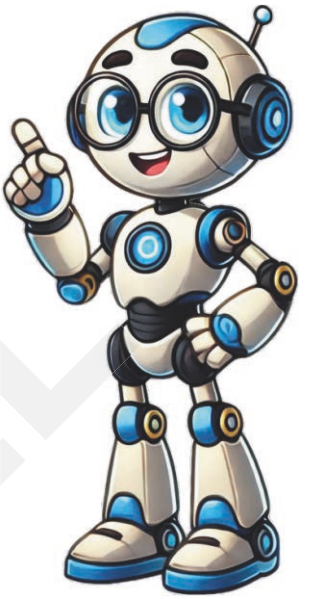
10
Chapter

Integers: The Other Side of Zero

We'll cover the following key points:

→ Introduction to Integers
→ Addition of Integers
→ Subtraction of Integers
→ Integers in Other Places
→ A Hollow Integer Grid



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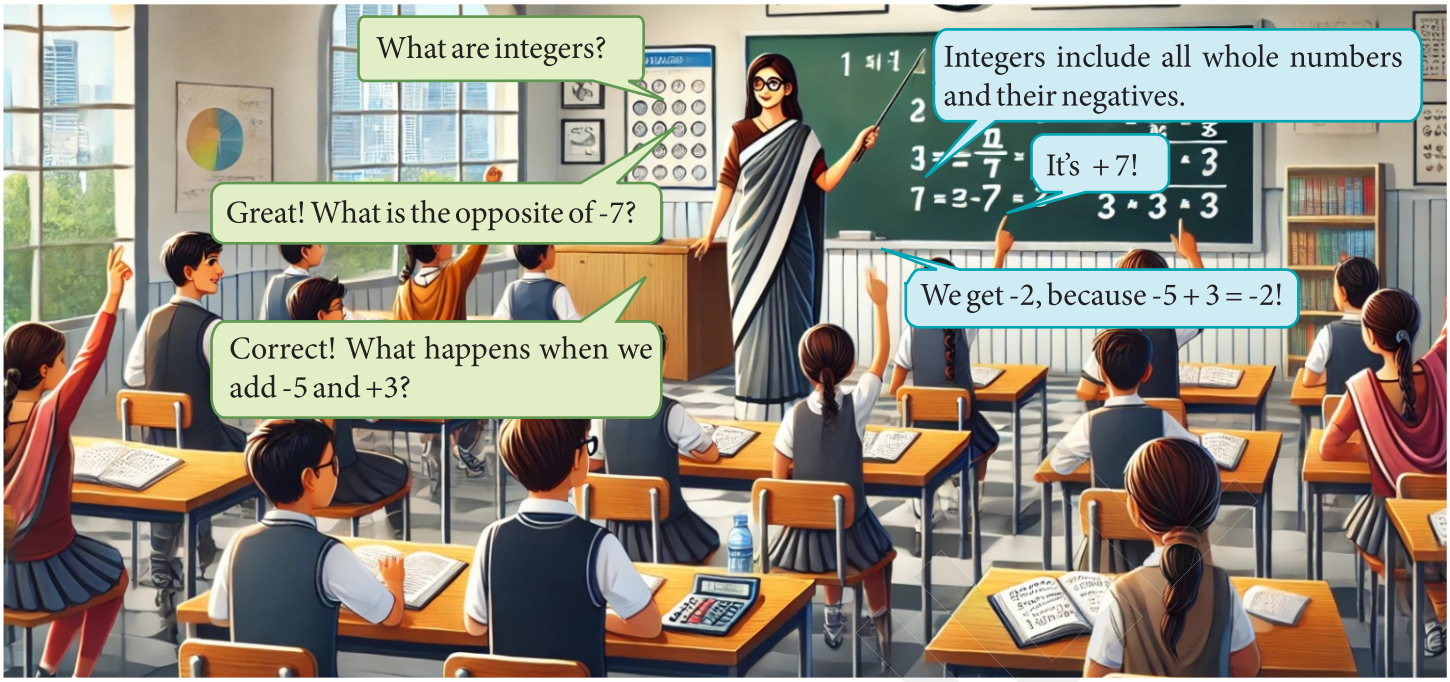
Learning Outcomes

By the end of this chapter, students will be able to:

- Understand and identify integers on a number line.
- Perform addition, subtraction, multiplication, and division of integers.
- Apply the concept of negative and positive integers in real-life situations.
- Solve problems involving integers in everyday contexts.
- Understand and use the concept of absolute value.
- Compare and order integers on a number line.
- Relate integers to the concept of temperature, elevation, and balance.
- Solve word problems involving integers and their operations.
- Analyze and recognize patterns in integer operations, including the roles of zero and one in multiplication and division.
- Develop and apply strategies for solving multi-step problems that involve a combination of integer operations and the order of operations.

Introduction



In mathematics, the very first numbers that we studied were the counting numbers. These counting numbers are called **NATURAL NUMBERS**.

Natural numbers are the set of positive numbers that are used for counting and ordering. These are the numbers you probably use every day to count objects, such as 1, 2, 3, 4, and so on. They form the basis of most of the math that we use in daily life.

In mathematics, **whole numbers** are a set of numbers that include all the **natural numbers** and **zero**. Whole numbers are essentially the set of numbers starting from **0** and going upwards without any fractions or decimals.

Key Points about Natural Numbers:

- **Natural Numbers:** The set of numbers starting from 1 and going onwards without any end. They are written as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...

Fig. 10.1

- **Starting Point:** Natural numbers start at 1, and there is no zero or negative number in the set of natural numbers.
- **Used for Counting:** Natural numbers are used when counting objects (e.g., 5 apples, 12 students).

Zero and Natural Numbers:

- Zero (0) is not considered a natural number.

- However, zero is a very important number in mathematics, and it is used to represent the absence of any quantity.

Relationship between Natural Numbers and Integers:

Natural numbers are a part of the broader group of integers. Integers include:

- Positive numbers (which are natural numbers),
- Negative numbers (e.g., -1, -2, -3),
- Zero (which is neither positive nor negative).

