Factorization When a Binomial is the Difference of Two Squares

Understanding of Factorization When a Binomial is the Difference of Two Squares

- A difference of two squares means an expression of the form $a^2 - b^2$.
- It can always be factored as $(a + b)(a - b)$.
- Both terms must be perfect squares and separated by a minus sign.
- This method simplifies binomials that look complicated into a product of two binomials.

Important Points

- **Standard identity:** $a^2 - b^2 = (a + b)(a - b)$
- Both terms must be perfect squares.
- If required, first rearrange or rewrite terms as squares.
- **Be careful:** this method applies only when the operation is subtraction, not addition.
- Always check factorization by multiplying back.

Examples with Solutions

Example: Simple Difference of Squares

➢ **Factorize $x^2 - 9$.**

Solution: $x^2 - 9 = (x + 3)(x - 3)$

Example: With Variables and Numbers

➢ **Factorize $4a^2 - 25b^2$.**

Solution: $4a^2 - 25b^2 = (2a + 5b)(2a - 5b)$

Example: Higher Powers

➢ **Factorize $16x^4 - 81y^4$.**

Solution: $16x^4 - 81y^4 = (4x^2 + 9y^2)(4x^2 - 9y^2)$

Now factorize further: $4x^2 - 9y^2 = (2x + 3y)(2x - 3y)$

Final Answer: $(4x^2 + 9y^2)(2x + 3y)(2x - 3y)$

Example: Common Factor First

- **Factorize $2(x^2 - 49)$.**

Solution: $x^2 - 49 = (x + 7)(x - 7)$

So, $2(x + 7)(x - 7)$

Example: Expression with Fractions

- **Factorize $\left(\frac{1}{4}\right)p^2 - \left(\frac{9}{16}\right)q^2$.**

Solution: $\left(\frac{1}{4}\right)p^2 - \left(\frac{9}{16}\right)q^2 = \left[\left(\frac{1}{2}\right)p + \left(\frac{3}{4}\right)q\right]\left[\left(\frac{1}{2}\right)p - \left(\frac{3}{4}\right)q\right]$

Summary Points

- Use $a^2 - b^2 = (a + b)(a - b)$ when two perfect squares are subtracted.
- Check that both terms are squares.
- Do not apply for addition ($a^2 + b^2$ cannot be factored like this).
- Simplify if there is a common factor first.
- Verify factorization by expansion.