WAVES

THE PRINCIPLE OF SUPERPOSITION OF WAVES

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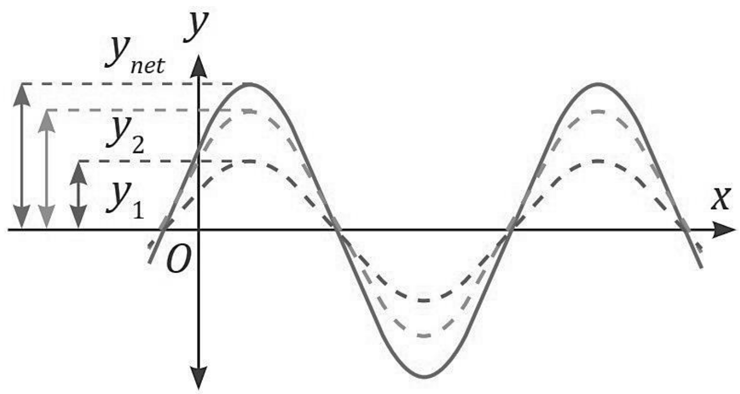
When two or more waves meet at a spot, the way something moves at that spot is like adding up the movements of each wave. We call this the "superposition of waves" principle.

Let's look at two graphs of waves, one called y1 and the other called y2, and see how they overlap.



In this case, you can see that the highest point of one wave is matching up with the highest point of the other wave. And the lowest point of one wave is matching up with the lowest point of the other wave.

The total movement of the combined wave that forms when these two waves overlap is shown like this:

$$\vec{y}_{net} = \vec{y}_1 + \vec{y}_2$$

For superposition of n waves,

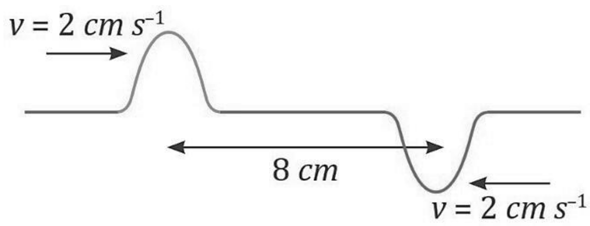
$$\vec{y}_{net} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots\ldots + \vec{y}_n$$

Question.

Imagine two identical bumps on a string. At first, they are 8 centimeters away from each other and moving towards each other, as shown in the picture. Each bump travels at a speed of 2 centimeters per second. After 2 seconds, what will be the total amount of energy in these bumps?

(A) Zero (B) Purely kinetic

(C) Purely potential (D) Partial kinetic and partial potential



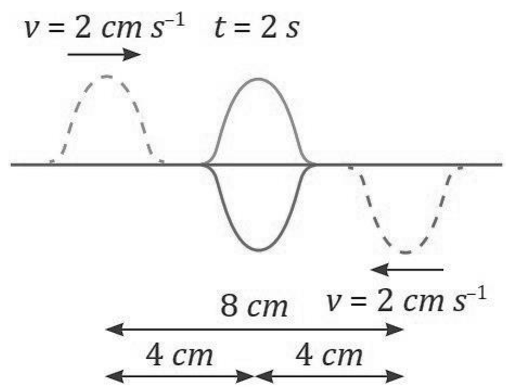
Solution.

The distance covered by the initial wave within a 2 s period is expressed as follows:

$$x_1 = v \times t$$
$$x_1 = 2 \times 2 = 4\ cm$$

The distance traversed by the second wave during a 2 s interval is presented as follows:

$$x_2 = V \times t$$
$$x_2 = 2 \times 2 = 4\ cm$$



After 2 s, both the waves will be at the same position.

When these waves come together, their combined movement will cancel out, making the total movement zero. Because there's no stretching happening, there's no stored energy at this moment.

Thus, PE = 0

However, the velocity of the wave pulses is not zero at that point. Therefore, the KE is greater than zero.

i.e., KE ≠ 0

Hence,
Total energy = Kinetic energy + Potential energy

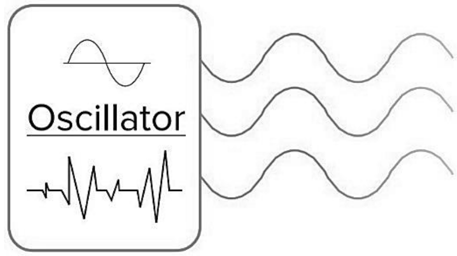
$$TE = KE + PE$$
$$TE = KE$$

At this point, the total energy is purely kinetic.
Thus, option (B) is the correct answer.

Interference of Waves

Coherent source

Sources that create waves with the same pitch, length, and a steady timing difference are called coherent sources.



Incoherent source

Sources that make waves with different pitches, lengths, and varying timing are called incoherent sources.

