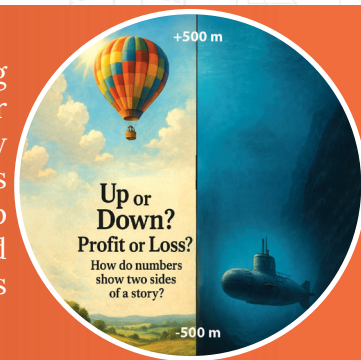




Integers

Why This Chapter Matters

Have you ever been in an elevator that goes to a basement parking lot? The buttons might show B1, B2. Or have you checked the weather in a very cold place like Leh, where the temperature drops below zero? How do we talk about numbers that are less than nothing? This is where a new, exciting set of numbers comes in: Integers. They help us describe opposites—like up and down, profit and loss, forward and backward. What is the opposite of winning 10 points in a game? Let's find out!



Meet EeeBee.AI



Hello, explorers! I'm EeeBee, your friendly robot guide. The world of numbers is bigger than you think! I was designed to explore deep space, where temperatures can be extremely cold—way below zero! I use integers all the time to calculate my position and manage my energy levels (gaining and losing power). I'll be here to share fun facts, give you hints, and ask tricky questions. Let's start our journey into the exciting world of integers together!



Learning Outcomes

By the end of this chapter, students will be able to:

- Define and identify positive integers, negative integers, and zero.
- Represent integers on a number line and compare their values.
- Calculate the absolute value of any integer.
- Perform addition and subtraction of integers using a number line and rules.
- Solve real-life word problems involving temperature, elevation, and money using integers.
- Apply the concept of additive inverse to simplify calculations.

From Last Year's Notebook

- In previous grade, you mastered Whole Numbers (0, 1, 2, 3, ...).
- Now, we'll expand our number family by introducing their opposites: Negative Numbers (-1, -2, -3, ...).
- Together, positive numbers, negative numbers, and zero form the set of Integers.
- Your skills in ordering and comparing numbers will be the perfect foundation for this new topic!

Real Math, Real Life

Integers are used in many real-world situations:

- **Science:** Meteorologists use them to report temperatures above and below zero.
- **Finance:** Bankers track credits (+) and debits (-) in accounts.
- **Geography:** Geographers measure heights of mountains and depths of oceans from sea level (0).
- **Entertainment:** Video games use them to show gaining or losing points and lives.

Quick Prep

1. What is the smallest whole number?
2. If you have 5 chocolates and you give away 5, how many are left?
3. Arrange these numbers from smallest to largest: 19, 2, 56, 0, 100.
4. What is $100 - 20$?
5. If you climb 10 steps up a ladder and then 3 steps down, how many steps are you from the ground?
6. Can you subtract a larger number from a smaller number (e.g., $5 - 8$) using only whole numbers? Why or why not?

Introduction

So far, your world of numbers probably started at zero and went up. But what happens when you owe your friend ₹10? Or when the temperature in winter drops below the freezing point? We need a new kind of number to describe these situations. In this section, we will officially meet the family of integers. We'll learn what they are, how to visualize them on a special line, and how to tell which integer is bigger or smaller.

Chapter Overview

- **Central Idea:** THE WORLD OF INTEGERS
- **What are Integers?:** Explore positive numbers, negative numbers, zero, and their visual representation on a number line.
- **Comparing & Ordering:** Use symbols ($>$, $<$), the 'right is greater' rule, and understand absolute value as the distance from zero.
- **Operations:** Master addition and subtraction using number line movements and key rules like 'adding the inverse'.
- **Real-World Applications:** See how integers are used in temperature, elevation, and finance.
- **Special Challenges:** Apply your knowledge to fun activities like the Hollow Integer Grid.

From History's Pages

Did you know that negative numbers were once considered “**absurd**”? For centuries, the idea of a number less than zero was confusing. It was the brilliant Indian mathematician Brahmagupta, over 1300 years ago, who finally gave clear rules for them. He described them in a simple way we can all understand: ‘**debts**’ (negative numbers) and ‘**fortunes**’ (positive numbers). This key idea helped the world accept these numbers, which are essential for mathematics today.

Integer

Imagine a straight line that goes on forever in both directions. In the exact middle, we place **Zero (0)**. All the whole numbers you know (1, 2, 3, ...) are placed to the right of zero at equal distances. These are called **Positive Integers**. But what about the left side? The left side is a mirror image. For every positive integer, there is an opposite number on the left side. These are the **Negative Integers** (-1, -2, -3, ...). Together, the positive integers, negative integers, and zero make up the set of **Integers**.

Sub-concepts to be covered

1. Natural numbers
2. **Integers:** Positive and Negative Integers
3. Representing Integers on a Number Line
4. Ordering of Integers
5. Absolute Value

Natural numbers

In mathematics, the very first numbers that we studied were the counting numbers. These counting numbers are called NATURAL NUMBERS.

Natural numbers are the set of positive numbers that are used for counting and ordering. These are the numbers you probably use every day to count objects, such as 1, 2, 3, 4, and so on. They form the basis of most of the math that we use in daily life.

In mathematics, whole numbers are a set of numbers that include all the natural numbers and zero. Whole numbers are essentially the set of numbers starting from 0 and going upwards without any fractions or decimals.

Key Points about Natural Numbers:

- **Natural Numbers:** The set of numbers starting from 1 and going onwards without any end. They are written as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...



Fig. 10.1

- **Starting Point:** Natural numbers start at 1, and there is no zero or negative number in the set of natural numbers.
- **Used for Counting:** Natural numbers are used when counting objects (e.g., 5 apples, 12 students).

Zero and Natural Numbers:

- Zero (0) is not considered a natural number.
- However, zero is a very important number in mathematics, and it is used to represent the absence of any quantity.

Relationship between Natural Numbers and Integers:

Natural numbers are a part of the broader group of integers. Integers include:

- Positive numbers (which are natural numbers),
- Negative numbers (e.g., -1, -2, -3),
- Zero (which is neither positive nor negative).



Fig. 10.2

So, integers include natural numbers, negative numbers, and zero, while natural numbers are strictly positive numbers starting from 1.

Integers: Positive and Negative Integers

An integer is any whole number that can be positive, negative, or zero. Integers do not have fractions or decimal points. In mathematics, integers are a group of numbers that include all the whole numbers and their negative counterparts. They consist of:

- Positive integers: Numbers greater than zero (e.g., 1, 2, 3, 4, ...)
- Negative integers: Numbers less than zero (e.g., -1, -2, -3, -4, ...)
- Zero: The number that separates positive and negative integers.

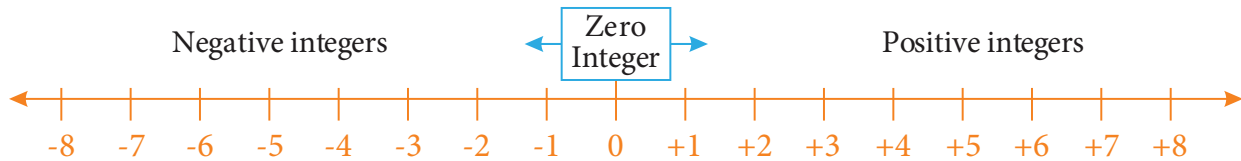


Fig. 10.3

Integers are used in many aspects of everyday life, such as in temperatures (e.g., $+5^{\circ}\text{C}$ and -5°C), financial transactions (e.g., gains and losses), and in measuring distances (e.g., above and below sea level).

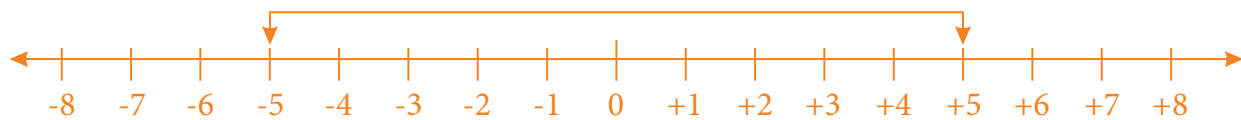


Fig. 10.4

Key Points:

- Integers can be positive or negative.
- Zero is neither positive nor negative; it is the middle point that divides the positive and negative integers.
- Integers are often represented on a number line where zero is at the center, and positive integers extend to the right while negative integers extend to the left.

Representing Integers on a Number Line

On the number line:

- Numbers to the right of zero (greater than zero) are positive integers.
- Numbers to the left of zero (less than zero) are negative integers.
- Two integers are opposites if they are the same distance from zero but are on opposite sides of it.

Example: 3 and -3 are opposites.

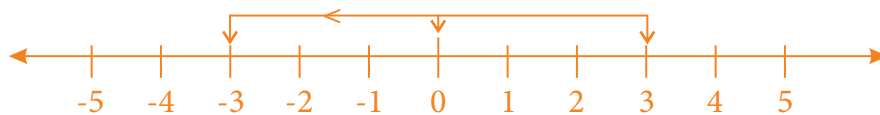


Fig. 10.5

In daily life:

- Positive numbers can represent gains, while negative numbers represent losses.
- A positive integer might represent a temperature increase, while a negative integer would represent a decrease.
- Moving forward is denoted by a positive integer, while moving backward is denoted by a negative integer.

Representation of Integers on a Number Line:

Now, we learn how to represent integers (positive and negative numbers) on a number line. Here's how it looks:

- Whole numbers (0, 1, 2, 3, etc.) are shown on the number line, starting from zero and going to the right.



Fig. 10.6

- Negative numbers (-1 , -2 , -3 , etc.) are shown to the left of zero.



Fig. 10.7

When we combine both whole numbers and negative numbers, we get integers. The number line for integers looks like this:

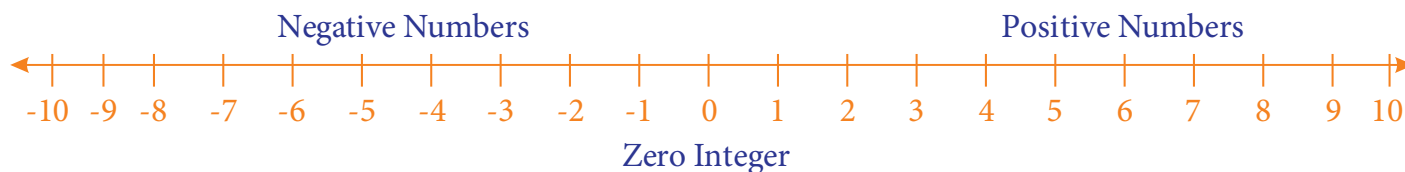


Fig. 10.8

Example 1 : Represent the following numbers on a number line.

