## YDSE- INTERFERENCE PATTERN DUE TO INSERTION OF SLAB AT SLIT, OBLIQUE INCIDENCE, AND BICHROMATIC COHERENT LIGHT SOURCE

### YDSE- Insertion of slab of different Refractive index at slit

**Ex.** A plate of thickness $t$ made of a material of refractive index $\mu$ is placed in front of one of the slits in a double slit experiment. Wavelength of light used is $\lambda$. ($\lambda << d$, $D >> d$) Find out the distance of central maxima from the central line.

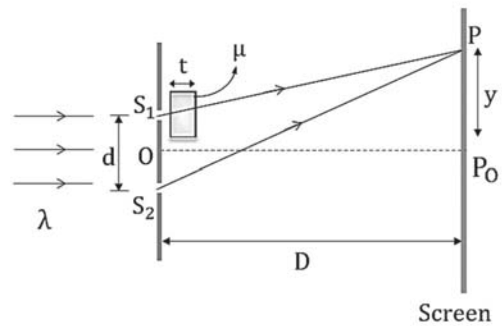
**Sol.** First of all, let us assume that the plate of thickness $t$ is absent. Thus, in absence of the plate, we can write geometrically:

$$S_1P_0 = S_2P_0$$

As soon as the slit is placed in the path of the ray coming from $S_1$, the optical path length gets changed and the effective path length becomes,

$$(S_1P_0)_{effective} = S_1P_0 + (\mu - 1)t$$

Since we know that '$t$ distance in a medium of R.I. $\mu$' is equivalent to '$\mu t$ distance in air', the effective difference or the optical path difference between them is $(\mu - 1)t$.




Therefore, the path difference at $P_0$ is,

$$(\Delta x)_{Effective} = (S_1P_0)_{effective} - S_1P_0$$

$$(\Delta x)_{Effective} = (\mu - 1)t$$

Since the glass slab is placed in the path of the ray coming from $S_1$, optical path length for the ray coming from $S_1$ will be greater than that coming from $S_2$. Therefore, to get optical path difference = 0, we have to move upwards from the central line $OP_0$ so that optical path length for the ray coming from $S_2$ becomes equal to that coming from $S_1$.

The central maxima will get shifted in upward direction from the central line.

Therefore, assume that the new central maxima will be located at point $P$, as shown in the figure

Therefore, at point $P$, the optical path length for both the rays coming from the slits becomes equal. So, we can write,

$$(S_1P)_{Effective} = S_2P$$

$$S_1P + (\mu - 1)t = S_2P$$

$$(\mu - 1)t = S_2P - S_1P$$

If the central maxima is shifted by a distance $y$ as shown, then we can write:

$$S_2P - S_1P = \frac{yd}{D}$$

Therefore, the distance of central maxima from the central line can be found out as follows:

$$\left.\begin{array}{c}(\mu - 1)t = S_2P - S_1P \\ S_2P - S_1P = \dfrac{yd}{D}\end{array}\right\} \quad (\mu - 1)t = \frac{yd}{D} \Rightarrow y = \frac{(\mu - 1)tD}{d}$$

### YDSE- path difference due to oblique incidence of light from coherent source

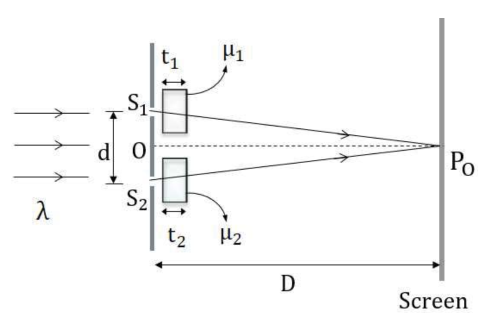
**Ex.** In a YDSE setup, slabs of ($t_1\mu_1$) and ($t_2$,$\mu_2$) are placed in front of slits $S_1$ and $S_2$, respectively. Wavelength of light used is $\lambda$. ($\lambda << d$, $D >> d$) Find out the path difference at the central point $P_0$

