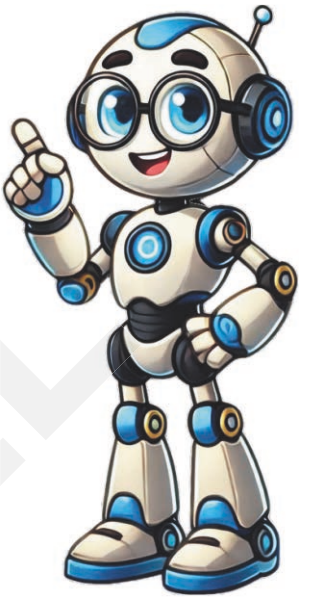
1
Chapter

# Patterns in Mathematics

**We'll cover the following key points:**

→ Introduction to Patterns in Mathematics
→ Numbers Patterns (with Visualising Number Sequences)
→ Relations among Number Sequences
→ Some More Patterns in Numbers
→ Patterns in Shapes



Hi, I'm EeeBee

**Do you Remember fundamental concept in previous class.**
In class 5th we learnt
→ Identification of Patterns in Square and Triangular Numbers
→ Relation in the Sequence of Odd Numbers between Consecutive Square Numbers

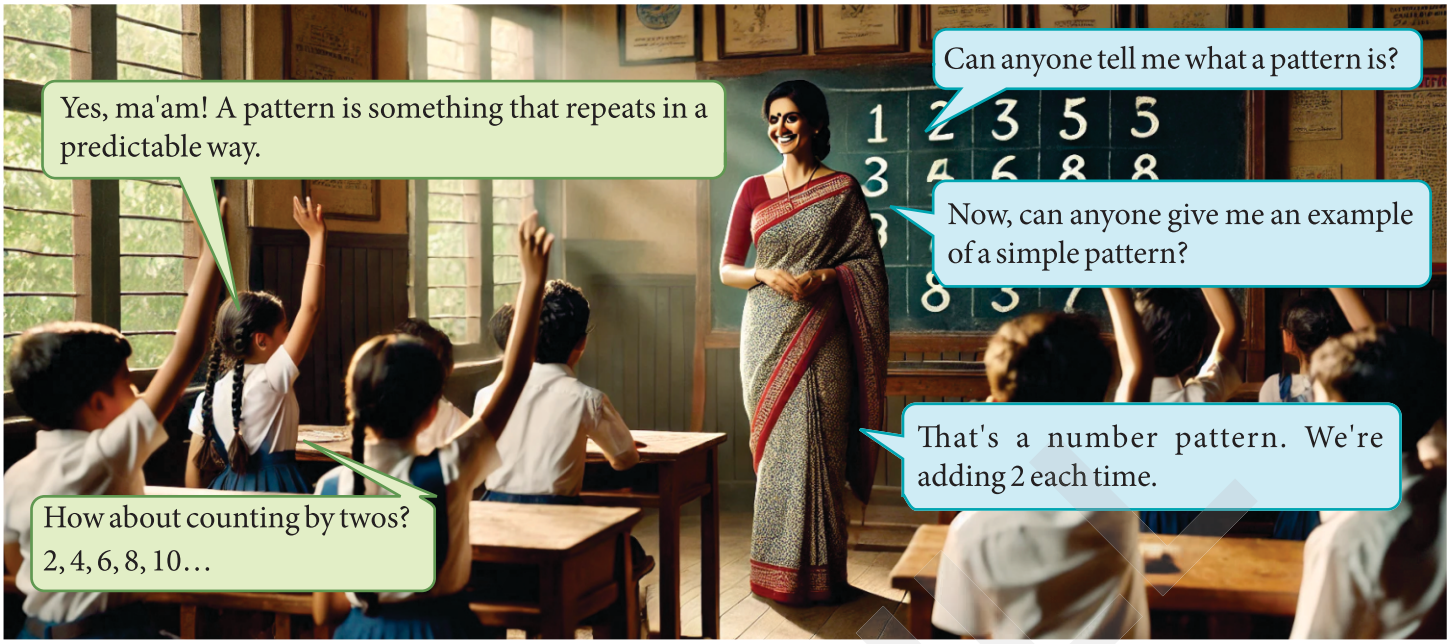
**Still curious?** Talk to me by scanning the QR code.

## Learning Outcomes

**By the end of this chapter, students will be able to:**

- Recognize and create number patterns based on addition, subtraction, multiplication, and division.
- Identify and extend simple number patterns and sequences.
- Understand the concept of arithmetic sequences and geometric patterns.
- Use patterns to solve problems and predict future numbers in sequences.
- Analyze the relationship between different terms in a pattern.
- Identify geometric patterns in shapes and designs.
- Solve real-world problems using patterns in numbers and shapes.
- Understand the significance of patterns in mathematical problem-solving.
- Explore mixed operations and complex sequences that integrate both arithmetic and geometric reasoning.
- Develop strategies for recognizing hidden or non-linear patterns in more advanced mathematical contexts.

## Introduction




Mathematics is not just about numbers and calculations; it is also the study of patterns and structures that help us make sense of the world around us. Patterns are arrangements of things (like numbers, shapes, or symbols) that follow a particular rule or sequence. They are everywhere—in nature, art, music, and even in our daily routines.

