1 Patterns in Mathematics

Why This Chapter Matters

Have you ever noticed that the petals of a flower, the scales of a pineapple, and the spiral of a galaxy all follow a secret code? This code is the language of patterns. Mathematics is not just about numbers; it's about being a detective, finding clues, and predicting what comes next. What if you could predict the next day's weather, the next move in a video game, or the next big trend, all by understanding patterns? This chapter will unlock the secrets of this mathematical language.

Meet EeeBee.AI

Hi, I'm EeeBee! I'm a friendly robot from a planet where everything runs on patterns. I love finding logic and order in the universe. Throughout this chapter, I'll be your guide. I'll pop up with helpful hints, ask tricky questions to make you think, and share some amazing facts about the patterns that shape our world.

Learning Outcomes

By the end of this chapter, you will be able to:

- Identify and describe arithmetic and geometric number patterns.
- Extend number and shape sequences by finding the rule.
- Visualize and explain special patterns like square, cube, and triangular numbers.
- Analyze relationships between different number sequences.
- Apply your understanding of patterns to solve real-world problems.

From Last Year's Notebook

- When you learned multiplication tables and how to skip count, you were actually learning about number patterns.
- **A Simple Example:** The multiplication table for 4 is a pattern: 4, 8, 12, 16... The rule is simple: add 4 to get the next number.
- We will move beyond just seeing patterns. Our new goal is to become "pattern detectives" and figure out the rule that creates each pattern.
- **Your Detective Tools:** You already have the tools you need! We will use addition, subtraction, multiplication, and division to build and decode even the most complex patterns.

Real Math, Real Life

Did you know that patterns are all around us, forming the foundation of many things we see and use every day?

- **In Architecture:** Architects use geometric patterns to design buildings that are not only beautiful but also strong and stable.
- **In Technology:** Computer programmers use logical patterns to write the code for your favorite games, apps, and websites.
- **In Music:** Musicians create catchy melodies and rhythms by arranging notes and sounds in specific patterns.
- **In Finance:** Financial analysts study number patterns in the stock market to understand trends and make predictions.

Quick Prep

1. What comes next? Circle, Square, Triangle, Circle, Square, ___?
2. **Fill in the blank:** 10, 20, 30, ___, 50.
3. If today is Monday, what day will it be in 10 days? How did you figure it out?
4. Look at the tiles on your classroom floor. Do you see a repeating pattern? Describe it.
5. What is the rule for this sequence: 50, 45, 40, 35...?
6. A frog jumps 2 steps forward and 1 step back. If it starts at 0, where will it be after 3 sets of jumps?

Introduction

Welcome to the world of patterns! A pattern is a series of numbers, shapes, or objects that repeat in a predictable way based on a specific rule. Think of it as a secret recipe. Once you figure out the recipe (the rule), you can continue the pattern forever! In this section, we will learn about the two most fundamental types of number patterns: Arithmetic and Geometric. Mastering these will give you the basic tools to become a master pattern detective.

Chapter Overview

In this chapter, we will become pattern detectives! Here's our mission:

- **Understanding Number Patterns:** We'll start by defining rules and sequences, then explore fun visual patterns like square and triangular numbers.
- **Relating Number Sequences:** We'll discover how patterns can be combined, like adding odd numbers to form perfect squares.
- **Advanced Numerical Patterns:** We'll crack tougher codes involving alternating operations (+5, -2) and growing steps (×2, ×3).
- **Patterns in Shapes:** We'll move beyond numbers to explore geometric patterns like rotations, tiling (tessellations), and amazing fractals.

From History

The study of number patterns is ancient. While early civilizations used them for practical tasks like construction, it was the Greek **Pythagoreans** (around 500 BCE) who first studied numbers for their own sake. They were fascinated by "figurate numbers," like triangular and square numbers, believing these patterns revealed the harmony of the universe. Their deep curiosity about the properties and relationships within numbers laid the essential groundwork for number theory, a major branch of mathematics we still explore today.

Types of Number Patterns

Number patterns are sequences where numbers are **connected by a rule**. To understand patterns, we need to find this rule. Is a number being added each time? Or multiplied? The two main types of patterns we'll explore are Arithmetic Patterns, which are based on addition or subtraction, and Geometric Patterns, which are based on multiplication or division. Understanding these two types is the first step to unlocking more complex and exciting mathematical sequences.

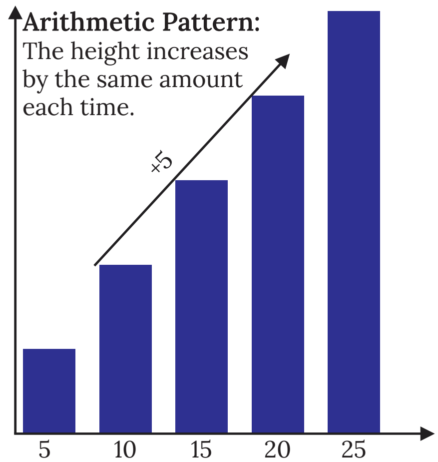


Fig. 1.1

A simple bar chart showing an arithmetic progression.

Sub-concepts to be covered

1. Arithmetic Patterns
2. Geometric Patterns

Mathematical Explanation

Arithmetic Patterns

An **arithmetic pattern** is like walking up or down a staircase, where every step is the exact same height. This "**step height**" is a fixed number that you add or subtract each time to get the next number in the pattern.

This fixed number is called the **common difference**.

1. **Increasing Pattern:** If you are adding the number, the pattern grows.
   - **Example:** 5, 10, 15, 20, ...
   - **Rule:** To find the common difference, we take any term and subtract the one before it: 10 - 5 = 5. The rule is "add 5".
   - **Next Term:** 20 + 5 = 25.
2. **Decreasing Pattern:** If you are subtracting the number, the pattern shrinks.
   - **Example:** 18, 15, 12, 9, ...
   - **Rule**: 15 - 18 = -3. The rule is "subtract 3".
   - **Next Term:** 9 - 3 = 6.

Geometric Patterns

A **geometric pattern** grows or shrinks by multiplying or dividing by the same number every time. Think of a tiny plant that doubles its height every day—it's following a geometric pattern!

This fixed multiplier is called the **common ratio**.

1. **Growing Pattern:** When you multiply by a number greater than 1, the pattern grows very quickly.
   - **Example:** 2, 6, 18, 54, ...
   - **Rule:** To find the common ratio, we divide any term by the one before it: 6 ÷ 2 = 3. The rule is "multiply by 3".
   - **Next Term:** 54 × 3 = 162.
2. **Shrinking Pattern:** When the multiplier is a fraction (like $\frac{1}{2}$ or $\frac{1}{4}$), the pattern gets smaller.
   - **Example:** 100, 50, 25, ...
   - **Rule:** 50 ÷ 100 = $\frac{1}{2}$. The rule is "divide by 2" or "multiply by $\frac{1}{2}$".
   - **Next Term:** 25 ÷ 2 = 12.5.

A diagram showing a geometric progression.

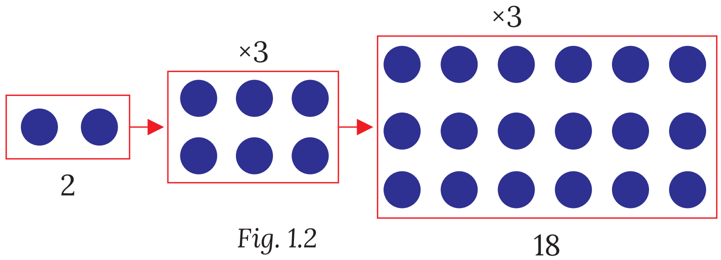


Fig. 1.2

Geometric Pattern: The number of dots is multiplied by the same amount each time.

Numbers Patterns (with Visualising Number Sequences)

A number pattern is a sequence of numbers that follows a specific **rule or logic**. These patterns help us recognize relationships between numbers and understand mathematical concepts better.

In **earlier classes**, we have explored different types of numbers and sequences. Some of the most basic and intriguing patterns are based on these number sequences. Below are examples of common number patterns, along with their visual representations to make them easier to understand.

Figurate Numbers and Basic Sequences

Some number sequences are famous because they can be represented by geometric shapes. These are called **figurate numbers**. We will revisit the fundamental sequences of **Natural**, **Odd**, and **Even numbers** that form the basis of all number patterns. By arranging dots or blocks, we can see why these numbers get their names and understand the rule that generates them.

Sub-concepts to be covered

1. Natural Numbers or Counting Numbers
2. Odd Numbers
3. Even Numbers
4. Cube Numbers
5. Triangular Number
6. Square Number
7. Hexagonal Numbers

Mathematical Explanation

Natural Numbers or Counting Numbers

"Natural numbers, also known as counting numbers. They are the numbers we use for counting objects." They start from 1 and go on infinitely. These numbers do not include zero or any negative numbers.

Set of Natural Numbers: Sequence: {1, 2, 3, 4, 5, 6...}

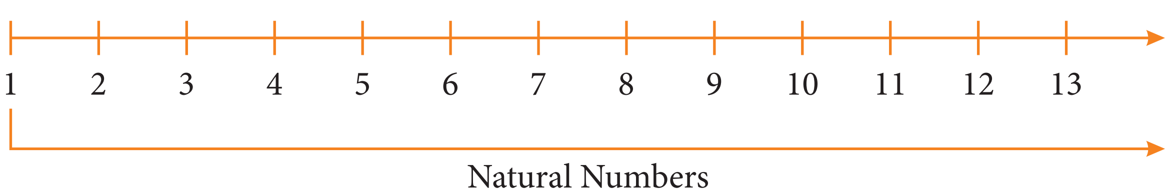


Fig. 1.3

Odd Numbers

Odd numbers are natural numbers that cannot be divided evenly by 2. In other words, they leave a remainder of 1 when divided by 2. These numbers have 1, 3, 5, 7, or 9 in their units place.

Set of Odd Numbers: Sequence: {1, 3, 5, 7, 9, 11, 13, ...}

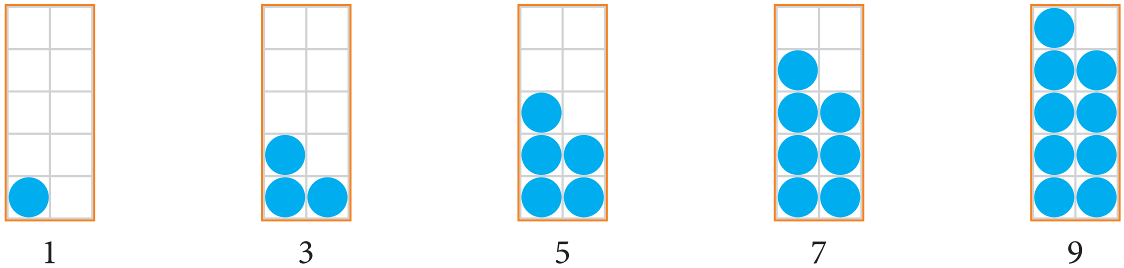


Fig. 1.4

Even Numbers

Even numbers are natural numbers that can be divided evenly by 2, meaning they leave no remainder. These numbers always have 0, 2, 4, 6, or 8 in their units place.