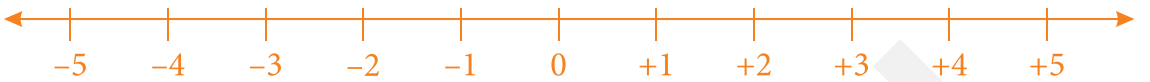
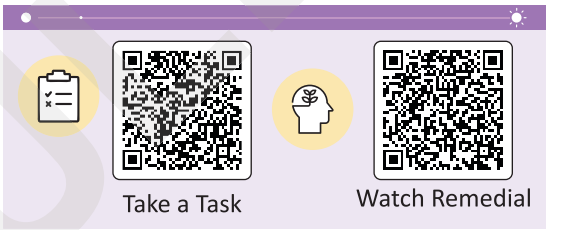


Fig. 10.2

So, integers include natural numbers, negative numbers, and zero, while natural numbers are strictly positive numbers starting from 1.



Integers:

An integer is any whole number that can be positive, negative, or zero. Integers do not have fractions or decimal points.In mathematics, integers are a group of numbers that include all the whole numbers and their negative counterparts. They consist of:

- **Positive integers:** Numbers greater than zero (e.g., 1, 2, 3, 4, ...)
- **Negative integers:** Numbers less than zero (e.g., -1, -2, -3, -4, ...)
- **Zero:** The number that separates positive and negative integers.

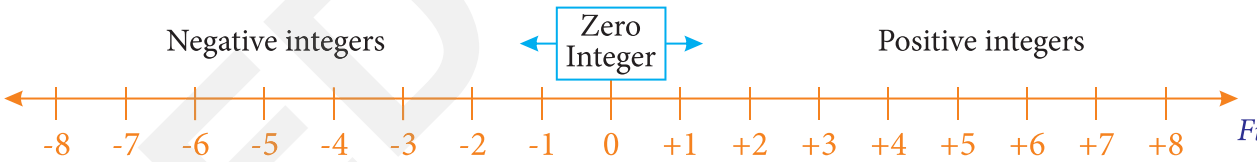


Fig. 10.3

Integers are used in many aspects of everyday life, such as in temperatures (e.g., +5°C and -5°C), financial transactions (e.g., gains and losses), and in measuring distances (e.g., above and below sea level).

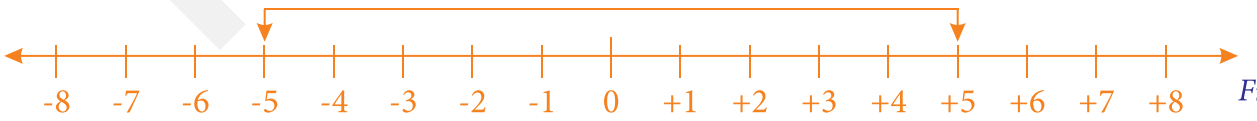


Fig. 10.4

KEY POINTS

- Integers can be positive or negative.
- Zero is neither positive nor negative; it is the middle point that divides the positive and negative integers.
- Integers are often represented on a number line where zero is at the center, and positive integers extend to the right while negative integers extend to the left.

The numbers located to the right of zero on the number line are called **positive numbers**, while those to the left are called **negative numbers**. Zero serves as a neutral point on the number line and does not belong to either the positive or negative set. The relationship between positive and negative numbers can be visualized as follows:

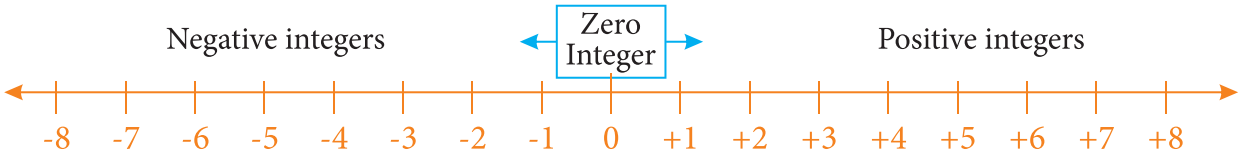


Fig. 10.5

Integers are the set that includes both positive and negative whole numbers, as well as zero. This can be defined as: **An integer is the set of natural numbers, their opposites, and zero.**

On the number line:

- Numbers to the right of zero (greater than zero) are positive integers.
- Numbers to the left of zero (less than zero) are negative integers.
- Two integers are opposites if they are the same distance from zero but are on opposite sides of it.

Example: 3 and –3 are opposites.

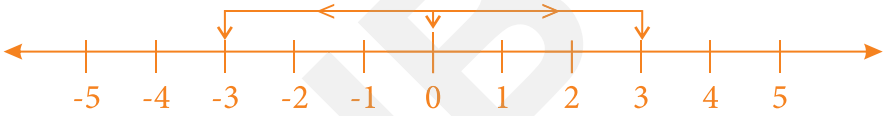


Fig. 10.6

In daily life:

- Positive numbers can represent gains, while negative numbers represent losses.
- A positive integer might represent a temperature increase, while a negative integer would represent a decrease.
- Moving forward is denoted by a positive integer, while moving backward is denoted by a negative integer.

Points to Remember

- All whole numbers and their opposites (negative numbers) make up integers.
- Zero is neither a positive nor a negative integer.
- Integers can be both positive and negative, but only positive integers are natural numbers.
- All natural numbers are integers but all integers are not natural numbers.
- All whole numbers are integers but all integers are not whole numbers.

Representation of Integers on a Number Line:

Now, we learn how to represent integers (positive and negative numbers) on a number line. Here's how it looks:

- Whole numbers (0, 1, 2, 3, etc.) are shown on the number line, starting from zero and going to the right.

Fig. 10.7

- Negative numbers (-1, -2, -3, etc.) are shown to the left of zero. image

Fig. 10.8

When we combine both whole numbers and negative numbers, we get integers. The number line for integers looks like this:

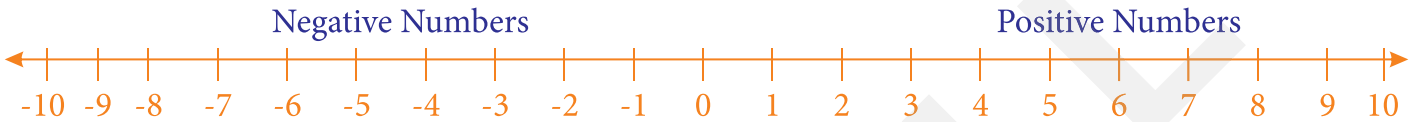


Fig. 10.9

Fun Fact

- There are **infinitely many positive** and **negative integers**.
- **Negative numbers** are always smaller than any **whole number** (e.g., -1 is smaller than 1).
- There is **no smallest or largest integer** because you can always go further in both directions.
- Every integer to the right of zero has an **opposite integer** to the left of zero. They are at the same distance from zero but have opposite signs.
- Each natural number and its **negative counterpart** are at the same distance from zero.

