3.56.

Example: Compare 5.002 and 5.02.

Solution: Both numbers have the same whole part (5).

Compare the tenths place: 0 (in 5.002) vs 0 (in 5.02).

Compare the hundredths place: 0 (in 5.002) vs 2 (in 5.02).

Since 2 > 0, 5.02 > 5.002.

3. Comparing Negative Numbers

Negative numbers are smaller as they move away from 0.

Example: Compare −5 and −8.

Solution: Since −5 is closer to 0, −5 > −8.

Example: Compare −12 and −7.

Solution: Since −7 is closer to 0, −7 > −12.

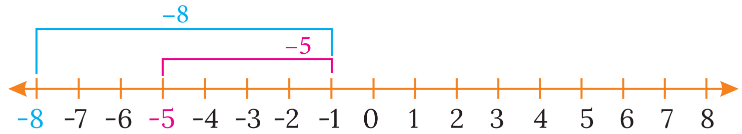


Fig. 3.5

Example 6 : Using the digits 1, 3, 5, 7, and 9: **(Digits cannot be repeated.)**

1. Write the smallest and largest odd 3-digit numbers you can form.
2. Write the smallest and largest even 3-digit numbers you can form.

Solution: **Odd 3-digit numbers**

To make a number odd, the last digit must be an odd digit.

Smallest odd 3-digit number: Use the smallest odd digit for the hundreds place, the next smallest for the tens, and the smallest remaining odd digit for the ones place:

Answer: 135

Largest odd 3-digit number: Use the largest odd digit for the hundreds place, the next largest for the tens, and the largest remaining odd digit for the ones place.

Answer: 975

Even 3-digit numbers

To make a number even, the last digit must be an even digit. However, the given digits are all odd (1, 3, 5, 7, 9), so it is not possible to form an even 3-digit number.

Ascending and Descending Order

Ascending Order

Steps to Arrange in Ascending Order

1. **Compare the numbers and find the smallest:**
   Look at the numbers one by one:
   32047, 652130, 254836, 7812345, 542781, 95124, 30842
   The smallest number is 30842.
2. **Find the next smallest number:**
   From the remaining numbers:
   32047, 652130, 254836, 7812345, 542781, 95124
   The next smallest number is 32047.
3. **Repeat the process:**
   From the remaining numbers: 652130, 254836, 7812345, 542781, 95124
   The next smallest number is 95124.
   - 254836
   - 542781
   - 652130

   The last number is 7812345.
   The numbers in ascending order are:
   30842, 32047, 95124, 254836, 542781, 652130, 7812345.

Descending Order

Steps to Arrange in Descending Order

1. **Compare the numbers and find the largest:**
   Look at all the numbers:
   10238, 752131, 4512368, 1385479, 652307, 230514, 786123
   The largest number is 4512368.
2. **Find the next largest number:**
   From the remaining numbers
   10238, 752131, 1385479, 652307, 230514, 786123
   The next largest number is 1385479.
3. **Continue finding the next largest:**
   From: 10238, 752131, 652307, 230514, 786123
   The next largest number is 786123.
   - 752131
   - 652307
   - 230514

   The last remaining number is 10238.
   The numbers in descending order are:
   4512368, 1385479, 786123, 752131, 652307, 230514, 10238.

Example 7 : Write all 3-digit numbers using the digits 2, 4, and 8, taking each digit only once. Arrange the numbers in ascending order.

Solution: Place each digit in the units place one by one and interchange the tens and hundreds places for the remaining digits as follows:

1. When 2 is at the units place: Numbers are 842 and 482.
2. When 4 is at the units place: Numbers are 824 and 284.
3. When 8 is at the units place: Numbers are 248 and 428.

So, the required numbers are:
842, 482, 824, 284, 248, and 428.
Numbers in ascending order:
248, 284, 428, 482, 824, and 842.

Example 8 : What is the difference between the greatest and smallest numbers that can be formed by rearranging the digits 3, 1, 5, and 9?

Solution: To find the greatest and smallest numbers that can be formed by rearranging the digits 3, 1, 5, and 9:

1. **Greatest Number:** Arrange the digits in descending order;
   9, 5, 3, 1 → The greatest number is 9531.
2. **Smallest Number:** Arrange the digits in ascending order;
   1, 3, 5, 9 → The smallest number is 1359.

Now, to find the difference between the greatest and smallest numbers:

9531 – 1359 = 8172

The difference is 8172.

Knowledge Checkpoint

- Which is smaller: 4,56,789 or 4,56,879?
- Arrange in ascending order: 5050, 5500, 5005.
- What is the largest 4-digit number you can make with the digits 0, 1, 2, 3?

Do It Yourself

If you could only use the digits 1 and 2, how many different 4-digit numbers could you create? Can you arrange them all in ascending order? Do you see a pattern?

Facts Flash

- The symbols for "**greater than**" (>) and "**less than**" (<) were first introduced by English mathematician Thomas Harriot in a book published in 1631. He might have gotten the idea from a symbol he saw on a Native American's arm!
- The alligator mouth analogy is a popular way to remember the symbols: the alligator always wants to eat the bigger number!

Activity

Human Number Line

Objective: To physically order large numbers.

Materials: Large cards, each with a different 6 or 7-digit number written on it.

Procedure:

1. Give one card to each of 10-12 students.
2. Ask the students to stand in a line at the front of the class.
3. **Task 1 (Ascending Order):** Instruct them to arrange themselves in a line from smallest number to largest number without talking, only by showing their cards to each other.
4. The rest of the class acts as verifiers, checking if the final order is correct.
5. **Task 2 (Descending Order):** Repeat the activity, but this time they must arrange themselves in descending order.

Key Terms

- **Comparing Numbers:** Determining if a number is greater than, less than, or equal to another.
- **Ascending Order:** Arranging numbers from smallest to largest.
- **Descending Order:** Arranging numbers from largest to smallest.
- **Forming Numbers:** Creating numbers using a given set of digits according to specific rules.

Mental Mathematics

- **Quick Compare:** When comparing large numbers, first glance at the number of digits. If they're different, you're done! If they're the same, scan from left to right until you find the first point of difference.
- **Forming Numbers:** To form the greatest number, just say the digits out loud from largest to smallest (e.g., for 8, 1, 9, say "nine, eight, one" → 981). For the smallest, say them from smallest to largest ("one, eight, nine" → 189), but remember the zero rule!

Gap Analyzer™ Homework

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Exercise 3.2

1. **Mark the numbers on a number line. What is the middle number in this set?**

   (a) 15, 20, 35, 50, 60 (b) 56, 59, 62, 67, 70

   (c) 60, 70, 65, 85, 90 (d) 30, 45, 25, 50, 60

2. **Compare the following numbers and write the correct symbol (<, >, =):**

(a)	562,384 ___ 562,438	(b)	823,690 ___ 829,603
(c)	100,345 ___ 103,450	(d)	45,130 ___ 45,130
(e)	769,580 ___ 759,608	(f)	73,891 ___ 73,198
(g)	45,672 ___ 45,672	(h)	239,581 ___ 239,815
(i)	872,540 ___ 875,430	(j)	3,465 ___ 3,456

3. **Arrange the following numbers in ascending order and then descending order.**

(a) 15682, 16200, 16100, 15700 (b) 120001, 120000, 119999, 120050, 119998

(c) 568, 423, 598, 347 (d) 5.367, 5.68, 5.34, 5.306, 5.74

4. **Write the numbers from the greatest to the smallest and from the smallest to the greatest using the digits 6, 3, 8, 4, and 2.**

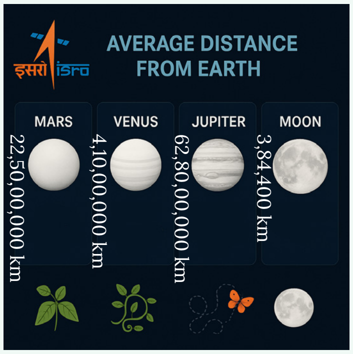
(a) Arrange the numbers in order from greatest to smallest.
(b) Arrange the numbers in order from smallest to greatest.
(c) By reversing the order of digits of the greatest number made by these five digits, what number do you get?

5. **Mountain Expedition Data (Four mountain peaks and their heights in metres.)**

- Mount Everest: 8,848 m
- K2: 8,611 m
- Kangchenjunga: 8,586 m
- Nanda Devi: 7,816 m

i. Which is the tallest mountain peak shown in the infographic?
ii. Arrange the heights of the four mountains in descending order.
iii. What is the difference in height between Mount Everest and Nanda Devi?
iv. Using the digits from the height of K2 (8, 6, 1, 1), what is the smallest possible 4-digit number you can form?

6. **An ISRO mission control dashboard displays the approximate average distance from Earth to four celestial bodies:**



- Mars: 22,50,00,000 km
- Venus: 4,10,00,000 km
- Jupiter: 62,80,00,000 km
- Moon: 3,84,400 km

i. Which of these celestial bodies is closest to Earth?
ii. Arrange the distances in ascending order.
iii. Write the distance to Jupiter in words using the International System of Numeration.
iv. Approximately how many times farther is Venus from Earth than the Moon? (Students can estimate by comparing 4,10,00,000 and 3,84,400).

7. **National Highway Signboard (A highway signboard showing the distances to four different cities.)**

- Mumbai: 1,485 km
- Chennai: 2,180 km
- Kolkata: 1,560 km
- Jaipur: 282 km

i. A family is driving from Delhi. Which city on the signboard is the farthest?
ii. Arrange the cities based on their distance from Delhi, from nearest to farthest.
iii. Using the digits from the distance to Kolkata (1, 5, 6, 0), what is the largest possible 4-digit number you can form?
iv. Compare the distances to Mumbai and Kolkata using the correct symbol (<, >, or =).