- **Conditions for interference**

  1. Two sinusoidal waves should be coherent.
  2. They should move along the same direction

Let us assume that two sources, S_1 andS_2, emit waves, and the waves pass through point P. We can observe that the wave from source S_2 has to travel some extra distance as compared to the wave from sourceS_1. This difference in path travelled by the waves causes phase difference.

Let the equation of wave from first source S_1 be the following:

$$y_1 = A_1 \sin(\omega t - kx)$$

The equation of wave from source S_2 is as follows:

$$y_2 = A_2 \sin(\omega t - kx + \phi)$$

From the principle of superposition, we get,

$$\vec{y}_{net} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots\ldots + \vec{y}_n$$

In this case,

$$y_{\text{net}} = y = y_1 + y_2$$
$$y = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx + \phi)$$
$$y = A_1 \sin(\omega t - kx) + A_2[\sin(\omega t - kx)\cos\phi + \cos(\omega t - kx)\sin\phi]$$
$$y = \left(A_1 + A_2 \cos\phi\right)\sin(\omega t - kx) + A_2 \cos(\omega t - kx)\sin\phi \quad \ldots.(i)$$
$$\text{Let}\left(A_1 + A_2 \cos\phi\right) = A\cos\alpha \quad \ldots.(ii)$$
$$\text{And, } A_2 \sin\phi = A\sin\alpha \quad \ldots(iii)$$

By substituting these values in equation (i), we get the following:

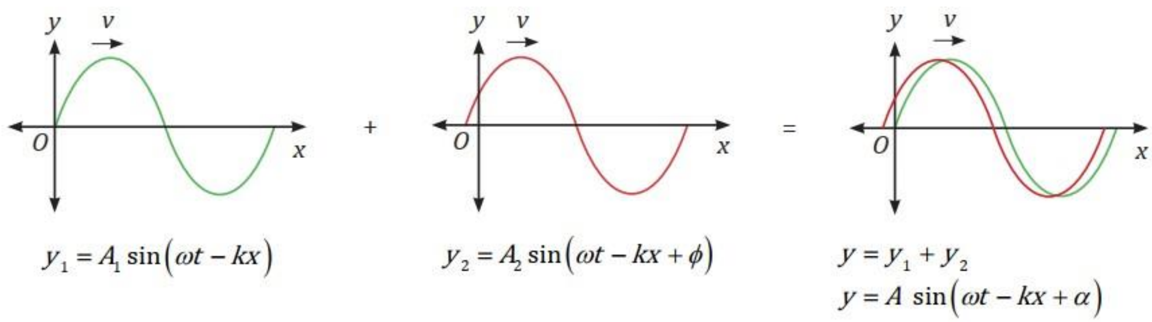
$$y = A\cos\alpha\sin(\omega t - kx) + A\sin\alpha\cos(\omega t - kx)$$
$$y = A[\cos\alpha\sin(\omega t - kx) + \sin\alpha\cos(\omega t - kx)]$$
$$y = A\sin(\omega t - kx + \alpha)(\because \sin(A+B) = \sin A\cos B + \cos A\sin B)$$

By squaring and adding equations (ii) and (iii), we get,

$$A^2\cos^2\alpha + A^2\sin^2\alpha = \left(A_1 + A_2\cos\phi\right)^2 + \left(A_2\sin\phi\right)^2$$
$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

By dividing equation (iii) by equation (ii), we get the following:

$$\tan(\alpha) = \frac{A\sin\alpha}{A\cos\alpha} = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$



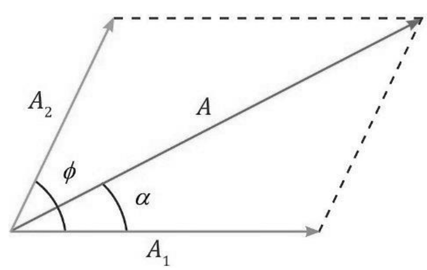
Geometrical Interpretation of Interference of Waves

Consider the following two waves:

$$y_1 = A_1\sin(\omega t - kx)$$
$$y_2 = A_2\sin(\omega t - kx + \phi)$$

Let's think of the size of the waves A1 and A2 like arrows with lengths A1 and A2, and they are at an angle Ø between them. The size of the combined wave is shown as:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$



In this way, we can think of A as what you get when you combine two arrows, A_1 andA_2, with an angle Ø between them. The size of the combined wave depends on how the two waves' timings match up or don't match up.

Question.

Two waves have equations, $x_1 = a\sin(\omega t - kx + \phi_1)$ and $x_2 = a\sin(\omega t - kx + \phi_2)$ If in the resultant wave, the frequency and amplitude remains equal to the amplitude of superimposing waves, then what is the phase difference between them ?

(A) $\frac{\pi}{8}$ (B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{10}$

Solution.

