# RATIO & PROPORTION

## RATIO

**Ratio :** A comparison by division is called ratio. A ratio is usually denoted by the symbol (:). If a and b (b ≠ 0) are two quantities of the same kind, then the fraction $\dfrac{a}{b}$ is called the ratio of a to b, we write it as a : b.

or $\dfrac{a}{b} = \dfrac{a \to \text{anticedent}}{b \to \text{consequent}}$

In the ratio a: b, the first term is 'a' and the second term is 'b'. A ratio is said to be in the simplest form if its two terms have no common factor other than 1.

**NOTE :**

(i)   The ratio of two numbers is usually expressed in its simplest form.

(ii)  In a ratio, we compare two quantities. The comparison becomes meaningless if the quantities being compared are not of the same kind i.e. they are not measured in the same units.

(iii) It is just meaningless to comeare 20 bags with 200 crows. Therefore, to find the ratio of two quantities, they must be expressed in the same units.

(iv)  Since the ratio of two quantities of the same kind determines how many times one quantity is contained by the other. So the ratio of any two quantities of the same kind is an abstract quantity. In other words, ratio has no unit or it is independent of the units used in the quantities compared.

(v)   The order of the terms in a ratio a : b is very important. The ratio 3 : 2 is different from the ratio 2 : 3.

(vi)  We can multiply or divide both the terms of the ratio by a non zero number which does not alter the value of the ratio.

## RATIO IN THE SIMPLEST OR LOWEST FORM

A ratio $\dfrac{a}{b}$ or a : b is said to be in its lowest or simplest form if a and b have no common factors except 1.

**For example,**

$$40 : 80 = \dfrac{40 \div 40}{80 \div 40} = \dfrac{1}{2} = 1 : 2$$

$$10000 : 8000 = \dfrac{10000 \div 2000}{8000 \div 2000} = \dfrac{5}{4} = 5 : 4$$

**STEPS :**

(a) Write the ratio as a fraction.

(b) Divide the numerator and the denominator by their HCF.

(c) The answer is a fraction in its lowest form ; so change it to ratio, which will be in the lowest form.

**Ex.** Express the following ratio in their simplest form :

(a) 150 : 400

(b) 85 : 225

**Sol.** (a) $150 : 400 = \dfrac{150 \div 50}{400 \div 50} = \dfrac{3}{8} = 3 : 8$

(b) $85 : 255 = \dfrac{85 \div 85}{255 \div 85} = \dfrac{1}{3} = 1 : 3$

**Ex.**   Find the ratio of the following :

(a) 36 minutes to 2 hours.

(b) 50 cm to 5 metres.

(c) 32 g to 1 kg

(d) 3 days to 1 years.

**Sol.**   (a) Change both 36 minutes and 2 hours to the same unit.

Now, 36 minutes = 36 minutes

2 hours = 2 × 60 minutes = 120 minutes

∴ Ratio of 36 minutes to 2 hours

$$36 : 120 = \frac{36 \div 12}{120 \div 12}$$

$$= \frac{3}{10} = 3 : 10$$

(b) First convert both into numbers with the same unit.

50 cm = 50 cm

5 metres = 500 cm

Hence ratio of 50 cm to 5 metes is

$$= 50 : 500 = \frac{50 \div 50}{500 \div 50}$$

$$= \frac{1}{10} = 1 : 10$$

(c) First convert both into numbers with the same unit.

32 g = 32 g

3kg = 3000 g

$$\text{Ratio} = 32 : 3000 = \frac{32 \div 8}{3000 \div 8} = \frac{4}{375}$$

$$= 4 : 375$$

(d) First, convert both into numbers with the same unit.

3 days = 3days

1 year = 365 days

Ratio = 3 : 365

**Comparison of Ratios :**

**1.**   Write the given ratios as fractions in the simplest form.

**2.**   Find the LCM of the denominators of the fractions.

**3.**   Convert them into like fractions with same denominators.

**4.**   Compare the numerators and arrange the fractions.

**5.**   Then respective ratios are also in the same order.

**Ex.**   Compare 5 : 12 and 3 : 6

**Sol.**   $5 : 12 = \dfrac{5}{12}$, $3 : 8 = \dfrac{3}{8}$

LCM of 8, 12 = 24

$5 : 12 = \dfrac{5}{12} \times \dfrac{2}{2} = \dfrac{10}{24}$

$3 : 8 = \dfrac{3}{8} \times \dfrac{3}{3} = \dfrac{9}{24}$

$\dfrac{9}{24} < \dfrac{10}{24}$

$\therefore \dfrac{3}{8} < \dfrac{5}{12}$ OR $3 : 8 < 5 : 12$

**Ex.** The ratio of the number of girls to the number of boys in a school is 5 : 8. In another school the ratio of the number of girl to the number of boys is 7 : 10. Which school has a higher ratio of girls?