Set of Even Numbers: Sequence: {2, 4, 6, 8, 10, 12 ...}

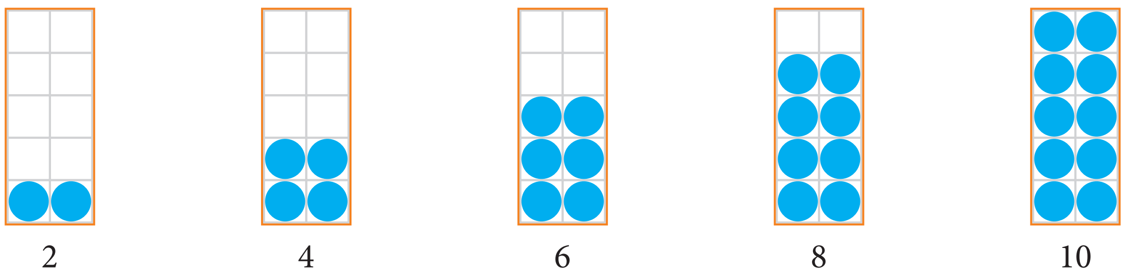


Fig. 1.5

Cube Numbers

A **cube number** is the result of multiplying a number by itself three times. In other words, it is a number raised to the power of 3 (n^3). Cube numbers are also called "**perfect cubes.**"

Formula: $n \times n \times n = n^3$

First Five Cube Numbers:

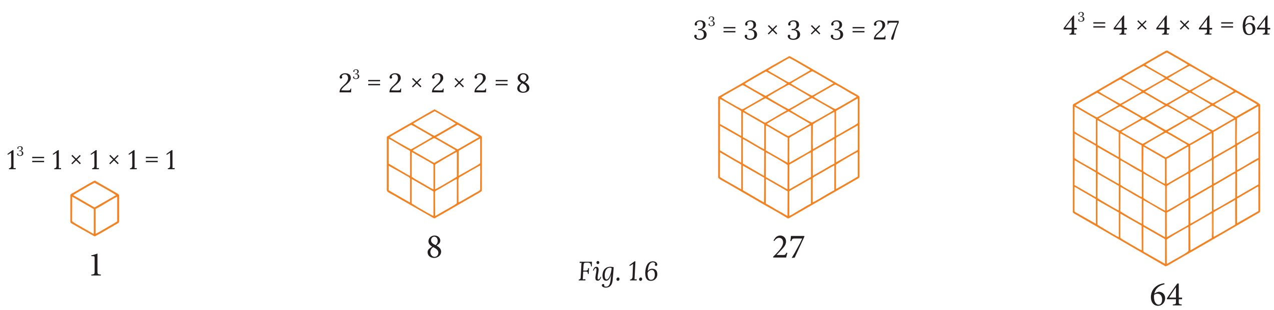


Fig. 1.6

Triangular Number Patterns

Triangular numbers are a sequence of numbers that can be arranged in the shape of an equilateral triangle. Each triangular number represents a pattern where each successive term is the total number of dots required to form a triangle.

If n is a natural number, the n^{th} triangular number is:

$$\textbf{Formula : } T_n = \frac{n(n+1)}{2}$$

- $T_1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$
- $T_2 = \frac{2(2+1)}{2} = \frac{6}{2} = 3$
- $T_3 = \frac{3(3+1)}{2} = \frac{12}{2} = 6$
- $T_4 = \frac{4(4+1)}{2} = \frac{20}{2} = 10$
- $T_5 = \frac{5(5+1)}{2} = \frac{30}{2} = 15$

So, the sequence of triangular numbers is: **1, 3, 6, 10, 15**

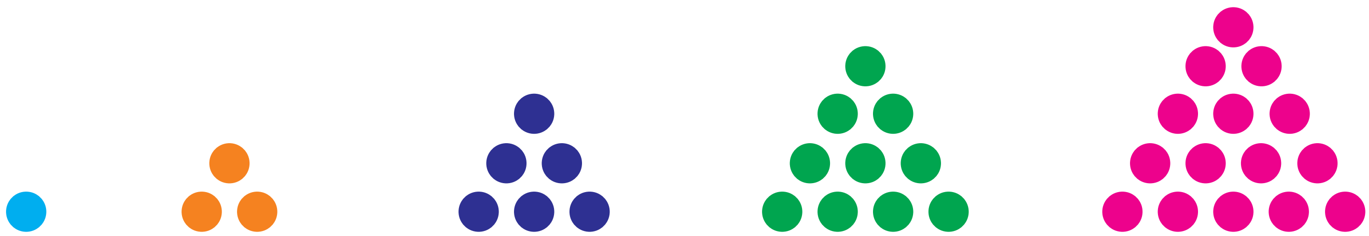


Fig. 1.7

- **The Pattern:** The sequence of triangular numbers is 1, 3, 6, 10, 15, ...
- **The Rule:** To find the next number, you add the next whole number in the sequence. After adding 5 to get 15, the next step is to add 6.
- **Next Term:** 15 + 6 = 21.

This pattern is a wonderful example of how numbers can create shapes!

Square Number Patterns

Square numbers are numbers that can be expressed as the product of an integer multiplied by itself. In other words, a square number is the result of squaring a number, or raising it to the power of 2.

Formula = $n \times n = n^2$

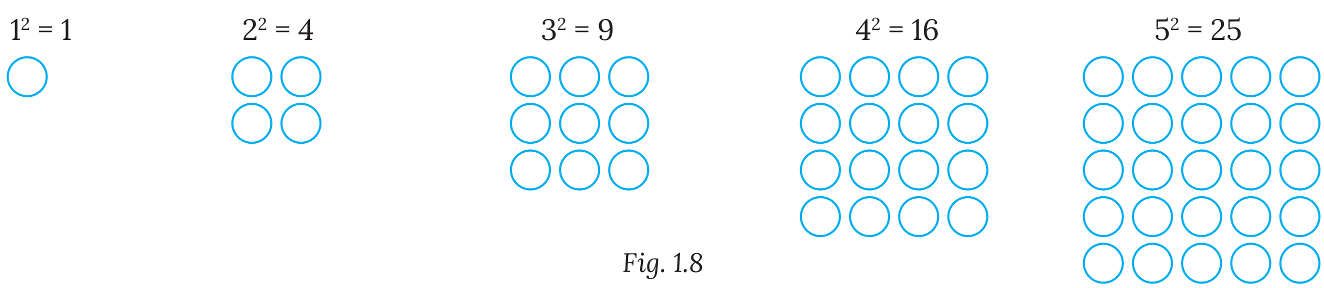


Fig. 1.8

- **The Pattern:** The sequence of square numbers is 1, 4, 9, 16, 25, ...
- **The Rule:** To find the nth square number, you simply multiply n by n.
- **Next Term:** The next number in the sequence is the 6^{th} term, so we calculate $6 \times 6 = 36$.

Hexagonal Numbers

Hexagonal numbers are a type of figurate number that represent a hexagon shape when arranged in a dot pattern. The nth hexagonal number is the number of dots required to form a hexagon with n layers.

Formula: $H_n = n(2n-1)$

$H_1 = 1(2\times1-1)$
$= 1(2-1)$
$= 1$

$H_2 = 2(2\times2-1)$
$= 2\times3$
$= 6$

$H_3 = 3(2\times3-1)$
$= 3(6-1)$
$= 3\times5$
$= 15$

$H_4 = 4(2\times4-1)$
$= 4(8-1)$
$= 4\times7$
$= 28$

$H_1 = 2(1)^2 - 1 = 1$ | $H_2 = 2(2)^2 - 2 = 6$ | $H_3 = 2(3)^2 - 3 = 15$ | $H_4 = 2(4)^2 - 4 = 28$

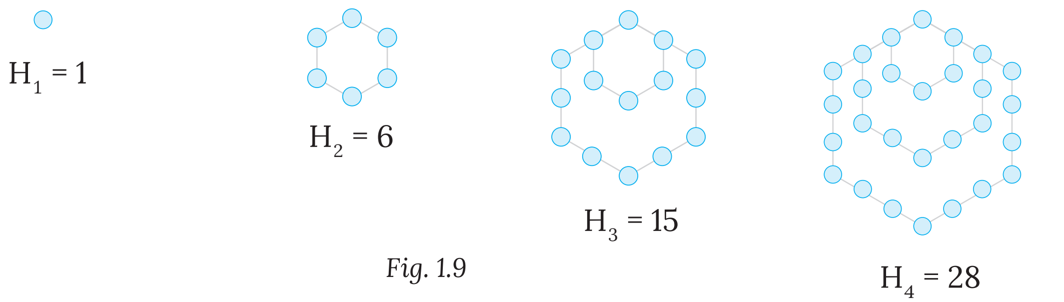


Fig. 1.9

Example 1 : Identify the rule and find the next two terms in the sequence: 4, 12, 36, ...

Solution: Step 1: Check for a common ratio. $\frac{12}{4} = 3$; $\frac{36}{12} = 3$ The common ratio is 3.

Step 2: The rule is "multiply by 3". This is a geometric pattern.

Step 3: Find the next terms.

$36 \times 3 = 108$

$108 \times 3 = 324$

Answer: The next two terms are 108 and 324.

Example 2 : A bus has 60 passengers. At the first stop, 5 passengers get off. At the second stop, another 5 get off. If this pattern continues, how many passengers are left after the 4th stop?

Solution: Step 1: This is a decreasing arithmetic pattern. The starting term is 60 and the rule is "**subtract 5**".

Step 2: After 1^{st} stop: 60 - 5 = 55

Step 3: After 2^{nd} stop: 55 - 5 = 50

Step 4: After 3^{rd} stop: 50 - 5 = 45

Step 5: After 4^{th} stop: 45 - 5 = 40

Answer: There are 40 passengers left.

Example 3 : A baker bakes 128 cakes on Monday. Each day, she bakes half the number of cakes from the previous day. How many cakes does she bake on Friday?

Solution: Step 1: This is a decreasing geometric pattern. The rule is "divide by 2".

