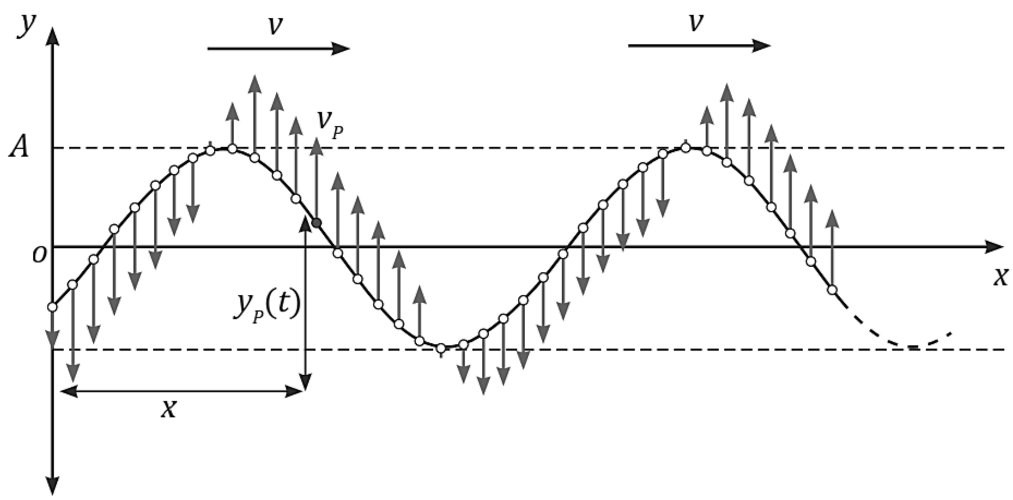
WAVES

THE SPEED OF A TRAVELLING WAVE

WAVE VELOCITY AND PARTICLE VELOCITY

Imagine a wave moving back and forth in a straight line to the right. The up-and-down motion of a particle at a spot called P, which is located at a distance x from the starting point, is described by this equation:

$$y = A\sin(\omega t - kx + \phi)$$



It is important to note that the velocity of the particle is a result of its SHM with respect to the equilibrium position, where $y = 0$. In the x-direction, the particle's displacement is zero. The speed of the wave or disturbance that propagates in the x-direction is v$\left(=\frac{\omega}{k}\right)$.

You can determine the velocity of the particle by looking at how the displacement y changes with time.

$$v_p = \frac{\partial y(x,t)}{\partial t} = \left[\frac{dy}{dt}\right]_{x=\text{ Constant}}$$

$$v_p = A\omega\cos(\omega t - kx + \phi) \quad \ldots\ldots(i)$$

Since the particle P is in SHM, v_p can also be written as follows:

$$v_p = \omega\sqrt{A^2 - y^2}$$

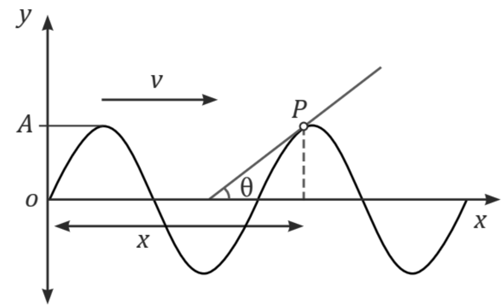
If we focus on a specific moment in time (t) in the wave equation, the graph of y-x shows us the shape of the wave at that instant. You can see one of these plots in the picture below.

When you calculate the partial derivative of the displacement y with respect to x, it tells you how steep or slanted the y-x plot is at the point (x, y).

$$\frac{\partial y(x,t)}{\partial x} = \left[\frac{d}{dx}(A\sin(\omega t - kx + \phi))\right]_{t=\text{ Constant}}$$

$$\frac{\partial y(x,t)}{\partial x} = -kA\cos(\omega t - kx + \phi)$$

$$\text{Slope} = -kA\cos(\omega t - kx + \phi) \quad .....(ii)$$



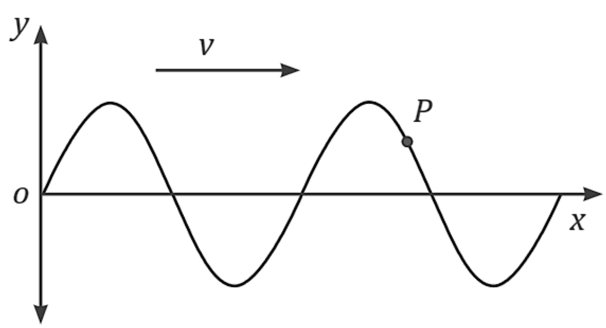
On dividing equation by (i) equation (ii), we get,

$$\frac{v_P}{\text{Slope}} = -\frac{\omega}{k} = -v$$

$$\frac{v_P}{v} = -\text{ Slope of the } y - x \text{ plot}$$

Example.

A wave is moving in the direction of the positive x-axis with a speed v. Your task is to determine which way the particle P is moving in the picture at that specific moment.



Solution.

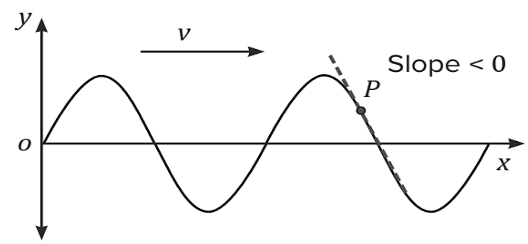
When the wave is traveling to the right positive x-direction, (v > 0), the slope (let's call it m) of the graph showing the position of particle P over time is negative.

The velocity of particle P is given by,

$$v_P = -v \times (\text{ Slope}) = -vm$$

$$v_P = -(+ve) \times (-ve)$$

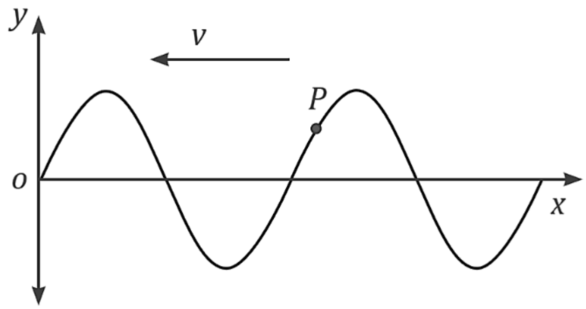
$$v_P > 0$$



Hence, particle P is moving in the upward direction at that instant.

