WAVES

REFLECTION OF WAVES

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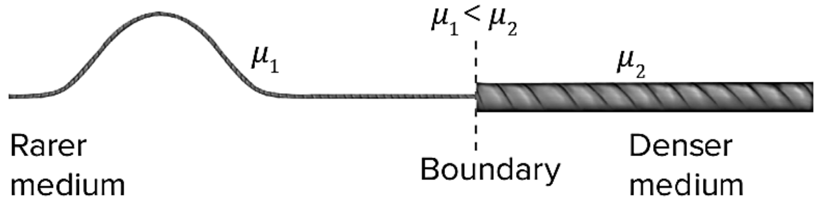
Wave Propagation between Two Strings

Imagine you have a thin string with a mass density μ_1 connected to a thick string with a mass density μ_2 . Because $\mu_1 < \mu_2$, we can think of the thin string as the less dense one, and the thick string as the denser one.



When a wave moves from a light to a heavy string, some of it goes into the heavy string, and some bounces back into the light string at the boundary.

While the wave switches from one string to the other, the point where it changes stays still. This means that the tension in both strings is equal.

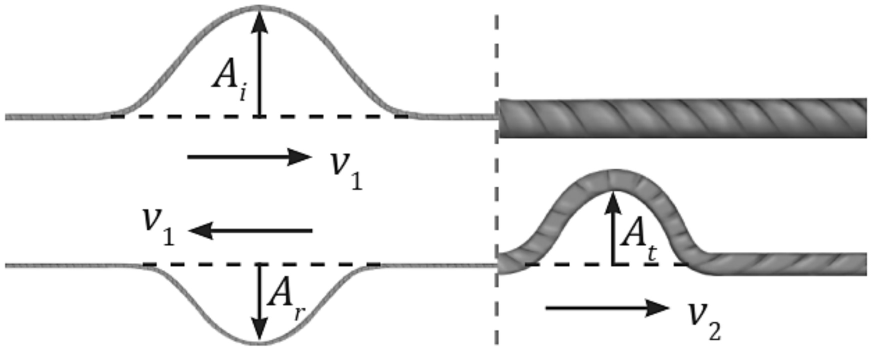
Further, the velocity of the wave in a medium is defined as $v = \sqrt{\frac{T}{\mu}}$.

So, the speeds of the incoming and bouncing waves in the lighter material are faster than the speed of the wave that goes into the denser material. We think of the denser material's ends as the ones that don't move, and the lighter material's ends as the free ones.

Keep in mind that how dense the materials are depends on the specific type of wave. When it comes to light waves, they slow down when they go from air to water. But when sound waves go from air to water, they actually speed up.

Amplitudes of Reflected and Transmitted Waves

Imagine a wave traveling from a light string (less dense material) to a heavy string (more dense material) with a certain size called Ai. Suppose the speed of the incoming and transmitted waves is v_1 andv_2, respectively. When the wave moves from the light string to the heavy string, we find that $v_1 > v_2$.



Now, let's say the size of the wave that bounces back is Ar, and the size of the wave that goes through is At. When the wave reaches the boundary, it divides into two parts – one part bounces back, and the other part goes forward.

Based on the principle of conserving energy, we can say that the total power of the incoming wave gets shared between the part that bounces back and the part that goes through.

$$< P_i > = < P_r > + < P_t > \quad \ldots\ldots(i)$$

The average power transmitted by a wave in a medium is given by

$$P = 2\pi^2 A^2 f^2 \mu v$$

$$P = \frac{\omega^2 A^2 \mu v}{2}$$

We know that the velocity of the wave on a string is, $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \mu = \frac{T}{v^2}$$

By putting the obtained value of μ in the expression of P, we get,

$$P = \frac{\omega^2 A^2 \times \frac{T}{v^2} \times v}{2}$$

$$P = \frac{T\omega^2 A^2}{2v}$$

By substituting the value of from the obtained equation in equation, we get,

$$\frac{T\omega^2 A_i^2}{2v_1} = \frac{T\omega^2 A_r^2}{2v_1} + \frac{T\omega^2 A_t^2}{2v_2}$$

$$\frac{A_i^2}{v_1} = \frac{A_r^2}{v_1} + \frac{A_t^2}{v_2} \quad \ldots..(ii)$$

Imagine a rope with waves on it. When these waves reach the point where the rope changes from thick to thin, the size of the waves on the thinner side is called At. The combined size of the waves on the thicker side is found by adding the size of the incoming wave and the size of the reflected wave, which is written as $(A_i + A_r)$.

Since both expressions represent the amplitude of a unique point, we can write,

$$A_i + A_r = A_t \ldots\ldots \text{(iii)}$$

Note that the amplitudes of the incident, transmitted, and reflected waves (A_i,A_t andA_r) are taken with appropriate signs, i.e., positive for $y > 0$ and negative for $y < 0$.
Let us rewrite equation (ii),

$$\frac{A_i^2}{v_1} = \frac{A_r^2}{v_1} + \frac{A_t^2}{v_2}$$

$$\frac{A_i^2 - A_r^2}{v_1} = \frac{A_t^2}{v_2}$$

$$\frac{(A_i + A_r)(A_i - A_r)}{v_1} = \frac{A_t^2}{v_2}$$

On substituting the value of $A_i + A_r$ from equation (iii) in the obtained equation, we get,

$$\frac{(A_i - A_r)A_t}{v_1} = \frac{A_t^2}{v_2}$$

$$(A_i - A_r) = \frac{v_1}{v_2} A_t \quad \ldots..(\text{ iv})$$

Upon adding equations (iii) and (iv), we get,

$$A_i - A_r + A_i + A_r = A_t + \frac{v_1}{v_2} A_t$$

$$2A_i = \left(\frac{v_1}{v_2} + 1\right) A_t$$

$$A_t = \frac{2v_2}{v_1 + v_2} A_i \quad \ldots..(v)$$

By subtracting equation (iv) rom equation (iii), we get

$$A_i + A_r - (A_i - A_r) = A_t - \frac{v_1}{v_2} A_t$$

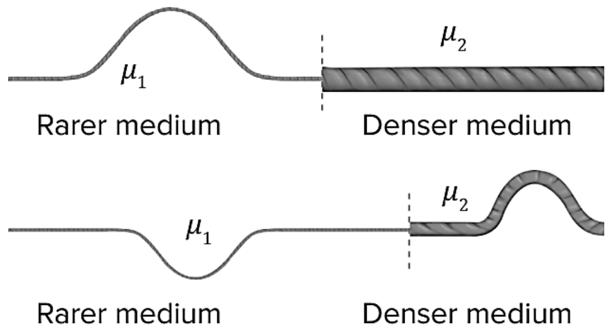
$$2A_r = \left(1 - \frac{v_1}{v_2}\right) A_t \quad \ldots..(vi)$$