Supercells

Have you ever played a game like "**King of the Hill**," where the person at the very top is higher than everyone else around them? In the world of numbers, we have a similar idea for numbers arranged in a grid. We call these "king of the hill" numbers Supercells.

A Supercell is a number in a grid that is greater than all of its immediate neighbours.

What are neighbours?

They are the numbers directly to the **left, right, top, and bottom** of a cell. A number in a corner or on an edge will have fewer neighbours, but the rule is the same!

Mathematical Explanation

A supercell can refer to a larger unit of a repetitive pattern or a structure. In geometry or grid-based problems, the term could be used to describe a large unit that contains smaller cells. A Supercell is a cell that contains a number larger than all its neighboring cells (cells directly next to it, either above, below, left, or right).

72	27	19	12	70	89	22	11

180	526	621	699	226	448	589	185

The shaded cells are called Supercells. Here, the number is shaded if the adjacent numbers are smaller than that number. For e.g., 72 is shaded as 27 < 72.89 is shaded as 70 < 89 and 22 < 89. Similarly, in the table, 621 < 699, 226 < 695, so 699 is shaded.

Example: A Single Row

Consider this simple row of numbers. Let's find the Supercells.

25	50	30

1. **Look at 25:** Its only neighbour is 50. Is 25 greater than 50? No. So, 25 is not a Supercell.
2. **Look at 50:** Its neighbours are 25 and 30. Is 50 greater than 25? Yes. Is 50 greater than 30? Yes. Since it's greater than all its neighbours, 50 is a Supercell!
3. **Look at 30:** Its only neighbour is 50. Is 30 greater than 50? No. So, 30 is not a Supercell.

In this row, only the number 50 is a Supercell.

Example: The Problem: Colour or mark the supercells in the table below.

6828	670	9435
3780	3708	7308
8000	5583	52

Solution Strategy

We need to check every number in the grid. For each number, we will identify its neighbours (the cells directly to its top, bottom, left, and right) and check if our number is greater than all of them.

Step-by-Step Analysis

1. **Check 6828:** Neighbours are 670 (right) and 3780 (bottom).
   - Is 6828 > 670? **Yes.**
   - Is 6828 > 3780? **Yes.**
   - **Conclusion:** 6828 is a Supercell.

2. **Check 670:** Neighbours are 6828 (left), 9435 (right), and 3708 (bottom).
   - Is 670 > 6828? **No.**
   - **Conclusion**: 670 is not a Supercell. (We can stop as soon as one check fails).
3. **Check 9435:** Neighbours are 670 (left) and 7308 (bottom).
   - Is 9435 > 670? **Yes.**
   - Is 9435 > 7308? **Yes.**
   - **Conclusion:** 9435 is a Supercell.
4. **Check 3780:** Neighbours are 6828 (top), 3708 (right), and 8000 (bottom).
   - Is 3780 > 6828? **No.**
   - **Conclusion:** 3780 is not a Supercell.
5. **Check 3708:** Neighbours are 670 (top), 3780 (left), 7308 (right), and 5583 (bottom).
   - Is 3708 > 670? **Yes.** But is 3708 > 3780? No.
   - **Conclusion:** 3708 is not a Supercell.
6. **Check 7308:** Neighbours are 9435 (top), 3708 (left), and 52 (bottom).
   - Is 7308 > 9435? **No.**
   - **Conclusion:** 7308 is not a Supercell.
7. **Check 8000:** Neighbours are 3780 (top) and 5583 (right).
   - Is 8000 > 3780? **Yes.**
   - Is 8000 > 5583? **Yes.**
   - **Conclusion:** 8000 is a Supercell.
8. **Check 5583:** Neighbours are 8000 (left), 3708 (top), and 52 (right).
   - Is 5583 > 8000? **No.**
   - **Conclusion:** 5583 is not a Supercell.
9. **Check 52:** Neighbours are 5583 (left) and 7308 (top).
   - Is 52 > 5583? **No.**
   - **Conclusion:** 52 is not a Supercell.

Final Answer:

The Supercells in this grid are 6828, 9435, and 8000. The final marked table would look like this:

6828	670	**9435**
3780	3708	7308
8000	5583	52

Do It Yourself

- Can you fill a 3 × 3 grid using the numbers 1 to 9 (without repeating any number) such that there are NO Supercells at all? Discuss your strategy.
- What is the maximum number of Supercells you can create in a 3 × 3 grid? Is there a pattern to how you must arrange the numbers to achieve this?

Knowledge Checkpoint

- Find all the Supercells in the grid below.

15	8	20
7	25	19
12	18	22

- **True or False:** The largest number in a grid is always a Supercell. Explain your answer.
- **True or False:** A number in a corner can never be a Supercell. Explain your answer.

Activity

Supercells Grid

Objective: A "Supercell" is a number in a grid that is greater than all its immediate neighbors (up, down, left, right).

Materials: A 4×4 grid drawn on paper.

Procedure: Fill the grid with any 2-digit numbers.

- Examine each cell (that is not on the edge).
- Compare the number in the cell with the numbers directly above, below, to its left, and to its right.
- If the number is greater than all four neighbors, circle it. It's a Supercell!

- **Inquiry Question:** Can you create a grid with the maximum possible number of Supercells? What strategy would you use?

Key Terms

- **Supercell:** A number in a grid that is greater than all of its immediate neighbours.
- **Neighbour / Adjacent Cell:** A cell that is directly to the top, bottom, left, or right of another cell. Diagonal cells are not considered neighbours in this puzzle.

Facts Flash

- **Real-World Supercells:** This idea is used in computer science! In digital image processing, a similar method is used to find the brightest pixel (a "supercell") in a part of a photo. What's in a Name? "Supercell" is a fun, descriptive name made for this puzzle. In advanced mathematics, this concept is called a "local maximum."

Mental Mathematics

A Supercell Spotting Trick!

Instead of checking every single number one by one, try this faster method:

- Scan the grid and find the largest number. Check if it's a Supercell.
- Then, find the next largest number and check it.
- Continue this with the biggest numbers. You are more likely to find Supercells quickly this way, because only numbers that are relatively large have a chance of being a Supercell!

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Exercise 3.3

1. **Colour or mark the supercells in the table below:**

a)

2758	5268	2458	6584	4587	6254	5249	1025	2548

b)

1105	2548	5428	9852	4258	3584	8524

c)

2257	6250	1189
8453	9852	5248
4321	9158	9999

d)

8524	5248	9562	2548
2542	52486	9546	6666
6584	4444	2547	1125
3254	7542	5924	5842

2. **Fill the table below with numbers from 100 to 1000 (without repetitions) so that we get as many supercells as possible.**

(a)

(b)

3. **Fill the tables below with 4-digit numbers such that the supercells are exactly the shaded cells.**

(a)

(b)

4. **In the table, if the number 76 is in the center, we check if it is greater than the numbers surrounding it: 54 (left), 23 (right), 88 (below), and 67 (above). If 76 is greater than all of these, then it is a Supercell. (Justify your answer)**

5. **In the following table, some numbers are shaded.**

	5428	
		9532
1248		8425

(a) Find all the supercells in the table.

(b) Identify all the supercells in the grid.

(c) Which is the biggest number in the table?

Playing with Digits, Divisibility and Number Patterns

Have you ever wondered if you can share 123 chocolates equally among 3 friends without cutting any? Or if a large number like 54,321 can be divided perfectly by 9 without actually doing the division? This is where the **magic of divisibility** rules comes in! In this section, we will learn simple tricks to test if a number is divisible by other numbers just by looking at its digits. This is a powerful tool for **simplifying calculations** and **solving number puzzles**.

Playing with Digits

"**Playing with Digits**" typically refers to a chapter that focuses on number patterns, place value, and operations with digits. This topic helps students understand the structure and properties of numbers, including:

1. **Place Value System:** Understanding how digits in a number represent different values based on their position. For example, in the number 348, the place value of 3 is 300, of 4 is 40, and of 8 is 8.
2. **Forming Numbers:** Creating different numbers by rearranging or adding digits. For example, using the digits 2, 5, and 7, you can form numbers like 257, 572, 725, etc.
3. **Patterns with Digits:** Identifying patterns in numbers, such as even and odd numbers, palindromes, or numbers formed using repetitive digits like 111, 222, etc.
4. **Operations on Numbers:** Applying addition, subtraction, multiplication, and division to numbers formed by different combinations of digits.
5. **Small Number Tricks:** Learning shortcuts or tricks for quickly calculating with numbers. For example, recognizing that numbers ending in 0 or 5 are divisible by 5.

Example: Identify whether the following numbers are divisible by 9 or not: 153, 274, 369, 846.

To determine if a number is divisible by 9, you can add up the digits of the number.

If the sum of the digits is divisible by 9, then the number itself is divisible by 9.

Let's check each number

1. **153:**

   Sum of digits: 1 + 5 + 3 = 9

   Since 9 is divisible by 9, 153 is divisible by 9.
2. **274:**

   Sum of digits: 2 + 7 + 4 = 13

   Since 13 is not divisible by 9, 274 is not divisible by 9.
3. **369:**

   Sum of digits: 3 + 6 + 9 = 18

   Since 18 is divisible by 9, 369 is divisible by 9.
4. **846:**

   Sum of digits: 8 + 4 + 6 = 18

   Since 18 is divisible by 9, 846 is divisible by 9.

   So, the numbers divisible by 9 are: 153, 369, and 846.

Divisibility Rules

Divisibility means a number can be divided by another number without leaving a remainder. Divisibility rules are shortcuts to check this. We will revisit factors, multiples, prime, and composite numbers, and then dive into the specific rules for checking divisibility by 2, 3, 5, 6, 9, 10, and 11.

Sub-concepts to be covered

1. Factors and Multiples
2. Prime and Composite Numbers
3. Tests for Divisibility

Factors and Multiples

A factor is a number that divides another number exactly. A multiple is the result of multiplying a number by an integer. (e.g., 4 is a factor of 12; 12 is a multiple of 4).