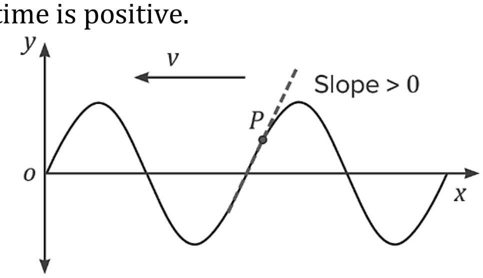
Example.

A wave is moving in the direction of the negative x-axis with a speed v. Your job is to determine in which direction the particle P is moving in the figure at that specific moment.



Solution.

When the wave is traveling to the left negative x-direction, (v < 0), the slope (let's call it m) of the graph showing the position of particle P over time is positive.



The velocity of particle P is given by

$$v_P = -v \times (\text{ Slope}) = -vm$$
$$v_P = -(-ve) \times (+ve)$$
$$v_p > 0$$

Hence, particle P is moving in the upward direction at that instant.

Differential Form of Wave Equation

Now, let's write down the formulas for the speed of the particle and how steep the graph of position over time is the slope of the y-x plot.

$$v_P = \frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx + \phi) \ldots\ldots (i)$$
$$\frac{\partial y}{\partial x} = -kA\cos(\omega t - kx + \phi) \ldots\ldots (ii)$$

By taking the partial differentials of equations (i) and (ii) with respect to t and x, respectively, we get,

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx + \phi) \ldots\ldots (iii)$$
$$\frac{\partial^2 y}{\partial x^2} = -k^2 A\sin(\omega t - kx + \phi) \ldots\ldots (iv)$$

Equation (iii) talks about how the particle accelerates, and equation (iv) describes how the slope of the y-x graph changes over time.

On dividing equation (iii) by equation (iv), we get,

$$\frac{\frac{\partial^2 y}{\partial t^2}}{\frac{\partial^2 y}{\partial x^2}} = \frac{\omega^2}{k^2} = v^2$$

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} .....(v)$$

Equation (v) is the math equation that describes the behavior of the wave. Any arrangements of $y = f(x, t)$ that make this equation true show us what wave motion looks like.

Example.

Find out the velocity of the wave represented by $y = \frac{2}{\left(t - \frac{x}{2}\right)^2 + 1}$

Solution.

The wave equation provided looks like y(x, t) = f (ωt - kx). This means the wave is moving in the direction of the positive x-axis. You can find the speed of the wave by using this formula:

$$v = \left| \frac{\text{Coefficient of } t}{\text{Coefficient of } x} \right|$$

$$v = \left| \frac{1}{-\frac{1}{2}} \right| = 2\ ms^{-1}$$

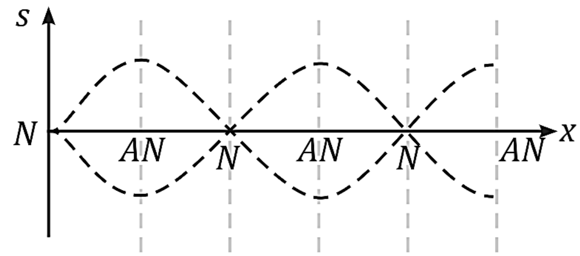
Standing Sound Waves

In our previous lessons, we talked about waves that don't seem to move on a string. Now, we're going to talk about similar waves, but in sound. These waves are like the ones we studied on the string. We describe how the particles move with the following formula:

$$s = 2s_0 \sin(kx + \phi_1)\sin(\omega t + \phi_2)$$

If we think of $\phi 1$ and $\phi 2$ as being equal to zero, then the movement looks like this:

$$s = 2s_0 \sin kx \sin \omega t$$

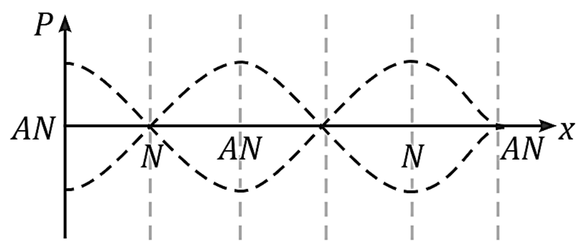


We can describe the changes in pressure in standing sound waves like this:

$$\Delta P = 2\Delta P_0 \sin\left(kx + \phi_1 + \frac{\pi}{2}\right)\sin\left(kx + \phi_2 + \frac{\pi}{2}\right)$$

Or

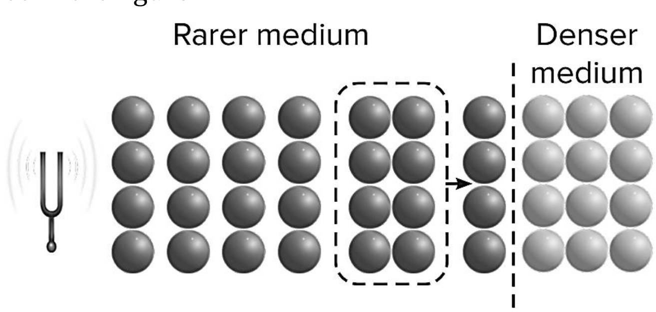
$$\Delta P = 2\Delta P_0 \cos(kx + \phi_1)\cos(kx + \phi_2)$$



Reflection of Sound Waves

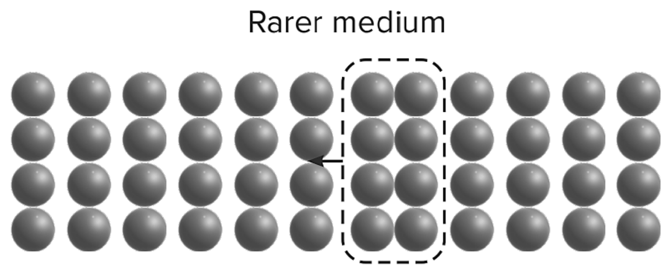
Reflection from a denser medium

Think about a compressed wave moving through a less dense material, bouncing back from a denser one, as you can see in the figure.