Step 2:

Monday: 128 Tuesday: $\frac{128}{2} = 64$ Wednesday: $\frac{64}{2} = 32$

Thursday: $\frac{32}{2} = 16$ Friday: $\frac{16}{2} = 8$

Answer: She bakes 8 cakes on Friday.

Example 4 : Think about how bowling pins are set up. They form a perfect triangle. This is a real-life example of a triangular number pattern.

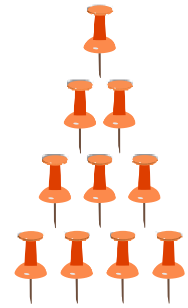


Fig. 1.10

Solution: **The Pattern:**

The 1st row has 1 pin.

The 2nd row has 2 pins, making a total of 1 + 2 = 3 pins.

The 3rd row has 3 pins, making a total of 3 + 3 = 6 pins.

The 4th row has 4 pins, making a total of 6 + 4 = 10 pins.

The total number of pins for each size of triangle is: 1, 3, 6, 10, ...

Example 5 : Imagine you are laying square tiles to make a square-shaped courtyard.

Solution: **The Pattern:**

To make a 1-tile by 1-tile square, you need 1 × 1 = 1 tile.

To make a 2-tile by 2-tile square, you need 2 × 2 = 4 tiles.

To make a 3-tile by 3-tile square, you need 3 × 3 = 9 tiles.

To make a 4-tile by 4-tile square, you need 4 × 4 = 16 tiles.

The sequence of the total number of tiles is: 1, 4, 9, 16, ..

Knowledge Checkpoint

- What is the rule for the sequence 9, 18, 27, 36? Is it arithmetic or geometric?
- Find the next number in the pattern: 160, 80, 40, ___.
- What is the difference between a common difference and a common ratio?
- What is the 10th square number?
- Is 25 a triangular number?

Activity

Number Patterns

- **Objective:** Identify and describe arithmetic, geometric, square, triangular, and alternating patterns using pictures and numbers.
- **Materials:** Grid paper, colored pencils, counters/tiles, ruler, sticky notes.
- **Starter:** On a 100-chart, quickly shade multiples of 4. Ask: "What pattern do you see?"
- **Activity :** In pairs, choose one pattern card:

  (A) Arithmetic +3 from 2

  (B) Geometric ×2 from 3

  (C) Squares

  (D) Triangular

  (E) Alternating +2, −1

  Build the first five terms with counters, sketch each term, and write the number sequence below. On a number line, show the jumps or multiplications. Then state the rule in words and, if possible, the nth-term.
- **Share:** Pairs gallery-walk. Each learner guesses the rule from visuals before reading the caption.
- **Check:** Extend two more terms for another pair's pattern and explain which type it is and why.

  What do you notice about the shape of a square number? How many counters do you add each time to get from one triangular number to the next?

Facts Flash

- The sum of the first 'n' odd numbers always results in a perfect square (n^2).
  For example, 1 + 3 + 5 = 9, which is 3^2.
- Any two consecutive triangular numbers add up to a square number! For example, the 3rd (6) and 4^{th} (10) triangular numbers add up to 16, which is the 4^{th} square number (4^2).
- The number 1 is the only number that is a square, cube, and triangular number!

Do It Yourself

We've seen square and triangular numbers. Can you imagine what a "pentagonal number" might look like? Try to draw the dot patterns for the first three pentagonal numbers. What would the rule be for finding the next one?

Key Terms

- **Arithmetic Pattern:** A sequence where a constant number is added or subtracted to get the next term.
- **Geometric Pattern:** A sequence where each term is multiplied or divided by a constant number.
- **Figurate Numbers**: Numbers that can be represented by a geometrical arrangement of equally spaced points.
- **Square Number:** A number that is the product of an integer with itself (n^2).
- **Cube Number:** A number that is the product of an integer with itself three times (n^3).
- **Triangular Number:** A number obtained by the continued summation of natural numbers.

Mental Mathematics

- Find the next term in an arithmetic sequence, quickly calculate the difference between the last two numbers you see and add it to the last number.
- For geometric patterns with a ratio of 2 or 1/2, practice quick mental doubling and halving.
- Imagine you are building squares with small blocks. The first, smallest square takes 1 block. The second square takes 4 blocks. The third square takes 9 blocks. How many blocks would you need to build the fifth square in this pattern?
- Here is a trickier pattern. Find the missing number in the middle. The sequence is: 4, 9, 19, ___, 79."

Gap Analyzer™ Homework

Watch Remedial

Exercise 1.1

1. **Look at the sequence: 88, 81, 74, 67, ...**
   a) What is the rule for this pattern? b) What are the next three numbers in the sequence?
2. **Create your own number pattern with the following conditions:**
   a) It must have at least 5 numbers.
   b) It must be a geometric pattern (uses multiplication or division).

3. **Riya starts a reading challenge. On Day 1, she reads 10 pages. On Day 2, she reads 13 pages. On Day 3, she reads 16 pages. She continues this pattern.**
   a) How many pages will she read on Day 5?
   b) On which day will she read exactly 25 pages?
   c) It must start with the number 80. Write down your sequence and the rule you used.
4. **A supermarket employee is stacking cans of juice in a triangular pyramid. The top layer has 1 can, the second layer has 3 cans, and the third layer has 6 cans. They want to continue this pattern to make the display bigger.**

   Question:
   a) Look at the pattern of cans: 1, 3, 6, ... This is a sequence of triangular numbers. How many cans will be in the fourth layer?
   b) If the display has a total of 5 layers, what is the total number of cans used in the entire pyramid?
5. Kavita is training for a school marathon. Her coach gives her a weekly running plan.
   - Week 1: Run a total of 5 km.
   - Week 2: Run a total of 7 km.
   - Week 3: Run a total of 10 km.
   - Week 4: Run a total of 14 km.

   a) Look at the increase in distance each week. What is the pattern in the increases?
   b) If Kavita continues this training pattern, what total distance will she run in Week 5?
6. **A large 10-litre bucket of water has a small leak. At 1:00 PM, it is full (10,000 ml). At 2:00 PM, a student measures the water and finds it has 9,850 ml. At 3:00 PM, it has 9,700 ml. The leak is constant.**
   a) What is the rule for this pattern? (How much water is lost each hour?)
   b) How much water will be in the bucket at 6:00 PM?
7. **A pattern is being created using small squares. Observe the first three figures in the sequence. Correction in image figure 3 will be 9- 1,5,9**

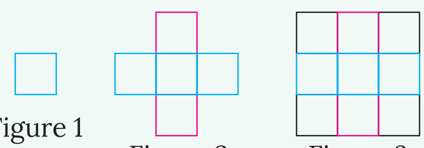


Figure 1 Figure 2 Figure 3

   a) Draw what Figure 4 would look like.
   b) Write the number of squares in the first four figures as a sequence. What is the rule?
   c) Without drawing, how many squares would be in Figure 6?

Relations among Number Sequences

In this section, we will investigate the relationships between different sequences. We'll see how adding odd numbers creates square numbers, and how adding two simple arithmetic sequences can result in a new one. This will help us see that mathematics is a web of interconnected ideas, not just a list of separate topics.

A picture often makes a concept easier to understand. Square numbers can be thought of as the total number of dots arranged in a perfect square grid.

Now imagine breaking this square grid into groups of dots that follow the odd numbers: 1, 3, 5, 7, ...

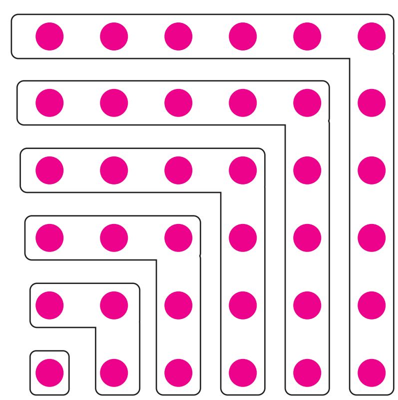


Fig. 1.11

This picture now makes it evident that: **1 + 3 + 5 + 7 + 9 + 11 = 36**

Combining and Summing Sequences

This concept explores two key ideas. **First**, we will see how performing an operation (like addition) on the terms of two different sequences can generate a new sequence with its own unique pattern. **Second**, we will investigate the patterns that emerge when we find the cumulative sum of the terms within a single sequence, such as adding up consecutive odd or even numbers.

Sub-concepts to be Covered

1. **Adding Two Arithmetic Sequences:** Combining two arithmetic sequences term-by-term.
2. **Sum of Consecutive Odd Numbers:** The relationship between adding odd numbers and generating square numbers.
3. **Sum of Consecutive Even Numbers:** The pattern formed by the cumulative sum of even numbers.

Mathematical Explanation

Adding Two Arithmetic Sequences

If you have two arithmetic sequences, A and B, you can create a new sequence C by adding their corresponding terms ($C_1 = A_1 + B_1$, $C_2 = A_2 + B_2$, etc.). The resulting sequence C will also be an arithmetic sequence. Its new common difference will be the sum of the common differences of sequences A and B.

Adding Two Arithmetic Sequences

Arithmetic Sequence 1	Arithmetic Sequence 2
Arithmetic Sequence 1: 2, 4, 6, 8, 10... This sequence has a common difference of 2 (since each term increases by 2).	**Arithmetic Sequence 2:** 1, 3, 5, 7, 9... This sequence also has a common difference of 2 (each term increases by 2).

- **Adding the Two Sequences:** We add the corresponding terms of both sequences. So, let's add the first few terms:
  - The 1st term of both sequences: 2 + 1 = 3
  - The 2nd term of both sequences: 4 + 3 = 7
  - The 3rd term of both sequences: 6 + 5 = 11
  - The 4th term of both sequences: 8 + 7 = 15
  - The 5th term of both sequences: 10 + 9 = 19

  So, the resulting sequence after adding the two arithmetic sequences is: **3, 7, 11, 15, 19...**

  This sequence itself forms an arithmetic sequence with a common difference of 4 (since 7 − 3 = 4, 11 − 7 = 4 etc.).

Sum of Consecutive Odd Numbers

A remarkable pattern emerges when you sum odd numbers starting from 1.

- 1 = 1 (which is 1^2)
- 1 + 3 = 4 (which is 2^2)
- 1 + 3 + 5 = 9 (which is 3^2)
- 1 + 3 + 5 + 7 = 16 (which is 4^2)

The sum of the first 'n' odd numbers is always n^2.

Sum of Consecutive Even Numbers

Adding consecutive even numbers also creates a pattern.