Prime and Composite Numbers

A prime number has exactly two factors: 1 and itself (e.g., 2, 3, 5, 7, 11). A composite number has more than two factors (e.g., 4, 6, 8, 9). The number 1 is neither prime nor composite.

Tests for Divisibility

- **by 2:** The last digit is even (0, 2, 4, 6, 8).
- **by 5:** The last digit is 0 or 5.
- **by 10:** The last digit is 0.
- **by 3:** The sum of the digits is divisible by 3.
- **by 9:** The sum of the digits is divisible by 9.
- **by 6:** The number is divisible by both 2 and 3.
- **by 11:** The difference between the sum of digits at odd places and the sum of digits at even places is 0 or a multiple of 11.

Playing with Number Patterns

Have you ever noticed the beautiful patterns on a butterfly's wings, the way tiles are arranged on a floor, or the repeating chorus in your favourite song? Patterns are all around us! In mathematics, numbers can also be arranged in fascinating patterns. A number pattern is a list of numbers, **called a sequence**, that follows a **certain rule**.

In this section, we will become pattern detectives! Our job is to look for clues, discover the hidden rule, and predict what comes next. Let's start our investigation!

Mathematical Explanation

Divisibility rules are not magic; they are logical consequences of our base-10 system.

- **Rule for 2, 5, 10:** These depend only on the last digit because 10 is divisible by 2 and 5. Any number can be written as (10 × rest of the number) + last digit. Since the first part is always divisible by 2 and 5, we only need to check the last digit.
- **Rule for 3, 9:** This works because of place value. For example, $345 = 3 \times 100 + 4 \times 10 + 5$. This can be rewritten as $3 \times (99 + 1) + 4 \times (9 + 1) + 5 = (3 \times 99 + 4 \times 9) + (3 + 4 + 5)$. The first part is always divisible by 9 (and 3), so we only need to check if the sum of the digits (the second part) is divisible by 9 or 3.
- **Rule for 11:** This rule also comes from place value properties, specifically how powers of 10 relate to 11 ($10 = 11 - 1$, $100 = 99 + 1$, $1000 = 1001 - 1$, etc.). This creates the alternating sum pattern.

Example: A number is divisible by 11 if the difference between the sum of the digits in odd positions and the sum of the digits in even positions is a multiple of 11 (or 0). Check if the number 531024 is divisible by 11.

To check if the number 531024 is divisible by 11, we will:

1. Separate the digits into odd and even positions.
2. Calculate the sum of the digits in odd positions and the sum of the digits in even positions.
3. Find the difference between these sums.

4. If the difference is divisible by 11 (or 0), the number is divisible by 11.

    Let's break down the number 531024:

    Odd positions: 5, 1, 2 (1st, 3rd, 5th digits)

    Even positions: 3, 0, 4 (2nd, 4th, 6th digits)

    Now, we'll calculate the sums and check the difference.

The difference between the sum of the digits in odd positions and the sum of the digits in even positions is 1, which is not divisible by 11. Therefore, the number 531024 is not divisible by 11.

Example 9 : Check if the number 789456 is divisible by 6.

Solution:

1. **Test for 2:** The last digit is 6 (even). So, it is divisible by 2.
2. **Test for 3:** Sum of digits = 7+8+9+4+5+6 = 39. Since 39 is divisible by 3 (39 : 3 = 13), the number is divisible by 3.
3. **Conclusion:** Since the number is divisible by both 2 and 3, it is divisible by 6.

Example 10 : Find the smallest digit to replace * in 54*23 to make it divisible by 9.

Solution: Sum of known digits = 5 + 4 + 2 + 3 = 14.

1. The next multiple of 9 after 14 is 18.
2. To make the sum 18, we need to add 18 − 14 = 4.
3. So, the missing digit is 4. The number is 54423.

Example 11 : Imagine you are crossing a river on stepping stones. Each stone has a number on it.

2, 6, 10, 14, ...

- What do you notice about these numbers?
- Can you guess what the next number will be?

Let's look closely. To get from the first stone (2) to the second stone (6), we add 4. 2 + 4 = 6

To get from the second stone (6) to the third stone (10), we again add 4. 6 + 4 = 10

It seems we've found the rule! The Rule: Add 4 to the previous number to get the next one.

So, the next stepping stone would be: 14 + 4 = 18

And the one after that? 18 + 4 = 22

Fig. 3.6

This is an addition pattern. We can also have patterns that use subtraction!

Example 12 : Find the sum of all the numbers in the above triangle pattern

(15 = 4 times), (25 = 4 times), (40 = 8 times)

Method 1: Multiply each number by its frequency and add the results.

Method 2: Sum in all triangles = 15 × 4 + 25 × 4 + 40 × 8 = 480

The other way, can be:

Sum in one triangle = 15 × 1+ 25 × 1 + 40 × 2 = 120

Sum of all numbers = 120 × 4 = 480

15	40	15	25
25	40	40	40
15	40	15	40
40	25	40	25

Fig. 3.7

Knowledge Checkpoint

- Is 555 divisible by 3?
- Is 12345 divisible by 5?
- Is 8888 divisible by 11?

Find the rule and write the next three numbers for each pattern.

- 7, 14, 21, 28, ___, ___, ___
  - **Rule:** ________________
- 105, 100, 95, 90, ___, ___, ___
  - **Rule:** ________________

Activity

Activity 1: The Pattern Challenge

1. **Player 1:** Secretly think of a simple rule (e.g., "add 7" or "multiply by 2").
2. Write down the first four numbers (terms) of your pattern on a slip of paper.
3. **Player 2:** Look at the numbers, figure out the rule, and write down the next three terms.
4. Discuss if the rule was guessed correctly.
5. Swap roles and play again! Try creating subtraction and division patterns too.

Activity 2: Matchstick Geometry Create a pattern by making a sequence of squares with matchsticks, as shown below.

1. Count the number of matchsticks needed for 1 square. (It's 4)
2. Count the number of matchsticks needed for 2 squares joined together. (It's 7)
3. Count the number of matchsticks for 3 squares. (It's 10)
4. Write down the sequence of numbers: 4, 7, 10, ...
5. What is the rule for this pattern? How many matchsticks would you need for 4 squares? And for 5 squares?

   This shows how patterns can connect numbers and shapes!

Key Terms

- **Divisibility:** When a number can be divided by another with a remainder of zero.
- **Prime Number:** A number with exactly two factors: 1 and itself.
- **Composite Number:** A number with more than two factors.
- **Prime Factorization:** Expressing a number as a product of its prime factors.
- **Pattern:** A set of numbers, shapes, or objects that are arranged according to a specific rule.
- **Sequence:** An ordered list of numbers that form a pattern.
- **Term:** Each individual number in a sequence. (e.g., in the sequence 2, 4, 6, the number 4 is the second term).

Do It Yourself

Can you come up with a divisibility rule for 4? Hint: Look at the last two digits. Why does it work? (Because 100 is divisible by 4).

Facts Flash

- 2 is the only even prime number.
- There is no largest **known prime number**; mathematicians are always finding new, bigger ones using supercomputers!

Mental Mathematics

- **Divisibility by 6:** Quickly check if the number is even. If it is, then add up the digits in your head to check for 3.
- **Divisibility by 9:** As you add the digits, you can "cast out nines." For 9,18,27, just ignore them. For 5436, 5+4=9 (ignore), 3+6=9 (ignore). The sum is a multiple of 9.
- The rule is "multiply by 2". If the second term is 12, what was the first term?
- Your friend's pattern is 100, 50, 25, ... What is the next number?
- A pattern starts at 1 and the rule is "add 3". Which of these numbers will be in the pattern: 10, 11, 12?

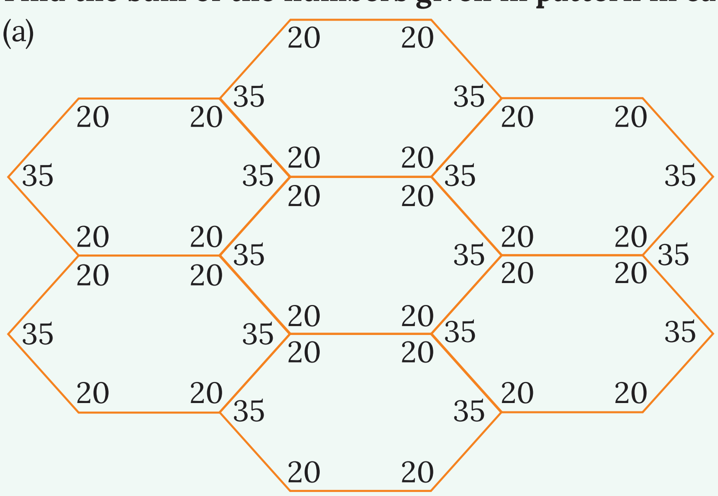
Gap Analyzer™ Homework

Watch Remedial

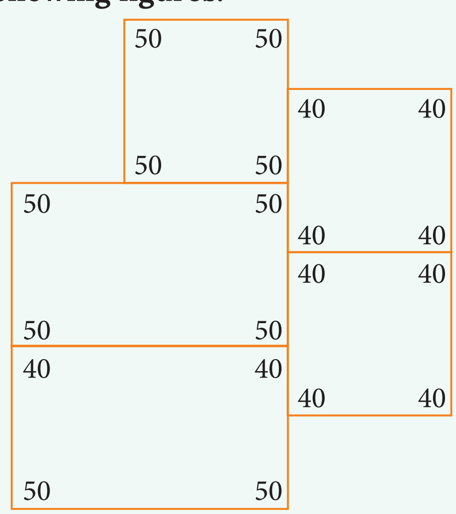
Exercise 3.4

1. Arrange the digits 3, 8, 1, 5, and 0 to form the largest and smallest possible numbers. What is the difference between the largest and smallest number?
2. **Your new digital locker requires a 5-digit passcode. The number pad provides you with the digits 1, 4, 7, _, 5. The final digit is a mystery, but the system gives you a clue: "The complete 5-digit passcode is divisible by 11."**
   i. Using the divisibility rule for 11, find the missing digit that replaces the asterisk (*).
   ii. What is the final 5-digit passcode? Check if this passcode is also divisible by 3.
   iii. Now, using all five digits of the final passcode, what is the greatest possible 5-digit number you can create?
   iv. What is the difference between the greatest number you formed in part (c) and the final passcode you found in part (b)?
3. **Find the sum of the numbers given in pattern in each of the following figures:**