Understanding patterns is an important part of mathematics because they help us:

- **Recognize relationships** between numbers and shapes.
- **Predict what comes next** in a sequence.
- **Solve real-world problems** by identifying regularities.

Take a Task

Watch Remedial

### Importance of Patterns

- Patterns make solving problems easier by helping us find shortcuts.
- They teach us to think logically and develop problem-solving skills.
- Patterns are the foundation for advanced mathematical concepts like algebra and geometry.

### Types of Number Patterns

#### 1. Arithmetic Patterns

In these patterns, numbers increase or decrease by the same amount each time.

- **Rule:** Add or subtract a fixed number.
- **Example:** Sequence: 2, 5, 8, 11, 14…..

**Rule:** Add 3 to get the next number. → **2 + 3 = 5, 5 + 3 = 8**

### 2. Geometric Patterns

In these patterns, numbers are multiplied or divided by a fixed number.

- **Rule:** Multiply or divide by a constant.
- **Example:** Sequence: 3, 6, 12, 24, 48...

**Rule:** Multiply by 2. → **$3 \times 2 = 6, 6 \times 2 = 12, 12 \times 2 = 24$**

### 3. Triangular Number Patterns

These patterns form triangular shapes when dots are arranged in a triangular grid.

- **Rule:** Add consecutive natural numbers.
- **Example:** Sequence: 1, 3, 6, 10, 15...

**Rule:** $n^{th}$ term $= \frac{n(n+1)}{2}$ where n is the term's position.


### 4. Square Number Patterns

These patterns involve numbers that are perfect squares.

- **Example:** Sequence: 1, 4, 9, 16, 25...

**Rule:** $n^{th}$ term $= n^2$ → **$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25$**

## Numbers Patterns (with Visualising Number Sequences)

A **number pattern** is a sequence of numbers that follows a specific rule or logic. These patterns help us recognize relationships between numbers and understand mathematical concepts better.

In earlier classes, we have explored different types of numbers and sequences. Some of the most basic and intriguing patterns are based on these number sequences. Below are examples of common number patterns, along with their visual representations to make them easier to understand.

### Natural Numbers or Counting Numbers

Natural numbers, also known as counting numbers and the numbers we use for counting objects. They start from 1 and go on infinitely. These numbers do not include **zero** or any **negative numbers**.

**Set of Natural Numbers:** Sequence: {1, 2, 3, 4, 5, 6…}

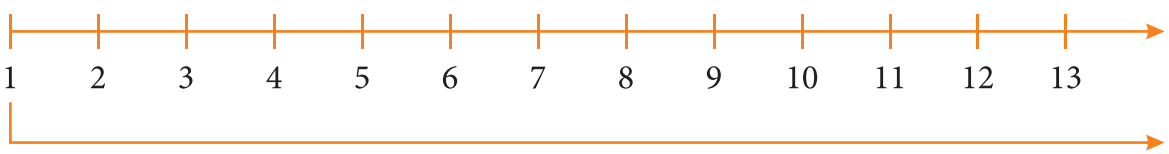



*Fig. 1.1* Natural Numbers

### Odd Numbers

Odd numbers are natural numbers that cannot be divided evenly by 2. In other words, they leave a

remainder of 1 when divided by 2. These numbers have 1, 3, 5, 7, or 9 in their units place.

**Set of Odd Numbers:** Sequence: {1, 3, 5, 7, 9, 11, 13, …}

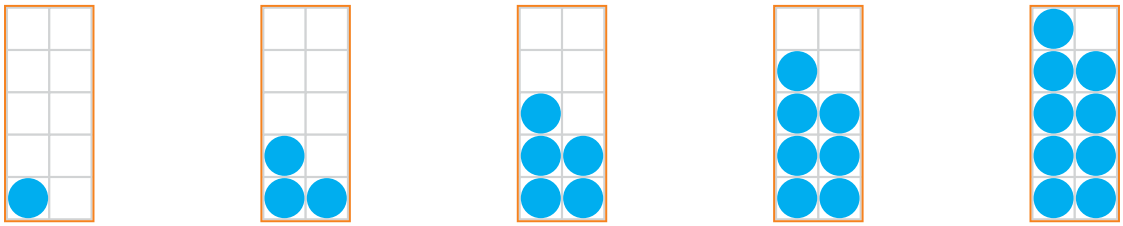



*Fig. 1.2*

## Even Numbers

Even numbers are natural numbers that can be divided evenly by 2, meaning they leave no remainder. These numbers always have 0, 2, 4, 6, or 8 in their units place.