- (i) 6 more than 4 (ii) 3 more than -6 (iii) 4 less than 7

Solution:

- (i) 6 more than 4

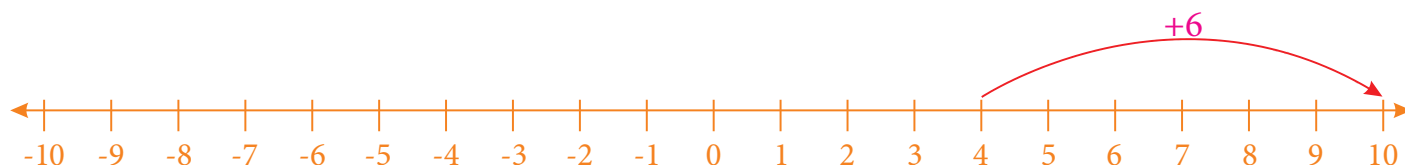


Fig. 10.9

- (ii) 3 more than -6

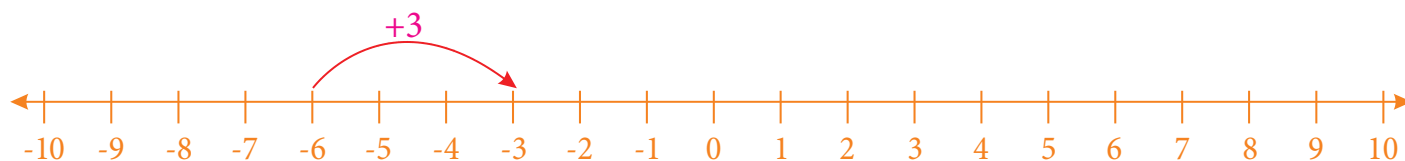


Fig. 10.10

- (iii) 4 less than 7

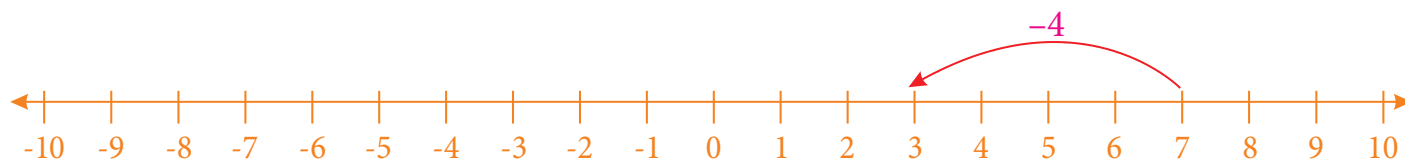


Fig. 10.11

Example 2 : Which numbers lie between -8 and -3 ? Which is the largest number and the smallest number among them?

Solution: The numbers between -8 and -3 are: -7 , -6 , -5 , -4

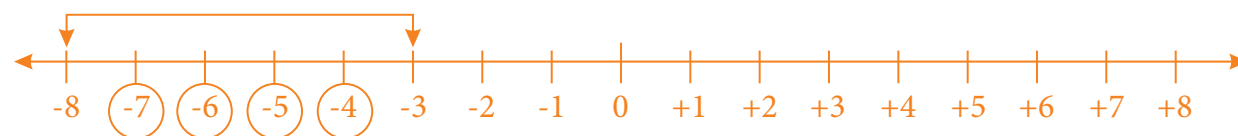


Fig. 10.12

Largest number: -4 (because -4 is closest to zero).

Smallest number: -7 (because -7 is the farthest from zero).

Ordering of Integers:

When we compare integers, we follow the rule that the number to the right on the number line is greater than the number to the left.

- 5 is greater than 4 (because 5 is on the right of 4).
- -2 is less than -1 (because -2 is to the left of -1).
- 0 is greater than -3, but less than 3.

Some important observations:

- Any positive integer is greater than any negative integer.
- Zero is neither positive nor negative. It is greater than all negative numbers and less than all positive numbers.

Common Error: Thinking that -10 is greater than -5 because 10 is greater than 5. This is incorrect. On the number line, -5 is to the right of -10, so $-5 > -10$.

Example: Which is greater, in each of the following pairs?

(i) 0 and 4

- 4 is greater than 0 because 4 is to the right of 0 on the number line.

(ii) -7 and -6

- -6 is greater than -7 because -6 is closer to zero on the number line (to the right of -7).

(iii) 177 and -83

- 177 is greater than -83 because 177 is a positive number and lies to the right of -83 on the number line.

Absolute Value

The absolute value of an integer is its distance from zero on the number line. The absolute value of a number is always positive or zero, regardless of whether the number itself is positive or negative.

The absolute value is written using vertical bars around the number, like this: $|a|$.

Example:

- The absolute value of 5 is $|5| = 5$.
- The absolute value of -5 is $|-5| = 5$.
- **Key Points:** The absolute value of any non-zero integer is always positive.

Example 3 : Write the absolute value of the following

i) 12 ii) -19 iii) -23

Solution:

i) $|12| = 12$ ii) $|-19| = 19$ iii) $|-23| = 23$



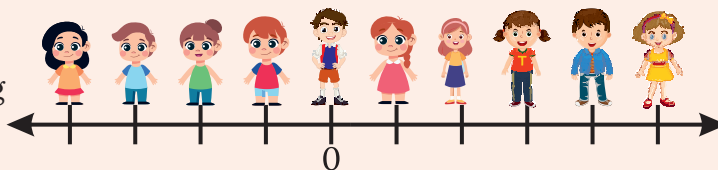
Knowledge Checkpoint

1. Is -100 a whole number? Is it an integer?
2. What is the opposite of -32?
3. Which is greater: -2 or 1?

Activity

Human Number Line

Objective: To physically understand the ordering and movement of integers.



Materials: Chalk or masking tape, index cards with integers written on them (e.g., -10 to 10).

Procedure:

1. Use chalk or tape to draw a large number line on the floor, with a clear mark for 0.
2. Ask 5-7 students to pick an integer card and stand on their number on the line.
3. Ask questions like: “Who is standing on the largest number?”, “Who is closest to zero?”, “Is the person on -3 to the left or right of the person on -1?”.
4. Give instructions like: “Person on 2, take 5 steps to the left. Where are you now?”. This introduces the idea of addition/subtraction.



Do It Yourself

If you are on a number line, and you can only take steps of size 3 or 5 (either forward or backward), can you reach every integer starting from 0? For example, to get to 1, you could go forward 2 steps of 3 (+6) and backward 1 step of 5 (-5), ending up at +1. What about reaching 2?

Key Terms

- **Integers:** The set of all positive and negative whole numbers, including zero. $\{\dots-3, -2, -1, 0, 1, 2, 3\dots\}$.
- **Positive Integers:** Integers greater than zero.
- **Negative Integers:** Integers less than zero.
- **Number Line:** A line used to visualize the order and relationship between numbers.
- **Absolute Value:** A number's distance from zero on the number line, always expressed as a non-negative value. Denoted by $||$.



Facts Flash

- The word “**integer**” comes from the Latin word integer, which means “**whole**” or “**untouched**”.
- If you add up all the numbers on a standard roulette wheel (0 to 36), you get 666. But if you consider a number system with negatives, you could create a balanced wheel that sums to 0!
- **Zero was a revolutionary idea!** It took centuries for it to be accepted as a number in its own right, rather than just a placeholder.



Mental Mathematics

- What is the opposite of -15?
- Arrange these in ascending order: -3, 5, -7, 0.
- Write the absolute value of -7.
- Which is smaller: -12 or -8?
- Which integer lies between -5 and -3?



Gap Analyzer™
Homework

Watch Remedial



Exercise 10.1

1. Complete the following sequences:

- a) -15, -13, -11, ..., ... b) -60, -70, -80, ..., ...
c) 4, 2, 0, -2, ..., ... d) -5, -3, -1, 1, ..., ...

2. Answer the following on the number line:

- a) Which number will we reach if we move 4 numbers to the right of -4?
b) Which number will we reach if we move 5 numbers to the left of 3?
c) Which number will we reach if we move 2 numbers to the right of 0 and then return 2 numbers to the left?
d) If we are at -6 on the number line, in which direction should we move to reach 7?
e) If we are at -10 on the number line, in which direction should we move to reach -5?

3. Represent the following numbers on a number line:

- a) 3, -4, 6 b) -2, -5, 5 c) 6, -2, -7

4. Using the number line, write the integer which is:

- a) 5 more than 2 b) 6 more than -4 c) 10 less than 8

5. Mark the integers on the number line and state which is greater in each of the following pairs:

- a) -3, 2 b) 4, -6 c) -8, -15 d) 0, -1 e) -12, -11



6. In each of the following pairs, which number is to the right of the other on the number line?

- a) 4, -5 b) -6, -2 c) -10, -11 d) 1, 0 e) -300, -200

7. Compare using $>$, $<$ or $=$

- a) $2 \square -3$ b) $-11 \square -10$ c) $6 \square -6$ d) $-15 \square 10$ e) $-4 \square -4$

8. Write 5 integers less than -9.

9. Write 6 positive integers greater than 10.

10. Write the absolute value of the following:

- a) -8 b) 0 c) 12 d) -19

11. In a multi-level parking garage, a car starts at the 3rd floor. It goes down 5 floors to pick up a passenger and then goes up 7 floors to find a new parking spot. If the ground floor is represented by 0, on which floor is the car now parked?



12. On a particular day, the temperatures (in °C) of four cities A, B, C, D are recorded at 6 a.m. and 6 p.m.

City	Temp (°C) at 6 a.m.	Temp (°C) at 6 p.m.
A	-4	3
B	-7	-1
C	5	-3
D	-2	-9

- a) In which city the temperature is maximum at 6 pm? In which city is it minimum at 6 pm?
b) In which city the temperature is maximum at 6 am? In which city is it minimum at 6 am?
c) At what time, the temperature is exactly between -3 and -1? For which city this temperature is recorded?
13. Imagine a long number line is drawn on the school ground. Each student is given a card with an integer written on it. They have to stand at the correct place on the number line according to the integer.
- Rahul has a card with -10.
 - Ayesha has -2.
 - Arjun has 5.
 - Meena has 15.
 - Karan has 2
- a) Name the students who will be standing on the left-side of Karan?
b) Name the students who will be standing on the right-side of Ayesha?
c) Which student is standing at 15?
d) Now, the teacher asked each student to shift 2 points towards left, at what number will each student be at now?
- Rahul _____ Arjun _____ Meena _____ Ayesha _____ Karan _____

Addition of Integers

Now that we know what integers are, it's time to learn how to work with them. What happens when you combine a profit with a loss? Or when the temperature, which is already below zero, drops even further? In this section, we will explore the rules for adding integers. We'll use the number line and a fun "token model" to make sure the process is crystal clear. Get ready to see how adding a negative number is like moving backward!