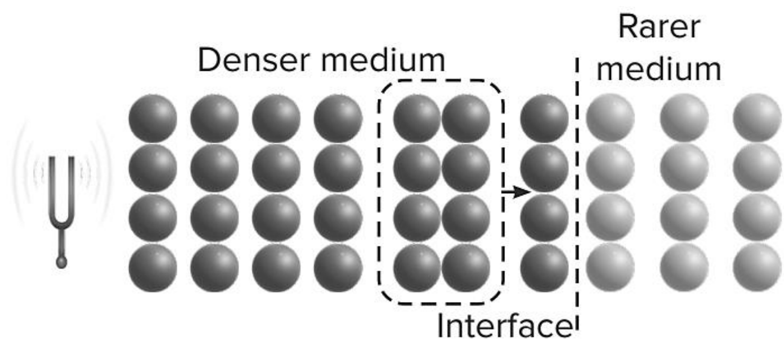
Right before the bounce, the particles in the thinner material next to the border get squeezed together and collide with the particles in the thicker material. After these collisions, the particles in the thinner material bounce back the same way they were right before the bounce. In simpler words, when they hit a fixed end (where they don't move), a squeezed part goes back as a squeezed part, and a stretched-out part goes back as a stretched-out part. So, the thicker material (the fixed end) stays still like a point that doesn't move.

Because the wave pulse retains its characteristics after reflecting from the denser medium, there is no alteration in the phase of the sound wave pulse following its reflection from the denser medium.

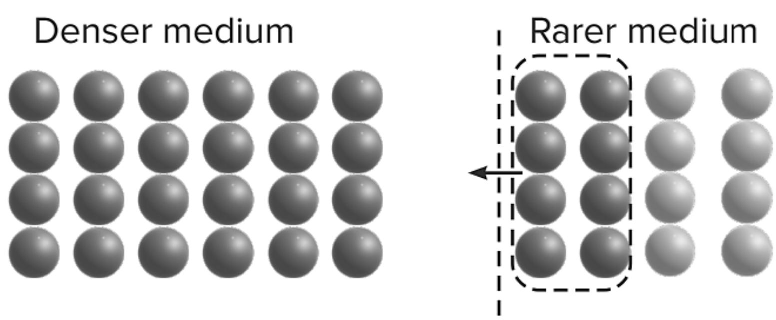


Reflection from a rarer medium

Let's think about what happens when a compressed wave moves from the denser stuff to the thinner stuff and hits the border. Here's a Figure to help you see it.



At this point, the place where they meet acts like a loose end (similar to how it works with string waves). Right after the collisions, the particles from the thicker material move into the thinner material and bounce back into the thicker one, but they bounce back as a stretch-out part. So, the thinner material (the loose end) acts like a point that moves a lot, as you can see in the picture.



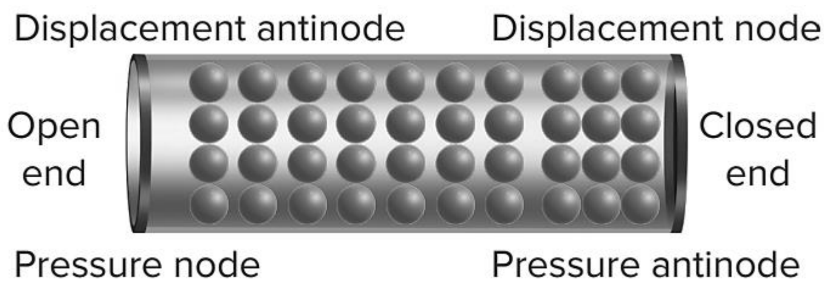
In simple terms, when a sound wave bounces off the thinner material, the squeezed part turns into a stretched-out part. So, we can say that the sound wave changes by half a circle (π) after bouncing off the thinner material.

Modes of Vibration in a Closed Organ Pipe

In string waves, if you fix the string at both ends, you get a standing wave between two points that don't move, called nodes. If you fix it at just one end, you get a standing wave between a point that doesn't move (node) and a point that moves a lot (antinode).

Similarly, in a closed organ pipe where one end is closed and the other is open, you get a standing wave between a point that doesn't move (node) and a point that moves a lot (antinode). In an open organ pipe where both ends are open, you get a standing wave between two points that don't move (pressure nodes).

In a closed organ pipe, which has a certain length l, the open end will have a point where the air pressure doesn't change (like a pressure node), and the closed end will have a point where the air doesn't move (like a displacement node). This is illustrated in the figure.



Fundamental mode

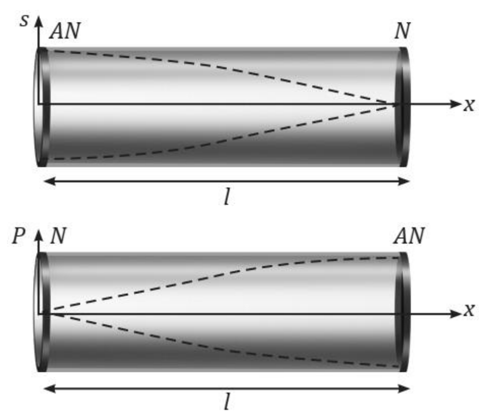
The fundamental mode is the simplest way to create standing sound waves at the lowest possible frequency. In the fundamental mode, something specific happens.

$$l = \frac{\lambda}{4}$$

Or

$$\lambda = 4l$$

$$\Rightarrow f = \frac{v}{4l} \quad (\because v = f\lambda)$$



There will be one and a half loop in between the ends as shown in the figure.