By putting the value of A_t from equation (v) in equation (vi), we get

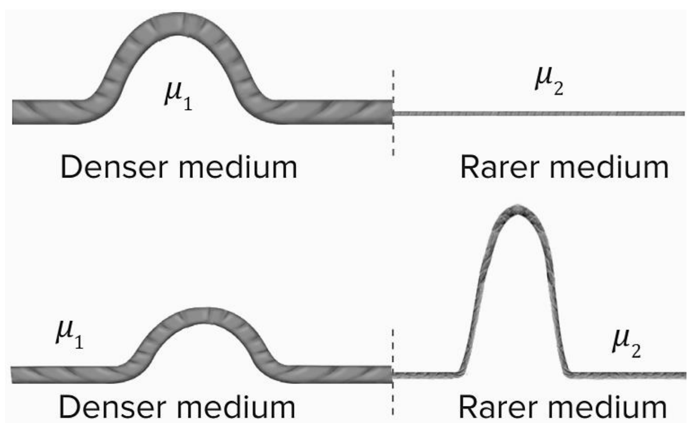
$$2A_r = \left(1 - \frac{v_1}{v_2}\right)\left(\frac{2v_2}{v_1 + v_2}\right)A_i$$

$$A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right)A_i \quad \ldots..(vii)$$

Because the wave in the light string moves faster (v_1) than in the heavy string (v_2), the size of the wave bouncing back from the boundary (A_r) becomes smaller and opposite in direction. So, when a wave is traveling on the light string and reaches the boundary with the heavy string, the reflected wave flips or changes by 180 degrees (phase change π).

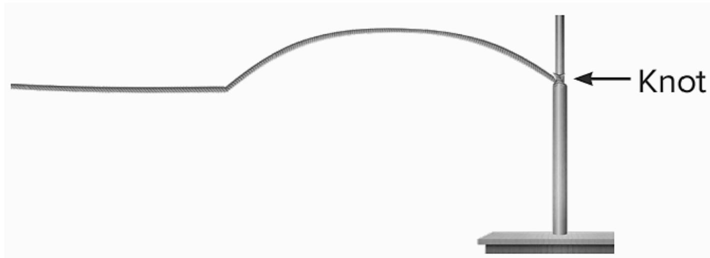


But, if the initial wave is in the thick string and it bounces back from where the thick and thin strings meet, the reflected wave stays the same (no flipping). In this case, the transmitted wave is always positive, so it doesn't flip either, no matter how thick or thin the string is.



Wave at an Interface
Reflection from a fixed end

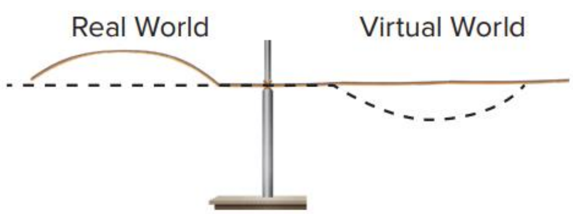
Imagine a string that's tied to something very heavy and can't move at one end. Now, picture sending a wave pulse (a sort of wave bump) along the string towards the end where it's tied tightly, which we call a 'knot.' The picture shows the exact moment when the wave pulse is getting close to the knot.

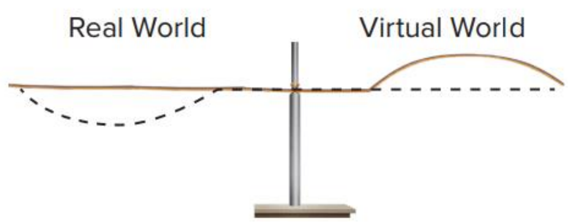


The amplitude of the wave during the reflection from the knot can be visualised as the superposition of the following waves:

(i) The incident wave of the string
(ii) A hypothetical wave of the same amplitude, travelling in the opposite direction with the same speed and inverted with respect to the incident wave

Two distinct waves originating from disparate realms are combined at the knot, where they overlap. The wave pulse stemming from the theoretical realm crosses into the physical world, while the wave from the actual world penetrates the hypothetical realm. Notably, these waves do not distort each other's shapes; following their superposition, the wave pulses retain their original forms.





The sizes of the waves that go through and bounce back are shown as,

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i = \left(\frac{2 \times 0}{v_1 + v_2}\right) A_i = 0 \quad [\because \mu_2 = \infty \text{ for rigid media}]$$

$$A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_i = \left(\frac{0 - v_1}{v_1 + 0}\right) A_i = -A_i$$

Because the fixed end causes the entire incoming wave to turn into the reflected wave, the size of the wave that goes through becomes zero. The size of the reflected wave is just like the initial wave, but it flips upside down.

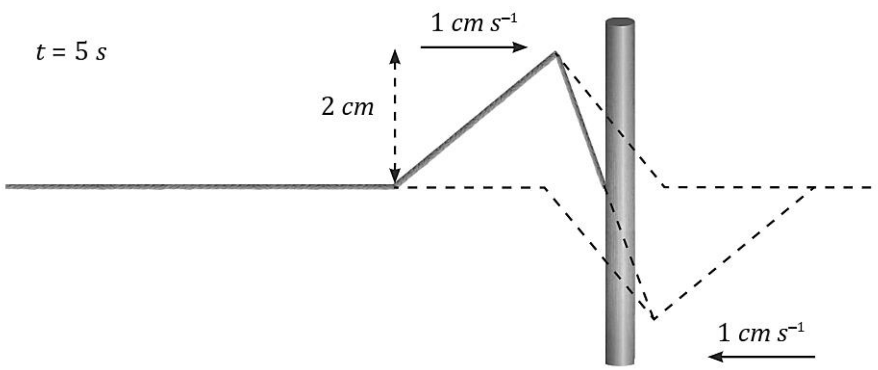
Question.

Determine the configuration of the triangular wave on a string, characterized by a height of 2 cm and a base of 4 cm, moving at a velocity of $1 cm\ s^{-1}$. This wave reflects off an unyielding surface, as illustrated, and we seek to understand its shape at various intervals.

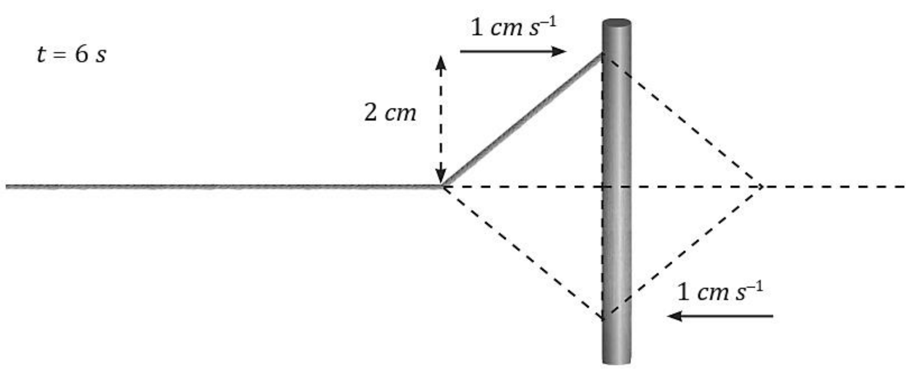
(a) $t = 5s$ (b) $t = 6s$

Solution.

