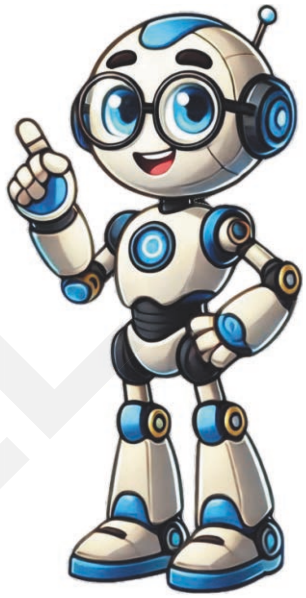
Chapter 5

Prime Time

We'll cover the following key points:

→ Introduction to Prime Time
→ Prime Factors
→ Common Factors and Common Multiples
→ Prime and Composite Numbers
→ Test for Divisibility of Numbers
→ Prime Factorisation
→ Highest Common Factor (HCF)



Hi, I'm EeeBee

Do you Remember fundamental concept in previous class.
In class 5^{th} we learnt
→ Highest Common Factor (HCF)
→ Lowest Common Multiple (LCM)
→ Relationship between the HCF and LCM

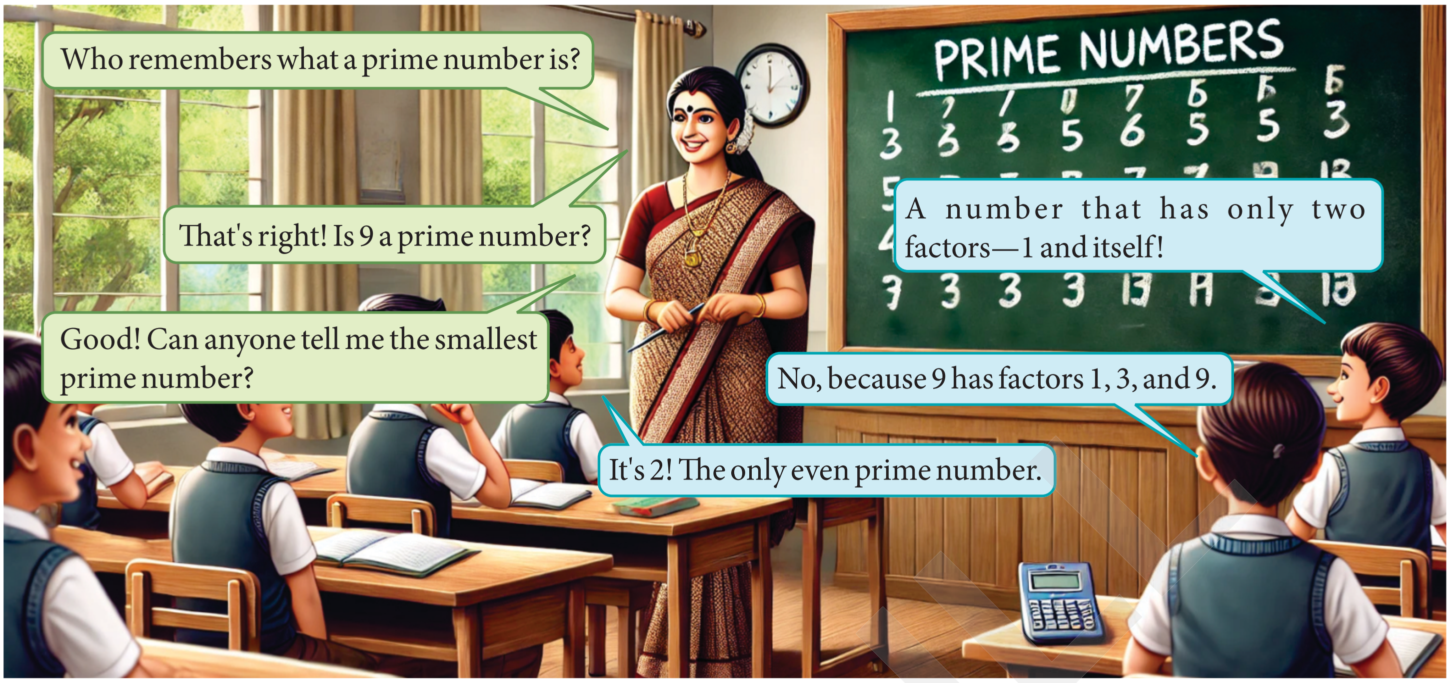
Still curious? Talk to me by scanning the QR code.

Learning Outcomes

By the end of this chapter, students will be able to:

- Identify and define prime and composite numbers.
- Use divisibility rules to determine if a number is prime or composite.
- Factorize numbers into prime factors.
- Understand the importance of prime numbers in number theory.
- Solve problems related to prime numbers, factors, and multiples.
- Recognize the role of prime numbers in encryption and real-life applications.
- Apply prime factorization to find the LCM and GCD of numbers.
- Solve word problems involving prime numbers.
- Analyze patterns and properties of primes, including concepts like twin primes and Mersenne primes.
- Develop and apply efficient algorithms for prime testing and factorization.
- Explore the historical evolution and proofs related to prime numbers to build deeper mathematical insight.

Introduction



In mathematics, prime numbers are very special! A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself. In other words, prime numbers cannot be divided evenly by any number other than 1 and the number itself.

For example:

- 2 is a prime number because it can only be divided by 1 and 2.
- 3 is also a prime number because it can only be divided by 1 and 3.
- However, 4 is not a prime number because it can be divided by 1, 2, and 4.

Take a Task

Watch Remedial

Prime Factors

Prime factors are the prime numbers that multiply together to give the original number. To find the prime factors of a number, we break it down into smaller prime numbers.

Prime Numbers

A prime number is a number greater than 1 that can only be divided by 1 and itself.

Examples: 2, 3, 5, 7, 11, 13, 17, etc.

Steps to Find Prime Factors:

Step 1. Start with the number you want to factorize.

Step 2. Divide the number by the smallest prime number (usually 2) and check if the result is a whole number.

Step 3. If it is, continue dividing by the same prime number until it is no longer divisible.

Step 4. Then, move on to the next prime number (3, 5, 7, etc.) and repeat the process.

Step 5. Continue until you can no longer divide by any prime number.

Example : Prime Factors of 18

Step 1. Start with 18.

Step 2. Divide by 2 (the smallest prime number):

$18 \div 2 = 9$

So, $18 = 2 \times 9$.