Given,

$x_1 = a\sin(\omega t - kx + \phi_1)$

$x_2 = a\sin(\omega t - kx + \phi_2)$

Resultant amplitude,

$A = a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\Delta\phi)}$

Also, $a_1 = a_2 = a$

$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\Delta\phi)}$

$a = \sqrt{a^2 + a^2 + 2a^2\cos(\Delta\phi)}$

$a = \sqrt{2a^2(1+\cos(\Delta\phi))}$

$\sqrt{1+\cos(\Delta\phi)} = \frac{1}{\sqrt{2}}$

$1+\cos(\Delta\phi) = \frac{1}{2}$

$\cos(\Delta\phi) = -\frac{1}{2}$

The phase difference is given as follows:

$\Delta\phi = \frac{2\pi}{3}$

Thus, option (B) is the correct answer.

Constructive Interference

We can calculate the total size of the wave when two consistent waves with sizes A_1 and A_2 come together like this:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

If $\cos\phi = 1$,

$$\phi = 2n\pi$$
$$A = A_{max} = A_1 + A_2$$

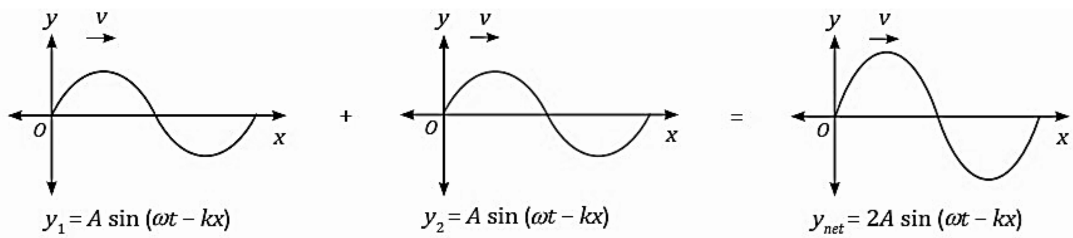
When waves combine to make the biggest possible size, we call it constructive interference.

Path difference

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times (\text{ Path difference})$$

$$2n\pi = \frac{2\pi}{\lambda} \times (\Delta x)$$

$$\text{Path difference} = \Delta x = n\lambda$$



Destructive Interference

We can figure out the combined size of two waves that match up well, with sizes A_1 andA_2, like this:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Amplitude is minimum for, $\cos\phi = -1$,

$$\phi = (2n+1)\pi$$
$$A = A_{min} = A_1 - A_2$$

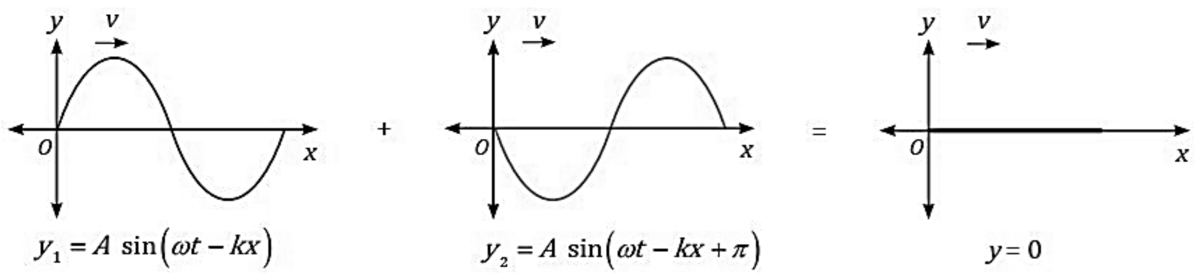
When waves combine in a way that makes the smallest possible size, it's called "destructive interference."

Path difference

Phase difference $= \frac{2\pi}{\lambda} \times (\text{Path difference})$

$$2n+1)\pi = \frac{2\pi}{\lambda} \times (\Delta x)$$

Path difference $= \Delta x = (2n+1)\frac{\lambda}{2}$



Intensity of Resultant Wave

We know that the intensity of any sinusoidal wave is proportional to the square of its amplitude, i.e,

$$I \propto A^2$$
$$I = cA^2$$

We know, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

So, the total strength or power changes like this:

$$I_{\text{net}} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$$

If $I_1 = I_2 = I,$

$$I_{\text{net}} = 2I(1+\cos\phi)$$

$$I_{\text{net}} = 4I\cos^2\frac{\phi}{2}$$

For constructive interference, we have,

$$\cos\phi = 1$$

$$I_{\text{net}} = I_{\text{max.}} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}$$

$$I_{\text{net}} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

If $I_1 = I_2 = I$

$$I_{\text{max}} = 4I$$

For destructive interference, we have,

$$\cos\phi = -1$$
$$I_{net} = I_{\min} = I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2}$$
$$I_{net} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$
If $I_1 = I_2 = I$,
$$I_{\min} = 0$$