(a) At $= 5\ s$, the top part of both waves (the real one and the imaginary one) is 1 cm away from the wall. We can figure out their combined size by using the rule that helps us add up waves, like in the picture.

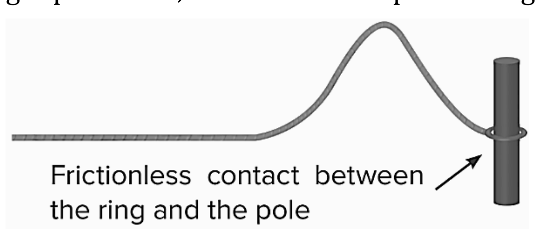


(b) Similarly, at t = 6 s, both the pulses overlap. The resultant shape of the wave is shown in the figure.

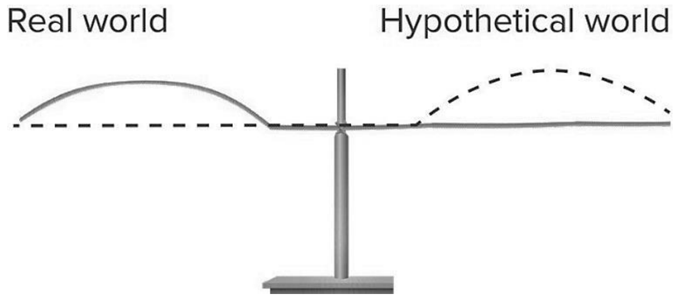


Reflection from a free end

Imagine a small, smooth ring attached to one end of the string. The string is not moving, and it's firmly attached to a tall, straight pole. Now, we send a wave pulse along the string towards the ring, as shown in the picture.

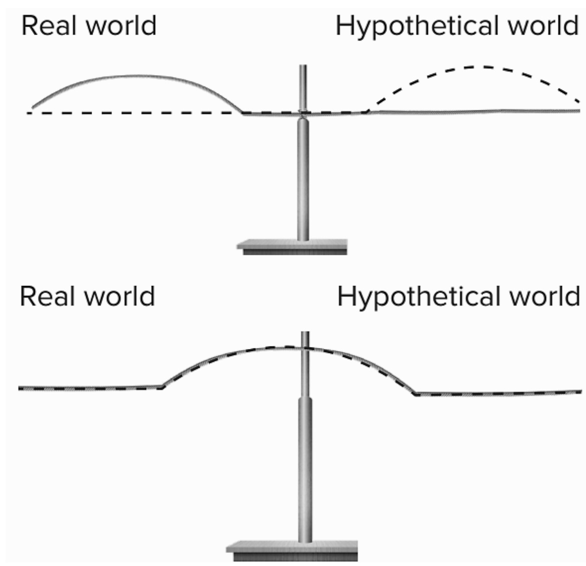


In this situation, the very last piece of the string doesn't experience a balanced force because the pole can't push with enough strength due to the ring being smooth and not causing friction. As a result, there's an overall upward force on that last part of the string.



Here, the very end of the string isn't in balance because the pole can't push hard enough. This happens because there's a smooth ring on the string. So, the last part of the string feels an upward force.

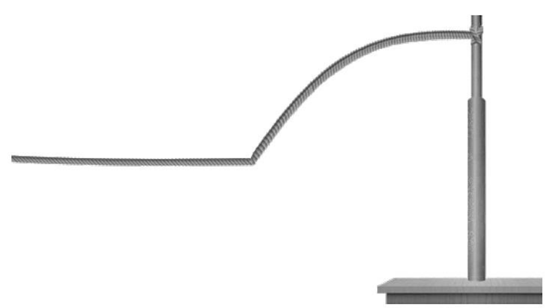
Imagine two wave bumps from different worlds, one being real and the other imaginary. They have the same timing and are both heading toward the pole, where they combine. Importantly, they don't change each other's shapes. This entire situation is shown in the picture.



The amplitudes of the transmitted and reflected waves are given by,

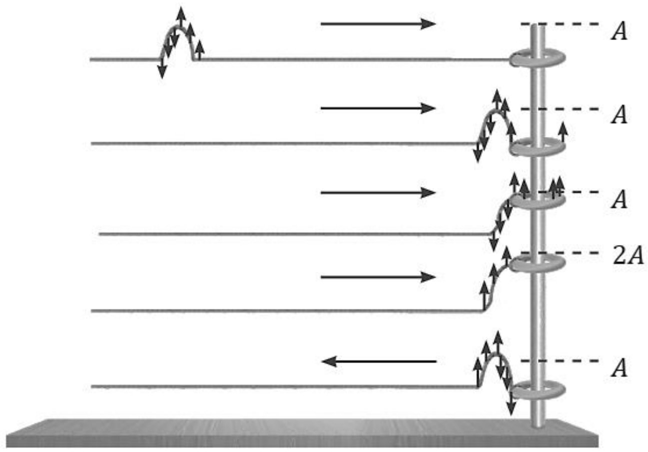
$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i = \left(\frac{2v_2}{v_2}\right) A_i = 2A_i$$

$$\left[\because v_2 \gg v_1 \text{ since } \mu_2 = 0 \text{ for a free end}\right]$$



$$A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_i = \left(\frac{v_2}{v_2}\right) A_i = A_i$$

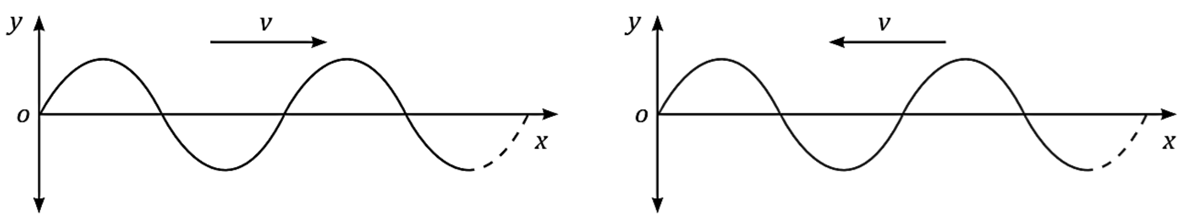
The picture shows how fast the parts of the wave are moving and where the ring is at various points in the process.



Therefore, the form of the string-wave pulse does not get inverted after reflection from the free end.