Suppose you are in a lift of a tall building. The ground floor is considered as "0". The floors above the ground floor are positive numbers, and the floors below the ground floor are negative numbers. The lift goes up by pressing the "+" button and goes down by pressing the "-" button.

Example 4 :

- If you are on the 2nd floor (i.e., +2), and you press the "-" button 3 times, which floor will you reach?
- If you are on the 5th floor (i.e., +5), and you press the "+" button 2 times, which floor will you reach?

Solution:

- Starting floor: +2
Movement: -3

Calculation:

$$(+2) + (-3) = -1$$

So, you will reach the -1st floor.

2. Starting floor: +5

Movement: +2

Calculation:

$$(+5) + (+2) = +7$$

So, you will reach the 7th floor.

Starting floor + Movement = Target floor

Example 5 : Using the concept, evaluate the following expressions:

i) $(+4) + (-5)$

ii) $(-3) + (-6)$

iii) $(+2) + (+3)$

Solution:

We know: Starting floor + Movement = Target floor

i) $(+4) + (-5)$

Starting with + 4 floor, if we press -5, we go down and reach -1 floor.

$$(+4) + (-5) = -1$$

So, the target floor is - 1.

ii) $(-3) + (-6)$

Starting with -3 floor, if we press -6, we go further down and reach -9 floor.

$$(-3) + (-6) = -9$$

So, the target floor is -9.

iii) $(+2) + (+3)$

Starting with +2 floor, if we press +3, we go up and reach +5 floor.

$$(+2) + (+3) = +5$$

So, the target floor is +5.

5
4
3
2
1
Ground Floor
-1
-2
-3
-4
-5

Fig. 10.13

Combining Integers

Adding integers is like combining movements or values. A positive integer means moving right (or a gain), and a negative integer means moving left (or a loss). We will learn how to find the final position or total value after two or more such changes happen.

Sub-concepts to be covered

1. Addition on a Number Line
2. Rules for Adding Integers
3. Additive Inverse
4. Successor and Predecessor

Addition on a Number Line

Understanding number patterns on a number line helps students visualize how numbers increase or decrease. Here's a simplified explanation, focusing on common patterns found on a number line:

Steps for Adding Integers on a Number Line:

1. Identify the first number (starting point):

- Begin by locating the first number on the number line. If the number is positive, move to the right from zero; if the number is negative, move to the left from zero.

2. Move the second number (number to be added):

- If the second number is positive, move to the right from your current position.
- If the second number is negative, move to the left from your current position.

3. Stop at the final position:

- The point where you land is the result of the addition.

Example: $-5 + 2$

1. Start at -5 (move 5 units to the left from 0).
2. Add 2 (move 2 units to the right).
3. Result: You will land at -3 .

So, $-5 + 2 = -3$

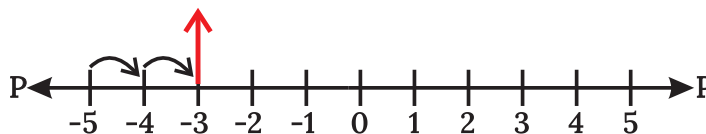


Fig. 10.14

Example: $3 + (-4)$

1. Start at 3 (move 3 units to the right from 0).
2. Add -4 (move 4 units to the left).
3. Result: You will land at -1 .

So, $3 + (-4) = -1$

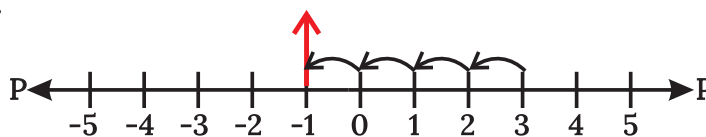


Fig. 10.15

Rules for Adding Integers

Definition: Instead of drawing a number line every time, we can use simple rules.

- **Rule 1 (Same Signs):** If the integers have the same sign (both positive or both negative), add their absolute values and keep the common sign.
- **Rule 2 (Different Signs):** If the integers have different signs, subtract the smaller absolute value from the larger absolute value. The result takes the sign of the integer with the larger absolute value.

Examples:

- Same Signs: $(+10) + (+5) = +15$. $(-10) + (-5) = -15$.
- Different Signs: $(+10) + (-5)$. Subtract $10 - 5 = 5$. The sign of $+10$ (larger absolute value) is positive. So, the answer is $+5$.
- Different Signs: $(-10) + (+5)$. Subtract $10 - 5 = 5$. The sign of -10 (larger absolute value) is negative. So, the answer is -5 .

Additive Inverse

The additive inverse of a number is the number that, when added to the original number, results in zero.

- For any integer a its additive inverse is $-a$.
- This means: $a + (-a) = 0$
or $(-a) + a = 0$

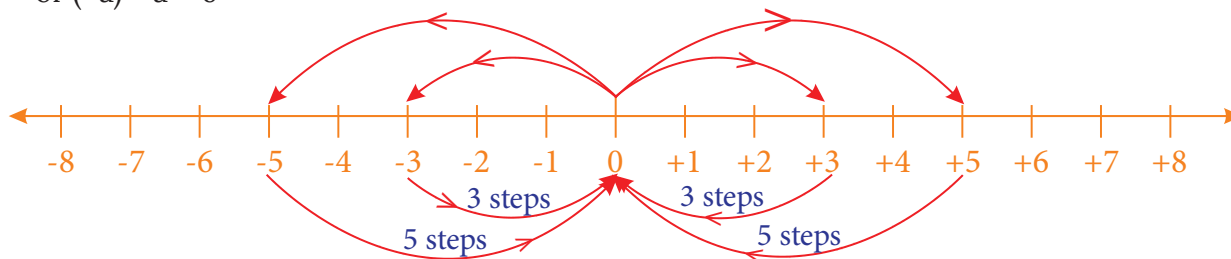


Fig. 10.16

- The additive inverse of 5 is -5 , because: $5 + (-5) = 0$
- The additive inverse of -3 is 3, because: $(-3) + 3 = 0$
- **Key Point:** Additive inverses are also called “opposites”. They are the same distance from zero on the number line but on opposite sides.

Successor and Predecessor

The successor of a number is the number that comes immediately after it when counting forward (increasing).

- To find the successor of a number, simply add 1 to the number.

Examples:

- The successor of 3 is $3 + 1 = 4$
- The successor of -5 is $-5 + 1 = -4$
- The successor of 0 is $0 + 1 = 1$

Predecessor of an Integer:

The predecessor of a number is the number that comes immediately before it when counting backward (decreasing).

- To find the predecessor of a number, simply subtract 1 from the number.

Examples:

- The predecessor of 7 is $7 - 1 = 6$
- The predecessor of -3 is $-3 - 1 = -4$
- The predecessor of 0 is $0 - 1 = -1$

The Token Model

Using Tokens for Addition

In Tara's Tower of Games, the elevator operator finds a creative way to pass time. She keeps a box filled with **positive tokens (green)** and **negative tokens (purple)**. Each time she presses the '+' button, she takes a positive token and places it in her pocket. Similarly, each time she presses the '-' button, she takes a negative token and places it in her pocket.

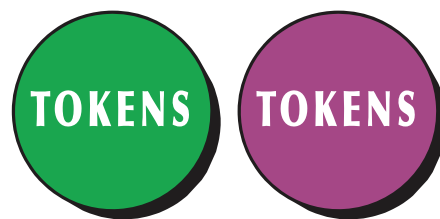


Fig. 10.17

She starts on the ground floor (Floor 0) with an empty pocket. After some time, she checks her pocket and finds 7 positive tokens and 4 negative tokens.

Question: On which floor is she now?

She pressed the '+' button 7 times and the '-' button 4 times. The total is calculated as:

$$(+7) + (-4) = +3.$$

Thus, she is now on Floor +3.



Fig. 10.18

Another Way to Visualize the Calculation:

A positive token and a negative token cancel each other out, forming a zero pair because their combined value is zero.

- In her pocket, there are 7 positive tokens and 4 negative tokens.
- Removing 4 zero pairs (4 green tokens and 4 purple tokens cancel out), she is left with 3 green tokens.

$$\text{So, } (+7) + (-4) = +3.$$

Example 6 : Add without using a number line.



Fig. 10.19

i) $25 + 18$

ii) $14 + (-32)$

iii) $(-45) + 75$

iv) $(-150) + (-350)$

v) $(-50) + (-125) + 60$

Solution:

- i) Both integers are positive.

$$25 + 18 = 43$$

- ii) One integer is positive, and the other is negative. Subtract the absolute values.

$$32 - 14 = 18 \text{ (sign of the larger number is negative, so)} 14 + (-32) = -18$$

- iii) One integer is positive, and the other is negative. Subtract the absolute values.

$$75 - 45 = 30 \text{ (sign of the larger number is positive, so)} (-45) + 75 = 30$$

- iv) Both integers are negative. Add the absolute values and keep the negative sign.

$$150 + 350 = 500 \text{ so } (-150) + (-350) = -500$$

- v) $(-50) + (-125) + 60$

$$[(-50) + (-125)] + 60 = -175 + 60 = -115$$

Example 7 : Write five distinct integers whose sum is -4 .

Solution: $-4 + 6 + (-6) + 2 + (-2) = -4 + \{6 + (-6)\} + \{2 + (-2)\} = -4 + 0 + 0 = -4$

Hence, the five integers are $-4, 6, -6, 2, -2$.

Example 8 : Calculate the sum:s

$$3 + (-3) + 3 + (-3) + 3 + (-3)$$

- i) If the total number of terms is 251.

- ii) If the total number of terms is 120.

