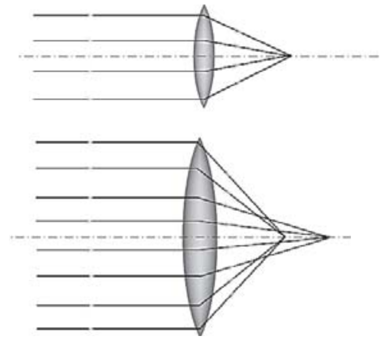
REFLECTING TELESCOPE AND RESOLVING POWER



Drawbacks

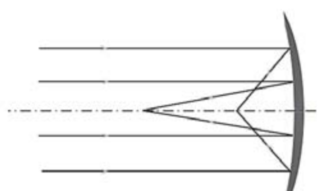
Drawback of Galilean Telescope
Difficult and expensive to constant.
Difficult to handle
Suffers from chromatic aberration
⇓
It is an inability of a lens to focus all colour at a particular point.
Large aperture ⇒ marginal rays ⇒ spherical aberration

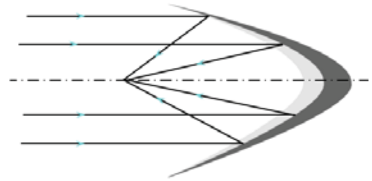
Solution of drawbacks

By utilizing a mirror, particularly a parabolic mirror, instead of a lens, it is possible to overcome all the drawbacks.

Advantages of using mirror

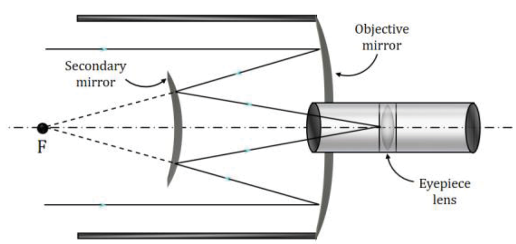
Free from chromatic aberration
High resolving power
Cost effective
Weights less
Chance of spherical aberration reduce using parabolodial mirror.
These are also the benefits of opting for a reflecting telescope over an astronomical telescope.


Large spherical mirror

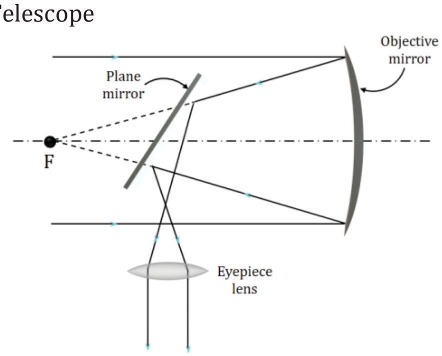

Parabolic mirror

Reflecting Telescope

In this arrangement, the objective lens is substituted with a spherical mirror.
The primary limitation of this configuration is that both the observer and the eyepiece obstruct the incoming light, resulting in a decrease in the intensity of the incident light.
The issue is addressed in the Cassegrain reflecting telescope by incorporating an additional spherical mirror into the setup.
Cassegrain Reflecting Telescope



Newtonian Reflecting Telescope



Binoculars

Binoculars consist of two small telescopes functioning together, providing a sense of depth perception that is not achievable with a single telescope.

With telescopes, the tube length is typically close to the focal length of the objective lens, and in most practical scenarios, the focal length of the objective lens (fo) is significantly greater than the focal length of the eyepiece (fe). To utilize telescopes as binoculars, it's necessary to decrease the tube length.

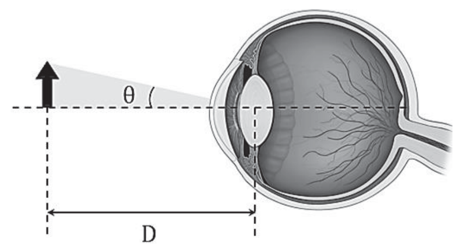
To achieve this objective, two isosceles right-angle prisms are employed in each of the two telescopes. This configuration allows light to follow a path as depicted in the figure, adhering to total internal reflection.

