QUESTIONS BASED ON BOHR MODEL

Ex. Find the radius of ion in its ground state assuming Bohr's model to be valid.

Sol. Radius of the n^{th} orbit for hydrogen like atom is given by: $r_n = 0.53Å\frac{n^2}{z}$

For ground state $n = 1$ and for Li, $Z = 3$. Now, Li^{++} ion is hydrogen like atom. Therefore, the radius of Li^{++} ion in its ground state will be:

$$r_1 = 0.53Å \times \frac{1}{3}$$
$$= 0.1767Å$$
$$\approx 18pm$$
$$1pm = 10^{-12}\ m$$

Ex. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is bohr radius.

(a) $\frac{h^2}{4\pi^2 ma_0^2}$ (b) $\frac{h^2}{16\pi^2 ma_0^2}$ (c) $\frac{h^2}{32\pi^2 ma_0^2}$ (d) $\frac{h^2}{64\pi^2 ma_0^2}$

Sol. Radius of the n^{th} orbit of a hydrogen like atom is given by $r_n = 0.53Å\frac{n^2}{z}$

The radius of the first orbit of a hydrogen atom is called the Bohr radius (a_0)

$$r_1 = 0.53Å = a_0$$
$$r_n = a_0\frac{n^2}{z}$$

The K.E. of electron: $V = \frac{1}{2}mv^2 \Rightarrow V = \frac{p^2}{2m}$

Where, p = linear momentum

If m be the mass of electron and v be the velocity of electron in nth orbit, then angular momentum of electron:

$$L = mvr = \frac{nh}{2\pi} \Rightarrow mv = p = \frac{nh}{2\pi r}$$

Substituting the value of P in the expression of K.E. of electron, we get

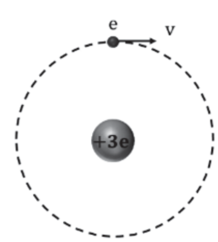
$$V = \frac{n^2h^2}{4\pi^2 r^2 \times 2m}$$

We know, $r = a_0\frac{n^2}{z}$ Substituting the value of r in the above expression of K, we get,

$$k = \frac{n^2h^2z^2}{8\pi^2 m}a_0^2 n^4$$
$$K = \frac{n^2h^2z^2}{8\pi^2 ma_0^2 n^4} \qquad Z = 1;\ n = 2;$$
$$\frac{h^2}{8\pi^2 ma_0^2 4}$$
$$\frac{h^2}{32\pi^2 ma_0^2}$$

Hence, option (C) is correct.

Ex. A particle known as μ – meson, has a charge equal to that of an electron and mass 208 times the mass of the electron. It moves in a circular orbit around a nucleus of charge $+3e$. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system,



(a) Derive an expression for radius of the nth Bohr orbit.

Sol. Let the charge of the nucleus be Ze. The electrostatic force between the positively charged nucleus and the negatively charged particle provides the centripetal force.

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} = \frac{mv^2}{r} \Rightarrow \frac{1}{4\pi\varepsilon_0}Ze^2 = mv^2 r$$

Multiplying both side of the equation by mr, we get,

$$mr\frac{Ze^2}{4\pi\varepsilon_0} = m^2v^2r^2 = (mvr)^2$$
$$mr\frac{Ze^2}{4\pi\varepsilon_0} = \frac{n^2h^2}{4x^2\pi} \qquad \text{Since } L = mvr = \frac{nh}{2\pi}$$
$$r = \frac{n^2h^2\varepsilon_0}{\pi me^2 z} \qquad \dots(1)$$

For μ-meson $m = 208m_e$ and $Z = 3$

$$r = \frac{n^2h^2\varepsilon_0}{208\pi m_e e^2 z} = \frac{n^2h^2\varepsilon_0}{624\pi m_e e^2}$$

$$r = \frac{n^2h^2\varepsilon_0}{624\pi m_e e^2}$$

(b) Find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for a hydrogen atom.

Sol. Radius of the n^{th} Bohr orbit: $r = \frac{n^2h^2\varepsilon_0}{\pi m e^2 z}$

For μ – meson $m = 208m_e$ and $R = 3$. If the radius of $n\ th$ orbit for μ – meson becomes equal $\Rightarrow n \approx 25 \therefore n$ must be an integer to that of the first Bohr orbit for a hydrogen atom, then, we have:

$r = \frac{n^2h^2\varepsilon_0}{\pi\times 208m_e e^2\times 3} == \frac{n^2h^2\varepsilon_0}{\pi m_e e^2\times 1} \rightarrow$ Radius of 1st orbit for Hydrogen atom

$$n^2 = 3\times 208 = 624$$

$$n \approx 25 \because \text{n must be an integer}$$

(c) Find the wavelength of the radiation emitted when the μ – meson jumps from the third orbit to the first orbit.