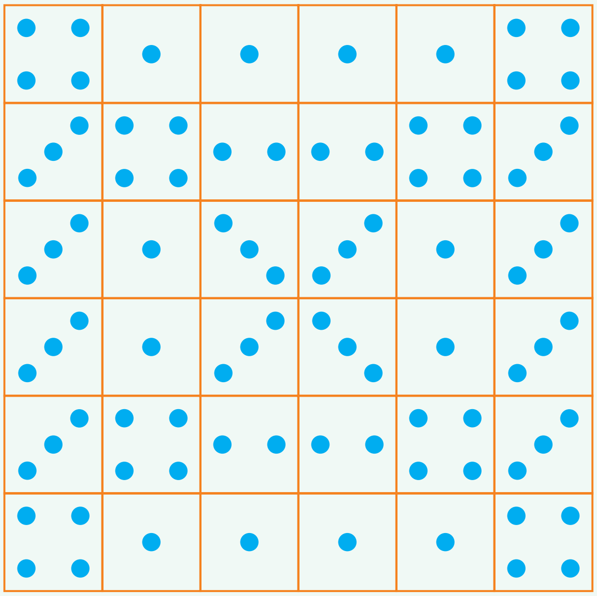
(a)



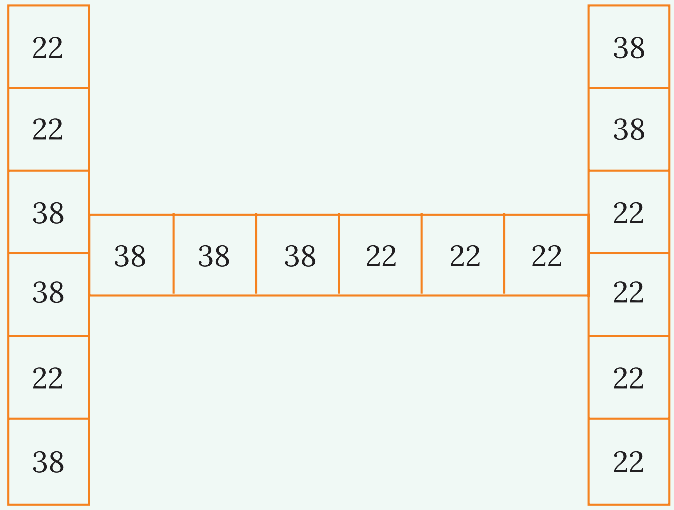
(b)



(c)



(d)



4. What is the remainder when the number 987654321 is divided by 9?
5. Find the sum of all numbers formed by arranging the digits 2, 4, 6, and 8 in all possible 3-digit combinations without repetition of digits.
6. Using the digits 2, 4, and 6, form all the possible three-digit numbers. How many different numbers can you form
7. **A gamer achieved a high score of 1,486,2_0. The final digit of the score is smudged, but bonus points are awarded based on divisibility rules.**
   i. To get the "Nifty Nine" bonus, the score must be divisible by 9. What must the smudged digit (_) be?
   ii. To get the "Elegant Eleven" bonus, the score must be divisible by 11. What must the smudged digit be in this case?
   iii. To get the "Super Six" bonus, the score must be divisible by 6. What is the highest possible value the smudged digit could be to qualify for this bonus?
   iv. Using the digits from the "Super Six" bonus score you identified in part (c), what is the greatest possible 7-digit number you can form?

Operations on Large Numbers

Arithmetic with Large Numbers The principles of addition, subtraction, multiplication, and division are the same for large numbers as for small ones. The key is to be systematic and organized, paying close attention to place value, carrying over in addition, borrowing in subtraction, and careful alignment in multiplication and division.

Sub-concepts to be covered

1. Addition of Large Numbers
2. Subtraction of Large Numbers
3. Multiplication of Large Numbers
4. Division of Large Numbers

Mathematical Explanation

All these operations are built on the foundation of place value.

- **Addition/Subtraction:** When we "**carry a 1**" from the tens to the hundreds column, we are really carrying a value of 100 (ten 10s). When we "borrow 1" from the thousands place, we are borrowing 1000 and adding it to the hundreds place as ten 100s.
- **Multiplication:** When we multiply 345 by 23, we are actually calculating (345 × 3) + (345 × 20). The zero we add in the second line of multiplication is a placeholder to ensure we are multiplying by 20, not just 2.
- **Division:** Long division is essentially a process of repeated subtraction. When we divide 86 by 20, we are asking, "How many times can I subtract 20 from 86?" The answer is 4, with 6 left over.

Addition of large numbers

When teaching the addition of large numbers, it's important to focus on a structured approach. Here's how to add large numbers step-by-step:

Steps for Adding Large Numbers

1. **Align the Numbers:** Write the numbers vertically, one under the other, ensuring the digits are aligned by place value (ones, tens, hundreds, thousands, etc.).
2. **Start from the Right:** Begin adding the numbers from the ones place (rightmost digit).
3. **Carry Over:** If the sum of any column exceeds 9, carry over the extra value to the next place value. For example, if you add 7 and 5, the result is 12. You write 2 in the ones place and carry over 1 to the tens place.
4. **Move to the Next Column:** After adding the ones place, move to the tens, hundreds, and continue similarly to the left.
5. **Final Result:** Continue adding each column and carry over when necessary until the final sum is obtained.

Example: Add 4457 and 6789

	4	4	5	7
+	6	7	8	9
	11		4	6

Subtraction of large numbers

Subtraction of large numbers involves a similar process to addition, but this time you'll be taking digits away. Here's a step-by-step guide on how to subtract large numbers:

Steps for Subtracting Large Numbers:

1. **Align the Numbers:** Write the numbers vertically, one under the other, ensuring that the digits are aligned by place value (ones, tens, hundreds, thousands, etc.).
2. **Start from the Right:** Begin subtracting from the rightmost digit (the ones place).
3. **Borrow if Necessary:** If the top digit is smaller than the bottom digit (i.e., you can't subtract directly), you need to borrow from the next higher place value.

   For example, if you need to subtract 7 from 3 in the ones place, you borrow 1 from the tens place, making the 3 become 13 and the tens place decrease by 1.

4. **Move to the Next Column:** After subtracting one column, move to the next (tens, hundreds, etc.), remembering to borrow when needed.
5. **Final Result:** Continue until you've subtracted all the digits.

Example: Let's subtract 9367 from 87582:

	8	7	5	8	2
-		9	3	6	7
	7	8	2	1	5

Multiplication of large numbers

Multiplying large numbers involves breaking down the process into manageable steps, using the multiplication of smaller numbers. Here's how you can multiply large numbers step-by-step:

Steps for Multiplying Large Numbers

1. **Write the Numbers Vertically:** Place the numbers vertically, one under the other, ensuring proper alignment by place value (ones, tens, hundreds, etc.).
2. **Multiply Each Digit of the Second Number by the First Number:** Start by multiplying the first digit of the second number by each digit of the first number (from right to left).

   Write the result below the line, aligning it according to the place value of the digit you're multiplying by.
3. **Carry Over if Necessary:** Just like addition or subtraction, if the product of any multiplication exceeds 9, carry over the extra value to the next column.
4. **Move to the Next Digit:** Move to the next digit of the second number and repeat the multiplication for all digits.

   When multiplying with the next digit, remember to shift the results to the left by one place to account for the tens, hundreds, etc.
5. **Add the Partial Products:** After completing the multiplication for each digit of the second number, add the results (partial products) together to get the final product.

Example: Let's multiply 8945 by 67.

			8	9	4	5
			×		6	7
		6	2	6	1	5
+	5	3	6	7	0	
	5	9	9	3	1	5

Division of Large Numbers

Dividing large numbers follows a structured approach similar to multiplication but involves repeated subtraction. Here's a step-by-step guide on how to divide large numbers:

Steps for Dividing Large Numbers

1. **Set Up the Division:** Write the division in the long division format. The larger number (dividend) goes inside the division box, and the smaller number (divisor) goes outside.

2. **Divide the First Few Digits:** Start with the leftmost digits of the dividend (the number inside the division box).

   Determine how many times the divisor can fit into the selected digits without exceeding them. This is the quotient for that portion.

3. **Multiply and Subtract:** Multiply the divisor by the quotient you found and write the result below the selected digits of the dividend.

   Subtract the result from the selected digits of the dividend.

4. **Bring Down the Next Digit:** Bring down the next digit of the dividend and repeat the process.

   Continue dividing, multiplying, and subtracting until all the digits of the dividend have been used.

5. **Final Result:** Once you've worked through all the digits, the final quotient is the answer to the division problem.

   If there is a remainder, write it as "remainder X" or as a decimal (depending on the problem).

Example: Let's divide 8745 by 23.

```
     3 8 0
23 ) 8 7 4 5
 -   6 9
 -----------
     1 8 4
 -   1 8 4
 -----------
 -       0 5  Remainder
 -----------
```

Example 13 : A charity event sold 1,050 tickets on the first day, 1,280 on the second day, and 1,540 on the third day. How many more tickets were sold on the third day compared to the first day?

To find how many more tickets were sold on the third day compared to the first day, subtract the number of tickets sold on the first day from the number of tickets sold on the third day.