This arrangement not only decreases the length of the tube but also enhances the observer's field of view.

A single prism cannot be utilized alone because it results in an inverted image. However, if our sole objective is to minimize the telescope's tube length, then a single isosceles right-angle prism can be employed.

Visibility of an Eye

If at distance D, the angle θ is greater than 1', then the object will be discernible to the eye.



If not, then beyond this point, the restriction on the ciliary muscles to alter the focal length becomes a limitation.

Now, let's contrast the process of image formation on the retina via the eye's lens with the formation of a diffraction pattern through a circular aperture.

Lens of the eye = Circular aperture

The image of the object 12 will be created on the retina, similar to how a diffraction pattern forms on the screen.

The eye can distinguish object 12 if the diffraction pattern of the object (specifically, the circular central maxima of 1 and 2) satisfies specific resolution criteria.

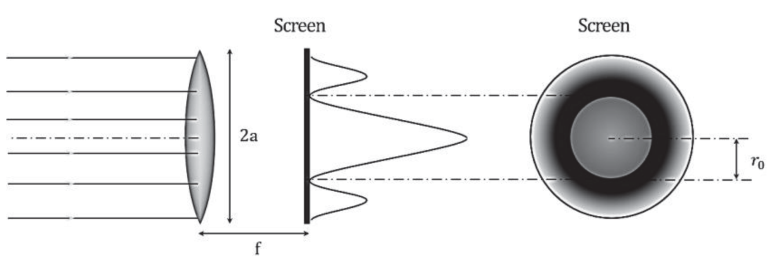
The minimum resolution criterion is: "Two images are considered just resolvable when the center of the diffraction pattern of one aligns directly over the first minimum of the diffraction pattern of the other.

In simpler terms, if the radius of the central maxima of 1 and 2 exceeds the lateral dimension of object 12, then our eye cannot effectively distinguish them.

Image formed by the objective lens of the telescope

a = Radius of aperture of objective lens

r_0 The diameter of the central bright region of the diffraction pattern created on the screen.



Based on our understanding of diffraction through a circular aperture, we're aware that for the initial dark ring,

$$\theta = \frac{1.22\lambda}{b}$$

Where b is the diameter of circular aperture.

Here, the diameter of circular aperture is $2a$. Therefore $\theta = \frac{1.22\lambda}{2a} = \frac{0.61\lambda}{a}$

The resolving angle is: $\Delta\theta = \frac{0.61\lambda}{g}$ It is also known as "Limit of resolution"

Any distant object which makes an angle less than $\Delta\theta$ can't be resolved by the telescope.
Since the screen is placed at the focal plane of the lens, the radius of the central bright region will be: $r_0 = \theta f = \frac{0.61\lambda f}{a}$

Ex. Assume that light of wavelength 6000 Å is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 100 *inch*?

Sol. Given:
The wavelength of light: $\lambda = 6000\text{Å} = 6000 \times 10^{-10}\text{m}$
The diameter of the objective lens:

$$2a = 100 \text{ inch } = 100 \times 2.54\text{cm} = 2.54 \text{ m}$$
$$a = 1.27\text{m}$$

The limit of resolution of telescope is given by,

$$\Delta\theta = \frac{0.61\lambda}{a}$$

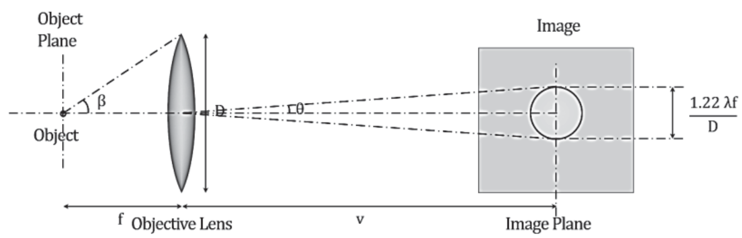
Substituting the values, we get, $\Delta\theta = \frac{0.61\times6000\times10^{-10}}{1.27}$

$$\Delta\theta = \frac{0.61\times6000\times10^{-10}}{1.27}$$
$$\Delta\theta = \frac{3.66\times10^{-7}}{1.27}$$
$$\Delta\theta = 2.88\times10^{-7} \approx 2.9\times10^{-7}\text{rad}$$
$$\Delta\theta = 2.9\times10^{-7} \text{ radians}$$