**Sol.** The path difference at the central point $P_0$:

$$\Delta x_{\ Effective} = |s_1p_{Effective} - s_2p_{Effective}|$$

$$s_1p + (\mu_1 - 1)t_1 - s_2p - (\mu_2 - 1)t_2$$

$$|(\mu_1 - 1)t_1 - (\mu_2 - 1)t_2|$$




Sometimes, it is given that, $t_1 = t_2 = t$. In this case, the path difference at $P_0$ becomes, $\Delta x_{effective} = |(\mu_1 - \mu_2\ t|$. If $\mu_1 > \mu_2$, the maxima will be shifted to upper side of the central line $OP_0$, otherwise it will be shifted lower side of $OP_0$

**Ex.** Monochromatic light of wavelength $\lambda$ is used in a Young's double slit experiment. One of the slits is covered by a transparent sheet of thickness $t$ made of a material of refractive index $\mu$. How many fringes will shift due to the introduction of the slit ? ($\lambda << d$, $D >> d$)

**Sol.** The fringe width for an ideal YDSE setup is:

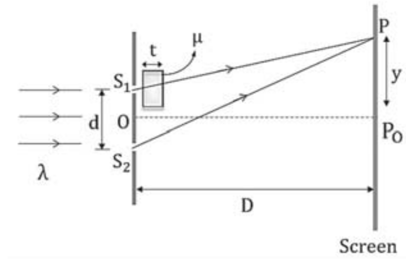
$$\beta = \frac{yD}{d}$$

The shift of central maxima is:

$$y = (\mu - 1)\frac{tD}{d}$$

Therefore, the number of fringes shifted will be:

$$n = \frac{y}{\beta} = \frac{(\mu-1)tD}{d\lambda D} = \frac{(\mu-1)t}{\lambda}$$



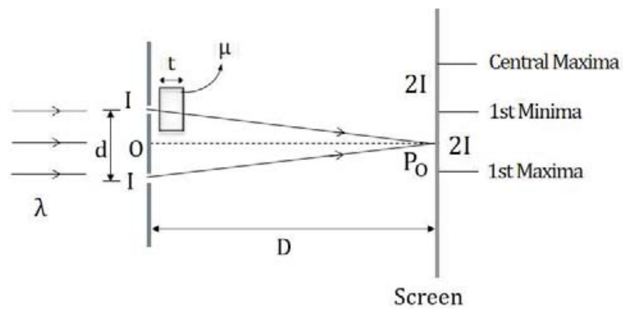

**Ex.** In a YDSE setup slab of $(t, \mu)$ is placed in front of one of the slits. If the intensity of light is $I$ and the intensiy at point $P_0$ is $2I$ as shown then find out the thickness of the slab in terms of $\lambda$,$d$,$D$ and $\mu$.($\lambda << d$, $D >> d$)

**Sol.** The path difference between the rays at $P_0$ is,

$$\Delta x = (\mu - 1)t$$

Therefore, the phase difference between the rays at $P_0$ will be,

$$\Delta\phi = \frac{2\pi}{J}\cdot\Delta x \Rightarrow \frac{\Delta\phi}{2} = \frac{\pi}{d}\cdot\Delta x$$




Since the intensity of both the rays is $I$, the net intensity at any point on the screen will be,

$$I_{net} = 4ICos^2\frac{\Delta\phi}{2} \xrightarrow[r=90^\circ - i_B]{\text{At } P_0,\ \frac{\Delta\phi}{2} = \frac{\pi}{2}\cdot\Delta x} 2I = 4ICos^2\frac{\pi}{\lambda}\cdot\Delta x \Rightarrow Cos\frac{\pi}{\lambda}\cdot\Delta x = \pm\frac{1}{\sqrt{2}}.$$

Since $P_0$ is the second point from the central maxima whose intensity is $2I$, we should have:

$$\frac{\pi}{\lambda}\Delta x = \frac{3\pi}{4} \Rightarrow \Delta x = \frac{3\lambda}{4}$$

Therefore, we can write:

$$\Delta x = (\mu - 1)t = \frac{3\lambda}{4}$$

$$t = \frac{3\lambda}{4(\mu - 1)}$$

### Oblique Incidence in YDSE

If the parallel rays are incident obliquely on the slits, then the sources $S_1$ and $S_2$ remains coherent but the initial phase difference between them is not zero. $d$