**Set of Even Numbers:** Sequence: {2, 4, 6, 8, 10, 12 …}

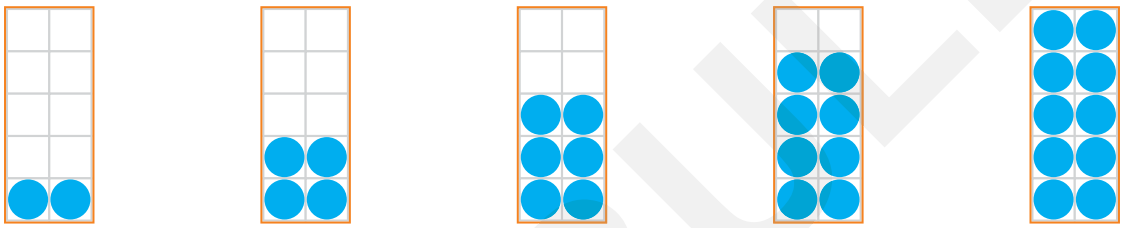



*Fig. 1.3*

## Square Numbers

Square numbers are numbers that can be expressed as the product of an integer multiplied by itself. In other words, a square number is the result of squaring a number, or raising it to the power of 2. Square numbers form a square shape when represented visually.

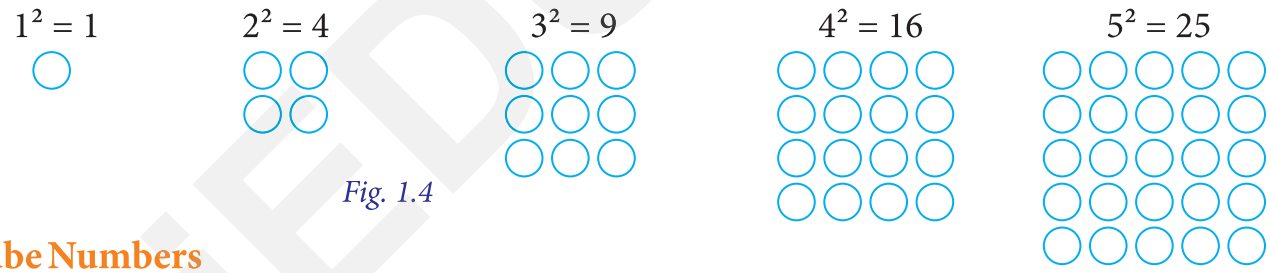



*Fig. 1.4*

## Cube Numbers

A cube number is the result of multiplying a number by itself three times. In other words, it is a number raised to the power of 3 ($n^3$). Cube numbers are also called "**perfect cubes.**"

**First Five Cube Numbers:** $1^3 = 1 \times 1 \times 1 = 1$ $2^3 = 2 \times 2 \times 2 = 8$ $3^3 = 3 \times 3 \times 3 = 27$

$4^3 = 4 \times 4 \times 4 = 64$ $5^3 = 5 \times 5 \times 5 = 125$

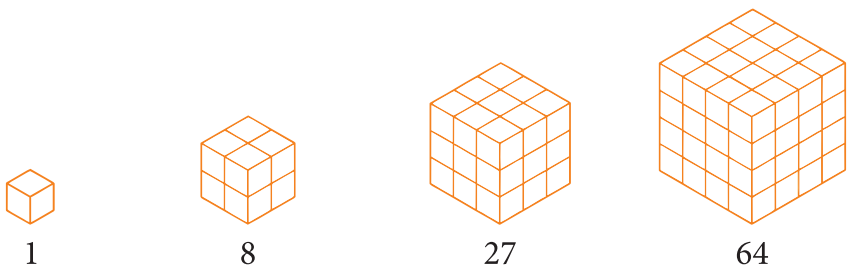



*Fig. 1.5*

## Triangular Numbers

Triangular numbers are a sequence of numbers that can be arranged in the shape of an equilateral triangle. Each triangular number represents a pattern where each successive term is the total number of dots required to form a triangle.

**Sequence:** 1, 3, 6, 10, 15

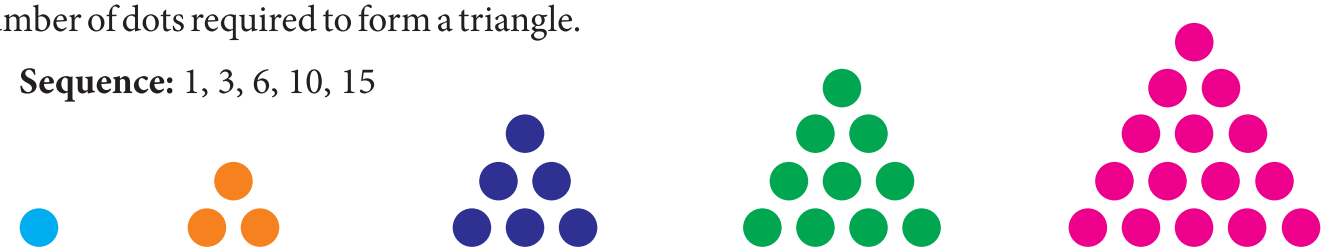



*Fig. 1.6*

## Hexagonal Numbers

Hexagonal numbers are a type of figurative number that represent a hexagon shape when arranged in a dot pattern. The $n^{th}$ hexagonal number is the number of dots required to form a hexagon with n layers.

