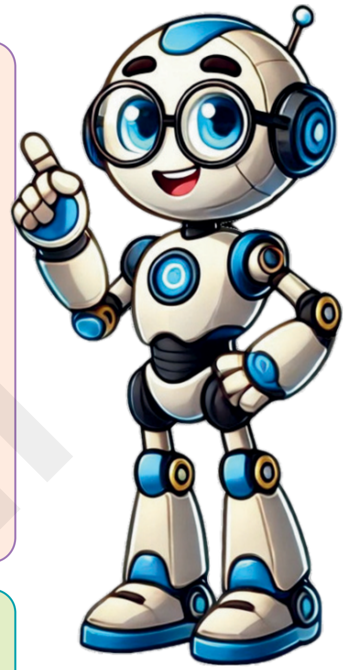
8 Symmetry and Pattern

We'll cover the following key points:

→Reflection Symmetry / Line Symmetry
→Slides, Flips and Turns
→Identification of Patterns in Square and Triangular Numbers
→Patterns in Triangular Numbers
→Pascal's Triangle
→Observations Based on Pascal's Triangle
→Relation in the Sequence of Odd Numbers Between Consecutive Square Numbers



EeeBee

Do you Remember fundamental concept in previous class.
In class 4th we learnt

→Symmetry (Line Symmetry)

In class 3rd we learnt

→Number Patterns
→Even and Odd Numbers

Still curious? Talk to me by scanning the QR code.

Learning Outcomes

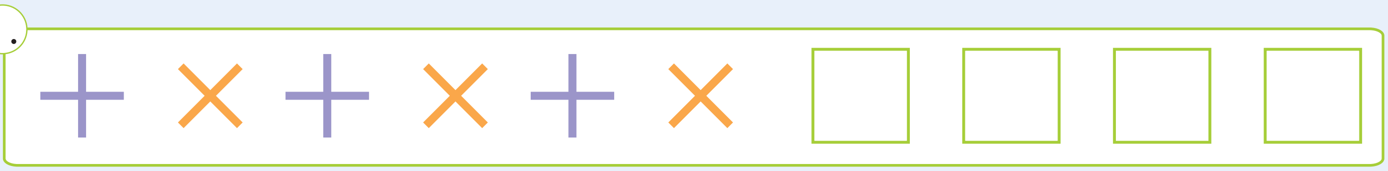
By the end of this chapter, students will be able to:

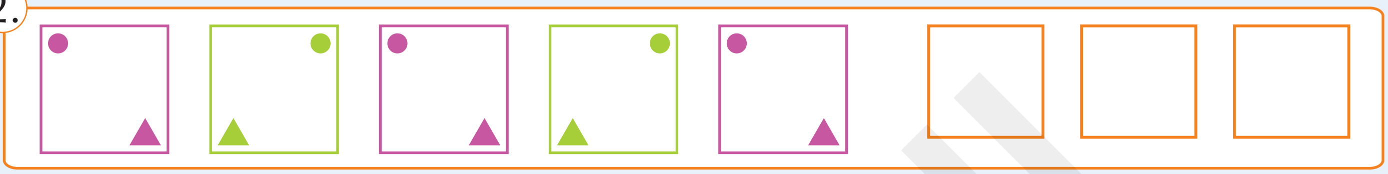
- Identify symmetrical objects and figures in the environment.
- Understand the concept of line of symmetry and identify it in given shapes.
- Draw lines of symmetry for various 2D shapes.
- Create symmetrical designs and patterns using rulers and freehand.
- Recognize and extend patterns involving shapes, colors, and numbers.
- Differentiate between symmetrical and asymmetrical figures.
- Understand rotational symmetry and identify the order of rotation for given figures.
- Solve problems involving completion of incomplete symmetrical shapes.
- Create creative patterns by repeating or reflecting designs.

Warm Up

Experiential Learning

Complete the given patterns.

1.


2.


3.


Choose the correct option:

1. The number of lines of symmetry of a circle is

   (a) 0 (b) 1 (c) 2 (d) infinity

2. The letter of English alphabet which has only vertical line of symmetry is

   (a) D (b) P (c) R (d) V

3. The number of lines of symmetry in the English alphabet 'H' is

   (a) 2 (b) 3 (c) 4 (d) 1

4. The number of lines of symmetry in the English alphabet 'A' is

   (a) 0 (b) 1 (c) 2 (d) 3

Reflection Symmetry / Line Symmetry

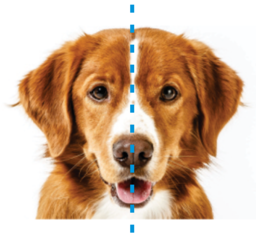
Take a Task

Watch Remedial

A figure is said to be symmetric when it can be folded and matched.

