Chapter 14

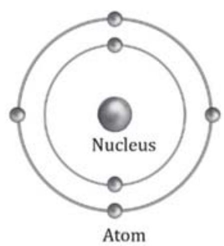
Nuclei

- Introduction to nucleus
  - Atoms and nuclei
  - Nuclei properties
  - Nuclear density, shape, size
  - Nuclear force
  - Binding energy
- Radioactivity
  - Q value of a reaction , nuclear stability
  - Radioactivity
  - Alpha decay
  - Beta decay
  - Gamma decay
- Law of radioactive decay
  - Law of radioactive decay
  - Half-life, mean life
  - Questions based on law of radioactive decay
- Radioactive series
  - Radioactive equilibrium
  - Radioactive series
- Nuclear fission and fusion
  - Nuclear fission
  - Uranium fission
  - Chain reaction
  - Nuclear reactor
  - Nuclear fusion
  - Stellar life cycle

INTRODUCTION TO NUCLEUS

Nucleus

In an atom, the mass and positive charge are found at the center in a tiny space called the nucleus.
The size of the nucleus is very small when compared to the overall size of the atom.



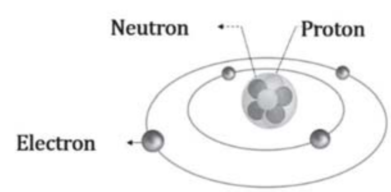
$r_{atom} \approx 10^{-10}$ m

$r_{nucleus} \approx 10^{-15}$ m

Atom

Nucleus is made up of neutrons and protons.
Both protons and neutrons present in the nucleus are collectively called as nucleons.

Subatomic Particles
Electron
Proton
Neutron



Subatomic Particles	Absolute charge (C)	Mass (Kg)	Symbol
Proton	1.602 × 10-19 (+1e)	1.6727 × 10-27 kg (≈ 1 amu)	$^{1}_{1}p$ +
Neutron	Zero	1.6749 × 10-27 kg (≈ 1 amu)	$^{1}_{0}n$

Atomic Number

Atomic number (Z) = Number of protons present in the nucleus
= Number of electrons in the neutral atom

To uphold electrical balance, the count of electrons within an atom matches the count of protons, also known as the atomic number (Z).

Mass Number

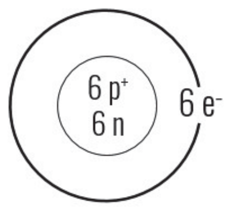
Mass number (A) = Number of nucleons
= No. of neutrons (N) + No. of protons (Z)

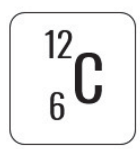
The nucleus carries a positive charge because of its protons, and its mass comes from both protons and neutrons, collectively known as nucleons.

Atomic mass unit

The weight of an atom is exceedingly tiny when compared to a kilogram. Consequently, the kilogram isn't practical for measuring such minuscule amounts. Hence, a distinct unit, known as the atomic mass unit (amu), is employed for indicating atomic masses.

$$1\text{amu} = \frac{1}{12} \times \text{ mass of one } {}^{12}_{6}\text{C atom.}$$





$$1\text{amu} = 1.66 \times 10^{-27}\text{ kg}$$

$$1 \text{ mole of C-atoms } = 12\text{ g}$$

Let N_A represents the Avogadaro's number.

$$\therefore N_A \text{ atoms of C } = 12\text{g}$$

$$1 \text{ atom of C} = \frac{12}{N_A}\text{g}$$

$$1\text{ amu} = \frac{1}{12} \times \frac{12}{N_A}\text{g} = \frac{1}{N_A}\text{g}$$

$$\frac{1}{N_A} \times 10^{-3}\text{kg} = \frac{10^{-3}}{N_A}\text{ kg}$$

Nuclei properties

Nuclear density, shape, size

For all practical purposes, shape of the nucleus is regarded as "Spherical"

Volume of nucleus ∝ No. of nucleons in nucleus (A)

$$\frac{4}{3}\pi R^3 \propto A \Rightarrow R^3 \propto A$$

$$R \propto A^{\frac{1}{3}} \Rightarrow R = R_0 A^{\frac{1}{3}}$$

R_0 is Constant for all nuclei.

$$R_0 = 1.2 \times 10^{-15}\text{ m} = 1.2\text{fermi}$$

Radius of nucleus in fermi

Density of nucleus

Let m be the mass of each nucleon and A be the mass no. or the no. of nucleons

$$f = \frac{M}{V} = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

$$R = R_0 A^{\frac{1}{3}}$$

As, the density of nucleus is independent of mass number . Therefore, density of all the nuclei is constant and is equal to:

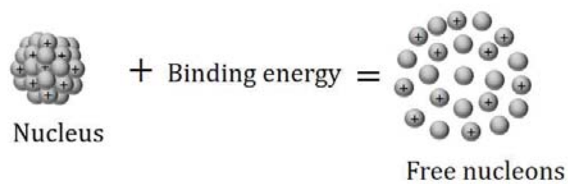
$$\rho \approx 2 \times 10^{17}\ \text{kg/m}^3$$

Nuclear force

- The force that keeps nucleons bound together, overcoming the repulsion between protons, is termed the nuclear force.
- It is a short-ranged force and is operative only over the size of nucleus 10^{-15} .
- It is not a central force
- It is independent of charge, i.e. the nuclear force between $n - n$, $p+ - n$ and $p^+ - p^+$ have approximately same value.
- Nuclear force favors pairing of 2 protons and 2 neutrons together.
- In every quantum state, there can be a maximum of two protons (with opposite spins) and two neutrons (also with opposite spins). The nuclear force between nucleons with the same spin is stronger compared to that between nucleons with opposite spins.
- Nuclear force is the strongest force in nature.