Step 3. Now divide 9 by 3 (next prime number):

$9 \div 3 = 3$

So, $9 = 3 \times 3$.

Step 4. Now, 3 is a prime number, so the prime factors of 18 are:

$2 \times 3 \times 3$ or 2×3^2.

Points to Remember

Prime numbers are infinite!
The number 1 is NOT a prime number because it has only one factor (1 itself).
All prime numbers (except 2 and 5) end in 1, 3, 7, or 9 in the decimal system.
The largest known prime number has over 24 million digits (found using computers)!

Example : Prime Factors of 30

Step 1. Start with 30.

Step 2. Divide by 2:

$30 \div 2 = 15$

So, $30 = 2 \times 15$.

Step 3. Now divide 15 by 3 (the next smallest prime):

$15 \div 3 = 5$

So, $15 = 3 \times 5$.

Step 4. 5 is a prime number, so the prime factors of 30 are: $\mathbf{2 \times 3 \times 5}$.

Common Factors and Common Multiples

In mathematics, understanding common factors and common multiples is important for solving problems involving numbers. These concepts help us work with numbers that are shared by two or more numbers. Let's take a closer look at what they mean!

Common Factors

Common factors are numbers that are factors of two or more numbers. In other words, common factors are numbers that divide two or more numbers exactly (without leaving a remainder).

Example : Common Factors of 12 and 18

- Factors of 12: 1, 2, 3, 4, 6, 12
- Factors of 18: 1, 2, 3, 6, 9, 18

The common factors of 12 and 18 are: 1, 2, 3, 6.

Example : Common Factors of 15 and 20

- Factors of 15: 1, 3, 5, 15
- Factors of 20: 1, 2, 4, 5, 10, 20

The common factors of 15 and 20 are: 1, 5.

Common Multiples

Common multiples are numbers that are multiples of two or more numbers. In other words, common multiples are numbers that can be divided by both numbers evenly.

Example : Common Multiples of 4 and 6

- Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, …
- Multiples of 6: 6, 12, 18, 24, 30, …

The common multiples of 4 and 6 are: 12, 24, …

Points to Remember

Common Factors: Factors that two or more numbers share.

Common Multiples: Multiples that two or more numbers share.

Greatest Common Factor (GCF)

The greatest common factor is the largest number that divides both numbers. For example, the GCF of 12 and 18 is 6 (the greatest factor they share).

Least Common Multiple (LCM)

The least common multiple is the smallest multiple that both numbers share. For example, the LCM of 4 and 6 is 12 (the smallest common multiple).

Jump Treasure

A number line up to 30 was shown to students in a class. They were asked to make jumps of equal size starting from 0 and land exactly on 30. The jump sizes chosen by some students are:

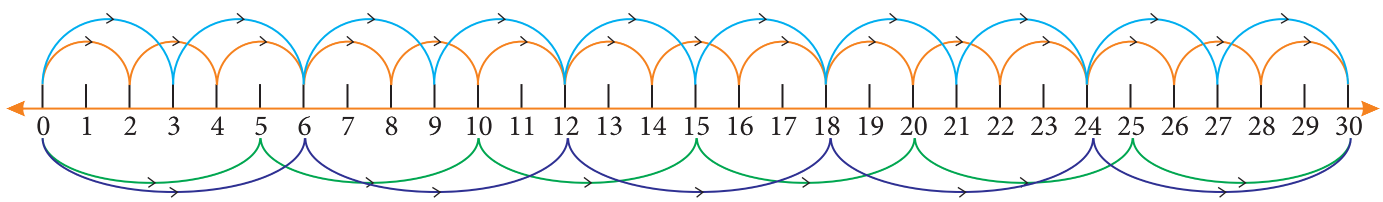


Fig. 5.1

- Raj: "I will take jumps of 2."
- Riya: "I will take jumps of 3."
- Aman: "Let me use jumps of 5."

- Sonal: "Oh! I can use jumps of 6."

Solution:

Let's analyze the jumps made by the students:

- **Raj :** Jumps of size 2 (2 → 4 → 6 → 8 → 10 → 12 → 14 → 16 → 18 → 20 → 22 → 24 → 26 → 28 → 30).
- **Riya:** Jumps of size 3 (3 → 6 → 9 → 12 → 15 → 18 → 21 → 24 → 27 → 30).
- **Aman:** Jumps of size 5 (5 → 10 → 15 → 20 → 25 → 30).
- **Sonal:** Jumps of size 6 (6 → 12 → 18 → 24 → 30).

Thus, the factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30.

Example : **Determine the common factors of: (I) 16 and 40 (II) 24, 36, and 48.**

(I) 16 and 40

Factors: Factors of 16: 1, 2, 4, 8, 16

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

Common factors: 1, 2, 4, 8

(II) 24, 36, and 48

Factors: Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Common factors: 1, 2, 3, 4, 6, 12

Example : **Write the first 5 multiples of: 12 and 14**

Multiples of 12: $12 \times 1 = 12$ $12 \times 2 = 24$ $12 \times 3 = 36$ $12 \times 4 = 48$ $12 \times 5 = 60$

Multiples of 12: 12, 24, 36, 48, 60

Multiples of 14: $14 \times 1 = 14$ $14 \times 2 = 28$ $14 \times 3 = 42$ $14 \times 4 = 56$ $14 \times 5 = 70$

Multiples of 14: 14, 28, 42, 56, 70

Example : **Find the Common Factors of 24 and 36 and Determine the Greatest Common Factor (GCF).**

Step 1: List the factors of 24 and 36.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Step 2: Identify the common factors.

The common factors of 24 and 36 are the numbers that appear in both lists: 1, 2, 3, 4, 6, 12

Step 3: Find the Greatest Common Factor (GCF).

- The GCF is the largest number that is a common factor.
- In this case, the largest common factor is 12.
- Therefore, the GCF of 24 and 36 is 12.

Exercise 5.1

Knowledge Application

1. Find the common factors of:

(a) 12 and 20 (b) 16 and 24 (c) 9 and 27 (d) 6 and 15

(e) 45, 60, and 90 (f) 16, 20, and 24 (g) 40, 60, and 80 (h) 18, 27, and 36

2. Write the first five multiples of:

(a) 9 (b) 6 (c) 13 (d) 8

(e) 7 (f) 12 (g) 16 (h) 20

3. Write all the factors of the following numbers:

(a) 36 (b) 18 (c) 50 (d) 48

(e) 28 (f) 13 (g) 100 (h) 7

4. Find the first three common multiples of:

(a) 3 and 5 (b) 4 and 10 (c) 6 and 9 (d) 12 and 15

(e) 4, 5, and 8 (f) 2, 3, and 7 (g) 6, 9, and 12 (h) 5, 7, and 10

5. Decide whether the following statements are true or false:

(a) The common factors of 18 and 24 are 2, 3, and 6.