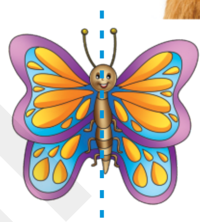
Reflection symmetry (sometimes called line symmetry or mirror symmetry) is easy to recognise, because one half is the reflection of the other half.

Line of Symmetry



Here, the face of the dog has been made perfectly symmetrical.

The dotted line drawn on the centre is called the **line of symmetry**.

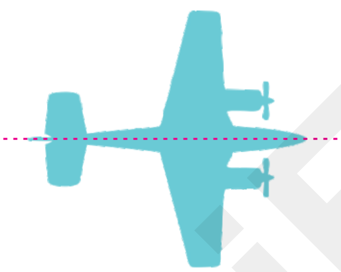


- Now, look at this beautiful butterfly.
- It is folded in two halves.
- See that its two halves match exactly.

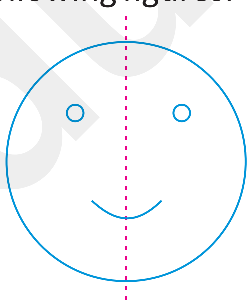
This particular line created by folding the butterfly is called the line of symmetry or the line of reflection.

When the butterfly is folded at the line of symmetry such that the two sides match each other exactly, then we can say the two sides are congruent and the butterfly is symmetrical.

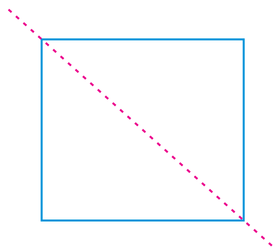
Let us draw the line of symmetry in the following figures.



Horizontal line of symmetry



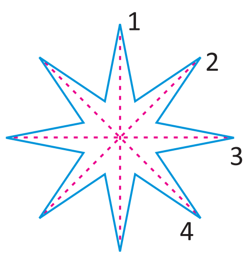
Vertical line of symmetry



Diagonal line of symmetry

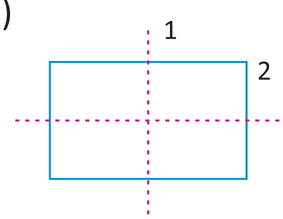
Many shapes also have multiple lines of symmetry or no line of symmetry.

(i)



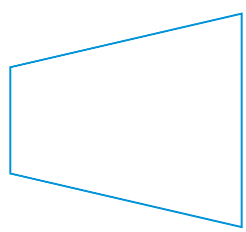
The figure has 4 lines of symmetry.

(ii)



The figure has 2 lines of symmetry.

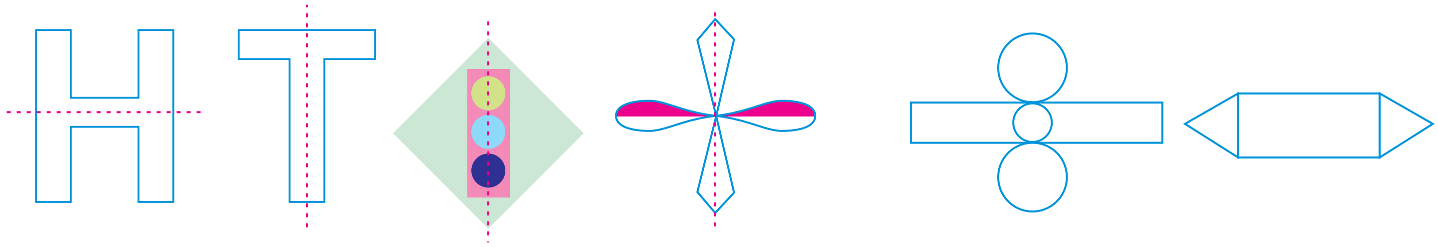
(iii)



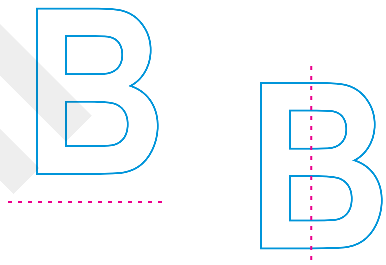
The figure has no line of symmetry.

Symmetric Objects Around Us

Now, we will see some examples of symmetric objects around us, i.e., the letters of English alphabets, traffic-signs, geometric shapes and the other images.



Symmetry exists in architecture all around the world. The best known example of this is the Taj Mahal.



Line symmetry can also be seen in the letters of English alphabets.

The capital letter B given alongside has only one horizontal line **symmetry**.