$H_1 = 2(1)^2 - 1 = 1$ $H_2 = 2(2)^2 - 2 = 6$ $H_3 = 2(3)^2 - 3 = 15$ $H_4 = 2(4)^2 - 4 = 28$

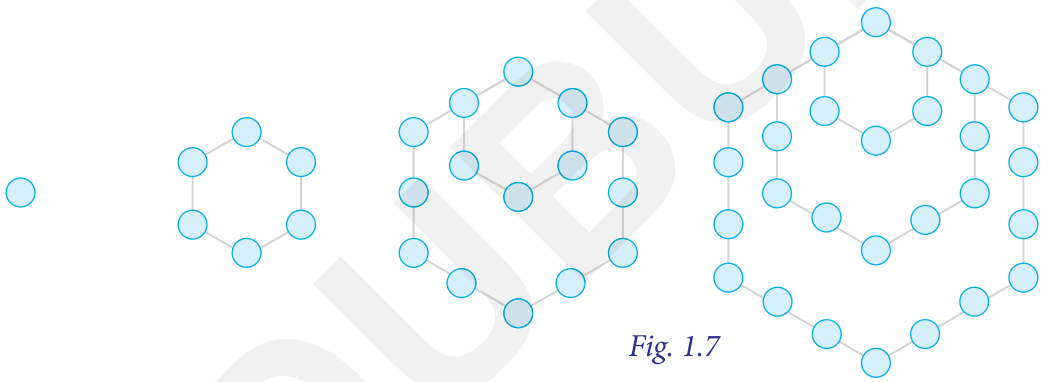


*Fig. 1.7*

$H_1 = 1$ $H_2 = 6$ $H_3 = 15$ $H_4 = 28$

## Relations among Number Sequences

In mathematics, different number sequences often have fascinating relationships with each other. These relationships allow us to understand how one sequence is connected to another and help us uncover patterns and properties that might not be immediately obvious. Here are some important relations among common number sequences:

### Adding Two Arithmetic Sequences

**Arithmetic Sequence 1:**

2, 4, 6, 8, 10…

This sequence has a common difference of 2 (since each term increases by 2).

**Arithmetic Sequence 2:**

1, 3, 5, 7, 9…

This sequence also has a common difference of 2 (each term increases by 2).

## Adding the Two Sequences:

We add the corresponding terms of both sequences. So, let's add the first few terms:

- The 1st term of both sequences: $2 + 1 = 3$
- The 2nd term of both sequences: $4 + 3 = 7$
- The 3rd term of both sequences: $6 + 5 = 11$
- The 4th term of both sequences: $8 + 7 = 15$
- The 5th term of both sequences: $10 + 9 = 19$

So, the resulting sequence after adding the two arithmetic sequences is: **3, 7, 11, 15, 19…..**

This sequence itself forms an arithmetic sequence with a common difference of 4 (since $7 - 3 = 4$, $11 - 7 = 4$ etc.).

**Let us discuss some more examples.**

**Examples :** Which sequence do we get when we add up even numbers?

**Solution:** $S_1 = 2$

$S_2 = 2 + 4 = 6$

$S_3 = 2 + 4 + 6 = 12$

$S_4 = 2 + 4 + 6 + 8 = 20$

$S_5 = 2 + 4 + 6 + 8 + 10 = 30$

**The sequence formed by adding the even numbers is:** 2, 6, 12, 20, 30…..

This is a quadratic sequence where each term is given by the formula $Sn = n (n + 1)$

**Examples :** What happens when you start to add up powers of 3 starting with 1, i.e., take 1, 1 + 3, 1 + 3 + 9, 1 + 3 + 9 + 27, ...? Now add 1 to each of these numbers—what numbers do you get? Why does this happen?

**Solution:** Now, we sum them up progressively:

First sum: $1 + 1 = 2$

Second sum: $1 + 1 + 3 = 5$

Third sum: $1 + 1 + 3 + 9 = 14$

Fourth sum: $1 + 1 + 3 + 9 + 27 = 41$

Fifth sum: $1 + 1 + 3 + 9 + 27 + 81 = 122$

**So, after adding 1 to each sum, the new sequence is:** 2, 5, 14, 41, 122….

The reason this pattern occurs is because each term is based on the sum of powers of 3, and adding 1 simply shifts the results.

**Examples :** Which sequence is obtained on adding the odd numbers up and down?

**Solution:** The sequence obtained by adding consecutive odd numbers in order is **the sequence of perfect squares**.

Odd numbers: 1, 3, 5, 7, 9…

Adding them consecutively:

$1 = 1$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

**Points of Rules**

**Identifying Patterns in Number Sequences**

**Look for a rule:**

- Addition, subtraction, multiplication, or division.
- Check if the difference between terms is constant (arithmetic).
- Check if each term is multiplied by the same number (geometric).
- Look for squares, cubes, or Fibonacci numbers.

Thus, the sequence obtained is 1, 4, 9, 16, 25….

So, on adding the odd numbers up and down, we get sequence of square number. This can now be illustrated visually, as shown in the following images.