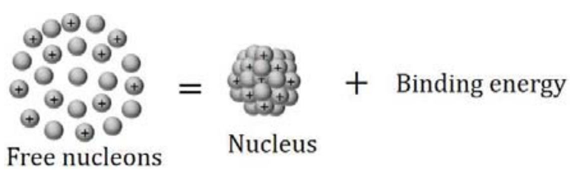
Binding energy

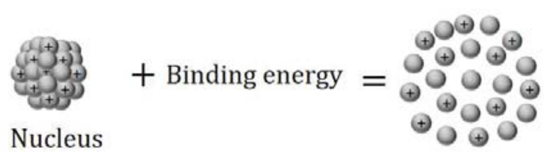
It is the minimum energy required to separate the constituent nucleons to large distances.



or

Amount of energy released during the formation of nucleus by its nucleons if brought from infinite separation.





[Atomic number =Z, Mass number = A] Free nucleons

M_{atom} = Mass of atom
(Calculated practically by mass spectroscopy)

[Protons=Z, Neutrons = A-Z]

$M_{Nucleus} = M_{atom} - Z_{me}$

$M_{Nucleons} = Zm_p + (A\text{-}Z)\ m_n$

When the nucleons are brought together to form the nucleus, some of the mass gets converted into the binding energy, that's why the mass of the nucleus is less than the sum of the masses of the nucleons.

The difference in mass of a nucleus and its constituents, is called the mass defect Δm.

According to the mass energy equivalence $E = mc^2$, Δm mass gets converted into the binding energy

$$\Delta m = [Zm_p + (A - Z)m_n] - (M_{nucleus})$$

Mass defect in terms of mass of the atom,

$$\Delta m = [(Zm({}_1^1H) + (A - Z)m_n) - m({}_Z^AX)]$$

Binding energy

$$E_b = \Delta mc^2$$
$$E_b = \Delta m(\text{ in amu}) \times 931.5\text{MeV/amu}$$

Binding energy per nucleon

Ratio of the binding energy E_b to the number of the nucleons of a nucleus is called the binding energy per nucleon.

$$E_{bn} = \frac{E_b}{A} \quad \text{Where, } A = \text{Mass number}$$

If binding energy per nucleon is more for a nucleus, then it is more stable. For example

$$\left(\frac{E_{b1}}{A_1}\right) > \left(\frac{E_{b2}}{A_2}\right)$$

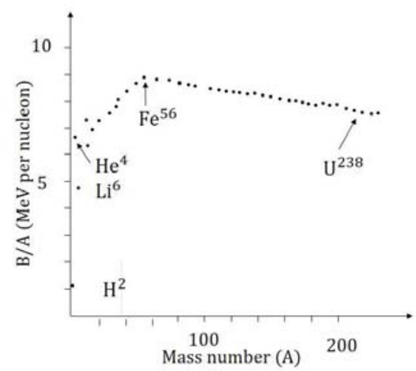
Nucleus 1 is more stable

Binding energy per nucleon

For deuterium, E_{bn} is 1.11MeV.

Fe^{56} has the highest E_{bn} of 8.7MeV.

Nuclei in the intermediate region ($A \approx 50 - 80$) are the most stable.
For U^{238}, E_{bn} drops to 7.5MeV



Isobars and Isotones

Isobars are nuclides characterized by having the same mass number (A), despite differences in their atomic numbers (Z). For instance, consider isotopes $^{31}{}_{1}\text{H}$ and $^{32}{}_{1}\text{H}$, both falling under the category of isobars due to their shared mass number.

Isotones, on the other hand, represent nuclides possessing an identical number of neutrons (N), yet exhibiting distinct atomic numbers (Z). An illustrative example involves the isotones $^{198}{}_{80}\text{Hg}$ and $^{197}{}_{79}\text{Hg}$, where the neutron count remains constant at 118, while the atomic numbers differ.

Example.

In the realm of stable lithium isotopes, specifically $^6{}_3\text{Li}$ and $^7{}_3\text{Li}$, their respective abundances are documented as 7.5% and 92.5%. These isotopes carry individual masses of 6.01512 atomic mass units (u) and 7.01600 u, correspondingly. The task at hand is to determine the atomic mass of lithium.

Solution.

Mass of the first lithium isotope $^6{}_3\text{Li} = m_1 = 6.01512\text{u}$
Mass of the second lithium isotope $^7 3\text{Li} = m_2 = 7.01600\text{u}$
Abundance of $^6{}_3\text{Li} = n_1 = 7.5\%$
Abundance of $^7{}_3\text{Li} = n_2 = 92.5\%$
The atomic mass (m) of lithium is
$m = (m_1n_1 + m_2n_2) / (n_1 + n_2)$
$= \{(6.01512\text{u} \times 7.5) + (7.01600\text{u} \times 92.5)\} / (7.5 + 92.5)$
$= 6.940934\text{u}$
Hence, the atomic mass of lithium is 6.940934u.