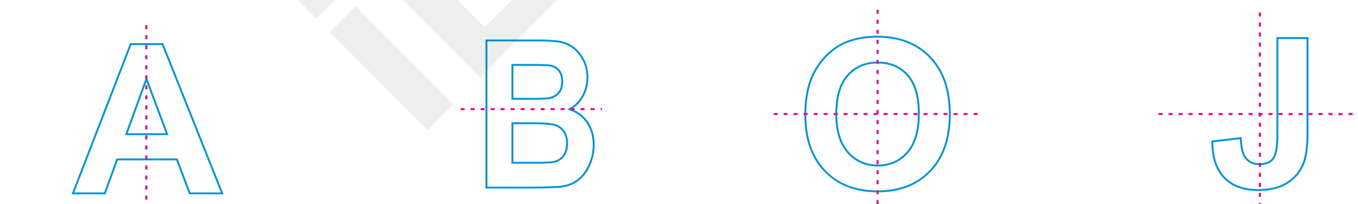
If we fold this B from the horizontal line, its top and bottom parts match each other exactly.

But, the letter B does not have vertical line symmetry.

When we draw a line vertically, the left and right parts of B do not match each other exactly.

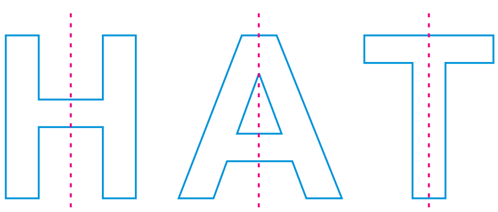
The capital letter P given below has no line symmetry.

Look at the few letters of the English alphabets

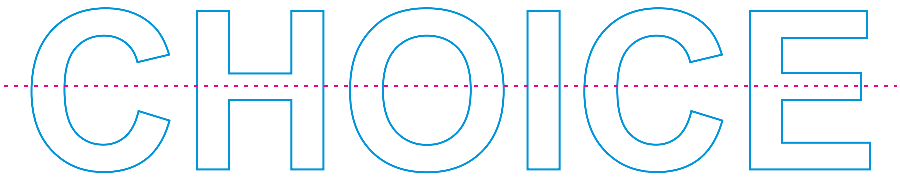


Vertical axis of Symmetry	Horizontal axis of Symmetry	Both axis of Symmetry	No axis of Symmetry

Let us try to make a word using letters with vertical axis of symmetry.

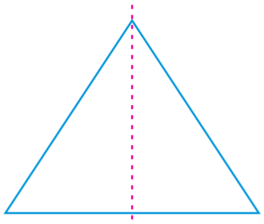


Let us now make a word using letters with horizontal axis of symmetry.

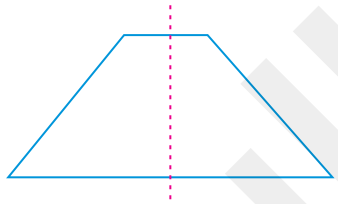


Figures with more than two lines of symmetry

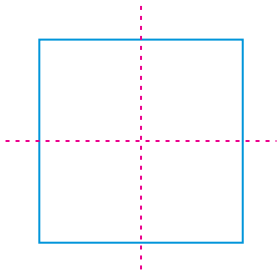
Look at the below given figures. You will observe more than two lines of symmetry.



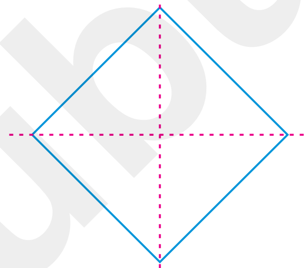
One line of symmetry



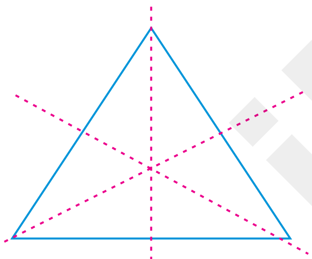
One line of symmetry



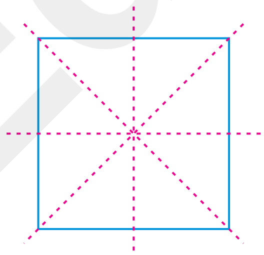
Two lines of symmetry



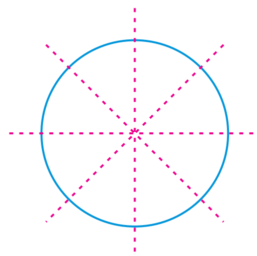
Two lines of symmetry



Three lines of symmetry

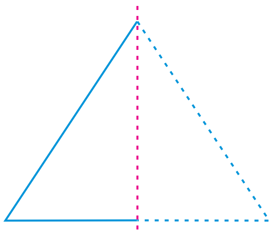


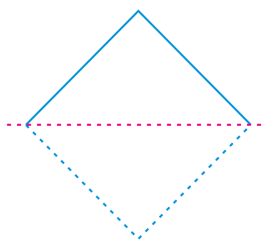
Four lines of symmetry



A circle has infinite lines of symmetry

When a figure is symmetrical, the other half of the figure can be drawn by looking at the given half.