Introduction to Standing Waves

Think about two exactly the same waves going in opposite directions. These waves have the same number of ups and downs, the same distance between each up and down, and the same size. Because they mix together, they create waves that look like they're not moving at all, called standing waves or stationary waves.



Suppose the equations of the waves in consideration are as follows:

$$y_1 = A\sin(kx - \omega t)$$
$$y_2 = A\sin(kx + \omega t)$$

Using the idea that you can add up waves, we can figure out how much a particle at a certain spot and time is moved, and this is shown as,

$$y = y_1 + y_2$$
$$y = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$
$$y = A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

Since we know that, $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$$y = A\left[2\sin\left(\frac{(kx-\omega t)+(kx+\omega t)}{2}\right)\cos\left(\frac{(kx-\omega t)-(kx+\omega t)}{2}\right)\right]$$

$$y = 2A\sin kx\cos(-\omega t)$$

We know that cosine is an even function $\cos(-\omega t) = \cos\omega t$

$$y = 2A\sin kx\cos\omega t \quad \ldots\text{..(iii)}$$

The final equation for the wave can't be written like f (ωt - kx). The equation we got shows a standing wave instead.

If we denote 2A sin kx = A(x), then the equation takes the following form: y = A(x) cos ωt

This equation represents the collective equation of particles executing SHM with the same angular frequency and of different amplitudes depending on their locations.

Antinodes

If the places (x) of the particles make sin kx equal to either +1 or -1, then those particles will have the biggest up and down movement compared to the others, which is 2A.

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots = (2n+1)\frac{\pi}{2}$$

Where n = 0, 1, 2…

$$\frac{2\pi}{\lambda}x = (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\lambda}{4} = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

All such particles that execute SHM with the maximum amplitude are known as the antinodes.

Nodes

If the places (x) of the particles make sin kx = 0, then those particles don't move up or down at all. Their position in the y-direction stays at zero, meaning y = 0.

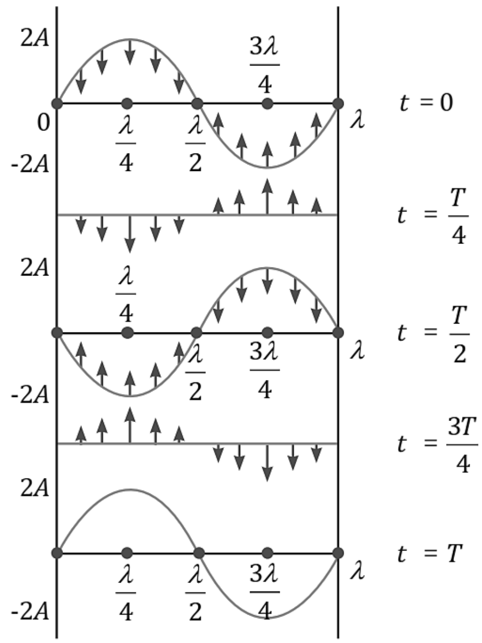
$$kx = 0, \pi, 2\pi, \ldots = n\pi$$

Where n = 0, 1, 2…

$$\frac{2\pi}{\lambda}x = n\pi$$

$$x = \frac{n\lambda}{2} = 0, \frac{\lambda}{2}, \lambda, \ldots$$

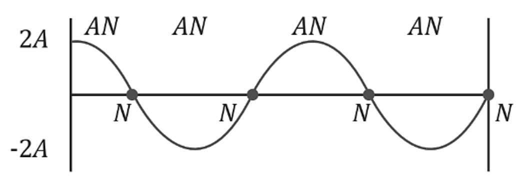
We call all the points that hardly move at all, with their amplitude being zero, nodes.



From simple back-and-forth motion, we know that it takes a particle $\frac{T}{4}$ time to go a quarter of the way in its cycle, where T is the time it takes for one complete cycle. Now, let's change the equation for the standing wave we found earlier.

$$y = 2A\sin kx\cos\omega t$$

The plot of displacement (y) vs location (x) at different instances of t is as shown in the above figure.



N and AN represent the nodes and antinodes in the figure, respectively.

As the particles located at nodes maintain a constant zero displacement, they become ineffective in the transfer of energy. Consequently, the energy becomes restricted within the particles positioned between two successive nodes, often referred to as a 'wave packet.' Within these packets, the overall mechanical energy fluctuates between the highest kinetic and potential energy levels, as each particle undergoes SHM.

So, based on what we've learned, we can describe what a standing or stationary wave is like this:

When two waves with the same strength and shaking speed move towards each other in something like water and come together, they make a wave that doesn't seem to move, and we call it a standing wave.
Example: The strings in a guitar produce standing waves.

The table below shows how traveling waves and standing waves are different.

Travelling waves	Standing waves
All particles of the medium oscillate with the same frequency and amplitude.	All particles except nodes oscillate with the same frequency but different amplitudes.
The phase difference between two particles can be any value between zero and 2π.	The phase difference between any two particles is either zero or π.
These waves transmit energy from one place of the medium to the other.	These waves do not transmit energy, provided both the superposing waves carry the same amplitude
There is no instant when all the particles are at the mean positions together	All the particles cross their mean positions together.

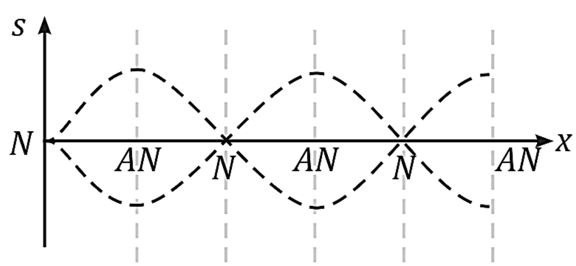
Standing Sound Waves

In our previous lessons, we talked about waves that don't seem to move on a string. Now, we're going to talk about similar waves, but in sound. These waves are like the ones we studied on the string. We describe how the particles move with the following formula:

$$s = 2s_0 \sin(kx + \phi_1)\sin(\omega t + \phi_2)$$

If we think of $\phi 1$ and $\phi 2$ as being equal to zero, then the movement looks like this:

$$s = 2s_0 \sin kx \sin \omega t$$

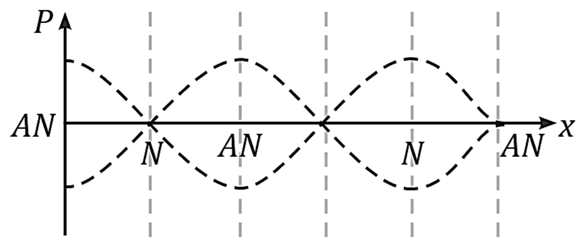


We can describe the changes in pressure in standing sound waves like this:

$$\Delta P = 2\Delta P_0 \sin\left(kx + \phi_1 + \frac{\pi}{2}\right)\sin\left(kx + \phi_2 + \frac{\pi}{2}\right)$$