The frequency for the first overtone or third harmonic is given by,

$$\Rightarrow f_1 = \frac{3v}{4l} = 3f_0$$

Second overtone or fifth harmonic:

For the fifth harmonic,

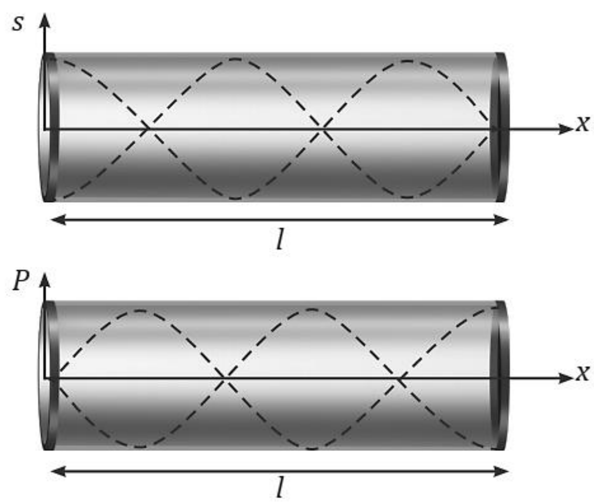
$$l = \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{5\lambda}{4}$$

Or

$$\lambda = \frac{4l}{5}$$

$$f_2 = \frac{5v}{4l}$$

There will be two and a half loop in between the ends as shown in the figure.



The frequency for the second overtone or fifth harmonic is given by,

$$\Rightarrow f_2 = \frac{5v}{4l} = 5f_0$$

n^{th} Overtone or $(2n + 1)^{th}$ harmonic:

For the $(2n + 1)^{th}$ harmonic, (n = 0, 1, 2, 3…)

$$l = \frac{n\lambda}{2} + \frac{\lambda}{4} = (2n+1)\frac{\lambda}{4}$$

Or

$$\lambda = \frac{4l}{2n+1}$$

There will be $n+\frac{1}{2}$ loops in between the ends of the organ pipe. The corresponding frequency is given by,

$$f_n = (2n+1)\frac{v}{4l}$$

Sound Wave

- A sound wave is a kind of wave that moves through the air, and it's like a three-dimensional back-and-forth vibration. The air particles jiggle in the same way the wave is moving. These waves are made when something vibrates, like a guitar string, vocal cords, a tuning fork, or a speaker's diaphragm.

- Every mechanical wave, including sound waves, requires a medium with attributes like inertia and elasticity to be able to travel through it. This is a fundamental requirement for mechanical waves, such as sound waves, to propagate, just like any other type of mechanical wave.

- Sound waves transmit through a given medium by inducing a regular pattern of pressure compressions and rarefactions, all of which are generated by the oscillations of the sound source.

Condition for a longitudinal wave

The oscillations of the particles within a medium align with the direction of the wave's propagation. Consequently, the vectors representing the velocity of the wave and the velocity of the particles are parallel to each other.

$$\vec{v}_w \parallel \vec{v}_p$$

Or

$$\vec{v}_w \times \vec{v}_p = 0$$

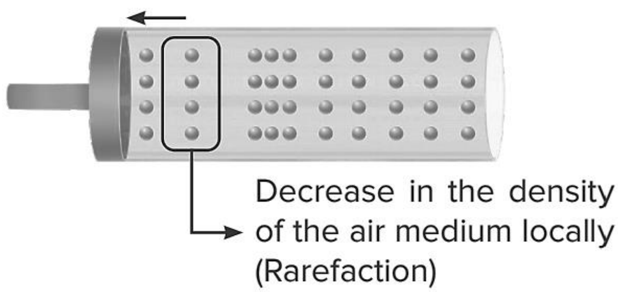
Propagation of a Sound Wave

Let's think about a vibrating source, like a piston, making sound waves. Before the piston starts to vibrate, the air molecules in the medium (in this case, the air) are calm and not moving.

When the piston moves to the right, it pushes the air in front of it, which squeezes the air and makes the pressure go up just a bit.

The area where the pressure goes up is called a "compression pulse," and it moves away from the piston at the speed of sound.

Once the piston creates the compression pulse, it changes its movement and moves outward. This action pulls some air away from the area in front of it, making the pressure drop slightly below the usual level. This area with lower pressure is called a rarefaction pulse.

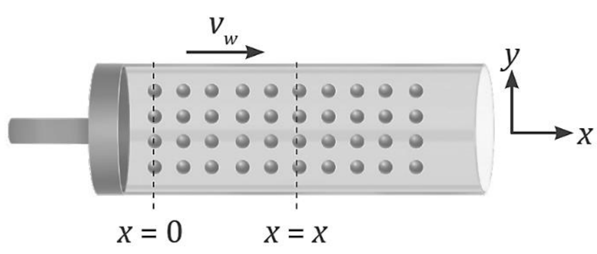


Right after the compression pulse, another pulse, called the rarefaction pulse, goes away from the piston just as fast as sound travels. This keeps repeating, and that's how the sound wave keeps moving forward.

Equation of Displacement of a Particle

A sound wave comprises variations in certain properties of the medium through which it propagates. These properties include:

(i) Position of the particle
(ii) Localised pressure



As sound travels through a substance, the particles in that substance move back and forth in a regular way, kind of like swinging in a straight line. This makes the position of the particles change in the following manner:

$$s = s_0 \sin(kx - \omega t + \phi)$$

In this context, the symbol s signifies the momentary displacement of the particle positioned at a distance x from the piston (the source of vibration).

Example.

The equation representing a sound wave's propagation along the x-axis is defined as $s = 6.0\ sin\ (600t\ -\ 1.8x)$, with the measurement of s in units of$10^{-5}m$, t in seconds, and x in meters.

(a) Find the ratio of the displacement amplitude of the particles to the wavelength of the wave

(b) Find the ratio of the velocity amplitude of the particles to the wave speed

Solution.