Exercise 8.1

Knowledge Application

1. Multiple Choice Questions (MCQs)

Choose the correct option.

(a) Which letter has the vertical line symmetry ?

(i) J ☐ (ii) T ☐ (iii) K ☐ (iv) P ☐

(b) Which letter has the horizontal line symmetry ?

(i) C ☐ (ii) G ☐ (iii) L ☐ (iv) O ☐

2. Decide whether the given pictures have vertical, horizontal or no line symmetry:

Put tick mark on the corresponding answer.

(a) Vertical symmetry ☐

Horizontal symmetry ☐

No symmetry ☐

(b) Vertical symmetry ☐

Horizontal symmetry ☐

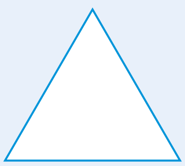
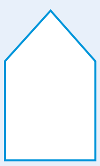
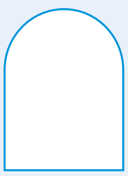
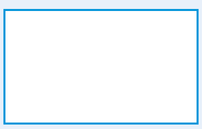
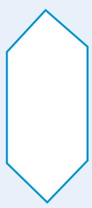
No symmetry ☐

(c) Vertical symmetry ☐

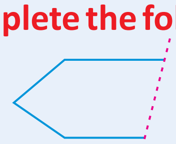
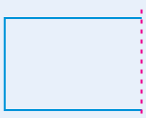
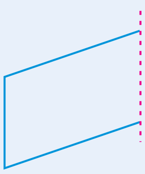
Horizontal symmetry ☐

No symmetry ☐

3. Draw the line (s) of symmetry for each of the following:

(a)  (b)  (c)  (d)  (e) 

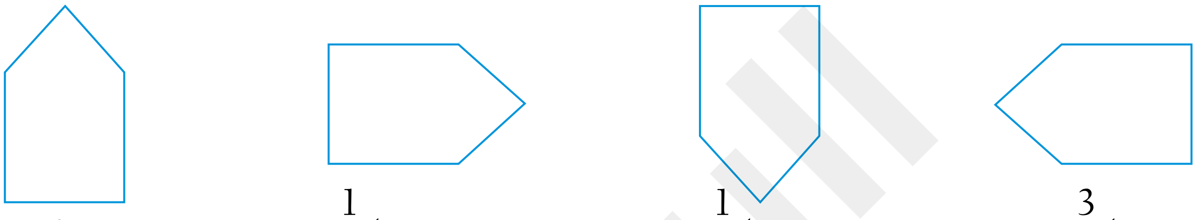
4. Complete the following symmetrical shapes:

(a)  (b)  (c) 

Slide : A slide takes place when a shape moves in one direction from one place to another.

Flip : A flip takes place when a shape is flipped across a line and faces the opposite direction.

Turn : A turn takes place when a shape is turned through an angle.



Original position $\frac{1}{4}$ turn $\frac{1}{2}$ turn $\frac{3}{4}$ turn

1. Draw a tessellation with pentagons or hexagons.
2. Draw a tessellation by combining two plane shapes.
3. Write the letters D, P, M, Y and Z and read their image through a mirror. How does each letter look like?

Original position $\frac{1}{4}$ turn $\frac{1}{2}$ turn $\frac{3}{4}$ turn

Identification of Patterns in Square and Triangular Numbers

Take a Task

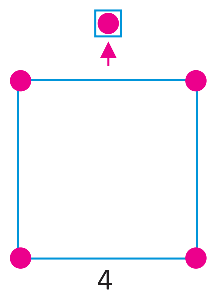
Watch Remedial

Patterns often occur in science and mathematics. They are formed by groups of shapes, objects, diagrams or numbers. Patterns are the part of daily-life and are very useful in mathematics as they can help us to solve some problems.

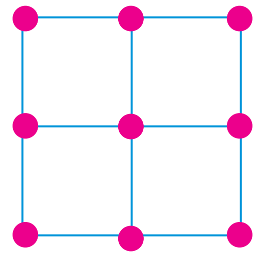
Patterns in Square Numbers

A square number is a number that can be arranged in a square pattern. The patterns of the first three square numbers are depicted below.

The first square number 1 is shown as a square containing a single dot. There is 1 row containing 1 dot.



