Integrals

Methods of Integration

Integration by substitution :

If we substitution $\phi(x) = t$ in an integral then

(i) everywhere x will be replaced in terms of new variable t.

(ii) dx also gets converted in terms of dt.

Ex.1 Evaluate : $\int \frac{\sec^2 x}{3+\tan x} dx$

Sol. $I = \int \frac{\sec^2 x}{3+\tan x} dx$

Let $3 + \tan x = t$

$\Rightarrow \quad \sec^2 x dx = dt = \int \frac{dt}{t} = \lambda n\ t + C = \lambda n|(3 + \tan x)| + C$

Ex.2 Evaluate : $\int \frac{1}{1+e^{-x}} dx$

Sol. $I = \int \frac{1}{1+e^{-x}} dx$

$= \int \frac{e^x}{e^x+1}$

$= \int \frac{\frac{d}{dx}\left(e^x+1\right)}{(e^x+1)}$

$= \log_e|e^x + 1| + C$

Ex.3 Evaluate : $\int \tan^4 x\, dx$

Sol. $\int \tan^4 x\, dx = \int \tan^2 x . \tan^2 x\, dx$

$= \int \tan^2 x(\sec^2 x - 1) dx$

$= \int \tan^2 x \sec^2 x\ dx - \int \tan^2 x\, dx$

$$= \int \tan^2 x \sec^2 x dx - \int (\sec^2 x - 1) dx$$

$$= \frac{\tan^3 x}{3} - \tan x + x + C$$

Ex.4 Evaluate : $\int \frac{x}{x^4 + x^2 + 1} dx$

Sol. We have,

$I = \int \frac{x}{x^4 + x^2 + 1} dx = \int \frac{x}{(x^2)^2 + x^2 + 1} dx$ {Put $x^2 = t \Rightarrow x.dx = \frac{dt}{2}$ }

$$\Rightarrow \quad I = \frac{1}{2} \int \frac{1}{t^2 + t + 1} dt$$

$$= \frac{1}{2} \int \frac{1}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dt$$

$$= \frac{1}{2} . \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C.$$