Given,

$s_0 = 6\times10^{-5}$ m (Since s is measured in$10^{-5}m$), $\omega = 600\ \text{s}^{-1}, k = 1.8\ \text{m}^{-1}$

(a) You can find the ratio of how much the particles move and the length of the wave by using this formula:

$$\frac{s_0}{\lambda} = \frac{s_0}{\left(\frac{2\pi}{k}\right)} = \frac{s_0 k}{2\pi}$$

$$\frac{s_0}{\lambda} = \frac{6\times10^{-5}\times1.8}{2\pi}$$

$$\frac{s_0}{\lambda} = 1.7\times10^{-5}$$

(b) You can figure out the relationship between how fast the particles move back and forth and the speed of the wave using this formula:

$$\frac{(v_p)_{\max}}{v_w} = \frac{s_0\omega}{v}$$

$$= s_0 k$$

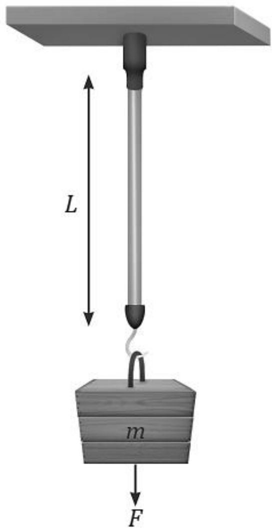
$$= 1.8\times6\times10^{-5}$$

$$\frac{(v_p)_{\max}}{v_w} = 1.1\times10^{-4}$$

Young's Modulus

Think of a long rod hanging from the ceiling, and there's a block attached to the bottom of the rod. The block is pulling the rod downwards with a force called F. Similar to a spring, the rod also has an inner force that wants to make the rod go back to its original length, which is L. This inner force in the rod is also F.

If we know how much the rod has stretched, which is called ΔL, we can use this information to calculate two things: the stress and the strain.



Stress

The force inside the stretched material for each unit of its cross-sectional area is called stress. When this force is pushing or pulling straight in and out of the material, we call it longitudinal stress.

The expression of longitudinal stress (tensile or compressive) is given by,

$$\sigma_n = \frac{\text{Internal restoring force}}{\text{Cross-sectional area}} = \frac{F}{A}$$

Strain

Strain is a measure of how much the size of an object changes because of the stress applied to it. We calculate the change in size compared to the original size. The formula for this kind of strain, where the stress is acting straight in and out, is given by:

$$\varepsilon_l = \frac{\Delta L}{L}$$

Hooke's law tells us that when we slightly stretch or squeeze something, how much it moves is directly related to how much force we use. Since we're assuming the size of the object doesn't change much, we can express it like this:

$\sigma_n \propto \varepsilon_l$

Or

$\sigma_n = Y\varepsilon_l$

The letter Y stands for something called Young's modulus or the modulus of elasticity. It's like a measure of how good a material is at handling stretching or squeezing in a straight line without changing its shape too much.

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{\sigma_n}{\varepsilon_l} = \frac{FL}{A\Delta L}$$

Bulk Modulus

Pressure means the force that's pushing straight down on a specific area. To calculate pressure, you use this formula:

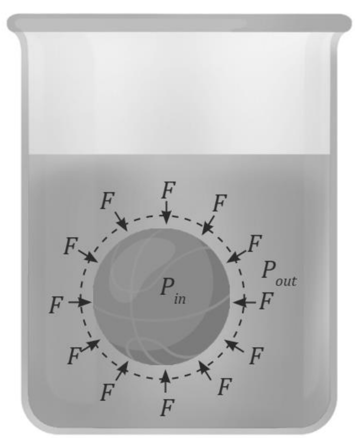
$$P = \frac{F_\perp}{A}$$

Think about a ball filled with gas and placed in a liquid, like the picture shows. If the pressure inside the ball at that moment is P_{in} and the pressure of the liquid isP_{out} , you can find the extra pressure using this formula:

$$\Delta P = P_{\text{out}} - P_{\text{in}}$$

If the ball started with a certain volume V and, because of the extra pressure, it gets a bit smaller by ΔV, we can define something called the bulk modulus like this:

$$\beta = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = -V\frac{dP}{dV}$$



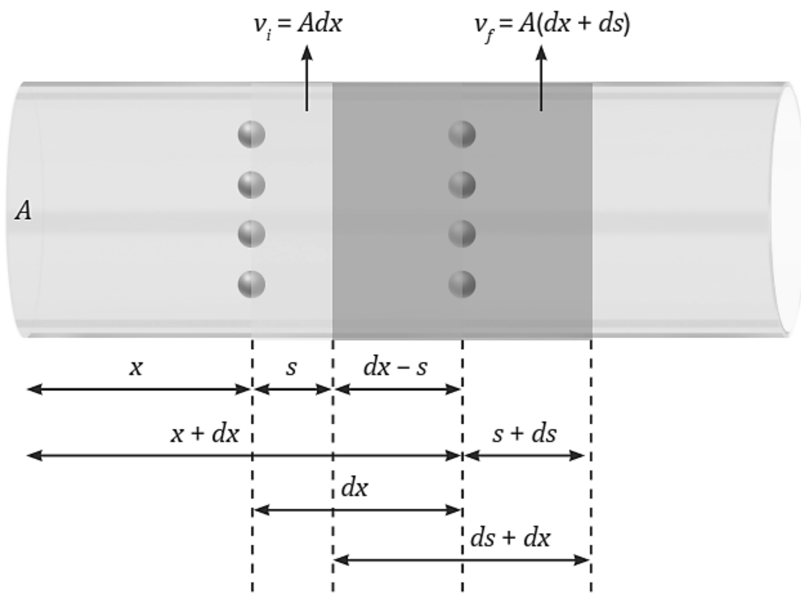
The bulk modulus is a measure of how a substance responds to pressure. It's significant when we're dealing with fluids like liquids and gases. The bulk modulus is always a positive number, and the negative sign in its formula shows that when the pressure goes up, the substance tends to get a bit smaller in response.