Solution: Given: Tickets sold on the first day = 1,050

Tickets sold on the third day = 1,540

1,540 – 1,050 = 490

So, 490 more tickets were sold on the third day compared to the first day.

Example 14 : A concert was held for five days. The number of tickets sold on each day was: 1,236, 1,472, 1,830, 2,150, and 2,640. Find the total number of tickets sold.

To find the total number of tickets sold over the five days, we add the number of tickets sold each day.

Solution: Given:

Day 1: 1,236

Day 2: 1,472

Day 3: 1,830

Day 4: 2,150

Day 5: 2,640

1,236 + 1,472 + 1,830 + 2,150 + 2,640

= 9,328

So, the total number of tickets sold is 9,328.

Example 15 : A factory produces 2,150 toys a day. How many toys will it produce in 30 days?

To find out how many toys the factory will produce in 30 days, multiply the number of toys produced per day by 30.

Solution: **Given:**

Toys produced per day = 2,150

Number of days = 30

2,150 × 30 = 64,500

So, the factory will produce 64,500 toys in 30 days.

Example 16 : A factory produces 15,000 cookies in a month. If each packet contains 50 cookies, how many full packets will be made, and how many cookies will remain unpacked?

To find how many full packets will be made and how many cookies will remain unpacked, we divide the total number of cookies by the number of cookies per packet.

Solution: **Given:** Total cookies = 15,000

Cookies per packet = 50

Step 1: Find the number of full packets by dividing the total cookies by the number of cookies per packet:

15,000 : 50 = 300 full packets

Step 2: Find how many cookies remain unpacked by calculating the remainder:

15,000 – (300 × 50) = 15,000 – 15,000 = 0

So, 300 full packets will be made, and there will be no cookies remaining unpacked.

Example 17 : A warehouse has 50,000 products. 9,500 are packed in large cartons, 12,000 in medium cartons, and 10,500 in small cartons. How many products are not packed yet?

To find how many products are not packed yet, we need to calculate the total number of products packed and subtract it from the total number of products in the warehouse.

Solution: **The number of products packed in each carton type is:**

Large cartons: 9,500

Medium cartons: 12,000

Small cartons: 10,500

Now, let's add the number of products packed in all cartons:

Total packed products = 9,500 + 12,000 + 10,500 = 32,000

Now, subtract the total packed products from the total number of products in the warehouse:

Products not packed = 50,000 – 32,000 = 18,000

So, 18,000 products are not packed yet.

Knowledge Checkpoint

- Add: 5,67,890 + 1,23,456.
- Multiply: 4567 × 12.
- Divide 1250 by 25.

Activity

The Shopping Spree

- **Materials:** A fictional catalogue of items with large price tags (e.g., Car: ₹8,50,000; Laptop: ₹75,990; Phone: ₹48,500).
- **Procedure:**
  1. Give each group a budget, for example, ₹10,00,000.
  2. Ask them to "buy" as many items as they can without exceeding the budget.
  3. They must use addition to find the total cost and subtraction to find the remaining balance.
- **Inquiry Question:** What is the most expensive combination of items you can buy within the budget?

Facts Flash

- Multiplying any number by 11 has a trick. For a 2-digit number like 43, split them (4__3) and add them (4+3=7) to put in the middle: 473. It gets a bit trickier with carrying, but the pattern holds!
- The word "**division**" comes from the Latin "**dividere**," which means "**to force apart**."

Do It Yourself

Why is division by zero undefined? Think about what division means. 12 : 4 asks "what number multiplied by 4 gives 12?". So, 12 : 0 asks "what number multiplied by 0 gives 12?". Is there such a number?

Key Terms

- **Sum:** The result of an addition.
- **Difference:** The result of a subtraction.
- **Product:** The result of a multiplication.
- **Quotient:** The result of a division.
- **Remainder:** The amount left over after a division.

Mental Mathematics

- Multiplying by 10, 100, 1000: Just add the required number of zeros to the end.
- **Addition by Compensation:** To add 598, add 600 and then subtract 2. For 345 + 598, think (345 + 600) - 2 = 945 - 2 = 943.

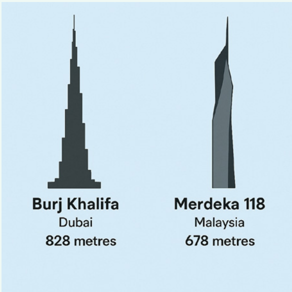
Gap Analyzer™ Homework

Watch Remedial

Exercise 3.5

1. During a book fair, the number of tickets sold on each day for a week were as follows: 1,520, 2,190, 2,760, 3,110, 3,450, 4,080, and 4,320. How many tickets were sold in total for the week?
2. There are 35,000 workers in a factory. Out of these, 4,500 workers attend their duty by car, 10,000 by bus, and 7,200 by train. How many workers attend by foot?
3. A company packs 24,000 books in boxes. If each box can hold 120 books, how many boxes will be needed and how many books will be left unpacked?
4. A warehouse has 50,000 products. 9,500 are packed in large cartons, 12,000 in medium cartons, and 10,500 in small cartons. How many products are not packed yet?
5. **World's Tallest Buildings (2023)**
   - Burj Khalifa, Dubai: 828 metres
   - Merdeka 118, Malaysia: 678 metres

   i. Find the difference in their heights.
   ii. Find the total height of both buildings.
   iii. If Burj Khalifa was reduced by 250 m (due to floors closed), what would be the new height?
   iv. How much taller is Burj Khalifa compared to Merdeka 118?



6. The NITI Aayog report stated that 13,50,00,000 (13.5 crore) people moved out of multidimensional poverty in India over a 5-year period (2016-2021). The top three contributing states were:

Uttar Pradesh	3,42,72,000 people
Bihar	2,25,32,000 people
Madhya Pradesh	1,35,76,000 people

   i. What is the total number of people who moved out of poverty from these three states combined?
   ii. How many people who moved out of poverty were from states other than Uttar Pradesh, Bihar, and Madhya Pradesh?
   iii. By how much did the number of people exiting poverty in Uttar Pradesh exceed the number in Bihar?
   iv. What was the average number of people who moved out of poverty per year across all of India during this 5-year period?

7. India's successful Chandrayaan-3 mission was celebrated worldwide. The approximate cost of the entire mission was ₹6,15,00,00,000 (615 crore). The distance from the Earth to the Moon is approximately 3,84,400 km.

   i. Write the cost of the mission in words using the International System of Numeration.
   ii. To send the spacecraft from Earth to the Moon and receive data back, a signal travels the distance twice (to the Moon and back). What is this total distance?
   iii. If the total cost of the mission (₹6,15,00,00,000) were to be funded equally by 300 large companies, how much would each company contribute?
   iv. The rover, Pragyan, was designed to travel on the lunar surface. If it traveled an average of 95 meters per day, how many meters would it travel in 14 days?

Estimation and its Applications

Do you need to know the exact number of people in a cricket stadium, or is "about 50,000" good enough? When you're shopping, do you calculate the bill to the last paisa, or do you round the prices to get a quick total? This is estimation—the art of finding a close-enough answer quickly. It's one of the most practical math skills you'll ever learn. In this section, we will master the technique of rounding numbers to estimate sums, differences, and products.

The Power of Approximation

Estimation is about finding a value that is close to the exact answer. The key technique is rounding. By rounding numbers to the nearest ten, hundred, or thousand, we can simplify complex calculations and perform them mentally.

Sub-concepts to be covered

1. Rounding to the Nearest Tens, Hundreds, and Thousands
2. Estimating Sums and Differences
3. Estimating Products and Quotients
4. Using Estimation in Real Life

Mathematical Explanation

Rounding simplifies numbers to their nearest "landmark" values (multiples of 10, 100, etc.), making mental arithmetic possible. The "5 or more, round up" rule is a convention to ensure consistency. When we estimate, we trade a little bit of accuracy for a lot of speed and convenience. The choice of what to round to (tens, hundreds) depends on the context. For a bill of ₹587 + ₹213, rounding to the nearest hundred (600+200=800) gives a rough idea. Rounding to the nearest ten (590 + 210 = 800) gives a more accurate estimate but is slightly harder to calculate mentally.

Rounding to the Nearest Tens, Hundreds, and Thousands

Estimation to the Nearest Tens: Estimating to the nearest ten is a simple and useful skill where we round numbers to the closest multiple of 10. This technique makes large numbers easier to work with, especially in mental math and real-life situations like shopping, measuring, or budgeting.

How to Estimate to the Nearest Tens:

Look at the ones place (the last digit)

- If the ones digit is 5 or more, round the number up to the next multiple of 10.
- If the ones digit is less than 5, round the number down to the previous multiple of 10.

Example:

Round 27 to the nearest ten.

The ones digit is 7, which is 5 or more, so round up to 30.

Estimation to the Nearest Hundreds: Estimating to the nearest hundred is a method of rounding numbers to the closest multiple of 100. This helps simplify larger numbers and makes calculations easier, especially when an exact value is not necessary.

Steps for Estimation to the Nearest Hundred:

1. Look at the tens digit (the second-last digit).
2. If the tens digit is 5 or more, round the number up to the next multiple of 100.
3. If the tens digit is less than 5, round the number down to the previous multiple of 100.

Example:

Round 472 to the nearest hundred.

The tens digit is 7, which is 5 or more, so we round up to 500.

Estimation to the Nearest Thousands: Estimating to the nearest thousand is a method of rounding numbers to the nearest multiple of 1,000. This makes it easier to perform quick calculations and work with large numbers when precision is not essential.

Steps for Estimation to the Nearest Thousand:

1. Look at the hundreds digit (the third digit from the right).
2. If the hundreds digit is 5 or more, round the number up to the next multiple of 1,000.
3. If the hundreds digit is less than 5, round the number down to the previous multiple of 1,000.