Ordering of Integers:

When we compare integers, we follow the rule that the number to the right on the number line is greater than the number to the left.

- 5 is greater than 4 (because 5 is on the right of 4).
- -2 is less than -1 (because -2 is to the left of -1).
- 0 is greater than -3, but less than 3.

Some important observations:

- Any positive integer is greater than any negative integer.
- Zero is neither positive nor negative. It is greater than all negative numbers and less than all positive numbers.

Absolute Value of an Integer:

The absolute value of an integer is its distance from zero on the number line. The absolute value of a number is always positive or zero, regardless of whether the number itself is positive or negative.

The absolute value is written using vertical bars around the number, like this: |a|.

Example:

- The absolute value of 5 is |5| = 5.
- The absolute value of -5 is |-5| = 5.

This means that the absolute value measures how far a number is from zero, but without considering whether the number is positive or negative.

Example: Which is greater, in each of the following pair?

(i) 0 and 4

- 4 is greater than 0 because 4 is to the right of 0 on the number line.

(ii) -7 and -6

- -6 is greater than -7 because -6 is closer to zero on the number line (to the right of -7).

(iii) 177 and -83

- 177 is greater than -83 because 177 is a positive number and lies to the right of -83 on the number line.

Example: Represent the following numbers on a number line.

(i) 6 more than 4 (ii) 3 more than -6 (iii) 4 less than 7

Solution:

(i) 6 more than 4

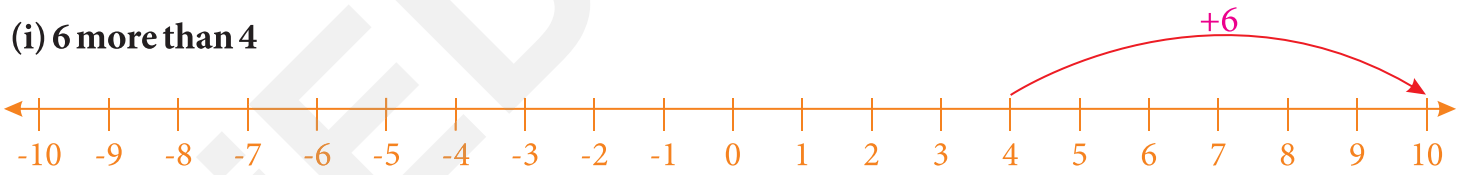


Fig. 10.10

(ii) 3 more than -6

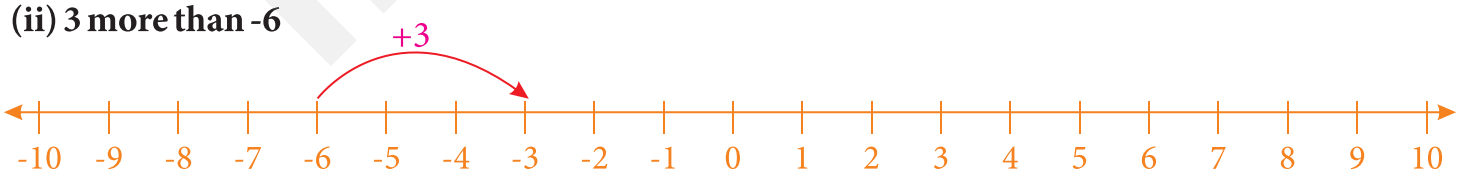


Fig. 10.11

(iii) 4 less than 7

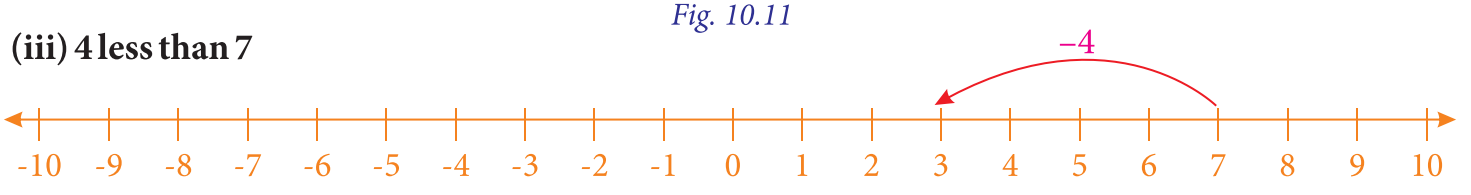


Fig. 10.12

Example: Which numbers lie between -8 and -3? Which is the largest number and the smallest number among them?

Solution: The numbers between -8 and -3 are: -7, -6, -5, -4

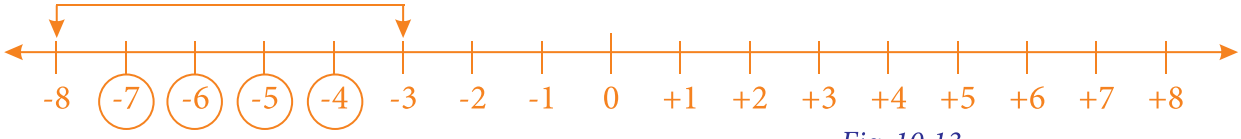


Fig. 10.13

Largest number: -7 (because -7 is closest to zero).

Smallest number: -8 (because -8 is the farthest from zero).

Example: Write the absolute value of the following

(i) 12 (ii) -19 (iii) -23

Solution:

(i) $|12| = 12$ (ii) $|-19| = 19$ (iii) $|-23| = 23$

Exercise 10.1

Knowledge Application

1. Complete the following sequences:

(a) -15, -13, -11, ..., ... (b) -60, -70, -80, ..., ...

(c) 5, 2, 0, -2, ..., ... (d) -3, -2, 0, 1, 3, ..., ...

2. Answer the following on the number line:

(a) Which number will we reach if we move 4 numbers to the right of -4?

(b) Which number will we reach if we move 5 numbers to the left of 3?

(c) Which number will we reach if we move 2 numbers to the right of 0 and then return 2 numbers to the left?

(d) If we are at -6 on the number line, in which direction should we move to reach 7?

(e) If we are at -10 on the number line, in which direction should we move to reach -5?

