YDSE- INCOHERENT SOURCES AND OPTICAL PATH LENGTH

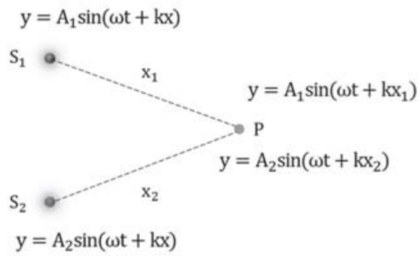
YDSE- Coherent and incoherent Sources

Coherent Sources

The phase difference at a certain point (here, point P) stays the same as time passes.

$$\Delta\phi = k(x_2 - x_1)$$

When the phase difference between two sources remains constant (or is zero) at a given point over time, those two sources are considered coherent.



The phase difference between waves happens solely because of the difference in their paths.

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

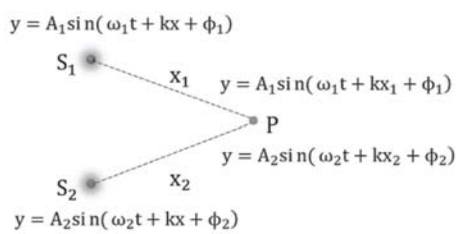
$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

At a particular location, the difference in phase doesn't change as time passes. As a result, the brightness stays the same. That's why we see a distinct pattern of bright and dark lines on the screen in Young's Double Slit Experiment when using two coherent sources.

Incoherent Sources

The phase difference between the waves at point *P* will be:

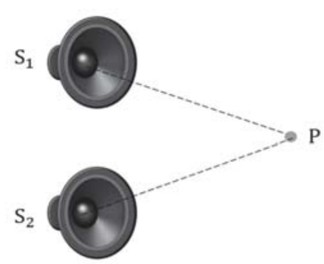
$$\Delta\phi = \underbrace{(\omega_2 - \omega_1)t}_{\text{Phase difference due to difference in frequency with time}} + \underbrace{k(x_2 - x_1)}_{\text{due to path difference}} + \underbrace{(\phi_2 - \phi_1)}_{\text{Phase difference due to initial phase}}$$



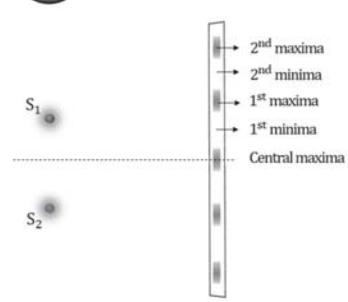
$$I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

The phase difference and intensity vary over both time and position. This characteristic defines these sources as incoherent sources.

Coherent and incoherent Sources

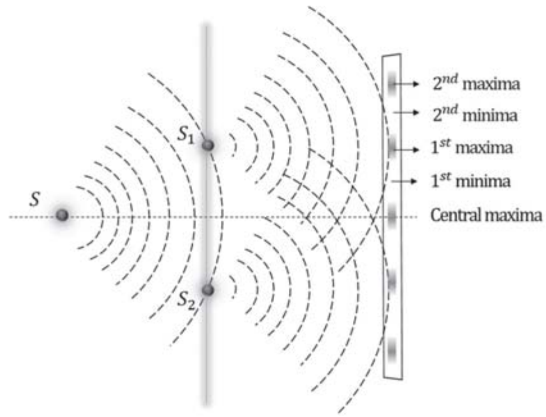


- Beats are produced when similar sound sources are used together.
- At a specific location, two incoherent sources create peaks and troughs, known as beats.
- The number of peaks formed at a certain location per second is called the beat frequency.
- Beats are only noticeable when the beat frequency is less than 10.
- In our daily lives, we don't observe the phenomenon of constructive or destructive interference when two identical bulbs are turned on. This is because even though they're identical, it's challenging to make them emit light in a synchronized manner, making them incoherent. As a result, they produce waves with varying phase differences over time, causing peaks and troughs to form rapidly, which we perceive as a steady brightness.



Generation of coherent sources

- According to Huygens' principle, every point on a secondary wavefront acts as a secondary source of light.
- The secondary sources (S1 and S2) created from a single primary source (S) behave as coherent sources.



For Maxima (Constructive interference):

$$y = \frac{n\lambda D}{d} \text{ Where, } n = 0, \pm1, \pm2, \pm3 \ldots$$

For Minima (Destructive interference)

$$\begin{cases} (2n-1)\dfrac{\lambda D}{2d}, & n = 1,2,3 \ldots \\ (2n+1)\dfrac{\lambda D}{2d}, & n = -1,-2,-3 \ldots \end{cases}$$

Fringe Width

$$\beta = \frac{\lambda D}{d}$$

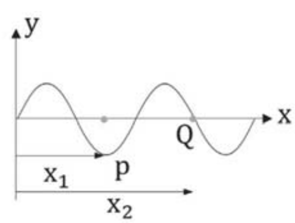
Conditions For Good Interference,

- Sources should be coherent.
- Fringe width (β) should be coherent.
- Angular fringe width (Δθ) should be coherent.
- $\frac{I_{min}}{I_{max}}$ should be small.