- 2 = 2
- 2 + 4 = 6
- 2 + 4 + 6 = 12
- 2 + 4 + 6 + 8 = 20

This sequence (2, 6, 12, 20, 30...) is known as the pronic or oblong numbers. Each term is the product of two consecutive integers: 1 × 2, 2 × 3, 3 × 4, 4 × 5, 5 × 6... The formula for the nth term is n(n + 1).

Example 7 : Sequence A is 3, 6, 9, 12... and Sequence B is 1, 3, 5, 7... Find the first four terms of the sequence formed by adding them.

Solution: **Term 1:** 3 + 1 = 4

Term 2: 6 + 3 = 9

Term 3: 9 + 5 = 14

Term 4: 12 + 7 = 19

Answer: The new sequence is 4, 9, 14, 19...

(It is also arithmetic with a common difference of 5).

Example 8 : Which sequence do we get when we add up even numbers?

Solution: $S_1 = 2$

$S_2 = 2 + 4 = 6$

$S_3 = 2 + 4 + 6 = 12$

$S_4 = 2 + 4 + 6 + 8 = 20$

$S_5 = 2 + 4 + 6 + 8 + 10 = 30$

The sequence formed by adding the even numbers is: 2, 6, 12, 20, 30.....

This is a quadratic sequence where each term is given by the formula

$S_n = n\ (n + 1)$

Example 9 : What happens when you start to add up powers of 3 starting with 1, i.e., take 1, 1 + 3, 1 + 3 + 9, 1 + 3 + 9 + 27, ...? Now add 1 to each of these numbers—what numbers do you get? Why does this happen?

Solution: Now, we sum them up progressively:

First sum: 1 + 1 = 2

Second sum: 1 + 1 + 3 = 5

Third sum: 1 + 1 + 3 + 9 = 14

Fourth sum: 1 + 1 + 3 + 9 + 27 = 41

Fifth sum: 1 + 1 + 3 + 9 + 27 + 81 = 122

So, after adding 1 to each sum, the new sequence is: 2, 5, 14, 41, 122...

The reason this pattern occurs is because each term is based on the sum of powers of 3, and adding 1 simply shifts the results.

Example 10 : Which sequence is obtained by adding consecutive odd numbers?

Solution: The sequence obtained by adding consecutive odd numbers in order is the sequence of perfect squares.

Odd numbers: 1, 3, 5, 7, 9...

Adding them consecutively:

1 = 1

1 + 3 = 4 = 22

1 + 3 + 5 = 9 = 32

1 + 3 + 5 + 7 = 16 = 42

1 + 3 + 5 + 7 + 9 = 25 = 52

Thus, the sequence obtained is 1, 4, 9, 16, 25....

So, on adding the odd numbers consecutively, we get the sequence of square numbers. This can now be illustrated visually, as shown in the following images.

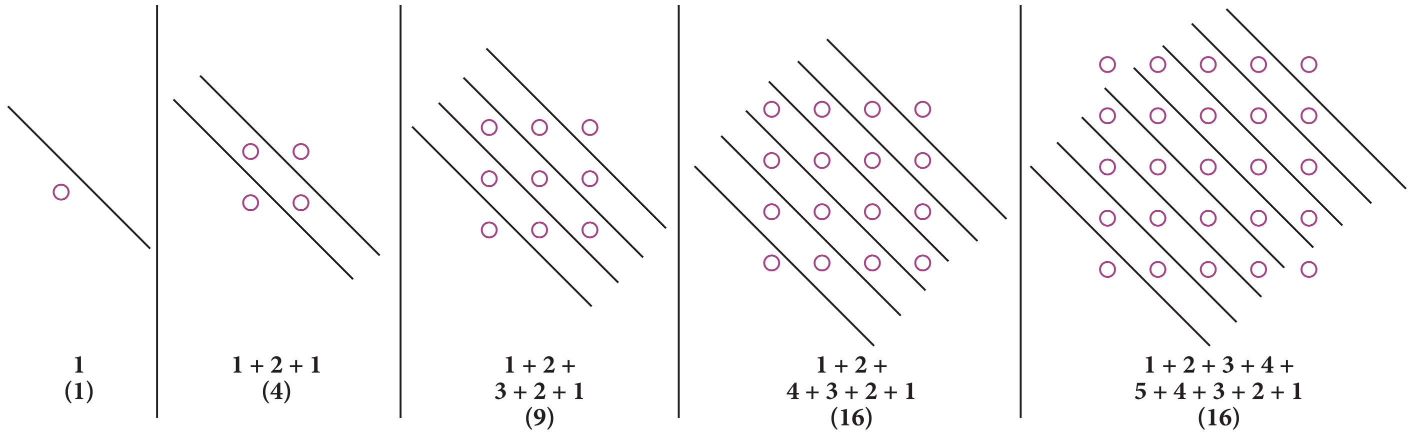


Fig. 1.12

Knowledge Checkpoint

- What is the sum of the first 3 odd numbers (1 + 3 + 5)? Which square number is this?
- What is the sum of the first 3 even numbers (2 + 4 + 6)?
- Sequence A is 1, 2, 3... Sequence B is 5, 10, 15... What are the first three terms of the sequence A + B?

Activity

Building Numbers

- **Objective:** To build and visualize figurate numbers.
- **Materials:** Square counters, blocks, or even dried beans.
- **Procedure:**
  1. Ask students to take 1 counter. Then ask them to add counters to make the next triangular number (3). Then the next (6).
  2. Challenge them to rearrange the 6 counters. Can they make a rectangle?
  3. Now, ask them to use the counters to build the first four square numbers (1, 4, 9, 16).
  4. If using blocks, challenge them to build the first three cube numbers (1, 8, 27).
- **Inquiry Questions:**
  What do you notice about the shape of a square number? How many counters do you add each time to get from one triangular number to the next?

Key Terms

- **Sum of a Sequence (Series):** The result of adding the terms of a sequence up to a certain point.
- **Consecutive:** Numbers that follow each other in order, without gaps.
- **Pronic Number (or Rectangular Number):** A number that is the product of two consecutive integers, $n(n + 1)$.

Facts Flash

- The sum of the first 'n' cube numbers ($1^3 + 2^3 + ... + n^3$) is equal to the square of the nth triangular number! For example, $1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$. The 3rd triangular number is 6, and $6^2 = 36$.
- Every perfect number (a number that is the sum of its proper divisors, like 6 = 1 + 2 + 3) is also a triangular number.

Do It Yourself

We've seen that adding consecutive odd numbers gives square numbers. What happens if you add consecutive triangular numbers?

- 1 + 3 = 4
- 3 + 6 = 9
- 6 + 10 = 16
- 10 + 15 = 25

What pattern do you see in the results? Why do you think this happens?

Mental Mathematics

1. **Imagine two secret codes are being generated at the same time.**
- Code A starts at 10 and adds 10 each time: 10, 20, 30, 40, ...
- Code B starts at 90 and subtracts 5 each time: 90, 85, 80, 75, ...
- A new sequence is formed by adding the matching terms from Code A and Code B. What will the 5th term of this new sequence be?

2. **Look at this jumbled-up sequence. It's actually two simple patterns woven together. Can you figure them out and find the next three numbers? 5, 100, 10, 90, 15, 80, ___, ___, ___**

Gap Analyzer™ Homework

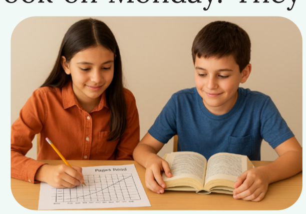
Watch Remedial

Exercise 1.2

1. **Complete the table**

Term Number	Rule P: (Term × 2) + 5	Value P	Rule Q: (Term × 3) + 2	Value Q	Difference (Q - P)
i	(1 × 2) + 5 = 7	7	(1 × 3) + 2 = 5	5	5 - 7 = -2
ii	(2 × 2) + 5 = 9	9	(2 × 3) + 2 = 8	8	?
iii	?	?	?	?	?
iv	?	?	?	?	?

2. Two friends, Anika and Ben, decide to start reading the same 300-page book on Monday. They read every day and track their progress differently.



- Anika's Plan: She reads 15 pages on the first day and increases her reading by 2 pages every day after that.
- **Ben's Plan:** He reads 5 pages on the first day and decides to double his reading every day.

a) Write down the number of pages each friend reads for the first 4 days.

Anika's sequence: __, __, __, __ **Ben's sequence:** __, __, __, __

b) On which day does Ben first read more pages than Anika?

c) **Challenge:** Who do you think will finish the 300-page book first? Explain your reasoning. (A full calculation is not required, just the reasoning).

3. **Finding the n^{th} Term of Number Patterns:**

Type	n^{th}-term a_n	a_5	a_{10}
i. Natural numbers	a_n = _____	_____	_____
ii. Odd numbers	a_n = _____	_____	_____
iii. Square numbers	a_n = _____	_____	_____
iv. Cube numbers	a_n = _____	_____	_____
v. Hexagonal numbers	a_n = _____	_____	_____

4. **A grid is being filled with shaded and unshaded squares in a checkerboard pattern.**

Question:

a) Draw or describe Figure 4, which is a 5 × 5 grid. How many shaded and unshaded squares does it have?

b) Write down the first four terms for three different sequences:

- Total Squares: __, __, __, __
- Shaded Squares: __, __, __, __
- Unshaded Squares: __, __, __, __

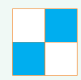


Fig. 1

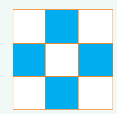


Fig. 2

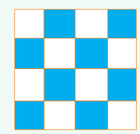


Fig. 3

c) What is the relationship between the "Shaded Squares" sequence and the "Unshaded Squares" sequence in any figure?

5. **Consider the powers of 2 starting from 1: 1, 2, 4, 8, 16...What happens when you add the first n terms and subtract 1 from each of these sums? What sequence emerges?**

Some More Patterns in Numbers

Now that you are a pro at arithmetic and geometric patterns, let's explore some trickier sequences! Not all patterns follow a simple rule of adding or multiplying by the same number. Some patterns use a mix of operations, or the rule itself changes at each step. This section will challenge you to think more flexibly and look for more complex relationships between numbers, like patterns that alternate or patterns based on place value.

Advanced Numerical Patterns

This concept moves beyond single-rule patterns. We will investigate Alternating Patterns, where the operation switches back and forth (e.g., add, then subtract, then add again). We will also look at Place Value Patterns, which demonstrate the power of our base-10 system by showing how a digit's value changes as it shifts to the left or right.

Sub-concepts to be covered

1. **Alternating Addition and Subtraction Patterns:** Sequences where the rule alternates between adding and subtracting.
2. **Shifting Digits in Place Value:** Patterns created by multiplying a digit by increasing powers of 10.