Solution:

- i) When the total number of terms is 251 (odd):

$$3 + [(-3) + 3] + [(-3) + 3] + \dots$$

$$= 3 + 0 + 0 + \dots = 3$$

- ii) When the total number of terms is 120 (even):

$$[3 + (-3)] + [3 + (-3)] + \dots$$

$$= 0 + 0 + \dots = 0$$

Example 9 : Calculate the value of the following:

a) $A + B$

b) $C + E$

c) $F + A + B$

Solution: a) Point $A = -1$

Point $B = -4$

So, $A + B = -1 + (-4) = -1 - 4 = -5$

b) Point $C = -7$

Point $E = 7$

So, $C + E = -7 + 7 = 0$

c) Point $F = 10$

Point $A = -1$

Point $B = -4$

So, $F + A + B = 10 + (-1) + (-4) = 10 - 1 - 4 = 5$



Fig. 10.20

Examples 10 : Multiple Additions: Find the sum: $(-10) + 5 + (-20) + 12$.

Solution:

- Step 1: Group the positive and negative integers together. $(5 + 12) + ((-10) + (-20))$
- Step 2: Add the positive integers. $5 + 12 = 17$
- Step 3: Add the negative integers. $(-10) + (-20) = -30$
- Step 4: Add the results from Step 2 and Step 3. $17 + (-30) = -13$.
- Final Answer: -13.

Examples 11 : Find the successor of -99 and the predecessor of -1.

Solution:

- Successor of -99 = $-99 + 1 = -98$.
- Predecessor of -1 = $-1 - 1 = -1 + (-1) = -2$.

Knowledge Checkpoint

1. What is $(-1) + 1$?
2. Is the sum of two negative integers positive or negative?
3. What is $100 + (-50) + 80 + (-38) - 50$?

Activity

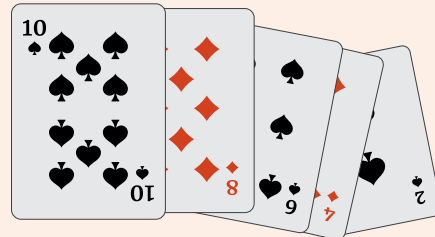
Integer Card Game

Objective: To practice adding integers quickly and strategically.

Materials: A deck of playing cards (red cards = positive, black cards = negative; A = 1, J = 11, Q = 12, K = 13).

Procedure:

1. Shuffle the deck and deal 5 cards to each player.
2. In each round, every player plays one card.
3. The players add the values of the cards played. For example, if a red 7 (+7) and a black 5 (-5) are played, the sum for the round is +2.
4. Keep a running total of the score.
5. The player with the highest score after 5 rounds wins. This encourages quick mental addition of positive and negative numbers.



Do It Yourself

Is the sum of any three consecutive integers always divisible by 3? Try it out:

- $1 + 2 + 3 = 6$ (Yes, $6/3 = 2$)
- $(-2) + (-1) + 0 = -3$ (Yes, $-3/3 = -1$)
- $(-5) + (-4) + (-3) = -12$ (Yes, $-12/3 = -4$) Why do you think this happens? (Hint: Let the middle integer be 'n'. Then the three integers are n-1, n, and n+1).

Key Terms

- **Sum:** The result of an addition operation.
- **Additive Inverse:** The opposite of a number. When added to the number, the sum is zero.
- **Successor:** The integer that is one greater than a given integer.
- **Predecessor:** The integer that is one less than a given integer.



Facts Flash

- A “**zero-sum game**” is a situation where one person’s gain is exactly equal to another person’s loss. The total change in wealth is $(+X) + (-X) = 0$. Poker and chess are examples.
- The concept of adding opposites to get zero is a form of symmetry, a very important idea in higher mathematics and physics.



Mental Mathematics

1. Use a number line to find $-5 + (-2)$ $-5 + (-2)$ $-5 + (-2)$.
2. On a number line, starting from -3 , move 3 steps to the left. Where do you land?
3. $18 + (-9) = ?$ $18 + (-9) = ?$ $18 + (-9) = ?$
4. What is the successor of 999?
5. Find the additive inverse of -100



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Exercise 10.2

1. **Simplify and find the sum:**
a) $5 + (-7) + 12 + (-3) + 25$ b) $(-20) + 35 + (-15) + (-30) + 50$ c) $48 + (-22) + (-11) + 9 + (-35)$
2. **Find the sum of:**
a) $213 + (-132) + 18 + (-56) + 75$ b) $(-10,000) + (-1200) + 4500 + (-340)$
c) $1892 + (-1923) + (-108) + 1234 + (-500)$
3. **Find the successor of the following integers:**
a) -15 b) 0 c) 89 d) -1000
4. **Find the predecessor of the following integers:**
a) 20 b) -45 c) -1 d) 400
5. **Find the additive inverse of each of the following integers:**
a) 10 b) -25 c) 99 d) -2000 e) 13
6. **Fill in the blanks with $<$, $>$, or $=$:**
a) $(-6) + (-4)$ _____ $(-4) + (-6)$ b) $(-5) + (-8)$ _____ $(-7) + (-6)$
c) $(-10) + (-15)$ _____ $(-30) + 5$ d) $(-25) + 30$ _____ $(-10) + 20$
e) $(-50) - (-25)$ _____ $(-25) - 50$

7. Write five distinct integers whose sum is -5 .

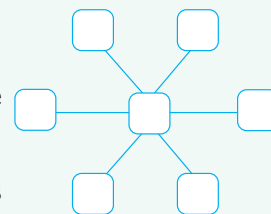
8. Write pairs of integers whose sum is:

a) 5

b) -3

c) 0

9. Use the numbers 2, 3, -7 , -8 , -10 , 14 and -19 only once, and fill in the space so that the sum is -15 along each line.

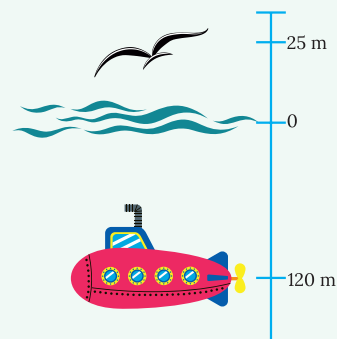


10. In a video game, you gain 20 points for collecting a gem but lose 35 points for hitting an obstacle. If you collect a gem and then immediately hit an obstacle, what is the total change in your score? (Represent this as an addition of integers).

What if in the complete game you collected 18 gems and hit 12 obstacles.

What will be your final score?

11. A remote-controlled submarine is at a depth of 120 meters below the surface of a lake. A seagull is flying directly above it at a height of 25 meters above the lake's surface. What is the vertical distance between the submarine and the seagull?



12. Complete the given addition table:

+	-3	$+1$	$+6$	-4
-3				
$+1$				
$+6$				
-4				

Subtraction of Integers

If adding a negative number is like moving backward, what does it mean to subtract a negative number? This can be one of the trickiest ideas when you first learn about integers, but we'll uncover a simple secret that makes it easy. In this section, we will learn how to subtract any integer from another. We'll see how subtraction is just a special kind of addition in disguise!

Example : $8 - 5$ can be stated as a problem in two different ways:

- I have 8 apples. I gave 5 to my friend. How many apples are left with me?
- My friend has 5 apples, and I have 8 apples. How many more apples do I have than my friend?

In general, subtraction tells us how much more or less one quantity is compared to another. Let us consider a scenario on a temperature scale. If the temperature at noon is $+6^{\circ}\text{C}$ and by evening, it drops to -3°C , what is the change in temperature?

To find this, we calculate:

Evening Temperature $-$ Noon Temperature

$=$ Change in Temperature

So, $-3 - (+6) = -9$.

This means the temperature dropped by 9°C



Fig. 10.21

Example 12 : In a parking garage, the floors are numbered as negative and positive levels. Use subtraction to determine the movement needed between floors.

- If your car is parked on floor -2 and you need to reach floor -6, what is the movement needed?
- If you are currently on floor +4 and your target is floor -1, what is the movement needed?

Solution:

- To move from floor -2 to -6, we need to go down 4 steps.
Movement needed is -4.

Expression:

$$(-6) - (-2) = -4$$

- To move from floor +4 to -1, we need to go down 5 steps.
Movement needed is -5.

Expression:

$$(-1) - (+4) = -5$$

Sub-concepts to be covered:

- Subtraction on a Number Line
- Subtraction as the Addition of the Inverse

Subtraction on a Number Line

Subtraction can also be visualized using a number line. To subtract integers:

Step 1: Start at the first number on the number line.

Step 2: Move left if subtracting a positive number or move right if subtracting a negative number.

Step 3: The point where you land is the result.

Example: Subtract $5 - 3$:

- Start at +5 on the number line.
 - Move 3 steps to the left because 3 is positive.
 - You land at +2.
- So, $5 - 3 = 2$

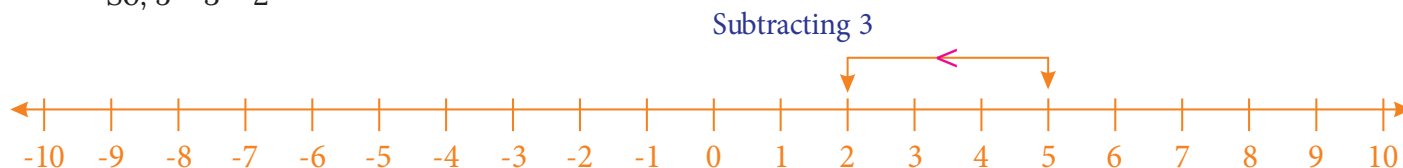


Fig. 10.23

Example: i) $3 - 9$:

- Start at +3
 - Move 9 steps to the left (because you are subtracting a positive number).
 - You land at -6
- So, $3 - 9 = -6$

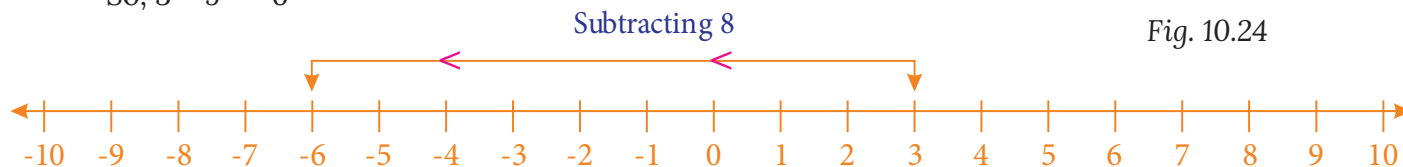


Fig. 10.24

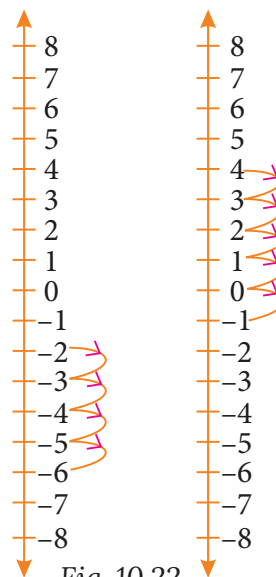


Fig. 10.22

ii) $-4 - 6$:

- Start at -4
- Move 6 steps to the left (since you are subtracting a positive number).
- You land at -10 .