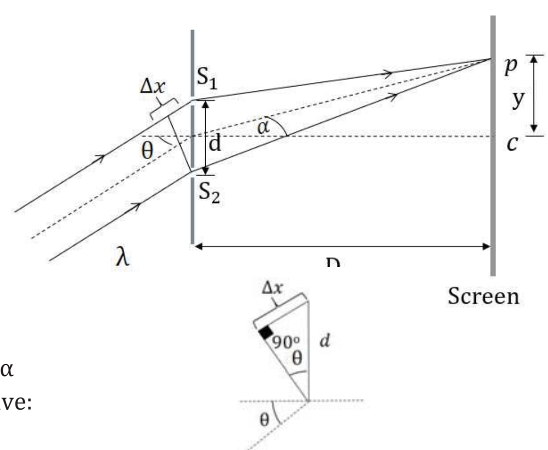
The initial path difference between the rays becomes, $\Delta x = d\sin\theta$

Due to this initial path difference, let the central maxima gets shifted from $C$ to $P$. Thus,

$$(S_1P)_{Effective} = S_2P$$

$$S_1P + d\sin\theta = S_2P$$

$$d\sin\theta = S_2P - S_1P \Rightarrow d\sin\theta = d\sin\alpha$$

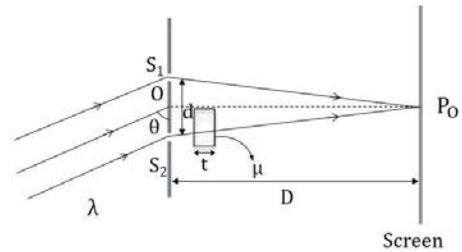



If the assumption "$\alpha$ is small" holds, then, we will have:

$$d\sin\theta = d\sin\alpha \approx d\tan\alpha = \frac{yd}{D}$$

**Ex.** In a YDSE setup, slab of $(t, \mu)$ is placed in front of one of the slits as shown. Wavelength of light used is $\lambda$. $(\lambda << d, D >> d)$ Find $t$ for central maxima at point $P_0$.

**Sol.** The introduction of the slab in the path of the ray coming from $S_2$ will shift the central maxima to lower side of the central line $OP_0$ whereas the initial path difference (extra path is associated with the ray incident on $S_1$ ) due to the oblique incident on slits will try to shift the central maxima to the upper side of $OP_0$.



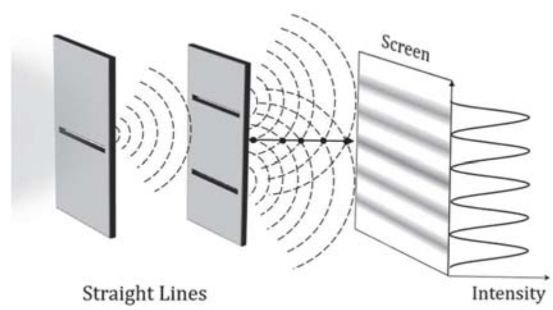

For central maxima at central point $P_O$:

$$S_1P_{\text{Effective}} = S_2P_{\text{Effective}}$$
$$s_1p + d\sin\theta = s_2p + (\mu - 1)t.$$
$$d\sin\theta = (\mu - 1)t$$
$$t = \frac{d\sin\theta}{(\mu - 1)}$$

**Fringe pattern due to different shape of slits**

**Young's double slit Experiment**

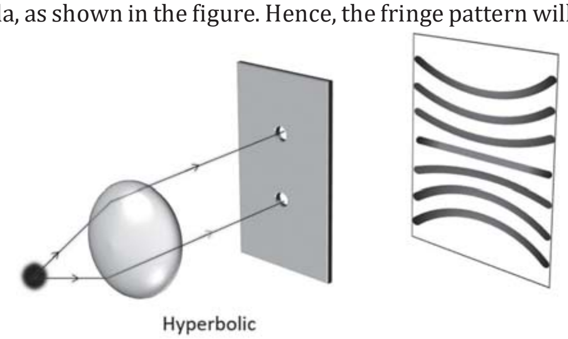
What will be the shape of interference fringes in Young's double slit experiment?