Mathematical Explanation

Alternating Addition and Subtraction Patterns

These sequences do not have a single common difference or ratio. The key is to look at the operation between each pair of terms. For example, in 10, 15, 12, 17, 14...

- 10 to 15 is +5
- 15 to 12 is -3
- 12 to 17 is +5
- 17 to 14 is -3

The pattern is "**add 5, then subtract 3**". To find the next term, you would add 5 to 14, which is 19.

Shifting Digits in Place Value

These are a special type of geometric pattern where the common ratio is a power of 10.

- **Sequence:** 7, 70, 700, 7000...
- This is a geometric pattern with a rule of "**multiply by 10**".
- It shows the digit 7 moving from the ones place, to the tens place, to the hundreds place, and so on. Each step increases its value tenfold.

A number line with a frog jumping. The frog starts at 5, jumps forward 6 units to 11 (+6), then jumps backward 3 units to 8 (-3), then forward 6 units to 14 (+6).

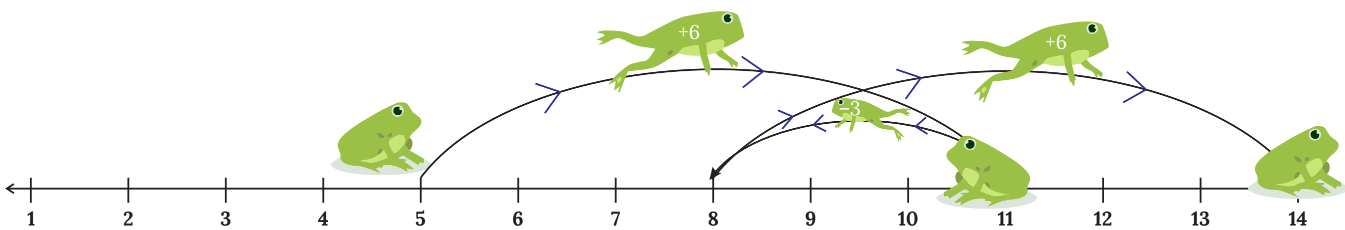


Fig. 1.13 Alternating patterns can hop back and forth!

Example 11 : Observe the following Alternating Addition and Subtraction Pattern:

General Approach to Solving Alternating Addition/Subtraction Patterns:

1. Identify the pattern of addition and subtraction (e.g., adding 3, then subtracting 3, or adding and subtracting progressively larger numbers).
2. Follow the alternating sequence step-by-step to determine the next term.
3. Check if the numbers are increasing or decreasing and whether the addition and subtraction follow a consistent pattern or changing increments.

Consider the sequence: 6, 9, 3, 6, 2, ...

Find the next term in the sequence.

Solution: **Start with 6, add 3**: 6 + 3 = 9

Subtract 6 from 9: 9 − 6 = 3

Add 3 to 3: 3 + 3 = 6

Subtract 4 from 6: 6 − 4 = 2

Identify the pattern: Add 3, Subtract 6, Add 3, Subtract 4, Next should be Add 3

The next term should be: 2 + 3 = 5

Example 12 : Observe the following patterns of Shifting Digits in Place Value

8, 80, 800, 8000, 80000...

Solution: This pattern involves shifting the digit 8 through increasing powers of 10.

- The place value of 8 increases by one order of magnitude at each step.
- The pattern is: 8×10^0, 8×10^1, 8×10^2, 8×10^3, 8×10^4....

- Each term is 10 times larger than the previous one.

This pattern demonstrates the effect of shifting the digit 8 through different powers of 10. It starts with 8 × 100 and continues by multiplying by increasing powers of 10. Here's the breakdown:

1. $8 \times 10^0 = 8$
2. $8 \times 10^1 = 80$
3. $8 \times 10^2 = 800$
4. $8 \times 10^3 = 8000$
5. $8 \times 10^4 = 80000$

At each step, the value increases by a factor of 10, shifting the digit 8 one place to the left in the decimal system. This pattern follows the rule: 8 × 10n where n starts from 0 and increases by 1 with each new term.

Knowledge Checkpoint

- Write the next four terms for the following sequence.
  - Start at 7. The rule is Add 6, then Subtract 1.
  - 7, ___, ___, ___, ___
- Identify the two-step rule for the sequence below.
  - 2, 12, 9, 19, 16, 26, ...
  - The rule is: _______________
- What is the shuffling rule for this sequence?
  - 4567 → 5476 → 4567 → 5476
  - The rule is: _______________

Activity

The Pattern Dance

- **Objective:**
  - To identify and describe patterns that involve more than one operation.
  - To apply a two-step rule to continue a sequence.
  - To strengthen mental math and working memory.
- **Materials:**
  - A whiteboard or large sheet of paper for demonstration.
  - Notebooks and pencils.
- **Procedure:** "Hello, Pattern Detectives! You have become experts at finding patterns with one simple rule. But some patterns are trickier—they like to dance! They take one step forward and one step back. Let's learn their moves."

  Part A: Following the Dance Steps

  1. **Demonstrate:** On the board, write down a starting number and a two-part rule.
  - **Start at:** 10
  - **Rule:** Add 5, then Subtract 2.

2. **Model the First Few Terms:**
- "Our first number is 10."
- "The first dance step is 'Add 5'. So, 10 + 5 = 15."
- "The second dance step is 'Subtract 2'. So, 15 - 2 = 13."
- "Now we repeat the dance! Start from 13. Add 5 → 18. Subtract 2 → 16."

3. **Write the Sequence:** The sequence looks like this: 10, 15, 13, 18, 16, ...
4. **Student Practice:** Give students a new starting point and rule.
- **Start at:** 20
- **Rule:** Add 10, then Subtract 3.
- Ask them to find the first 5 terms in their notebooks.

Part B: Discovering the Dance

1. **Challenge:** Write a sequence on the board and ask students to work in pairs to figure out the "dance steps" (the two-part rule).
- **Sequence 1:** 5, 10, 8, 13, 11, 16, ...
- **Sequence 2:** 50, 45, 47, 42, 44, ...

- **Discussion Questions:**
  - How are these patterns different from the ones we saw last time?
  - What two things do you need to keep in your mind to solve these? (The two operations and which one comes next).
  - Can you find the rule for Sequence 1?

Facts Flash

Secret Codes: Alternating patterns are a simple form of cryptography! You can create a secret code by assigning each letter a number (A = 1, B = 2) and then applying a two-step rule to "**scramble**" your message. To decode it, your friend just has to reverse the rule!

Key Terms

- **Alternating Pattern:** A sequence that follows a rule with two or more steps that repeat in a cycle, such as "add 5, then subtract 2".
- **Place Value:** The value of a digit based on its position in a number (e.g., in 246, the '2' represents 200).
- **Digit:** A single symbol used to make numerals (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).
- **Cyclic Pattern:** A sequence in which the terms repeat themselves in the same order after a certain number of steps.

Do It Yourself

Have you ever counted the petals on a flower? Many flowers follow a famous pattern called the Fibonacci Sequence.The sequence is 1, 1, 2, 3, 5, 8, 13, ...

- **The Rule:** You get the next number by adding the two numbers before it (e.g., 3 + 5 = 8).

Mental Mathematics

- My pattern starts at 3. The rule is "+10, -1". What are the first four terms? (Pause for thought) → (3, 13, 12, 22)
- **Listen to the sequence:** 100, 90, 95, 85, 90... What is the rule? (Pause for thought) → (-10, +5)

- Take the number 92. Swap the digits. What is the new number? (Pause for thought) → (29)
- Take the number 123. Move the first digit to the end. What is the new number? (Pause for thought) → (231)

Gap Analyzer™ Homework

Watch Remedial

Exercise 1.3

1. **What is the next number in the sequence? Consider the sequence:**

   (a) 10,7,10,6,10.... (b) 50,48,53,51,56,... (c) 20,18,20,16,20,... (d) 7,11,8,12,9,...

2. **Given the pattern:** $20 \times 0 - 15 = -15$

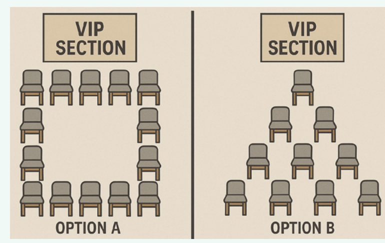
   $20 \times 1 - 16 = 4$

   $20 \times 2 - 17 = 23$

   What is the result of:

   (a) $20 \times 3 - 18 = ?$ (b) $20 \times 4 - 19 = ?$

3. A school is setting up chairs for its annual day function. The event manager suggests two seating arrangement options for a special VIP section.



   - **Option A:** A large square formation.
   - **Option B:** A large triangular formation (like the bowling pins).

   a) If the manager creates a square formation with 9 rows of 9 chairs each, how many chairs are used in total? What is this number called in pattern language?

   b) The manager has 55 chairs available for the triangular formation. Can all the chairs be used to form a perfect triangle? Explain your reasoning.

   c) If you have 49 chairs, which formation (Square or Triangle) could you create without any chairs leftover?

4. **Find the following patterns:**

   a) 1, 4, 9, 16, 25, ...What comes next?

   b) 2, 6, 12, 20, 30, ...What is the 7th term?

   c) 5, 10, 20, 40, 80, ...Write the next 3 terms.

   d) 1, 1, 2, 3, 5, 8, ...Fill in the next 4 numbers.

   e) 9, 18, 27, 36, ...Find the 15th term of this pattern.

   f) 4, 8, 12, 16, 20, ...Fill in the next 4 numbers.

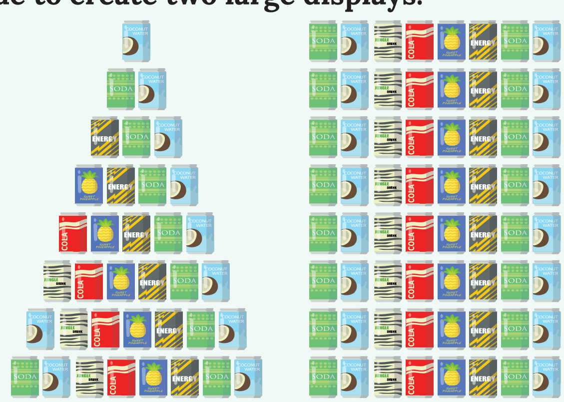
5. **Your class is collecting cans for a food drive. You decide to create two large displays.**

   - **Display A:** A triangular stack that is 8 layers high.
   - **Display B:** A square base that is 8x8 cans.

   a) How many cans do you need for the triangular Display A?

   b) How many cans do you need for the square Display B?

   c) You have already built a 8-layer triangular stack. How many more cans do you need to turn it into the 9-layer triangular stack for Display A?