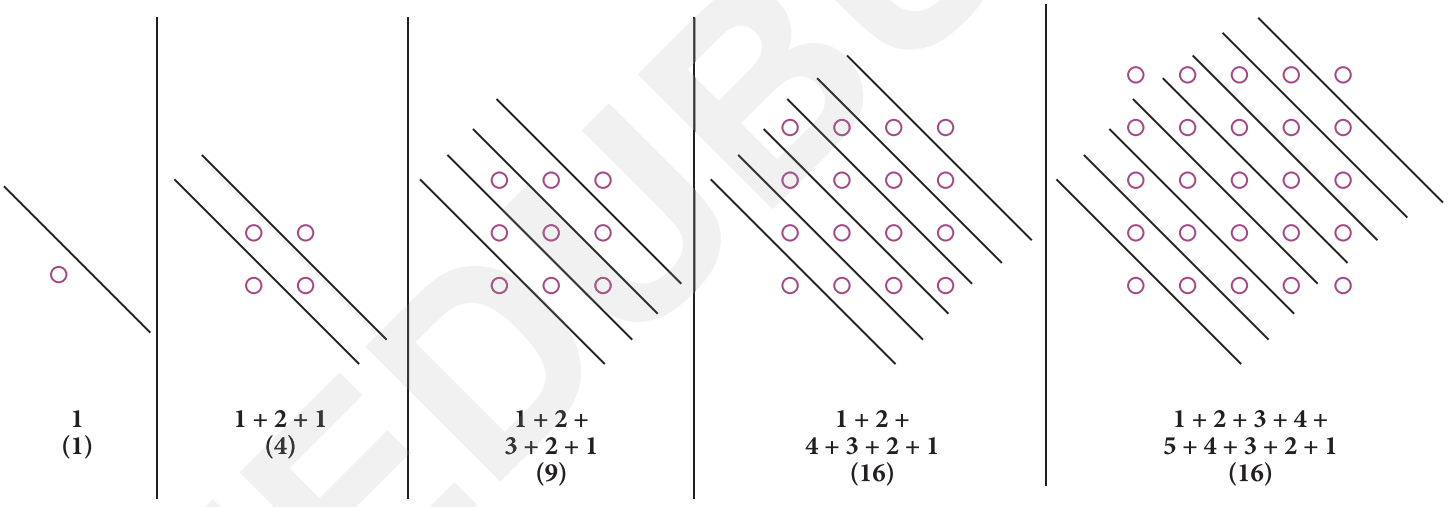



*Fig. 1.8*

## Exercise 1.1

Knowledge Application

1. **What is the sequence formed by adding the first 1, 2, 3, and 4 natural numbers?**
2. **Identify the pattern in the following number sequence and determine the next three numbers:**

(a) 3, 6, 9, 12, ___, ___, ___

(b) 1, 4, 9, 16, 25, ___, ___, ___

(c) 2, 5, 10, 17, ___, ___, ___

(d) 1, 1, 2, 3, 5, 8, ___, ___, ___

3. **Which sequence do you get when you add the first 4 triangular numbers up and down?**
4. **Consider the powers of 2 starting from 1: 1, 2, 4, 8, 16…What happens when you add the first n terms and subtract 1 from each of these sums? What sequence emerges?**
5. **Which sequence do you get when you add the first 5 square numbers up and down? Explain through picture.**
6. **Provide the answer each of the following questions below.**
   (a) What are the first five even numbers?
   (b) What are the first three perfect cubes?
   (c) What are the first five triangular numbers?
   (d) What are the first four powers of 4?
7. **What sequence do you get when you add the first 6 even numbers up and down? Explain through picture.**
8. **What happens when you multiply the cube numbers by 3 and subtract 2?**
9. **What is the next number in the pattern: 3, 6, 12, 24, ___?**
10. **What sequence is obtained by adding odd numbers in an incremental pattern?**

## Some More Patterns in Numbers

**Examples :** **Observe the following Alternating Addition and Subtraction Pattern:**

**General Approach to Solving Alternating Addition/Subtraction Patterns:**

1. Identify the pattern of addition and subtraction (e.g., adding 3, then subtracting 3, or adding and subtracting progressively larger numbers).
2. Follow the alternating sequence step-by-step to determine the next term.
3. Check if the numbers are increasing or decreasing and whether the addition and subtraction follow a consistent pattern or changing increments.

   **Consider the sequence: 6, 9, 3, 6, 2, …**

   Find the next term in the sequence.

**Solution:** Start with 6, add 3: 6 + 3 = 9

Subtract 6 from 9: 9 – 6 = 3

Add 3 to 3: 3 + 3 = 6

Subtract 4 from 6: 6 – 4 = 2

The next term should be: 2 + 3 = 5

The next term is 5.

**Examples :** Observe the following patterns Shifting Digits in Place Value

8, 80, 800, 8000, 80000…

**Solution:**

- This pattern involves shifting the digit 8 through increasing powers of 10.
- The place value of 8 increases by one order of magnitude at each step.
- The pattern is: $8 \times 10^0, 8 \times 10^1, 8 \times 10^2, 8 \times 10^3, 8 \times 10^4$….
- Each term is 10 times larger than the previous one.

This pattern demonstrates the effect of shifting the digit 8 through different powers of 10. It starts with 8×100 and continues by multiplying by increasing powers of 10. Here's the breakdown:

1. $8 \times 10^0 = 8$
2. $8 \times 10^1 = 80$
3. $8 \times 10^2 = 800$
4. $8 \times 10^3 = 8000$
5. $8 \times 10^4 = 80000$

At each step, the value increases by a factor of 10, shifting the digit 8 one place to the left in the decimal system. This pattern follows the rule: $\mathbf{8 \times 10^n}$ where n starts from 0 and increases by 1 with each new term.