(b) The first common multiple of 4 and 6 is 12.

(c) The highest common factor (HCF) of 20 and 30 is 10.

(d) The least common multiple (LCM) of 9 and 15 is 30.

(e) The number 1 is a common factor of every pair of numbers.

6. Provide the missing information in the blanks.

(a) The HCF of 36 and 48 is ______________.

(b) The first common multiple of 7 and 9 is ______________.

(c) Two numbers whose only common factor is 1 are called ______________.

(d) The LCM of 8 and 12 is ______________.

(e) The greatest factor of any number is the ______________ itself.

Prime and Composite Numbers

Take a Task

Watch Remedial

Prime Numbers

- A prime number is a number that has exactly two distinct positive divisors: 1 and itself.
- In other words, a prime number can only be divided by 1 and the number itself without leaving a remainder.

Examples:

✦ 2 is prime because it can only be divided by 1 and 2.

✦ 3 is prime because it can only be divided by 1 and 3.

✦ 5 is prime because it can only be divided by 1 and 5.

- The smallest prime number is 2, and it is the only even prime number. All other prime numbers are odd.

Composite Numbers

- A composite number is a number that has more than two distinct divisors.
- In other words, a composite number can be divided by 1, itself, and at least one other number.

Examples:

✦ 4 is composite because it can be divided by 1, 2, and 4.

✦ 6 is composite because it can be divided by 1, 2, 3, and 6.

✦ 9 is composite because it can be divided by 1, 3, and 9.

Key Differences:

- Prime Numbers have only 2 divisors: 1 and the number itself.
- Composite Numbers have more than 2 divisors.

For example:

- 2, 3, 5, 7, 11, 13, etc. is prime.
- 4, 6, 8, 9, 10, 12, etc is composite.

Fun Fact

Did you know that 2 is the only even prime number?

KEY POINTS

Prime Numbers:

- 2 is the only even prime number.
- All other prime numbers are odd.
- Prime numbers are greater than 1.

Composite Numbers:

- Composite numbers have more than two divisors.
- The smallest composite number is 4.
- All even numbers (except 2) are composite numbers because they have at least three factors (1, 2, and themselves).

Twin Primes

- Twin primes are pairs of prime numbers that have a difference of 2.
- In other words, if you subtract the smaller prime number from the larger one in the pair, the result is 2.

Examples:

- ✦ (3, 5): 5 - 3 = 2
- ✦ (5, 7): 7 - 5 = 2
- ✦ (11, 13): 13 - 11 = 2
- ✦ (17, 19): 19 - 17 = 2

- Twin primes are rare, and as numbers get larger, they become less frequent.

Prime Triplets

- A prime triplet consists of three prime numbers that are close together, usually differing by just 2 or 4 between them.

Examples :

- ✦ (3, 5, 7): These are three consecutive prime numbers.
- ✦ (5, 7, 11): These are three prime numbers, with the difference between 5 and 7 being 2, and between 7 and 11 being 4.
- ✦ Note that prime triplets are quite rare compared to twin primes.

Co-primes (or Relatively Prime Numbers):

- Co-primes (or relatively prime numbers) are two numbers that do not have any common divisor other than 1.
- In other words, the greatest common divisor (GCD) of two co-prime numbers is 1.
- Co-primes may or may not be prime numbers themselves, but they are always numbers whose only common factor is 1.

Examples :

- ✦ (8, 15): GCD of 8 and 15 is 1, so they are co-primes.
- ✦ (9, 28): GCD of 9 and 28 is 1, so they are co-primes.
- ✦ (14, 25): GCD of 14 and 25 is 1, so they are co-primes.

The Sieve of **Eratosthenes** is a simple and efficient way to find **all prime numbers** up to a given number **N**. It was invented by the ancient Greek mathematician **Eratosthenes**.

Steps to Find Prime Numbers Using the Sieve of Eratosthenes

1. Write down all numbers from 2 to N.
2. Start with the first number (2) and mark all its multiples (except itself).
3. Move to the next unmarked number (3) and mark all its multiples.
4. Repeat this process for the next number 5 and mark all its multiples..
5. Continue this process for the next unmark number 7 and mark all its multiples
6. The unmarked numbers left are prime numbers.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	~~17~~	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Fig. 5.2

Example: Find Prime Numbers Up to 30

1. Write numbers from 2 to 30.
2. Start with 2 (smallest prime) and mark 4, 6, 8, 10, 12, 14, …, 30.
3. Move to the next unmarked number 3 and mark 6, 9, 12, 15, 18, …, 30.
4. Next, 5 and mark 10, 15, 20, 25, 30.
5. Next, 7 and mark 14, 21, 28.
6. The remaining unmarked numbers are:
   2, 3, 5, 7, 11, 13, 17, 19, 23, 29 → These are prime numbers.

Rules to Check Whether a Number is Prime

Case 1: If a number lies "between" 1 to 100

- Use the definition of a prime number: A number is prime if it has exactly two factors: **1 and itself**.
- Alternatively, use the Sieve of Eratosthenes to check if the number is encircled (**Prime**) or crossed out (**Composite**).

Case 2: If a number lies "between" 100 to 200

- Check if it is divisible by any prime number less than 15 (i.e., 2, 3, 5, 7, 11, 13).
- If it is divisible by any of these, it is not prime; otherwise, it may be prime.

Case 3: If a number lies "between" 200 to 400

- Check if it is divisible by any prime number less than 20 (i.e., 2, 3, 5, 7, 11, 13, 17, 19).
- If it is not divisible by any of these, then it is prime.

Example : Which of the following numbers are prime?

(a) 29 (b) 45 (c) 37 (d) 91

Solution:

(a) 29: Factors of 29 are only 1 and 29. Hence, 29 is a prime number.

(b) 45: 45 is divisible by 1, 3, 5, 9, 15, and 45. Hence, 45 is a composite number.

(c) 37: Factors of 37 are only 1 and 37. Hence, 37 is a prime number.