Sol. We know that, $V = -E = -\frac{U}{2}$

Potential energy of the particle: $U = \frac{-Ze^2}{4\pi\varepsilon_0 r}$

Substituting $r = \frac{n^2h^2\varepsilon_0}{\pi m e^2 z}$ in the expression of U, we get,

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2\times\pi m e^2 z}{n^2h^2\varepsilon_0}$$

$$U = \frac{-me^4z^2}{4h^2\varepsilon_0^2n^2}$$

As $E = \frac{U}{2}$, total energy of the particle: $E = -\frac{me^4z^2}{8h^2\varepsilon_0^2n^2}$

Now, total energy of electron in n^{th} orbit:

$$E = -\frac{me^4Z^2}{8h^2\varepsilon_0 n^2} = -13.6\times\frac{Z^2}{n^2}\text{eV}$$

$$\frac{me^4}{8h^2\varepsilon_0} = 13.6\text{eV}$$

For μ-meson $$E = -2088\frac{m_e e^4}{8h^2\varepsilon_0^2}\frac{z^2}{x^2}$$

$$E_\mu = -208\times 13.6\frac{z^2}{n^2}$$

Total energy of μ – meson in third orbit:

$$E_3 = -208\times 13.6\frac{z^2}{3^2}$$

Total energy of μ – meson in first orbit:

$$E_1 = -208\times 13.6\frac{z^2}{1^2}$$

Therefore, the difference between energies at 3^{rd} and 1^{st} orbit:

$$E_3 - E_1 = 208\times 13.6\times 2^2[1-\tfrac{1}{9}]$$

$$= 208\times 13.6\times 9\times\frac{8}{9}$$

$$= 208\times 13.6\times 8\text{eV}$$

Hence, the wavelength of the radiation emitted is,

$$E_3 - E_1 = \frac{hc}{\lambda}$$

$$\lambda = \frac{1240\text{eV}-\text{nm}}{208\times 13.6\times 8\text{eV}} = 0.0548\text{ nm} \approx 55\text{pm}$$

$$\lambda \approx 55\text{pm}$$

Ex. Consider a neutron and an electron bound to each other due to gravitational force. Assuming Bohr's quantization rule for angular momentum to be valid in this case, derive an expression for the energy of the neutron electron system.

Sol. Let m_n and m_e be the mass of neutron and electron respectively.

The gravitational force between the neutron and the electron provides the centripetal force.

$$Fg = \frac{Gm_n m_e}{r^2} = \frac{m_{ev^2}}{r}$$

$$Gm_n m_e = m_e v^2 r$$

$$Gm_n m_e^2 r = (m_e vr)^2$$

$$Gm_n m_e^2 r = \frac{n^2h^2}{4\pi^2} \qquad m_e v_r = \frac{nh}{2\pi}$$

$$r = \frac{n^2h^2}{4\pi^2 Gm_n m_e^2}$$

We know that, $V = -E = -\frac{U}{2}$

Now, the gravitational potential energy of the neutron electron system is,

$$U = \frac{Gm_n m_e}{r}$$

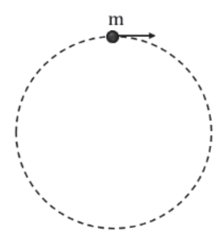
Substituting the obtained value of r in the expression of U:

$$U = -\frac{Gm_n m_e}{n^2h^2} \times 4\pi^2 Gm_n m_e^2$$

$$U = \frac{-4\pi^2 G^2 m_n^2 m_e^3}{n^2h^2}$$

As $E = \frac{U}{2}$, total energy of the system : $E = -\frac{2\pi^2 G^2 m_n^2 m_e^3}{n^2h^2}$

Ex. A small particle of mass m moves in such a way that the potential energy $U = \frac{1}{2}m^2\omega^2 r^2$ where ω is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, show that radius of the nth allowed orbit is proportional to $\sqrt{n}$.



Sol. Given, $U = \frac{1}{2}n^2\omega^2 r^2$

Assuming the force on the particle is conservative, we can write:

$$F = -\frac{dU}{dr}$$

$$-\frac{1}{2}m^2\omega^2 \times 2r$$

Here, this negative sign tells us that the force is attractive in nature and this attractive force will provide the centripetal force. Thus,

$$m^2\omega^2 r = \frac{mv^2}{r}$$

$$m\omega^2 r^2 = v^2 \qquad \dots (1)$$

Multiplying both sides equation (1) by m^2r^2, we get,

$$m^2r^2 \times m\omega^2 r^2 = (mvr)^2$$

$$m^3\omega^2 r^4 = \frac{n^2h^2}{4\lambda^2}$$

$$r^4 = vn^2 4\lambda^2$$

$$r = v^{1/4}\sqrt{n}$$

Therefore, the radius of the nth allowed orbit is proportional to $\sqrt{n}$.