So, $-4 - 6 = -10$.



Fig. 10.25

iii) $-7 - (-4)$:

- Start at -7 .
- Subtracting -4 is the same as adding 4 (because subtracting a negative number is like adding a positive one).
- Move 4 steps to the right.
- You land at -3 .

So, $-7 - (-4) = -3$.



Fig. 10.26

Subtraction as the Addition of the Inverse

Definition (The Golden Rule): To subtract an integer, add its additive inverse. $a - b = a + (-b)$, $a - (-b) = a + b$

How it works:

1. Keep the first number the same.
2. Change the subtraction sign to an addition sign.
3. Change the sign of the second number (find its opposite).
4. Follow the rules for addition.

Examples:

- $8 - 12$ becomes $8 + (-12)$. Now use addition rules: $8 + (-12) = -4$.
- $(-5) - 10$ becomes $(-5) + (-10)$. Now use addition rules: $(-5) + (-10) = -15$.
- $6 - (-9)$ becomes $6 + 9$. Now use addition rules: $6 + 9 = 15$.
- $(-7) - (-3)$ becomes $(-7) + 3$. Now use addition rules: $(-7) + 3 = -4$.

The Token Model for Subtracting Integers

In Tara's Tower of Games, Tara also uses the Token Model to handle subtraction. Just as she uses positive (green) and negative (purple) tokens for addition, she uses the same tokens for subtraction, but with a twist.

Using Tokens for Subtraction

Let's say Tara has 5 purple tokens (negative) but she wants to have only 3 purple tokens (negative) with her.

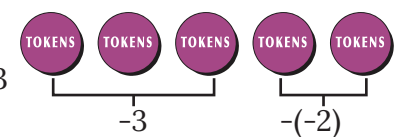


Fig. 10.27

That means, she wants to subtract (-2) . To subtract the number -2 (which is equivalent to adding positive tokens), she would need to remove tokens from her pocket.

What Happens in Tokens?

- Adding +2 means she removes 2 purple tokens from her pocket.
- This is because purple tokens represent negative numbers, and removing a negative is the same as adding a positive.

Number Sentence

$$-5 - (-2) = -5 + 2 = -3$$

Floor Analogy

If she were moving between floors:

- Start at Floor 0 \rightarrow Subtract 5 floors (purple tokens) \rightarrow Floor -5
- Subtract -2 floors (go up 2 floors) \rightarrow Floor -3

Let us understand through some examples now:

Example 13 : i) $8 - (-12)$ ii) $(-45) - 32$
 iii) $(-15) - (-30)$ iv) $-120 + 45 - 37 + 60 - 90 + 125$

Solution:

i) $8 - (-12)$

Since subtracting a negative number is the same as adding a positive number,

$$8 - (-12) = 8 + 12 = 20$$

ii) $(-45) - 32$

Since both are negative, we add the absolute values and keep the negative sign:

$$(-45)-32=(-45)+(-32)=-77$$

iii) $(-15) - (-30)$

Since subtracting a negative number is the same as adding a positive number,

$$(-15) - (-30) = (-15) + 30 = 15$$

iv) $-120 + 45 - 37 + 60 - 90 + 125$

Group the negative and positive integers:

$$[-120 - 37 - 90] + [45 + 60 + 125] = -247 + 230 = -17$$

Example 14 : Ram wrote some integers on the staircase. He asks his brother to write the subtraction statement and stand on the final staircase, it will lead to:

- If you start at -3 and move 5 steps down, where will you reach?
- Starting at 1, move 7 steps down. What number will you land on?
- If you are standing at -1 and want to reach -7 , how many steps must you move?

Solution: a) Starting from -3 and moving 5 steps down:

He will reach at -8.

b) Starting from 1 and moving 7 steps down:

He will reach at -6.

c) Starting from -1 and to reach -7:

He will have to take 6 steps downwards

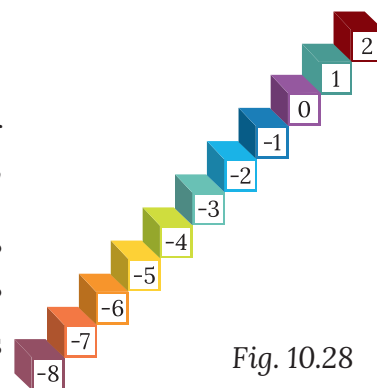


Fig. 10.28

Example 15 : Find the difference of two integers if their sum is 75 and one of them is -45.

Solution: We are given the sum as 75 and one of the integer as - 45.

To find other integer, we subtract 75 and (-45).

Required integer = $75 - (-45) = 75 + 45 = 120$

So, the other integer is 120.

Now, Difference = $120 - (-45) = 120 + 45 = 165$

Example 16 : A scuba diver is swimming 21 meters below sea level. He then descends 13 meters further. What will be his new position with respect to sea level?

Solution: Scubadiver's initial position = -21 m (as he is below the sea level)

He goes down 13 m more

So, new position = $-21 - 13 = -33$ m

Therefore, he is 33 m below the sea level.

Example 17 : Find the result of the following expression: $15 - 4 + (-7) - 3 + 12 - (-10)$

Solution: Group positive and negative integers:

$$= 15 - 4 - 7 - 3 + 12 + 10$$

$$= (15 + 12 + 10) - (4 + 7 + 3)$$

$$= 37 - 14$$

$$= 23$$

So, the result is 23.

Example 18 : Solve: $-3 - 6$ using the addition of inverse.

Solution:

$$-3 - 6 = -3 + (-6)$$

$$= -9$$



Knowledge Checkpoint

1. What is $0 - 10$?
2. What is $5 - (-5)$?
3. To subtract -20, you should add ____.

Activity

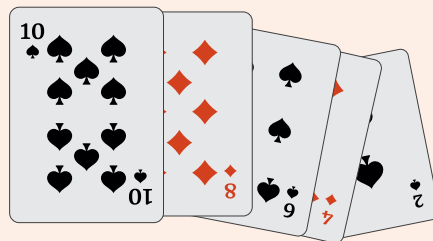
Subtraction War

Objective: To master the rule for subtracting integers.

Materials: Deck of cards (as in the addition game).

Steps:

1. Players split the deck. In each round, both players flip a card.
2. Player 1's card is the first number, and Player 2's card is the second.
3. Both players must calculate the difference: (Player 1's card) - (Player 2's card).
4. For example, if Player 1 flips a red 8 (+8) and Player 2 flips a black 6 (-6), the problem is $8 - (-6)$.
5. The first player to correctly say the answer (14) wins both cards.
6. The player with all the cards at the end wins.





Do It Yourself

Is subtraction of integers commutative?

That is, is $a - b$ the same as $b - a$? Try with a few numbers like 5 and 3. Then try with 5 and -3. What do you notice about the results of $a - b$ and $b - a$? (They are opposites!).

Key Terms

- **Difference:** The result of a subtraction operation.
- **Inverse Operation:** An operation that undoes another. Subtraction is the inverse operation of addition.



Facts Flash

- The minus sign (-) is thought to have originated from a bar that merchants in the 15th century used to mark barrels that were underweight.
- Before the minus sign was common, accountants would use the letter 'm' or 'M' for 'minus' or 'subtrahendus'.



Mental Mathematics

1. On a number line, starting from -3, move 8 steps to the left. Where do you land?
2. Starting at -8 on the number line, move 3 steps to the right. What number do you reach?
3. Rewrite $-9 - (-6)$ as an addition and solve.
4. Starting at 6 on the number line, move 5 steps to the left. What number do you reach?
5. Rewrite $15 - 20$ as an addition and solve



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Exercise 10.3

1. Find the following using a number line:

- a) $(5) - (7)$ b) $(-10) - (-15)$ c) $(12) - (-8)$ d) $(-25) - (10)$

2. Represent the following on the number line and fill in the blanks:

a) $10 - (-4) = \text{-----}$

b) $-8 - 3 = \text{-----}$

c) $11 + (-8) = \text{-----}$

3. Compute the following questions:

- a) $6 - (-10) + 20 - (-15)$ b) $-100 - (-200) + (-150) - 250$
c) $-25 + 30 - (-5) - 10 + (-40)$ d) $0 - (-8)$

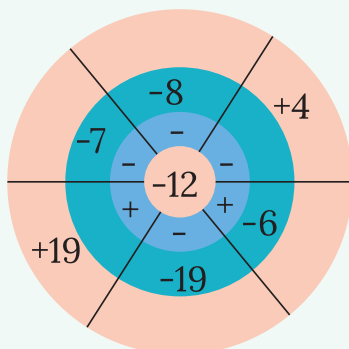
4. Observe the pattern and calculate:

$(5) + (-5) + (5) + (-5)$ repeated for 50 terms.

5. Change one sign to '+' in the expression $15-8-5-2$ to make the sum equal to 10.

6. An elevator is on the 17th floor of a building. It then moves down to the basement level at -6 . What is the total number of floors it traveled?

7. Complete the following according to the operation given below:



8. Observe the number line and answer the following:



a) + - =

b) + - =

c) + - =

d) + - =

9. If $a = -25$ and $b = 15$:

a) Find $a - b$.

b) Find $b - a$.

10. A building has floors marked as $-3, -2, -1, 0, +1, +2, +3, +4$.

- If Tina is on Floor $+3$ and moves 5 floors down, where will she be?
- If Arjun is on Floor -2 and moves 4 floors up, where will he be?
- If Lisa is on Floor $+4$ and moves 8 floors down, what will her position be?

11. Does the following statement stand True (T) or False (F):

- $a - b$ is always positive when $a > b$.
- The sum of two negative integers is always negative.
- The difference of a positive and a negative integer is always positive.