3. Represent the following numbers on a number line:

(a) 3, -4,6 (b) -2,-5,5 (c) 6, -2,-7

4. Using the number line, write the integer which is:

(a) 5 more than 2 (b) 6 more than -4 (c) 10 less than 8

5. Which is greater, in each of the following pairs:

(a) -3, 2 (b) 4, -6 (c) -8, -15 (d) 0, -1 (e) -12, -17

6. **In each of the following pairs, which number is to the right of the other on the number line?**

(a) 4, -5 (b) -6, -2 (c) -10, -11 (d) 1, 0 (e) -300, -200

7. **Replace by < or > sign:**

(a) 2, -3 (b) -11, -10 (c) 7, -6 (d) -15, 10 (e) -4, 0

8 **Write 5 integers less than -9.**

9. **Write 6 positive integers greater than 10.**

10. **Write the absolute value of the following:**

(a) -8 (b) 0 (c) 12 (d) -19

Addition of Integers

Suppose you are in a lift of a tall building. The ground floor is considered as "0". The floors above the ground floor are positive numbers, and the floors below the ground floor are negative numbers. The lift goes up by pressing the "+" button and goes down by pressing the "-" button.

1. If you are on the 2nd floor (i.e., +2), and you press the "-" button 3 times, which floor will you reach?
2. If you are on the 5th floor (i.e., +5), and you press the "+" button 2 times, which floor will you reach?

Solution:

1. Starting floor: +2

   Movement: -3

Calculation:

$(+2) + (-3) = -1$

So, you will reach the -1st floor.

2. Starting floor: +5

   Movement: +2

Calculation:

$(+5) + (+2) = +7$

So, you will reach the 7th floor.

5
4
3
2
1
Ground Floor
–1
–2
–3
–4
–5

Fig. 10.14

Starting floor + Movement = Target floor

Example : Using the concept, evaluate the following expressions:

(i) (+4) + (-5) (ii) (-3) + (-6) (iii) (+2) + (+3)

Solution:

We know: Starting floor + Movement = Target floor

(i) (+4) + (-5)

Starting with + 4 floor, if we press -5, we go down and reach -1 floor.

$(+4)+(-5)=-1$

So, the target floor is -1.

(ii) (-3) + (-6)

Starting with -3 floor, if we press -6, we go further down and reach -9 floor.

$(-3)+(-6)=-9$

So, the target floor is -9.

(iii) (+2) + (+3)

Starting with +2 floor, if we press +3, we go up and reach +5 floor.

$(+2)+(+3)=+5$

So, the target floor is +5.

Addition of Integers on a number line

Steps for Adding Integers on a Number Line:

1. **Identify the first number (starting point):**
   - Begin by locating the first number on the number line. If the number is positive, move to the right from zero; if the number is negative, move to the left from zero.
2. **Move the second number (number to be added):**
   - If the second number is positive, move to the right from your current position.
   - If the second number is negative, move to the left from your current position.
3. **Stop at the final position:**
   - The point where you land is the result of the addition.

Example : $-5+2$

1. Start at -5 (move 5 units to the left from 0).
2. Add 2 (move 2 units to the right).
3. Result: You will land at -3.

So, $-5+2=-3$

Example : 3 + (−4)

1. Start at 3 (move 3 units to the right from 0).
2. Add -4 (move 4 units to the left).
3. Result: You will land at -1.

So, 3+(−4)=−1

Additive Inverse of an Integer:

The additive inverse of a number is the number that, when added to the original number, results in zero.

- For any integer a itsadditive inverse is −a.
- This means: a + (−a) = 0

  or (−a) + a = 0

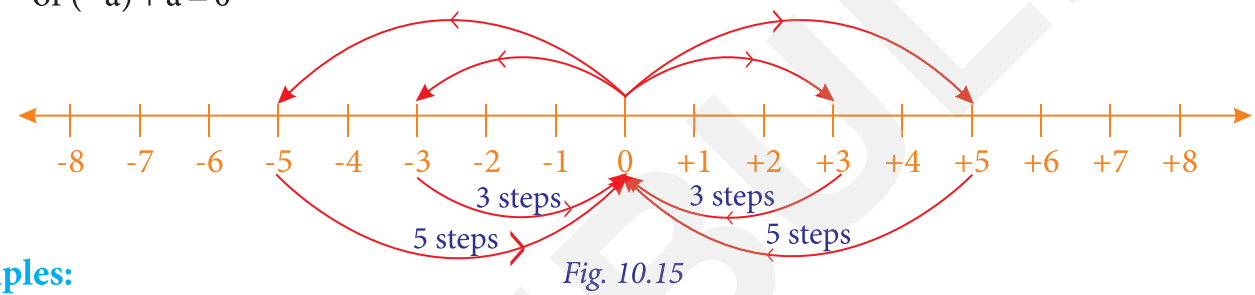


Fig. 10.15

Examples:

- The additive inverse of 5 is −5, because: 5+(−5)=0
- The additive inverse of −3 is 3, because: (−3) + 3=0

Successor of an Integer:

The successor of a number is the number that comes immediately after it when counting forward (increasing).

- To find the successor of a number, simply add 1 to the number.

Examples:

- The successor of 3 is 3+1=4
- The successor of −5 is −5+1=−4
- The successor of 0 is 0+1=1

Predecessor of an Integer:

The predecessor of a number is the number that comes immediately before it when counting backward (decreasing).

- To find the predecessor of a number, simply subtract 1 from the number.

Examples:

- The predecessor of 7 is $7 - 1 = 6$
- The predecessor of -3 is $-3 - 1 = -4$
- The predecessor of 0 is $0 - 1 = -1$

The Token Model

Using Tokens for Addition

In Tara's Tower of Games, the elevator operator finds a creative way to pass time. She keeps a box filled with **positive tokens (green)** and **negative tokens (purple)**. Each time she presses the '+' button, she takes a positive token and places it in her pocket. Similarly, each time she presses the '–' button, she takes a negative token and places it in her pocket.

She starts on the **ground floor (Floor 0)** with an empty pocket. After some time, she checks her pocket and finds **7 positive tokens** and **4 negative tokens**.

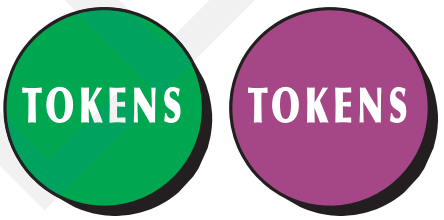


Fig. 10.16

Question: On which floor is she now?

She pressed the '+' button 7 times and the '–' button 4 times. The total is calculated as:

$(+7) + (-4) = +3.$

Fig. 10.17

Thus, she is now on Floor +3.

Another Way to Visualize the Calculation:

A positive token and a negative token cancel each other out, forming a zero pair because their combined value is zero.