Variation of Excess Pressure

Imagine a sound wave traveling through a material, where the position of a particle at a certain spot x changes over time like this:

$$s = s_0 \sin(\omega t - kx) \quad \ldots..(i)$$

Think about the particles at two nearby spots, one at a distance x and the other a tiny bit further at x + dx. As the wave moves, the particle at x has a position described as s, and the one at x + dx has a position described as s + ds.



The volume enclosed between the two particles separated by a distance dx is given by,

V = Adx..... (ii)

The change in the volume due to a small displacement ds is given by,

$$\Delta V = A(ds + dx) - Adx$$
$$\Delta V = Ads$$

Upon dividing the obtained expression of ΔV by equation (ii), we get,

$$\Rightarrow \frac{\Delta V}{V} = \frac{Ads}{Adx} = \frac{ds}{dx} \quad \ldots..(iii)$$

From the definition of bulk modulus

$$\beta = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

$$\Rightarrow \Delta P = -\beta \frac{\Delta V}{V} = \text{ Excess pressure}$$

By substituting the value of $\frac{\Delta V}{v}$ from equation (iii) in the above equation, we get,

$$\Delta P = -\beta \frac{ds}{dx} \quad .....(\text{ iv})$$

Because s depends on both x and t, we'll use a special way of writing the differentiation with respect to x (or t) called partial differential notation.

Previously, we assumed the displacement of the particle as follows:

$$s = s_0 \sin(\omega t - kx)$$
$$\frac{\partial s}{\partial x} = -s_0 k \cos(\omega t - kx)$$

By substituting the obtained value of $\frac{\partial s}{\partial x}$ in equation (iv), we get,

$$\Delta P = -\beta \times -s_0 k \cos(\omega t - kx)$$
$$\Delta P = \beta k s_0 \cos(\omega t - kx) \quad .....(\text{ iv})$$

Equation (iv) shows us how much extra pressure there is in the material where the wave is moving through.

$$\Delta P = \Delta P_0 \cos(\omega t - kx)$$

Where, $\Delta P_0 = \beta k s_0$ is the amplitude of the excess pressure

Equations of sound propagation

The way sound waves travel can be expressed in two different forms, which are shown below:

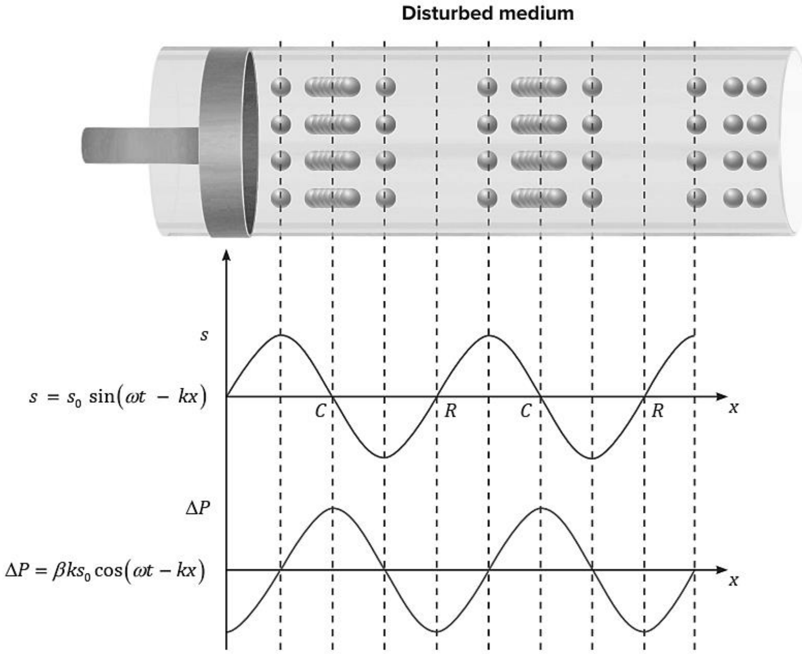
1. Through the displacement of the particles

$$s = s_0 \sin(\omega t - kx) \quad .....(i)$$

2. Through the change in the pressure of the medium

$$\Delta P = \Delta P_0 \cos(\omega t - kx) = \Delta P_0 \sin\left(\omega t - kx + \frac{\pi}{2}\right)$$

The picture below displays how displacement and extra pressure change over time.

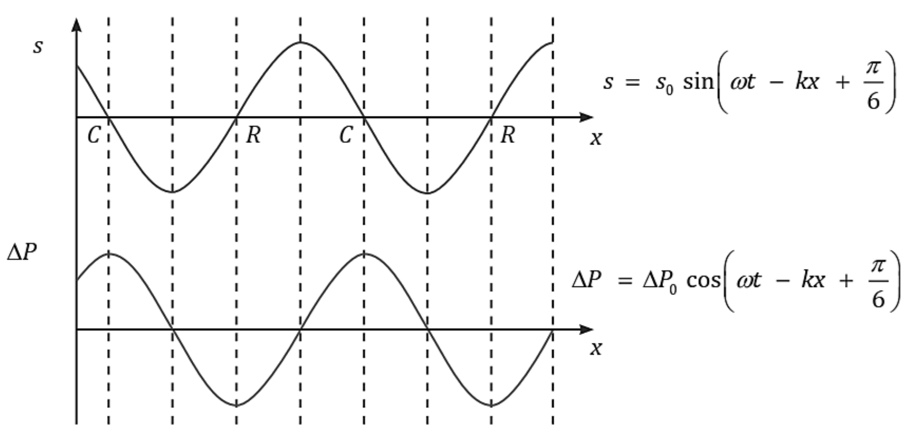


At the points of compression, the slope of $\frac{\partial s}{\partial x}$ is negative. It implies that the excess pressure according to equation (iv) is positive. Further, the second partial differential of s with respect to x is a multiple of sin (ωt - kx), and therefore, it is equal to zero. The obtained information implies that the slope $\frac{\partial s}{\partial x}$ and hence, the excess pressure has achieved its maximum value. Whereas in rarefaction, the slope of $\frac{\partial s}{\partial x}$ is positive. Hence, the excess pressure is the minimum.