Display A Display B

6. You are planting a new vegetable garden and have 36 small saplings. You want to arrange them in a neat, geometric pattern.
    a) Can you plant all 36 saplings in a perfect square grid? If so, what would be the dimensions (rows × columns)?
    b) Can you plant all 36 saplings in a perfect triangular formation? If so, how many rows would it have?
    c) Based on your answers, if you have 36 saplings, what two different types of "special number" patterns can you form?

Patterns in Shapes

Patterns are not just in numbers; they are all around us in the beautiful and intricate shapes of the world. From the repeating pattern of tiles on a floor to the way a leaf grows, geometry is full of patterns. In this section, we will explore patterns made of shapes. We will look at patterns that repeat, patterns that grow, and fascinating patterns that can tile a surface without any gaps. This will connect our logical thinking with our spatial and creative skills.

Geometric and Spatial Patterns

This concept focuses on patterns you can see and draw. We will explore Repeating Patterns, where a core group of shapes repeats, and Growing Patterns, where the shape changes systematically at each step (e.g., by adding more components). We will also dive into two special types of shape patterns: Tessellations, which are repeating tiles covering a surface, and Fractals, which are "self-similar" patterns that look the same at any level of magnification.

Sub-concepts to be covered

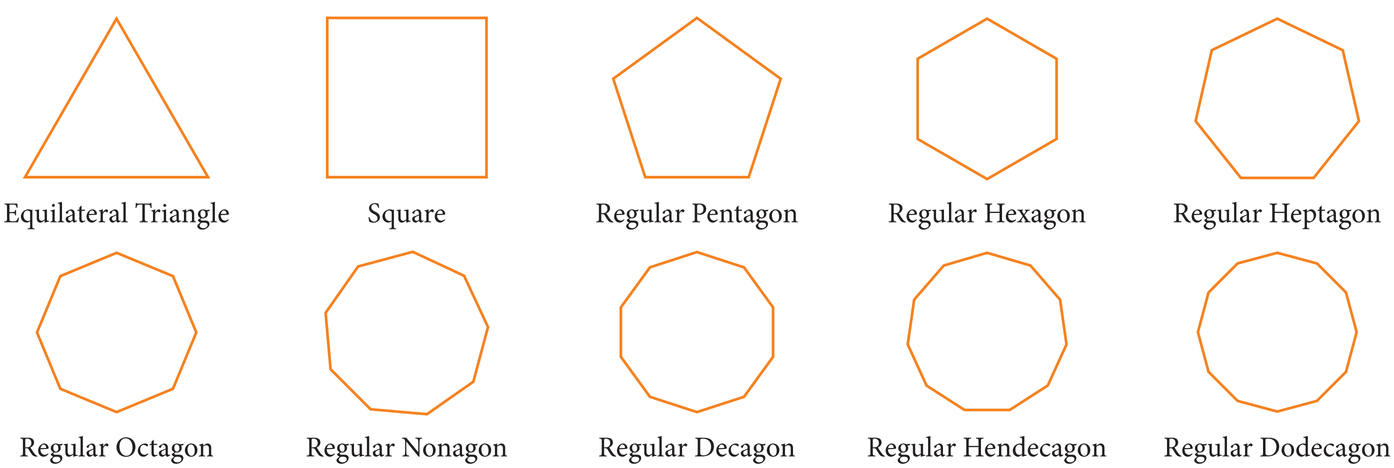
1. Regular polygons
2. Stacked triangles
3. Stacked Squares
4. Complete Graphs
5. Koch Snowflake (Fractal)

Mathematical Explanation

Regular polygons

A **regular polygon** is a polygon where all the sides and angles are equal. The most common types of regular polygons include triangles, squares, pentagons, hexagons, and so on.

Here are examples of a few regular polygons:



Stacked triangles

Stacked triangles typically involve arranging multiple equilateral triangles in a pattern, either by placing them side-by-side or on top of each other to create a larger triangular structure.

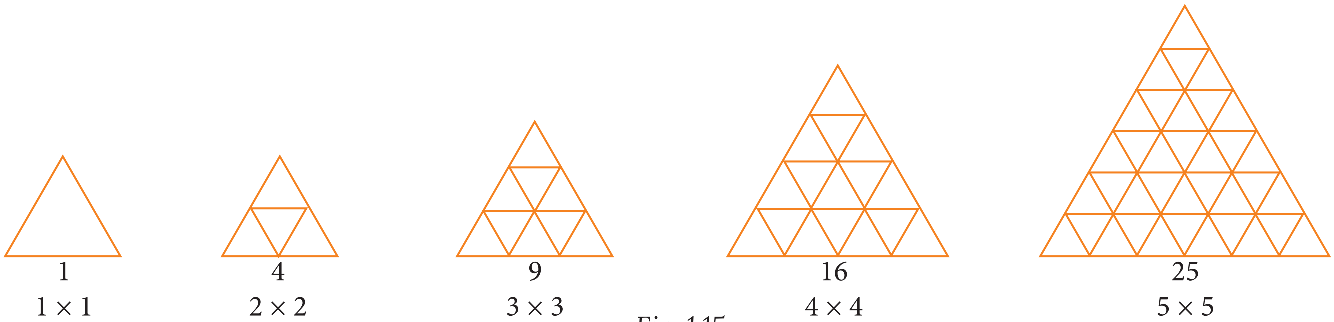


Fig. 1.15

Stacked Squares

Stacked squares typically involve arranging squares in a vertical or grid-like pattern. This can result in a simple column of squares or more complex arrangements where multiple rows of squares are stacked on top of each other.

Formula: $S_n = n \times n = n^2$

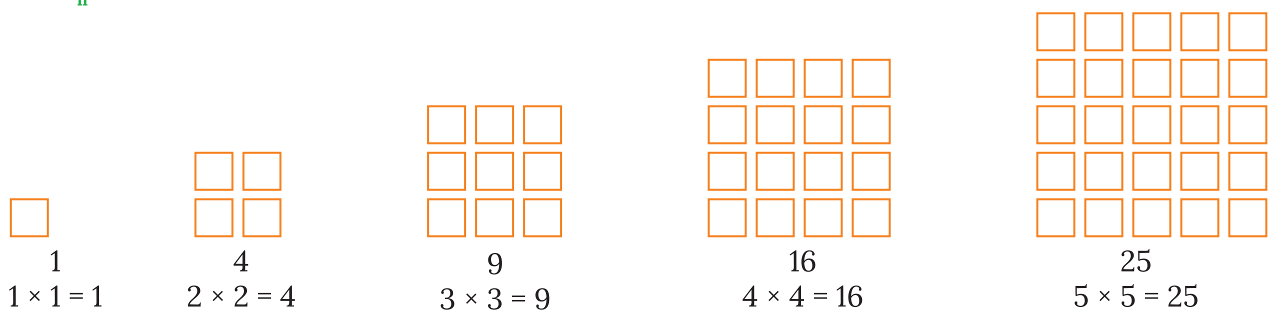


Fig. 1.16

Complete Graphs

A **complete graph** is a type of graph in graph theory where every pair of distinct vertices is connected by a unique edge. In simpler terms, each vertex is directly connected to every other vertex.

If a complete graph has n vertices (points), then the number of edges (lines) is:

Formula: $E = \frac{n(n-1)}{2}$

Examples

- 2 vertices (K_2) → 1 edge
- 3 vertices (K_3) → 3 edges (a triangle)
- 4 vertices (K_4) → 6 edges (all corners connected)
- 5 vertices (K_5) → 10 edges

Sequence of edges: 1, 3, 6, 10, 15, ... (which are triangular numbers!).

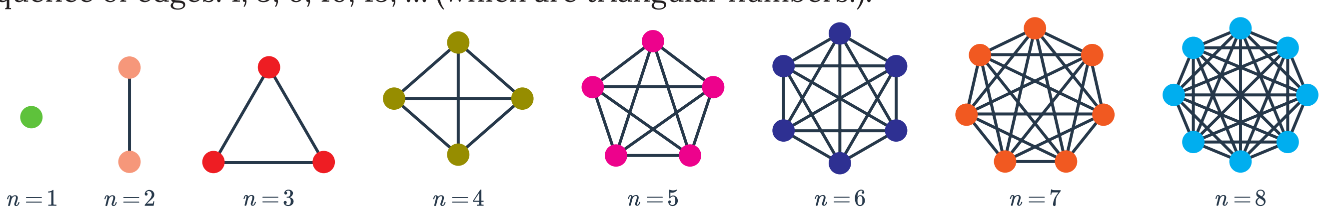


Fig. 1.17

Koch Snowflake (Fractal)

The **Koch Snowflake** is a famous fractal curve and one of the most well-known examples of a self replicating pattern in mathematics. It is constructed starting with an equilateral triangle, and then each side of the triangle is divided into smaller segments, with a smaller equilateral triangle "sticking out" at the middle of each side. The process is repeated indefinitely, which creates a snowflake-like pattern.

Steps to Construct the Koch Snowflake

1. Start with an equilateral triangle.
2. Divide each side into three equal parts.
3. On the middle part, draw another equilateral triangle pointing outward.
4. Remove the base line of that small triangle.
5. Repeat this process for every new side, again and again.

   Each step makes the figure more detailed, but the overall shape still looks like a snowflake.

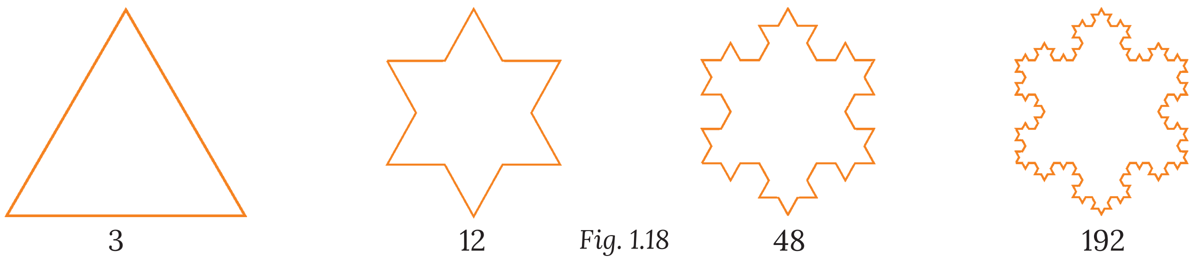


Fig. 1.18

Example 13 : A pattern is made of squares. Figure 1 has 1 square. Figure 2 has 3 squares. Figure 3 has 5 squares. How many squares will be in Figure 5?

