RADIOACTIVITY

Q value of a reaction, nuclear stability

Q-Value

The difference between the rest mass energy of the initial constituents and that of the final products in a nuclear reaction is called Q-value of the reaction.

$$\underset{m_A}{A} + \underset{m_B}{B} \longrightarrow \underset{m_C}{C} + \underset{m_D}{D}$$

Initial rest mass energy,

$$u_i = (m_A + m_B)c^2$$

Final rest mass energy,

$$U_f = (m_c + m_D)c^2$$

$$\therefore Q = U_i - U_f$$

$$Q = (m_A + m_B - m_C - m_D)c^2$$

$Q = +\text{ve}$: Energy is released

$Q = -\text{ve}$: Energy is absorbed

Q-value of a reaction

The difference between the rest mass energy (Ui) of the initial constituents and that of the final products (Uf) is called the Q-value of a reaction.

$$Q = U_i - U_f$$

- It is valid for any nuclear reaction.
- This energy is made available as the kinetic energy of the products.

Nuclear Stability

For atoms with $Z \leq 20$

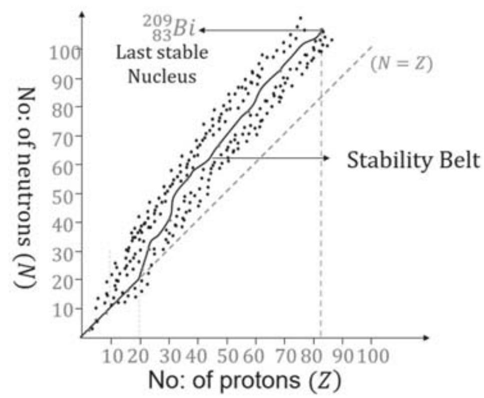
For stability: $\frac{n}{p} = 1$

Eg: ${}^{4}_{2}He$, ${}^{12}_{6}C$, ${}^{14}_{7}N$, ${}^{40}_{20}Ca$

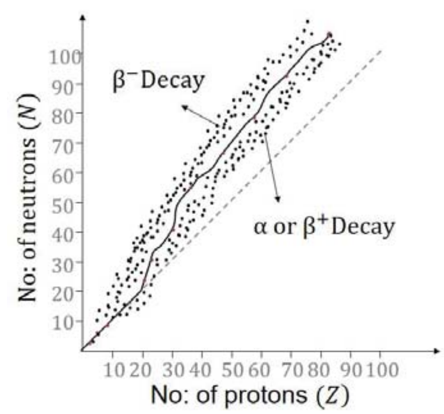
For atoms with $Z > 20$

For stability: $1 \leq \frac{n}{p} \leq 1.6$

Eg: ${}^{56}_{26}Fe$, ${}^{108}_{47}Ag$



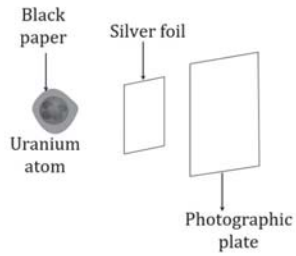
- Nuclear force is effective in the short range, hence it is sufficient to overcome Coulombic repulsion in smaller nuclei.
- For larger nuclei, nuclear force decreases between nucleons at larger distances, hence nuclei with less protons are stable.
- Nuclei with very large no. of neutrons are also unstable, as there are more unpaired nucleons.
- Nucleus above stability belt undergo $\beta-$ *decay* to reduce the n/p ratio because they have more no. of neutrons compared to the no. required for stability.
- Nucleus below stability belt undergo α or β^+ *decay* to increase the $\frac{n}{p}$ ratio because they have less no. of neutrons compared to the no. required for stability.



Radioactivity

- Amongst 1500 known nuclei, less than 260 are stable.
- Unstable nuclei become stable by releasing either α-particles, β-particles, or γ-electromagnetic waves. This process is known as radioactivity.

- Radioactivity is a nuclear occurrence. The speed at which α, β, and γ emissions happen relies solely on the concentration of nuclei and isn't influenced by factors like temperature, pressure, or other physical conditions.



Henry Becquerel

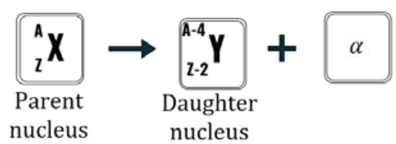
- Becquerel placed uranium salt beneath a black paper and shielded a photographic plate with a layer of silver foil. However, despite these precautions, the photographic plate still became exposed. This led him to infer the existence of certain invisible radiations capable of penetrating through objects and affecting the photographic plate.

Alpha decay

α-particle is $^{4}_{2}He$ nucleus.

α −particle

Particles	Charge (C)	Mass	Symbol
α −particle	$(+2e)$	$\approx 4\ amu$	He



- The daughter nucleus is shown with a different symbol because when two protons are taken away, the atomic number shifts, leading to a change in the element itself.
- Sum of protons and neutrons on both sides in decay must be equal.