- In her pocket, there are 7 positive tokens and 4 negative tokens.
- Removing 4 zero pairs (4 green tokens and 4 purple tokens cancel out), she is left with 3 green tokens.

So, $(+7)+(-4)=+3$.

Fig. 10.18

Example : Add without using a number line.

(i) $25 + 18$ (ii) $14 + (-32)$ (iii) $(-45) + 75$ (iv) $(-150) + (-350)$

(v) $(-50) + (-125) + 60$

Solution:

(i) Both integers are positive.

$25 + 18 = 43$

(ii) One integer is positive, and the other is negative. Subtract the absolute values.

$32 - 14 = 18$(sign of the larger number is negative, so) $14 + (-32) = -18$

(iii) One integer is positive, and the other is negative. Subtract the absolute values.

$75 - 45 = 30$ (sign of the larger number is positive, so) $(-45) + 75 = 30$

(iv) Both integers are negative. Add the absolute values and keep the negative sign.

$150 + 350 = 500$ so$(-150) + (-350) = -500$

(v) $(-50)+(-125)+60$

$[(-50) + (-125)] + 60 = -175 + 60 = -115$

Example : Write five distinct integers whose sum is -4.

Solution: $-4 + 6 + (-6) + 2 + (-2) = -4 + \{6 + (-6)\} + \{2 + (-2)\} = -4 + 0 + 0 = -4$

Hence, the five integers are $-4, 6, -6, 2, -2$.

Example : Calculate the sum:

$3+(-3)+3+(-3)+3+(-3)$

(i) If the total number of terms is 251.

(ii) If the total number of terms is 120.

Points of Rules

- Two negative numbers multiply to give a positive result.
- The sum of an integer and its additive inverse is always zero.
- Use a number line for addition and subtraction of integers.

Solution:

(i) When the total number of terms is 251 (odd):

$3 + [(-3) + 3] + [(-3) + 3] + \ldots$

$= 3 + 0 + 0 + \ldots = 3$

(ii) When the total number of terms is 120(even):

$[3 + (-3)] + [3 + (-3)] + \ldots$

$= 0 + 0 + \ldots = 0$

Example : In a building, if Floor A = -8, Floor C = -4, and Floor E = $+2$, find the numbers for Floors B,D and F. If you start on Floor +2 and press the lift button to go up +7 floors, where will you reach?

Solution: The pattern between floors seems to be increasing by +2 each step.

1. Floors in the building:

   B= -6, D = -2, F = $+6$.

2. Starting floor = +2, movement = +7:

   Target floor = $(+2) + (+7) = +9$

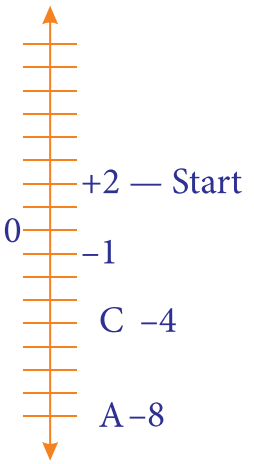


Fig. 10.19

Exercise 10.2

Knowledge Application

1. Simplify and find the sum:

(a) 5+(−7)+12+(−3)+25 (b) (−20)+35+(−15)+(−30)+50

(c) 48+(−22)+(−11)+9+(−35)

2. Find the sum of:

(a) 213+(−132)+18+(−56)+75 (b) (−10,000)+(−1200)+4500+(−340)

(c) 1892+(−1923)+(−108)+1234+(−500)

3. Find the successor of the following integers:

(a) −15 (b) 0 (c) 89 (d) −1000

4. Find the predecessor of the following integers:

(a) 20 (b) −45 (c) −1 (d) 400

5. Find the additive inverse of each of the following integers:

(a) 10 (b) −25 (c) 99 (d) −2000 (e) 13

6. Fill in the blanks with <, >, or =:

(a) (−6) + (−4) _____ (−4) + (−6)

(b) (−5) + (−8) _____ (−7) + (−6)

(c) (−10) + (−15) _____ (−30) + 5

(d) (−25) + 30 _____ (−10) + 20

(e) (−50) − (−25) _____ (−25) − 50

7. Write five distinct integers whose sum is −5.

8. Write pairs of integers whose sum is:

(a) 5 (b) −3 (c) 0

9. Solve the following equations:

(a) The sum of −5 and a number is 15. Find the number.

(b) Subtract −20 from 50.

(c) Add −200, 1000, and −800.

10. Write pairs of integers whose difference is:

(a) 10 (b) −5 (c) 20

Subtraction of Integers

Subtraction can also be seen as finding the difference or the number needed to adjust one quantity to match another.

Example : 8 – 5 can be stated as a problem in two different ways:

(i) I have 8 apples. I gave 5 to my friend. How many apples are left with me?

(ii) My friend has 5 apples, and I have 8 apples. How many more apples do I have than my friend?

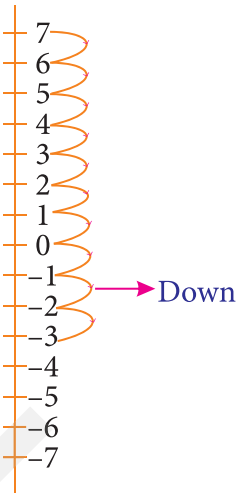


Fig. 10.20

In general, subtraction tells us how much more or less one quantity is compared to another.

Let us consider a scenario on a temperature scale. If the temperature at noon is +6°C and by evening, it drops to –3°C, what is the change in temperature?

To find this, we calculate:

Evening Temperature – Noon Temperature

= Change in Temperature

So, –3 – (+6) = –9.

This means the temperature dropped by 9°C

Example : In a parking garage, the floors are numbered as negative and positive levels. Use subtraction to determine the movement needed between floors.

(i) If your car is parked on floor -2 and you need to reach floor -6, what is the movement needed?

(ii) If you are currently on floor +4 and your target is floor -1, what is the movement needed?

Solution:

(i) To move from floor -2 to -6, we need to go down 4 steps.

Movement needed is -4.

Expression:

(–6) – (–2) = –4

(ii) To move from floor +4 to -1, we need to go down 5 steps.

Movement needed is –5-.