Example: Round 3,472 to the nearest thousand.
The hundreds digit is 4, which is less than 5, so we round down to 3,000.

Estimating Sums and Differences

When adding or subtracting, you can round the numbers first, then perform the operation.

Example: Estimate 87 + 68 by rounding both numbers to the nearest 10:
90 + 70 = 160

Estimating Products and Quotients

When multiplying or dividing, round the numbers to a convenient place value and then perform the operation.

Example: Estimate the product of 42 × 56 by rounding to the nearest 10:
40 × 60 = 2400

Using Estimation in Real Life

Estimation helps in everyday situations like shopping, cooking, or measuring distances where exact numbers may not be necessary but a close approximation is useful.

Example 18 : Estimate the following to the nearest tens.

(i) 56, (ii) 139, (iii) 222

Solution: (i) **56**: The ones digit is 6, which is greater than 5, so we round up.56 rounds to 60.

(ii) **139:** The ones digit is 9, which is greater than 5, so we round up.139 rounds to 140.

(iii) **222:** The ones digit is 2, which is less than 5, so we round down.222 rounds to 220.

Example 19 : Estimate the following numbers to the nearest hundreds

(i) 745 (ii) 1,262

Solution: (i) Round 745 to the nearest hundred:

The tens digit is 4, which is less than 5, so we round down.745 rounds to 700.

(ii) Round 1,262 to the nearest hundred:

The tens digit is 6, which is greater than 5, so we round up.1,262 rounds to 1,300.

Example 20 : Estimate each of the following to the nearest thousands

(i) 4,123 (ii) 3,758

Solution:

(i) Round 4,123 to the nearest thousand:
The hundreds digit is 1, which is less than 5, so we round down.
4,123 rounds to 4,000.

(ii) Round 3,758 to the nearest thousand:
The hundreds digit is 7, which is greater than 5, so we round up.
3,758 rounds to 4,000.

Example 21 : If you need to estimate the cost of 47 pencils at ₹6 each, round the number of pencils to the nearest 10 and then estimate the total cost.

Solution: To estimate the cost of 47 pencils at ₹6 each by rounding the number of pencils to the nearest 10:

1. **Round 47 pencils to the nearest 10:**
   - The ones digit is 7, which is greater than 5, so we round up.
   - 47 rounds to 50 pencils.
2. **Estimate the total cost:**
   - The cost of each pencil is ₹6.
   - Multiply the rounded number of pencils (50) by the cost of each pencil (₹6)
     ₹50 × ₹6 = ₹300.
     So, the estimated total cost for 47 pencils, when rounded to the nearest 10, is ₹300.

Example 22 : Estimate the sum of 12,874 and 5,399 by rounding to the nearest thousand.

Solution:

1. Round 12,874 to the nearest thousand. The hundreds digit is 8 (> = 5), so round up. ₹13,000.
2. Round 5,399 to the nearest thousand. The hundreds digit is 3 (< 5), so round down. ₹5,000.
3. Estimated Sum = 13,000 + 5,000 = 18,000.

Example 23 : Estimate the product of 489 and 321.

Solution:
1. Round 489 to the nearest hundred 500.
2. Round 321 to the nearest hundred 300.
3. Estimated Product = 500 × 300 = 150,000.

Knowledge Checkpoint

- Round 5,432 to the nearest thousand.
- Estimate the difference: 879 - 212 (round to nearest hundred).
- Estimate the product: 78 × 23 (round to nearest ten).

Do It Yourself

If you estimate 48 × 52 by rounding to the nearest ten (50 × 50 = 2500), is your estimate higher or lower than the actual answer (2496)? Why? Think about what you added to one number and subtracted from the other.

Activity

Estimation Jar

- **Materials:** A large clear jar filled with a known quantity of marbles or beans (e.g., 876).
- **Procedure:**
  1. Show the jar to the class for 10 seconds.
  2. Ask each student to write down their estimate of how many items are in the jar.
  3. Discuss the strategies used: Did they try to count a small section and multiply? Did they just guess?
  4. Reveal the actual number and see whose estimate was closest.
- **Inquiry Question:** Is it better to overestimate or underestimate in this situation? What about when estimating the time needed to finish homework?

Facts Flash

Fermi problems are famous estimation challenges named after physicist Enrico Fermi. A classic example is "How many piano tuners are there in Chicago?" Solving it requires making a series of logical estimations, not knowing the exact answer.

Key Terms

- **Estimation:** Finding an approximate value.
- **Rounding**: Simplifying a number to its nearest multiple of 10, 100, 1000, etc.
- **Approximate:** A value that is close to the actual value.

Mental Mathematics

- Round 4,678 to the nearest hundred.
- Estimate the sum: 3,245 + 4,678, by rounding each number to the nearest thousand.
- Estimate the product: 412 × 89, by rounding each number to the nearest ten.

Gap Analyzer™ Homework

Watch Remedial

Exercise 3.6

1. **Consider the following questions:**
   (a) Estimate the sum of 468 and 657 by rounding both numbers to the nearest hundred.
   (b) Estimate the product of 76 and 59 by rounding both numbers to the nearest ten.
   (c) Round 92 and 384 to the nearest ten and then estimate the sum.
   (d) Estimate the difference between 538 and 172 by rounding both numbers to the nearest ten.
2. If a pack of pencils costs ₹34 and you buy 8 packs, estimate the total cost by rounding the price of each pack to the nearest ten.
3. Round 897, 1,235, and 2,194 to the nearest hundred, and then find their sum.
4. Round 9,876, 4,251, and 1,732 to the nearest thousand, then find their sum.
5. If a book costs ₹162 and you buy 3 books, estimate the total cost by rounding the price of each book to the nearest ten.

6. A company produces 5,632 items in one month and 3,874 items the next month. Estimate the total number of items produced by rounding each number to the nearest thousand.
7. A wedding planner is expecting 483 guests. For seating, should she estimate to the nearest 10 or 100? What is the estimate?
8. Estimate the cost of 42 chairs if each chair costs ₹895.
9. The monthly electricity bills for three months are ₹1,876, ₹2,145, and ₹980. Estimate the total bill for the three months by rounding each to the nearest hundred.
10. **A city council has a budget of ₹75,00,000 to build a new park. The estimated costs for different sections are:**
    - Landscaping and Lawns: ₹28,45,500
    - Playground Equipment: ₹19,89,000
    - Walking Tracks: ₹12,15,200
    - Benches and Lighting: ₹8,75,000

    i. Round each of the four cost items to the nearest lakh.
    ii. Using the rounded numbers from part (a), what is the estimated total cost of the park?
    iii. Based on your estimation, is the project within the city council's budget of ₹75,00,000? By approximately how much is it over or under the budget?
    iv. The actual total cost is ₹69,24,700. What is the difference between the actual cost and your estimated cost from part (b)?
11. **A municipality in a large city was granted ₹5,45,80,000 for a cleanliness drive. They spent ₹1,89,65,500 on waste management trucks and ₹78,12,000 on hiring sanitation workers.**

    Questions:
    i. Round ₹1,89,65,500 and ₹78,12,000 to the nearest ten lakh and estimate the total spent.
    ii. Using the grant ₹5,45,80,000 and your estimated expenditure, estimate the money left.
    iii. If each dustbin costs about ₹2,000, estimate how many can be bought with ₹40,00,000.
    iv. Should the accounts department use exact or estimated numbers in the final report? Why?
12. **The Nile River is about 6,650 km long, the Amazon River is 6,400 km, and the Ganga River is about 2,525 km. By estimating, we can quickly compare their lengths and understand how vast they are compared to each other.**

    Questions:
    i. Round the length of each river to the nearest hundred.
    ii. Estimate the total length of the three rivers together.
    iii. Estimate the difference between Nile and Amazon.
    iv. Which is the shortest river, and approximately by how much?

Number Patterns and Puzzles

Mathematics isn't just about calculations; it's also about discovering beautiful and mysterious patterns. In this special section, we will become number detectives, exploring fascinating puzzles and sequences. We will look at numbers that read the same forwards and backwards (**palindromes**), uncover the secret of **Kaprekar's magic number 6174**, and investigate the unsolved mystery of the **Collatz Conjecture**. Get ready to see a playful and creative side of mathematics!

The Fun Side of Numbers This section moves beyond standard arithmetic to explore recreational mathematics. We will investigate numbers with special properties and processes that lead to surprising results.

Sub-concepts to be covered

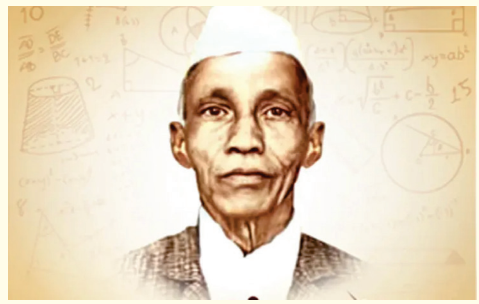
1. The Kaprekar Constant (6174)
2. Palindromic Numbers
3. Clock and Calendar Numbers in Mathematics
4. The Collatz Conjecture

Mathematical Explanation

The Kaprekar Constant (6174)

The Kaprekar Constant, named after the Indian mathematician **D.R. Kaprekar**, is a fascinating concept in mathematics. It is best demonstrated using a four-digit number, and the process always leads to the magic number 6174 in at most 7 iterations (excluding numbers with all identical digits).

D. R. Kaprekar



Dattaraya (Dattatreya) Ramchandra Kaprekar (1905–1986) was an Indian recreational mathematician from Maharashtra. A lifelong schoolteacher at Devlali, he made striking discoveries in number theory—despite having no postgraduate training—including the famous Kaprekar constant (6174) and families like Kaprekar numbers, Harshad (Niven) numbers, self/Devlali numbers, and Demlo numbers.