(d) 91: 91 is divisible by 7 ($91 \div 7 = 13$). Hence, 91 is a composite number.

Example : Write three pairs of prime numbers less than 30 whose sum is divisible by 10.

Solution: The required pairs are:

1. 3, 7 : $3 + 7 = 10$ (divisible by 10)
2. 11, 19 : $11 + 19 = 30$ (divisible by 10)
3. 13, 17 : $13 + 17 = 30$ (divisible by 10)

Example : Find all prime numbers between 40 and 60.

Solution: The prime numbers between 40 and 60 are: 41, 43, 47, 53, 59

These numbers have no divisors other than 1 and themselves.

Example : Which of the following numbers are co-prime pairs?

(a) 8,15 (b) 16,28 (c) 9,28 (d) 14,35

Solution:

(a) 8 and 15: The GCD of 8 and 15 is 1. Hence, they are co-prime.

(b) 16 and 28: The GCD of 16 and 28 is 4. Hence, they are not co-prime.

(c) 9 and 28: The GCD of 9 and 28 is 1. Hence, they are co-prime.

(d) 14 and 35: The GCD of 14 and 35 is 7. Hence, they are not co-prime.

Exercise 5.2

Knowledge Application

1. Which of the following numbers are prime numbers?

(a) 17 (b) 21 (c) 31 (d) 29 (e) 44 (f) 49 (g) 101

(h) 143 (i) 113 (j) 67 (k) 121 (l) 83

2. Provide the missing information in the blanks.

(a) The sum of the first three prime numbers is ___________.

(b) A number that is not divisible by any number except 1 and itself is called a ___________.

(c) The smallest two-digit prime number is ___________.

(d) The number ___________ is the first composite number.

3. **Determine if the following statements are true (T) or false (F) and provide justification.**
   (a) All even numbers greater than 2 are composite.
   (b) The number 51 is a prime number.
   (c) 89 is a prime number.
   (d) Every composite number is divisible by at least three numbers.
   (e) There are more prime numbers than composite numbers between 1 and 20.

4. **Sara was given a list of numbers: 15, 23, 30, 31, and 37. She has to identify which numbers are prime and which are composite. Can you help Sara?**

5. **Multiple Choice Questions (MCQs):**
   (a) Which of the following numbers is not a prime number?
   (i) 17 ☐ (ii) 23 ☐ (iii) 39 ☐ (iv) 29 ☐
   (b) How many prime numbers are there between 50 and 70?
   (i) 2 ☐ (ii) 3 ☐ (iii) 4 ☐ (iv) 5 ☐
   (c) What is the greatest prime number between 1 and 50?
   (i) 43 ☐ (ii) 47 ☐ (iii) 49 ☐ (iv) 50 ☐
   (d) Which pair contains one prime number and one composite number?
   (i) 7, 9 ☐ (ii) 11, 13 ☐ (iii) 15, 17 ☐ (iv) 21, 23 ☐
   (e) What are the prime factors of 60?
   (i) 2, 3, 5 ☐ (ii) 3, 5, 10 ☐ (iii) 2, 3, 6 ☐ (iv) 3, 4, 5 ☐

Test for Divisibility of Numbers

Take a Task

Watch Remedial

1. Divisibility by 2:

- A number is divisible by 2 if its last digit is even (0, 2, 4, 6, or 8).

Example: 246 is divisible by 2 because its last digit is 6 (an even number).

2. Divisibility by 3:

- A number is divisible by 3 if the sum of its digits is divisible by 3.

Example: 123 → 1 + 2 + 3 = 6, and 6 is divisible by 3, so 123 is divisible by 3.

3. Divisibility by 4:

- A number is divisible by 4 if the last two digits form a number divisible by 4.

Example: 312 → The last two digits are 12, and 12 is divisible by 4, so 312 is divisible by 4.

4. **Divisibility by 5:**

- A number is divisible by 5 if its last digit is either 0 or 5.

Example: 205 is divisible by 5 because it ends in 5.

5. **Divisibility by 6:**

- A number is divisible by 6 if it is divisible by both 2 and 3.

Example: 72 → 72 is divisible by 2 (last digit is 2) and by 3 ($7 + 2 = 9$, divisible by 3), so 72 is divisible by 6.

6. **Divisibility by 7:**

- There isn't a simple rule like the others, but a quick way is to double the last digit, subtract it from the rest of the number, and see if the result is divisible by 7.

Example: 203 → Double the last digit ($3 \times 2 = 6$). Subtract 6 from 20 → $20 - 6 = 14$. Since 14 is divisible by 7, 203 is divisible by 7.

7. **Divisibility by 8:**

- A number is divisible by 8 if the last three digits form a number divisible by 8.

Example: 1,312 → The last three digits are 312, and 312 is divisible by 8 ($312 \div 8 = 39$), so 1,312 is divisible by 8.

8. **Divisibility by 9:**

- A number is divisible by 9 if the sum of its digits is divisible by 9.

Example: 459 → $4 + 5 + 9 = 18$, and 18 is divisible by 9, so 459 is divisible by 9.

9. **Divisibility by 10:**

- A number is divisible by 10 if its last digit is 0.

Example: 550 is divisible by 10 because it ends in 0.

10. **Divisibility by 11:**

- A number is divisible by 11 if the difference between the sum of its digits in odd positions and the sum of its digits in even positions is divisible by 11 (or if the difference is 0).

Example: 2728 → $(2 + 2) - (7 + 8) = 4 - 15 = -11$, which is divisible by 11, so 2728 is divisible by 11.

Example : Check the divisibility of 5,624 by 2, 3 and 5.

Solution: Divisibility by 2:

- A number is divisible by 2 if its units digit is even.
- The units digit of 5,624 is 4, which is even.
- So, 5,624 is divisible by 2.

Divisibility by 3:

- A number is divisible by 3 if the sum of its digits is divisible by 3.
- Sum of the digits of 5,624 is 5 + 6 + 2 + 4 = 175 + 6 + 2 + 4 = 175 + 6 + 2 + 4 = 17.
- Since 17 is not divisible by 3, 5,624 is not divisible by 3.

Divisibility by 5:

- A number is divisible by 5 if its units digit is either 0 or 5.
- The units digit of 5,624 is 4, which is neither 0 nor 5.
- So, 5,624 is not divisible by 5.

Example : What is the smallest number that must replace * to make the number 5*2 divisible by 2?