So, there is $1 \times 1 = 1$ dot in this square number.



9

The second square number 4 is shown as a square with 2 dots in each side. It is clear that there are 2 rows each containing 2 dots.

So, there are $2 \times 2 = 4$ dots in this square number.

The third square number 9 is shown as a square with 3 dots in each side. It is clear that there are 3 rows each containing 3 dots.

So, there are $3 \times 3 = 9$ dots in this square number.

From the above discussion, we can make a series of square numbers

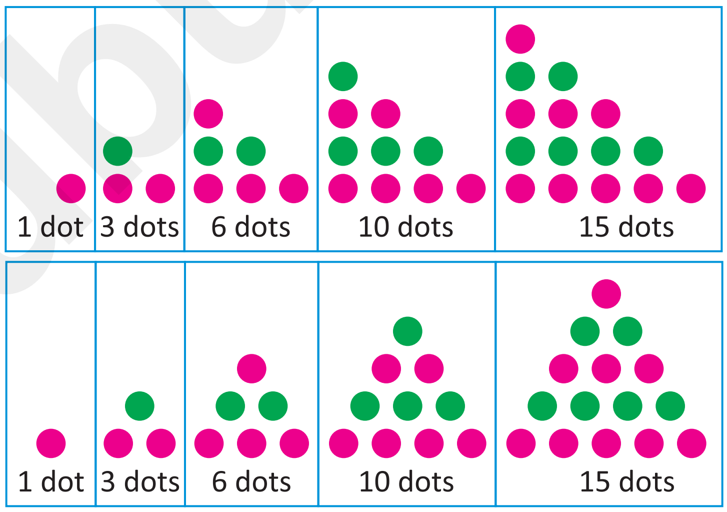
1, 4, 9, 16, 25, 36, 49, ...

Observing the above patterns, we find that a square number is made by squaring it where it is in a pattern.

For example, the second number is 2 squared i.e., $2 \times 2 = 4$.

The eighth number 8 is squared i.e., $8 \times 8 = 64$.

Patterns in Triangular Numbers



Triangular numbers are those numbers which can be formed by counting the number of objects used in making a triangle.

Look at the following for other triangular patterns :

From the above discussion, we can make a series of triangular numbers

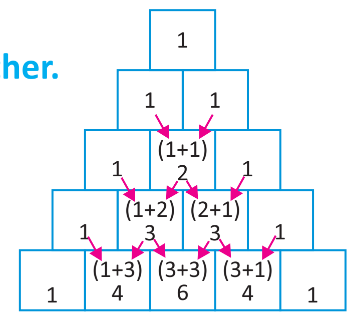
1, 3, 6, 10, 15, 21, 28, 36,..........

Pascal's Triangle

One of the most interesting number pattern is Pascal's Triangle (named on Blaise Pascal, a famous French Mathematician and Philosopher).

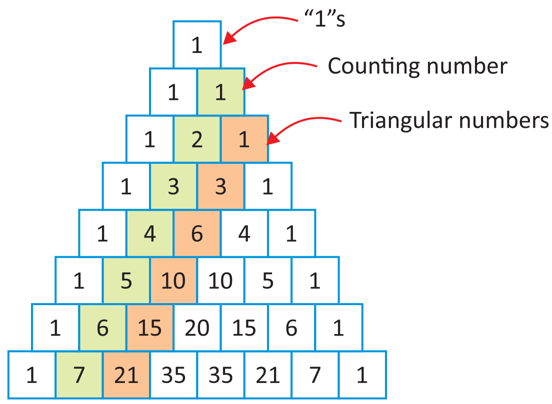
To build the triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern.

Each number is just the two numbers above it added together.



Observations Based on Pascal's Triangle

Diagonals

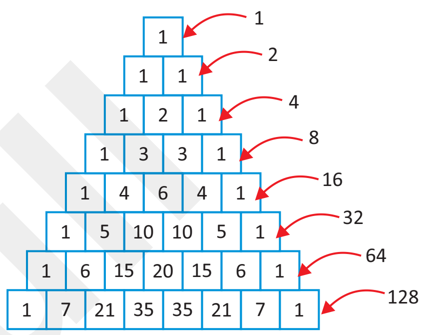


Of course, the first diagonal is just '1' s, and the second diagonal has the counting numbers i.e., 1, 2, 3, 4, 5, etc.

The third diagonal has the triangular numbers, i.e., 1, 3, 6, 10, 15, etc.

(The fourth diagonal, not highlighted, has the tetrahedral numbers. i.e., 1, 4, 10, 20, 35, etc.)