His work reached a global audience after Martin Gardner spotlighted it in Scientific American in 1975. Kaprekar was a self-taught Indian mathematician who made significant contributions to number theory and recreational mathematics.

Kaprekar's Process

1. Choose a four-digit number (at least two digits must be different).
2. Arrange the digits in descending order and ascending order to form two numbers.
3. Subtract the smaller number from the larger number.
4. Repeat the process with the result until you reach the magic number 6174.

Example: Let's start with the number 3524

i. **Arrange digits:**

Descending: 5432

Ascending: 2345

ii. **Subtract:** 5432–2345 = 3087

iii. **Repeat**

Descending: 8730

Ascending: 0378

Subtract: 8730 – 0378 = 8352

iv. **Repeat:**

Descending: 8532

Ascending: 2358

Subtract: 8532 – 2358 = 6174

Once 6174 is reached, repeating the process will always give 6174.

Special Cases

- If the number has all identical digits (e.g., 1111), the result will always be 0.
- For numbers with fewer than 4 digits, leading zeros must be added (e.g., 21 becomes 0021).

Palindromic Numbers

In mathematics, palindromes are numbers that read the same **forward and backward**. These numbers are simple and fun to work with, making them an exciting topic for young learners.

The process of reversing and adding often leads to a palindrome because it systematically balances the digits from both ends.

Examples of Palindromic Numbers

- Two-digit palindromes: 11, 22, 33, 44, 55, 66, 77, 88, 99
- Three-digit palindromes: 101, 121, 131, 141, 151, 161, 171, 181, 191
- Four-digit palindromes: 1001, 1221, 1331, 1441, 1551, 1661, 1771, 1881, 1991

Properties of Palindromic Numbers

1. **Symmetry:** A palindromic number looks the same when reversed.
2. **Patterns:** Palindromic numbers often follow specific patterns, making them easy to identify.
3. **Addition and Palindromes:** Sometimes, adding a number to its reverse can form a palindrome.

Activities for Students:

1. **Identify Palindromes:** List all palindromic numbers between 1 and 1000.
2. **Create Your Own Palindrome:** Take a number, reverse its digits, and add them. Repeat the process until you get a palindrome.

   Example: Start with 56 → Reverse: 65 → Add: 56 + 65 = 121 (a palindrome!).
3. **Odd and Even Palindromes:** Identify palindromes that are odd (e.g., 121, 131) and even (e.g., 44, 1221).
4. **Word Palindromes in Numbers:** Numbers can also be written in words to form palindromes.

   Example: "One" and "121" are both palindromes when reversed.

Clock and Calendar Numbers in Mathematics

Clock and calendar numbers are introduced as fun and practical concepts to help students understand **patterns, arithmetic**, and the **passage of time**. The numbers on a digital clock and a calendar page are full of fascinating patterns and puzzles. These topics are often connected to real-life scenarios, making them relatable and engaging.

Clock Numbers

Clock numbers deal with understanding time and its cyclical nature.

Key Concepts

1. **12-Hour and 24-Hour Format**
   - In a 12-hour clock, numbers repeat after 12 (e.g., 13:00 is 1:00 PM).
   - In a 24-hour clock, times are written without repetition (e.g., 1:00 PM is 13:00).
2. **Addition and Subtraction on a Clock**
   - If it's 07:00 now, what time will it be in 5 hours?
   - 7 + 5 = 12, so it will be 12:00.
   - **For subtraction:** If it's 9:00, what time was it 4 hours ago?
   - 9 – 4 = 5, so it was 5:00.
3. **Angles on a Clock**
   - Each hour on the clock represents 30° (360° divided by 12).
   - Example: At 03:00, the hands form a 90° angle.

Calendar Numbers

Calendar numbers involve understanding dates, months, years, and days of the week.

Key Concepts

1. **Days in Months:** Some months have 30 days (e.g., April, June, September, November), others have 31 days (e.g., January, March, May), and February has 28 or 29 days (leap years).

2. **Leap Years:** A leap year occurs every 4 years. A year is a leap year if it is divisible by 4 but not by 100, unless it is also divisible by 400.

   Example: 2020 was a leap year, but 1900 was not.

3. **Days of the Week:** If today is Monday, what day will it be 10 days from now?

   Days repeat in a cycle of 7. 10 mod 7 = 3, so it will be Thursday.

The Collatz Conjecture

In 1937, German mathematician Lothar Collatz proposed a fascinating conjecture that remains unsolved to this day. It shows that even simple rules can lead to incredibly complex behavior. According to his rule, starting with any positive whole number and following the steps below, the sequence will always eventually reach 1:

1. If the number is even, divide it by 2.
2. If the number is odd, multiply it by 3 and add 1.

This seemingly simple rule has captivated mathematicians because, despite extensive testing on countless numbers, no one has been able to prove that the sequence always ends in 1 for every positive integer.

Lothar Collatz — the mind behind the "3n + 1" puzzle

Lothar Collatz (1910–1990) was a German mathematician whose work bridged pure ideas and real-world computation. He's best known for proposing the deceptively simple Collatz conjecture (1937), the still-unsolved "3n + 1" problem that captivates students and researchers alike.

Beyond that famous riddle, Collatz contributed to numerical analysis and linear algebra—most notably the Collatz–Wielandt formula linked to the Perron–Frobenius eigenvalue—and, with Ulrich Sinogowitz, helped launch spectral graph theory in a 1957 paper.

Lothar Collatz was awarded many honors and prizes for his contributions to this field of mathematics.

Collatz Conjecture: Here's how it works:

1. Pick any whole number (positive number).
2. If the number is even, divide it by 2.

   Example: $8 \rightarrow 4$
3. If the number is odd, multiply it by 3 and add 1.

   Example: $5 \rightarrow 5 \times 3 + 1 = 16$
4. Repeat the steps with the new number.

   Keep going until you reach 1.

   Example: Starting number: 10

   i. 10 is even $\rightarrow 10 : 2 = 5$

   ii. 5 is odd $\rightarrow 5 \times 3 + 1 = 16$

   iii. 16 is even $\rightarrow 16 : 2 = 8$

   iv. 8 is even $\rightarrow 8 : 2 = 4$

   v. 4 is even $\rightarrow 4 : 2 = 2$

   vi. 2 is even $\rightarrow 2 : 2 = 1$

Example 24 : Start with the number 68. Find the palindrome by the reverse-and-add method.

Solution: 1. 68 + 86 = 154
2. 154 + 451 = 605
3. 605 + 506 = 1111 (A palindrome!)

Example 25 : Show the Kaprekar process for the number 2024.

Solution: 1. **Largest:** 4220. **Smallest:** 0224 (or 224).
2. 4220 - 224 = 3996.
3. **Largest:** 9963. **Smallest**: 3699.
4. 9963 - 3699 = 6264.
5. **Largest:** 6642. **Smallest:** 2466.
6. 6642 - 2466 = 4176.
7. **Largest:** 7641. **Smallest:** 1467.
8. 7641 - 1467 = 6174.

Knowledge Checkpoint

- Is 54345 a palindrome?
- What is the first step of the Kaprekar process for the number 5381?
- What is the next number in the Collatz sequence after 12?

Activity

Palindrome Hunt

- **Materials:** Pen and paper.
- **Procedure:**
  1. Challenge students to find a 2-digit number that takes the most steps to become a palindrome (**Hint:** try numbers in the 80s and 90s).
  2. The number 89 is famous for taking 24 steps!
- **Inquiry Question:** Do all numbers eventually become palindromes through this process? (This is another unsolved problem!)

Facts Flash

- **The date February 2nd, 2020, was a universal palindrome:** 02/02/2020. It reads the same in DD/MM/YYYY and MM/DD/YYYY formats!
- The number 6174 is sometimes called the "black hole" of numbers because the Kaprekar process always gets "sucked" into it.

Do It Yourself

Does the Kaprekar process work for 3-digit numbers? Try it with a few examples (e.g., 582). Do you always reach the same number? (**Hint:** you will always reach 495).

Key Terms

- **Palindrome:** A number that reads the same forwards and backwards.
- **Conjecture:** A mathematical statement that is believed to be true but has not been formally proven.
- **Sequence:** An ordered list of numbers.

Mental Mathematics

- **Palindrome Check:** To quickly check if a number is a palindrome, compare the first and last digits, then the second and second-to-last, and so on, moving inwards.
- **Collatz Steps:** For even numbers, halving is easy. For odd numbers, think of $3n + 1$ as $(2n + n) + 1$. For 15, it's $(2 \times 15 + 15) + 1 = 30 + 15 + 1 = 46$.

Gap Analyzer™ Homework

Watch Remedial

Exercise 3.7

1. **a) What happens when you square 297? Is it a Kaprekar number? Show the process.**
   b) Check if 9999 is a Kaprekar number. Show the steps to prove your answer
   c) What is the Kaprekar number of 45? Explain the steps to get the answer. (*Hints*: 45^2)
2. The number 6174 is known as Kaprekar's Constant. Let's test the process with the starting number 4817.
   a) Arrange the digits of 4817 to form the largest and smallest possible numbers. What is the difference between them?
   b) Take the result from part (a) and repeat the Kaprekar process. What is the new result?
   c) Continue the process until you reach 6174. How many steps did it take in total?
   d) What happens if you try to apply the Kaprekar process to the number 3333? Explain why it doesn't work.
3. A palindromic number reads the same forwards and backwards (like 121). We can try to create a palindrome by taking a number, reversing its digits, and adding the two together. Let's start with the number 78.
   a) Reverse the digits of 78 and add it to the original number. What is the sum?
   b) The result from part (a) is not a palindrome. Repeat the "reverse-and-add" process with this new number.
   c) Continue the process until you find a palindrome. What is the final palindromic number, and how many steps did it take to find it?
   d) Find a 3-digit number that becomes a palindrome in just one step. Show your work.
4. The Collatz Conjecture follows a simple set of rules: if a number is even, divide it by 2; if it's odd, multiply by 3 and add 1. The conjecture states that any starting number will eventually reach 1. Let's take a journey starting with the number 19.
   a) Is 19 even or odd? Apply the correct rule to find the next number in the sequence.
   b) Write out the entire sequence of numbers starting from 19 until you reach 1.
   c) What was the highest number reached during this journey before it started to decrease towards 1?
5. A secret number has left behind a set of clues. Use your detective skills to identify the number.
   - **Clue 1:** I am a 4-digit number.
   - **Clue 2:** I am a palindrome.
   - **Clue 3:** My thousands digit is 7.
   - **Clue 4:** The sum of all my digits is 24.

   a) Based on the clues, what is the secret number?
   b) Is your secret number from part (a) divisible by 11? Use the divisibility rule for 11 to prove your answer.
   c) If you apply the first step of the Kaprekar process to this secret number, what is the result?