Solution:

- We know that a number is divisible by 2 if its units digit is even.
- In the number 5*2, the units digit is 2, which is even.
- Therefore, any number placed in ** * ** will make the number divisible by 2.
- The smallest number to replace ** * ** is 0.
- So, the smallest number that must replace * is 0, and the number becomes 502.

Example : Test the divisibility of: (i) 124 by 2 (ii) 543 by 3 (iii) 1250 by 5

Solution: (i) We know that a number is divisible by 2 if its units digit is divisible by 2.

In the given number 124, its units digit is 4.

Since 4 is divisible by 2, 124 is divisible by 2.

(ii) We know that a number is divisible by 3 if the sum of its digits is divisible by 3.

In the given number 543, the sum of its digits is 5+4+3=125 + 4 + 3 = 12, which is divisible by 3.

So, 543 is divisible by 3.

(iii) We know that a number is divisible by 5 if its units digit is either 0 or 5.

In the given number 1250, the units digit is 0.

So, 1250 is divisible by 5.

Exercise 5.3

Knowledge Application

1. **Complete the blanks .**

   (a) A number is divisible by 2 if its last digit is ___________.

   (b) A number is divisible by 3 if the sum of its digits is divisible by ___________.

   (c) A number is divisible by 4 if the number formed by its last __________ digits is divisible by 4.

   (d) A number is divisible by 5 if its last digit is ___________ or ___________.

   (e) A number is divisible by 6 if it is divisible by both ___________ and ___________.

2. **Provide the missing information in the blanks.**

   (a) A number is divisible by 2 if its last digit is ___________, by 3 if the sum of its digits is divisible by ___________, and by 5 if its last digit is ___________ or ___________.

   (b) A number is divisible by 4 if the number formed by its last ___________ digits is divisible by ___________, by 6 if it is divisible by both ___________ and ___________.

   (c) A number is divisible by 9 if the sum of its digits is divisible by ___________, and divisible by 10 if its last digit is ___________.

   (d) A number is divisible by 7 if the result of dividing it by 7 gives a remainder of ___________, and divisible by 11 if the alternating sum of its digits is divisible by ___________.

   (e) A number is divisible by 12 if it is divisible by both ___________ and ___________, and divisible by 15 if it is divisible by both ___________ and ___________.

3. **Rearrange the digits of the number 543 to make it divisible by 6.**

4. **Can you give an example of a number that is divisible by...**

   (a) Divisible by 2 but not by 5 ______________________

   (b) Divisible by 4 but not by 8 ______________________

   (c) Divisible by 6 but not by 9 ______________________

   (d) Divisible by 3 but not by 6 ______________________

   (e) Divisible by 5 but not by 10 ______________________

   (f) Divisible by 8 but not by 4 ______________________

5. **Examine whether the following numbers are divisible by 2:**

   (a) 142 (b) 895 (c) 620 (d) 1321 (e) 4780

   (f) 2048 (g) 3999 (h) 2112

6. Examine whether the following numbers are divisible by3:

(a) 396 (b) 5514 (c) 1587 (d) 1278 (e) 6315

(f) 2319 (g) 2991 (h) 4572

7. Examine whether the following numbers are divisible by4:

(a) 1120 (b) 7856 (c) 567 (d) 1632 (e) 4552

(f) 5432 (g) 947 (h) 8256

8. Examine whether the following numbers are divisible by5:

(a) 175 (b) 570 (c) 9000 (d) 3670 (e) 5225

(f) 4885 (g) 9340 (h) 125

9. Examine whether the following numbers are divisible by6:

(a) 132 (b) 429 (c) 846 (d) 675 (e) 444

(f) 543 (g) 1152 (h) 8976

10. Examine whether the following numbers are divisible by 9:

(a) 612 (b) 1098 (c) 1575 (d) 7341 (e) 5127

(f) 3819 (g) 7410 (h) 8889

11. Examine whether the following numbers are divisible by 10:

(a) 230 (b) 4200 (c) 56710 (d) 8900 (e) 10020

(f) 130 (g) 1140 (h) 9990

Prime Factorisation

Prime factorisation is the process of breaking down a number into its smallest prime numbers that, when multiplied together, give the original number. A prime number is a number that is greater than 1 and has only two factors: 1 and itself. For example, 2, 3, 5, 7, and 11 are prime numbers.

Take a Task

Watch Remedial

Steps for Prime Factorization:

1. Start with the smallest prime number (2) and divide the given number.
2. Keep dividing the number by 2 until it is no longer divisible by 2.
3. Move to the next prime number (3, 5, 7, etc.), and repeat the process.
4. When you can no longer divide, the remaining numbers are prime factors.

Example : Prime factorisation of 36.

1. Start with 36.
2. Divide by 2 (the smallest prime number):
   - $36 \div 2 = 18$
3. Divide 18 by 2 again:
   - $18 \div 2 = 9$
4. Now divide 9 by the next smallest prime number, 3:
   - $9 \div 3 = 3$
5. Finally, divide 3 by 3:
   - $3 \div 3 = 1$

2	36
2	18
3	9
3	3
	1

So, the prime factorisation of 36 is: $36 = 2 \times 2 \times 3 \times 3$

1. Division Method

This is the most common method, where we divide the number by prime numbers until we reach 1.

Steps:

1. Start with the given number.
2. Divide it by the smallest prime number (2).
3. Keep dividing the quotient by the smallest prime until it is no longer divisible.
4. Move to the next prime number (3, 5, 7, etc.) and continue dividing.
5. When the quotient is 1, the prime factorization is complete.

Example : Prime factorisation of 48 using the division method:

2	48
2	24
2	12
2	6
3	3
	1

So, the prime factorisationof 36 is: $48 = 2 \times 2 \times 2 \times 2 \times 3$

2. Factor Tree Method

In this method, you create a factor tree, where you break the number down into factors step by step until you reach prime numbers.

Steps:

1. Write the number at the top.
2. Split it into any two factors (preferably prime factors).
3. Continue breaking down the factors until all of them are prime numbers.
4. The prime factors, when multiplied together, will give the original number.