If we replace slits by pin holes in YDSE, then what will be the shape of interference?

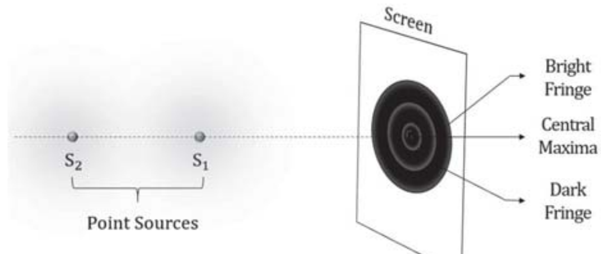
Since pin holes are used instead of slits, the perpendicular distance of each point on the screen from the pin holes will be varying.

Suppose for 1st maxima, as we go on the either side (right or left) of the screen, the perpendicular distance ($D$) of each point on the screen from the pin holes increases. Thus, the fringes get the shape of hyperbola, as shown in the figure. Hence, the fringe pattern will be "Family of hyperbolas".




What will be the shape of interference fringes in the following case of YDSE?

The locus of point $P$ on the screen for which the distance $S_1P$ and $S_2P$ remains fixed is a circle. Thus, the fringe pattern in the screen for this case will be "concentric circular fringes".




Concentric Circular Fringes

**YDSE- Bichromatic light interference**

**Mono Chromatic**

Only one wavelength or frequency.

**Bi Chromatic**

Two wavelengths or frequencies.

**Poly Chromatic**

Large number of wavelengths or frequencies.

**Ex.** A bi-chromatic light of wavelength $\lambda_1 = 4500$ Å and $\lambda_2 = 6000$ Å passes through slits as shown. Find out the minimum value of $y$ so that maxima's of $\lambda_1$ and $\lambda_2$ coincides.($D = 1\ m$, $d = 1\ mm$)

**Sol.** Since at the location of central maxima, the path difference between the rays becomes zero (so, no role of wavelength), the central maxima for both of the wavelength will be at same location. Now, since any other maxima depends on the wavelength of the light,

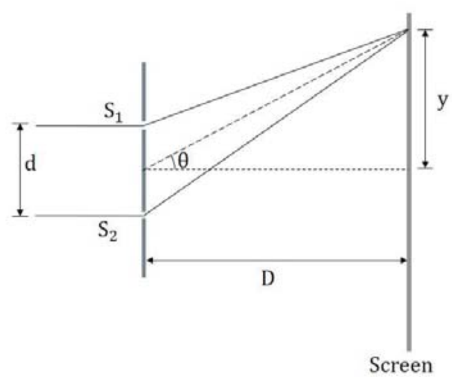
Assume that $n\ th$ maxima of $\lambda_1 = 4500$ Å coincides with $m^{th}$ maxima of $\lambda_2 = 6000$ Å, where $n$ and $m$ are integer. Therefore,

$$\frac{n\lambda_1\phi}{d} = \frac{m\lambda_2\phi}{d} \Rightarrow n\lambda_1 = m\lambda_2 \xrightarrow[\lambda_2=600\ \text{Å}]{\lambda_1 = 4500\ \text{Å}} \frac{n}{m} = \frac{6080}{4500} = \frac{4}{3}$$

Therefore, $4^{th}$ maxima of $\lambda_1 = 4500$ Å coincides with $3^{rd}$ maxima of $\lambda_2 = 6000$ Å,

Therefore, the minimum value of $y$ for which the above phenomenon will be taken place is given by,

$$y_{min} = \frac{n\lambda_1 D}{d}$$




We have:

$n = 4$, $\lambda_1 = 4500$ Å, $D = 1\ m$ and $d = 1\ mm = 10^{-3}$ m

Hence

$$y_{min} = \frac{4 \times 4500 \times 10^{-10} \times 1}{1 \times 10^{-3}}$$

$$y = 1.8\ \text{mm}$$