Or

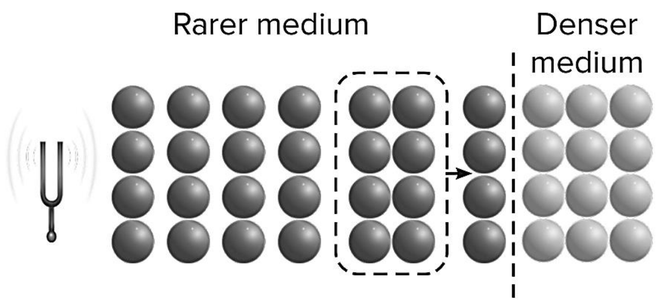
$$\Delta P = 2\Delta P_0 \cos(kx + \phi_1)\cos(kx + \phi_2)$$



Reflection of Sound Waves

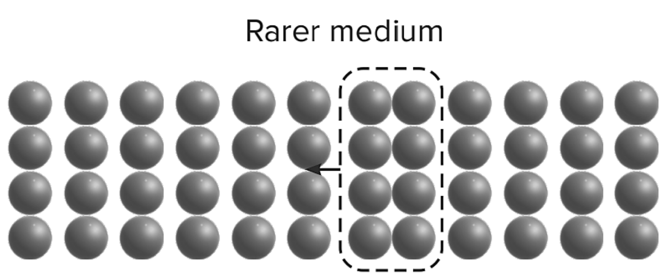
Reflection from a denser medium

Think about a compressed wave moving through a less dense material, bouncing back from a denser one, as you can see in the figure.



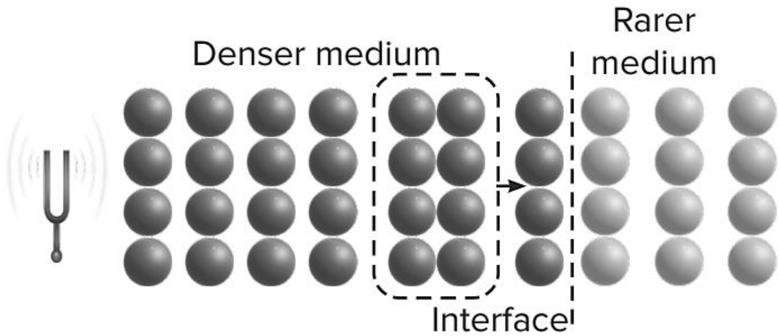
Right before the bounce, the particles in the thinner material next to the border get squeezed together and collide with the particles in the thicker material. After these collisions, the particles in the thinner material bounce back the same way they were right before the bounce. In simpler words, when they hit a fixed end (where they don't move), a squeezed part goes back as a squeezed part, and a stretched-out part goes back as a stretched-out part. So, the thicker material (the fixed end) stays still like a point that doesn't move.

Because the wave pulse retains its characteristics after reflecting from the denser medium, there is no alteration in the phase of the sound wave pulse following its reflection from the denser medium.

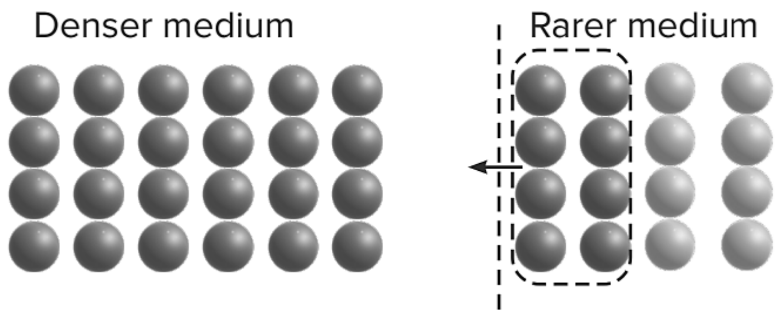


Reflection from a rarer medium

Let's think about what happens when a compressed wave moves from the denser stuff to the thinner stuff and hits the border. Here's a Figure to help you see it.



At this point, the place where they meet acts like a loose end (similar to how it works with string waves). Right after the collisions, the particles from the thicker material move into the thinner material and bounce back into the thicker one, but they bounce back as a stretch-out part. So, the thinner material (the loose end) acts like a point that moves a lot, as you can see in the picture.



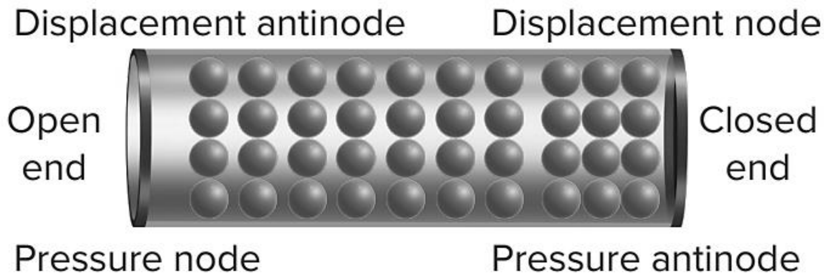
In simple terms, when a sound wave bounces off the thinner material, the squeezed part turns into a stretched-out part. So, we can say that the sound wave changes by half a circle (π) after bouncing off the thinner material.

Modes of Vibration in a Closed Organ Pipe

In string waves, if you fix the string at both ends, you get a standing wave between two points that don't move, called nodes. If you fix it at just one end, you get a standing wave between a point that doesn't move (node) and a point that moves a lot (antinode).

Similarly, in a closed organ pipe where one end is closed and the other is open, you get a standing wave between a point that doesn't move (node) and a point that moves a lot (antinode). In an open organ pipe where both ends are open, you get a standing wave between two points that don't move (pressure nodes).

In a closed organ pipe, which has a certain length l, the open end will have a point where the air pressure doesn't change (like a pressure node), and the closed end will have a point where the air doesn't move (like a displacement node). This is illustrated in the figure.



Fundamental mode

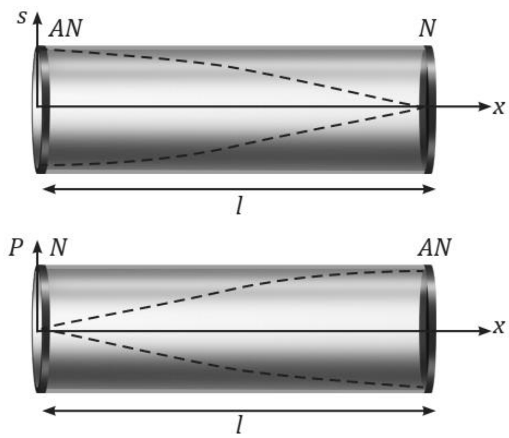
The fundamental mode is the simplest way to create standing sound waves at the lowest possible frequency. In the fundamental mode, something specific happens.

$$l = \frac{\lambda}{4}$$

Or

$$\lambda = 4l$$

$$\Rightarrow f = \frac{v}{4l} \quad (\because v = f\lambda)$$



There will be one and a half loop in between the ends as shown in the figure.