Example: Prime factorisation of 48 using the factor tree method:

Prime factorisation of 48 in different ways as:

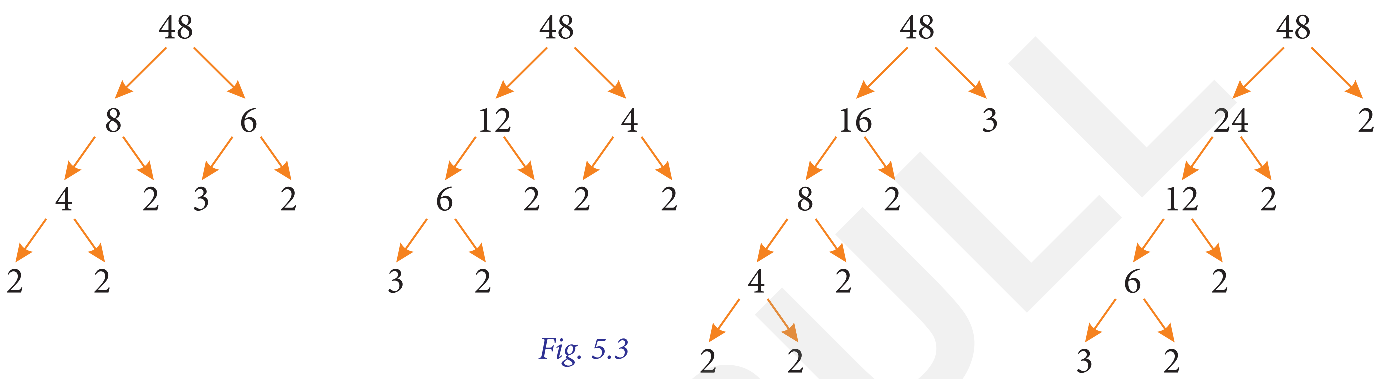


Fig. 5.3

Example : Find the prime factorisation of the following numbers:

(i) 60 **(ii) 144**

(i)

2	60
2	30
3	15
5	5
	1

Prime Factorization: $60 = 2 \times 2 \times 3 \times 5$

(ii)

2	144
2	72
2	36
2	18
3	9
3	3
	1

Prime Factorization: $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Example : Find the smallest number having four different prime factors.

Solution: The smallest four distinct prime numbers are 2, 3, 5 and 7.

The required number is: $2 \times 3 \times 5 \times 7 = 210$

Example : Which of the following numbers is the product of exactly four distinct prime numbers: 210,420,231,1155,462?

Solution:

1. $210=2\times3\times5\times7$ (Four distinct primes: 2, 3, 5, 7)

2. $420=2\times2\times3\times5\times7$ (Four primes, but 2 is repeated)
3. $231=3\times7\times11$ (Three distinct primes: 3, 7, 11)
4. $1155=3\times5\times7\times11$ (Four distinct primes: 3, 5, 7, 11)
5. $462=2\times3\times7\times11$ (Four distinct primes: 2, 3, 7, 11)

Thus, 210, 1155, and 462 are products of exactly four distinct prime numbers.

Exercise 5.4

Knowledge Application

1. **Provide the missing information in the blanks:**
   (a) The prime factorization of 18 is ________________.
   (b) The number 48 can be written as the product of prime numbers: ________________.
   (c) The prime factorization of 45 is ________________.
   (d) The smallest prime factor of 30 is ________________.
   (e) A prime number has only two factors: ________________ and ________________.

2. **Provide the missing information in the blanks:**
   (a) The factor tree of 24 starts with ______________ and its prime factorization is ___________.
   (b) The factor tree of 36 gives the prime factorization ________________.
   (c) To find the prime factorization of 60 using a factor tree, we start with ________________ and break down the factors until we reach prime numbers.

3. **Which of the following expressions has the correct prime factorization?**
   (a) $48 = 2\times2\times2\times2\times3$ (b) $120 = 2\times2\times3\times5\times5$ (c) $81 = 3\times3\times3\times3$
   (d) $56 = 2\times2\times2\times3\times7$ (e) $45 = 3\times3\times5$ (f) $72 = 2\times2\times2\times3\times3$

4. **Write the smallest 5-digit number and express it in the form of its prime factors.**
5. **Write the greatest 5-digit number and express it in terms of its prime factors**
6. **Which number has more prime factors: 36 or 48? Use a factor tree to find out.**
7. **A gardener has 72 flowers. He arranges them into rows, with each row having the same number of flowers. The number of flowers in each row is a prime number. What are the possible number of flowers in each row?**
8. **Find the smallest number having five different prime factors.**
9. **Which of the following numbers is the product of exactly three distinct prime numbers?**
   30, 60, 105, 210, 660

Highest Common Factor (HCF)

The Highest Common Factor (HCF) is the largest number that divides two or more numbers exactly (without leaving any remainder). In simple words, it is the biggest number that is a **factor of all the given numbers**.

Key Terms:

- **Factor:** A factor is a number that divides another number exactly, leaving no remainder.
- **Common Factor:** A common factor is a number that divides two or more numbers exactly.

Example:

Let's say you have two numbers: 12 and 18.

- The factors of 12 are: 1, 2, 3, 4, 6, 12
- The factors of 18 are: 1, 2, 3, 6, 9, 18

The common factors of 12 and 18 are: 1, 2, 3, 6

The highest common factor (HCF) is the largest of these common factors, which is 6.

So, the HCF of 12 and 18 is 6.

The Highest Common Factor (HCF) is the largest number that divides two or more numbers without leaving a remainder. To find the HCF of two or more numbers, follow these steps:

Method 1 : Listing the Factors

1. **List the factors of each number:** Factors of a number are the numbers that divide it exactly (without leaving a remainder).
2. **Find the common factors:** Look for the numbers that appear in both lists.
3. **Choose the highest factor:** The largest of the common factors is the HCF.

Example : Find the HCF of 12 and 18.

1. List the factors:
   - Factors of 12: 1, 2, 3, 4, 6, 12
   - Factors of 18: 1, 2, 3, 6, 9, 18
2. Find the common factors:
   - Common factors: 1, 2, 3, 6
3. Choose the highest factor:
   - HCF of 12 and 18 is 6.

Take a Task

Watch Remedial

Method 2 : Prime Factorization

1. Write the prime factorization of each number.
2. Identify the common prime factors with the smallest power.
3. Multiply these common factors to get the HCF.

Example: Find the HCF of 12 and 18 using prime factorization.

1. Prime factorization:
   - $12 = 2 \times 2 \times 3$
   - $18 = 2 \times 3 \times 3$
2. Common prime factors:
   - Common factors: 2 and 3
3. HCF:
   - $HCF = 2 \times 3 = 6$

These are the two common methods to find the HCF of two or more numbers.

