FRINGE WIDTH, ANGULAR FRINGE AND INTENSITY VARIATION IN YDSE

YDSE - Path difference, phase difference, Fringe width, angular fringe, Intensity Variation

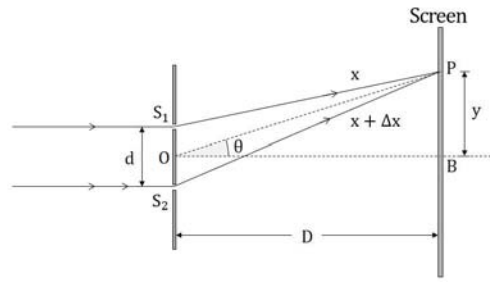
Path difference

$\Delta x = S_2P - S_1P$

$\Delta x = \sqrt{(y+\frac{d}{2})^2 + D^2} - \sqrt{(y-\frac{d}{2})^2 + D^2}$

For maxima, $\Delta x = n\lambda$

$$\sqrt{(y+\frac{d}{2})^2 + D^2} - \sqrt{(y-\frac{d}{2})^2 + D^2} = n\lambda$$



Thus, we can find positions of maxima and minima but it is difficult to solve this equation. Generally, we apply the assumption $d << D$ to solve the problems

$$\Delta x = \lambda/2 = 1^{st} \text{ Minima}$$
$$\Delta x = 3\lambda/2 = 2^{nd} \text{ Minima.}$$
$$\vdots$$
$$\Delta x = (2n-1)\lambda/2 = n^{th} \text{ minima}$$

$$\Delta x = 0 \rightarrow \text{Central Maxima}$$
$$\Delta x = \lambda \rightarrow 1^{st} \text{ Maxima}$$
$$\vdots$$
$$\Delta x = n\lambda \rightarrow n^{th} \text{ Maxima}$$

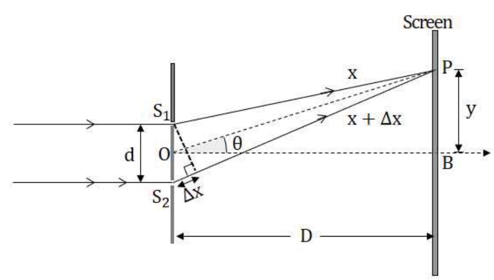
Approximation 1: $D \gg d$

For $D \gg d$, rays S_1P and S_2P will be approximately parallel.

The path difference between the rays S_1P and S_2P will be:

$$\Delta x = d\sin\theta = \frac{d \cdot y}{\sqrt{y^2 + D^2}}.$$

[This is not very easy to solve]



Approximation 2: θ is small

$\sin\theta \approx \tan\theta \Rightarrow \Delta x = d\sin\theta \approx d\tan\theta = \frac{d \cdot Y}{D}$

$$\Delta x = \frac{yd}{D}$$

[This is very easy to solve]

For Maxima (Constructive interference):

$d\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d} \Rightarrow \theta$ is small when $\lambda \ll d$.

Therefore, the statement "θ is small" is equivalent to "$\lambda \ll d$".

When both the approximations i.e., (1) $D \gg d$ and (2) θ is small (or, $\lambda \ll d$) are satisfied, the condition of constructive interference becomes:

$\Delta x = \frac{yd}{D} = n\lambda \Rightarrow y = \frac{n\lambda D}{d}$ Where, $n = 0, \pm1, \pm2, \pm3 \dots\dots$

$n = 0$ corresponds to the central maxima

$n = \pm1$ corresponds to the 1^{st} maxima

$n = \pm2$ corresponds to the 2^{nd} maxima and so on..

The difference between any two consecutive maximas is $\frac{\lambda D}{d}$.

For Minima (Destructive interference):

When both the approximations i.e., (1) $D \gg d$ and (2) θ is small (or, $\lambda \ll d$) are satisfied, the condition of destructive interference becomes:

$$\Delta x = \begin{cases} (2n-1)\frac{\lambda}{2}, & n = 1,2,3 \ldots \\ (2n+1)\frac{\lambda}{2}, & n = -1,-2,-3 \ldots \end{cases} \xrightarrow{\Delta x = \frac{yd}{D}} x = \begin{cases} (2n-1)\frac{\lambda D}{2d}, & n = 1,2,3 \ldots \\ (2n+1)\frac{\lambda D}{2d}, & n = -1,-2,-3 \ldots \end{cases}$$

$n = \pm 1$ corresponds to the 1^{st} minima

$n = \pm 2$ corresponds to the 2^{nd} minima and so on.