12. In a quiz with negative marking, Priya's score was -10 points. In the final "buzzer round," she answered a question incorrectly, and the rules state that 5 points must be subtracted from her score. What is Priya's final score?

13. Arjun had ₹2,000 in his bank account. He wrote a cheque for ₹2,500 to pay his rent, which the bank cleared using an overdraft facility. What is the new balance in Arjun's account?

Integers in Real-World Contexts

Mathematics is not just about numbers on a page; it's a tool for describing the world. Now that you are a pro at adding and subtracting integers, let's see them in action! In this section, we will travel from the coldest places on Earth to the highest mountains, dive deep into the ocean, and even manage a bank account. You'll solve real problems and see just how useful positive and negative numbers are in everyday life.

Integers in Action

We will now apply our knowledge of integer operations to solve multi-step problems based on realistic scenarios. This involves translating words into mathematical expressions with integers and then calculating the result.

Sub-concepts to be covered:

1. Temperature
2. Height and Depth (Elevation)
3. Credit and Debit (Banking)
4. Profit and Loss

Temperature

- **Positive Integers:** Represent temperatures above zero (e.g., $+10^{\circ}\text{C}$ means 10 degrees above freezing).
- **Negative Integers:** Represent temperatures below zero (e.g., -5°C means 5 degrees below freezing).

In news, we hear that the temperature in Jaipur during summer is 42°C , meaning it is 42°C higher than the freezing point of water (0°C). Meanwhile, the temperature in Shimla during winter is -5°C , meaning it is 5°C below 0°C . Note that Celsius ($^{\circ}\text{C}$) is a unit for measuring temperature.

Example 19 : The temperature of a place at 6:00 AM was -2°C . It increased by 4°C by 8:00 AM and then decreased by 3°C by 10:00 AM. What was the temperature at 10:00 AM?

Solution:

Initial temperature at 6:00 AM = -2°C

Increase by 4°C = $-2 + 4 = 2^{\circ}\text{C}$

Decrease by 3°C = $2 - 3 = -1^{\circ}\text{C}$

Temperature at 10:00 AM = -1°C

Height and Depth

- **Positive Integers:** Represent height above sea level (e.g., $+200$ m means 200 meters above sea level).
- **Negative Integers:** Represent depth below sea level (e.g., -50 m means 50 meters below sea level).

We measure the height of Mount Everest as $+8,848$ m above sea level, which is considered 0 m. Similarly, the depth of the Mariana Trench is $-10,984$ m, meaning it is 10,984 m below sea level. Positive numbers represent heights above sea level, and negative numbers represent depths below sea level.

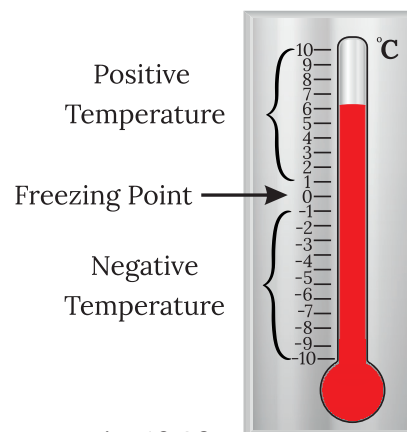


Fig. 10.29



Fig. 10.30

Example 20 : The height of a hill above sea level is +2,500 m. A climber descends 600 m and then climbs up 1,200 m. What is their final position relative to sea level?

Solution:

Initial height = +2,500 m
 After descending 600 m = $2,500 - 600 = 1,900$ m
 After climbing up 1,200 m = $1,900 + 1,200 = 3,100$ m
 Final position = +3,100 m

Credit and Debit (Banking)

- **Positive Integers:** Represent credit or money added to an account.
- **Negative Integers:** Represent debit or money withdrawn from an account.

If a person deposits ₹1,200 in their bank account, it is recorded as a credit and written as +₹1,200. If the person withdraws ₹450, it is recorded as a debit and written as -₹450. The balance is calculated as the total of credits and debits. For example:

$(+₹1,200) + (-₹450) = +₹750$, which means ₹750 remains in the account.

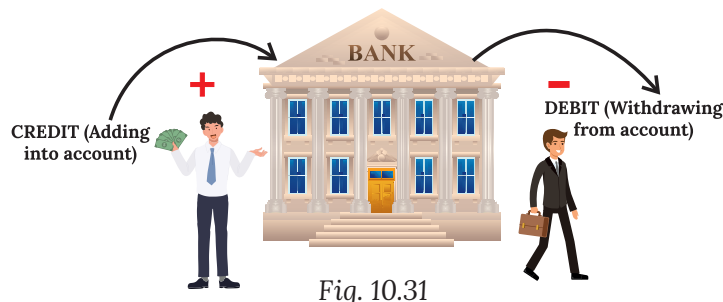


Fig. 10.31

Example 21 : A person starts with ₹1,000 in their bank account. They deposit ₹2,500 and then withdraw ₹1,800. What is the balance in their account after the transactions?

Solution:

Initial balance = ₹1,000
 After deposit = $1,000 + 2,500 = 3,500$
 After withdrawal = $3,500 - 1,800 = 1,700$
 Balance = ₹1,700

Profit and Loss

- **Positive Integers:** Represent profit (money gained).
- **Negative Integers:** Represent loss (money lost).

Suppose a trader sells one product and earns ₹200, which is a profit, written as +₹200. On another product, they incur a loss of ₹60, written as -₹60. The net income is calculated as:

$(+₹200) + (-₹60) = +₹140$, meaning the trader has an overall profit of ₹140.



Fig. 10.32

Example 22 : Rohini sold a toy car and earned a profit of ₹ 32 and on selling a teddy incurred a loss of ₹ 11. If she sold 5 such toy cars and 2 teddy bears. What will be the net profit she earned?

Solution: Profit on 1 toy car = +₹ 32

Profit on 5 toy cars = ₹ 32 × 5 = +₹ 160

Loss on 1 teddy bear = - ₹ 11

Loss on 2 teddy bears = - ₹ 11 × 2 = - ₹ 22

Net profit = +₹ 160 - ₹ 22 = +₹ 138



Fig. 10.33

Knowledge Checkpoint

1. A rise of 5°C followed by a fall of 8°C is a net change of ____.
2. A credit of ₹100 and a debit of ₹100 results in a balance change of ____.
3. Is the distance between -10m and +10m equal to 20m or 0m?

Activity

Budget Challenge

Objective: To help students understand integers through profit (+) and loss (-) situations by enacting as shopkeepers and customers.

Materials: Food Items (like apple, milk packet, bread loaf, banana, chocolate bar), Play money notes and a balance sheet chart for recording transactions.

Procedure

- **Shop Setup**

1. One student enacts the **Shopkeeper**.
2. Others are **Customers** who randomly select item cards to “buy.”

- **Transactions Begin**

1. Customer comes and says:
“I want to buy an Apple. The cost price was ₹ 50, but you are selling at ₹ 70.”
2. Shopkeeper announces profit/loss:
“I made a profit of ₹ 20 on this sale!”
3. If item is stale/near expiry, customer bargains and shopkeeper sells at lower price, recording a **loss (negative integer)**.

- **Accountant Records**

1. Each transaction is noted in the Profit-Loss Balance Sheet.
2. Example:
 - Apple → +20
 - Milk Packet → -5
 - Banana → -15

- **Game Rule**

1. The **shopkeeper's goal** is to maximise total profit by choosing how to price/sell items.
2. At the end, all integers (profits +, losses -) are added to find **Net Profit or Net Loss**.





Do It Yourself

Imagine a game where you start at 0. On your turn, you can either add 5 or subtract 3. Can you reach the number 1? What about the number 7? Are there any numbers you can never reach? This is a simple version of a problem in a field of math called Diophantine Equations.

Key Terms

- **Credit/Deposit:** An amount added to an account (positive).
- **Debit/Withdrawal:** An amount removed from an account (negative).
- **Profit:** A financial gain (positive).
- **Loss:** A financial deficit (negative).
- **Elevation:** Height above or below a reference point, usually sea level.



Facts Flash

- The concept of a “**negative**” quantity is used in photography. A photographic negative is an image where the light and dark areas are reversed. When “**added**” to the printing process, it produces the final, positive image.
- In video production, timecodes can run into negative numbers before the official start of a program (e.g., for countdowns).



Mental Mathematics

- The temperature in Leh was -8°C . It increased by 5°C . What is the new temperature?
- A submarine is at -120 m. It rises by 50 m. What is its new position?
- A mountaineer is at 400 m above sea level and climbs 200 m higher. Where is he now?
- The bank balance is ₹ -250 . A deposit of ₹300 is made. What is the new balance?
- A lift is on floor -2 . It goes up 5 floors. Which floor is it on now?



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Exercise 10.4

1. Provide the missing information in the blanks:

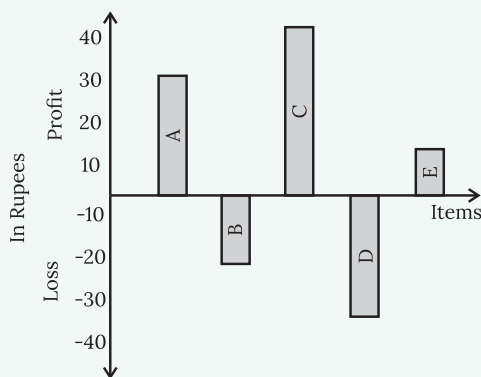
- If the submarine is at -300 meters, then it is at a depth of ____ meters below sea level.
- The temperature of a place decreased from 12°C to 7°C , so the temperature dropped by _____ $^{\circ}\text{C}$.
- If you start with ₹1,000 in your bank account and withdraw ₹500, your new balance is _____.
- The height of Mount Everest is _____ meters above sea level.
- A shopkeeper earns a profit of ₹150 and incurs a loss of ₹50, so the total profit or loss is _____.

2. Does the following statement stand True (T) or False (F):

- a) A gain in weight is represented as a negative number.
- b) The temperature of -2°C is higher than 3°C .
- c) The height of a mountain above sea level is always a positive number.
- d) A deposit of ₹1,000 in a bank account is represented as a negative number.
- e) The depth of a mine is represented by a negative integer.