Expression:

(–1) – (+4) = –5

Subtraction of Integers on a Number Line

Subtraction can also be visualized using a number line. To subtract integers:

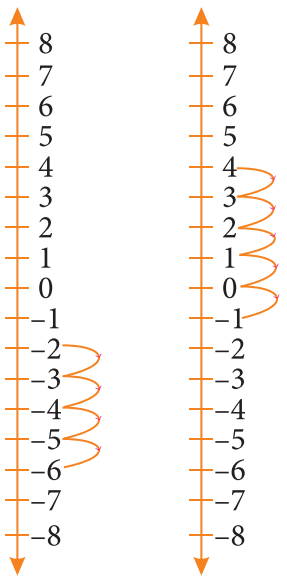


Fig. 10.21

Step 1: Start at the first number on the number line.

Step 2: Move left if subtracting a positive number or move right if subtracting a negative number.

Step 3: The point where you land is the result.

Example: Subtract 5 – 3:

- Start at +5 on the number line.
- Move 3 steps to the left because 3 is positive.
- You land at +2.

So, 5 – 3 = 2

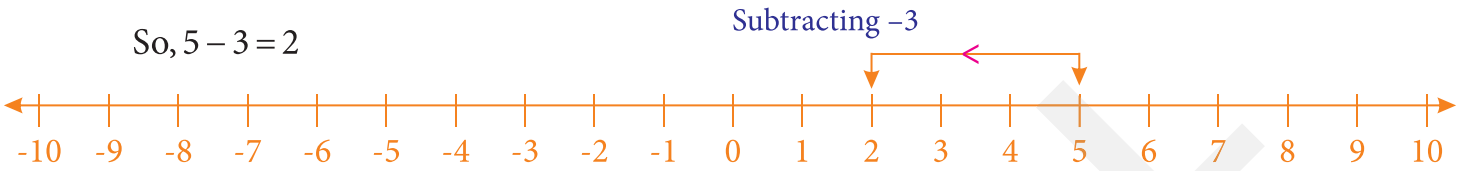


Fig. 10.22

Example : (i) 3 – 9:

- Start at +3
- Move 9 steps to the left (because you are subtracting a positive number).
- You land at –6

So, 3 – 9 = –6

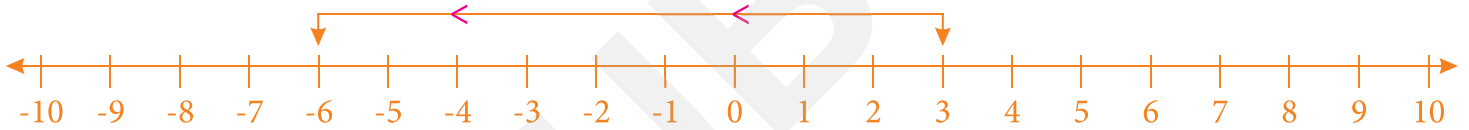


Fig. 10.23

(ii) –4–6:

- Start at –4
- Move 6 steps to the left (since you are subtracting a positive number).
- You land at –10.

So, –4 – 6 = –10.

Fig. 10.24

(iii) –7 – (–4):

- Start at –7.
- Subtracting –4 is the same as adding 4 (because subtracting a negative number is like adding a positive one).
- Move 4 steps to the right.
- You land at –3.

So, –7 – (–4) = –3.

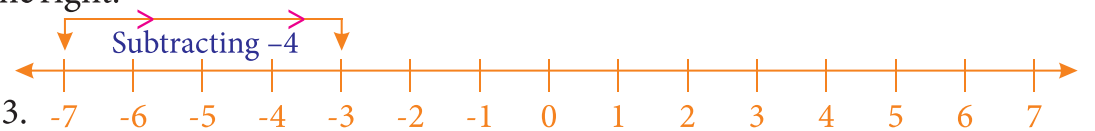


Fig. 10.25

The Token Model for Subtracting Integers

In Tara's Tower of Games, Tara also uses the Token Model to handle subtraction. Just as she uses positive (green) and negative (purple) tokens for addition, she uses the same tokens for subtraction, but with a twist.

Using Tokens for Subtraction

Let's say Tara starts on the ground floor (Floor 0) with an empty pocket. Each time she presses the "–" button, she adds a negative token (purple) to her pocket. Each time she presses the "+" button, she adds a positive token (green) to her pocket.

Now, let's assume she has a situation where she needs to subtract a number. For example, Tara has 3 green tokens (positive) and 5 purple tokens (negative) in her pocket. To subtract the number –2 (which is equivalent to adding positive tokens), she would need to remove tokens from her pocket.

Example Problem:

Tara's pocket contains 3 positive tokens and 5 negative tokens.

Step 1: Since subtracting a negative is the same as adding a positive, Tara needs to add 2 (positive) tokens to her pocket.

Step 2: If she adds 2 positive tokens, she now has:

- 5 positive tokens (3 from before + 2 added) (+) ⊕ ⊕ ⊕ ⊕ ⊕ ⊕
- 5 negative tokens (–) ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

Step 3: Tara can now cancel out 5 zero pairs (each pair consists of 1 positive and 1 negative token, totaling zero). After removing the 5 zero pairs, she's left with 2 positive tokens.

Thus, subtracting –2 from her current pocket results in 2 positive tokens left, which means Tara is now at Floor +2.

Another Way to Visualize the Calculation:

Each pair of positive and negative tokens cancels each other out. Tara starts with 3 positive tokens and 5 negative tokens. By subtracting –2, she ends up with 2 extra positive tokens remaining after canceling out the zero pairs.

Example : (i) $8 - (-12)$ (ii) $(-45) - 32$

(iii) $(-15) - (-30)$ (iv) $-120 + 45 - 37 + 60 - 90 + 125$

(i) 8 – (–12)

Solution: Since subtracting a negative number is the same as adding a positive number,

$8 - (-12) = 8 + 12 = 20$

(ii) (−45) − 32

Solution: Since both are negative, we add the absolute values and keep the negative sign:

$(-45)-32=(-45)+(-32)=-77$

(iii)(−15) − (−30)

Solution: Since subtracting a negative number is the same as adding a positive number,

$(-15)-(-30)=(-15)+30=15$

(iv) −120 + 45 − 37 + 60 − 90 + 125

Solution: Group the negative and positive integers:

$[-120-37-90]+[45+60+125]=-247+230=-17$

Example : Find the sum of the two integers if their sum is 75 and one of them is −45.

Solution : We are given the sum is 75 and one of the integers is −45.

To find the other integer, we subtract −45 from 75:

Required integer = 75− (−45) = 75 + 45 = 120

So, the other integer is 120.

Example : Compute: −64−(−23)

Solution : Since subtracting a negative number is the same as adding a positive number,

$-64-(-23)=-64+23=-41$

So, the result is −41.