The frequency for the first overtone or third harmonic is given by,

$$\Rightarrow f_1 = \frac{3v}{4l} = 3f_0$$

Second overtone or fifth harmonic:

For the fifth harmonic,

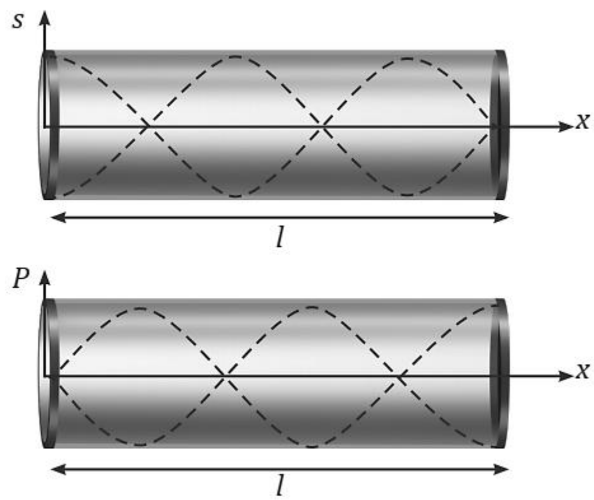
$$l = \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{5\lambda}{4}$$

Or

$$\lambda = \frac{4l}{5}$$

$$f_2 = \frac{5v}{4l}$$

There will be two and a half loop in between the ends as shown in the figure.



The frequency for the second overtone or fifth harmonic is given by,

$$\Rightarrow f_2 = \frac{5v}{4l} = 5f_0$$

n^{th} Overtone or $(2n + 1)^{th}$ harmonic:

For the $(2n + 1)^{th}$ harmonic, (n = 0, 1, 2, 3…)

$$l = \frac{n\lambda}{2} + \frac{\lambda}{4} = (2n+1)\frac{\lambda}{4}$$

Or

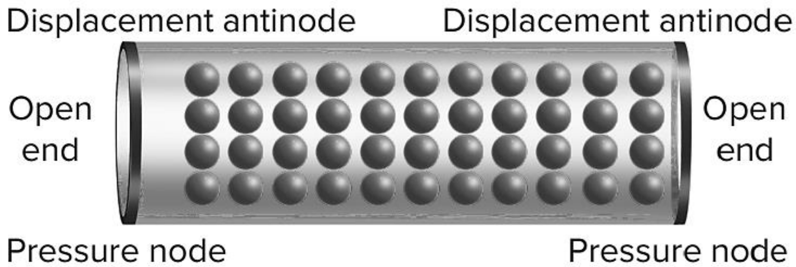
$$\lambda = \frac{4l}{2n+1}$$

There will be $n + \frac{1}{2}$ loops in between the ends of the organ pipe. The corresponding frequency is given by,

$$f_n = (2n+1)\frac{v}{4l}$$

Modes of Vibration of an Open Organ Pipe

In a long, open pipe, there are points where the air pressure doesn't change, and these points are at both ends of the pipe. You can see this in the figure.



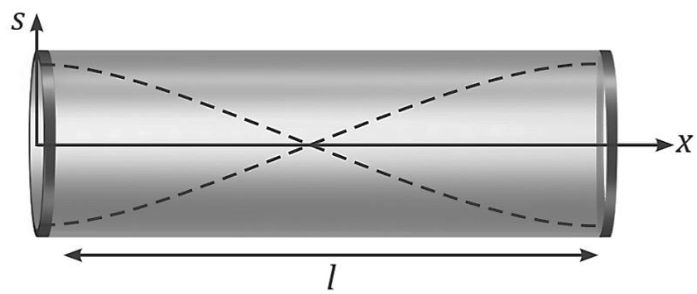
Fundamental mode or first harmonic:

When you have an open pipe, the lowest, most basic sound it can make happens when the length of the pipe is half the length of the sound wave.

$$l = \frac{\lambda}{2}$$

Or

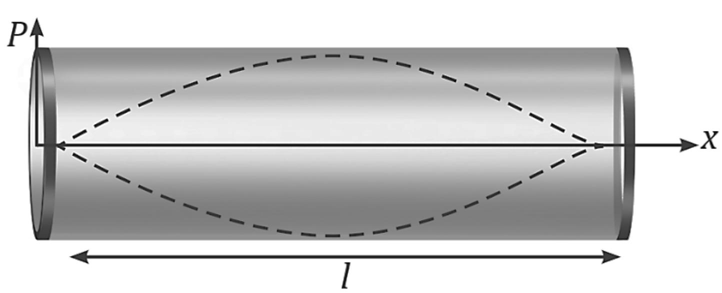
$$\lambda = 2l$$



From the displacement point of view, there will be two half loops between the open ends as shown in the figure.

The sound frequency of the basic mode for an open organ pipe is determined by this formula:

$$f_0 = \frac{v}{2l}$$



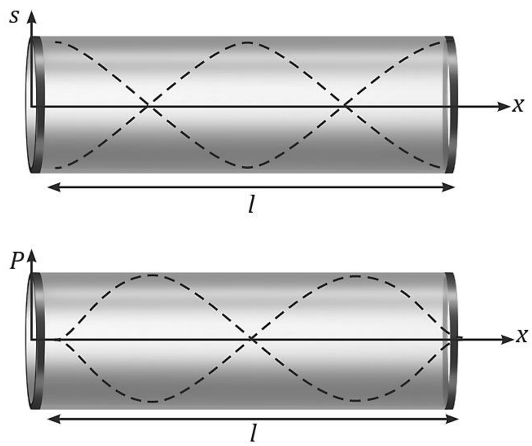
First overtone or second harmonic:

For the second harmonic,

$$l = \lambda$$

$$f_1 = \frac{2v}{2l} = \frac{v}{l}$$

Because there's one full curve and two half-curves between the fixed ends, there are two points where the air doesn't move much between the open ends, as you can see in the figure.