3. Represent the following situations as integers with appropriate signs:

- a) A balloon is flying 1,500 m above the ground.
 - b) A fish swims at a depth of 300 m below sea level.
 - c) A deposit of ₹7,000.
 - d) Losing a weight of 10 kg.
 - e) A temperature of 6°C below freezing point.
 - f) A loss of ₹250.
 - g) Decrease in speed by 12 km/h.
 - h) 4 km below sea level.
4. A shopkeeper earns a profit of ₹250 on Monday, a loss of ₹150 on Tuesday, and a profit of ₹100 on Wednesday. Find the net profit or loss for the three days.
5. A plateau is at a height of +2,400 m above sea level, while a trench is at -1,800 m below sea level. Which point is higher, and by how much?
6. The city of Delhi recorded a maximum temperature of 42°C in summer, while Leh recorded a minimum temperature of -15°C in winter. What is the difference between these two extreme temperatures?
7. A shopkeeper sold 5 different items in one day. The profit and loss (in ₹) are shown on the integer graph below:



- a) From the graph, which items were sold at a loss?
- b) Which item gave the highest profit?
- c) Find the difference between profit on Item C and loss on Item D.
- d) Calculate the net profit or loss made by the shopkeeper.
- e) If the shopkeeper manages to sell Item D at no loss (₹ 0 instead of -₹ 35), what would be his new net profit?

8. The following table shows the temperature changes in a hill station over 24 hours:

Time of Day	Temperature Change	Resulting Temperature
6:00 AM (morning)	Starting temperature = 5°C	?
12:00 Noon	Increased by 4°C	?
3:00 PM	Decreased by 7°C	?
9:00 PM	Further decreased by 6°C	?
Midnight (12 AM)	Increased by 3°C	?

Complete the table and determine the temperature at midnight.

9. Imagine a bank account statement showing the following transactions for Ravi in one week:

Day	Transaction Type	Amount (₹)
Monday	Credit (Salary)	+5000
Tuesday	Debit (Groceries)	-1200
Wednesday	Debit (Electricity Bill)	-2300
Friday	Credit (Gift)	+1500
Saturday	Debit (Movie & Food)	-1000

- Find Ravi's balance at the end of Saturday if he started with ₹ 0 in his account.
- On which day did he have the **lowest balance**?
- Represent credits as positive integers and debits as negative integers. Write the **expression** showing all his transactions.

The Hollow Integer Grid

A hollow integer grid is a rectangular arrangement of numbers where only the integers on the outer boundary are shown. Our task is to use our integer addition skills to analyze the properties of this grid by summing the numbers along its edges.

Example 23 : Example of a Hollow Integer Grid

In the grid, the sum of integers along the top row, bottom row, left column, and right column are same.

Solution:

- Top Row Sum = $(-5) + 3 + 6 = 4$
- Bottom Row Sum = $7 + 4 + (-6) = 4$
- Left Column (Including repeated corners) Sum = $(-5) + 2 + 7 = 4$
- Right Column (Including repeated corners) Sum = $6 + 4 + (-6) = 4$

-5	3	6
2		4
7	4	-6

Table 10.1

Example 24 :

-4	1	-3
-4		-2
2	-7	-1

Table 10.2

Check whether it follows the properties of hollow grid or not.

Solution:

1. **Find the sum of the top row:** Top Row Sum = $(-4) + 1 + (-3) = -6$.

2. **Find the sum of the right column:** Right Column Sum = $(-3) + (-2) + (-1) = -6$

3. **Find the sum of the bottom row:** Bottom Row Sum = $(2) + (-7) + (-1) = -6$

4. **Find the sum of the left column:** Left Column Sum = $(-4) + (-4) + 2 = -6$

Since, sum of all borders are same, so it is forming a Hollow Square Grid.

Let us see one more grid magic!

For any square or rectangular grid, circle any random number of your choice and strike out that particular row and column.

3	-4	1	-11	12	24
13	22	23	-8	-14	12
21	-12	-14	21	10	24
-20	-17	21	44	-1	19
-36	15	9	32	21	23

Table 10.3

Let us circle 23 and strike out its row and column.

3	-4	1	-11	12	24
13	22	23	-8	-14	12
21	-12	-14	21	10	24
-20	-17	21	44	-1	19
-36	15	9	32	21	23

Table 10.4

Similarly, circle another number which is not on any of the striked numbers.

Striking off 24 this time with its row and column.

3	-4	1	-11	12	24
13	22	23	-8	-14	12
21	-12	-14	21	10	24
-20	-17	21	44	-1	19
-36	15	9	32	21	23

Table 10.5

Continue the process until all numbers are striked off.

3	-4	1	-11	12	(24)
13	22	(23)	8	-14	12
21	(-12)	-14	21	10	24
-20	17	21	(44)	1	19
-36	15	9	32	(-21)	23

Table 10.6

Now, add all the circled numbers: $24 + 23 + (-12) + 44 + 21 = 100$

Example 25 : Circle the numbers one by one and strike off the particular rows and columns and check if sum of the circled numbers at end is less than 25 or not.

41	-14	11	-10	-1	2
-9	12	21	22	-3	33
12	-23	-8	-18	34	16
11	-4	-10	32	7	15
29	11	15	-5	-20	11
-30	1	9	14	-29	12

Table 10.7

Solution: Starting with a random number, circle the number one by one and strike off its row and column, until all numbers are covered.

(41)	-14	11	-10	-1	2
-9	12	21	(22)	3	33
12	(-23)	-8	-18	34	16
11	-4	(-10)	32	7	15
29	11	15	-5	(-20)	11
-30	1	9	14	-29	(12)

Table 10.8

* It can have multiple answers.

Finding sum = $41 + 22 + (-23) + (-10) + (-20) + 12 = 22$

Yes, the sum can be less than 25.



Knowledge Checkpoint

1. A horizontal line of numbers in a grid is called a _____.
2. What is the first step in finding the sum of a column?
3. If the top row sum is -5 and the bottom row sum is 5, what is the sum of these two rows?



Activity

Grid Creators

Objective: To apply integer addition in a creative, problem-solving context.

Materials: Paper, Pencils

Steps:

1. Challenge students to create a 3×3 hollow integer grid with specific properties.
2. **Challenge 1 (Easy):** Create a grid where the Top Row Sum is +10.
3. **Challenge 2 (Medium):** Create a grid where the Top Row Sum and the Bottom Row Sum are both 0.
4. **Challenge 3 (Hard):** Create a grid where the sum of all four sides (Top, Bottom, Left, Right) are equal. This is a variation of a magic square.
5. Students can swap their created grids with a partner to check the calculations.



Do It Yourself

Can you create a 3×3 hollow integer grid where the Border Sum is 0, but none of the integers on the grid are 0?

Key Terms

- **Grid:** A network of horizontal and vertical lines forming squares.
- **Row:** A horizontal line of entries in a grid.
- **Column:** A vertical line of entries in a grid.
- **Border Sum:** The sum of all the numbers on the outer edge of a grid.



Facts Flash

- Sudoku is a type of grid puzzle, but it uses rules of logic instead of arithmetic.
- A “**magic square**” is a grid where every row, column, and both main diagonals add up to the same number, called the “**magic constant**”. The Lo Shu Square, a 3×3 magic square using numbers 1–9, is an ancient Chinese legend.



Mental Mathematics

1. In a 3×4 hollow grid starting at 1 in the top-left and increasing by 1 each step, what is the top-right corner number?
2. In a hollow integer grid, if the bottom-left is -8 and numbers increase by 2 along the row, what is the bottom-right corner number in a row of 5 numbers?
3. A hollow grid has 12 numbers along its boundary. If the smallest number is -5 and each step adds 1, what is the largest number?
4. Two dice have faces $\{-10, -5, 0, 5, 10, 15\}$. What will be the smallest and largest sums?

Exercise 10.5



Gap Analyzer™
Homework

Watch Remedial



1. Provide the missing information in the blanks:

- The sum of integers along the top row of the grid $(-2, 5, -7)$ is _____.
- The sum of integers along the left column of the grid $(-2, 3, 8)$ is _____.
- The sum of integers along the bottom row of the grid $(8, -4, 1)$ is _____.
- The sum of integers along the right column of the grid $(-7, 6, 1)$ is _____.

-2	5	-7
3		6
8	-4	1

2. Consider the following hollow integer grid:

- Find the sum of integers along the top row.
- Find the sum of integers along the bottom row.
- Find the sum of integers along the left column (including repeated corners).
- Find the sum of integers along the right column (including repeated corners).

4	-2	5
3		-1
-3	2	6

3. Circle the numbers in the grid one by one, and each time strike out the entire row and column of the chosen number. Continue this process until no numbers are left to choose. At the end, find the sum of all the circled numbers. Check whether this sum is greater than 30 or not.

-12	33	21	10	-9	11
-4	3	-11	8	29	31
11	31	-7	5	21	12
63	23	10	-12	33	22
2	-21	4	-7	45	43
23	-31	9	7	26	-22

4. Fill in the missing numbers in the given number grids.

	4	
3		12
	1	

a) Border Sum = 3

	3	
11		4
	-10	

b) Border Sum = -7

5. A necklace has a total of 80 beads in the pattern given below. Here, if represents -1 and represents +2, then find the value of the necklace.



6. Circle the pairs of numbers horizontally, vertically or diagonally, which adds up to -3.

7	1	3	3	-6
3	0	2	1	4
4	-7	4	8	2
2	1	0	2	-5
1	-4	-2	5	-1

7. Two dice each have faces $\{-2, -2, -1, -1, 3, 3\}$.

- Determine the smallest and largest sums possible?
- Which sums are possible when these two dice are thrown together?
- Which integers in between are impossible?

Common Misconceptions

Misconception: “-8 is greater than -3.”

Correction: This is incorrect. On a number line, the number to the right is always greater. -3 is to the right of -8, so $-3 > -8$. Think about temperature: -3°C is warmer (a higher temperature) than -8°C .

Misconception: “To subtract, you just take the smaller number from the bigger one.”

Correction: This only works for simple whole number subtraction. For integers, you must follow the rule: Subtracting is adding the opposite. For example, in $5 - 10$, you can't do $10 - 5$. The correct way is $5 + (-10)$, which equals -5.

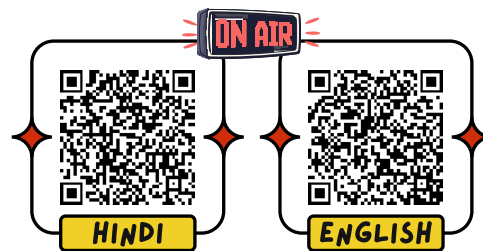
Misconception: “The absolute value of a number can be negative.”