Example : Find the result of the following expression: $15-4+(-7)-3+12-(-10)$

Solution : Group positive and negative integers:

$[15-4+12]+[(-7)-3+10]=23+0=23$

So, the result is 23.

Example : Compute the following:

$-50+25-70+40-(-15)$

Solution : First, simplify the expression: $[-50-70]+[25+40+15]=-120+80=-40$

So, the result is −40.

Fun Fact

- A number is greater if it is to the right on the number line.
- A number is smaller if it is to the left on the number line.

Exercise 10.3

Knowledge Application

1. **Find the following using a number line:**

   (a) (5) – (7) (b) (–10) – (–15) (c) (12) – (–8) (d) (–25) – (10)

2. **Calculate the given equations:**

   (a) 18 – 9 (b) –20 – 15 (c) –30 – (–25) (d) –14 – (7)

3. **Compute the following questions:**

   (a) 6 – (–10) + 20 – (–15) (b) –100 – (–200) + (–150) – 250

   (c) –25 + 30 – (–5) – 10 + (–40) (d) 0 – (–8)

4. **Observe the pattern and calculate:**

   (5) + (–5) + (5) + (–5) repeated for 50 terms.

5. **Change one sign to '+' in the expression 15–8–5–2 to make the sum equal to 10.**

6. **Write two distinct integers whose sum is greater than both integers.**

7. **Observe the vertical line given alongside. Complete the following based on the jumps:**

   - To reach +300from –100, we need to take a (+400) jump.
   - To reach –150 from +50, we need to take a (–200) jump.

   Complete:

   - To reach –50 from +150, the jump needed is: _____.
   - To reach +500 from –200, the jump needed is: _____.

8. **If a = –25 and b = 15:**

   (a) Find a – b. (b) Find b – a.

9. **A building has floors marked as –3, –2, –1, 0, +1, +2, +3, +4.**

   - If Tina is on Floor +3 and moves 5 floors down, where will she be?
   - If Arjun is on Floor –2 and moves 4 floors up, where will he be?
   - If Lisa is on Floor +4 and moves 8 floors down, what will her position be?

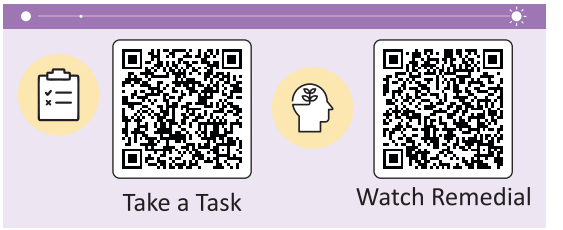
10. **Does the following statement stand True (T) or False (F):**

    (a) a–b is always positive when a > b.

    (b) The sum of two negative integers is always negative.

    (c) The difference of a positive and a negative integer is always positive.

Integers in Other Places



Integers are whole numbers, both positive and negative, including zero. They are widely used in real-life situations. Here are some examples:

1. Temperature

- **Positive Integers:** Represent temperatures above zero (e.g., +10°C means 10 degrees above freezing).
- **Negative Integers:** Represent temperatures below zero (e.g., -5°C means 5 degrees below freezing).

In news, we hear that the temperature in Jaipur during summer is 42°C, meaning it is 42°C higher than the freezing point of water (0°C). Meanwhile, the temperature in Shimla during winter is -5°C, meaning it is 5°C below 0°C. Note that Celsius (°C) is a unit for measuring temperature.

2. Height and Depth

- **Positive Integers:** Represent height above sea level (e.g., +200 m means 200 meters above sea level).
- **Negative Integers:** Represent depth below sea level (e.g., -50 m means 50 meters below sea level).

We measure the height of Mount Everest as +8,848 m above sea level, which is considered 0 m. Similarly, the depth of the Mariana Trench is -10,984 m, meaning it is 10,984 m below sea level. Positive numbers represent heights above sea level, and negative numbers represent depths below sea level.

3. Credit and Debit (Banking)

- **Positive Integers:** Represent credit or money added to an account.
- **Negative Integers:** Represent debit or money withdrawn from an account.

If a person deposits ₹1,200 in their bank account, it is recorded as a credit and written as +₹1,200. If the person withdraws ₹450, it is recorded as a debit and written as -₹450. The balance is calculated as the total of credits and debits. For example:

(+₹1,200) + (-₹450) = +₹750, which means ₹750 remains in the account.

4. Profit and Loss

- **Positive Integers:** Represent profit (money gained).
- **Negative Integers:** Represent loss (money lost).

Suppose a trader sells one product and earns ₹200, which is a profit, written as +₹200. On another product, they incur a loss of ₹60, written as -₹60. The net income is calculated as:

(+₹200) + (-₹60) = +₹140, meaning the trader has an overall profit of ₹140.

Example : The temperature of a place at 6:00 AM was -2°C. It increased by 4°C by 8:00 AM and then decreased by 3°C by 10:00 AM. What was the temperature at 10:00 AM?

Solution:

Initial temperature at 6:00 AM = -2°C

Increase by 4°C = -2 + 4 = 2°C

Decrease by 3°C = 2 - 3 = -1°C

Temperature at 10:00 AM = -1°C

Example : The height of a hill above sea level is +2,500 m. A climber descends 600 m and then climbs up 1,200 m. What is their final position relative to sea level?

Solution:

Initial height = +2,500 m

After descending 600 m = 2,500 - 600 = 1,900 m

After climbing up 1,200 m = 1,900 + 1,200 = 3,100 m

Final position = +3,100 m

Example : A person starts with ₹1,000 in their bank account. They deposit ₹2,500 and then withdraw ₹1,800. What is the balance in their account after the transactions?

Solution:

Initial balance = ₹1,000

After deposit = 1,000 + 2,500 = 3,500

After withdrawal = 3,500 - 1,800 = 1,700

Balance = ₹1,700

Activity

Conceptual Understanding

Integer War (Card Game)

Materials Needed: Playing cards labeled with integers (-10 to +10).

How to Play:

Each player gets two cards and must find the sum or difference.

The player with the highest result wins the round.

Play for multiple rounds and see who has the highest total.

Learning Outcome: Reinforces addition and subtraction of integers.

Exercise 10.4

Knowledge Application

1. **Provide the missing information in the blanks:**
   (a) A submarine is at a depth of _____ meters below sea level.
   (b) The temperature of a place decreased from 12°C to 7°C, so the temperature dropped by _____°C.
   (c) If you start with ₹1,000 in your bank account and withdraw ₹500, your new balance is _____.
   (d) The height of Mount Everest is _____ meters above sea level.
   (e) A shopkeeper earns a profit of ₹150 and incurs a loss of ₹50, so the total profit or loss is _____.