**Sol.** The ratios of girls to boys in the two schools are 5 : 8 and 7 : 10. Since the number of girls forms the numerator in both the cases, the school which has a greater ratio has a higher number of girls. We have two fractions $\dfrac{5}{8}$ and $\dfrac{7}{10}$. We can compare these fractions by converting both the fractions into fractions with same denominator.

The LCM of 8 and 10 is 40.

$\dfrac{5}{8} = \dfrac{5 \times 5}{8 \times 5} = \dfrac{25}{40}, \dfrac{7}{10} \times \dfrac{4}{4} = \dfrac{28}{40}$

$\dfrac{28}{40} > \dfrac{25}{40}$

So the second school with the ratio 7 : 10 has a higher ratio of girls.

**Ex.** Mr. Harry divided Rs. 84,630 between Shinchan and Nimavari in the ratio 3 : 4. How much did each of them get?

**Sol.** Ratio of money between Shinchan and Nimavari = 3 : 4

Sum of the terms of the ratio = 3 + 4 = 7

Sinchan's share = $\dfrac{3}{7}$ of total money

Nimavari's share = $\dfrac{4}{7}$ of total money

$\therefore$ The amount of money Shinchan's gets

$= \dfrac{3}{7} \times$ Rs. 84,630

$=$ Rs. 36, 270

The amount of money Nimavari gets

$= \dfrac{4}{7} \times$ Rs. 84,630

$=$ Rs. 48,360.

**Ex.** The number of stamps in the collections of Suniyo, Lobita, and Suzuka are in the ratio 3 : 4 : 5. If lobita has a collection of 108 stamps, find the number of stamps that Suniyo, and Suzuka each has.

**Sol.** The number of stamps in the collections of Suniyo Lobita, and Suzuka are in the ratio = 3 : 4 : 5.

Sum = 3 + 4 + 5 = 12

Let the number of stamps with Suniyo be 3x. Then Lobita will have 4x stamps and Suzuka will have 5x stamps.

Given that Lobita's 4x = 108 stamps

$$x = \frac{108}{4} = 27$$

Suniyo's 3x = 3 × 27 = 81 stamps

Suzuka's 5x = 5 × 27 = 135 stamps

∴ Suniyo's has 81 stamps and Suzuka has 135 stamps.

## EQUIVALENT RATIO

The two or more ratios are set to be equivalent if their simplest form is same.

e.g. $\frac{21}{35}$, $\frac{12}{20}$ are equivalent ratios

because simplest form of $\frac{21}{35}$, $\frac{12}{20}$ is $\frac{3}{5}$.

**Ex.** If a : b = 2 : 3 and b : c = 5 : 7, find a : c & b : c.

**Sol.** Given $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = 5 : 7$

$$\therefore \frac{a}{c} = \left( \frac{a}{b} \times \frac{b}{c} \right) = \left( \frac{2}{3} \times \frac{5}{7} \right) = \frac{10}{21}.$$

So, a : c = 10 : 21

Aslo, a : b = 2 : 3 and b : c = 5 : 7

LCM of 3 and 5 is 15

$$\therefore \quad \frac{a}{b} = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$$

$$\frac{b}{c} = \frac{5}{7} \times \frac{3}{3} = \frac{15}{21}$$

∴ a : b : c = 10 : 15 : 21.

**Ex.** Divide Rs. 4340 between A, B and C so that A's share : B's share : C' share = $\frac{1}{2}$ : $\frac{1}{3}$ : $\frac{1}{5}$

**Sol.** A, B and C so that A's share : B's share : C' share = $\frac{1}{2}$ : $\frac{1}{3}$ : $\frac{1}{5}$

L.C.M. 2, 3, 5 is 30

A, B and C so that A's share : B's share : C' share = $\frac{1}{2}$ : $\frac{1}{3}$ : $\frac{1}{5}$

$= \dfrac{1}{2} \times 30 : \dfrac{1}{3} \times 30 : \dfrac{1}{5} \times 30$

$= 15 : 10 : 6$

A share $= \dfrac{15}{15+10+6} = \dfrac{15}{15+10+6} \times 4340 = 2100$ Rs.

B share $= \dfrac{10}{15+10+6} = \dfrac{10}{15+10+6} \times 4340 = 1400$ Rs.

C share $= \dfrac{6}{15+10+6} = \dfrac{6}{15+10+6} \times 4340 = 840$ Rs.

## PROPORTION

An equality of two ratios is called a proportion.

For example, 3 : 5 = 9 : 15

The first and the fourth terms are called the extremes or extreme terms. The second and the third terms are called the middle terms or means.