Horizontal Sum

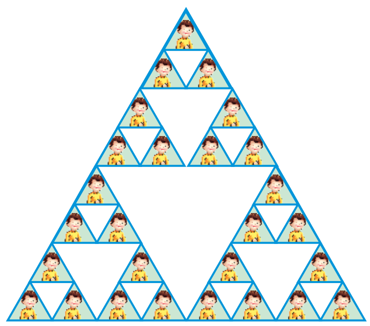


What do you notice about the horizontal sums ? Is there a pattern ? Isn't it amazing ?

It doubles each time.

Sierpinski Triangle

(An ever repeating pattern of triangles)



Let us learn how we can create a Sierpinski Triangle :

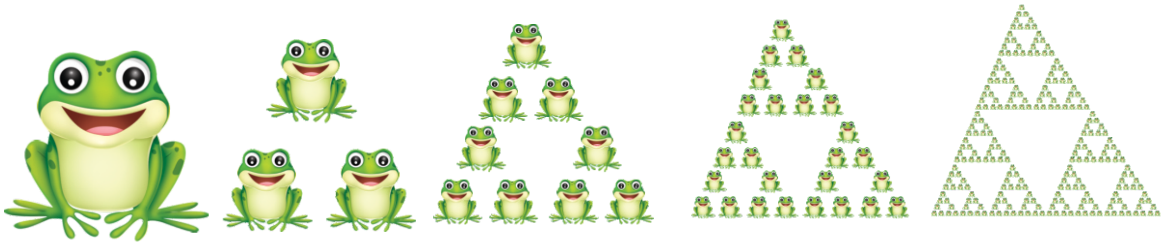
Step 1: Start with a triangle.

Step 2: Shrink the triangle to half, and put a copy in each of the three corners.

Step 3: Repeat Step 2 for the smaller triangles again and again, for ever !

First 5 steps in an infinite process.....

We can use any shape :



Make your own Number Patterns

You can make your own number patterns using coins or matchsticks. Here is an example using coins :

Size = 1 1 coin | **Size = 2** 6 coins | **Size = 3** 15 coins | **Size = 4** 28 coins

How many coins would you need when size is 5 ?

Can you make a formula that will tell you how many coins are needed for any size? The formula may look something like

Coins = size × size + ...

Example 1 : In a series of square numbers, find the square number next to :

(i) 121 (ii) 400

Solution : (i) Since $121 = 11 \times 11$, the number next to 121 is $12 \times 12 = 144$.

(ii) Since $400 = 20 \times 20$, the number next to 400 is $21 \times 21 = 441$.

Example 2 : In a series of square numbers, find the square number just before:

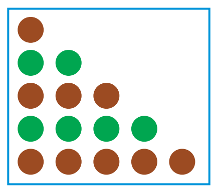
(i) 625 (ii) 196

Solution : (i) Since $625 = 25 \times 25$, the number just before 625 is $24 \times 24 = 576$.

(ii) Since $196 = 14 \times 14$, the number just before 196 is $13 \times 13 = 169$.

Example 3 : With the help of 15 dots, create a triangular pattern.

Solution : We create a triangular pattern using 15 dots as shown alongside :



Example 4 : What is next triangular number : 28, 36 ....?

Solution : Difference between 36 and 28 = 36 – 28 = 8

Therefore, the next triangular number is 36 + 9 = 45.

REMEMBER

Each side of the triangle has 5 dots.

Example 5 : What smallest number should be added to 396 to make it a square number ?

Solution : We have, $19 \times 19 = 361 < 396$

$20 \times 20 = 400 > 396$

Since $400 - 396 = 4$ and $396 + 4 = 400$, therefore 4 should be added to 396 to make it 400 (20×20), which is a square number.

Example 6 : Which of the following numbers are square numbers ?

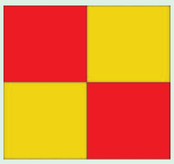
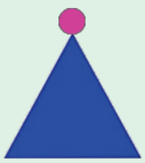
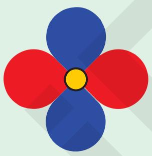
11, 12, 16, 32, 36, 49, 56, 81, 121

Solution : Numbers 16 (4×4), 36 (6×6), 49 (7×7), 81 (9×9) and 121 (11×11) are the square numbers.

Exercise 8.2

Knowledge Application

1. Give each shape a quarter turn to complete the pattern.

	Shape	$\frac{1}{4}$ turn	$\frac{1}{4}$ turn	$\frac{1}{4}$ turn	$\frac{1}{4}$ turn
(a)					
(b)					
(c)					

2. Generate the square and triangular patterns with the help of 36 dots.

[Note : 36 is a triangular as well as a square number].