## Exercise 1.2

Knowledge Application

**1. What is the next number in the sequence? Consider the sequence:**

(a) 10,7,10,6,10…. (b) 15,10,15,5,15,… (c) 20,18,20,16,20,…

(d) 7,11,8,12,9,… (e) 25,22,25,21,25,… (f) 50,48,53,51,56,…

**2. A sequence starts with the number 6 and follows the pattern of multiplying by increasing powers of 10. Write the first 6 terms of this sequence and identify the 4th term.**

**3. Given the pattern:** $6 \times 0 + 2 = 2$

$6 \times 1 + 3 = 9$

$6 \times 2 + 4 = 16$

**What is the result of:**

(a) $6 \times 3 + 5 = ?$ (b) $6 \times 4 + 6 = ?$

4. **Given the pattern:** $20 \times 0 - 15 = -15$

$20 \times 1 - 16 = 4$

$20 \times 2 - 17 = 23$

**What is the result of:**

(a) $20 \times 3 - 18 = ?$ (b) $20 \times 4 - 19 = ?$

5. **Evaluate the following:**

**if $45 \times 3 + 60 \div 5 = 147$; then solve:**

(a) $120 \div 4 \times 6 + 15$ (b) $(35 + 25) \times (18 - 13)$

(c) $(9 + 3) \times 7 - 20 \div 4$ (d) $(150 \div 3) \times (25 + 35) - 180 \div 9 + 12$

(e) $(60 \times 2) + (72 \div 8) \times (40 - 30)$

6. **Stop once you notice a pattern and complete the rest by following it."**

(a) $(90 - 18) \div 6 = \_\_\_$ (b) $(200 - 40) \div 20 = \_\_\_$

(c) $(-36 \div 12) \times \_\_\_ = 9$ (d) $(250 - \_\_\_) \div 25 = 8$

## Patterns in Shapes


**Patterns in Shapes** refers to the study of regularities and repetitions in geometric shapes. Understanding these patterns helps students develop a strong foundation in geometry and spatial reasoning. Here are some key concepts about patterns in shapes that students typically explore at this level:

1. **Regular polygon:** A regular polygon is a polygon where all the sides and angles are equal. The most common types of regular polygons include triangles, squares, pentagons, hexagons, and so on.

**Here are examples of a few regular polygons:**

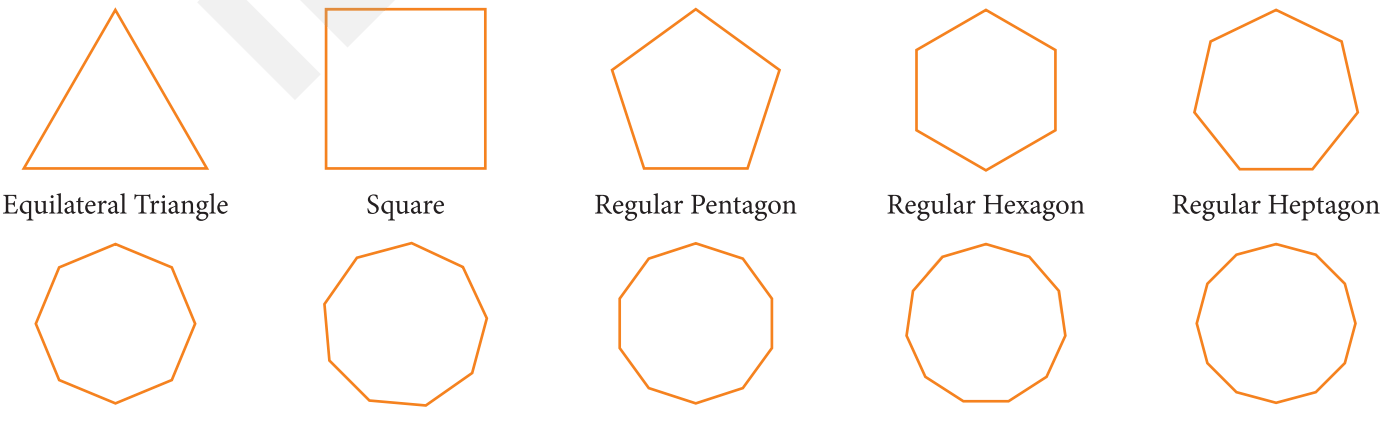



*Fig. 1.9*

2. **Stacked triangles:** Stacked triangles typically involve arranging multiple equilateral triangles in a pattern, either by placing them side-by-side or on top of each other to create a larger triangular structure.