2. **Does the following statement stand True (T) or False (F):**
   (a) A gain in weight is represented as a negative number.
   (b) The temperature of -2°C is higher than 3°C.
   (c) The height of a mountain above sea level is always a positive number.
   (d) A deposit of ₹1,000 in a bank account is represented as a negative number.
   (e) The depth of a mine is represented by a negative integer.

3. **Represent the following situations as integers with appropriate signs:**
   (a) A balloon is flying 1,500 m above the ground.
   (b) A fish swims at a depth of 300 m below sea level.
   (c) A deposit of ₹7,000.
   (d) Losing a weight of 10 kg.
   (e) A temperature of 6°C below freezing point.
   (f) A loss of ₹250.
   (g) Decrease in speed by 12 km/h.
   (h) 4 km below sea level.

4. **A shopkeeper earns a profit of ₹250 on Monday, a loss of ₹150 on Tuesday, and a profit of ₹100 on Wednesday. Find the net profit or loss for the three days.**

5. **A mountain is 3,200 m above sea level, while a valley is 600 m below sea level. What is the vertical distance between the mountain peak and the valley?**

6. **A plateau is at a height of +2,400 m above sea level, while a trench is at -1,800 m below sea level. Which point is higher, and by how much?**

7. **The temperature at noon was 7°C. It decreased by 5°C in the afternoon and further decreased by 4°C at night. What was the temperature at night?**

A Hollow Integer Grid

An integer grid is a way of representing integers in a two-dimensional space, with both positive and negative numbers. The grid has:

1. **Horizontal axis (x-axis):** Represents integers to the left (-) and right (+).
2. **Vertical axis (y-axis):** Represents integers above (+) and below (-).

A hollow integer grid highlights only the boundary of a rectangular grid, leaving the inside empty.

Steps to Create a Hollow Integer Grid

1. Draw a rectangle using grid points.
2. Mark the boundary points with their integer coordinates.
3. Leave the inner grid points unmarked.

Example of a Hollow Integer Grid

In the grid, find the sum of integers along the top row, bottom row, left column, and right column.

-5	3	6
2		-4
7	-3	-8

Fig. 10.26

Solution

1. Top Row
   Sum = $(-5) + 3 + 6 = 4$
2. Bottom Row
   Sum = $7 + (-3) + (-8) = -4$
3. Left Column (Excluding repeated corners)
   Sum = $(-5) + 2 + 7 = 4$
4. Right Column (Excluding repeated corners)
   Sum = $6 + (-4) + (-8) = -6$

Border Sum

- Combine the sums of the top row, bottom row, left column, and right column:
  Border Sum = $4 + (-4) + 4 + (-6) = -2$
  The Border Sum for this grid is -2.
  Uses of a Hollow Integer Grid
- To understand coordinates and their representation.
- To highlight boundary values while leaving the interior blank for specific purposes, like graph plotting.

Exercise 10.5

Knowledge Application

1. Provide the missing information in the blanks:

(a) The sum of integers along the top row of the grid (−2, 5, −7) is _____.

(b) The sum of integers along the left column of the grid (−2, 3, 8) is _____.

(c) The sum of integers along the bottom row of the grid (8, −4, 1) is _____.

(d) The sum of integers along the right row of the grid (−7, 6, 1) is _____.

-2	5	-7
3		6
8	-4	1

2. Does the following statement stand True (T) or False (F):

(a) The sum of integers along the top row is always positive in any hollow integer grid.

(b) The border sum can be zero.

(c) The sum of integers along the right column is the same as the sum of integers along the left column in a hollow integer grid.

(d) The sum of integers along the bottom row is calculated by adding integers from the first to the last element.

(e) In a hollow integer grid, the border sum is the sum of integers from all the outer boundary rows and columns.

3. Consider the following hollow integer grid:

4	-2	5
3		-1
-3	2	6

(a) Find the sum of integers along the top row.

(b) Find the sum of integers along the bottom row.

(c) Find the sum of integers along the left column (excluding repeated corners).

(d) Find the sum of integers along the right column (excluding repeated corners).

(e) What is the Border Sum for this grid?

4. For each pair of sums, state whether the sum on the left is greater than, less than, or equal to the sum on the right: <, >, =

(a) Top Row Sum (6, -1, 4) _____ Bottom Row Sum (3, 2, -5)

(b) Left Column Sum (-3, 5, 2) _____ Right Column Sum (6, -2, -4)

(c) Top Row Sum (5, -2, -1) _____ Right Column Sum (7, 3, -2)

(d) Left Column Sum (4, -1, 3) _____ Bottom Row Sum (2, -4, 6)

(e) Top Row Sum (1, 3, 7) _____ Left Column Sum (6, -2, -3)

(f) Right Column Sum (-1, -4, 8) _____ Bottom Row Sum (-6, 2, 3)

Think Tank

Gap Analyzer™
Take a Test

1. Tick (✓) the correct answer:

a. The integer to the left of –3 on the number line is:

(i) –2 ☐ (ii) –4 ☐ (iii) –1 ☐ (iv) 0 ☐

b. Which of the following is the smallest integer?

(i) 00 ☐ (ii) –5 ☐ (iii) –10 ☐ (iv) 5 ☐

c. Adding –7 and 4 results in:

(i) –11 ☐ (ii) –3 ☐ (iii) 3 ☐ (iv) 11 ☐

d. The additive inverse of –9 is:

(i) 9 ☐ (ii) –9 ☐ (iii) 0 ☐ (iv) –18 ☐

e. Which of the following pairs of integers sum up to 0?

(i) 2 and –3 ☐ (ii) –5 and 5 ☐ (iiii) –7 and –7 ☐ (iv) 4 and 2 ☐

2. Provide the missing information in the blanks:

a. The integer _____ lies between –5 and –3 on the number line.

b. The sum of –6 and 9 is _____.

c. _____ is the additive identity for integers.

d. The absolute value of –12 is _____.

e. When you subtract –8 from –3, the result is _____.

3. The temperature in City A was –6°C, and in City B, it was 5°C. What is the difference in temperature between the two cities?

4. Draw a number line showing integers from –10 to 10. Mark the points –7, –3, 4, and 9. Write their opposites below each number.

5. Match the Columns:

Column A	Column B
a) –5	i) 12
b) 0	ii) 5
c) –12	iii) 0
d) 7	iv) 7

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