In case of proportion, we can say that the product of the extreme terms = the product of the middle terms.

∴ Product of extremes = product of means

$$a : b = c : d \Rightarrow \dfrac{a}{b} = \dfrac{c}{d}$$

means / extremes

$\Rightarrow a \times d = b \times c$

**Ex.** If a : 30 :: 7 : 15, find the value of a.

**Sol.** Product of the extremes = 15a

product of the means = 30 × 7

Product of the extremes = product of the means

15a = 30 × 7

so $a = \dfrac{30 \times 7}{15} = 14$

**Ex.** Are 36, 49, 6, 7 in proportion.

**Sol.** We have,

Product of extremes = 36 × 7 = 252

Product of means = 49 × 6 = 294

Clearly, Product of extremes ≠ Product of means.

Hence, 36, 49, 6, 7 are not in proportion.

**Ex.** 80 students consume 720 kg of wheat in a month.

How many kilograms of wheat are required in a hostel with 150 students for a month?

**Sol.** Let wheat required be x kg.

Students : Students :: Quantity of Wheat : Quantity of wheat

80 : 150 :: 720 : x

Product of extremes = Product of means

80x = 150 × 720

$x = \dfrac{150 \times 720}{80} = 1{,}350$ kg.

**Ex.** Rajiv invest Rs. 9500 in a bank and earn interest Rs. 665. If Deepak invests in the same bank, at the same rate of interest, for the same time period, an amount of Rs.7,500, what will be the interest that he will earn?

**Sol.** Let the interest Deepak earns be Rs. x.

$$\begin{pmatrix} \text{Rajiv's} \\ \text{principal} \end{pmatrix} : \begin{pmatrix} \text{Deepak's} \\ \text{principal} \end{pmatrix} :: \begin{pmatrix} \text{Rajiv's} \\ \text{Interest} \end{pmatrix} : \begin{pmatrix} \text{Deepak's} \\ \text{Interest} \end{pmatrix}$$

9500 : 7500 :: 665 : x

9500x = 7500 × 665

$x = \dfrac{7500 \times 665}{9500}$ = Rs 525.

# UNITARY METHOD

Look at this problem.

If the cost of 3 pens is Rs. 12, what will be the cost of 8 pens?

The cost of 3 pens is Rs.12. So we know that the cost of one pen will be lesser than Rs. 12. We can find the cost of one pen by dividing Rs.12 by 3. The cost of one pen is Rs.12 ÷ 3 = Rs. 4. If we have to find the cost of 8 pens, we know that it will be more than the cost of one pen. So we multiply Rs.4 by 8 and we get Rs. 32. So the cost of 8 pens is Rs. 8 × 4 = Rs. 32.

In the process of our calculation we found out the value of one unit(in this case cost of one pen)

So this method of solving is called unitary method.

Steps to follow in unitary method

**1.** Identify the facts given.

**2.** Identify what is to be found out.

**3.** Find the value of a unit by dividing.

$$\text{Value of one} = \frac{\text{Given value}}{\text{Number of articles}}$$

**4.** Multiply this result with the required units. Value of many = value of one × required no of units.

**Ex.** If the cost of 6 chocolates is Rs. 210, then find the value of 4 chocolates.

**Sol.** Cost of 6 chocolates = Rs. 210

Cost of 1 chocolates = Rs. $\dfrac{210}{6}$ = Rs. 35

∴ Cost of 4 chocolates = Rs. 35 × 4 = Rs. 140

Thus, 4 chocolates cost Rs. 140.

**Ex.** A bus travels 240 km in 6 hours. How long will it take to travel 360 km?

**Sol.** Time taken for 240 km = 6 hours

Time taken for 1 km = $\dfrac{6}{240}$ hours = $\dfrac{1}{40}$ hours

∴ Time taken for 360 km = $\dfrac{1}{40}$ × 360 = 9 hours.

Thus, the bus takes 9 hours to travel 360 km.

**Ex.** Raj earns Rs. 1500 in 10 days. How much will he earn in 45 days ?

**Sol.** Earning in 10 days = Rs. 1500

Earning in 1 day = $\dfrac{1500}{10}$ = Rs. 150

∴ Earning in 45 days = Rs 150 × 45 = Rs. 6750

Thus, Raj earns Rs. 6750 in 45 days.

**Ex.** A family of 8 people is entitled to a ratio of 6,400 grams of sugar. What will be the sugar ratio for a family of 10 people?

**Sol.** Quantity of sugar for 1 person = $\dfrac{6400}{8}$ = 800 gm

∴ Quantity of sugar for 10 people = 800 × 10 = 8000 gm = 8 kg.