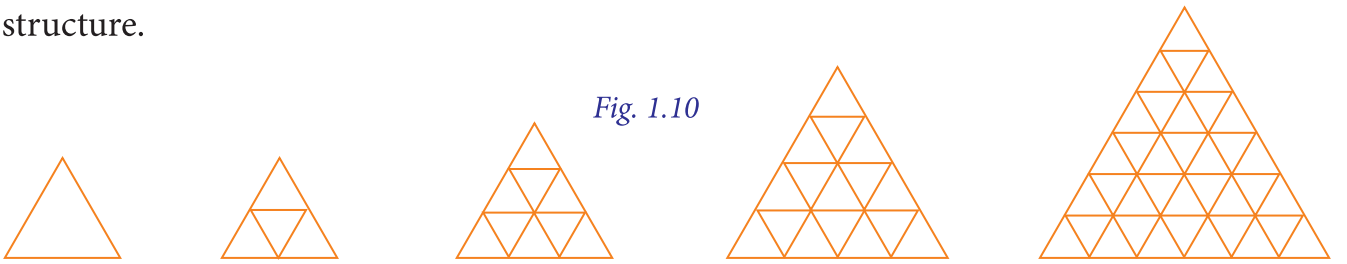


*Fig. 1.10*

| 1 | 2 | 9 | 16 | 25 |
|---|---|---|---|---|
| $1 \times 1$ | $2 \times 2$ | $3 \times 3$ | $4 \times 4$ | $5 \times 5$ |

3. **Stacked Squares:** Stacked squares typically involve arranging squares in a vertical or grid-like pattern. This can result in a simple column of squares or more complex arrangements where multiple rows of squares are stacked on top of each other.

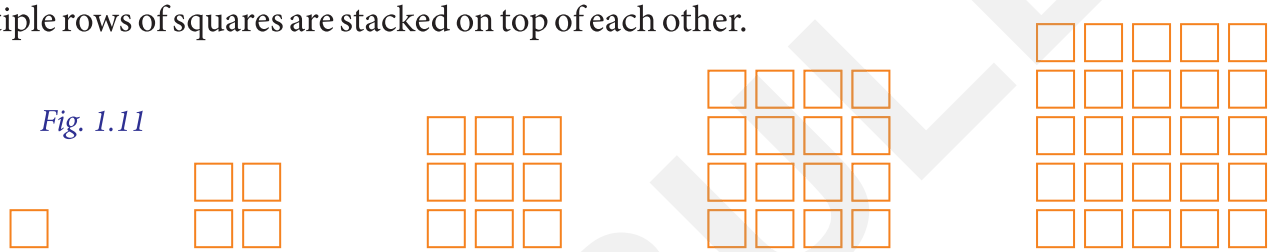
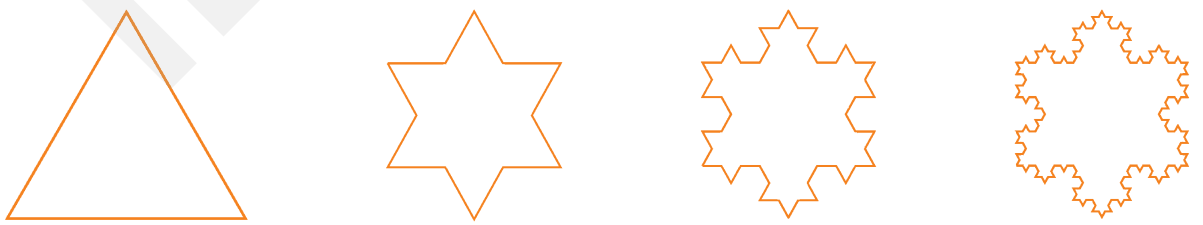


*Fig. 1.11*

| 1 | 4 | 3 | 4 | 5 |
|---|---|---|---|---|
| $1 \times 1 = 1^2$ | $2 \times 2 = 2^2$ | $3 \times 3 = 3^2$ | $4 \times 4 = 4^2$ | $5 \times 5 = 5^2$ |

4. **Complete Graphs:** A complete graph is a type of graph in graph theory where every pair of distinct vertices is connected by a unique edge. In simpler terms, each vertex is directly connected to every other vertex.

5. **Koch Snowflake:** The Koch Snowflake is a famous fractal curve and one of the most well-known examples of a self-replicating pattern in mathematics. It is constructed starting with an equilateral triangle, and then each side of the triangle is divided into smaller segments, with a smaller equilateral triangle "**sticking out**" at the middle of each side. The process is repeated indefinitely, which creates a snowflake-like pattern.



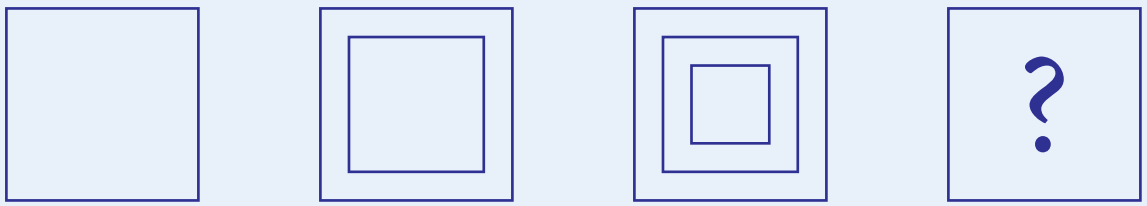
| 3 | 12 | 48 | 192 |
|---|---|---|---|

*Fig. 1.12*

# Exercise 1.3

Knowledge Application

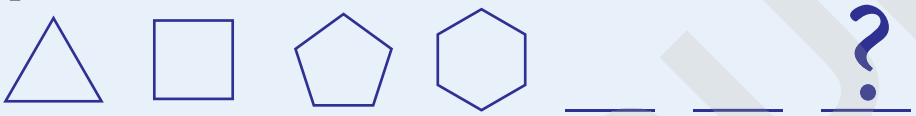
1. Identify the next figure in the sequence:



2. Look at the following sequence of figures:

If the sequence continues, which figure will appear 10th?

