# **Cube Root of a Rational Number**

#### **Understanding the Concept**

- A rational number is a number that can be written in the form <sup>p</sup>/<sub>q</sub> where p and q are integers and q is not equal to 0.
- The cube root of a rational number is found by taking the cube root of the numerator and the cube root of the denominator separately.

• Formula used is 
$$\sqrt[3]{\frac{p}{q}} = \frac{\sqrt[3]{p}}{\sqrt[3]{q}}$$
.

- This method works only when both numerator and denominator are perfect cubes.
- Useful for solving cube root problems involving fractions.

#### **Important Points**

- Find the cube root of the numerator and denominator separately.
- If numerator and denominator are perfect cubes the answer is a rational number.
- If either numerator or denominator is not a perfect cube the result may be irrational.
- Negative signs stay with the numerator when taking cube roots.
- Useful in simplifying fractions and solving real-world problems.

#### **Examples with Solutions**

**Example Easy Level** 

Find 
$$\sqrt[3]{\frac{8}{27}}$$

**Solution:** 
$$\frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

**Example Easy Level** 

Find 
$$\sqrt[3]{\frac{1}{64}}$$
  
Solution:  $\frac{\sqrt[3]{1}}{\sqrt[3]{64}} = \frac{1}{2}$ 

**Example Moderate Level** 

$$\succ$$
 Find  $\sqrt[3]{\frac{125}{8}}$ 

**Solution**:  $\frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \frac{5}{2}$ 

**Example Moderate Level** 

> Find 
$$\sqrt[3]{\frac{216}{343}}$$
  
Solution:  $\frac{\sqrt[3]{216}}{\sqrt[3]{343}} = \frac{6}{7}$ 

## **Example Word Problem**

A cube-shaped water tank has volume  $\frac{512}{27}$  cubic meters Find the side length Solution: Side length =  $\sqrt[3]{\frac{512}{27}}$ =  $\frac{\sqrt[3]{512}}{\sqrt[3]{27}}$ =  $\frac{8}{3}$  meters

### **Summary Points**

• Cube root of a rational number is cube root of numerator divided by cube root of denominator.

• 
$$\sqrt[3]{\frac{p}{q}} = \frac{\sqrt[3]{p}}{\sqrt[3]{q}}.$$

- Numerator and denominator must both be perfect cubes for easy calculation.
- Result is rational if both parts are perfect cubes.
- Negative signs remain in the numerator while finding cube roots.