Optical Path length

Consider a sinusoidal wave: $y = A\sin(\omega t + kx)$

The equation of motion of the particles located at P and Q on the wave are:

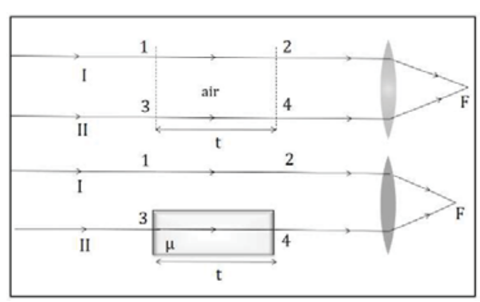


$$y_P = A\sin(\omega t + kx_1) \text{ and } y_Q = A\sin(\omega t + kx_2)$$

The phase difference:

$$\Delta\phi = [(\omega t + kx_2) - (\omega t + kx_1)] = k(x_2 - x_1) = \frac{2\pi}{\lambda}(x_2 - x_1)$$

In air (or vacuum), if we consider two particles on each of the given rays (I and II), then the relation between their phases will be as follows:

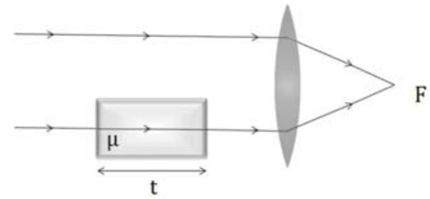


$\phi_1 = \phi_3, \phi_2 = \phi_4$ and $\phi_2 - \phi_1 = \phi_4 - \phi_3 = \frac{2\pi}{\lambda}t$

If a medium of thickness t and R.I. μ is placed in the path of ray II, then the relation between the phases of the particles becomes:

$$\phi_1 = \phi_3 \text{ and } \phi_2 - \phi_1 = \frac{2\pi}{\lambda}t$$

$$\phi_4 - \phi_3 = \frac{2\pi}{\lambda_\mu} t = \frac{2\pi}{\lambda} \mu t$$



$\lambda_\mu =$ Wavelength of light in the medium $(\mu) = \frac{\lambda}{\mu}$

$\lambda =$ Wavelength of light in air

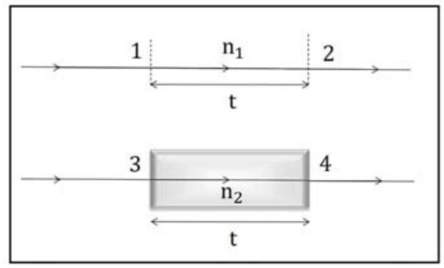
Optical Path Length $= \mu t$

In terms of phase difference, covering the distance t in medium of refractive index μ is equivalent to a distance μt in vacuum, which we call optical path.

Comparison of Optical Paths For Two Different Mediums

Let the wavelength of light in the medium of R.I. n_1 and n_2 be λ_1 and λ_2, respectively. Thus,

$$\frac{\lambda_2}{\lambda_1} = \frac{n_1}{n_2} \Rightarrow \lambda_1 = \frac{\lambda_2 n_2}{n_1}$$



The phase difference between point 3 and 4 is:

$$\phi_4 - \phi_3 = \frac{2\pi}{\lambda_2} t$$

The phase difference between point 1 and 2 is:

$$\therefore \phi_2 - \phi_1 = \frac{2\pi}{\lambda_1} t \Leftrightarrow \phi_2 - \phi_1 = \frac{2\pi}{\lambda_2} \frac{n_1 t}{n_2}$$

Effective Path Difference Between Two Parallel Waves Due To A Denser Medium

The geometrical path difference between point 1 and 2 as well as 3 and 4 is . Hence, geometrically, both the path 12 and 34 are same.

$$(\Delta x)_{\text{Geometric}} = 0$$

The phase of point 2 after covering t distance from point 1 in a medium of R.I. n_1 is

$$\phi_2 = \frac{2\pi}{\lambda_1} t$$

The phase of point 4 after covering t distance from point 1 in a medium of R.I. n_2 is,

$$\phi_4 = \frac{2\pi}{\lambda_2} t \Rightarrow \phi_4 = \frac{2\pi}{\lambda_1} \frac{n_2 t}{n_1}$$

Thus, point 4 covers an extra path in compared to point 1 by,

$$(\Delta x)_{\text{Effective}} = \left(\frac{n_2}{n_1} - 1\right)$$

If $n_1 = 1$ (for air) and $n_2 = \mu$ (for any medium denser than air), then

$$(\Delta x)_{\text{Effective}} = (\mu - 1)t$$