Similarly, for n^{th} overtone or $(n-1)^{th}$ harmonic,

$$\lambda = \frac{2l}{n+1}$$

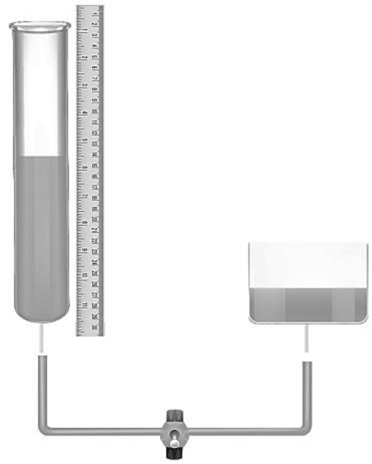
$$f_n = (n+1)\frac{v}{2l}$$

Resonance Column Method

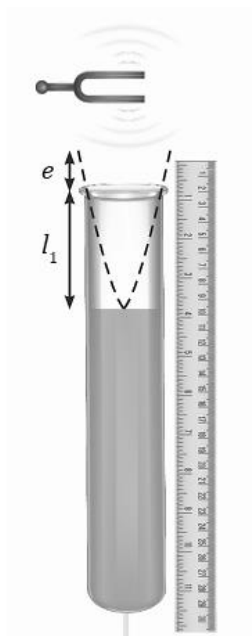
The resonance column method is a way to figure out how fast sound moves through the air. For this experiment, you need a long glass tube called a resonance tube and a container to hold water. Both the tube and the container are filled with water, and you can change the water level inside the tube using a cork that's connected to both of them. There's a scale on the glass tube to help measure the water level. The picture in the figure shows a simplified drawing of how this setup looks.

The air inside the long glass tube is trapped between one end that's open and another end that's closed because of the water level. This makes it act like a closed pipe for making sounds. To create stationary waves or standing waves in this air, the length of the air column needs to meet a certain condition that $L = (2n+1)\left(\frac{\lambda}{4}\right)$. The corresponding frequency of the tuning fork would be

$f_n = (2n+1)\left(\frac{v}{4L}\right) = (2n+1)\,f_0$, where f_0 the frequency of the fundamental mode.



You can change the water level in the tube to make the air inside it produce the first strong sound. This is called the fundamental sound mode, and you can see it in the figure.



A spot with no air pressure, which we call a pressure node, is created at a distance e above the open end of the tube. If the length of the air inside the tube for the first strong sound isl_1, then the condition for this to happen is:

$$l_1 + e = \frac{\lambda}{4} \ldots .(i)$$

Next, we adjust the water level to find the point where the air column creates a second strong sound. At this point, the length of the air column is labeled asl_2. This is the second time the air inside the column resonates strongly, and we call it the first overtone.

Thus, the condition for the second resonance is as follows:

$$l_2 + e = \frac{3\lambda}{4} \ldots.(ii)$$

By subtracting equation (i) from equation (ii), we get the following:

$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4}$$

$$l_2 - l_1 = \frac{\lambda}{2}$$

$$\lambda = 2(l_2 - l_1) \quad \ldots..(iii)$$

$$v_s = 2(l_2 - l_1) f \quad (\because v = f\lambda)$$

By substituting the value of λ from equation (iii) in equation (i), we get,

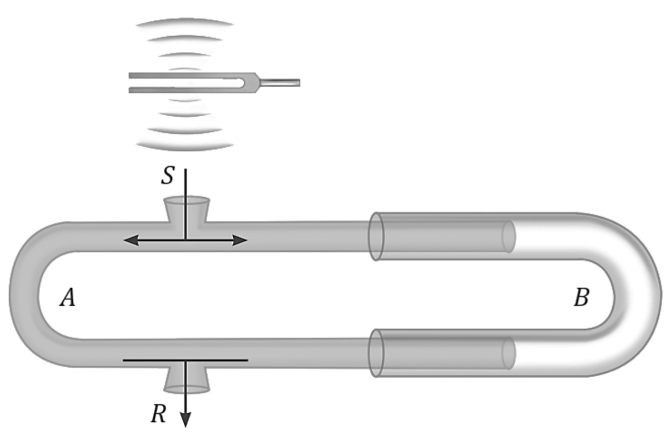
$$l_1 + e = \frac{l_2 - l_1}{2}$$

$$e = \frac{l_2 - 3l_1}{2} \approx 0.6r \quad \text{(Observed value)}$$

So, if we know the length of the air column for different strong sound points and the frequency of the tuning fork, we can figure out how fast sound travels through the air.

Quincke's Tube Method

Quincke's tube is made up of two U-shaped tubes, one called A and the other called B. Tube B can move up and down along tube A, as you can see in the picture.



Tube A has two openings: one at the top where the sound source S is placed to create sound waves, and another at the bottom where the receiver R (or detector) is located.

The sound waves coming from the source S divide in tube A and come together at the receiver. Because both of these beams come from the same source, they are in sync or coherent. This is why you see interference happening at the receiver.

When we move tube B to the right by a distance x from where it started, the extra distance that one of the sound waves travels is 2x. So, the difference in distance between the two sound waves becomes $\Delta x = 2x$, which causes a difference in their timing or phase. This difference in phase creates patterns of strong and weak sounds, like peaks and valleys. For instance, if you shift tube B by x to get consecutive peaks and valleys, then:

$$\Delta x = 2x = \frac{\lambda}{2} \quad \left[\text{Since distance between consecutive maxima and minima is } \frac{\lambda}{2} \right]$$

$$x = \frac{\lambda}{4}$$

By looking at how much x changes, we can figure out the length of a sound wave, which is its wavelength. With this information and knowing the frequency of the tuning fork, we can calculate how fast sound travels through the air using the formula $v = f\lambda$.

Example.

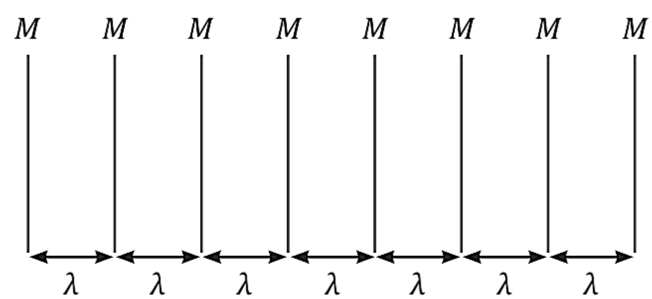
A Quincke's tube experiences a displacement of 10 centimeters. Both the initial and final positions of the tube coincide with points of maximum sound intensity. Within this span, six additional points of maximum sound intensity are encountered. Given a sound velocity of $330 ms^{-1}$, the objective is to determine the corresponding frequency.

Solution.

Given,
the shift in the movable tube, $x = 10\ cm$

Hence, the path difference between the waves, $\Delta x = 2 \times 10 = 20cm$

We know that there are six points of maximum sound intensity between the beginning and the end of the tube.