Properties of HCF (Highest Common Factor)

1. HCF of 1 and any number is 1.
2. HCF of two numbers is always less than or equal to the smaller number.
3. HCF of a number and itself is the number.
4. HCF divides both the numbers exactly.
5. HCF of two co-prime numbers is 1.

Example : Find the HCF of 18 and 42.

Step 1: List the factors of each number.

- Factors of 18: 1, 2, 3, 6, 9, 18
- Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

Step 2: Find the common factors. The common factors of 18 and 42 are: 1, 2, 3, 6

Step 3: Choose the highest common factor. The highest common factor (HCF) is 6.

Final Answer: HCF = 6

Example : A baker has two trays, one weighing 56 kg and the other 98 kg. What is the maximum weight of dough that can be measured on both trays exactly?

To find the HCF (Highest Common Factor) of 56 and 98, we can use the factorization method or listing the factors. Let's solve this step-by-step.

Step 1: Find the factors of 56 and 98.

- Factors of 56: 56 = 1, 2, 4, 7, 8, 14, 28, 56
- Factors of 98: 98 = 1, 2, 7, 14, 49, 98

Step 2: Identify the common factors.

The common factors of 56 and 98 are: 1, 2, 7, 14

Step 3: Choose the highest common factor.

The highest common factor (HCF) is 14.

The maximum weight of dough that can be measured on both trays exactly is 14 kg.

Lowest Common Multiple (LCM)

The Lowest Common Multiple (LCM) of two or more numbers is the smallest multiple that is divisible by all of them. It is a key concept in number theory and is widely used in solving problems that involve fractions, ratios, and divisibility.

- For example, the multiples of 4 are: 4, 8, 12, 16, 20, 24 ...
- The multiples of 6 are: 6, 12, 18, 24, 30 ...

The LCM of 4 and 6 is the smallest number that appears in both lists, which is 12.

Steps to Find the LCM:

1. List the multiples: Start by writing down the first few multiples of each number.
2. Find the smallest common multiple: Look for the smallest number that appears in both lists of multiples.

There are following methods for finding LCM of two or more numbers: (i) Prime Factorisation Method (ii) Common Division Method (iii) Listing Multiples Method (Alternative Method).

Method 1: Prime Factorisation Method

This method involves breaking down each number into its prime factors (the smallest numbers that can multiply together to give the original number). After finding the prime factors, we multiply them back together, taking the highest power of each factor that appears.

To calculate the LCM of two or more numbers, follow these steps:

1. **Obtain the Numbers :** Identify the numbers for which the LCM is required.
2. **Prime Factorization :** Perform the prime factorization of each number.

For example:

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$60 = 2 \times 2 \times 3 \times 5$

3. List All Prime Factors

Write down all the prime factors of the numbers. For the LCM, include each prime factor the maximum number of times it occurs in any single number's factorization.

In the example:

2 occurs a maximum of three times (from 72),

3 occurs a maximum of two times (from 72),

5 occurs a maximum of one time (from 60).

4. Multiply the Factors

Multiply all the factors together to find the LCM : $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

So, the LCM of 72 and 60 is 360.

Method 2: Common Division Method (or Division Ladder Method)

This method involves repeatedly dividing the numbers by common divisors, and recording the quotients. You keep dividing until no more common divisors can be found. Follow these steps:

1. Arrange the given numbers in a row, separated by commas.

Example: 72, 60.

2. Choose a number that divides at least two of the numbers exactly. Write the quotients of the division below each number. If a number is not divisible, carry it down unchanged.

Example:

- Divide by 2: 72, 60 → 36, 30
- Divide by 2: 36, 30 → 18, 15
- Divide by 3: 18, 15 → 6, 5
- Divide by 2: 6, 5 → 3, 5

2	72, 60
2	36, 30
3	18, 15
2	6, 5
	3, 5

3. Continue dividing until no two numbers have a common factor.
4. Multiply all the divisors and any numbers left in the final row to find the LCM.

Example:

Divisors: 2, 2, 3, 2, 3, 5.

LCM $= 2 \times 2 \times 3 \times 2 \times 3 \times 5 = 360$.

So, the LCM of 72 and 60 is 360.

This approach provides two systematic methods to calculate the LCM effectively.

Properties of LCM

1. LCM of Two Numbers is Always Greater Than or Equal to the Larger Number
2. LCM of Two Numbers is Divisible by each of the Numbers
3. LCM of a Number and Itself is the Number
4. LCM of Two Prime Numbers is Their Product
5. LCM of Two Numbers May Not Always Be Their Product
6. Relationship between LCM and HCF
   - For any two numbers, the product of their LCM and HCF is equal to the product of the two numbers.

   Formula:

   HCF $\times$ LCM = Product of the Two Numbers

   Example:

   Numbers : 4 and 6.

   HCF = 2, LCM = 12.

   $2 \times 12 = 4 \times 6 = 24$
7. The product of the HCF and the LCM of two numbers is always equal to their product. For example, if 5 and 20 are two numbers then, HCF $\times$ LCM $= 5 \times 20$ or, HCF $= \frac{5 \times 20}{\text{LCM}}$ or, LCM $= \frac{5 \times 20}{\text{HCF}}$

Example : Determine the Least Common Multiple (LCM) of the given numbers using the Prime Factorization Method.

1. 24 and 56.　　　　2. 50, 75 and 100.

Solution: Finding the LCM Using the Prime Factorization Method

1. LCM of 24 and 56

Prime Factorization of

- $24 = 2 \times 2 \times 2 \times 3$
- $56 = 2 \times 2 \times 2 \times 7$

We see the maximum number of term 2 coming in factorisation of 24 and 2 coming in factorisation of 56

LCM $= 2 \times 2 \times 2 \times 3 \times 7 = 8 \times 3 \times 7 = 168$

LCM of 42 and 56 is 168.

2. LCM of 50, 75, and 100

Prime Factorization of

- 50=2×5×5
- 75=3×5×5
- 100=2×2×5×5

We see that 5 comes 2 times, 2 comes 2 time, and 3 one time .

LCM = $2 \times 2 \times 3 \times 5 \times 5 = 4 \times 3 \times 25 = 300$

LCM of 50, 75, and 100 is 300.

Example : Calculate the Least Common Multiple (LCM) of using the Common Division Method.

1. 72 and 108 2. 18, 24, and 30

LCM of 72 and 108 using the Common Division Method:

2	72, 108
2	36, 54
3	18, 27
3	6, 9
	2, 3

Hence. the LCM of 72 and 108

$2 \times 2 \times 2 \times 3 \times 3 \times 3$

Now, multiply all the prime divisors together to get the LCM:

LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216$

Thus, the LCM of 72 and 108 is 216.