Resolving Power

Real image formed by the objective lens of the microscope

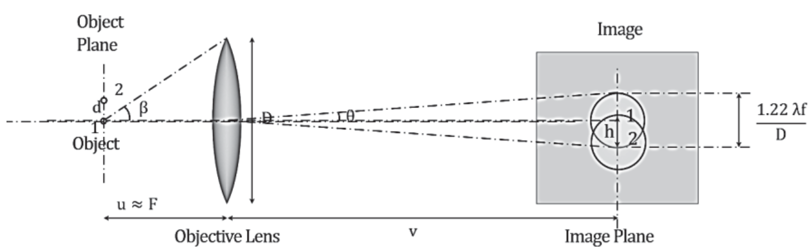
Typically, the object is positioned close to the focal point of the microscope's objective lens to achieve a magnified image.
The lens serves as the circular aperture through which light from the object passes, resulting in a diffraction pattern appearing on the screen positioned at a distance v from the lens.
From the figure, we can write: $\tan\beta = \frac{(D/2)}{f} \Rightarrow \frac{D}{f} = 2\tan\beta$ ... (1)

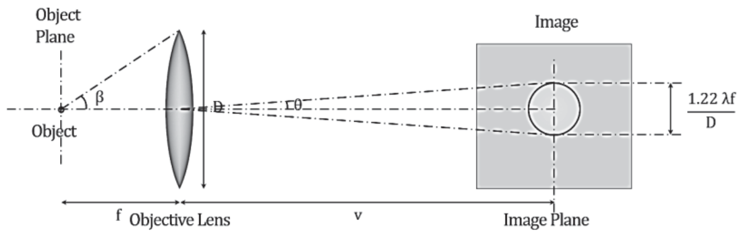


For central bright of diffraction pattern, we also know that: $\theta = \frac{1.22\lambda}{D}$ ... (2)
Therefore, the radius of central bright spot is given by, $r_0 = \theta v$ ... (3)
Now, consider two objects separated by a distance d, with the distance between the centers of the central bright spot corresponding to the objects being h'.
Therefore, magnitude of the magnification will be: : $m = \frac{h'}{d} = \frac{v}{u}$ ... (4)
Now, the object will be resolved perfectly if:
From equation (1) and (2), we have: $\frac{D}{f} = 2\tan\beta$ and $\theta = \frac{1.22\lambda}{D}$.

$$d > \frac{1.22\lambda}{2\tan\beta}$$

Condition of perfect resolution





Therefore, the minimum distance which can be resolved by using the microscope is,

$$d_{min} = \frac{1.22\lambda}{2\tan\beta} \xrightarrow[\text{image } \beta \text{ is small}]{\text{For application of shape}} \tan\beta \approx \sin\beta \Rightarrow d_{min} = \frac{1.22\lambda}{2\sin\beta}$$

The resolving power of microscope is, Resolving power $= \frac{1}{d_{min}}$

For a good microscope, d_{min} should always be small.

The resolving power of microscope can be increased further if both the object and the objective lens is immersed in transparent oil having R.I. μ. In this case, d_{min} becomes:

$$d_{min} = \frac{1.22\lambda}{2\mu\sin\beta} \Rightarrow \text{ Resolving power } = \frac{1}{d_{min}} = \frac{2\mu\sin\beta}{1.22\lambda}$$

"$\mu\sin\beta$"is known as “Numerical aperture”