Correction: Absolute value means distance from zero. Distance can never be negative. It is always positive or zero. $|-5| = 5$, not -5.



Real-Life Applications: Integers

- Integers help us describe situations that have an opposite or go below a starting point (zero). Here's how we use them:
- Temperature:** We describe weather using integers. A hot day in the city might be $+40^{\circ}\text{C}$, while a freezing day in the mountains could be -2°C . Zero represents the freezing point of water.
- Money and Finance:** Bank accounts use integers. A deposit of ₹500 is a positive change (+500), while a withdrawal of ₹100 is a negative one (-100). Profit is positive; a loss is negative.
- Elevation and Depth:** The height of land is measured relative to sea level (0). A mountain peak is at a positive altitude, while a submarine explores the ocean at a negative depth.
- Lifts and Buildings:** In a building, the ground floor is 0. Floors above ground are +1, +2, etc., while basement levels are represented by -1, -2.





Gap Analyzer™
Take an Exam

EXERCISE



A. MCQs (Multiple Choice Questions)

- Which of the following is the smallest integer?
a) -1 ☐ b) -100 ☐ c) 0 ☐ d) -50 ☐
- The sum of an integer and its additive inverse is:
a) 1 ☐ b) -1 ☐ c) 0 ☐ d) The integer itself ☐
- What is the result of $(-15) - (-25)$?
a) -40 ☐ b) 40 ☐ c) -10 ☐ d) 10 ☐
- The predecessor of -99 is:
a) -98 ☐ b) -100 ☐ c) 98 ☐ d) 100 ☐
- A diver is at -20m. He swims up 5 m. His new position is:
a) -25 m ☐ b) -15 m ☐ c) 15 m ☐ d) 25 m ☐

Assertion & Reason

Instructions: Each question has two statements, Assertion (A) and Reason (R). Choose the correct option:

- A) Both A and R are true, and R is the correct explanation of A.
B) Both A and R are true, but R is not the correct explanation of A.
C) A is true, but R is false.
D) A is false, but R is true.

1. **Assertion:** $(-5) - (-8) = 3$.

Reason: Subtracting an integer is equivalent to adding its additive inverse.

2. **Assertion:** The successor of -1 is 0.

Reason: The successor of any integer n is $n + 1$.

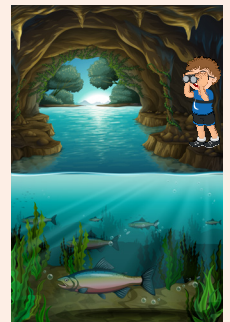
3. **Assertion:** Any positive integer is greater than any negative integer.

Reason: On a number line, positive integers are to the right of negative integers

Case Study

Scenario: An explorer is mapping a newly discovered cave system. Sea level is the reference point (0 m). The cave entrance is at +30 m.

- Day 1: He descends 50 m from the entrance.
- Day 2: He descends another 40 m.
- Day 3: He finds an underwater river and dives 15 m into it.
- Day 4: He ascends 60 m to set up a camp.



Questions:

1. What is the explorer's elevation at the end of Day 1?
2. What is his elevation at the end of Day 3?
3. What is his final elevation at the camp set up on Day 4?
4. What is the total vertical distance between the cave entrance and his lowest point?

Project

My Monthly Pocket Money Tracker

Objective: To apply integer operations to manage a personal budget over one month.

Task:

1. Create a "Transaction Diary" for one month (you can use real or imagined data).
2. Start with your opening balance (or an assumed balance of +₹500).
3. Record every instance of receiving money as a positive integer (e.g., "Pocket Money: +₹500", "Gift: +₹200").
4. Record every instance of spending money as a negative integer (e.g., "Snacks: -₹80", "Book: -₹150", "Movie: -₹250").
5. At the end of each week, calculate your weekly subtotal.
6. At the end of the month, write a final integer expression adding your opening balance and all transactions to find your closing balance.

Presentation:

Create a final report with a table of your transactions and a summary paragraph explaining your spending habits. Did you end with a profit (savings) or a loss (deficit)? What was your biggest expense? This project connects integers directly to financial literacy.

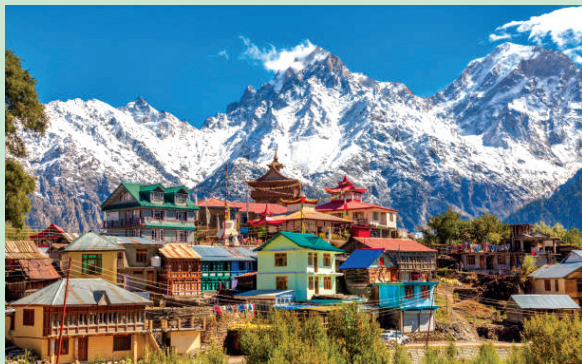
Source-Based Question

Shimla (Himachal Pradesh), December 25, 2024 (ANI):

Severe cold gripped Himachal Pradesh in the past 24 hours, with Shimla recording a minimum of 2°C . High-altitude areas like Kinnaur, Lahaul-Spiti, Kullu, Chamba, and Dalhousie fell to sub-zero levels, while plains such as Hamirpur, Bilaspur, Sirmour, Solan, and Kangra also reported biting cold.

The lowest temperature was in Tabo (Lahaul-Spiti) at -10.6°C , followed by Kukumseri -8.2°C , Kaza -6.9°C , Kalpa (Kinnaur) -4.0°C , Narkanda -2.8°C , and Rekong-Peo -0.9°C . Manali recorded -0.3°C , Kufri 0.1°C , while Dharamshala was 5.02°C .

Residents reported hardships with frozen water and difficult commutes. Local resident Rajesh said coping with the severe cold was becoming "challenging," though the Shimla administration has stepped up snow clearance and safety measures.



Adapted from reports published in Babushahi News (2024) on Shimla and surrounding areas' temperature records.

Deputy Commissioner Anupam Kashyap confirmed that major roads around Shimla have been reopened, with extra manpower deployed in heavy-snowfall areas. With Christmas and New Year approaching, Shimla has been divided into traffic zones, parking is restricted on smaller roads, and additional staff deployed for smooth movement.

With Christmas and New Year celebrations approaching, the administration has taken several steps to ensure smooth traffic and safety. Shimla city has been divided into different zones for better traffic management, and authorised parking has been restricted on smaller roads to prevent congestion.

DC Kashyap explained, “Given the Winter Carnival and the influx of tourists, we have deployed additional resources to manage traffic. Advisories have been issued, and we appeal to tourists to follow them. Traffic is being diverted according to the weather conditions to ensure smooth movement.”

Kashyap underlined the importance of cooperation from both locals and tourists. “I hope for the cooperation of everyone. Tourists visiting Shimla for the New Year should leave with fond memories. With everyone’s support, we can ensure smooth operations and a pleasant experience for all,” the DC added.

The administration has issued weather advisories, urging visitors to exercise caution while travelling in the region. Efforts are underway to ensure a hassle-free experience for both tourists and locals during the peak winter season.

As Shimla prepares for the festive season amid the intensifying cold, authorities remain focused on addressing the challenges posed by heavy snowfall and the surge in tourist activity.

Directions:

The state of Himachal Pradesh recently experienced severe cold conditions. Weather reports highlighted that several regions recorded sub-zero temperatures, while others faced near-zero or slightly above-zero conditions. The table below summarises some recent data.

Location (Himachal Pradesh)	Temperature Recorded
Tabo (Lahaul-Spiti)	-10.6 °C
Kukumseri	-8.2 °C
Kaza	-6.9 °C
Kalpa (Kinnaur)	-4.0 °C
Narkanda	-2.8 °C
Rekong-Peo (Kinnaur)	-0.9 °C
Manali	-0.3 °C
Kufri	0.1 °C
Shimla (capital city)	2.0 °C
Dharamshala	5.02 °C

Questions on the Data

1. Which place recorded the lowest temperature in Himachal Pradesh?
2. What is the difference in temperature between Shimla (+2.0 °C) and Tabo (-10.6 °C)?
3. Write the temperatures of Kufri and Dharamshala as integers (approximate to the nearest whole number).
4. Which of the given locations had temperatures below zero? Write their names.
5. If the temperature in Shimla drops by 5 °C the next day, what will its new minimum temperature be?

Integers

Introduction to Integers

- ❖ Meaning of integers (positive, negative, zero)
- ❖ Representation of integers on number line
- ❖ Ordering of Integers
- ❖ Absolute Value

Subtraction of Integers

- ❖ Subtraction on a Number Line
- ❖ Subtraction as the Addition of the Inverse

Hollow Integer Grid

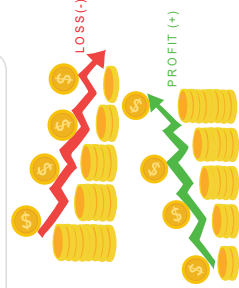
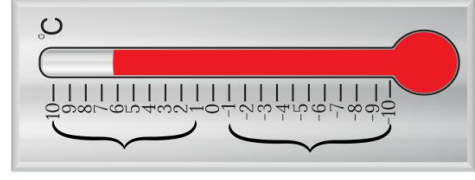
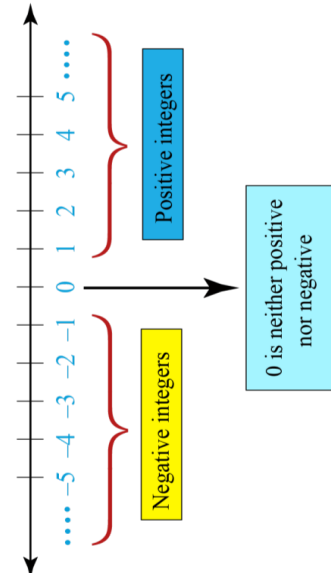
- ❖ Understanding position of integers on a grid.
- ❖ Practice locating integers in a 2D grid.
- ❖ Visualizing integer operations using grid.

Addition of Integers

- ❖ Addition on a Number Line
- ❖ Rules for Adding Integers
- ❖ Additive Inverse
- ❖ Successor and Predecessor

Integers in Other Places

- ❖ Temperature (above/below zero)
- ❖ Bank transactions (credit/debit)
- ❖ Heights and Depth levels (above/below sea level)
- ❖ Profit and Loss



Mind Map