LCM of 18, 24, and 30 using the Common Division Method:

2	18, 24, 30
3	9, 12, 15
2	3, 4, 5
	3, 2, 5

Hence. the LCM of18, 24, and 30

$2 \times 2 \times 2 \times 3 \times 3 \times 5$

Now, multiply all the prime divisors together to get the LCM:

LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Thus, the LCM of 18, 24, and 30 is 360.

Exercise 5.5

Knowledge Application

1. Find the LCM by Prime Factorisation Method:

(a) 30 and 45 (b) 48 and 72 (c) 24 and 32 (d) 80 and 100

(e) 14, 35, and 49 (f) 56, 84, and 126

2. **Word Problem on HCF and LCM:**

   (a) Two numbers have HCF 6 and LCM 180. If one number is 36, find the other number.

   (b) A school bell rings every 15 minutes. Another bell rings every 20 minutes. They ring together at 8:00 AM. At what time will they next ring together?

   (c) Two friends are walking around a circular track. One completes a lap every 12 minutes, and the other completes a lap every 15 minutes. They start walking at the same time. After how many minutes will they meet again at the starting point?

3. **Find the LCM by Common Division Method:**

   (a) 60 and 90 (b) 20 and 50 (c) 18, 24, and 30 (d) 72, 84, and 126

   (e) 36, 48, and 72 (f) 120, 150, and 180 (g) 250 and 375

4. **Solve for HCF and LCM using the relationship:**

   If the product of two numbers is 360 and their HCF is 12, find their LCM.

   (a) Product of two numbers = 144, HCF = 24

   (b) Product of two numbers = 240, HCF = 8

5. **The LCM of two numbers is 96, and their HCF is 8. Find the product of the two numbers.**

6. **If the LCM of two numbers is 180 and their HCF is 15, find the numbers if one of the numbers is 45.**

7. **True/False Statements with Explanations:**

   (a) The LCM of two numbers is always greater than or equal to the HCF.

   (b) If the LCM of two numbers is 1, the two numbers must be co-prime.

   (c) The product of the LCM and HCF of two numbers is always equal to the product of the numbers.

   (d) If two numbers are co-prime, their LCM is their product.

   (e) If the HCF of two numbers is the same as their LCM, then both numbers are the same.

8. **A person is planting trees in rows. In each row, there are 12 trees. In each column, there are 16 trees. What is the total number of trees if the person arranges them in such a way that the number of rows and columns is the same?**

9. **A cyclist and a pedestrian are traveling on a circular path. The cyclist completes one round in 18 minutes, and the pedestrian completes one round in 24 minutes. If they start together at 6:00 AM, at what time will they meet again at the starting point?**

Think Tank

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Take a Test

1. **Tick (✓) the correct answer:**

a. Which of the following is a prime number?

(i) 9 ☐ (ii) 15 ☐ (iii) 17 ☐ (iv) 20 ☐

b. The smallest prime number is:

(i) 1 ☐ (ii) 2 ☐ (iii) 3 ☐ (iv) 5 ☐

c. Which of the following numbers is divisible by 2 and 3 but not by 5?

(i) 6 ☐ (ii) 12 ☐ (iii) 18 ☐ (iv) 15 ☐

d. Which of the following is not a prime number?

(i) 3 ☐ (ii) 5 ☐ (iii) 11 ☐ (iv) 9 ☐

e. Which number is divisible by both 7 and 13?

(i) 91 ☐ (ii) 77 ☐ (iii) 104 ☐ (iv) 91 and 77 ☐

2. **Provide the missing information in the blanks.**

a. A prime number is a natural number greater than 1 that has exactly _______ divisors.

b. The first prime number is _______.

c. A number is divisible by 2 if its last digit is _______.

d. The product of two prime numbers 3 and 5 is _______.

e. 2 is the only _______ prime number.

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3. **A group of students are playing a game where each player can only choose a number that is prime. How many prime numbers can a player choose from between 1 and 30? List them.**

4. **If you multiply two prime numbers together and then subtract 1, will the result always be divisible by both prime numbers? Explain with examples.**

5. **Consider the number 42. Can it be factored into two prime numbers? If so, what are they?**

6. **Match the Columns:**

Match the number with its classification.

Column A	Column B
a) Composite number	i) A number that has more than two factors.
b) Prime factors of 60	ii) 2, 2, 2, 2, 7
c) Prime number	iii) A number that has exactly two positive factors.
d) Prime factors of 82	iv) 2 , 2, 3, 5

Assertion and Reason

Experiential Learning

Each question has two statements, Assertion (A) and Reason (R). Choose the correct option:

A: Both A and R are true, and R is the correct explanation of A.

B: Both A and R are true, but R is not the correct explanation of A.

C: A is true, but R is false.

D: A is false, but R is true.

1. **Assertion (A):** 2 is the only even prime number.
   Reason (R): All other even numbers are divisible by 2, and therefore are not prime.
2. **Assertion (A):** Prime numbers cannot be divisible by any number other than 1 and themselves.
   Reason (R): A prime number has exactly two divisors: 1 and itself.
3. **Assertion (A):** 4 is a prime number.
   Reason (R): Prime numbers have only two divisors: 1 and the number itself.
4. **Assertion (A):** All numbers ending in 5 are divisible by 5.
   Reason (R): Any number ending in 5 is divisible by 5.
5. **Assertion (A):** 1 is considered a prime number.
   Reason (R): Prime numbers must have exactly two divisors.

HOTS (Higher Order Thinking Skills)

Critical Thinking

1. Consider the number 25. Can it be factored into two prime numbers? If so, what are they?
2. What is the largest prime number less than 100? How can you prove that it's prime?
3. A prime number is divisible by only 1 and itself. If we subtract 1 from a prime number, will the result always be divisible by the prime number's factors? Justify your answer.

Case Study

Critical Thinking

In a game, players must pick a number that is prime. The range of numbers from which they can choose is 1 to 20. The numbers are: **{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}**.

Questions:

1. List all the prime numbers between 1 and 20.
2. How many prime numbers are there between 1 and 20?
3. What is the sum of the prime numbers between 1 and 20?
4. Can 16 be classified as a prime number? Justify why or why not.
5. Which prime number is the largest in the range from 1 to 20?