The difference between any two consecutive minimas is also $\frac{\lambda D}{d}$.

Fringe width

Fringe width is the distance between two successive maxima or two successive minima on one side of the central maxima.

1^{st} minima: $y_1 = \frac{\lambda D}{2d}$

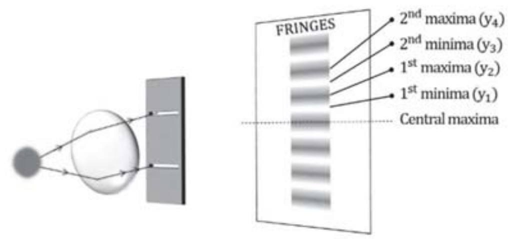
2^{nd} minima: $y_3 = \frac{3\lambda D}{2d}$

$\Big\}\ y_3 - y_1 = \frac{\lambda D}{d}$

1^{st} maxima: $y_2 = \frac{\lambda D}{d}$

2^{nd} maxima: $y_4 = \frac{2\lambda D}{d}$

$\Big\}\ y_4 - y_2 = \frac{\lambda D}{d}$



Fringe Width, $\beta = \frac{\lambda D}{d}$

Therefore, the fringe width of YDSE fringe pattern is, $\beta = \frac{\lambda D}{d}$. . To see the fringe pattern properly on the screen, β should be a significant value.

Resolving power of eyes

Our eyes can tell two things apart when they look like they're at a bigger angle than a certain smallest angle.

For human eye, the minimum angle for which we can resolve two distant objects as different objects is 1 min or $\left(\frac{1}{60}\right)^{\circ} \cdot (\because 1^{\circ} = 60 \text{ min})$

As the eye moves farther from an object, the angle it makes with the object gets smaller bit by bit.

Angular fringe width

The angular fringe width is the distance between the angles created by two neighboring bright or dark bands on one side of the middle bright band at the midpoint (P) between two slits.

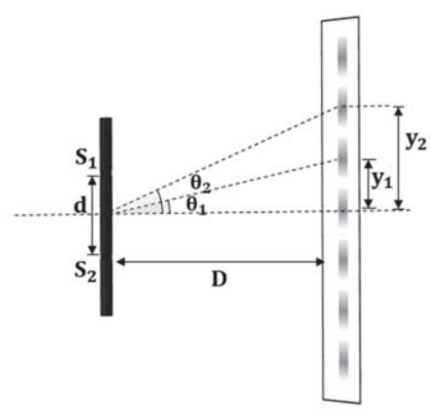
The linear distance between two successive maxima or two successive minima on one side of the central maxima is known as Fringe width and it is given by,

$$\text{Fringe Width, } \beta = \frac{\lambda D}{d}$$

The angular fringe width is given by $\Delta\theta$.

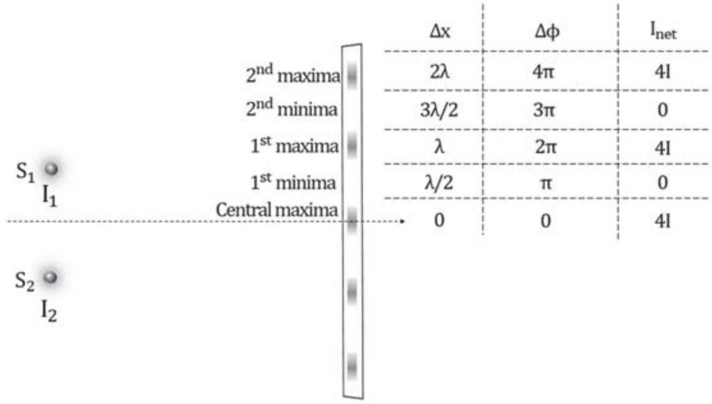
$$\Delta\theta = \theta_2 - \theta_1 \Rightarrow \Delta\theta = \frac{y_2 - y_1}{D}$$

Here, θ_1 and θ_2 are the angles made by two successive maxima on one side of central maxima at point P and $y_2 - y_1$ is the fringe width.



$$\Delta\theta = \frac{\beta}{D} \xrightarrow{\beta = \frac{\lambda D}{d}} \Delta\theta = \frac{\lambda}{d}$$

Intensity Variation



$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$
$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$
$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

When, $I_1 = I_2 = I$

$$I_{net} = 4I\cos^2\frac{\Delta\phi}{2}$$
$$I_{max} = 4I$$
$$I_{min} = 0$$

The ratio, $\frac{I_{min}}{I_{max}}$ is important because it tells how good is the interference. For good interference, the value of this ratio must be small.