3. The pattern is as follows:



What will be the number of sides of the 7th figure in this sequence?

4. In a stacked triangle arrangement where each row has one more triangle than the row above it, how many triangles will there be in 4 rows?
5. If you stack 4 squares, one on top of the other, with each square having a side length of 3 cm, what is the total height of the stacked squares?
6. A regular polygon has 6 sides. What is the name of the polygon, and what is the measure of each interior angle?
7. A square is rotated 90 degrees clockwise. If the square is rotated 270 degrees clockwise from its original position, which direction will the top of the square point towards?



## Think Tank

Gap Analyzer™
Take a Test

1. **Tick (✓) the correct answer:**

a. Which of the following is an example of a growing pattern?

(i) 2, 4, 8, 16 ☐ (ii) 10, 8, 6, 4 ☐ (iii) 1, 1, 1, 1 ☐ (iv) 5, 5, 5, 5 ☐

b. What is the next number in the pattern: 5, 10, 20, 40, …?

(i) 50 ☐ (ii) 60 ☐ (iii) 80 ☐ (iv) 100 ☐

c. If a shape pattern goes Circle, Square, Triangle, Circle, Square, ..., what will be the 9th shape?

(i) Triangle ☐ (ii) Square ☐ (iii) Circle ☐ (iv) Hexagon ☐

d. Which of the following patterns is arithmetic?

(i) 2, 4, 6, 8, 10 ☐ (ii) 3, 9, 27, 81 ☐ (iii) 5, 10, 20, 40 ☐ (iv) 7, 21, 49, 84 ☐

e. What is the rule for the pattern: 1, 4, 9, 16, 25?

(i) Multiply by 2 ☐ (ii) Add 3 ☐

(iii) Square the number ☐ (iv) Subtract 1 ☐

**2. Provide the missing information in the blanks:**

a. In the pattern 3, 6, 12, 24, ..., the next number is _____.

b. The pattern rule for 2, 5, 10, 17, 26 is _____.

c. If the pattern of shapes is Circle, Triangle, Square, ..., the 7th shape is _____.

d. In an arithmetic pattern with a rule of "add 4," the next number after 15 is _____.

e. The missing number in the sequence 10, 15, _____, 25, 30 is _____.

**3. Identify the number that does not belong to the pattern: 5, 10, 15, 22, 25, 30.**

**4. Match the Columns:**

| Column A | Column B |
|---|---|
| a) 2, 4, 6, 8, 10 | i) Geometric |
| b) 1, 3, 9, 27, 81 | ii) Arithmetic |
| c) 10, 9, 8, 7, 6 | iii) Squares |
| d) 1, 4, 9, 16, 25 | iv) Descending |


## Assertion and Reason

Experiential Learning

Each question has two statements, Assertion (A) and Reason ®. Choose the correct option:

A: Both A and R are true, and R is the correct explanation of A.

B: Both A and R are true, but R is not the correct explanation of A.

C: A is true, but R is false.

D: A is false, but R is true.

1. **Assertion (A):** A pattern of squares like 1, 4, 9, 16, 25 follows a mathematical rule.
**Reason (R):** The numbers in this pattern are the cubes of natural numbers.

2. **Assertion (A):** The pattern 2, 4, 8, 16 is a geometric progression.
   **Reason (R):** In a geometric progression, each term is obtained by multiplying the previous term by a fixed number.
3. **Assertion (A):** The pattern 1, 3, 5, 7, 9 is an arithmetic progression.
   **Reason (R):** In an arithmetic progression, the difference between consecutive terms is constant.
4. **Assertion (A):** A repeating pattern of shapes (e.g., Circle, Square, Triangle) is called a geometric pattern.
   **Reason (R):** Repeating patterns involve changes in numbers, not shapes.
5. **Assertion (A):** If the rule of a pattern is "add 5," the next number after 15 is 20.
   **Reason (R):** Adding 5 to 15 gives 20.

## HOTS (Higher Order Thinking Skills)

Critical Thinking

1. A staircase has 10 steps. The first step is 5 cm high, the second step is 10 cm high, the third step is 15 cm high, and so on. Find the height of the 10th step.
2. A pattern of shapes has the rule: "Add one more side to each new shape." If the first shape is a triangle, what will the 5th shape be?

## Case Study

Ravi noticed a unique pattern in a garden's layout. The first row had 1 tree, the second row had 3 trees, the third row had 5 trees, and so on. The total number of rows was 15. Ravi wanted to find the following:

**Questions:**

1. What is the rule for the pattern in the number of trees per row?
2. How many trees are there in the 10th row?
3. What is the total number of trees in all 15 rows?
4. Which type of pattern is this: arithmetic, geometric, or neither?

| | |
|---|---|
| 1 |  |
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