Therefore,

$$7\lambda = 20$$

$$\lambda = \frac{20}{7}\ cm = \frac{1}{35}\ \text{m}$$

Hence, the frequency of the sound is n given by

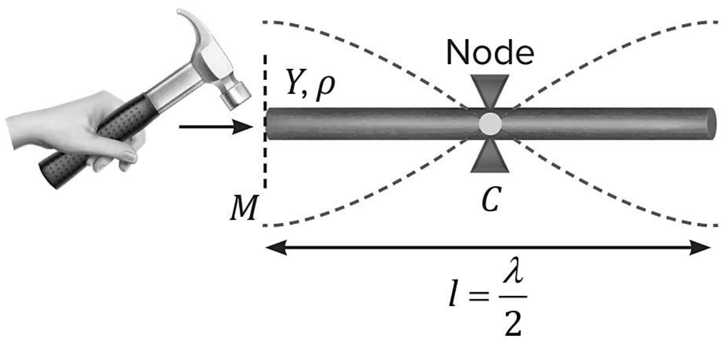
$$f = \frac{v}{\lambda}$$

$$f = \frac{330}{\left(\frac{1}{35}\right)}$$

$$f = 11550\ Hz$$

Kundt's Tube Method

Contemplate a solid rod with consistent density ρ and possessing Young's modulus Y, securely fastened or clamped at its midpoint, as illustrated in the diagram.



If you hit one end of this rod with a hammer, the middle of the rod won't move. It will stay still like a fixed point for the basic sound pattern, which we call the first harmonic or fundamental mode.

$$l = \frac{\lambda}{2}$$

Or

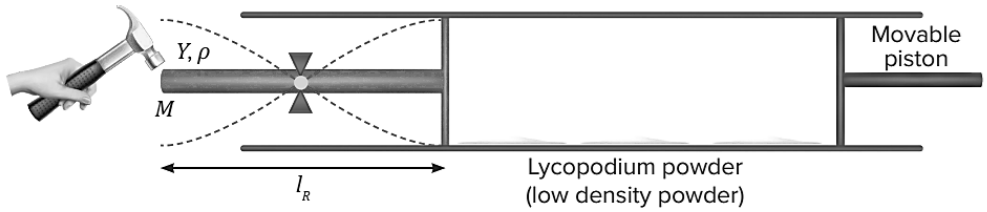
$$\lambda = 2l$$

Recall that the speed of sound in solid materials is given by $\sqrt{\frac{Y}{\rho}}$, where Y is Young's modulus of the material.
Hence, the fundamental frequency of the wave can be found as follows:

$$f = \frac{v}{\lambda}$$

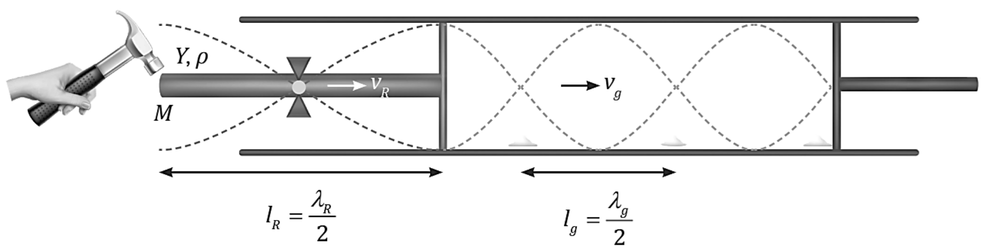
$$f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

Think about a tube filled with gas, and it's closed at both ends. One end is sealed with a movable, solid piston, and the other end is blocked by a plate where a solid rod of length l_r is attached. This rod is secured right in the middle. In the chamber, there's also some low-density lycopodium powder scattered around, as you can see in the figure.



If we don't move the piston, we can make the rod vibrate back and forth by hitting it with a hammer. Just like we saw before, the point where the rod is held still doesn't move, and this is shown in the figure.

Now, think about how the back-and-forth vibration of the rod is sent into the air inside the tube. This causes special kinds of waves, called standing waves, to appear in the air. These waves make the low-density powder gather in piles at the points where the air doesn't move, as you can see in the figure.



Imagine there are little piles of powder in a row, and the distance between any two neighboring piles is calledl_g. Also, the speed at which the standing waves travel in the air inside the tube and the rod is called v_g andv_R, respectively.

Consider the fundamental mode of longitudinal vibration in the rod.

Since the distance between two antinodes is $\frac{\lambda_R}{2}$,

$$l_R = \frac{\lambda_R}{2}$$

If is the frequency of vibration of the rod, then,

$$l_R = \frac{v_R}{2f}$$

$$f = \frac{v_R}{2l_R} \quad .....(i)$$

Because the frequency of the sound is determined by the source, when the sound travels from the solid into the air, the frequency stays the same, which is f.

The distance between two consecutive nodes inside the air column is given by,

$l_g = \frac{\lambda_g}{2}$ (Where is the wavelength of the standing wave in the tube)

$l_g = \frac{v_g}{2f}$

$f = \frac{v_g}{2l_g}$ .....(*ii*)

By equating the values of f obtained from equations (i) and (ii), we get,

$$\frac{v_R}{2l_R} = \frac{v_g}{2l_g}$$

$$\Rightarrow v_g = \frac{l_g}{l_R} v_R \quad \text{Or} \quad v_g = \frac{l_g}{l_R}\sqrt{\frac{Y}{\rho}}$$

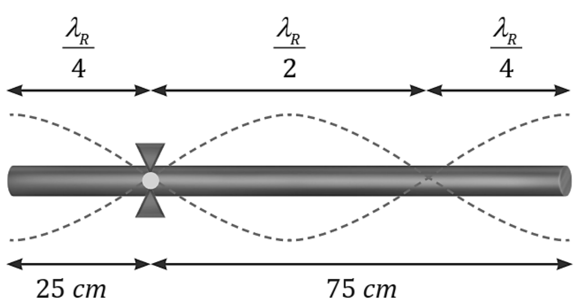
Example.

A Kundt's tube setup uses a copper rod that's 1m long, with one end clamped 25 cm from that end. Inside the tube, there's air, and sound travels in it at a speed of $340ms^{-1}$. The heaps of powder inside are 5 cm apart. We need to determine the speed of sound in copper.

Solution.

Given,
Velocity of sound in air, $v_g = 340ms^{-1}$
Length of the copper rod, $l_R = 1$ m
Distance between the consecutive heaps (node), $l_g = 5\ cm = 0.05\ m$

The copper rod is securely held at a point, called a "displacement node," which is 25 cm away from the end that can move freely.



We know that the distance between a consecutive node and an antinode is $\frac{\lambda_R}{4}$. Thus, we can write it as:

$$\frac{\lambda_R}{4} = 25$$

$$\lambda_R = 100\ cm = 1\ \mathrm{m}$$

Because the frequency of the standing waves doesn't change in both materials,

$$f_R = f_g$$

$$\frac{v_R}{\lambda_R} = \frac{v_g}{\lambda_g}$$

$$v_R = \lambda_R \times \frac{v_g}{2 \times l_g}$$

$$v_R = 1 \times \frac{340}{2 \times 0.05}$$

$$v_R = 3400\ ms^{-1}$$

Hence, the speed of the sound waves in the copper rod comes out to be3400 ms^{-1}.