Solution: **Step 1:** The sequence of squares is 1, 3, 5... This is an arithmetic pattern with the rule "add 2".
Step 2: Figure 4 will have 5 + 2 = 7 squares.

Step 3: Figure 5 will have 7 + 2 = 9 squares.

Answer: Figure 5 will have 9 squares.

Example 14 : The Growing House Pattern: This pattern involves shapes that grow in a predictable way. Our goal is to find the rule for how it grows.

The Pattern: Imagine you are building houses with triangles.

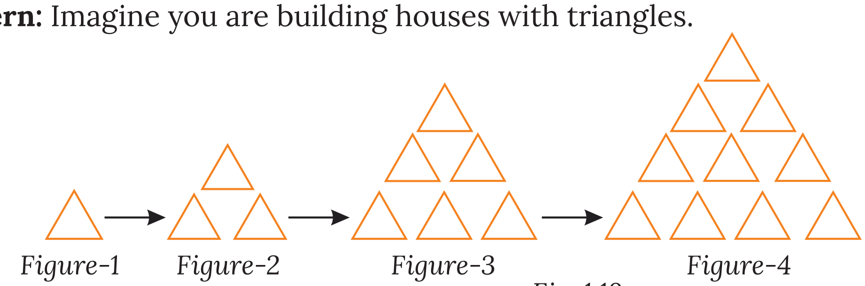


Fig. 1.19

Let's Analyze the Pattern (The Explanation)

To understand a growing pattern, it's very helpful to count and organize our findings in a table.

Observe and Count:

- **Figure 1** has 1 triangle.
- **Figure 2** has 1 + 2 = 3 triangles.
- **Figure 3** has 3 + 3 = 6 triangles.

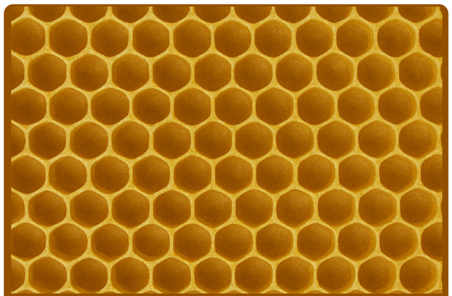
- **Find the Rule:** Look at the "How We Get the Total" column. To get the total number of triangles in any figure, we are adding the figure number to the previous total.

- For Figure 2, we added 2 triangles.
- For Figure 3, we added 3 triangles.

So, the rule is: "Add a new row of triangles at the bottom. The number of triangles you add is equal to the Figure Number."

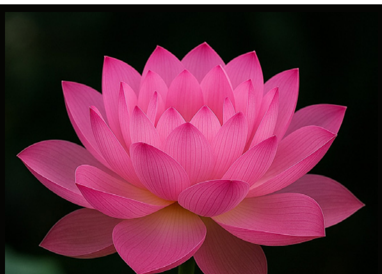
- **Predict the Next Step:** Using our rule, to get Figure 4, we must take Figure 3 (which has 6 triangles) and add 4 more triangles to the base.
  - Total triangles in Figure 4 = 6 + 4 = 10 triangles.
  - **The calculation would be:** 1 + 2 + 3 + 4 = 10.

Ant colonies
The number of ants doubles rapidly → 2, 4, 8, 16, 32...



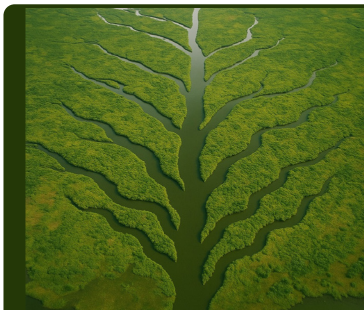
Beehive cells grow in hexagonal patterns that repeat infinitely.

Coral polyps
Colonies increase by branching → 1, 2, 4, 8, 16...



Ant colonies
A lotus flower can have petals in patterns like 4, 8, 16... (doubling).

Leaves on a stem
Often appear in fixed intervals (2, 4, 6, 8... or 3, 6, 9...).



River deltas often branch:
1 → 2 → 4 → 8, a doubling pattern of distributaries.

Knowledge Checkpoint

- The sum of angles in a regular Hexagon (6 sides) is 720°. Following the pattern, what is the sum of angles in a regular Heptagon (7 sides) (pattern rule $(n - 2) \times 180°$)?
- Look at the "Stacked Triangles" pattern (1, 4, 9, ...). What would be the number of small triangles in Stage 4?
- How many lines (handshakes) would you need to connect 6 dots in a complete graph? (**Hint**: The sequence is 1, 3, 6, 10, ...)
- In your own words, what is the first step you take when starting with a triangle to create a Koch Snowflake?

Activity

The Pattern Architect!

- **Objective:**
  - Patterns involving translation (slides), rotation (turns), and growth.
  - To identify and articulate the "rule" of a given pattern.
  - To collaborate and challenge classmates.

- **Materials Needed:** Grid paper (or plain paper), Pencils, erasers, and coloured pencils/crayons, A ruler

  The Rule Maker and Challenger

  Now for the most creative part! You get to invent your own secret pattern rule.

  1. **Choose Your Shape:** Start with a simple shape or a combination of shapes (e.g., a square with a triangle on top).
  2. **Create Your Secret Rule:** Your rule must involve at least two changes. Here are some ideas for your rule:
  - Add a shape in each step.
  - Change the colour in a repeating sequence (e.g., Red, Blue, Green, Red, Blue, Green...).
  - Rotate the entire design by a quarter-turn (90°) in each step.
  - Flip the design (reflection) in each step.

  Example Rule: "Start with a blue arrow. In each step, add a small circle to the base and rotate the whole thing a quarter-turn to the right."

  3. **Draw Your Pattern:** On a sheet of paper, draw the first three steps of your pattern based on your secret rule. Make it neat and clear.
  4. **Challenge a Friend:** Swap your pattern with a partner. Can they:
  - Figure out your secret rule?
  - Draw Step 4 and Step 5 of your pattern correctly?
- **Discussion and Reflection:**
  - What was the most interesting rule you saw today?
  - Was it easier to create a pattern or to figure out someone else's rule? Why?
  - Look around the classroom or outside the window. Find one pattern that involves rotation and one that is a tessellation. Sketch them!

Facts Flash

- Fractals are Everywhere! The Koch Snowflake isn't just a mathematical curiosity. Fractals are nature's design language! You can see fractal patterns in lightning bolts, river networks, the leaves of a fern, Romanesco broccoli, and even the coastline of a country.
- **The Handshake Problem:** The "**Connect the Dots**" activity is a famous puzzle. If you have 100 people in a room and everyone shakes hands with everyone else exactly once, there will be 4,950 handshakes! It's much easier to find this with the triangular number pattern than by counting!

Do It Yourself

- Have you ever noticed that the world around you is full of repeating designs?
  - Look at the beautiful rangoli your family makes during a festival.
  - Think about the tiles on the floor in your school or home.
  - Even a beehive, a pineapple’s skin, or a sunflower’s seeds have a special arrangement!
- What do they all have in common? They all follow a pattern! A pattern is simply a rule that is repeated over and over again.

Key Terms

- **Regular Polygon:** A polygon with all sides of equal length and all interior angles of equal measure.
- **Interior Angle:** An angle inside a shape.
- **Complete Graph:** A set of dots (vertices) where every dot is connected to every other dot by a line (edge).
- **Fractal:** A complex, never-ending pattern. Fractals are self-similar, meaning they look the same at any scale you zoom in on. The Koch Snowflake is a famous example.
- **Self-Similarity:** The property of a shape looking the same as a part of itself.

Mental Mathematics

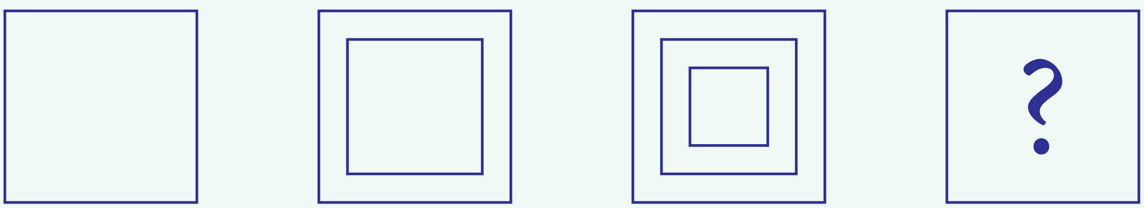
- A regular pentagon has 5 sides. A regular hexagon has 6. How many sides does a regular octagon have?
- If you Triangular number to get the sequence 1, 3, 6, 10... what is the next number?
- The pattern of small triangles in a bigger triangle is 1, 4, 9, 16... What is the next number?
- What shape do you start with to build a Koch Snowflake? (Pause for thought)

Gap Analyzer™ Homework

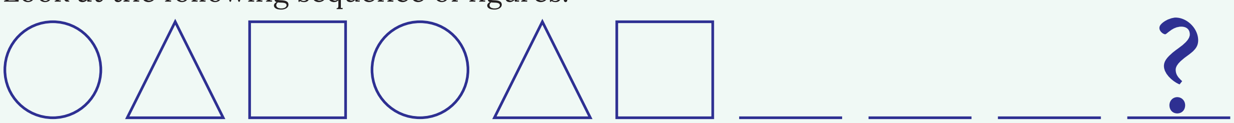
Watch Remedial

Exercise 1.4

1. **Identify the next figure in the sequence:**

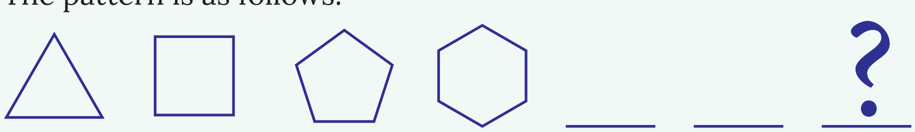


2. Look at the following sequence of figures:



If the sequence continues, which figure will appear 10th?

3. The pattern is as follows:



What will be the number of sides of the 7th figure in this sequence?

4. In a stacked triangle arrangement where each row has one more triangle than the row above it, how many triangles will there be in 4 rows?
5. If you stack 4 squares, one on top of the other, with each square having a side length of 3 cm, what is the total height of the stacked squares?
6. A regular polygon has 6 sides. What is the name of the polygon, and what is the measure of each interior angle?