Example.

The displacement equation of a sound wave is given as $s = s_0 \sin\left(\omega t - kx + \frac{\pi}{6}\right)$. Find the Equation and draw the graph of the pressure wave.

Solution.



Density Wave

Density is described as the amount of matter in a substance contained within a specific space. In the case of a substance with a total mass of M enclosed within a volume V, the density (symbolized as ρ) is calculated as follows:

$$\rho = \frac{M}{V}$$

By differentiating both the sides with respect to, we get,

$$\frac{d\rho}{dV} = -\frac{M}{V^2}$$
$$d\rho = -\frac{M}{V} \times \frac{dV}{V}$$
$$d\rho = -\rho \frac{dV}{V}$$

From the definition of the bulk modulus, we can substitute $-\frac{dV}{V}$ with $\frac{\Delta P}{\beta}$.

$$\Delta\rho = \rho \frac{\Delta P}{\beta}$$

$$\Delta\rho = \frac{\rho}{\beta} \times \beta k s_0 \cos(\omega t - kx)$$
$$\Delta\rho = \Delta\rho_0 \cos(\omega t - kx)$$

Where, $\Delta\rho_0 = \rho k s_0$

It means that the density of the medium varies in the same manner as pressure. The density of the medium fluctuates between $\rho + \rho k s_0$ and $\rho - \rho k s_0$ as the wave propagates.

Speed of Sound Wave

Imagine a closed space with an area (like the size of the opening) A and a small thickness (or width) dx, located at a distance x. The total pressures at the points x and $x + dx$ are $P_0 + \Delta P$ and $P_0 + \Delta P + d(\Delta P)$, as you can see in the picture.

The net force acting on the enclosed volume is given by,

$$F_{\text{net}} = \left(P_0 + \Delta P\right)A - \left(P_0 + \Delta P + d(\Delta P)\right)A$$
$$F_{\text{net}} = -d(\Delta P)A = dm \times a$$
$$-d(\Delta P)A = \rho A dx \frac{\partial^2 s}{\partial t^2}$$
$$-\frac{\partial}{\partial x}(\Delta P) = \rho \frac{\partial^2 s}{\partial t^2}$$

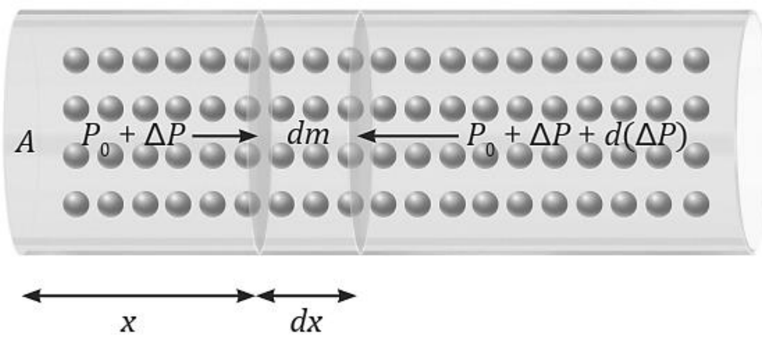
By substituting ΔP with $-\beta \frac{\partial s}{\partial x}$, we get,

$$\frac{\partial}{\partial x}\left(\beta \frac{\partial s}{\partial x}\right) = \rho \frac{\partial^2 s}{\partial t^2}$$

$$\beta \frac{\partial^2 s}{\partial x^2} = \rho \frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial^2 s}{\partial t^2} = \frac{\beta}{\rho} \frac{\partial^2 s}{\partial x^2}$$

$$\frac{\partial^2 s}{\partial t^2} = v_w^2 \frac{\partial^2 s}{\partial x^2}$$



Which is a second-order differential equation representing the wave motion. Hence, the speed of wave is given by,

$$v_w = \sqrt{\frac{\beta}{\rho}}$$

Where β and ρ are the bulk modulus and density of the medium, respectively.

Newton’s Theory of Sound Propagation

We found out that when sound travels through stuff, it creates areas where the stuff gets squeezed together (compression) and areas where it spreads out (rarefaction). When stuff gets squished together, it gets hotter, so it releases extra heat to stay at the right temperature. When stuff spreads out, it gets cooler, so it takes in some heat from the surroundings to stay at the right temperature.

So, when sound travels, the temperature of the stuff it's traveling through doesn't change. This means that sound moves in a way that keeps the temperature the same throughout its journey.

Derivation of Newton's formula

Assumption: Newton thought that when sound travels, it stays at the same temperature the whole time.

It means the following:
$\Delta T = 0$
For an ideal gas,
$PV = nRT$
As T is constant, we get the following:
$PV =$ Constant
On differentiating both the sides partially, we get

$$PdV + VdP = 0$$

$$PdV = -VdP$$

$$P = -\frac{dP}{\left(\frac{dV}{V}\right)}$$

Also, the bulk modulus for an isothermal process is given by,

$$B_{\text{isothermal}} = -\frac{dP}{\frac{dV}{V}}$$

$$\therefore P = B_{\text{isothermal}}$$

The speed of sound is given by,

$$v = \sqrt{\frac{B_{\text{isothermal}}}{\rho}}$$

$$v = \sqrt{\frac{P}{\rho}}$$

At STP,

$P = P_{atm} = 1.013 \times 10^5\ Pa$ And $\rho = \rho_{\text{air}} = 1.292 kgm^{-3}$

By substituting the values for the properties of air, the speed of sound, as determined by Newton's formula, is as follows:

$$= v = 280.0\ ms^{-1}$$

Newton believed that sound travels at 280 ms^{-1} at $0°C$ because he thought sound stays at the same temperature while traveling. But when scientists tested it, they found that sound actually goes at 331.5 ms^{-1} at that temperature. This means Newton's idea was not right.

