Extension of Laws of Exponents

Understanding of Extension of Laws of Exponents

- The extension of laws of exponents applies the same rules even when exponents are negative numbers, fractions, or involve variables.
- These laws make it easier to simplify more complex algebraic expressions.
- Extended laws help when bases are rational numbers, negative numbers, or expressions with powers.

Important Points

- $a^m \times a^n = a^{m+n}$ even if m and n are negative or fractions.
- $a^m \div a^n = a^{m-n}$ even if m and n are negative or fractions.
- $(a^m)^n = a^{mn}$ works for any rational exponents.
- $(ab)^m = a^m b^m$ even if a or b are negative or fractions.
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ for all rational numbers a and b ≠ 0.

Examples with Solutions

Example: Product Law with Negative Exponents

➢ **Simplify $2^{-3} \times 2^2$.**

Solution: $2^{-3} \times 2^2 = 2^{-3+2} = 2^{-1} = \frac{1}{2}$

Example: Quotient Law with Fractions

➢ **Simplify $\left(\frac{3}{4}\right)^5 \div \left(\frac{3}{4}\right)^2$**

Solution: $\left(\frac{3}{4}\right)^5 \div \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^{5-2} = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

Example: Power of a Power with Negative Exponent

➢ **Simplify $(5^{-2})^3$.**

Solution: $(5^{-2})^3 = 5^{-2 \times 3} = 5^{-6} = \frac{1}{5^6} = \frac{1}{15625}$

Example: Power of a Product with Fractional Bases

➢ **Simplify** $\left(\frac{2}{5}\right)^2$

Solution: $\left(\frac{2}{5}\right)^2 = \frac{2^2}{5^2} = \frac{4}{25}$

Example: Power of a Quotient with Negative Exponents

➢ **Simplify** $\left(\frac{7}{2}\right)^{-2}$.

Solution: $\left(\frac{7}{2}\right)^{-2} = \left(\frac{2}{7}\right)^2 = \frac{4}{49}$

Summary Points

- Extended laws work for negative, positive, and fractional exponents.
- Multiply or divide exponents as per the basic laws even for complex numbers.
- Change negative exponents to positive by taking reciprocal.
- Simplify expressions carefully step-by-step.
- Always express final answers in simplest form.