Eg: $^{238}_{92}U \rightarrow \, ^{234}_{90}Th + \, ^{4}_{2}He$

Mass of parent nucleus,

$$m_X = m(\,^{A}_{Z}X) - Zm_e$$

Mass of daughter nucleus,

$$m_Y = m(\,^{A-2}_{Z-2}Y) - (Z-2)m_e$$

Mass of α −particles,

$$m_\alpha = m(\,^{4}_{2}He) - 2m_e$$

∴ Q-value of the α- decay

$$Q = [m_X - (m_Y + m_\alpha)]c^2$$

$$\Rightarrow Q = [m(\,^{A}_{Z}X) - m(\,^{A-2}_{Z}Y) - m(\,^{4}_{2}He)]c^2$$

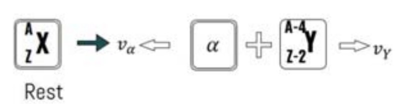
The energy released is made available as the kinetic energy of the products.

Momentum conservation

$P_i = 0$ No external force $P_f = 0$

$$P_f = m_\alpha v_\alpha - m_Y v_Y = 0$$

$$m_\alpha v_\alpha = m_Y v_Y$$



The α-particle and the daughter nucleus moves in opposite direction to each other so that the momentum is conserved

K.E of emitted α - particle

$$Q = K \cdot E_\alpha + K \cdot E_Y$$

$$K \cdot E_\alpha = \frac{Qm_Y}{m_\alpha + m_Y} \qquad K.E_Y = \frac{Qm_\alpha}{m_\alpha + m_Y}$$

Beta decay

β –Decay	Charge (C)	Mass	Symbol
β^-Decay	$(-e)$	9.1×10^{-31} kg	${}^{0}_{-1}e$ or ${}^{0}_{-1}\beta$
β^+Decay	$(+e)$ positron	9.1×10^{-31} kg	${}^{0}_{+1}e$ or ${}^{0}_{+1}\beta$

β^- Decay

$$ {}^{A}_{Z}X \Rightarrow {}^{A}_{Z+1}Y + \beta^- + \bar{\nu} $$

β^- decay is the conversion of a neutron to a proton inside the nucleus

$$ {}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}e + \bar{v} $$

${}^{0}_{-1}e(\beta^-)$ is generated at this moment inside nucleus

Eg: ${}^{14}_{6}C \rightarrow {}^{0}_{-1}e + {}^{14}_{7}N + \bar{v}$

Antineutrino accounts for the Energy & Spin momentum conservation.

Mass of the parent nucleus,

$$ m_X = m({}^{A}_{Z}X) - Zm_e $$

Mass of the daughter nucleus,

$$ m_Y = m({}_{Z+1}{}^{A}Y) - (Z+1)m_e $$

Mass of the β – particles,

$$ m_{\beta^-} = m_e $$

∴ Q-value of the β – decay,

$$ Q = [m({}^{A}_{Z}X) - m({}^{A}_{Z+1}Y)]c^2 $$

Due to the large mass of the residual nucleus, almost all the energy of the reaction is shared by the beta particle and the antineutrino.

β^+ Decay

$$ {}^{A}_{Z}X \Rightarrow {}^{A}_{Z-1}Y + \beta^+ + v $$

$$ {}^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{+1}e + v $$

${}^{0}_{+1}e(\beta^+)$is generated at this moment inside nucleus

β^+ Decay is the conversion of a proton to a neutron inside the nucleus.

Eg: ${}^{12}_{7}N \rightarrow {}^{0}_{+1}e + {}^{12}_{6}C + v$

Q-value of the $\boldsymbol{\beta}^+$decay,

$$ Q = [m({}^{A}_{Z}X) - m({}^{A}_{Z-1}Y) - 2m_e]c^2 $$

An isolated proton doesn't beta decay into a neutron, as Q value for such a process would be negative.

Electron-Capture

$$ {}^{A}_{Z}X \Rightarrow e^- + {}^{A}_{Z-1}Y + v $$

Proton captures K shell $e^-$and converts in to a neutron

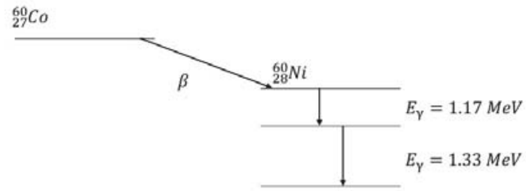
$$ {}^{1}_{1}p + {}^{0}_{-1}e \rightarrow {}^{1}_{0}n + v $$

Eg: ${}^{26}_{13}Al + {}^{0}_{-1}e \rightarrow {}^{26}_{12}Mg + v$

Q-value of the reaction,

$$ Q = [m({}^{A}_{Z}X) - m({}^{A}_{Z-1}Y)]c^2 $$

Gamma decay



When a nucleus emits α-particle or β-particle, the daughter nucleus formed is in an excited state. The excited nucleus tries to return to its ground state and while doing so, it emits highly energetic radiation called γ - radiation.

Properties of nuclear radiation

- Penetration power $\alpha < \beta < \gamma$
- α and β rays can be deflected by electric field and magnetic field but not γ rays.
- Ionizing power $\alpha > \beta > \gamma$

Uses of nuclear radiation:

- To detect and treat diseases
- Carbon Dating