3. Find three square numbers which are greater than 100.

4. Find the square numbers lie between 85 and 198.

5. Find the square number just before 256.

6. In a series of square numbers, find the square number just before :

(i) 289 (ii) 529

7. What smallest number should be added in 71 to make it a square number ?

Relation in the Sequence of Odd Numbers Between Consecutive Square Numbers

Square numbers have several interesting properties.

Let us see the following patterns for various properties :

Pattern A

$1 \times 1 = 1 = 1$ [The first odd number is 1]

$2 \times 2 = 4 = 1 + 3$ [Sum of first two odd numbers]

$3 \times 3 = 9 = 1 + 3 + 5$ [Sum of first three odd numbers]

$4 \times 4 = 16 = 1 + 3 + 5 + 7$ [Sum of first four odd numbers]

$5 \times 5 = 25 = 1 + 3 + 5 + 7 + 9$ [Sum of first five odd numbers]

By observing the above pattern, we can conclude that the square number formed by $n \times n$ is equal to sum of the first n odd numbers.

Pattern B

$1 = 1 = (1 \times 1)$

$1 + 3 = 4 = (2 \times 2)$

$4 + 5 = 9 = (3 \times 3)$

$9 + 7 = 16 = (4 \times 4)$

$16 + 9 = 25 = (5 \times 5)$

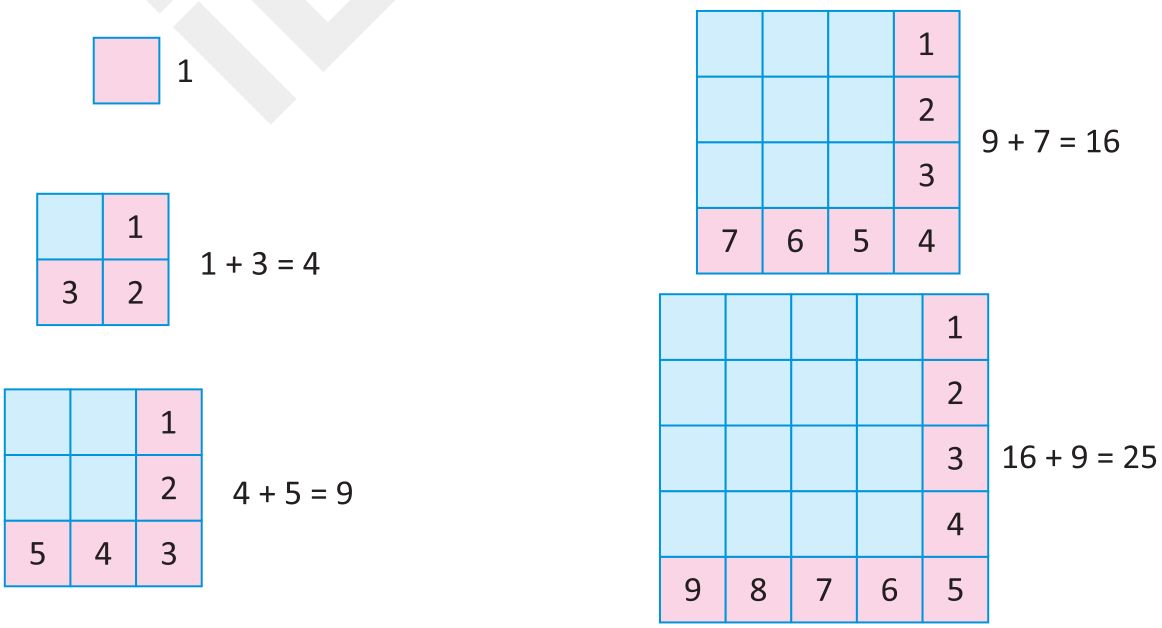
$25 + 11 = 36 = (6 \times 6)$

$36 + 13 = 49 = (7 \times 7)$

Take a Task

Watch Remedial

The above pattern can be represented pictorially as follows :



Pattern C

Look at the following pattern which relates a square number with another square number just before it :

$$7 \times 7 - (2 \times 7 - 1) = 6 \times 6$$

$$6 \times 6 - (2 \times 6 - 1) = 5 \times 5$$

$$5 \times 5 - (2 \times 5 - 1) = 4 \times 4$$

Can you extend this pattern further ?

Pattern D

See the following pattern carefully and write three more such relations between square numbers :

$$3 \times 3 + 2 \times 3 + 1 = 4 \times 4$$

$$4 \times 4 + 2 \times 4 + 1 = 5 \times 5$$

$$5 \times 5 + 2 \times 5 + 1 = 6 \times 6$$

$$6 \times 6 + 2 \times 6 + 1 = 7 \times 7$$

Example 8 : Express the following square numbers as the sum of odd numbers :

(i) 49 (ii) 81

Solution : (i) Since 49 = 7 × 7, therefore 49 can be expressed as the sum of first 7 odd numbers.