Laplace's Correction

Following the inadequacy of Newton's hypothesis, positing that sound wave propagation adheres to isothermal conditions, Laplace postulated an alternative perspective. Laplace asserted that sound wave propagation exhibits an adiabatic nature rather than an isothermal one. According to Laplace, the processes of compression and rarefaction transpire at a swift pace. Given that air is a relatively poor conductor of heat, it effectively precludes the rapid exchange of heat with the surrounding environment within such brief time intervals.

For an adiabatic process, $\Delta Q = 0$

Further,

$$PV^{\gamma} = \text{Constant}$$

On differentiating partially, we get the following:

$$P\gamma V^{\gamma-1}dV + V^{\gamma}dP = 0$$
$$\gamma PV^{\gamma-1}dV = -V^{\gamma}dP$$
$$\gamma P = -\frac{V^{\gamma}dP}{V^{\gamma-1}dV}$$
$$\gamma P = -\frac{dP}{\left(\frac{dV}{V}\right)}$$
$$\gamma P = B_{\text{adiabatic}}$$

At STP, by substituting the values ofγ, p and ρ of air, the corrected speed of sound according to Laplace's formulation is as follows:

$$= v = 331.7\ ms^{-1}$$

The speed of sound calculated this way is very, very close to the real speed of 331.5 ms^{-1}.This means that Laplace's ideas were right.

Effect of Different Factors on the Speed of a Sound Wave

1. Effect of pressure

In the event of an increment in the pressure of the medium, the gas particles tend to draw nearer to one another, consequently leading to an augmentation in the medium's density. This mutual escalation in pressure and density, bearing a proportional relationship($\frac{P}{\rho} = Constant$), is such that the speed of sound remains unaffected. Conversely, when the pressure diminishes, it exerts no influence on the speed of sound.

2. Effect of humidity

We understand that the speed of sound is linked to how tightly packed the air is. When there's water vapor in the air, it makes the air less dense because water vapor is lighter compared to the main stuff in the air, like oxygen and nitrogen. This effect is more obvious when it's warm. So, when there's more humidity, the speed of sound goes up.

3. Effect of temperature

For an ideal gas,

$$PV = nRT$$
$$PV = \frac{m}{M}RT \quad \left(\because n = \frac{m}{M}\right)$$

$$P = \frac{m}{V}\frac{RT}{M}$$

$$P = \frac{\rho RT}{M}$$

$$\frac{P}{\rho} = \frac{RT}{M} = \text{ Constant}$$

When we plug in the value we found for $\frac{P}{\rho}$ into the speed of sound formula, we get this:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v \propto \sqrt{T}$$

Here, T is in kelvin.

To find the speed of sound at a particular temperature t (°C), we get

$$\frac{v_t}{v_0} = \sqrt{\frac{273+t}{273}}$$

$$\frac{v_t}{v_0} = \left(1+\frac{t}{273}\right)^{\frac{1}{2}} \text{.....}(i)$$

$$\frac{v_t}{v_0} = 1+\frac{1}{2}\times\frac{t}{273} \quad \left(\because (1+x)^n = 1+nx, \text{ when } x << 1\right)$$

$$v_t = v_0\left(1+\frac{t}{546}\right)$$

Where,

v_0 = Speed of sound at 0 °C

v_t = Speed of sound at t °C

Parameter	**Temperature**	**Pressure/Density**	**Humidity**
Speed of sound (in terms of)	$v \propto \sqrt{T}$	$v = \sqrt{\frac{\gamma P}{\rho}}$	$\rho \propto \frac{1+H}{1+1.6H}$
Effect	The speed of sound increases with an increase in temperature.	There is no effect on the speed of sound.	The speed of sound increases as the humidity increases.

Example.

The constant value of γ applies universally, both to oxygen and hydrogen, standing at 1.40. Given that the speed of sound in oxygen registers at 470 ms^{-1} under identical temperature and pressure conditions, what shall be the corresponding speed of sound in hydrogen?

Solution.

Given, $(\gamma)_{H_2} = (\gamma)_{O_2} = 1.4$ and $(v)_{O_2} = 470\ ms^{-1}$

From the temperature dependence of the speed of sound, we can write,

$$(v)_{O_2} = \sqrt{\frac{\gamma RT}{M_{O_2}}} \text{ and } (v)_{H_2} = \sqrt{\frac{\gamma RT}{M_{H_2}}}$$

Hence,

$$\frac{(v)_{O_2}}{(v)_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$\frac{(v)_{O_2}}{(v)_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

$$(v)_{H_2} = 4 \times (v)_{O_2}$$

$$(v)_{H_2} = 4 \times 470$$

$$(v)_{H_2} = 1880\ ms^{-1}$$

Intensity of Sound Wave

Sound waves carry and pass on energy and push things when they move through stuff.

The expressions delineating the pressure and displacement characteristics of a sound wave are articulated thus:

$$s = s_0 \sin(\omega t - kx)$$

$$\Delta P = \Delta P_0 \cos(\omega t - kx)$$

Intensity is a measure of how much power spreads out over a certain space. In mathematical terms, it is expressed as follows:

$$I = 2\pi^2 s_0^2 f^2 \rho v$$

By substituting $s_0 = \frac{\Delta P_0}{\beta k}$ and $f = \frac{\omega}{2\pi}$, we get the following:

$$I = 2\pi^2 \left(\frac{\Delta P_0}{\beta k}\right)^2 \times \frac{\omega^2}{4\pi^2} \times \rho v$$

$$I = \frac{1}{2}\frac{\left(\Delta P_0\right)^2}{\beta^2} \times \frac{\omega^2}{k^2} \times \rho v$$

$$I = \frac{1}{2}\frac{\left(\Delta P_0\right)^2}{\beta} \times \frac{\rho}{\beta} \times v^3 \quad \left(\because \frac{\omega}{k} = v\right)$$

$$I = \frac{1}{2}\frac{\left(\Delta P_0\right)^2}{\beta} \times \frac{v^3}{v^2} \quad \left(\because v = \sqrt{\frac{\beta}{\rho}}\right)$$

$$I = \frac{\left(\Delta P_0\right)^2 v}{2\beta}$$