7. A square is rotated 90 degrees clockwise. If the square is rotated 270 degrees clockwise from its original position, which direction will the top of the square point towards?
8. Aisha's Pyramid of Blocks Aisha is building a pyramid with her square blocks. The base of the pyramid is a square, and each layer above it is also a square.
    - The top layer (Layer 1) is just 1 block.
    - The layer below it (Layer 2) is a 2 × 2 square of blocks.
    - The layer below that (Layer 3) is a 3 × 3 square of blocks.

    a) How many blocks will Aisha need for Layer 4 and Layer 5?
    b) Aisha finds a box with 50 blocks. What is the tallest complete pyramid she can build?
9. **The Community Hall Seating A new community hall has a V-shaped seating arrangement for its front section.**
    - Row 1 has 1 chair.
    - Row 2 has 3 chairs.
    - Row 3 has 5 chairs. The pattern continues, with each row forming a wider 'V'.

    a) How many chairs are in Row 4 and Row 5?
    b) Can you find the total number of chairs in the first 4 rows?
10. **At a school exhibition:**
    - Beside it, a stacked triangle of books is made with 15 layers.
    - Another display uses stacked squares, the largest being 30×30 tiles.
    - The computer club sets up a complete graph network of 50 laptops.

    Tasks:
    i. Find the total number of books in the 15-layer triangular stack.
    ii. Calculate the number of tiles in the 30 × 30 square.
    iii. Find the total number of connections in the 50 computer network.
11. **You are walking down a street and notice that different traffic signs have very specific, regular shapes. A shape is "regular" if all its sides are equal in length and all its angles are equal.**

    A collage of three common street signs.
    Question:

    a) Identify the mathematical names of the three regular polygons used for these signs.
    b) Name the regular polygons used for each sign: STOP (red), YIELD (yellow), SPEED LIMIT (white).
    c) Why do you think governments use distinct, regular shapes for important signs like STOP and YIELD? How does this pattern help drivers?

Common Misconceptions

Misconception: All patterns must involve adding or multiplying by the same number.

Correction: This is only true for basic arithmetic and geometric patterns. Many patterns, like alternating or growing shape patterns, have more complex rules. The pattern 1, 4, 9, 16... doesn't add or multiply by the same number, but it has a clear rule: squaring the term number.

Misconception: To find the next term in a sequence, I just look at the last two numbers.

Correction: While this works for simple patterns, it can be misleading. For the sequence 1, 2, 4, 8, 16... the difference between 8 and 16 is 8, but adding 8 to 16 gives 24, which is wrong. You must identify the overall rule (multiply by 2) to get the correct next term, 32. Always test your rule on at least three terms.

Real-Life Pattern in Mathematics: Mathematical Applications

Real-Life Applications: Patterns in Mathematics

- **Nature's Architect:** Observing patterns in nature, like the repeating hexagonal cells in a honeycomb, the spiral of a snail's shell, or the arrangement of petals on a flower. This shows that math is a blueprint for the natural world.
- **Creative Designer:** Creating a Rangoli or analysing the border of a sari or bedsheet. They can identify the core design that repeats to create the full, beautiful pattern, linking mathematics directly with art and culture.
- **Rhythm & Beats:** Listening to a simple song and clapping out the repeating beat. The chorus in a song is also a recurring pattern. This helps them understand that patterns can be auditory and time-based, not just visual.
- **Building Blocks:** Looking at a tiled floor or a brick wall. Students can identify the basic shape (square, rectangle) and how it repeats to cover a large area, a simple introduction to the concept of tessellation.

Gap Analyzer™
Complete Chapter Test

EXERCISE

A. Choose the correct answer.

1. What is the next term in the sequence: 81, 27, 9, 3, ...?

   (a) 1 ☐ (b) 0 ☐ (c) $\frac{1}{3}$ ☐ (d) −3 ☐

2. Which of these is a triangular number?

   (a) 9 ☐ (b) 12 ☐ (c) 15 ☐ (d) 16 ☐

3. The pattern in a pinecone's scales is often related to which famous sequence?

   (a) Square Numbers ☐ (b) Prime Numbers ☐

   (c) Fibonacci Sequence ☐ (d) Even Numbers ☐

4. A pattern's rule is "multiply by 3, then subtract 2". If a term is 10, what is the next term?

   (a) 28 ☐ (b) 32 ☐ (c) 24 ☐ (d) 30 ☐

5. Which regular polygon will NOT tessellate by itself?

(a) Equilateral Triangle ☐ (b) Square ☐

(c) Regular Hexagon ☐ (d) Regular Pentagon ☐

Assertion & Reason

In each of the following questions, an Assertion (A) and a corresponding Reason (R) supporting is given. Study both the statements and state which of the following is correct:

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

1. **Assertion (A):** The sum of the first 5 odd numbers is 25.
   Reason (R): The sum of the first 'n' odd numbers is given by the formula n^2.
2. **Assertion (A):** The sequence 2, 3, 5, 8, 12 is an arithmetic sequence.
   Reason (R): The difference between consecutive terms is not constant.
3. **Assertion (A):** A regular hexagon can tessellate the plane.
   Reason (R): The interior angle of a regular hexagon is 120°, and $3 \times 120° = 360°$.

Case Study

Priya is designing a patio with grey and white tiles.

- Row 1: 1 grey tile.
- Row 2: 1 grey tile, 2 white tiles.
- Row 3: 1 grey tile, 4 white tiles.
- Row 4: 1 grey tile, 6 white tiles.

She continues this pattern.

1. How many white tiles will be in Row 7?
2. What is the total number of tiles (grey and white) in Row 10?
3. Describe the pattern of the number of white tiles. Is it arithmetic, geometric, or something else?

Project

The Patterned Budget

Scenario: You have just received ₹1,000 as a birthday gift, and you want to make it last. You decide to create a spending plan based on a mathematical pattern.

Task: Choose ONE of the following plans and create a day-by-day budget for 10 days. Show your calculations in a table and determine how much money you will have left after 10 days.

- **Plan A (Arithmetic Decay):** On Day 1, you spend ₹20. Each following day, you spend ₹5 more than the day before. (Day 2: ₹25, Day 3: ₹30, etc.).
- **Plan B (Geometric Growth):** On Day 1, you spend a tiny amount: ₹2. Each following day, you spend double the amount from the previous day. (Day 2: ₹4, Day 3: ₹8, etc.).
- **Plan C (Your Own Pattern):** Create your own rule for spending (it could be an alternating pattern, or based on triangular numbers, etc.). Your rule must be clearly explained.

Presentation:

1. Create a table with columns: Day, Amount Spent, Total Spent, Money Remaining.
2. Write a short paragraph explaining which plan you think is the most sustainable and why.
3. Create a simple bar graph showing the amount spent each day for your chosen plan.

Source-Based Question

Ministry of New and Renewable Energy (MNRE), Government of India.

Directions: India is harnessing the power of the sun! The government has been working hard to increase the amount of electricity we generate from solar energy. This helps our environment and powers our nation's growth. The table below shows how India's solar power capacity has grown over recent years.

Source Text

India's Installed Solar Power Capacity (Approximate)

Year (as of 31st March)	Installed Solar Capacity (in Gigawatts - GW)
2018	22 GW
2019	28 GW
2020	35 GW
2021	40 GW
2022	54 GW
2023	67 GW

(**Note:** 1 Gigawatt (GW) is equal to 1,000 Megawatts (MW). This is a huge amount of power!)

(Source: Ministry of New and Renewable Energy (MNRE), Government of India.)

Questions

1. Look at the "Installed Solar Capacity" column in the table. What is the main pattern you observe in the numbers from 2018 to 2023?
2. How much solar capacity was added between March 2018 and March 2019? How much was added between March 2021 and March 2022?
3. The amount of new capacity added each year is not exactly the same, but we can see a pattern. If we wanted to create a simple rule like "Add 'x' GW each year," what would be a reasonable number for 'x' to get from 22 GW in 2018 to 67 GW in 2023? Show your thinking.
4. Based on the pattern of strong growth you see in the table, what would be your prediction for India's installed solar capacity by March 2024? Explain the reason for your prediction.
5. If you were to draw a bar graph for this data, with the years on the horizontal axis and the capacity on the vertical axis, what would the tops of the bars look like? Would they form a straight line, a downward curve, or an upward-climbing pattern? Why?

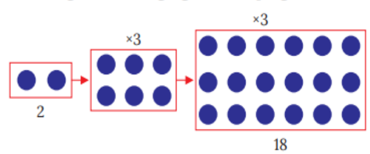
Mind Map

Patterns in Mathematics

Types of Number Patterns

- **Arithmetic Patterns:** Add/subtract a fixed number (common difference).
  - ✓ Increasing (e.g., 5, 10, 15... +5), Decreasing (18, 15, 12... −3).
- **Geometric Patterns:** Multiply/divide by a fixed number (common ratio).
  - ✓ Growing (2, 6, 18... ×3), Shrinking (100, 50, 25... ÷2).

A diagram showing a geometric progression.



Number Patterns (Visualising Sequences)

- **Basic:** Natural (1, 2, 3...), Odd (1, 3, 5...), Even (2, 4, 6...).
- **Figurate Numbers:**
  - ✓ Square (n^2 → 1, 4, 9, 16...)
  - ✓ Cube (n^3 → 1, 8, 27, 64...)
  - ✓ Triangular ($T_n = \frac{n(n+1)}{2}$ → 1, 3, 6, 10...)
  - ✓ Hexagonal (n(2n−1) → 1, 6, 15, 28...)

Relations Among Number Sequences

- **Sum of Consecutive Odd Numbers** → 1, 1+3, 1+3+5
  - ✓ Square Numbers (1, 4, 9...).
  - ✓ Rule: Sum of first n odd = n^2.
- **Sum of Consecutive Even Numbers** → 2, 2+4, 2+4+6 ...
  - ✓ Pronic Numbers (2, 6, 12...).
  - ✓ Rule: n (n+1).

Some More Patterns in Numbers

- **Alternating Patterns**
  - ✓ Rule: The operation changes in a cycle.
  - ✓ **Example:** "Add 5, then subtract 3" (e.g., 10, 15, 12, 17...).
- **Place Value Patterns**
  - ✓ Rule: A digit's value changes by multiplying by powers of 10.
  - ✓ **Example:** 7, 70, 700, 7000... (Rule: ×10).
- **Fibonacci Sequence**
  - ✓ Sequence: 1, 1, 2, 3, 5, 8...
  - ✓ Rule: Add the two previous numbers to get the next one.

Patterns in Shapes

- **Repeating** (🔴,🟦,△) & **Growing** (adding blocks).
- **Regular Polygons:** Equal sides & angles.
- **Tessellations:** Repeating tiles with no gaps.
- **Fractals (Koch Snowflake):** Self-similar, repeat at scale.
- **Complete Graphs:** Every vertex connects → Triangular Numbers (1, 3, 6, 10...).