6. An explorer has discovered two ancient numerical sequences.
   - Pattern A: 3, 7, 11, 15, ...
   - Pattern B: 5, 10, 20, 40, ...

   a) What is the simple rule for generating the next number in Pattern A?
   b) What is the simple rule for generating the next number in Pattern B?
   c) Write the next three numbers for both Pattern A and Pattern B.
   d) Which pattern do you think will grow faster? Explain your reasoning in one or two sentences.

Common Misconceptions

Misconception: (Place Value) "The number 100,000 is 'one hundred thousand' in both Indian and International systems."

Correction: While it is 'one hundred thousand' in the International system, it is correctly called 'one lakh' in the Indian system. The names change even if the value is the same.

Misconception: (Forming Numbers) "To form the smallest number with digits 5, 0, 2, 8, I just arrange them in ascending order: 0258."

Correction: A number cannot start with zero if it's meant to be a 4-digit number. 0258 is actually the 3-digit number 258. You must place the next smallest digit (2) first, then the zero. The correct smallest 4-digit number is 2058.

Misconception: (Estimation) "Estimation gives a wrong answer, so it's not useful."

Correction: Estimation gives an approximate answer, not a wrong one. Its purpose is speed and convenience, not perfect accuracy. It's extremely useful for checking if an exact answer is reasonable or for making quick decisions when precision isn't needed.

Real Life Number Play: Mathematical Applications

- **Event Scheduling:** To figure out when two friends, who visit a library every 3 days and 5 days respectively, will meet there again, you find the Lowest Common Multiple (LCM). This is used in planning and scheduling recurring events.
- **Creating Equal Groups:** A teacher with 20 boys and 24 girls wants to form the largest number of identical teams. The Highest Common Factor (HCF) will tell her exactly how many teams she can make with the same mix of boys and girls.
- **Fair Sharing:** Divisibility rules are mental shortcuts. You can instantly know if 126 chocolates can be shared equally among 3, 6, or 9 friends without counting, making sure everything is fair.
- **Digital Security:** Prime numbers are the secret heroes of the internet! They are used to create strong codes (cryptography) that keep our online messages, passwords, and bank details safe from others.

Gap Analyzer™
Complete Chapter Test

EXERCISE

A. Choose the correct answer.

1. The number 'Fifty crore, twenty-three lakh, forty-seven' in figures is:

   (a) 50,23,047 ☐ (b)50,23,00,047 ☐ (c) 5,23,47,000 ☐ (d) 50,00,23,047 ☐

2. Which of the following numbers is divisible by 9?

   (a) 12345 ☐ (b) 56789 ☐ (c) 98765 ☐ (d) 45678 ☐

3. The estimated product of 592 and 205 is:

   (a) 120000 ☐ (b) 100000 ☐ (c) 12000 ☐ (d) 150000 ☐

4. The difference between the place value and face value of 7 in 9,75,432 is:

   (a) 69,993 ☐ (b) 70,000 ☐ (c) 69,999 ☐ (d) 0 ☐

5. Which of these is NOT a prime number?

   (a) 23 ☐ (b) 41 ☐ (c) 51 ☐ (d) 61 ☐

Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting it is given.
Study both the statements and state which of the following is correct:

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

1. **Assertion (A):** The number 4560 is divisible by 6.
Reason (R): A number ending in 0 is divisible by both 2 and 3.

2. **Assertion (A):** The smallest 5-digit number formed by 0,1,2,3,4 is 10234.
Reason (R): To form the smallest number, digits must be arranged in ascending order.

3. **Assertion (A):** 1 is a prime number.
Reason (R): 1 is only divisible by itself.

Case Study

Scenario: The "Clean Your City" campaign has a budget of ₹2,50,00,000. They spend ₹85,45,500 on new garbage trucks, ₹42,10,800 on hiring cleaning staff for a year, and ₹15,00,000 on awareness programs.

1. What is the total expenditure of the campaign?
2. How much money is left in the budget?
3. If they want to buy new dustbins that cost ₹2,500 each, estimate how many dustbins they can buy with the remaining money.

Project

Plan a Dream Holiday

- **Objective:** To use all the skills learned in this chapter (large numbers, operations, estimation) to plan a fictional family holiday.
- **Instructions:**
  1. **Choose a Destination:** Pick a country you want to visit.
  2. **Budget:** Your family has a total budget of ₹8,00,000.
  3. **Research & Calculate:**
  - ♦ **Flights:** Find the approximate cost of a round-trip flight ticket for one person. Multiply it by the number of family members (e.g., 4). (e.g., ₹68,500 × 4)
  - ♦ **Accommodation:** Find the cost of a hotel per night. Multiply it by the number of nights you plan to stay (e.g., 7 nights). (e.g., ₹8,750 × 7)
  - ♦ **Activities:** Plan 3-4 activities (like visiting a theme park, museum, etc.) and find their ticket costs. Calculate the total for the family.

4. **Create a Budget Sheet:**
   - ♦ List all your calculated expenses.
   - ♦ Calculate the total exact cost of your trip.
   - ♦ Calculate the money remaining from your ₹8,00,000 budget.
   - ♦ Write the total cost and remaining money in words (Indian System).
5. **Estimation Check:** Before calculating exactly, create an "Estimated Budget" by rounding all costs to the nearest thousand. Compare your estimated total with your actual total. Was your estimate close?
   - ♦ **Final Product:** A one-page report titled "My Dream Holiday to [Country]" with your budget sheet, calculations, and a short paragraph on whether your trip was within budget.

Source-Based Question

India on the Move

Directions: Read the following text and analyze the data table about new vehicle registrations in India for the years 2021 and 2022. Then, answer the questions that follow.

India is one of the largest and fastest-growing automobile markets in the world. Every year, millions of new vehicles are registered across the country. These include two-wheelers like scooters and motorcycles, passenger cars, commercial buses, and goods vehicles like trucks. The Ministry of Road Transport and Highways (MoRTH) keeps a record of these registrations, which helps us understand the country's growth and transportation trends.

Table: New Vehicle Registrations in India (2021 vs. 2022)

Vehicle Category	Registrations in 2021	Registrations in 2022
Two-Wheelers	1,34,66,421	1,58,62,087
Cars, Jeeps, Taxis	25,78,901	30,99,712
Buses	45,678	51,205
Goods Vehicles	7,16,532	8,88,409

(Source: Data adapted from annual reports of the Ministry of Road Transport and Highways (MoRTH), Government of India.)

New Vehicle Registrations

Type	Registrations in 2021	Registrations in 2022
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Buses	45,678	51,205
Goods Vehicles	7,16,532	8,88,409

Questions

1. Write the number of Two-Wheelers registered in 2022 in words, according to both the Indian and the International systems of numeration.
2. How many more Goods Vehicles were registered in 2022 compared to 2021? Calculate the exact difference.
3. What was the total number of 'passenger vehicles' (which includes 'Cars, Jeeps, Taxis' and 'Buses') registered in the year 2022?
4. Estimate the total number of Two-Wheelers and Cars, Jeeps, Taxis registered in 2021 combined. First, round both numbers to the nearest lakh, and then find the estimated sum.
5. Using the digits from the number of Buses registered in 2021 (4, 5, 6, 7, 8) only once, what is the greatest and the smallest 5-digit number you can form?

Mind Map

Number Play

Place Value Systems

- Understanding large numbers
- **Indian System** → Ones, Thousands, Lakhs, Crores
- **International System** → Ones, Thousands, Millions, Billions
- **Place Value vs. Face Value**
- Writing numbers in Expanded Form

Operations on Large Numbers

- **Addition (carrying)**
- **Subtraction (borrowing)**
- **Multiplication (long method)**
- **Division (quotient & remainder)**

Comparing & Ordering Numbers

- **Rules of Comparison**
  - ✓ **Rule 1:** Count the digits
  - ✓ **Rule 2:** Compare from left to right
- **Ordering Numbers**
  - ✓ Ascending (small → large)
  - ✓ Descending (large → small)
- **Forming Numbers**
  - ✓ Greatest possible number
  - ✓ Smallest possible number (special case with 0)
- **Number Line visualization**

Supercells

- **Definition:** Number greater than its neighbors in a grid
- 25 50 30 → 50 is a Supercell
- Use in puzzles & computer science

Estimation & Applications

- **Rounding** → nearest 10, 100, 1000
- **Estimating Operations** → sums, differences, products, quotients

Playing with Digits & Divisibility

- **Foundations:** Factors, Multiples, Prime, Composite
- **Divisibility Rules:**
  - ✓ By 2, 5, 10 → last digit
  - ✓ By 3, 9 → sum of digits
  - ✓ By 6 → check 2 & 3
  - ✓ By 11 → alternating sum

Number Patterns & Puzzles

- Kaprekar Constant (6174)
- **Palindromes & Reverse**-Add process
- Clock & Calendar numbers in math
- **Collatz Conjecture** → Even ÷2, Odd ×3+1