Hence, $49 = 1 + 3 + 5 + 7 + 9 + 11 + 13$

(ii) Since 81 = 9 × 9, therefore 81 can be expressed as the sum of first 9 odd numbers.

Hence, $81 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$

Example 9 : Without actual addition, find the following sum :

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$$

Solution : Given sum is the sum of the first 11 odd numbers.

Therefore, $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = 11 \times 11 = 121$

Exercise 8.3

Knowledge Application

1. Express the following square numbers as the sum of odd numbers :

(a) 25 (b) 36 (c) 121 (d) 144 (e) 64 (f) 100

2. Without actual adding find the following sums :

(a) $1+3+5+7+9+11+13$

(b) $1+3+5+7+9+11+13+15+17+19$

3. Using a pattern, show that the difference between the two consecutive square numbers will always be an odd number.

4. By observing the following patterns, generate the next three lines of numbers :

(a) $4\times4=1+3+5+7$

$5\times5=1+3+5+7+9$

____

____

____

(b) $9+7=16$

$16+9=25$

$25+11=36$

____

____

____

(c) $81-17=64$

$64-15=49$

$49-13=36$

____

____

____

Think Tank

1. Tick (✓) the correct answer.

(a) The letter 'O' of the English alphabet has __________ lines of symmetry.

(i) 2 ☐ (ii) 1 ☐ (iii) 3 ☐ (iv) Many ☐

(b) The line which divides a shape into two identical halves is called the axis of ____________.

(i) proportion ☐ (ii) symmetry ☐

(iii) equality ☐ (iv) none of these ☐

(c) Which of the following alphabet is symmetrical ?

(i) G ☐ (ii) P ☐ (iii) A ☐ (iv) R ☐

2. Consider all the capital (or upper case) letters of the alphabet and classify them under the headings below.

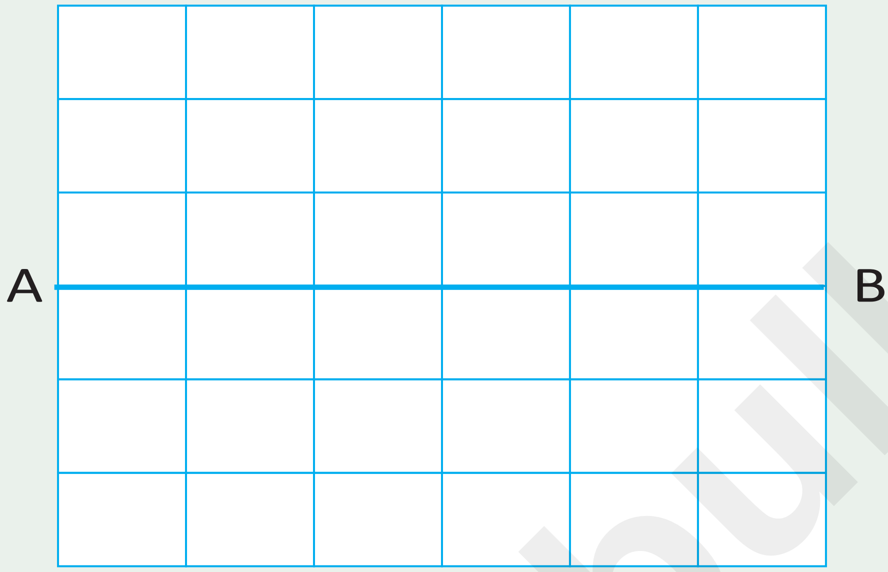
Lines of Symmetry

Horizontal only	Vertical only	Both Horizontal & Vertical	Neither Horizontal nor Vertical

Fun Time Activity

Problem Solving

Now, make a pattern on the top half of the grid and reflect it about the horizontal line AB.



Maths Lab Activity

Conceptual Learning

Material required : Used paper / Newspaper and plant leaves.

Method

- Teacher make an aeroplane, paper boat with the help of newspaper. Ask students to observe the aeroplane, and paper boat.
- Teacher show the line of symmetry in aeroplane, paper boat and plant leaves. Now ask the students to write the five names of plant leaves, fruits, and architecture, animals, alphabets which have line/reflection symmetry:
  - ❖ Symmetry in plants leaves ______________________
  - ❖ Symmetry in fruits ______________________
  - ❖ Symmetry in architecture ______________________
  - ❖ Symmetry in animals ______________________
  - ❖ Symmetry in Alphabets ______________________
- Now explain about reflection in water of all these objects of papers.