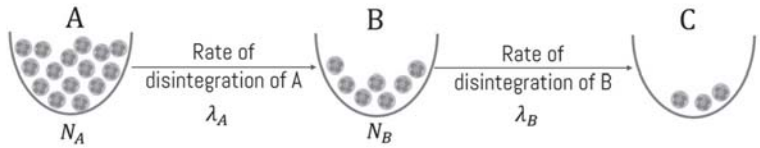
RADIOACTIVE SERIES

Radioactive equilibrium



Rate of disintegration of element A, $R_A = \lambda_A N_A$
Rate of disintegration of element B. $R_B = \lambda_B N_B$
Presently, the speed at which element *A* breaks down matches the speed at which element *B* forms. When the speed of formation matches the speed of breakdown for element *B*, it's termed as radioactive equilibrium.

$$\therefore R_A = R_B$$
$$\Rightarrow \lambda_A N_A = \lambda_B N_B$$

Radioactive series

Accumulation of a Radioactive Element in Radioactive Series

$$\underset{N_1}{A_1} \xrightarrow{\lambda_1} \underset{N_2}{A_2} \xrightarrow{\lambda_2} A_3$$

Let at time $t = 0$, N_0 be the number of active nuclei of A_1 and no. of active nuclei of A_2 is zero.
The no. of active nuclei of A_2 at time t is

$$N_1 = N_0 e^{-\lambda_1 t}$$

Decay rate of $A_1 =$ Production rate of $A_2 = \lambda_1 N_1$
Decay rate of $A_2 = \lambda_2 N_2$
Accumulation rate of A_2

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$\lambda_1 =$ Decay constant of A_1
$\lambda_2 =$ Decay constant of A_2

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$
$$dN_2 + \lambda_2 N_2 at = \lambda_1 N_1 dt$$
$$dN_2 + \lambda_2 N_2 dt = \lambda_1 N_0 e^{-\lambda_1 t} dt \ (\because N_1 = N_0 e^{-\lambda_1 t})$$

Multiplying both sides by $e^{\lambda_2 t}$

$$dN_2 e^{\lambda_2 t} + \lambda_2 e^{\lambda_2 t} N_2 dt = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} dt$$
$$d(N_2 e^{\lambda_2 t}) = \int \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} dt$$
$$N_2 e^{\lambda_2 t} = \lambda_1 N_0 \frac{e^{(\lambda_2 - \lambda_1)t}}{\lambda_2 - \lambda_1} + C$$

at $t = 0; N_2 = 0$

$$0 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} + c$$
$$C = \frac{\lambda_1 \lambda_0}{\lambda_1 - \lambda_2}$$
$$N_2 e^{\lambda_2 t} = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2} e^{(\lambda_2 - \lambda_1)t} \frac{+\lambda_1 N_0}{\lambda_1 - \lambda_2}$$
$$N_2 e^{\lambda_2 t} = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2} [1 - e^{(\lambda_2 - \lambda_1)t}]$$
$$N_2 = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2} \left[\frac{1}{e^{\lambda_2 t}} - \frac{e^{(\lambda_2 - \lambda_1)t}}{e^{\lambda_2 t}}\right]$$
$$N_2 = \frac{\lambda_1 N_0}{\lambda_1 - \lambda_2} [e^{-\lambda_2 t} - e^{(\lambda_2 - \lambda_1 - \lambda_2)t}]$$
$$N_2 = \frac{N_0 \lambda_1}{\lambda_1 - \lambda_2} [e^{-\lambda_2 t} - e^{-\lambda_1 t}]$$
$$N_2 = \frac{\lambda_1 N_0}{(\lambda_1 - \lambda_2)} (e^{-\lambda_2 t} - e^{-\lambda_1 t})$$

at t $= 0; N_2 = 0$
at $t \to \infty; N_2 = 0$
Variation of N_2 with time t